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**Working Paper**

# Incidence and Growth of Patent Thickets - The Impact of Technological Opportunities and Complexity

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GOVERNANCE AND THE EFFICIENCY OF ECONOMIC SYSTEMS
GESY

Discussion Paper No. 356

# Incidence and Growth of Patent Thickets - The Impact of Technological Opportunities and Complexity

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# Incidence and Growth of Patent Thickets - The Impact of Technological Opportunities and Complexity

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**Abstract**

We investigate incidence and evolution of patent thickets. Our empirical analysis is based on a theoretical model of patenting in complex and discrete technologies. The model captures how competition for patent portfolios and complementarity of patents affect patenting incentives. We show that lower technological opportunities increase patenting incentives in complex technologies while they decrease incentives in discrete technologies. Also, more competitors increase patenting incentives in complex technologies and reduce them in discrete technologies. To test these predictions a new measure of the density of patent thickets is introduced. European patent citations are used to construct measures of fragmentation and technological opportunity. Our empirical analysis is based on a panel capturing patenting behavior of 2074 firms in 30 technology areas over 15 years. GMM estimation results confirm the predictions of our theoretical model. The results show that patent thickets exist in 9 out of 30 technology areas. We find that decreased technological opportunities are a surprisingly strong driver of patent thicket growth.
JEL: L13, L20, O34.

Keywords: Patenting, Patent thickets, Patent portfolio races, Complexity, Technological Opportunities.

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# 1 Introduction

Strong increases in the level of patent applications have been observed at the United States Patent and Trademark Office (USPTO) (Kortum and Lerner, 1998, Hall, 2005a) and the European Patent Office (EPO) (von Graevenitz et al., 2007). These "patent explosions" pose serious challenges for existing patent systems and also for competition authorities.[1]

Explanations for this shift in patenting behavior focus on changes in the legal environment and management practices, the complexity of some technologies, greater technological opportunities and increased strategic behavior on the part of firms. While it has been shown that most of these factors play a role empirically, there are no explanations of patenting behavior that integrate the effects of these determinants.[2] This paper provides such an integrative model and an empirical test of its predictions. We introduce a new measure of complexity of blocking relationships, allowing us to quantify the extent of patent thickets. With this measure we show that patent thickets exist in 9 of the 30 technology areas making up the patent system and are growing to include more firms. Our empirical results also reveal that patenting responds surprisingly strongly to variation in technological opportunities.

Kortum and Lerner (1998, 1999) investigated the explosion of patenting at the USPTO which began around 1984 (Hall, 2005a). By a process of elimination they argue that increased patenting mainly results from changed management practices making R&D more applied and raising the yield of patents from R&D. Kortum and Lerner (1998, 1999) and Hall and Ziedonis (2001) also explore whether enhanced fertility of R&D led to an increase in patent filings, but cannot find systematic evidence for this. Hall and Ziedonis (2001) provide evidence that the patenting surge is a strategic response to an increased threat of hold-up specifically in complex technologies in which products depend on the combination of large numbers of patents. Complexity of a technology implies that patents are natural complements, and therefore hold-up arises easily if patent ownership is dispersed (Shapiro, 2001, Ziedonis, 2004). Hall (2005a) also shows that the patenting surge is driven by firms whose main technologies are complex.

Our model of patenting juxtaposes patenting incentives in complex and discrete technologies. In the model patenting incentives arise from the interaction of technological opportunity (fertility) and the complexity of technologies. To model the joint effect of these determinants of patenting, we posit a two dimensional matrix of technological opportunities and of patentable innovations within each such opportunity. We refer to the latter as facets. Firms choose between pursuing new technological opportunities and deepening protection of specific opportunities by patenting of more facets. Analysis of the model shows that firms' actions are strategic substitutes in discrete technologies but become strategic complements in

[1] For extensive discussions of the policy questions surrounding current functioning of the patent systems in the United States and in Europe refer to National Research Council (2004), Federal Trade Commission (2003), Jaffe and Lerner (2004), von Graevenitz et al. (2007) and Bessen and Meurer (2008).

[2] Formal models of patenting abound, for a survey of this literature refer to Scotchmer (2005) or Gallini and Scotchmer (2002). Formal models of patenting in patent thickets do not attempt to span both complex and discrete technologies as we do here: Bessen (2004), Clark and Konrad (2008) and Siebert and von Graevenitz (2010). These models build on the patent race literature pioneered by Loury (1979), Lee and Wilde (1980), Reinganum (1989) and Beath et al. (1989).

sufficiently complex technologies. In a complex technology firms patent *less* in response to increasing technological opportunities and *more* if more other firms compete for patents. Both effects result from strategic interaction of firms in a complex technology: greater technological opportunities reduce pressures on firms to defend their stake in existing technologies by patenting heavily, whereas greater competition increases this pressure.

Predictions derived from the model are tested using a comprehensive data set based on EPO patent data. It comprises information on patenting behavior between 1987 and 2002 spanning complex and discrete technologies. We measure the complexity of blocking in a technology area using information specific to European patents. Patent examiners at the EPO indicate which prior patents block or restrict the breadth of the patent application under review. We count how often three or more firms apply for mutually blocking patents within a three year period. This gives rise to a count of mutually blocking firm *triples*. The measure captures complex blocking relationships which arise even if patent ownership remains relatively concentrated. This new measure is validated by showing that greater incidence of complex blocking relationships is correlated with the classifications of technological complexity suggested by Cohen et al. (2000).

Patenting behavior is known to be highly persistent, due to the long term nature of firms' R&D investment decisions. We control for the persistence of patenting by including a lagged dependent variable in the empirical model. The model is estimated using systems GMM estimators (Blundell and Bond, 1998, Arellano, 2003, Alvarez and Arellano, 2003) to control for endogeneity of the lagged dependent variable. Additionally, we treat measures of technological opportunity, complexity and fragmentation as predetermined. Results from OLS, fixed effects and GMM regressions support our theoretical predictions. In particular, decreasing technological opportunities and increasing complexity lead to more patent filings. Thus, our paper suggests a new rationale for the rise of patent filings since the mid-1980s.

The remainder of this paper is structured as follows. Section 2 provides a theoretical model of patenting which explains firms' patenting strategies. We derive three hypotheses from this model that are empirically testable. In Section 3 we describe our data set and the variables we employ to analyze firms' patenting behavior. As there is little cross industry evidence of patenting trends at the EPO, Section 4 provides a descriptive analysis of these trends, focusing particularly on our own measure and alternative measures of complexity. Section 5 provides the empirical model and results and Section 6 concludes.

## 2 A Model of Patenting

Here we present a model of patenting behavior. This model shows how technological opportunity and complexity of technology affect the levels of patenting set by firms. Technological opportunity and complexity are assumed to be fixed in the short- to medium term.[3] First,

[3]In the long run technological opportunity may be affected by firms' patenting efforts. Unravelling this question will require a separate study with data on firms' R&D activities over a very long period.

we motivate the model and discuss assumptions. Then, we provide a number of definitions. Next we solve the model, presenting several predictions. These underpin the empirical results presented in Sections 4 and 5 below.

## 2.1 Motivation

The following model captures patenting incentives in discrete and complex technologies. We characterize how the degree of technological opportunity and the complexity of a technology determine the level of patenting of that technology.

We recognize that the patent system covers a multitude of different technology areas. Within these we posit distinct technological opportunities that derive from separate research efforts. Each technological opportunity consists of one or more patentable facets. Every facet corresponds to a potential patent. Technologically related facets are grouped together in technological opportunities because they derive from the same knowledge and science base.

The underlying model of R&D and of the patent office is kept as simple as possible: firms select how many technological opportunities to research and how many facets of each to seek to patent. Facets and opportunities are chosen randomly by each firm. Where more than one applicant applies for a facet, that is randomly assigned to one applicant. Patent allocation is the sole function of the patent office in the model. These assumptions capture competition between firms seeking to make enough applications to ensure that some result in granted patents. This model can be presented as a matrix of patents that firms compete to patent:

Figure 1: Complexity and the number of patentable facets per technological opportunity.

This figure shows different matrices corresponding to technology areas with growing levels of complexity and varying levels of technological opportunity. Complexity increases with the number of facets. With higher complexity it is increasingly probable that ownership of patents in a technological opportunity becomes dispersed. We assume that the value of owning patents

in technological opportunities with more than one facet depends on the share of patents in each technological opportunity that firms own. This captures the interdependence of patents in complex technologies and the possibility for hold-up within them. In this model hold-up arises within technological opportunities but not between them.

Consider two examples: first, one patent generally suffices for the applicant to protect an ethical drug effectively against attempts to invent around the patent. This is the case of a discrete technology in which each patent covers one technological opportunity. Second, laser technology is used in a very wide range of applications such as eye surgery (e.g. LASIK) or pollution monitoring and forestry management (LIDAR). This is the case of a complex technology area within the field of optics.[4] Each application of laser technology can be thought of as a technological opportunity requiring a range of different patentable inventions that are combined in a functioning product. A product using laser technology will usually also embody some patents relating to different technological areas outside of optics, but this is a point our model abstracts from. Due to the complexity of the technology hold-up may arise: in the case of LASIK there has was a string of court cases between VISX Inc. and Nidek Inc. after 1998 regarding infringement of VISX patents on LASIK. The companies finally settled their disputes world wide in April of 2003.

We do not explicitly model such hold-up or its resolution. The literature on patent thickets shows that several institutional arrangements allow firms to disentangle overlapping property rights - these include licensing, patent pools, standard setting as well as litigation (Shapiro, 2001, Scotchmer, 2005). There is some evidence that firms holding a large share of patents within a given technology benefit substantially from their patent portfolio and may be able to reduce the likelihood of hold-up (Grindley and Teece, 1997, Shapiro, 2001, Ziedonis, 2004). This is attributed to an increase in bargaining power. Additional patents also reduce marginal legal costs as the share of patents grows: firms with a large share of patents on a technological opportunity will need to cross-license or litigate less (Lanjouw and Schankerman, 2004).

Our model abstracts from the link between patents and the product market, assuming only that the technological opportunities firms are patenting are valuable. In complex technologies patents may be complements: then the value of the entire technology grows as more components of the technology are patented. The process of patenting is also modeled simply: once firms invest in R&D for a technological opportunity the number of facets they seek to patent is limited only by the costs of maintaining granted patents. The probability of obtaining a granted patent on a facet falls with the number of rivals also seeking to patent that facet.

This model allows us to capture competition for patents in discrete technologies and in complex technologies. As we show next the nature of competition depends on the complexity of a technology and the complementarity of facets in each opportunity.

[4]To further clarify the definitions of opportunities, facets and technology areas we discuss the example of LED technology at greater length in Appendix D.

## 2.2 Assumptions

We study a setting in which a technology area is characterized by $(O)$ technological opportunities each of which consists of patentable facets. A technological opportunity is an independent source of profit to a firm and each facet is a separate patentable invention which is part of the opportunity. The total number of patentable inventions (facets) offered by a technological opportunity is $F$. Thus a technology area is discrete if $F = 1$. We assume that:[5]

*All technological opportunities in a technology area are symmetrical; they offer the same number of facets, and costs of R&D and of patenting are identical.* $(S)$

The total set of patentable inventions in a technology consists of $\Omega = O \times F$ facets. As $F$ grows the underlying technology grows more complex. If there is more technological opportunity, $O$ grows. Variation in the two dimensions of the set of available patents $\Omega$ arises for different reasons. Current efforts in basic R&D open additional new opportunities in the future raising $O$. The number of facets which are patentable on a given opportunity depends mainly on the nature of technology but also on institutional and legal factors.

