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Deterministic versus Stochastic Mechanisms in Principal-Agent Models

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Discussion Paper No. 26

Deterministic versus Stochastic Mechanisms in Principal–Agent Models

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September 2004

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Deterministic versus Stochastic Mechanisms in Principal–Agent Models

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September 13, 2004

Abstract

This paper shows that, contrary to what is generally believed, decreasing concavity of the agent's utility function with respect to the screening variable is not sufficient to ensure that stochastic mechanisms are suboptimal. The paper demonstrates, however, that they are suboptimal whenever the optimal deterministic mechanism exhibits no bunching. This is the case for most applications of the theory and therefore validates the literature's usual focus on deterministic mechanisms.

Keywords: principal-agent theory, mechanism design, deterministic mechanisms, randomization, bunching.
JEL Classification No.: D82

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1 Introduction

In the past decades economic literature has seen many fruitful applications of mechanism design in the theory of nonlinear pricing, monopoly regulation, taxation, and insurance. Most of these applications, however, restrict their analysis to deterministic mechanisms. By itself the restriction is problematic, because it is well known that in general deterministic mechanisms are suboptimal (e.g. Stiglitz [6], Arnott and Stiglitz [1]). It is therefore a priori not clear that the obtained mechanisms are truly optimal and do indeed represent optimal ways to deal with asymmetric information.[1] This has prompted authors to find additional conditions that guarantee an optimality of deterministic mechanisms.

For principal–agent problems with quasi–linear utility such a condition is seen in a decreasing concavity of the agent's utility function with respect to the screening variable (e.g., Laffont and Tirole [4,p.119], or Fudenberg and Tirole [2,p.306], Laffont and Martimort [3,p.65]). This paper presents a counter example which contradicts this claim. It provides the intuition that, contrary to deterministic mechanisms, stochastic mechanisms are able to implement allocations that are non–monotonic in expected terms. Consequently, stochastic mechanisms may be optimal when non–monotonic schemes are desirable.

The paper further shows that when the optimal *deterministic* mechanism does not involve bunching, then it is also optimal with respect to *stochastic* mechanisms. As "no–bunching" is the norm in most applications of the theory, the result validates the literature's focus on deterministic mechanisms.[2] I demonstrate this result by considering the optimality of stochastic mechanisms directly. The approach therefore differs from the literature's more indirect procedure of replacing each random mechanism with a corresponding deterministic one. Instead, my approach is more related to Myerson [5], who shows that also in auctions the implementation problem can be reduced to an unconstrained maximization problem which is linear in probability.

2 The Example

An existing argument that demonstrates the suboptimality of random mechanisms in principal–agent settings with quasi–linear utilities may be found in standard text books (e.g., [2,p.306], [4,p.119], [3,p.65])). The logic behind the ar-

[1] See [6] for a more elaborate discussion of this problem.

[2] Hence, apart from the fact that the analysis of bunching is more involved, an additional reason to focus on settings without bunching is the optimality of deterministic mechanisms.

gument is to show that the players gain by replacing a random mechanism with one that implements the expected allocation of the mechanism deterministically. Indeed, if this can be done for *any* random mechanism, then random mechanisms are suboptimal. However, contrary to deterministic mechanisms, random mechanisms are able to implement allocations that are non–monotonic in expected terms. As is well known, such schedules cannot be implemented by deterministic mechanisms.

This suggests that if non–monotonic schedules are desirable, stochastic mechanisms may outperform deterministic ones. In this case, the principal faces a trade–off between the desirability of non–monotonic schedules versus the introduction of additional risk. Consequently, this section demonstrates the beneficial role of stochastic mechanisms in a specific principal agent problem in which the principal and the efficient type do not mind risky allocations. It therefore illustrates the positive effect of stochastic mechanisms in an extreme way; stochastic mechanisms are able to implement the optimal non–monotonic deterministic mechanism arbitrarily closely.

