Precontractual Investigation and Sequential Screening

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Precontractual Investigation
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Abstract

Should contract design induce an agent to conduct a precontractual investigation even though, in any case, the agent will become fully informed after the signing of the contract? This paper shows that imperfect investigations might be encouraged. The result stands in contrast to previous studies, which focus on perfect investigations. The contrast exists because if precontractual investigation is perfect, the benefits of sequential screening vanish.

Keywords: Principal agent, information acquisition, sequential screening

JEL Codes: D82, D83, D86

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1 Introduction

This paper analyzes profit-maximizing contracts in a principal-agent model of bilateral trade with the following information structure: Initially, both parties do not know the agent’s preferences over trade agreements. While deliberating whether to accept a contract, the agent can spend resources to investigate the state. In any case, he perfectly learns his preferences at some date when the contract has been signed, but not yet carried out. The principal, on the other hand, can neither observe whether the agent conducts a precontractual investigation, nor verify any reports that the agent might submit about his private information. A good example for a trade relationship with this information structure is procurement of goods that need to be customized. Here, the contractor typically does not know his operating costs before inspecting the designs, but can make a forecast based on experience from related projects.

Should the principal encourage the agent to conduct a precontractual investigation? In a seminal paper, Crémer and Khalil (1992) address this question under the assumption that the agent would obtain through an investigation the very same, perfect information about his preferences which he otherwise receives at no cost after the signing of the contract. They find that the principal is unambiguously better off if the agent has no incentive to acquire precontractual information. The result does not hold if multiple agents compete for a single, bilateral contract; the principal might then encourage information gathering to find a suitable candidate (Compte and Jehiel 2008, see also Craswell 1988). Apart from this qualification, Crémer and Khalil’s finding suggests the following conclusion: an agent should not be encouraged to investigate his preferences before entering into a contract if he will learn them anyway afterwards, before the transaction takes place. If true, this conclusion would be astonishing. It
implies, for instance, that a customer should not be encouraged to examine test reviews before buying a returnable good—which seems to be at odds with the prevalence of test reviews and generous return rights. The purpose of this paper is to show that the conclusion is false. According to my analysis, the principal induces information gathering if and only if she benefits from sequential screening.

I address the question of whether the principal should encourage the agent to conduct a precontractual investigation under the assumption that an investigation does not remove all uncertainty about the agent’s preferences. Either way, the terms of trade optimally depend on communication, because the agent obtains private information during the interaction. By the revelation principle for multistage games (Myerson 1986), the principal can without loss of generality design the contract so that the agent is willing to report each piece of private information truthfully as soon as he obtains it. Thus, in particular, contracts that deter the agent from conducting an investigation have a static screening mechanism, and contracts that induce him to do so have a sequential screening mechanism. Now, sequential screening is advantageous, because it forces the agent to make a decision about deviations from truthful reporting when the exact gains thereof are still uncertain—unless precontractual investigation removes all uncertainty about the agent’s preferences, as Crémer and Khalil (1992) assume. I show that if the agent’s investigation costs are low, optimal contracts induce information gathering.

The paper is related to the growing literature that explores scope and design of dynamic screening mechanisms in scenarios where agents gradually receive private information over time (e.g., Battaglini 2005; Boleslavsky and Said 2013; Courty and Li 2000; Esö and Szentes 2007; Krähmer and Strausz 2011; Pavan...
et al. 2013). That literature usually imposes stringent regularity conditions which allow to fully characterize optimal contracts.\textsuperscript{1} To maintain generality, I do not follow this approach. My analysis only verifies the key property: optimal contracts might involve a sequential screening mechanism, rather than a static one. Most closely related within the literature is the seminal article on advanced ticket sales by Courty and Li (2000). In their model, the agent freely obtains private information about his valuation for a ticket both before and after the contract has been signed. Conceptually, the present framework adds a moral hazard issue to that setting, as precontractual investigation entails costs and cannot be observed. A polar scenario, with postcontractual investigation, has been analyzed by Krähmer and Strausz (2011). There, the agent’s incentives to acquire information differ, since he cannot quit the contract afterwards. Relevant is finally recent work by Krähmer and Strausz (2012). They show under general conditions that dynamic screening mechanisms cannot improve over static ones if agents are protected by limited liability. According to my analysis, the principal will then discourage information gathering.

Various papers analyze profit-maximizing contracts for related scenarios with endogenous precontractual information.\textsuperscript{2} The use of sequential screening mechanisms has not been explored yet. Indeed, Crémer et al. (1998a), Kessler (1998), Lewis and Sappington (1997), Shi (2012), and Szalay (2009) consider situations in which agents do not receive additional information once the contract has been signed. Crémer et al. (1998b), Crémer and Khalil (1994), Matthews and Persico (2005), and Terstiege (2012), on the other hand, assume

\textsuperscript{1}See Battaglini and Lamba (2012) and Pavan et al. (2013) for details and discussions of the regularity conditions.

\textsuperscript{2}See Bergemann and Välimäki (2006) for a comprehensive survey of the literature on mechanism design with endogenous information.
like Crémer and Khalil (1992) that precontractual investigation yields perfect information about the unknown state. One of the literature’s key objectives is to examine comparative statics with respect to investigation costs and timing. This paper, in contrast, effectively performs a comparative static analysis with respect to the *quality* of precontractual investigation: I show that a principal might only induce an imperfect investigation.

The paper is organized as follows. The next section presents the model. Section 3 derives the main result, according to which optimal contracts might induce precontractual information acquisition. Section 4 concludes. Proofs appear in the appendix.

2 Model

I use a variant of the procurement model by Crémer and Khalil (1992). Specifically, a principal seeks to purchase units of some good which an agent can produce. Given output $q \geq 0$, marginal disutility of production $\beta$, and transfer $t \in \mathbb{R}$, the agent’s payoff is $t - \beta q$. The principal’s payoff is $V(q) - t$, where $V$ is strictly concave, continuously differentiable, and satisfies $\lim_{q \to 0} V'(q) = \infty$ and $\lim_{q \to \infty} V'(q) = 0$.