Each technological opportunity is associated with a maximal total value $V(F)$ and an actually attained value $V(\tilde{F})$. The attained value depends on the number of facets actually patented by all firms $\tilde{F}$, which may be less or equal to the number of available facets $F$. Firms appropriate a share $s$ of the attained value by acquiring patents. To capture the complementarity of inventions in complex technologies we assume that the value of the technological opportunity increases in the number of facets of that opportunity patented by all firms $\tilde{F}$:[6]

$$V(0) = 0 \qquad \text{and} \qquad \frac{\partial V}{\partial \tilde{F}} > 0 \qquad . \tag{CI}$$

There are $N + 1$ firms active in a given technology area. Each can apply for patent protection for all facets of a technological opportunity. A firm's strategy consists of the number of opportunities $o_k$ ($o_k \in [0, O]$) it invests in and the number of facets $f_k$ ($f_k \in [0, F]$) per opportunity which it seeks to patent. Subscripts index the firm. Each firm can only make one patent application per facet and it can only patent in technological opportunities which it has researched . It trades off patenting more facets per opportunity and patenting in more different technological opportunities. While patenting additional facets is assumed to be costless,[7] a maintenance fee is payable ($C_a$) on granted patents. Additionally, firms must undertake costly R&D ($C_o$) on each technological opportunity they turn to. Finally, costs of coordinating sepa-

[5]Note that this assumption rules out aspects of complexity that may be quite important in practice. Thus we rule out that some facets may belong to more than one technological opportunity, making patents on them particularly valuable blocking patents. We leave this aspect of complexity for future work.

[6]A similar assumption is made by Lerner and Tirole (2004).

[7]We make this assumption in order to simplify the model, but it can be shown that it does not affect our results if patent filing costs are sufficiently low in comparison to the costs of maintenance. In practice, initial application and examination fees for patents are indeed much lower than post-grant translation and renewal fees, since most patent offices cross-subsidize the initial stages in order to encourage patent filing.

rate research projects ($C_c$) are generally viewed as significant in the literature (Roberts, 2004). To summarize:

i *Per opportunity a firm invests in, it faces costs of R&D:* $C_o$.

ii *Per granted patent a firm faces costs of maintaining that patent:* $C_a$.

iii *The coordination of R&D on* different *technological opportunities imposes costs* $C_c(o_k)$. *Therefore, we assume that* $\frac{\partial C_c}{\partial o_k} > 0$. $(FVC)$

As the number of facets per technological opportunity grows, so does the probability that different firms own patents belonging to one opportunity. Hold-up becomes increasingly likely. Then, firms need to disentangle ownership rights, giving rise to legal costs $(LC)$. These encompass the costs of monitoring, licensing, and negotiating settlements as well as court fees. As noted above, a greater share of patents per technological opportunity reduces marginal costs of resolving hold-up. If we define the expected share of patents granted to firm $k$: $s_k \equiv p_k f_k / \tilde{F}$ Therefore, we assume:

$$\frac{\partial L}{\partial s_k} > 0 \quad , \quad \frac{\partial^2 L}{\partial {s_k}^2} < 0 \quad , \tag{LC}$$

where $s_k$ is the expected share of granted patents obtained in each technological opportunity.

Note that the terms of licensing deals or any other arrangements that resolve hold-up will depend on the size of firms' patent portfolios and on the relatedness of patents (Siebert and von Graevenitz, 2010). We capture this in a very reduced form approach, using the notion of the share of patents owned per technological opportunity to keep the model manageable.

We assume throughout that the levels of $N$, $O$, $F$ and $V$ are known by all patenting firms.

## 2.3 Definitions

This subsection sets out a number of definitions that follow from our previous assumptions. Given that the number of firms $N$ is common knowledge, firms can compute the expected number of rivals active within a technological opportunity, the expected number of facets on which patents are granted and the likelihood of obtaining a patent grant.

The expected number of rivals ($N_O$) competing for patents within a technological opportunity is derived in Appendix A.1. It depends on technological opportunity ($O$), the overall number of firms in a technology area $N$ and each rival's investments in R&D ($o_j$). We show that:

$$\frac{\partial N_O}{\partial O} < 0 \quad \text{and} \quad \frac{\partial N_O}{\partial o_j} > 0 \quad . \tag{1}$$

To simplify notation we define the share of facets each firm $k$ applies for per technological opportunity as $\phi_k \equiv f_k / F$. Given our simplified model of the patent application process the

expected number of facets per technological opportunity on which patents are granted is:

$$\tilde{F}(f_k, \mathbf{f}_{\not k}, F, N_O(O, \mathbf{o}_{\not k}, N)) = F\left[1 - (1-\phi_k)\prod_{j\neq k, j=1}^{N_O}(1-\phi_j)\right] \quad , \tag{2}$$

where $\mathbf{f}_{\not k}, \mathbf{o}_{\not k}$ are vectors containing the choices of the number of facets and the number of opportunities to invest in made by all rival firms. This expression results from the assumptions that firms randomly choose facets and that the patent office randomly selects which application to grant. This model of patenting captures coordination failure and duplication of applications by firms. Then, the proportion of facets covered by at least one applicant is one minus the number of facets attracting no applications. In Appendix A.2 we show that the number of facets covered increases in the complexity of the technology, in the number of rivals investing in a technological opportunity and also in the number of facets each firm invests in:

$$\frac{\partial \tilde{F}}{\partial F} > 0 \quad , \quad \frac{\partial \tilde{F}}{\partial N_O} > 0 \quad \text{and} \quad \frac{\partial \tilde{F}}{\partial f_k} > 0 \quad , \quad \frac{\partial \tilde{F}}{\partial f_j} > 0. \tag{3}$$

We assume that the patent office will grant each application for a patent on a facet with equal probability, but only grants one patent overall on the facet.Then the probability of patenting a facet depends on the expected number of rivals seeking to patent each facet and the probability with which the particular number of rivals occurs. In Appendix A.3 we show that the probability that firm $_k$ obtains a patent on a given facet is:

$$p_k(\mathbf{f}_{\not k}, F, N_O(O, \mathbf{o}_{\not k}, N)) = \sum_{l=0}^{N_O}\frac{1}{l+1}\binom{N_O}{l}\prod_{i=0}^{N_O-l}(1-\phi_i)\prod_{j=N_O-l}^{N_O}\phi_j \quad . \tag{4}$$

This expression shows that the probability of obtaining a patent on an application is a sum of weighted probabilities. Each element of the sum consists of the weighted probability of obtaining a patent $1/(1+l)$ given the number of rival firms also seeking a patent on the facet $l$. The weight captures the probability of observing a given number of rivals. In Appendix A.3 we show that the probability of obtaining a patent decreases in the level of facets rival firms seek to patent and in the number of rival firms per technological opportunity:

$$\frac{\partial p_k}{\partial \phi_j} < 0 \quad \text{and} \quad \frac{\partial p_k}{\partial N_O} < 0 \quad . \tag{5}$$

Finally, define the expected share of patents granted to firm $k$: $s_k \equiv p_k f_k / \tilde{F}$ and the elasticity of the value of a technological opportunity $(V)$ with respect to covered patents $(\tilde{F})$: $\mu \equiv \frac{\partial V}{\partial \tilde{F}}\frac{\tilde{F}}{V}$.

## 2.4 Results

In this section we set out a firm's objective function and the patenting game it is involved in. We analyze this game, show when it is supermodular and derive comparative statics results.

Given symmetry of technological opportunities (Assumption $S$) the expected value of patenting for firm $k$ in a technology area is:

$$\pi_k(o_k, f_k) = o_k \left( V(\tilde{F}) s_k - L(s_k) - C_o - f_k p_k C_a \right) - C_c(o_k) \quad . \tag{6}$$

Firms derive revenues from each technological opportunity and face costs of coordinating R&D across different technological opportunities ($C_c$). Profits per technological opportunity depend on the share of patents granted ($s_k$), legal costs ($L$) as well as costs of R&D on the technological opportunity ($C_o$) and costs of maintaining granted patents ($C_a$).

Define a game $G$ in which:

- There are $N + 1$ firms.
- Each firm simultaneously chooses the number of technological opportunities $o_k \in [0, O]$ and the number of facets applied for per opportunity $f_k \in [0, F]$, to maximize the payoff function $\pi_k$. Firms' strategy sets $S_n$ are elements of $R^2$.[8]
- Firms' payoff functions $\pi_k$, defined in equation (6), are twice continuously differentiable and depend only on rivals' aggregate strategies.

Firms' payoffs depend on their rivals' aggregate strategies because the probability of obtaining a patent on a given facet is a function of all rivals' patent applications. Note that the game is symmetric as it is exchangeable in permutations of the players. This implies that symmetric equilibria exist if the game can be shown to be supermodular (Vives, 2005).[9]

In this game firms compete for granted patents on a technological opportunity. They pick a certain number of technological opportunities and apply for patents on a share of the facets in each opportunity. As rival firms' applications increase, the probability of receiving a patent grant decreases. However, rivals' patent applications can be complementary to own applications as they can raise the overall value of technological opportunities in complex technologies. These two effects counteract one another: where the effect of rivalry dominates the game is one of strategic substitutes, where the effect of the complementarity dominates the game becomes a game of strategic complements. In particular we can show that:

**Proposition 1**

*The game $G$ is smooth supermodular if the technology is sufficiently complex, if there are enough patenting firms and if the value of the marginal patent grant outweighs its administrative cost.*

This proposition contains three conditions for supermodularity: the first regards the number of patentable facets per technological opportunity and the second the number of firms competing for patents. We find that supermodularity is more likely to hold as both of these factors

[8] We treat $o_k$ and $f_k$ as continuous real numbers in the paper. Both determine probabilities: that a firm will invest in specific technological opportunities in case of $o_k$ or facets in case of $f_k$. These probabilities are defined in Appendix A.

[9] Note also that only symmetric equilibria exist as the strategy spaces of players are completely ordered.

increase. The third condition regards the marginal value of additional patents relative to administrative costs. If the marginal gains are large enough the game is supermodular.

Where the game is supermodular we can characterize its comparative statics. In most cases where the game is not supermodular the comparative statics will be difficult to characterize in general.[10] There is one important exception, that of a discrete technology. In that case there is only one facet $(F = 1)$ per technological opportunity. We characterize this important special case at the end of this section. It can be shown that the comparative statics for that case differ from those of the equilibria of the supermodular game.

To prove Proposition 1 we show in Appendix B that firms' profit functions are supermodular (i) in their own actions and (ii) in every combination of their own actions with those of rival firms (Milgrom and Roberts, 1990, Vives, 1999, 2005, Amir, 2005). This is the case if the cross-partial derivatives between own as well as own and rival actions are positive, indicating that all of these actions are strategic complements. We provide three intermediate results to clarify conditions under which firms' actions are strategic complements. The proofs of the following lemmas is given in Appendix B. The first result is negative. We show that:

**Lemma 1**

*In the absence of either administrative or legal costs game $G$ is not supermodular.*

The lemma clarifies that both administrative and legal costs provide a moderating effect for firms' patenting efforts. In the absence of either of these costs the game is not one of strategic complements, in fact some of the firms' actions may be strategic substitutes.[11] However, these costs do exist in all patent systems. In this case we find that:

**Lemma 2**

*Given positive administrative and legal costs firms' actions are strategic complements, if the marginal value of additional patents exceeds their administrative costs.*

This lemma shows that sufficiently high administrative costs can prevent strategic complementarity if the marginal value of a granted patent decreases in the number of covered patents. Additionally, analysis of effects of rivals' actions on the number of facets a firm seeks to patent[12] reveals:

**Lemma 3**

*As the complexity of the technology increases and the number of rival firms grows, it is more likely that firms' actions are strategic complements.*

This last lemma shows that the number of firms competing for patents on each technological opportunity and the complexity of each technology affect how likely it is that firms' actions are strategic complements in the game $G$.

[10] In the model we analyze simultaneous optimization over two parameters. In the absence of supermodularity a general characterization of comparative statics leads to the analysis of multiple implicit relations. We do not pursue this line of analysis as it will require a host of additional assumptions.

[11] Our analysis shows that absent administrative or legal costs rivals' actions are strategic substitutes for the number of technological opportunities invested in. This emerges from analysis of equations (26) and (27).

[12] Compare equations (28) and (29) in Appendix B.

There are two main implications that we can take away from these results. First, the model shows that simultaneous competition for patents on various technological opportunities is not necessarily characterized by strategic complementarities. We do not pursue those cases in which there are no strategic complementarities, save for the important special case of discrete technologies. Secondly, the conditions under which strategic complementarity is likely to arise in our model fit our current understanding of settings in which patent thickets arise very well. These are settings in which technologies are highly complex, in which many firms seek to build large patent portfolios and in which the combination of multiple parties' technologies yields the best standards and products.