Suppose the principal has to implement some allocation $x \in \mathbb{R}_+$ and may specify a transfer $w \in \mathbb{R}$. There are two types of agents $\theta \in \{1, 2\}$ with quasi–linear utility functions

$$u_1(w, x) = w - x^2 \quad \text{and} \quad u_2(w, x) = \begin{cases} w - x^2/2 & \text{for } x < 2; \\ w - 2x + 2 & \text{for } x \geq 2. \end{cases}$$

Note that the utility function of type 2 is differentiable at $x = 2$ and satisfies the single crossing condition $\partial u_1(w, x)/\partial x \leq \partial u_2(w, x)/\partial x$ for all (w, x). The specific feature of type 2's utility function is that it does not exhibit any risk aversion for allocations $x \geq 2$. Observe however that his utility function is less concave with respect to x than type 1's; a property that a part of the literature views as a guarantee for the optimality of deterministic mechanisms.

Let the principal consider the two types of agents equally likely. The principal's utility associated with an allocation (w, x) depends on the agent's type as follows

$$V_1(w, x) = 10x - w; \quad V_2(w, x) = x - w.$$

The first best allocations, which maximize $V_i(w, x) + U_i(w, x)$, are $x_1^{fb} = 5$ and $x_2^{fb} = 1$. Thus, a decreasing schedule is socially desirable. Under asymmetric information the optimal direct, deterministic mechanism $((w_1^{sb}, x_1^{sb}), (w_2^{sb}, x_2^{sb}))$ is a solution to

$$\max \quad V_1(w_1, x_1)/2 + V_2(w_2, x_2)/2$$

$$\text{s.t.} \quad U_1(w_1, x_1) \geq U_1(w_2, x_2) \text{ and } U_2(w_2, x_2) \geq U_2(w_1, x_1); \tag{1}$$

$$U_1(w_1, x_1) \geq 0 \text{ and } U_2(w_2, x_2) \geq 0; \tag{2}$$

where (1) represents the incentive constraints and (2) the individual rationality constraints. Following standard procedure, we first neglect the individual rationality constraint of the efficient type 2 and the incentive constraint of the inefficient type 1. This procedure yields the schedule $\hat{x} = (\hat{x}_1, \hat{x}_2) = (3, 1)$ with wages $\hat{w} = (\hat{w}_1, \hat{w}_2) = (9, 11/2)$. However, from the two incentive constraints and the single crossing property it follows that a deterministic schedule is only implementable if it is weakly increasing. Hence, the obtained schedule is not implementable; the solution $(\hat{x}, \hat{w})$ violates the incentive constraint of type 1. Consequently, the optimal deterministic mechanism involves bunching, i.e., $x_1 = x_2 = x$, and maximizes $V_1(w, x)/2 + V_2(w, x)/2$ subject to $U_1(w, x) \geq 0$. Straightforward calculations yield the solution $(w^*, x^*) \equiv (121/16, 11/4)$. It follows that the optimal deterministic mechanism is $(w_1^{sb}, x_1^{sb}) = (w_2^{sb}, x_2^{sb}) = (w^*, x^*)$ and yields the principal a payoff of 121/16.

Note that the second best allocation $x_2^{sb} = 11/4$ lies in the range for which type 2 is risk neutral with respect to the allocation x. That is, within this range type 2 is only interested in the expected allocation and is not affected by randomness. Since also the principal is only interested in the expected allocation, we can introduce some randomness at no costs. Yet, by introducing randomness concerning type 2's allocation, it becomes less attractive to type 1. Randomness may therefore be introduced to relax the incentive constraint for type 1. As a concrete example consider the following stochastic direct mechanism. Type 1 is offered the original contract $(121/16, 11/4)$. Type 2 is offered a contract with a deterministic wage 119/16 but after acceptance the contract randomizes between the allocations $x = 2$ and $x = 10/3$ with equal probability. Note that the expected allocation $1/2 * 2 + 1/2 * 10/3$ is smaller than the allocation 11/4 which is meant for type 1. Straightforward calculations show that, despite this feature, the stochastic mechanism is both individual rational and incentive compatible. It yields the principal an expected payoff of 91/12, which exceeds the payoff 121/16 from the optimal deterministic contract.