When the principal offers the contract, neither party knows the value of the disutility parameter $\beta$. Both believe that $\beta$ equals $\beta_i$, $i \in I \triangleq \{1, \ldots, n\}$, with respective prior probability $\gamma_i > 0$. Suppose $1 < n < \infty$, and let $0 < \beta_1 < \cdots < \beta_n < \infty$. The agent learns the true value of $\beta$ before production takes place, but only after the date at which he must decide about his participation in the contract.
At investigation costs $e \geq 0$, the agent can acquire information about the disutility parameter while deliberating whether to accept the contract. Both parties assume that such precontractual information gives rise to one of the posterior probability distributions $(\gamma_{ji})_{i=1}^{n}$ of $\beta$,

$$j \in J \triangleq \{1, \ldots, m\},$$

with respective probability $\pi_j > 0$. Suppose $1 < m < \infty$, and let the possible posterior distributions be ordered in terms of first-order stochastic dominance: for any $i < n$, the cumulative posterior probability that $\beta_i$ obtains,

$$\Gamma_{ji} \triangleq \sum_{k=1}^{i} \gamma_{jk},$$

strictly decreases in $j$. By Bayes’ rule, it furthermore holds that the expected posterior probability $\sum_{j=1}^{m} \pi_j \gamma_{ji}$ equals the prior probability $\gamma_i$.

I depart from Crémer and Khalil (1992) and assume that precontractual information does not reveal the value of the disutility parameter perfectly. In particular, let $\{i \in I : \gamma_{ji} > 0\} = I$ for all $j$, so that the agent still deems each level of $\beta$ possible upon acquiring information. Crémer and Khalil, on the other hand, effectively assume $m = n$ and $\{i \in I : \gamma_{ji} > 0\} = \{j\}$, so that information gathering removes all uncertainty.

The principal cannot observe whether the agent acquires precontractual information. Moreover, she can neither explore the disutility of production herself, nor verify any reports that the agent might submit. To distinguish the two possible pieces of private information of the model, I refer to $j \in J$ as the agent’s posterior belief and to $i \in I$ as his type. Given $i, k \in I$ with $i < k$, I say that type $i$ is more efficient (less inefficient) than type $k$. Similarly, given $j, l \in J$ with $j < l$, I say that posterior belief $j$ is more optimistic (less pessimistic) than posterior belief $l$. 

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The timing of the events can be summarized as follows:

1. Principal offers contract
2. Agent can acquire precontractual information to obtain posterior belief
3. Agent accepts or rejects contract
4. If contract accepted: Agent learns type before producing output
   If contract rejected: Relationship ends without trade

Efficiency. — From an efficiency perspective, the agent should produce the quantity \( q^* \triangleq V^{r-1}(\beta) \). Precontractual information gathering is socially wasteful given that the agent anyway learns the true value of \( \beta \)—perfectly, at no costs, and before production takes place.

3 Analysis

I now study the contracting problem from the perspective of the principal, who wants to maximize her expected payoff. My aim is to examine whether contract design should provide incentives that induce the agent to acquire information. It will turn out that if investigation costs are low, the principal must make a trade-off between efficiency and surplus extraction, so that contracts which induce information gathering cannot be ruled out solely on the grounds that they are inefficient. On the contrary, such contracts possibly admit a more favorable trade-off.

3.1 Trade-off

The objective of this section is to show that if investigation costs are low, the principal must make a trade-off between efficiency and surplus extraction to find an optimal contract. Since precontractual information gathering is inefficient,
I focus on contracts which deter the agent from conducting an investigation. According to the revelation principle for multistage games (Myerson 1986), the analysis can additionally be restricted to direct, incentive-compatible contracts.

Henceforth, I refer to the direct contracts used to discourage precontractual information gathering as pooling contracts. A pooling contract, formally denoted by

$$C^P \triangleq (t_k, q_k)_{k \in I},$$

stipulates the terms of trade as follows. Once the agent learns his type, he is asked to announce it with a report $k \in I$. Given this report, the parties must exchange $(t_k, q_k)$. Pooling contracts are incentive-compatible if the agent finds it best to conduct no investigation and to report the disutility of production truthfully.

In detail, a pooling contract should thus satisfy the following conditions. First, the agent must submit an honest report about his type. Using for the agent’s payoff the notation

$$U_i \triangleq t_i - \beta_i q_i,$$

this condition reads:

$$U_i \geq U_k + (\beta_k - \beta_i) q_k \quad \forall i, k \in I. \quad (P1)$$

Second, the agent should be willing to join the contract. Since he does not yet know his type when the participation decision is due, this constraint only requires that the contract guarantees him a non-negative payoff in expectation, rather than for each particular type. Importantly, the expectation derives from the prior belief, because the agent is supposed to dispense with information gathering:

$$\sum_{j=1}^{m} \pi_j \sum_{i=1}^{n} \gamma_{ji} U_i \geq 0. \quad (P2)$$
Finally, the agent must indeed not acquire information before deciding whether to participate in the contract. Precontractual information is valuable to him if, with some posterior beliefs, the proposal yields a negative expected payoff. For if he could update his expectation, he would be able to avoid a likely loss by refusing to accept the contract offer. The constraint requires that the agent’s valuation for precontractual information is smaller than the level of investigation costs:

\[
\sum_{j=1}^{m} \pi_j \sum_{i=1}^{n} \gamma_{ji} U_i \geq \sum_{j=1}^{m} \pi_j \max \left\{ \sum_{i=1}^{n} \gamma_{ji} U_i, 0 \right\} - e. \quad (P3)
\]

Consider now the principal’s objective. By definition, her expected payoff from a pooling contract that satisfies the conditions listed above is

\[
\Delta P \triangleq \sum_{j=1}^{m} \pi_j \sum_{i=1}^{n} \gamma_{ji} [V(q_i) - t_i].
\]

Thus, the best pooling contracts are the solutions to

\[
\mathcal{P}^P : \quad \max_{C^P} \Delta P \quad s.t. \quad (P1)-(P3).
\]

The following lemma suggests a more transparent representation of the best pooling contracts.