**Comparative Statics of the Model**

Here we provide comparative statics assuming that Proposition 1 holds. Throughout *patenting efforts* refers to the choice of $f_k$ and $o_k$. All derivations are provided in Appendix B. There we begin with the following Corollary:

**Corollary 1**
*If game $G$ is supermodular, firms' patenting efforts increase in the number of competitors $(N)$.*

If firms' actions are strategic complements, then additional competitors raise the number of patents covered, increasing the expected value of all patents. At the same time the probability of success on any given patent application will fall. Both of these effects reinforce firms' patenting incentives and efforts. Additionally, we can show that:

**Proposition 2**
*If game $G$ is supermodular, firms' patenting efforts fall with technological opportunity $(O)$.*

If firms' actions are strategic complements, then greater technological opportunity reduces the number of patents granted per technological opportunity and the value of each opportunity while increasing the probability of success on any given patent application. Both of these effects reduce firms' patenting incentives and efforts. Finally, consider how greater technological complexity affects patenting:

**Proposition 3**
*If game $G$ is supermodular greater complexity increases firms' patenting efforts.*

Greater complexity of a technology has two effects. First, it increases the number of facets per technological opportunity, which makes it easier to patent. Second, it reduces the share of the value which a firm can secure with granted patents it already expects to hold. Both effects lead firms to step up their patenting efforts.

**Discrete Technologies** We turn now to the case of a discrete technology where - by definition - $F = 1$. Additionally, legal costs of defending and exploiting a patent right are not a function of the share of patents owned on a technological opportunity; this share is one by

definition. Similarly $V$ does not depend on the level of applications made: one granted patent application guarantees that a firm receives $V$. Then, firms' payoffs can be simplified to:

$$\pi_k = o_k V p_k - o_k L - o_k C_o - o_k p_k C_a - C_c(o_k) \quad . \tag{7}$$

Define game $G'$ with this payoff function. This game is no longer supermodular: firms' choices of the number of technological opportunities to invest in are strategic substitutes. Note that the number of opportunities to invest in is also the number of facets invested in, as $F = 1$. Therefore firms only have one choice variable here.

We can show that under the slightly stronger assumption that costs of coordinating technological opportunities ($C_c(o_k)$) are strictly convex in the number of opportunities firms invest in, we obtain a unique equilibrium for the game. We can demonstrate that:

**Proposition 4**
*In a discrete technology, greater technological opportunity increases firms' patenting efforts.*

In a discrete technology firms' choices of how many technological opportunities to invest in are strategic substitutes because the value of each opportunity is not a function of the overall level of patenting and because legal costs are constant. Then, greater technological opportunity reduces the costs of patenting by raising the probability of obtaining a granted patent. This increases patenting efforts. Notice that this result also implies that:

**Corollary 2**
*In a discrete technology firms' patenting efforts decrease in the number of competitors* $(N)$.

In this section we have shown that there can be countervailing patenting incentives in complex and discrete technologies. This results from the fact that patenting efforts are strategic substitutes in a discrete technology whilst they become strategic complements in a complex technology. Strategic complementarity arises if there are sufficient numbers of competing firms, if complexity is high enough and if additional patented facets of a technological opportunity add value. Our model implies that firms' patenting incentives in complex technologies and in discrete technologies differ profoundly. In particular, in complex technologies an increase of complexity raises patenting incentives, while increasing technological opportunity lowers them. In a discrete technology, richer technological opportunity leads to an increase in patenting activity.

# 3 Data set and Variables

The model developed in the previous section suggests that technological opportunity and complexity of technology jointly affect firms' patenting behavior. In order to test the predictions of the model developed above we derive measures of technological opportunities and complexity from European patent data. We exploit information on blocking patents provided in these data

to derive a new continuous measure of complexity of technologies. This information is also used to construct a measure of fragmentation.[13]

Our empirical analysis is based on the PATSTAT database ("EPO Worldwide Patent Statistical Database") provided by the EPO.[14] We extracted all patent applications filed at the EPO between 1980 and 2003: more than 1,5 million patent applications with about 4.5 million referenced documents. Patents are classified using the IPC classification, allowing us to analyze differences in patenting activities across different technologies. The categorization used is based on an updated version of the OST-INPI/FhG-ISI technology classification[15] which divides the domain of patentable technologies into 30 distinct technology areas.[16] We also classify all technology areas as discrete or complex as suggested by Cohen et al. (2000).

Below we discuss measures of patenting, technological opportunities and complexity. These are the most important variables needed to test the theoretical model. Additionally, we discuss variables that are used as controls in the empirical model presented in Section 5.

### Measures of Patenting, Complexity and Technological Opportunity

**Number of Patent Applications** We compute the number of patent applications $A_{kat}$ filed by applicant $k$ and year $t$ separately for all of the 30 OST-INPI/FhG-ISI technology areas $a$. To aggregate patent applications to the firm level two challenges must be overcome: firm names provided in PATSTAT are occasionally misspelled, or different acronyms are used for parts of the firm names. Moreover, subsidiaries of larger firms are not identified in the data set. Therefore, we clean applicant names and consolidate ownership structures.[17] The aggregation of patent applications are based on these consolidated applicant identities. The variables discussed below are also based on this consolidation. Due to the skew distribution of patent applications as measured by $A_{kat}$ we transform the variable logarithmically to derive a dependent variable for the empirical analysis.

**Technological Opportunity** In our model, we establish a clear relationship between firms' patenting levels in complex technologies and the extent of technological opportunities. Unfortunately, a direct (and time-variant) measure of technological opportunities does not exist. To fill this gap, we use a proxy measure that is based on the number of non-patent literature references in the search report of the patent. In the search report, the EPO examiner lists patent and non-patent references which allow her to assess the degree novelty and of inventive step of the invention described in the patent application. Non-patent literature consists largely

[13] The effects of fragmentation do not emerge directly from our model. We discuss the rationale of controlling for this variable below.

[14] We use the September 2006 version of PATSTAT.

[15] See OECD (1994), p. 77

[16] These are listed in Table 8 in the appendix.

[17] We would like to thank Bronwyn Hall for providing us with code for name consolidation. Ownership information was extracted from the Amadeus database and other sources. Detailed information on the cleaning and aggregation algorithms can be obtained from the authors upon request.

of scientific papers. A high number of such references reflects strong science-based research efforts, and a significant inflow of new technological opportunities, leading to a relatively high level of such opportunities for invention processes. The number of non-patent references can thus be used as a good proxy for the strength of the science link of a technology as a number of studies have pointed out (Meyer, 2000, Narin and Noma, 1985, Narin et al., 1997). Callaert et al. (2006) show that EPO patents contain a high proportion of scientific articles among non-patent references, making European patent data a good source for this measure of technological opportunity. We use the average number of non-patent references (NPR) per patent in a technology area as a proxy for the position of a technology area in the technology cycle and hence as a measure of technological opportunities.

In the theoretical model an increase in technological opportunity reduces competition for remaining facets in complex technologies. This has the effect of reducing the level of patenting. The measure of technological opportunity presented here will capture this effect as long as the number of patents that can be obtained from older technological opportunities does not change significantly and systematically in the opposite direction to the level of non-patent references. We are not aware of any reason to expect such systematic changes.[18]

**Complexity of Technology Areas** The distinction between discrete and complex technologies is widely accepted in the literature (Cohen et al., 2000, Kusonaki et al., 1998, Hall, 2005a). Discrete technologies are characterized by a relatively strong product-patent link (pharmaceuticals or chemistry) whereas in complex technology industries products incorporate technology protected by many patents. Due to the multiplicity of relevant patents hold-up is much more likely in complex technologies than in discrete ones (Shapiro, 2001).

Despite the widely used notion of technological complexity there is no direct measure of it nor is there an indirect construct related to complexity. Kusonaki et al. (1998) and Cohen et al. (2000) (footnote 44) provide schemes which classify industries as discrete or complex based on ISIC codes. These classification schemes are based on qualitative evidence gathered by the authors from various sources in order to separate different industrial sectors into complex or discrete areas. A major drawback of a classification based on prior information from industry codes is that is does not allow to analyze the influence of different levels of complexity but only to distinguish between discrete and complex industries.

An ideal measure of complexity should link patents to characteristics of products, showing how many patents are incorporated in each product and how frequently products incorporate patents of rival firms. This measure would yield precise information about overlapping patent portfolios and the potential for hold-up. The measure should also cover products that do not reach the market due to hold-up. The information necessary for such a measure is only very rarely available and not available consistently across technology areas and through time. How-

[18] In fact, the time-series graph of non-patent references in semiconductors closely mirrors, but anticipates, the time series graph of various measures of the speed of technological advance in semiconductors that are provided by Aizcorbe et al. (2008). This indicates that non-patent references are a reliable indicator of technological opportunity for this very important technology.

ever, it is possible to come close to this ideal by measuring the similarity and overlap between patents in specific technology areas. Where the subject matter covered by patents overlaps, the potential for hold-up exists. This can be measured, albeit without information on the market value of each case of overlap. To achieve this we use blocking dependencies among firms. If patents containing prior art critical to the patentability of new inventions in a technology area are held by two firms, each firm can block its rival's use of the technology. Each firm can only commercialize the technology if it receives a license to use the other's blocking patents. In technology areas in which products draw on many patents -complex technologies- we expect to observe a larger number of such dependencies. In discrete technologies the inverse is true.

In our theoretical model the potential for hold-up exists as soon as each technological opportunity consists of more than one facet and there are enough competitors relative to technological opportunities that dispersed ownership of patents within each technological opportunity is probable. Our model predicts more dispersed ownership of patents in technological opportunities as competition increases and technology becomes more complex.

The examiners at the EPO determine and record the extent to which existing prior art limits patentability of an invention in a search report which is typically released 18 months after the priority date of the patent application (Harhoff et al., 2006). *Critical* documents containing conflicting prior art are classified as X or Y references by the EPO patent examiner.[19]

If the patentability of a firm A's inventions is frequently limited by existing patents of another firm B, it is reasonable to assume that appropriation of rents by A can be blocked by B to a certain degree. If the inverse is also true, A and B are in a mutual blocking relationship which we call a blocking *pair*. If more than two firms own mutually blocking patents the complexity of blocking relationships increases and resolution of blocking becomes increasingly costly. To capture more complex structures of blocking we compute the number of *triples* in which three firms mutually block each other's patents for each technology area. Figure 2 illustrates this measure.

The algorithm we use to calculate the pairs and triples is discussed in more detail in von Graevenitz et al. (2009). There we also show that the level of triples in complex and discrete technologies as defined by Cohen et al. (2000) is not driven by the level of patenting. Thus the measure is not distorted by the different rates of patenting that have previously been documented for complex and discrete technologies (Hall, 2005a, von Graevenitz et al., 2007). It is important to note that our measure is very weakly correlated ($0.044$) with measures of dispersion of patent references such as the Fragmentation index discussed next.

**Fragmentation of Prior Art** Ziedonis (2004) shows that semiconductor firms increase their patenting activities in situations where firms' patent portfolios are fragmented. Ziedonis'

[19] A search report contains different types of references – not all of them are critical. Often, related patents which are not critical are also included in the search report in order to describe the general state of the art in the respective technology. These are then classified as A-type references. X-type references point to prior patents that on their own cast doubt on the patent's inventive step or novelty; Y-type references do the same, but only in conjunction with additional documents. We have found that for our purposes the distinction between X and Y references is not important and we aggregate them in our empirical analysis.

Figure 2: Identification of our measures of a technology field's complexity.

fragmentation index has predominantly been studied in complex industries (Ziedonis, 2004, Schankerman and Noel, 2006) where increasing fragmentation raises firms' patent applications. This is attributed to firms' efforts to reduce potential hold-up by opportunistic patentees owning critical or blocking patent rights – a situation which is associated with the existence of *patent thickets*. This index does not measure hold-up potential as precisely as the complexity measure we discuss above.