However, in this extreme example where type 2 is risk neutral for any $x > 2$, the principal may do even better and implement the decreasing schedule $\hat{x} = (3, 1)$ with wages $\hat{w}$ arbitrarily closely. To see this, consider the deterministic contract $\gamma_1 = (w_1, x_1) = (9, 3)$ and a contract $\gamma_2(\alpha)$ with a deterministic wage $w_2(\alpha) = 5 + \alpha/2 + \sqrt{2(1-\alpha)(8+\alpha)}$ that randomizes between the allocation $x_{21} = 1$ and $x_{22}(\alpha) = 1 + \sqrt{8+\alpha}/\sqrt{2-2\alpha}$ with probability α and $1 - \alpha$ respectively. By

construction the menu $(\gamma_1, \gamma_2(\alpha))$ is individual rational and incentive compatible for any $\alpha \in (0,1)$. Note that as α goes to one, the mechanism implements $(\hat{x}, \hat{w})$ ever more closely.

The example shows that the optimality of stochastic mechanisms is related to a desirability of non–monotonic allocation schedules. One may provide the following intuition for this result: When the principal prefers a non–monotonic schedule, a separation of types on the basis of *different degrees of efficiency* is not appropriate, since such separation demands that schedules are monotonic. In contrast, non–monotonic separation is possible on the basis of *different risk attitudes* by choosing a schedule of increasing risks for the less risk averse types. Hence, the example *suggests* that deterministic mechanisms are optimal if non–monotonic schedules are undesirable. The rest of the paper shows that this reasoning is correct; whenever the optimal deterministic mechanism does not involve bunching, stochastic mechanisms are indeed suboptimal.

3 A General Principal–Agent Setup

Consider a contracting problem between a principal and an agent. The principal has no private information. The agent, however, is privately informed about his type $\theta_i \in \Theta$, where the number of types $n \equiv |\Theta|$ is finite.[3] The agent's type θ is drawn from some objective distribution $p = (p_1, p_2, \ldots, p_n)$. The principal's problem consists of selecting a monetary transfer $t \in \mathbb{R}$ to the agent and an allocation $x \in X \subset \mathbb{R}$. The principal's and agent's payoffs are quasi–linear and satisfy

$$V(t, x|\theta_i) = v_i(x) - t \quad \text{and} \quad U(t, x|\theta_i) = u_i(x) + t,$$

respectively. I make three assumptions concerning the agent's utility that are standard for these types of principal–agent problems: First, higher types derive a higher utility from an allocation x than lower types. I.e.,

$$u_i(x) > u_{i-1}(x), \ \forall \theta_i \in \Theta \backslash \{\theta_1\}, \ \forall x \in X. \tag{3}$$

Second, the agent's preferences satisfy a *single crossing condition* in that for all $x, y \in X$ with $x > y$ it holds

$$u_i(x) - u_i(y) \geq u_{i-1}(x) - u_{i-1}(y), \ \forall \theta_i \in \Theta \backslash \{\theta_1\}. \tag{4}$$

Third, the agent's reservation utility is type independent and normalized to zero.

[3] To limit technicalities, I restrict attention to finite, but arbitrarily large type sets.

In order to solve her contracting problem, the principal may use a mechanism to elicit the private information from the agent. I distinguish between two types of mechanisms: deterministic and stochastic ones. For deterministic mechanisms, the implemented allocation $x \in X$ depends on the agent's supply of information in a deterministic way, whereas for stochastic mechanisms the relationship may be stochastic. As is well know (e.g., [2,p.306]), the quasi–linear payoff structure implies that there is no gain in randomizing with respect to the transfer t. Consequently, without loss of generality the transfer is assumed deterministic.[4]

More precisely, a *deterministic, direct mechanism* $\Delta = (t, x)$ specifies a transfer schedule $t : \Theta \to \mathbb{R}$ and an implementation function $x : \Theta \to X$. Thus, when the agent announces he is of type θ_i, he receives a transfer t_i and the deterministic allocation x_i is implemented.