**Lemma 1.** For each pooling contract that satisfies (P1)–(P3), there is a pooling contract with identical expected payoffs for both parties that satisfies

\[
U_i - U_{i+1} = (\beta_{i+1} - \beta_i) q_{i+1} \quad \forall i \in \{1, \ldots, n - 1\}, \quad (*)
\]

\[
q_i - q_{i+1} \geq 0 \quad \forall i \in \{1, \ldots, n - 1\}, \quad (1)
\]

\[
\sum_{j=1}^{m} \pi_j \left[ U_n + \sum_{i=1}^{n-1} \Gamma_{ji} (\beta_{i+1} - \beta_i) q_{i+1} \right] \geq 0, \quad (2)
\]

\[
\sum_{j=1}^{m} \pi_j \left[ U_n + \sum_{i=1}^{n-1} \Gamma_{ji} (\beta_{i+1} - \beta_i) q_{i+1} \right] + e \geq 0 \quad \forall l \in \{2, \ldots, m\}. \quad (3)
\]

Moreover, (*) and (1)–(3) together imply (P1)–(P3).
Proof. Follows from Claims A1–A3 in the appendix. □

According to (*), the agent earns a rent for not exaggerating the disutility of production. By (1), on the other hand, reporting a more efficient type obliges him to produce more output. These two, standard constraints replace (P1) to make sure that truthful reporting is incentive-compatible.

(2) is the participation constraint, which replaces (P2). Observe that the agent’s expected payoff equals his payoff with the most inefficient type, $U_n$, plus the expected rent. Henceforth, I refer to $-U_n$ as a participation fee. The agent finds the contact acceptable if, given the prior belief, the participation fee does not exceed the expected rent.

(3), finally, guarantees instead of (P3) that the agent does not acquire information. Precontractual information is valuable to the agent if, with some posterior belief, the participation fee exceeds the rent which he can expect, so that his expected payoff given that posterior belief is negative. At this point, the first-order stochastic dominance ranking of the posterior beliefs becomes relevant: By (*), the rent decreases in the type. Consequently, the ranking implies that an agent with a more optimistic posterior belief expects to earn more rent, because he is more confident to have a low disutility of production. The formulation of (3) uses the converse implication that if the participation fee exceeds the expected rent with some posterior belief $j$, then so it does for all posterior beliefs $l > j$.

Condition (*) can be inserted directly into the principal’s objective function, which I now state as the difference between the expected surplus and the agent’s expected payoff:

$$\Delta P \triangleq \sum_{j=1}^{m} \pi_j \sum_{i=1}^{n} \gamma_{ji} [V(q_i) - \beta_i q_i] - \sum_{j=1}^{m} \pi_j \left[ U_n + \sum_{i=1}^{n-1} \Gamma_{ji}(\beta_{i+1} - \beta_i) q_{i+1} \right].$$
Thus, Lemma 1 allows to represent the best pooling contracts as the solutions to

$$\hat{\mathcal{P}}: \max_{(q_i)_{i=1}^n, -U_n} \hat{\Delta}^P \quad s.t. \quad (1)-(3).$$

The contracting problem involves the following trade-off. To maximize the expected surplus, the principal should discourage information gathering. Hence, she should offer a pooling contract, the stipulated output level being efficient given the prevailing type. To minimize the agent’s expected payoff, on the other hand, the principal can exploit the agent’s initial ignorance regarding the disutility of production. Specifically, she should appropriate the expected rent with a participation fee. In sum, the contract should thus feature:

$$(q_i)_{i=1}^n = (q_i^*)_{i=1}^n$$

$$-U_n = \sum_{j=1}^m \pi_j \sum_{i=1}^{n-1} \Gamma_{ji}(\beta_i+1 - \beta_i)q_i^*.$$  

This scheme satisfies the monotonicity constraint, (1), as well as the participation constraint, (2). But if the agent acquired information and updated to a pessimistic posterior belief, he would expect to earn less rent than initially assumed, and hence find the contract unprofitable. Accordingly, precontractual information is valuable to him. If information gathering entails low costs, condition (3) thus forces the principal to make a trade-off between efficiency and surplus extraction. Contracts which induce the agent to conduct an investigation can then not be ruled out solely on the grounds that they are inefficient.

3.2 Benchmark

As a benchmark, it is instructive to review the contracting problem under the assumption that precontractual information reveals the disutility of production
perfectly. Given this assumption, my model is equivalent to the one studied by Crémer and Khalil (1992). Their analysis shows that optimal contracts never induce information acquisition.

Crémer and Khalil’s (1992) finding can be explained by the following thought experiment. Consider a contract that induces information acquisition. Imagine now a different timing, according to which this contract is offered only after the date at which the agent learns his type for free. Since precontractual investigation exposes the disutility of production perfectly, the agent has the same information as under the original timing when making the participation decision and choosing a reporting strategy. By “revealed preferences”, the contract therefore implements the same terms of trade. But the agent has clearly no incentive to acquire information in advance.

The thought experiment suggests that for each contract which induces information acquisition, one can construct a pooling contract that yields both parties the same payoffs as the original contract. In fact, the principal can do better and reduce the transfers of the pooling contract, as the agent saves on investigation costs. I want to find out whether Crémer and Khalil’s (1992) finding extends to the current model, in which precontractual information is imperfect.

### 3.3 Main result

This section presents the main result, according to which optimal contracts might induce precontractual information acquisition. Again, I apply the revelation principle for multistage games (Myerson 1986) and restrict attention to direct, incentive-compatible contracts.