The complexity measure combines information on actual blocking relationships within technological opportunities which the fragmentation index does not. The fragmentation index captures the number of potential rivals across all technological opportunities in a technology area. Therefore, the measures complement one another: triples capturing complexity, the fragmentation index capturing the intensity of competition.[20]

We construct the index of fragmentation of patent ownership for each firm copying the fragmentation index proposed by Ziedonis (2004):

$$Frag_{iat} = 1 - \sum_{j=1}^{n} s_{ijt}^2 \tag{8}$$

where $s_{ijt}$ is firm $i$'s share of critical references pointing to patents held by firm $j$. Following Ziedonis (2004) we correct the index for a bias arising if firms have few patents (Hall (2005b)).

This index is based on the Herfindahl index of concentration. Small values of the fragmentation index indicate that prior art referenced in a firm's patent portfolio is concentrated among few rival firms and vice versa. For instance the measure takes the value zero, if all references of one firm point to just one other firm. If the references of a firm are many and highly dispersed, then the index approaches the value one. The more firms patent actively on

[20]In unreported results we find that the number of firms patenting in a technology area has a strong positive correlation with the fragmentation index conditional on year and area fixed effects.

the same technological opportunities the greater the index is likely to be. Therefore, the index proxies intensity of competition in a technology area ($N$ in the theoretical model).

Unlike previous studies of patenting in complex technologies relying on USPTO patent data (Ziedonis, 2004, Schankerman and Noel, 2006, Siebert and von Graevenitz, 2010) we compute the fragmentation index solely from critical references which are classified as limiting the patentability of the invention to be patented (X and Y references). This distinction is not available in the USPTO data. Computing the fragmentation index based on critical references will yield a more precise measure of direct competition for similar technologies.

### Control Variables

**Technological Diversity of R&D Activities** A firm's reaction to changing technological or competitive characteristics in a given technology area might be influenced by its opportunities to strengthen its R&D activities in other fields. For example, if a firm is active in two technology areas it might react by a concentration of its activities in one area if competition in the other area is increasing. If a firm is active in only one technology area, it does not possess similar possibilities to react to increases in competitive pressure. In order to control for potential effects of opportunities to shift R&D resources we measure the total number of technology areas ($\text{Areas}_{i,t}$) with at least one patent application filed by firm $i$ in year $t$.

**Size Dummies.** While we do not explicitly model the influence of firm size on patenting behavior, it seems reasonable to assume that the cost of obtaining and upholding a patent depends on the size of a firm. In particular, larger firms might face lower legal cost due to economies of scale, increased potential to source in legal services and accumulation of relevant knowledge which in turn might lead to a different patenting behavior than smaller firms. For instance Somaya et al. (2007), find that the size of internal patent departments positively influences firms' patenting propensity.

If the economies-of-scale argument holds, the cost of patenting should not be directly related to size characteristics such as a firm's number of employees, its total revenues or sales. Rather, the cost of patenting can be assumed to be a function of the total patents filed by a firm. Therefore, we include a 'size dummy' variable based on the number of patents filed by a firm in a technology area in a given year in our regressions. We distinguish between small and large patentees based on annual patent applications by area $a$. Firms belonging to the upper half of the distribution of patentees in a given year are coded as large firms.

# 4 Descriptive Analysis of Patenting in Europe

In this section we provide descriptive aggregate statistics on patenting trends at the EPO. We show that descriptive evidence on patenting supports the theoretical model. Also, the measure of complexity is validated by a comparison with existing measures.

Figure 3: Annual number of patent applications filed at the EPO by priority year. Note: Black line (diamonds) indicates total patent applications. Blue line indicates patent applications in complex technology areas. Red line (starred) indicates patent applications in discrete technology areas.

Figure 3 presents annual patent applications filed at the EPO between 1978 and 2003. We distinguish applications filed in complex and discrete technology areas using the categorization of Cohen et al. (2000). Patenting grew strongly over the period we plot, with the main contribution coming from technology areas classified as complex. This development is comparable to trends at the USPTO. Hall (2005a) shows that the strong increase in patent applications is is driven by firms patenting in the electrical, computing and instruments area all of which are complex technology areas by the classification of Cohen et al. (2000).

Figure 4: Average fragmentation index. Note: Blue line indicates average level of fragmentation index in complex technology areas. Red line indicates average level of fragmentation index in discrete technology areas.

Now consider explanations for the strong growth in patenting. First, in a complex technology area fragmentation of patent rights is likely to raise firms' transactions costs as they must bargain with increasing numbers of rivals in order to prevent hold-up of their products. Ziedonis (2004) and Schankerman and Noel (2006) show that increased fragmentation of patents leads to greater patenting efforts in the semiconductor and software industries respectively.

Figure 4 provides annual averages of the fragmentation index at the EPO for the years 1980 to 2003.[21] Two observations derived from Figure 4 are striking: First, fragmentation of ownership rights increases steadily over the sample period (1988-2002). Second, the difference in the fragmentation index in complex and discrete technology areas is negligible.

Both observations raise the question whether the growth in patent applications can be attributed to fragmentation alone. While the development of fragmentation in complex and discrete areas is almost identical we observe striking differences in the growth of patent applications between complex and discrete technology areas.

Figure 5: Average number of triples identified. Note: The blue line indicates average number of triples in complex technology areas. The red line (starred) indicates average number of triples in discrete technology areas.

We can derive two separate explanations for the increase in patenting at the EPO from our theoretical model: firstly, firms build patent portfolios to strengthen their bargaining positions if complex bargaining situations are more likely to arise and secondly, the pressure to obtain patents becomes more intense as technological opportunity declines.

First, the measure of mutual blocking between three and more firms (triples) captures the degree to which complex blocking arises. In Figure 5 this measure is presented. The figure presents annual averages of the number of triples in complex and in discrete areas.[22] We observe very different developments of the count of triples in these technology areas. The number of triples is stable at values well under 10 in discrete technology areas, while it increases strongly in complex technology areas. It is reassuring to see that this measure capturing complex bargaining situations is greater in complex technologies as previously defined by Cohen et al. (2000).

Table 1 below provides additional information on the distribution of triples across the 30 technology areas. It shows the significant hold-up potential, measured by triples, within ICT technologies. There are between five and six times as many triples there as in other industries such as Handling, Printing which still exhibit significant complexity by this measure.

[21] The precise definition of this measure is given in Section 3 above.

[22] We distinguish complex and discrete using the classification suggested by Cohen et al. (2000) here.

Table 1: The Distribution of Triples Between 1988 and 2002

| Technology area | Mean | Median | Std. dev. | Minimum | Maximum |
|---|---|---|---|---|---|
| Electrical machinery, Electrical energy | 24.23 | 20 | 8.99 | 10 | 42 |
| Audiovisual technology | 116.48 | 120 | 17.68 | 74 | 148 |
| Telecommunications | 99.64 | 93 | 39.17 | 27 | 166 |
| Information technology | 57.16 | 59 | 10.71 | 28 | 73 |
| Semiconductors | 62.84 | 63 | 17.89 | 26 | 91 |
| Optics | 57.30 | 58 | 12.02 | 42 | 77 |
| Analysis, Measurement, Control | 6.61 | 4 | 6.31 | 0 | 21 |
| Medical technology | 4.10 | 3 | 2.16 | 1 | 8 |
| Nuclear engineering | 0.95 | 1 | 1.17 | 0 | 4 |
| Organic fine chemistry | 3.77 | 2 | 4.03 | 0 | 15 |
| Macromolecular chemistry, Polymers | 16.00 | 14 | 8.17 | 4 | 32 |
| Pharmaceuticals, Cosmetics | 3.47 | 4 | 2.68 | 0 | 8 |
| Biotechnology | 0.00 | 0 | 0.00 | 0 | 0 |
| Agriculture, Food chemistry | 0.07 | 0 | 0.26 | 0 | 1 |
| Chemical and Petrol industry | 11.16 | 10 | 5.49 | 4 | 22 |
| Chemical engineering | 1.35 | 1 | 0.87 | 0 | 3 |
| Surface technology, Coating | 3.48 | 3 | 2.82 | 0 | 9 |
| Materials, Metallurgy | 2.41 | 2 | 2.12 | 0 | 6 |
| Materials processing, Textiles, Paper | 3.92 | 3 | 2.73 | 1 | 9 |
| Handling, Printing | 20.26 | 16 | 13.55 | 4 | 50 |
| Agricultural and Food processing, | 0.35 | 0 | 0.71 | 0 | 2 |
| Environmental technology | 3.23 | 0 | 4.73 | 0 | 15 |
| Machine tools | 1.91 | 1 | 1.57 | 0 | 5 |
| Engines, Pumps and Turbines | 21.72 | 15 | 21.10 | 3 | 69 |
| Thermal processes and apparatus | 0.37 | 0 | 0.62 | 0 | 2 |
| Mechanical elements | 2.33 | 2 | 2.14 | 0 | 7 |
| Transport | 16.54 | 14 | 12.00 | 2 | 50 |
| Space technology, Weapons | 0.00 | 0 | 0.00 | 0 | 0 |
| Consumer goods | 0.72 | 0 | 1.05 | 0 | 4 |
| Civil engineering, Building, Mining | 0.00 | 0 | 0.00 | 0 | 0 |

Second, consider the development of technological opportunities as an explanation of the overall patenting trends. Proposition 2 indicates greater technological opportunity in a complex technology should lower the pressure to patent. As noted in Section 3 we measure technological opportunity using changes in the rate of references to non patent literature within a technology area. This measure provides information about variation in technological opportunities between and across technology areas. The left panel of Figure 6 below shows a hump shaped pattern for technological opportunities in complex technology industries. In contrast, technological opportunities in discrete technologies also level off, but at a later date then in complex technologies. Note that technological opportunities in complex technology areas began to decline just after 1992, which coincides with the date at which the growth in patent applications at the EPO picked up as Figure 3 shows. The right panel of the Figure shows

that average non patent references in complex technology areas mask considerable variation across and especially within technologies.

Figure 6: The left panel presents average non patent references per patent for complex (blue line) and discrete (red line, starred) technology areas. The right panel presents average non patent references per patent for several complex technology areas.

# 5 The Empirical Model and Results

In this section we set out empirical results. To begin with we provide a discussion of our empirical model and discuss descriptives for the sample used. Then we turn to the results from estimation and a discussion of their implications.

## 5.1 An Empirical Model of Patenting

Building on the results of Section 2 we estimate a reduce form model predicting the level of patent applications filed by a firm in a given year at the EPO. Patent applications are highly persistent as they reflect long term investments in R&D capacity. Therefore, we include a lagged dependent variable in our model. We estimate the following dynamic relationship:[23]

$$A_{i,t} = \beta_0 + \beta_A A_{i,t-1} + \beta_{AC} A_{i,t-1} C_{i,t} + \beta_O O_{i,t} + \beta_C C_{i,t} + \beta_{OC} O_{i,t} C_{i,t} + \beta_F F_{i,t} + \beta_{FC} F_{i,t} C_{i,t} + \boldsymbol{\beta}_X{}' \boldsymbol{X}_{i,t} + \Upsilon_i + \zeta_{i,t} \ , \tag{9}$$

where:

$A_{i,t}$ – ln(Patent Applications) $O_{i,t}$ – Technological Opportunity: Non Patent References

$C_{i,t}$ – Complexity: Triples $F_{i,t}$ – Fragmentation index: Concentration

$\boldsymbol{X}_{i,t}$ – Control variables: Area count, Size

$\Upsilon_i$ – Firm area fixed effects $\zeta_{i,t}$ – Error term.

[23] Our model did not explicitly account for dynamic aspects of firms' strategic decisions. However, it seems appropriate to take the persistent nature of patenting decision into account when analyzing patenting over time.

With this specification we capture effects of technological opportunity $\beta_O$, complexity $\beta_C$ and competition $\beta_F$ as well as the effects of complexity and competition in complex technologies ($\beta_{OC}, \beta_{FC}$). We also allow the effect of the lagged dependent variable to differ in complex and discrete technology areas ($\beta_{AC}$).

In an extension of this basic specification we also include interaction terms that allow us to distinguish the patenting behavior of large and small firms in complex and discrete technologies. Our theoretical model indicates that firms' patenting behavior will depend on the share of patents they expect to receive on a given technological opportunity which may differ systematically between large and small firms.