In order to introduce stochastic mechanisms, let $\mathcal{X}$ denote the Borel σ-algebra on X.[5] A *stochastic, direct mechanism* $\Sigma = (t, \mu)$ specifies a transfer schedule $t : \Theta \to \mathbb{R}$ and an implementation function $\mu : \Theta \to Q$, where Q is the set of probability measures on $\mathcal{X}$. Thus, $\mu_i(H)$ with $H \in \mathcal{X}$ denotes the probability that an allocation which lies in H is implemented, when the agent reports he is of type θ_i. Deterministic mechanisms are a special case of stochastic mechanisms. In particular, a deterministic mechanism $\Delta = (t, x)$ is equivalent to the stochastic mechanism $\Sigma^\Delta = (t, \mu^\Delta)$ with

$$\mu_i^\Delta(x(\theta_i)) = 1, \ \forall \theta_i \in \Theta.$$

Alternatively, a stochastic mechanism (t, μ) is equivalent to a deterministic mechanism, whenever it is degenerated. That is, whenever it holds

$$\mu_i(H) \in \{0, 1\}, \quad \forall H \in \mathcal{X}. \tag{5}$$

4 Two Optimization Problems

Since the principal operates under perfect commitment, the revelation principle holds. Consequently, there is no loss of generality by focusing on direct mechanisms Σ that are incentive compatible. The following maximization problem yields such an optimal mechanism:

$$P^\Sigma : \quad \max_{\Sigma=(t,\mu)} \quad V_\sigma(\Sigma) \equiv \sum_i^n \left\{ \int p_i v_i(x) d\mu_i - p_i t_i \right\} \tag{6}$$

[4]When also the screening variable enters only linearly, deterministic mechanisms are optimal by the same argument.

[5]All functions, such as v and u, are therefore assumed measurable on $(X, \mathcal{X})$.

$$\text{s.t.} \quad \int u_i(x)d\mu_i + t_i \geq \int u_i(x)d\mu_j + t_j, \quad \forall \theta_i, \theta_j \in \Theta; \tag{7}$$

$$\int u_i(x)d\mu_i + t_i \geq 0, \quad \forall \theta_i \in \Theta; \tag{8}$$

where (7) represent the incentive compatibility constraints and (8) the individual rationality constraints. Let $\Sigma^* = (t^\Sigma, \mu^\Sigma)$ with value $V^\Sigma = V_\sigma(\Sigma^*)$ denote a solution to problem P^Σ. Throughout this paper I assume that optimal mechanisms exist. Clearly, V^Σ is unique, while there may be multiple solutions Σ^*.

Most applications of mechanism design do not study problem P^Σ directly. Rather, they first restrict attention to deterministic mechanisms of the type Δ. In a second step, they then search for conditions that imply a suboptimality of stochastic mechanisms. Indeed, given a suboptimality of stochastic mechanisms, the revelation principle indirectly implies that there exists an optimal mechanism that is direct, deterministic, and incentive compatible. Hence, an optimal deterministic mechanism is a solution to

$$P^\Delta : \max_{\Delta=(t,x)} \quad V_\delta(\Delta) \equiv \sum_i p_i \{v_i(x_i) - t_i\} \tag{9}$$

$$\text{s.t.} \quad u_i(x_i) + t_i \geq u_i(x_j) + t_j, \quad \forall \theta_i, \theta_j \in \Theta; \tag{10}$$

$$u_i(x_i) + t_i \geq 0, \quad \forall \theta_i \in \Theta; \tag{11}$$

where (10) and (11) represent the incentive compatibility and individual rationality constraints, respectively. Let $\Delta^* = (t^\Delta, \mu^\Delta)$ with value $V^\Delta = V(\Delta^*)$ denote a solution to problem P^Δ.