I refer to the direct contracts used to encourage information acquisition as
separating contracts. A separating contract, generally denoted by

\[ C^S \triangleq ((t_{lk}, q_{lk})_{k \in I})_{l \in J}, \]

stipulates the terms of trade as follows. First, the agent must submit a report \( l \in J \) about his posterior belief, where \( J \subset J \) denotes the set of posterior beliefs for which the agent is supposed to join the contract. Importantly, the revelation principle demands that the report is due before the agent learns his type. Later on, the agent has to submit a report \( k \in I \) about the type. Given some sequence of reports \( l k \), the parties must exchange \((t_{lk}, q_{lk})\). A separating contracts is incentive-compatible if the agent prefers to gather information, to join the contract if and only if \( j \in J \), and to submit a sequence of two truthful reports.

In fact, the analysis may ignore separating contracts with \( J \neq J \), which the agent possibly rejects upon acquiring information. For if, given some posterior belief \( j \), the agent rejects a given incentive-compatible contract \( \tilde{C}^S \), he would accept a contract \( \hat{C}^S \) which only differs from \( \tilde{C}^S \) in that \( \hat{J} = \tilde{J} \cup \{j\} \) and \((\hat{t}_{jk}, \hat{q}_{jk})_{k \in I} = (0, 0)\). By “revealed preferences”, \( \hat{C}^S \) is incentive-compatible, too, and consequently yields both parties the same payoffs as \( \tilde{C}^S \). Henceforth, I therefore assume \( J = J \) and denote separating contracts by

\[ C^S \triangleq ((t_{lk}, q_{lk})_{k \in I})_{l \in J}. \]

Observe that the “revealed preferences” argument would not apply if the agent had to report the posterior belief after he learns his type, rather than already before: Once the agent knows the true disutility of production, the posterior belief is irrelevant to him. Thus, a contract which the agent would accept regardless of the posterior belief and which does not require any report before the agent learns his type could not induce information gathering. Crémer
and Khalil (1992) assume that communication is impossible before the agent freely learns his type. Consequently, they do not require $\mathcal{J} = J$.

In detail, a separating contract should thus satisfy the following conditions. First, the agent must submit a truthful report about his type, provided he reported the posterior belief honestly before. The proviso can in fact be dropped: As argued above, the posterior belief is irrelevant to the agent once he knows his type. Hence, if he announces the type truthfully after an honest report about the posterior belief, he will also announce it truthfully after a dishonest report. Using for the agent’s payoff from a separating contract the notation

$$U_{ji} \triangleq t_{ji} - \beta_i q_{ji},$$

the constraint reads:

$$U_{ji} \geq U_{jk} + (\beta_k - \beta_i)q_{jk} \quad \forall i, k \in I; j \in J. \quad (S1)$$

Second, the agent must submit an honest report about the posterior belief, provided he will later on announce the type truthfully. Since he does not yet know his type when the report about the posterior belief is due, this constraint only requires that honesty is more profitable to him in expectation, rather than for each particular type:

$$\sum_{i=1}^{n} \gamma_{ji} U_{ji} \geq \sum_{i=1}^{n} \gamma_{ji} U_{li} \quad \forall j, l \in J. \quad (S2)$$

Third, the agent should be willing to join the contract. Since, as with pooling contracts, the agent also does not yet know his type when the participation decision is due, this constraint only requires that the contract guarantees him a non-negative payoff in expectation, rather than for each particular type. In contrast to pooling contracts, however, the expectation derives from the posterior
beliefs, because the agent is supposed to gather information:

\[ \sum_{i=1}^{n} \gamma_{ji} U_{ij} \geq 0 \quad \forall j \in J. \tag{S3} \]

Finally, the agent must indeed acquire information. As indicated above, precontractual information can be valuable to him for two reasons if being offered a separating contract. On the one hand, it allows to take the participation decision contingent on the posterior belief. On the other hand, given that the report about the posterior belief is already due before the agent learns the true disutility of production, precontractual information allows to identify the report which yields the largest expected payoff. Now, by (S3), the first motive does not apply. The agent’s valuation for precontractual information is consequently larger than the level of investigation costs if and only if

\[ \sum_{j=1}^{m} \pi_j \sum_{i=1}^{n} \gamma_{ji} U_{ji} - e \geq \max \left\{ \sum_{j=1}^{m} \pi_j \sum_{i=1}^{n} \gamma_{ji} U_{li}, 0 \right\} \quad \forall l \in J. \tag{S4} \]

Consider now the principal’s objective. By definition, her expected payoff from a separating contract that satisfies the conditions listed above is

\[ \Delta^S \triangleq \sum_{j=1}^{m} \pi_j \sum_{i=1}^{n} \gamma_{ji} [V(q_{ji}) - t_{ji}]. \]

Thus, the best separating contracts are the solutions to

\[ \mathcal{P}^S : \max_{C^S} \Delta^S \quad s.t. \quad (S1)-(S4). \]

Separating contracts involve a sequential screening mechanism: the agent must report the posterior belief before he learns his type. Incentive constraints for dynamic screening mechanisms generally lack useful characterizations (e.g., Pavan et al. 2013). Therefore, I will not derive the optimal contract but show directly that separating contracts might improve over the pooling ones. Let
$W^P(e)$ and $W^S(e)$ denote the principal’s expected payoff from the best pooling and separating contracts, respectively, depending on the level of investigation costs. For a given cost level $e$, optimal contracts are separating contracts if $W^S(e) > W^P(e)$ and pooling contracts if $W^S(e) < W^P(e)$. Claim A4 in the appendix confirms that the functions $W^P$ and $W^S$ are indeed defined. Lemma 2 provides a partial comparison.

**Lemma 2.** The functions $W^P$ and $W^S$ have the following properties:

1. They are non-decreasing and non-increasing, respectively.
2. They are continuous.
3. They have a unique intersection.

*Proof.* See Claims A5–A8 in the appendix. □

The first and the last statement of the lemma are intuitive. With high investigation costs, the principal does not need to make a trade-off between efficiency and surplus extraction if she offers a pooling contract. Hence, separating contracts are clearly inferior. With low investigation costs, on the other hand, any disadvantage of separating contracts must be negligible. In particular, with zero costs, the agent virtually has precontractual information anyway, so that at least in that case, offering a separating contract cannot generally be suboptimal.