Estimates of this specification provide a test of the following hypotheses. These reflect Propositions 2- 4 and Corollaries 1- 2:

H1 Greater complexity of technologies raises patent applications, $\beta_C > 0$ (Proposition 3);

H2 Competition raises patent applications in complex technologies, $\beta_{FC} > 0$ (Corollary 1);

H3 Technological opportunity reduces patent applications in complex technologies, $\beta_{OC} < 0$ (Proposition 2);

H4 Competition reduces patent applications in discrete technologies, $\beta_F < 0$ (Corollary 2);

H5 Technological opportunity raises patent applications in discrete technologies, $\beta_O > 0$ (Proposition 4).

Hypotheses 1-3 capture the effects of complexity, competition and technological opportunity in complex technologies. Proposition 1 shows that greater complexity of a technology is more likely to render firms' actions in a patenting game strategic complements. The reverse is true in a discrete technology, here firms actions are strategic substitutes and the comparative statics with respect to competition and technological opportunity are exactly reversed. By interacting complexity with the number of competing firms and our measure of technological opportunity in Hypotheses 2 and 3 we separate the two types of equilibria.

## 5.2 Descriptive Statistics for the Sample

Our data set contains observations of patent applications by firms in specific technology areas and covers the period between 1978 when the EPO began operating and 2003. We intend to study patent applicants patenting over a prolonged period and possibly across several technology areas. Therefore, we excluded small patentees from the sample. Two criteria were used: first, we excluded all those patentees with fewer than 100 patent applications between 1980 and 2002. Second, we excluded those patentees who had fewer than three years of positive patent applications in a technology area in the fifteen years after 1987.

These criteria result in a sample containing 173,448 observations of patenting activity by a firm in a technology area. Table 2 shows that these patent applications are due to 2074

distinct firms. The average size of these firms' patent portfolios in 2002 was 628 patents resulting from an average of 37 patent applications per firm and year across all technology areas. 34% of observations in the data set contain a zero patent application count but only 0.05% of observations belong to firms that have no patent applications at all in a given year. The lower half of Table 2 shows that our sample covers on average 55.8% of the annual mean of 2594 patent applications filed within an average technology area. As the sample focuses on large patentees the share of firms we covered by the sample is smaller: on average 1077 firms patent per area per year and 24.8% of these are included in the sample.[24]

Table 2: Panel Descriptives for the Sample

| **Firm level (2074 firms)** | **Mean** | **Median** | **SD** |
|---|---|---|---|
| Total patents | 628.27 | 205 | 1944.94 |
| Total patents (annual) | 37.02 | 12 | 111.65 |
| Technological areas (annual) | 5.54 | 4 | 4.56 |
| | | | |
| **Area-Year level (650 area-year observations)** | **Mean** | **Median** | **SD** |
| Total patents in area | 2594.23 | 2310 | 1778.87 |
| Total patents in area and sample | 1449.35 | 1012 | 1695.86 |
| Total firms in area | 1077.62 | 893 | 668.14 |
| Total firms in area and sample | 266.84 | 263 | 253.71 |
| Triples | 14.67 | 2 | 27.69 |
| Non Patent References | 0.98 | 0.75 | 0.75 |
| Fragmentation | 0.05 | 0.05 | 0.03 |

Firms operating in several technology areas are treated as distinct in each area. Hence, our panel structure is not defined over firms' total patent applications per year (firm-years) but over firms' annual patent applications within specific technology areas (firm-area-years). We do this to control for area specific patenting behavior of individual firms and its relation to area characteristics like complexity.[25] Where we use panel data, the panel is unbalanced due to entry and exit of firms into technology areas.

Table 3 presents descriptive statistics at the firm-area-year level. Most firms in the sample patent relative broadly across technology areas. While the number of patent applications within a given technology area is relatively low with 5.43 application per year firms are active in 8 or 9 different technology areas. The average technology area contained about 18.5 triples in a given year – however the distribution is skew with a median of 5 and a maximum of 166 triples (observed in Telecommunications in 2000). The level of non patent references in the average technology area is 1.151. Table 3 also contains information about sample statistics

[24] We have experimented with alternative sample selection rules and found our results to be robust.

[25] We find that firms in more complex technologies are very slightly more likely to be active in more technology areas than the average firm, with a very weak positive correlation of 0.04.

for the year 1992, after which patent applications increased markedly as Figure 3 shows. A comparison of sample means (upper part of Table 3) and means for 1992 (lower part of 3) shows that firms patent in more areas, face more complexity (triples) and generate fewer non patent references after 1992 than before. This confirms what we showed previously.

Table 3: Descriptive Statistics for the Sample (1988-2002)

| **Variable** | **Aggregation level** | **Mean** | **Median** | **Standard deviation** | **Minimum** | **Maximum** |
|---|---|---|---|---|---|---|
| Patent applications per area | Firm | 5.431 | 1.000 | 18.594 | 0.000 | 752.000 |
| log Patent applications per area | Firm | 1.051 | 0.693 | 1.052 | 0.000 | 6.624 |
| Areas | Firm | 8.751 | 7.000 | 6.027 | 0.000 | 30.000 |
| Large dummy | Firm | 0.504 | 1.000 | - | 0.000 | 1.000 |
| Non Patent References | Area | 1.151 | 0.894 | 0.827 | 0.174 | 4.532 |
| Triples | Area | 18.480 | 5.000 | 30.085 | 0.000 | 166.000 |
| Fragmentation | Firm | 0.210 | 0.000 | 0.427 | 0.000 | 1.961 |

**Observations** = 173,448

Sample statistics for 1992

| | | | | | | |
|---|---|---|---|---|---|---|
| Patent applications per area | Firm | 4.235 | 1.000 | 14.024 | 0.000 | 387.000 |
| log Patent applications p/a | Firm | 0.923 | 0.693 | 0.990 | 0.000 | 5.961 |
| Areas | Firm | 7.746 | 6.000 | 5.563 | 0.000 | 27.000 |
| Large dummy | Firm | 0.438 | 0.000 | - | 0.000 | 1.000 |
| Non Patent References | Area | 1.205 | 0.970 | 0.747 | 0.290 | 3.554 |
| Triples | Area | 15.761 | 3.000 | 25.348 | 0.000 | 104.000 |
| Fragmentation | Firm | 0.175 | 0.000 | 0.389 | 0.000 | 1.935 |

**Observations** = 11,325

## 5.3 Results

In this section we present results from estimation of the empirical model (Equation 9) using GMM. The lagged dependent variable and several explanatory variables which may be expected to be endogenous are instrumented. We show that the predictions of the theoretical model in Section 2 are supported by the data.

We use panel estimators to avoid misspecification of the empirical model arising from unobserved heterogeneity, such as variation in managerial ability. To capture persistence in patenting we introduce a lagged dependent variable into our models, which introduces an additional source of misspecification. This renders fixed and random effects estimators inconsistent in short panels such as ours (Arellano, 2003). Instead, we employ system GMM estimators which also allow us to address the potential endogeneity of some of our regressors.

Table 4: GMM Models for Patent Applications

| **Variable** | SGMM A | SGMM B | SGMM C | SGMM D | SGMM E |
|---|---|---|---|---|---|
| log Patentcount$_{t-1}$ | 0.720*** | 0.509*** | 0.426*** | 0.688*** | 0.749*** |
| | (0.042) | (0.039) | (0.039) | (0.122) | (0.093) |
| log Patentcount$_{t-1}\times$ Triples | | | | -0.018*** | -0.017*** |
| | | | | (0.003) | (0.003) |
| Non Patent References (NPR) | 0.226*** | 0.121*** | -0.152* | 1.700*** | 1.553*** |
| | (0.030) | (0.032) | (0.061) | (0.344) | (0.254) |
| NPR $\times$ Triples | | | 0.001 | -0.043*** | -0.036*** |
| | | | (0.001) | (0.007) | (0.006) |
| NPR $\times$ Triples $\times$ Large | | | | | 0.007*** |
| | | | | | (0.002) |
| NPR $\times$ Large | | | | | -0.366*** |
| | | | | | (0.081) |
| Fragmentation | | 0.641*** | 0.793*** | -0.489* | -0.474** |
| | | (0.065) | (0.064) | (0.213) | (0.170) |
| Fragmentation $\times$ Triples | | | | 0.010 | 0.006 |
| | | | | (0.006) | (0.006) |
| Triples | -0.000 | 0.000 | -0.001 | 0.071*** | 0.055*** |
| | (0.000) | (0.000) | (0.001) | (0.012) | (0.010) |
| Areas | 0.067*** | 0.068*** | 0.073*** | 0.105*** | 0.096*** |
| | (0.007) | (0.006) | (0.006) | (0.016) | (0.012) |
| Large | -0.079** | -0.117*** | -0.183*** | 0.010 | 0.342** |
| | (0.026) | (0.025) | (0.025) | (0.078) | (0.117) |
| Year dummies | YES | YES | YES | YES | YES |
| Primary area dummies | YES | YES | YES | YES | YES |
| Constant | -0.403*** | -0.221*** | 0.044 | -1.700*** | -1.443*** |
| | (0.038) | (0.043) | (0.062) | (0.314) | (0.319) |
| N | 173448 | 173448 | 173448 | 173448 | 173448 |
| m1 | -25.041 | -23.908 | -23.413 | -7.934 | -10.860 |
| m2 | 18.356 | 13.590 | 10.836 | 3.131 | 4.739 |
| m3 | -1.707 | -2.230 | -2.285 | 1.606 | .896 |
| Hansen | 525.187 | 412.714 | 456.374 | 19.221 | 10.988 |
| p-value | 9.1e-115 | 3.90e-89 | 2.08e-95 | .004 | .052 |
| Degrees of freedom | 2 | 3 | 6 | 6 | 5 |

* p<0.05, ** p<0.01, *** p<0.001

1. Asymptotic standard errors, asymptotically robust to heteroskedasticity are reported in parentheses
2. m1-m3 are tests for first- to third-order serial correlation in the first differenced residuals.
3. Hansen is a test of overidentifying restrictions. It is distributed as $\chi^2$ under the null of instrument validity, with degrees of freedom reported below.
4. In all cases GMM instrument sets were collapsed and lags were limited.

We instrument potentially endogenous variables such as the number of areas a firm is active in using lagged values. Exogeneity of these instruments is tested using difference in Hansen tests (Roodman, 2006).

Table 4 presents results of system GMM estimators using forward deviations transformations (Blundell and Bond, 1998, Arellano and Bover, 1995, Alvarez and Arellano, 2003).[26] Reported standard errors are based on two step estimators using the correction suggested by Windmeijer (2005). Tests for first, second and third order serial correlation (m1-m3) indicate presence of first and second order serial correlation. In all specifications we instrument predetermined variables with third order lags and endogenous variables with fourth order lags.

Instrument sets are collapsed[27] in order to reduce the number of instruments used. Throughout we rely on the Hansen test to determine whether the instruments we use are truly exogenous. Where the statistic indicates that this is not the case we reject the models.

Specification SGMM A contains the lagged dependent variable, measures of technological opportunity (*Non Patent References (NPR)*), complexity (*triples*), the breadth of a firms' activities within the patent system (*Areas*), a dummy for the size of a firms' patent portfolio (*Large*) and dummies for year and main technology area. Specification SGMM B adds the measure of fragmentation suggested by Ziedonis (2004). This is adjusted as proposed by Hall (2005b). In specification SGMM C we add interactions of the complexity measure (*triples*) with the measure of technological opportunity (*NPR*). Hansen tests for these simple specifications reject their validity, indicating that the instruments used are not exogenous.

In specification SGMM D *triples* are interacted with the lagged dependent variable, to capture the possibility that firms adjust their levels of patenting differently in complex and discrete technologies. This specification performs better than SGMM A-C, the $\chi^2$ statistic being significantly lower than for those specifications. Finally, specification SGMM E also includes interactions which test the effects of firm size on non patent references. This specification performs best, the Hansen test does not reject the model. We now focus on this model.