Since deterministic mechanisms are degenerated stochastic mechanisms, problem P^Δ is more constrained than problem P^Σ. More precisely, problem P^Δ is equivalent to P^Σ with the additional restriction (5) so that $V^\Delta \leq V^\Sigma$.

The usual approach to solving problem P^Δ is to focus on the *local downward* constraints. In particular, disregarding the other constraints, one concentrates on the *relaxed* maximization problem:

$$\max_{\Delta=(t,x)} \quad V_\delta(\Delta) \quad \text{s.t. } u_i(x_i) + t_i \geq u_i(x_{i-1}) + t_{i-1}, \quad \forall \theta_i \in \Theta, \tag{12}$$

where $u_1(x_0) = t_0 = 0$. As is well known, the single crossing condition implies that a solution to the relaxed problem (12) coincides with the solution to problem P^Δ if it satisfies the following *monotonicity* conditions:

$$x_i \geq x_{i-1}, \quad \forall \theta_i \in \Theta \backslash \{\theta_1\}. \tag{13}$$

5 Optimality of Deterministic Mechanisms

This section demonstrates that whenever a solution to (12) exists that satisfies the monotonicity constraint (13), then this implies that deterministic mechanisms are optimal. To arrive at this result, I first relax problem P^{Σ} in a similar way as in the deterministic problem and focus on the relaxed problem

$$\max_{\Sigma=(t,\mu)} \quad V_\sigma(\Sigma) \quad \text{s.t.} \int u_i(x)d\mu_i + t_i \geq \int u_i(x)d\mu_{i-1} + t_{i-1}, \quad \forall \theta_i \in \Theta, \tag{14}$$

where $t_0 = 0$ and $\mu_0(H) = 0$ for all $H \in \mathcal{X}$. Since problem (14) is less constrained than the original problem P^{Σ}, its solution, V^r, is weakly greater than V^{Σ}. In addition, the problem is also less constrained than the relaxed deterministic program (12). Indeed, program (12) is equivalent to program (14) with the additional constraint (5).[6]

Clearly the constraints in program (14) must be binding at the optimum, since otherwise one could lower the respective payment t_i and increase the objective function without violating any constraints. Hence, the transfers t_i may be solved recursively as

$$t_i = t_i^r(\mu) \equiv -\sum_{j=1}^{i} \left\{ \int u_j(x)d\mu_j - \int u_j(x)d\mu_{j-1} \right\}.$$

Substitution of t_i and a rearrangement of terms leads to the maximization problem

$$\max_{\mu} \quad V_\sigma(t_i^r(\mu), \mu) = \sum_{i}^{n} \left\{ \int c_i(x)d\mu_i \right\}, \tag{15}$$

with

$$c_i(x) \equiv p_i(v_i(x) + u_i(x)) - \sum_{j=i+1}^{n} p_j(u_{i+1}(x) - u_i(x)).$$

A solution of (15) is straightforward.[7] For each type θ_i, it puts all probability mass on an allocation x that maximizes $c_i(x)$. More precisely, define x_i^r as a maximizer of $c_i(x)$, i.e., $x_i^r \in \arg\max_x c_i(x)$. Moreover, define $\mu^r = (\mu_1^r, \ldots, \mu_n^r)$ as

$$\mu_i^r(H) \equiv \begin{cases} 1 & \text{if } x_i^r \in H \\ 0 & \text{if } x_i^r \notin H, \end{cases}$$

for all $\theta_i \in \Theta$ and $H \in \mathcal{X}$.

[6]Note that there exist no straightforward conditions similar to (13) which guarantee that the solution of problem (14) coincides with the solution of the original problem (6).

[7]Existence is guaranteed when X is compact and $c_i(x)$ is upper semicontinuous.