Lemma 3 completes the comparison.

**Lemma 3.** It holds that $W^S(0) > W^P(0)$.

*Proof.* See Claim A9 in the appendix. □

The lemma derives from the observation that the posterior belief determines the agent’s marginal rate of substitution between rent and participation fee with
pooling contracts. The basic idea is well-known in the sequential screening literature and features, in similar form, for instance in the analysis by Battaglini (2005). To illustrate it, suppose there are just two possible posterior beliefs and types. Moreover, assume zero investigation costs, so that the agent virtually has precontractual information anyway. If the principal offers a pooling contract, the trade-off between efficiency and surplus extraction then results in an inefficiently low output level $q_2 < q_2^*$ for the agent with the high disutility of production. The key insight is that the terms of trade should never be distorted for both posterior beliefs in this fashion: With the optimistic posterior belief, the agent has a larger valuation for additional rent, because he is more confident to earn it. Hence, if—with the optimistic posterior belief—the agent was to produce efficiently and pay an extra participation fee that fully extracts the extra surplus, whereas—with the pessimistic posterior belief—he was to stick with the original contract, the agent would comply. This scheme amounts to a separating contract that outperforms all pooling ones. The proof extends the reasoning to the original model, where the numbers of posterior beliefs and types are arbitrary, and shows that to each pooling contract there corresponds a separating one which exhibits “no distortion at the top” and provides the agent with the same expected payoff.

Combined, Lemmas 2 and 3 yield the main result.

**Proposition 1.** There exists a cutoff level of investigation costs $\hat{e} > 0$ such that optimal contracts are separating contracts if $e < \hat{e}$ and pooling contracts if $e > \hat{e}$.

Intuitively, the benefit of separating contracts results from a relaxed incentive-compatibility condition. With both pooling and separating contracts, truthful reporting must yield the agent at least the same payoff as the best
possible deviation strategy. This condition is relaxed with separating contracts because they oblige the agent to report a piece of private information before he fully learns the disutility of production. Thus, screening involves choice under uncertainty for the agent, and he might ultimately not be able to secure those terms of trade in the contract that he likes best given his true preferences. This feature vanishes if precontractual investigation removes all uncertainty.

4 Conclusion

This paper provides a new perspective on contract design with endogenous information. It can be optimal to induce a precontractual investigation even though, in any case, the agent will become fully informed after the signing of the contract. Previous studies reached a different conclusion because they focused on perfect investigations, so that the benefits of sequential screening vanished.

The result might have an implication for contracting problems in which the principal can disclose, without observing, a source of private information to the agent and the agent can explore some other, less informative source, which the principal does not control. A good example for such a contracting problem is again the procurement-of-customized-goods example mentioned in the introduction. Here, it seems plausible that the project owner can to some extent influence when, and in which form, the contractor inspects the designs of the good. The recent literature on disclosure rules in optimal auctions has not studied the case in which agents can conduct precontractual investigations, but the analyses suggests that the principal should disclose her source of private information only after the signing of the contract, so as to be able to charge participation fees (e.g., Bergemann and Pesendorfer 2007; Esö and Szentes
The information structure would then be identical to the one in the present paper. Thus, the principal possibly induces the agent to conduct a precontractual investigation even though she herself can costlessly disclose precontractual information.

Appendix

Claim A1. (P1) is satisfied if and only if it holds that

\[ U_i - U_{i+1} \in [(\beta_{i+1} - \beta_i) q_{i+1}, (\beta_{i+1} - \beta_i) q_i] \quad \forall i \in \{1, \ldots, n-1\} \quad (A.1) \]

\[ q_i - q_{i+1} \geq 0 \quad \forall i \in \{1, \ldots, n-1\} \quad (1) \]

Proof. The proof is standard, and therefore omitted. \(\square\)

Claim A2. If \(C^P\) satisfies (P1) and (P2), condition (P3) holds as well if and only if

\[ \sum_{j=1}^{m} \pi_j \sum_{i=1}^{n} \gamma_{ji} U_i + e \geq 0 \quad \forall l \in \{2, \ldots, m\}. \quad (A.2) \]

Proof. I first derive some auxiliary results.

(I) First, straightforward algebra shows that (P3) is equivalent to

\[ \sum_{j=1}^{m} \pi_j \min \left\{ 0, \sum_{i=1}^{n} \gamma_{ji} U_i \right\} + e \geq 0. \]

Second, according to Claim A1, (P1) implies \(U_i \geq U_{i+1}\) for all types \(i \in \{1, \ldots, n-1\}\) and hence, by the first-order stochastic dominance ranking of the posterior probability distributions,

\[ \sum_{i=1}^{n} \gamma_{ji} U_i \geq \sum_{i=1}^{n} \gamma_{j(i+1)} U_i \quad \forall j \in \{1, \ldots, m-1\}. \quad (A.3) \]

Third, given (A.3) a necessary condition for (P2) is \(\sum_{i=1}^{n} \gamma_{ij} U_i \geq 0\).
The claim can now be proved. Suppose $C^P$ satisfies (P1) and (P2) but violates (P3). By (I), there is a posterior belief $l \in \{2, \ldots, m\}$ such that
\[
\sum_{j=1}^{m} \pi_{j} \min \left\{ 0, \sum_{i=1}^{n} \gamma_{ji} U_{i} \right\} + e = \sum_{j=l}^{m} \pi_{j} \sum_{i=1}^{n} \gamma_{ji} U_{i} + e < 0;
\]
hence, (A.2) does not hold. Suppose $C^P$ satisfies (P1) and (P2) but violates (A.2). Let $l \in \{2, \ldots, m\}$ be the smallest posterior belief $j$ such that
\[
\sum_{i=1}^{n} \gamma_{ji} U_{i} < 0.
\]
By (I),
\[
0 > \sum_{j=l}^{m} \pi_{j} \sum_{i=1}^{n} \gamma_{ji} U_{i} + e = \sum_{j=1}^{m} \pi_{j} \min \left\{ 0, \sum_{i=1}^{n} \gamma_{ji} U_{i} \right\} + e,
\]
that is, (P3) does not hold. \[\square\]

**Claim A3.** Suppose $C^P$ satisfies (A.1), (1), (P2), and (A.2). Then, there is a pooling contract which satisfies (1), (P2), (A.2),
\[
U_{i} - U_{i+1} = (\beta_{i+1} - \beta_i) q_{i+1} \quad \forall i \in \{1, \ldots, n-1\},
\]
and results in identical expected payoffs for both parties.