We find that greater technological opportunities (NPR) raise patenting levels showing that we cannot reject Hypothesis 5. The effect of technological opportunity is highly significant across almost all estimated specifications (see models (A) to (E) of Table 4). The inclusion of the interaction between our measure of complexity (triples) and technological opportunities shows that the effect differs in discrete - and complex technologies. In particular, if the number of triples in a technology area is larger than 39 (in specification (D) ) or larger than 43 in specification (E) of Table 4, the overall effect from increasing technological opportunities is negative as $\beta_O + \beta_{OC} \times C_{i,t} < 0$. The negative coefficient on the interaction of complexity and non patent references supports Hypothesis 3: increasing technological opportunities reduce patenting efforts in more complex technology areas. Additionally, the significant positive coefficient on the effects of complexity alone supports Hypothesis 1.

Table 1 shows the average number of triples for 5 technology areas in our sample is greater

[26] All models were estimated with xtabond2 in Stata 9.2 . This package is described in Roodman (2006).

[27] Collapsing instrument sets reduces the number of moment conditions used for GMM (Roodman (2006)).

than 43. For Audiovisual technology and Optics triples are always above 43. This indicates that increased technological opportunities always or almost always reduce patenting efforts in these areas.

With regard to the effects of the number of competitors blocking a specific firm in technology space we fail to reject either Hypothesis 4, i.e. more competition (greater fragmentation) reduces patenting efforts in discrete technologies. The coefficient on the interaction of fragmentation and complexity is not significant. However, the joint effect of fragmentation and complexity is significant. Thus we have weak evidence that increased competition raises patenting efforts in complex technologies (Hypothesis 2).

Finally, our results on the interaction of the lagged dependent variable with *triples* indicate that persistence of patenting decreases as technology areas become more complex. Persistence is entirely absent in very complex technologies. This shows that patentees are more responsive to their competitors' patenting behavior and to technological opportunity in complex technology areas than in discrete technology areas.

Table 5 below provides effects of changes in complexity (*triples*), technological opportunities (*Non patent references*) and *Fragmentation* for patenting rates in nine technology areas.[28] The table presents effects for small and large firms where appropriate and contains mean and median results. Five of the technology areas presented are highly likely complex as the mean and median levels of triples are clearly above 43 in these areas (viz. Table 1). They are Audiovisual Technology, Telecommunications, Information Technology, Semiconductors and Optics. We also present results for four additional areas. These are more likely discrete by this measure: Medical Technology; Electrical Machinery; Analysis, Measurement, Control; and Pharmaceuticals.

Table 5 shows that in all discrete technologies an increase in technological opportunity raises patenting, while in all complex technology areas it lowers patenting. These results fit the predictions of Hypotheses 5 and 3 respectively. Most importantly the effects of a one standard deviation change in technological opportunity are comparatively large in the complex technologies. This is a surprising finding that indicates that technological opportunity is an important determinant of firms' patenting efforts.

Hypothesis 1 states that increases in the complexity of a technology will raise firms' levels of patenting in the technology is complex. Table 5 shows this result generally holds at the median and at the mean for large firms in complex technology areas apart from Semiconductors.[29]

Interestingly, Table 5 also shows that the effect of *Fragmentation* on firms' patenting efforts in very complex technology areas is positive as predicted by Hypothesis 2. Also, *Fragmentation* has a negative effect on patenting in discrete technology areas, as predicted in Hy-

[28]These effects are calculated taking account of the logarithmic transformation of the dependent and the lagged dependent variable.

[29]The precise delineation of the areas for Information Technology and Semiconductors in the classification we use is not clear. In von Graevenitz et al. (2007) we find that a large proportion of patents from semiconductor firms are patented within the Information Technology area.

pothesis 4. The positive effects for complex technology areas support the findings of Ziedonis (2004), Schankerman and Noel (2006) who find that additional fragmentation of patent ownership increases patenting efforts in semiconductors and software in the United States. Note however, that *Fragmentation* has small negative effects on patenting in the moderately complex technologies included in Table 5. In discrete technology areas fragmentation has a very strong negative effect so that overall we confirm the prediction that firms are more likely to patent more as fragmentation increases if technology areas are more complex.

Table 5: Mean and Median Percentage Changes in Patent Applications in Complex and Discrete Technologies

| **Technology area** | **Applications growth 1990-2000** | **Triples** | **Triples SD change** | | **Non patent references SD change** | | **Fragmentation** | |
|---|---|---|---|---|---|---|---|---|
| | | | **Small** | **Large** | **Small** | **Large** | **Unit change** | **SD change** |
| **Complex Technologies** | | | | | | | | |
| Audiovisual | 52% | 118 | -2.76% | 10.13% | -51.59% | -45.21% | 26.14% | 10.65% |
| Technology | | 120 | 6.29% | 20.47% | -52.66% | -46.19% | 27.89% | 11.32% |
| Telecom- | 253% | 103 | -22.38% | 4.80% | -34.87% | -30.11% | 15.25% | 6.83% |
| munications | | 93 | -11.40% | 18.14% | -30.18% | -26.08% | 8.76% | 3.99% |
| Semiconductors | 63% | 62 | -33.51% | -17.04% | -23.19% | -21.10% | -9.48% | -4.14% |
| | | 63 | -31.09% | -13.51% | -23.83% | -21.62% | -9.15% | -3.99% |
| Information | 174% | 58 | -8.11% | 0.04% | -9.75% | -9.03% | -11.63% | -4.99% |
| Technology | | 59 | -3.83% | 4.69% | -10.12% | -9.33% | -11.31% | -4.85% |
| Optics | 41% | 57 | -11.07% | -0.49% | -7.02% | -6.54% | -12.02% | -5.55% |
| | | 58 | -6.16% | 5.02% | -7.18% | -6.67% | -11.84% | -5.46% |
| **Discrete Technologies** | | | | | | | | |
| Analysis, | 75% | 25 | -1.48% | 3.87% | 10.07% | 7.53% | -35.19% | -17.83% |
| Measurement, | | 20 | 1.24% | 6.63% | 11.15% | 8.38% | -36.62% | -17.87% |
| Control | | | | | | | | |
| Electrical | 91% | 7 | 7.97% | 15.59% | 4.28% | 3.02% | -27.83% | -13.57% |
| Machinery | | 3 | 12.24% | 20.27% | 5.39% | 3.90% | -29.81% | -14.64% |
| Pharmaceu- | 221% | 4 | -15.60% | -11.26% | 55.06% | 39.60% | -36.38% | -19.43% |
| ticals | | 4 | -16.29% | -11.93% | 54.41% | 39.13% | -36.24% | -19.49% |
| Medical | 148% | 4 | 6.00% | 6.54% | 5.81% | 4.38% | -36.16% | -17.99% |
| Technology | | 4 | 6.62% | 7.19% | 5.84% | 4.41% | -36.24% | -18.02% |

This table reports **means** (upper row) and **medians** (lower row) for each technology area. We report changes in patent applications in response to standard deviation (**SD**) changes in each variable. For *Triples* and *Non patent references* we report effects for **small** and **large** firms.

## 5.4 Robustness of the Results

In a next step, we test the robustness of our results using alternative GMM estimators. Results from these tests are reported in Table 6. We vary size of the instrument set and the estimator

Table 6: Robustness Checks for Patent Applications Estimates

| | Allowing correlation with fixed effects | | | | Assuming no correlation with fixed effects | |
|---|---|---|---|---|---|---|
| **Variable** | SGMM F | SGMM E | DGMM G | SGMM H | SGMM I | SGMM J |
| log Patentcount$_{t-1}$ | 0.675*** | 0.749*** | 0.935*** | 0.617*** | 0.742*** | 0.879*** |
| | (0.102) | (0.093) | (0.110) | (0.067) | (0.047) | (0.054) |
| log Patentcount$_{t-1}\times$ Triples | -0.021*** | -0.017*** | -0.014*** | -0.011*** | -0.009*** | -0.011*** |
| | (0.004) | (0.003) | (0.003) | (0.002) | (0.001) | (0.001) |
| Non Patent References (NPR) | 1.880*** | 1.553*** | 1.502*** | 0.654*** | 0.475*** | 1.306*** |
| | (0.361) | (0.254) | (0.258) | (0.112) | (0.039) | (0.173) |
| NPR $\times$ Triples | -0.042*** | -0.036*** | -0.035*** | -0.019*** | -0.016*** | -0.031*** |
| | (0.008) | (0.006) | (0.006) | (0.003) | (0.002) | (0.004) |
| NPR $\times$ Triples $\times$ Large | 0.008*** | 0.007*** | 0.006*** | 0.006*** | 0.005*** | 0.004*** |
| | (0.002) | (0.002) | (0.001) | (0.001) | (0.001) | (0.001) |
| NPR $\times$ Large | -0.248* | -0.366*** | -0.224*** | -0.373*** | -0.484*** | -0.557*** |
| | (0.125) | (0.081) | (0.040) | (0.057) | (0.036) | (0.049) |
| Fragmentation | -0.558** | -0.474** | -0.490** | -0.091 | -0.007 | -0.183 |
| | (0.211) | (0.170) | (0.164) | (0.111) | (0.081) | (0.113) |
| Fragmentation $\times$ Triples | 0.009 | 0.006 | 0.000 | -0.004 | -0.007* | -0.000 |
| | (0.008) | (0.006) | (0.006) | (0.004) | (0.003) | (0.004) |
| Triples | 0.056*** | 0.055*** | 0.052*** | 0.031*** | 0.034*** | 0.059*** |
| | (0.013) | (0.010) | (0.008) | (0.006) | (0.004) | (0.007) |
| Areas | 0.097*** | 0.096*** | 0.036* | 0.098*** | 0.082*** | 0.090*** |
| | (0.014) | (0.012) | (0.016) | (0.009) | (0.008) | (0.007) |
| Large | 0.221 | 0.342** | 0.256*** | 0.349*** | 0.562*** | 0.562*** |
| | (0.165) | (0.117) | (0.065) | (0.083) | (0.060) | (0.080) |
| Year dummies | YES | YES | YES | YES | YES | YES |
| Primary area dummies | YES | YES | YES | YES | YES | YES |
| Constant | -1.311** | -1.443*** | | -0.703*** | -0.886*** | -2.000*** |
| | (0.466) | (0.319) | | (0.194) | (0.075) | (0.206) |
| N | 173448 | 173448 | 171380 | 173448 | 173448 | 173448 |
| m1 | -9.878 | -10.860 | -8.276 | -12.868 | -19.934 | -20.345 |
| m2 | 2.651 | 4.739 | 4.881 | 6.166 | 11.269 | 14.863 |
| m3 | 1.353 | .896 | .192 | -1.051 | -.951 | .3055 |
| Hansen | 4.101 | 10.988 | 5.177 | 57.671 | 197.573 | 52.158 |
| p-value | .129 | .052 | .270 | 3.76e-09 | 2.07e-38 | 5.43e-09 |
| Degrees of freedom | 2 | 5 | 4 | 9 | 8 | 7 |

* p<0.05, ** p<0.01, *** p<0.001

1. Asymptotic standard errors, asymptotically robust to heteroskedasticity are reported in parentheses
2. m1-m3 are tests for first- to third-order serial correlation in the first differenced residuals.
3. Hansen is a test of overidentifying restrictions. It is distributed as $\chi 2$ under the null of instrument

validity, with degrees of freedom reported below.

4. In all cases GMM instrument sets were collapsed and lags were limited.

used. All models reported in Table 6 are estimated using forward deviations and reported standard errors corrected as previously noted. The models differ in the number of overidentifying restrictions employed as well as assumptions about the correlation of the explanatory variables with fixed effects. Hansen tests are used to determine which of the models are reliable. These show that only the first three models reported in the table are not rejected.

The four models reported in the central part of Table 6 allow for correlation between all explanatory variables with fixed effects. In two specifications on the right side of the table we assume subsets of the explanatory variables are uncorrelated with fixed effects. The number of observations in our data set implies that $T/N \to 0$. Therefore, a systems GMM estimator (Blundell and Bond, 1998) using forward deviations is asymptotically consistent (Alvarez and Arellano, 2003, Hayakawa, 2006). We employ this estimator as the patenting series are highly persistent in our sample: the coefficient on the lagged dependent variable in an AR1 model with time and primary area dummies is $0.92$. Blundell and Bond (1998) note that difference GMM is affected by a weak instruments problem in this context which is not the case in the specificatio we report. However, the coefficient on the lagged dependent variable is somewhat above that reported for the comparable systems estimators. It is also significantly above the coefficients from the OLS regressions reported in Table 7. Therefore, we focus our analysis on the results from the system estimators.