Proposition 1 *The implementation function μ^r is a solution of (15).*

Proof: For any implementation function $\tilde{\mu}$ it holds

$$V_\sigma(t_i^r(\tilde{\mu}), \tilde{\mu}) = \sum_i^n p_i \left\{ \int c_i(x) d\tilde{\mu}_i \right\} \leq \sum_i^n p_i \left\{ \int \max_x c_i(x) d\tilde{\mu}_i \right\} =$$
$$\sum_i^n p_i \max_x c_i(x) = \sum_i^n p_i c_i(x_i^r) = \sum_i^n p_i \left\{ \int c_i(x) d\mu_i^r \right\} = V_\sigma(\Sigma^r),$$

with $\Sigma^r \equiv (t_i^r(\mu^r), \mu^r)$. Hence, μ^r yields at least as much as any other $\tilde{\mu}$ and is a maximizer. Q.E.D.

Since μ_i^r satisfies (5), it is a degenerated measure. Hence, the mechanism $\Sigma^r = (t^r(\mu^r), \mu^r)$ is equivalent to the deterministic mechanism $\Delta^r \equiv (t^r(\mu^r), x^r(\theta))$ with

$$x^r(\theta_i) \equiv x_i^r.$$

As Δ^r satisfies the constraints of the relaxed maximization problem (12), it must also solve (12). This reasoning leads to our main result:

Proposition 2 *If there exists a solution $(\bar{t}, \bar{x})$ of (12) that satisfies (13) then $V^\Delta = V^\Sigma$ and deterministic mechanisms are optimal.*

Proof: Suppose $(\bar{t}, \bar{x})$ solves (12) and satisfies (13) then $(\bar{t}, \bar{x})$ solves P^Δ so that $V^\Delta = V_\delta(\bar{t}, \bar{x})$. Thus, it holds $V_\delta(\bar{t}, \bar{x}) = V^\Delta \leq V^\Sigma \leq V_\sigma(\Sigma^r)$. But Δ^r satisfies the constraints in (12) so that it is feasible for this program. Since $(\bar{t}, \bar{x})$ solves (12), it must therefore hold $V_\delta(\Delta^r) \leq V_\delta(\bar{t}, \bar{x})$. Finally we have that $V_\sigma(\Sigma^r) = V_\delta(\Delta^r)$. Linking all these weak inequalities yields $V_\sigma(\Sigma^r) = V_\delta(\Delta^r) \leq V_\delta(\bar{t}, \bar{x}) = V^\Delta \leq V^\Sigma \leq V_\sigma(\Sigma^r)$. Hence, $V^\Delta = V^\Sigma$, so that a deterministic mechanism is optimal. Q.E.D.

6 Concluding Remarks

This paper shows that in principal–agent problems deterministic mechanisms are suboptimal even if the concavity of the agent's utility function with respect to the screening variable decreases with the agent's type. The advantage of stochastic mechanisms is that they are able to separate types due to a difference in risk attitudes with respect to the screening variable instead of efficiency alone. As a consequence, they are, in contrast to deterministic ones, able to implement allocations that are decreasing in expected terms. This may render deterministic mechanisms suboptimal.

Yet, whenever the optimal deterministic contract involves no bunching of types, it is also optimal with respect to stochastic mechanisms. This endogenous condition hands applications a practical tool for checking the optimality of deterministic contracts; normally one already checks for monotonicity for other reasons. Indeed, since in most applications the obtained optimal *deterministic* mechanisms do not involve bunching, it validates the literature's usual restriction to such mechanisms.

At first sight it may seem puzzling that the result does not depend on the concavity of the players' utility functions $v_i(x)$ and $u_i(x)$. That is, deterministic mechanisms are optimal, even when the principal and agent are risk–loving with respect to x and explicit randomization seems desirable. To understand that such an intuition is misleading, consider the special case of $n = 1$. In this case there is no asymmetric information and the principal simply maximizes the social surplus $v_1(x) + u_1(x)$. Clearly, randomization with respect to x does not yield a higher surplus. In particular, when v_1 and u_1 are both convex on the entire domain X, the optimal allocation is either a corner solution or does not exist. Finally, note that the result does not claim that, in general, there do not exist deterministic allocations $x \in X$ that are Pareto dominated by stochastic ones. Rather, if such allocations exist, they are suboptimal.

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