**Proof.** Suppose there exists a type $k \in I$ for which the original contract $C^P$ implies
\[
U_{k} - U_{k+1} = (\beta_{k+1} - \beta_k) q_{k+1} + x,
\]
where $x$ is such that (A.1) holds. The alternative contract, denoted by $\tilde{C}^P$, differs from $C^P$ only with respect to transfers. Specifically,
\[
\tilde{t}_i = \begin{cases} 
    t_i - \left(1 - \sum_{j=1}^{m} \pi_j \Gamma_{jk}\right) x & \text{if } i \in \{1, \ldots, k\} \\
    t_i + \sum_{j=1}^{m} \pi_j \Gamma_{jk} x & \text{if } i \in \{k+1, \ldots, n\}.
\end{cases}
\]
This difference implies
\[
\tilde{U}_{k} - \tilde{U}_{k+1} = (\beta_{k+1} - \beta_k) q_{k+1}
\]
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and
\[ \tilde{U}_i - \tilde{U}_{i+1} = U_i - U_{i+1} \quad \forall i \in \{1, \ldots, n-1\} \setminus k. \]
Furthermore,
\[ \sum_{j=1}^{m} \pi_j \sum_{i=1}^{n} \gamma_{ji} \tilde{t}_i = \sum_{j=1}^{m} \pi_j \sum_{i=1}^{n} \gamma_{ji} t_i, \]
so that the alternative contract satisfies (P2) and results in identical expected payoffs for both parties as the original one. Finally, since it holds for any posterior belief \( l \in \{2, \ldots, m\} \) that
\[ \sum_{j=l}^{m} \pi_j \sum_{i=1}^{n} \gamma_{ji} \tilde{U}_i + e = \sum_{j=l}^{m} \pi_j \left[ \sum_{i=1}^{n} \gamma_{ji} U_i - \Gamma_{jk} x + \sum_{j=1}^{m} \pi_j \Gamma_{jk} x \right] + e \]
\[ \geq \sum_{j=l}^{m} \pi_j \sum_{i=1}^{n} \gamma_{ji} U_i + e \geq 0, \]
where the first inequality follows from the first-order stochastic dominance ranking of the posterior probability distributions and the second one from the hypothesis, condition (A.2) is met as well.

\( \square \)

Claim A4. There exist solutions to both \( \mathcal{P}^P \) and \( \mathcal{P}^S \).

Proof. I will use the following definitions and the theorem below by Rockafellar (1970): A concave function \( h : \mathbb{R}^I \to \mathbb{R} \cup \{-\infty\} \) is proper if \( h(x) > -\infty \) for at least one \( x \), and it is closed if \( \{x : h(x) \geq \alpha\} \) is closed for every \( \alpha \in \mathbb{R} \). A vector \( y \neq 0 \) is a direction of recession of a convex set \( D \) if \( x + \lambda y \in D \) for every \( \lambda \geq 0 \) and every \( x \in D \). The directions of recession of a closed proper concave function \( h \) are the directions of recession of the sets \( \{x : h(x) \geq \alpha\}, \alpha \in \mathbb{R} \).

(II) (Rockafellar 1970, Thm. 27.3) Let \( h : \mathbb{R}^I \to \mathbb{R} \cup \{-\infty\} \) be a closed proper concave function, and let \( D \subseteq \mathbb{R}^I \) be a non-empty closed convex set over which \( h \) is to be maximized. If \( h \) and \( D \) have no direction of recession in common, then \( h \) attains its supremum over \( D \).
Consider first program $\mathcal{P}^P$. The objective function can be restated as follows:

$$h(C^P) = \begin{cases} \sum_{j=1}^m \pi_j \sum_{i=1}^n \gamma_{ji} [V(q_i) - t_i] & \text{if } q_i \geq 0 \forall i \in I \\ -\infty & \text{else.} \end{cases}$$

Due to the assumptions on $V$, $h$ is a closed proper concave function. Let $D$ be the set of all pooling contracts that satisfy (P1)–(P3). Those constraints can be described by closed level sets of affine functions, so that $D$ must be closed and convex. $D$ is furthermore non-empty; for instance, the pooling contract $C^P$ with $(t_i, q_i) = (0, 0)$ for all types $i \in I$ satisfies (P1)–(P3).

I now apply (II) and show that the function $h$ and the set $D$ do not have directions of recession in common. As $V'(\infty) = 0$, and because $h(C^P) = -\infty$ if $C^P$ contains some strictly negative $q_i$, any direction of recession of $h$ must satisfy

$$\sum_{j=1}^m \pi_j \sum_{i=1}^n \gamma_{ji} t_i \leq 0 \text{ and } q_i \geq 0 \forall i \in I.$$ 

But even the set of all pooling contracts that satisfy (P1) and (P2) cannot have such directions of recession.