In all models reported in Table 6 the instrument sets were collapsed and instrumenting lags were limited as described below. This was done as the Hansen test and difference in Hansen tests rejected the overall instrument sets as well as individual instruments where larger instrument sets were employed. Specification SGMM H illustrates how sensitive the Hansen test is to the size of the instrument set here. This specification is identical to SGMM E, we just allow for an extra lag on the instrument sets for the endogenous variables in this specification. The specification is rejected by the Hansen test.

All models reported in Table 6 contain the following explanatory variables: *Non patent references*, *triples*, *Fragmentation*, *Area count*, *Large dummy* and the lagged dependent variable as well as interactions of some of these variables. We consider *Fragmentation* and *Area count* to be endogenous as they reflect decisions about how widely and where to engage in research which may be contemporaneous with decisions determining the level of patent applications. We consider the remaining variables to be predetermined since they depend in large part on the aggregated decisions of rival firms. Finally note that we include only year and primary area dummies in the levels equation as it is likely that the fixed effects are correlated with differences in the remaining explanatory variables.

We estimate two models in which we treat Fragmentation (SGMM J) and Non patent references (SGMM I) as uncorrelated with fixed effects. Results from the Hansen tests for both specifications reported in Table 6 show that these models are clearly rejected.

Our preferred models are reported as SGMM F and SGMM E in Table 6. In SGMM F we

restrict the number of instruments such that the model is just overidentified. Hayakawa (2006) argues that such a minimum instruments specification is unbiased in settings where $T$ is fixed and $N \to \infty$. Specification SGMM E includes additional instruments for the endogenous variables. Results from these two specifications are statistically indistinguishable.

Additionally to the GMM results reported here, Table 7 (Appendix C) provides results from OLS on the pooled sample and from fixed effects regressions. These results are known to be biased due to inclusion of the lagged dependent variable. However, they provide lower and upper bounds on the values of the lagged dependent variable for GMM (Bond (2002)). Once we take account of the interaction of the lagged dependent variable with triples we find that the coefficients on the lagged dependent variable are within the range provided by OLS and fixed effects estimates for technology areas of average complexity.

# 6 Conclusion

Patent applications have been increasing steeply at the USPTO and the EPO since 1984 and 1992 respectively. In both cases these increases have raised questions about the operations of the affected patent offices as well as effects of these trends on economic activity more generally (Federal Trade Commission, 2003, National Research Council, 2004, von Graevenitz et al., 2007, Bessen and Meurer, 2008). Our paper makes a number of contributions towards a systematic explanation of these phenomena. There is strong evidence by now that patenting has increased in response to evolution of the legal environment, specifically in the United States, to changes in the management of R&D and patenting, and to increasing complexity of technology and more strategic behavior of patent applicants (Kortum and Lerner, 1998, Hall and Ziedonis, 2001, Ziedonis, 2004). But the contribution of technological opportunity to current patenting trends and its interaction with other determinants has been less well understood.

This latter effect is central to our analysis. Our model is the first to consider the effect of complexity and of technological opportunity *jointly*. Moreover, while other studies have focused on selected industries, our model and the empirical test encompass discrete and complex technologies, providing predictions for patenting behavior in both types of technology. We show theoretically that greater technological opportunity will raise patenting in discrete technologies but will lower it as technologies become increasingly complex. Additionally, we show that greater competition in R&D raises firms' patenting levels in complex technologies.

To test our model we derive a new measure of complexity of blocking relationships in patent thickets. This measure exploits information on critical references to capture mutual blocking between the patent portfolios of firms contained in European patent data. Using the measure we are able to confirm that blocking is a much more serious problem in technology areas previously identified as complex than in those previously identified as discrete.

Using data on patenting in Europe and these measures, we find that patenting behavior largely conforms to the predictions of our theoretical model. Most importantly, we find that variation in technological opportunity strongly affected firms' patenting levels. Our data show

that increased technological opportunity during the early 1990's counteracted the effects of growing complexity and retarded the onset of the patenting explosion observable after 1994. The patent explosion coincides with the decrease in technological opportunities after 1994. We also show - for the first time with European data - that greater fragmentation of patent ownership increases patenting in complex technologies (Ziedonis, 2004). We attribute this to a greater number of competing patent applicants as we control for the degree of hold-up potential with the triples measure of complexity of the technology.

Finally, our results show that as technology areas become more complex, firms patenting activities increase. As we use lagged values of complexity to instrument current complexity this finding is likely to reflect a causal mechanism - as firms encounter more complexity they respond by patenting more.

We are able to show that patent thickets exist in nine out of thirty technology areas at the EPO. The data we use indicate that the extent of patent thickets at the EPO has been increasing in recent years. These increases are concentrated in complex technology areas (Hall, 2005a, von Graevenitz et al., 2007). Resulting increases in transactions costs would therefore affect exactly those technologies that have been central to large productivity increases in the recent past (Jorgenson and Wessner, 2007). Extended "patent wars" may threaten this source of productivity gains in the long run. In future work we therefore intend to investigate whether strategic patenting has measurable effects on the productivity of firms' R&D investments and how the decision variables of patent offices (fees and administrative rules) might be used to influence patent filings.

Our findings on the effects of technological opportunity raise important questions about the relationship between patent breadth, the fecundity of research areas and firms' R&D investments. We find that the contest for patent rights becomes more intense as the level of technological opportunities decreases if a technology is complex. This raises the question how firms' incentives to patent more intensively interact with incentives to undertake basic research which might stem the reduced fecundity of these technologies. At a more fundamental level the findings indicate that research into the relationship between technological opportunities and R&D is important if we are to understand the welfare implications of recent patenting trends better.

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# Appendix

# A Technical Appendix for the Theoretical Model

In this section we derive several of the results which we make use of in deriving our theoretical predictions in Section 2. In particular we describe the functions describing the expected number of facets covered $\tilde{F}$ and the probability of patenting a facet $p_k$.

Note that below we also employ the following definitions:

$$\omega_k \equiv o_k/O \qquad \phi_k \equiv f_k/F \quad . \tag{10}$$

## A.1 The Expected Number of Rival Investors

Here we derive the expected number of rival firms $N_O$ that undertake R&D on the same technology opportunity as firm $k$. This expected number of rivals can be expressed as a sum of products. Each product gives the probability that a given number of rivals invest in the same technological opportunity. All of these probabilities are then summed to give the overall expected number of rival firms on a given technological opportunity:

$$\begin{aligned} N_O &= \binom{N}{1}\omega_j(1-\omega_j)^{N-1} + 2\binom{N}{2}\omega_j^2(1-\omega_j)^{N-2} + 3\binom{N}{3}\omega_j^3(1-\omega_j)^{N-3}... \\ &= \sum_{i=0}^{N} i\binom{N}{i}\prod_{j=0}^{N-i}(1-\omega_j)\prod_{l=0}^{i}\omega_l \end{aligned} \tag{11}$$

It can be shown that $N_O$ is increasing in $\omega$. First rewrite $N_O$ as a function of a firm $m$'s choice $\omega_m$ and $\omega_k$, $\omega_l$:

$$N_O = \sum_{i=0}^{N-1}\left[(1-\omega_m)i + \omega_m(i+1)\right]\binom{N-1}{i}\prod_{j=0}^{N-1-i}(1-\omega_j)\prod_{l=0}^{i}\omega_l \tag{12}$$

Next we take the derivative:

$$\frac{\partial N_O}{\partial \omega_k} = \sum_{i=0}^{N-1}\omega_m\binom{N-1}{i}\prod_{j=0}^{N-1-i}(1-\omega_j)\prod_{l=0}^{i}\omega_l > 0 \ . \tag{13}$$

An increase in the number of opportunities $o_j$ which other firms invest in, increases the expected number of rivals patenting facets on the same technological opportunity.

## A.2 The Expected Number of Facets Covered

The expected number of facets covered through the joint efforts of all firms investing in a technological opportunity is:[30]

$$\tilde{F} = F\Big[1 - (1-\phi_k)\prod_{j=1}^{N_O}(1-\phi_j)\Big] \tag{14}$$

As noted above, the derivatives of this expression with respect to $F$ and $f_k$ are important for the results there. Both of these can be shown to be positive:

$$\frac{\partial \tilde{F}}{\partial F} = 1 - (1-\phi_j)^{N_O}\,(1+\phi_j N_O) \geq 0\ ,\quad \frac{\partial \tilde{F}}{\partial f_k} = \prod_{j=1}^{N_O}(1-\phi_j) > 0\ , \tag{15}$$

where we impose symmetry in the choice of $f$ across firms in the derivative w.r.t. $F$. This derivative is used for comparative statics purposes, after first derivatives have been taken.

Finally note that the elasticities of $\tilde{F}$ with respect to $F$ and $\tilde{f}_i$ are:

$$\epsilon_{\tilde{F}f_k} = \phi_k \frac{\Big[\prod_{j=1}^{N_O}(1-\phi_j)\Big]}{1-(1-\phi_k)\prod_{j=1}^{N_O}(1-\phi_j)}\quad , \tag{16}$$

$$\epsilon_{\tilde{F}F} = \frac{1-(1-\phi_j)^{N_O}\,(1+\phi_j N_O)}{1-(1-\phi_j)^{(N_O+1)}}\quad , \tag{17}$$

which shows that $1 \geq \epsilon_{\tilde{F}F} \geq 0$ as the denominator in the fraction is always greater than the numerator. It is useful to observe that the upper bound of the elasticity $\epsilon_{\tilde{F}f_k}$ is decreasing in $N_O$. To see this note that in equilibrium the elasticity is defined as:

$$\epsilon_{\tilde{F}f_k} = \phi_j \frac{(1-\phi_j)^{N_O}}{1-(1-\phi_j)^{N_O+1}} = \frac{(1-\phi_j)^{N_O}}{(N_o+1)\left(1-\phi_j\frac{N_o}{2!}+\phi_j^2\frac{N_o(N_O-1)}{3!}...\right)}\ . \tag{18}$$

The second expression above makes clear that the upper bound of the ealsticity decreases in $N_O$: $\lim_{\phi_j\to 0}\epsilon_{\tilde{F}f_k} = 1/(N_O+1)$. Here we make use of the binomial expansion of $(1-\phi_j)^{N_o+1}$. From this expression it is also clear that the lower bound of the elasticity is always zero.

## A.3 The Probability of Patenting a Facet

Now turn to the probability of obtaining a patent on a facet given $N_O$:

$$p_k = \prod_{j=1}^{N_O}(1-\phi_j) + \frac{N_O}{2}\cdot\phi_j\prod_{j=1}^{N_O-1}(1-\phi_j) + \frac{(N_O)(N_O-1)}{6}\prod_{j=1}^{N_O-2}(1-\phi_j)\prod_{l=N_O-2}^{N_O}(\phi_l)...\ ,$$

[30] We are grateful for the help of Professor Helmut Küchenhoff and Mr. Fabian Scheipl in deriving this expression.

$$= \sum_{i=0}^{N_O} \frac{1}{i+1} \binom{N_O}{i} \prod_{j=0}^{N_O-i} (1-\phi_j) \prod_{l=0}^{i} \phi_l \tag{19}$$

The properties of this expression are not easily derived. Here we set out the derivative of $p_k$ w.r.t. $\phi$ and we show that $p_k$ decreases in $N_O$.