Consider now program $\mathcal{P}^S$. The set of all separating contracts that satisfy (S1)–(S4) is non-empty; for instance, any separating contract $C^S$ with

$$(t_{ji}, q_{ji}) = \begin{cases} (0, 0) \forall i \in I & \text{if } j \neq 1 \\ (t, q) \forall i \in I & \text{if } j = 1, \end{cases}$$

where $(t, q)$ fulfills

$$\pi_1 \left[ t - \sum_{i=1}^n \gamma_{1i} \beta_i q \right] - e \geq 0 \text{ and } t - \sum_{i=1}^n \gamma_{2i} \beta_i q \leq 0,$$

satisfies (S1)–(S4). Such separating contracts exist due to the first-order stochastic dominance ranking of the posterior probability distributions. Ex-
istence of solutions to program $P^S$ can now be proved analogously to program $P^P$.

\[ \square \]

Claim A5. The functions $W^P$ and $W^S$ are non-decreasing and non-increasing, respectively.

Proof. If $e$ increases, the feasible sets in $P^P$ and $P^S$ expand and contract, respectively. The corresponding objective functions, on the other hand, do not vary with $e$.

Claim A6. There exists a cutoff level of investigation costs $e' > 0$ such that, for any $e < e'$,

\[
W^P(e) = \max_{(q_i)_{i=1}^n} \sum_{j=1}^m \pi_j \sum_{i=1}^n \gamma_{ji} [V(q_i) - \beta_i q_i]
- \left[ -\frac{e}{\pi_m} + \sum_{j=1}^{m-1} \pi_j \sum_{i=1}^{n-1} (\Gamma_{ji} - \Gamma_{mi})(\beta_{i+1} - \beta_i)q_{i+1} \right] \quad \text{s.t. (1)}.\]


(III) There exists a value $q > 0$ such that, for any $e$, solutions to $\hat{P}^P$ satisfy $q_i \geq q$ for all $i \in I$.

Proof. In effect, this is Claim 1 of the appendix by Crémer and Khalil (1992). Their proof applies to the current setting.

The claim can now be proved. By (III) and the first-order stochastic dominance ranking of the posterior probability distributions, solutions to $\hat{P}^P$ satisfy

\[
\sum_{i=1}^{n-1} \Gamma_{ji}(\beta_{i+1} - \beta_i)q_{i+1} > \sum_{i=1}^{n-1} \Gamma_{(j+1)i}(\beta_{i+1} - \beta_i)q_{i+1} \quad \forall j \in \{1, \ldots, m - 1\}.\]
Since $U_n$ enters the objective function with negative sign, there must hence exist a cutoff level of investigation costs $e' > 0$ such that, for any $e < e'$, the principal chooses $U_n$ to satisfy

$$\pi_m \left[ U_n + \sum_{i=1}^{n-1} \Gamma_{mi}(\beta_{i+1} - \beta_i)q_{i+1} \right] + e = 0$$

and (2) holds with strict inequality. Inserting this value of $U_n$ into the objective function establishes the claim. \[\square\]

Claim A7. The functions $W^P$ and $W^S$ are continuous.

Proof. Since the objective functions and the constraints of $\mathcal{P}^P$ and $\mathcal{P}^S$ are concave in both the parameter $e$ and the choice variables, $W^P$ and $W^S$ must be concave (e.g., de la Fuente 2000, p. 313, Thm. 2.12). Hence, $W^P$ and $W^S$ are continuous on $(0, \infty)$. Claim A6 implies that $W^P$ must be continuous at $e = 0$. Suppose $W^S$ is not continuous at $e = 0$. By concavity, $\lim_{e \downarrow 0} W^S(e)$ exists and $\lim_{e \downarrow 0} W^S(e) > W^S(0)$. This inequality contradicts Claim A5. \[\square\]

Claim A8. The functions $W^P$ and $W^S$ have a unique intersection.

Proof. The following expression will be helpful:

$$W^* \triangleq \sum_{j=1}^{m} \pi_j \sum_{i=1}^{n} \gamma_{ji} [V(q^*_i) - \beta_i q^*_i].$$

Note that $W^P(e) \leq W^*$ and $W^S(e) \leq W^* - e$ for all $e$.

I first argue that intersections exist. At $e = 0$, to any pooling contract $C^P$ which satisfies (P1)–(P3) there corresponds a separating contract $C^S$ which satisfies (S1)–(S4) such that $(t_{ji}, q_{ji})_{i \in I} = (t_i, q_i)_{i \in I}$ for all $j \in J$. Thus, $W^S(0) \geq W^P(0)$. Suppose $W^S(0) > W^P(0)$, and define $\bar{e} = W^* - W^P(0)$.
As $W^P$ is increasing by Claim A5, it holds that $W^S(\tilde{e}) \leq W^P(\tilde{e})$. Since $W^S$ and $W^P$ are continuous by Claim A7, the intermediate value theorem implies that intersections exist.

I now argue that there cannot be several intersections. Any intersection must lie in the interval $[0, e^*)$, $e^* > 0$ being the cutoff level of investigation costs such that condition (3) binds in program $\mathcal{P}$ if and only if $e < e^*$. That is, $W^P(e) < W^*$ if $e < e^*$ and $W^P(e) = W^*$ if $e \geq e^*$. Recall from the proof of Claim A7 that $W^P$ is concave, and therefore differentiable almost everywhere. At points where it is differentiable, it holds that

$$\frac{dW^P(e)}{de} = \sum_{j=1}^{m-1} \kappa_j,$$

where the $\kappa_j$'s are non-negative Lagrange multipliers associated to condition (3). By definition, at each $e \in [0, e^*)$ there is at least one strictly positive multiplier. As the function $W^P$ is continuous by Claim A7, it must hence be strictly increasing on $[0, e^*)$. This implies that there cannot be several intersections, since the function $W^S$ is non-increasing by Claim A5.

\begin{claim}
It holds that $W^S(0) > W^P(0)$.
\end{claim}

\begin{proof}
I first establish two auxiliary results.

\textbf{(IV)} Suppose $e = 0$. Let $\mathcal{C}^P$ be one of the best pooling contracts that satisfy $(\star)$ and (1)--(3). Then, it holds that $\overline{q}_i < q^*_i$ for all types $i > 1$ and $\overline{q}_1 = q^*_1$.

\begin{proof}
By Claim A6,

$$\overline{q}_n \in \arg\max_q \sum_{j=1}^{m} \pi_j \gamma_{jn} [V(q) - \beta_n q] - \sum_{j=1}^{m-1} \pi_j \left( \Gamma_{j(n-1)} - \Gamma_{m(n-1)} \right) (\beta_n - \beta_{n-1}) q + \lambda_{n-1}(q_{n-1} - q),$$

\end{proof}

\end{proof}
\[ q_i \in \arg \max_q \sum_{j=1}^m \pi_j \gamma_{ji} [V(q) - \beta_i q] - \sum_{j=1}^{m-1} \pi_j (\Gamma_{j(i-1)} - \Gamma_{m(i-1)}) (\beta_i - \beta_{i-1}) q \]

\[ + \lambda_i (q - q_{i+1}) + \lambda_{i-1} (q_{i-1} - q) \quad \forall i \in \{2, \ldots, n-1\}, \]

and

\[ q_1 \in \arg \max_q \sum_{j=1}^m \pi_j \gamma_{1i} [V(q) - \beta_1 q] + \lambda_1 (q - q_2), \]

where the \( \lambda \)s are non-negative Lagrange multipliers associated to constraint (1). It follows that \( \overline{q}_1 = q_1^* \) and \( \overline{q}_n < q_n^* \). Consider any type \( i \in \{2, \ldots, n-1\} \), and suppose \( \overline{q}_{i+1} < q_{i+1}^* \). If \( \lambda_i = 0 \), it holds that \( \overline{q}_i < q_i^* \). If \( \lambda_i > 0 \), on the other hand, complementary slackness implies \( \overline{q}_i = \overline{q}_{i+1} \), so that \( \overline{q}_i < q_i^* \) by the induction hypothesis.

**(V)** Suppose \( e = 0 \). Let \( \overline{C}^P \) be a pooling contract that satisfies (\( \ast \)) and (1)--(3), and let \( \overline{C}^P \) be a pooling contract that satisfies (\( \ast \)) and (1). Moreover, suppose \( q_i \leq \overline{q}_i \) for all \( i \in I \) and

\[ U_n + \sum_{i=1}^{n-1} \Gamma_{1i} (\beta_{i+1} - \beta_i) q_{i+1} = U_n + \sum_{i=1}^{n-1} \Gamma_{1i} (\beta_{i+1} - \beta_i) \overline{q}_{i+1}. \quad (A.4) \]

Then, the separating contract \( \tilde{C}^S \), defined as follows, satisfies (S1)--(S4):

\[ (\tilde{t}_{1i}, \tilde{q}_{1i})_{i \in I} = (t_i, q_i)_{i \in I} \quad \text{and} \quad (\tilde{t}_{ji}, \tilde{q}_{ji})_{i \in I} = (\overline{t}_i, \overline{q}_i)_{i \in I} \quad \forall j \in \{2, \ldots, m\}. \]

**Proof.** Since both \( \overline{C}^P \) and \( \overline{C}^P \) satisfy (\( \ast \)) and (1), the separating contract \( \tilde{C}^S \) satisfies (S1) by Claim A1. I next verify that (S2) is met as well. By (A.4), the agent has no incentive to misrepresent the most optimistic posterior belief, \( j = 1 \). With all other posterior beliefs, the agent’s expected
gain from deviating is non-positive as well:

\[
\sum_{i=1}^{n} \gamma_{ij} [t_i - \beta_i q_i] - \sum_{i=1}^{n} \gamma_{ij} [\bar{t}_i - \beta_i \bar{q}_i] = U_n + \sum_{i=1}^{n-1} \Gamma_{ij} (\beta_{i+1} - \beta_i) q_{i+1} - U_n - \sum_{i=1}^{n-1} \Gamma_{ij} (\beta_{i+1} - \beta_i) \bar{q}_{i+1}
\]

\[
\sum_{i=1}^{n} (\Gamma_{ij} - \Gamma_{i1}) (\beta_{i+1} - \beta_i) (q_{i+1} - \bar{q}_{i+1}) \leq 0,
\]

where the inequality follows from the first-order stochastic dominance ranking of the posterior probability distributions. Condition (S3) holds since \( C_P \) satisfies (3) and since the agent has an incentive to report the posterior belief truthfully. Finally, given that investigation costs are zero, (S2) implies (S4).

The claim can now be proved. Let \( \tilde{C}_P \) be one of the best pooling contracts that satisfy \((*)\) and (1)–(3). Consider the pooling contract \( C_P \), defined as follows:

\[
(q_i)_{i=1}^{n} = (q_i^*)_{i=1}^{n},
\]

\[
(t_i)_{i=1}^{n-1} \quad \text{s.t.} \quad \bar{U}_i - U_{i+1} = (\beta_{i+1} - \beta_i) q_{i+1} \quad \forall i \in \{1, \ldots, n - 1\},
\]

\[
\bar{t}_n \quad \text{s.t.} \quad \bar{U}_n + \sum_{i=1}^{n-1} \Gamma_{i1} (\beta_{i+1} - \beta_i) q_{i+1} = U_n + \sum_{i=1}^{n-1} \Gamma_{i1} (\beta_{i+1} - \beta_i) \bar{q}_{i+1}.
\]

By (IV), \( q_i \geq \bar{q}_i \) for all \( i \in I \). Therefore, (V) ensures that the separating contract \( \tilde{C}_S \), defined as follows, satisfies (S1)–(S4):

\[
(t_{i1}, q_{i1})_{i \in I} = (t_i, q_i)_{i \in I} \quad \text{and} \quad (t_{ij}, q_{ij})_{i \in I} = (\bar{t}_i, \bar{q}_i)_{i \in I} \quad \forall j \in \{2, \ldots, m\}.
\]

By (IV), \( \tilde{C}_S \) generates strictly more expected surplus than the best pooling contract \( \bar{C}_P \). By construction, both contracts provide the agent with the same expected payoff. These two observations establish the claim. \( \square \)
References


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