Consider first the effects of an increase in $\phi_m$, i.e. an increase in the proportion of facets covered by firm $m$ on the probability that firm $k$ obtains a given facet. To investigate this we reexpress the probability of obtaining a facet as follows:

$$p_k = \left[ \sum_{i=0}^{N_O-1} \left[ (1-\phi_m) \frac{1}{i+1} + \phi_m \frac{1}{(i+2)} \right] \binom{N_O-1}{i} \prod_{j=0}^{N_O-1-i} (1-\phi_j) \prod_{l=0}^{i} \phi_l \right] \tag{20}$$

Then the derivative is:

$$\frac{\partial p_k}{\partial \phi_m} = \left[ \sum_{i=0}^{N_O-1} \left[ -\frac{1}{(i+1)(i+2)} \right] \binom{N_O-1}{i} \prod_{j=0}^{N_O-1-i} (1-\phi_j) \prod_{l=0}^{i} \phi_l \right] < 0 \quad . \tag{21}$$

Finally consider the effects of an increase in $N_O$ on the probability of patenting a facet:

$$\begin{aligned} p_i(N_O+1) - p_i(N_O) = & \sum_{i=0}^{N_O} \frac{1}{i+1} \binom{N_O}{i} \prod_{j=1}^{N_O-i} (1-\phi_j) \prod_{j=0}^{i} \phi_j \\ & - \sum_{i=0}^{N_O-1} \frac{1}{i+1} \binom{N_O-1}{i} \prod_{j=1}^{N_O-1-i} (1-\phi_j) \prod_{j=0}^{i} \phi_j \\ = & \left[ \sum_{i=0}^{N_O-1} (-\phi_j) \frac{1}{i+1} \binom{N_O-1}{i} \prod_{j=1}^{N_O-1-i} (1-\phi_j) \prod_{j=0}^{i} \phi_j \right] + \frac{1}{N_O+1} \phi_j^{N_O} \leq 0 \end{aligned} \tag{22}$$

We also plot the function, allowing $\phi$ and $N_O$ to vary.

# B Proofs

**Proof of Proposition 1** To show that the game $G$ is supermodular we derive the first order conditions that determine the number of facets ($\hat{f}_k$) and ($\hat{o}_k$) technological opportunities that each firm decides to pursue in equilibrium.

$$\frac{\partial \pi_k}{\partial o_k} = V s_k - L(s_k) - C_o - f_k p_k C_a - \frac{\partial C_c}{\partial o_k} = 0 \quad , \tag{23}$$

$$\begin{aligned} \frac{\partial \pi_k}{\partial f_k} &= o_k \left( V \frac{p_k}{\tilde{F}} - \frac{\partial L}{\partial s_k} \frac{p_k}{\tilde{F}} - p_k C_a + \frac{s_k}{\tilde{F}} \left[ \frac{\partial V}{\partial \tilde{F}} \tilde{F} - V + \frac{\partial L}{\partial s_k} \right] \frac{\partial \tilde{F}}{\partial f_k} \right) \\ &= \frac{o_k p_k}{\tilde{F}} \left( \left[ V - \frac{\partial L}{\partial s_k} - \tilde{F} C_a \right] - \left( V - \frac{\partial L}{\partial s_k} - V\mu \right) \epsilon_{\tilde{F} f_k} \right) = 0 \quad . \end{aligned} \tag{24}$$

Figure 7: Simulation of $p_i$ for $N_O \in 0, 100$ and $\phi \in 0, 1$.

Next, consider the cross-partial derivatives which must be positive if the game $G$ is supermodular. First, we derive the cross partial derivative with respect to firms' own actions:

$$\frac{\partial^2 \pi_k}{\partial o_k \partial f_k} = \left( V \frac{p_k}{\tilde{F}} - \frac{\partial L}{\partial s_k} \frac{p_k}{\tilde{F}} - p_k C_a + \frac{s_k}{\tilde{F}} \left[ \frac{\partial V}{\partial \tilde{F}} \tilde{F} - V + \frac{\partial L}{\partial s_k} \right] \frac{\partial \tilde{F}}{\partial f_k} \right) = 0 \tag{25}$$

This expression corresponds to the first order condition (24) for the optimal number of facets. Now consider effects of rivals' actions on firms' own actions:

$$\frac{\partial^2 \pi_k}{\partial o_k \partial o_j} = \frac{f_k}{\tilde{F}} \left[ \frac{p_k}{\tilde{F}} \left( \frac{\partial V}{\partial \tilde{F}} \tilde{F} - V + \frac{\partial L}{\partial s_k} \right) \frac{\partial \tilde{F}}{\partial o_j} + \left[ V - \frac{\partial L}{\partial s_k} - \tilde{F} C_a \right] \frac{\partial p_k}{\partial o_j} \right], \tag{26}$$

$$\frac{\partial^2 \pi_k}{\partial o_k \partial f_j} = \frac{f_k}{\tilde{F}} \left[ \frac{p_k}{\tilde{F}} \left( \frac{\partial V}{\partial \tilde{F}} \tilde{F} - V + \frac{\partial L}{\partial s_k} \right) \frac{\partial \tilde{F}}{\partial f_j} + \left[ V - \frac{\partial L}{\partial s_k} - \tilde{F} C_a \right] \frac{\partial p_k}{\partial f_j} \right], \tag{27}$$

$$\begin{aligned} \frac{\partial^2 \pi_k}{\partial f_k \partial o_j} = & \left[ \frac{\partial V}{\partial \tilde{F}} + \frac{\partial^2 V}{\partial \tilde{F}^2} \tilde{F} \epsilon_{\tilde{F} f_k} - C_a \right] \frac{\partial \tilde{F}}{\partial o_j} + \left( \frac{\partial V}{\partial \tilde{F}} \tilde{F} - V + \frac{\partial L}{\partial s_k} \right) \frac{\partial \epsilon_{\tilde{F} f_k}}{\partial o_j} \\ & + \left[ \frac{\partial \tilde{F}}{\partial o_j} \frac{p_k}{\tilde{F}} - \frac{\partial p_k}{\partial o_j} \right] \frac{\partial^2 L}{\partial {s_k}^2} \frac{f_k}{\tilde{F}} \left( 1 - \epsilon_{\tilde{F} f_k} \right) , \end{aligned} \tag{28}$$

$$\begin{aligned} \frac{\partial^2 \pi_k}{\partial f_k \partial f_j} = & \left[ \frac{\partial V}{\partial \tilde{F}} + \frac{\partial^2 V}{\partial \tilde{F}^2} \tilde{F} \epsilon_{\tilde{F} f_k} - C_a \right] \frac{\partial \tilde{F}}{\partial f_j} + \left( \frac{\partial V}{\partial \tilde{F}} \tilde{F} - V + \frac{\partial L}{\partial s_k} \right) \frac{\partial \epsilon_{\tilde{F} f_k}}{\partial f_j} \\ & + \left[ \frac{\partial \tilde{F}}{\partial f_j} \frac{p_k}{\tilde{F}} - \frac{\partial p_k}{\partial f_j} \right] \frac{\partial^2 L}{\partial {s_k}^2} \frac{f_k}{\tilde{F}} \left( 1 - \epsilon_{\tilde{F} f_k} \right) \quad . \end{aligned} \tag{29}$$

The game is supermodular if the equations (26)-(29) are non-negative. The following results show that the conditions noted in Proposition 1 must hold simultaneously if the game is supermodular.

**Proof of Lemma 1** We have already noted that:

$$\frac{\partial \tilde{F}}{\partial o_j} > 0, \qquad \frac{\partial \tilde{F}}{\partial f_j} > 0, \qquad \frac{\partial p_k}{\partial o_j} < 0, \qquad \frac{\partial p_k}{\partial f_j} < 0 \ . \tag{30}$$

Thus equations (26) and (27) are positive if and only if:

$$V - \frac{\partial V}{\partial \tilde{F}}\tilde{F} - \frac{\partial L}{\partial s_k} < 0 \qquad V - \frac{\partial L}{\partial s_k} - \tilde{F}C_a < 0 \tag{31}$$

Under the conditions of Lemma 1 these inequalities require that $\mu > 1$ and that $V < 0$. This second condition contradicts assumption ($CI$). Also, note that under these conditions the first order condition (24) implies that:

$$\left(1 - \epsilon_{\tilde{F}f_k}\right) + \mu\epsilon_{\tilde{F}f_k} = 0 \Leftrightarrow (1-\mu)^{-1} = \epsilon_{\tilde{F}f_k} \ . \tag{32}$$

We show in the appendix that $1 \geq \epsilon_{\tilde{F}f_k} \geq 0$ which would imply that $\mu < 0$ contradicting (31) above. This shows that in the absence of both types of costs the game $G$ is not supermodular.

We can also show that it is not enough if just one of these costs is positive since the right hand inequality at 31 can only be fulfilled if there are both legal and administrative costs. By definition it must be the case that $V > \tilde{F}C_a$ or expected profits from patenting would be negative. Also, note that the the first order condition (23) and assumption $LC$ jointly imply that $V > \frac{\partial L}{\partial s_k}$. Therefore, the right inequality at 31 can only be satisfied if there are both legal and administrative costs. This proves Lemma 1.

**Proof of Lemma 2** Rewriting the first order condition (24) allowing for legal and administrative costs implies:

$$\frac{V - \frac{\partial L}{\partial s_k} - \tilde{F}C_a}{V - \frac{\partial L}{\partial s_k} - V\mu} = \epsilon_{\tilde{F}f_k} \tag{33}$$

Notice that numerator and denominator of the fraction above correspond to the terms on the left of the inequalities at (31). Both terms must be negative if the inequalities at (31) hold. This expression shows that one inequality is a multiple of the other: either both or neither holds. We have also noted that $1 \geq \epsilon_{\tilde{F}f_k} \geq 0$. Therefore, the fraction above implies that:

$$\tilde{F}C_a < \mu V(\mu, \tilde{F}) \Leftrightarrow C_a < \frac{\partial V}{\partial \tilde{F}} \quad . \tag{34}$$

We have already noted that firms will not patent unless $V > \tilde{F}C_a$. If the elasticity of the value of a technological opportunity with respect to covered facets $(\mu)$ is less than one, the condition here provides an upper bound for the costs of maintaining granted patents $(C_a)$.

**Proof of Lemma 3** We have already shown that:

$$\frac{\partial \tilde{F}}{\partial o_j} > 0, \qquad \frac{\partial \tilde{F}}{\partial f_j} > 0, \qquad \frac{\partial \epsilon_{\tilde{F} f_k}}{\partial o_j} < 0, \qquad \frac{\partial \epsilon_{\tilde{F} f_k}}{\partial f_j} < 0 \,. \tag{35}$$

The terms in square brackets in equations (28) and (29) are positive if $\mu > 1$. If $1 > \mu > 0$, then the term is positive in the limit as $\epsilon_{\tilde{F} f_k}$ approaches zero. This happens as more firms compete for a given technological opportunity and $N_O$ increases. The second terms in equations (28) and (29) are negative if the inequalities at (31) are satisfied. The third terms in equations (28) and (29) are always negative by Assumption (LC).

Thus equations (28) and (29) can only be positive if the first positive term outweighs the negative terms. Greater complexity of the technology has two effects: it raises the number of covered facets reducing $f_k/F$ and thus making the third term in both equations smaller and it increases $\partial \tilde{F}/f_j$ strengthening the first positive term in equation (29). An increase in the number of competing firms reduces the elasticity of covered facets $\epsilon_{\tilde{F} f_k}$. This also has two effects: it reduces the derivatives multiplying the second terms in both equations and as noted it raises the likelihood that the first terms in both equations are positive if $\mu < 1$. These findings prove Lemma 3.

**Proof of Corrolary 1** This result arises because $\frac{\partial \tilde{F}}{\partial N_O}\frac{\partial N_O}{\partial N} > 0$, $\frac{\partial p_i}{\partial N_O}\frac{\partial N_O}{\partial N} < 0$ and $\frac{\partial \epsilon_{\tilde{F} \tilde{f}_i}}{\partial N_O}\frac{\partial N_O}{\partial N} < 0$ as we show in Appendices A.1 and A.2. Then, we can show that:

$$\frac{\partial^2 \pi_k}{\partial o_k \partial N} = \frac{f_k}{\tilde{F}}\left[\frac{p_k}{\tilde{F}}\left(\frac{\partial V}{\partial \tilde{F}}\tilde{N_O} - V + \frac{\partial L}{\partial s_k}\right)\frac{\partial \tilde{F}}{\partial N_O} + \left[V - \frac{\partial L}{\partial s_k} - \tilde{F} C_a\right]\frac{\partial p_k}{\partial N_O}\right]\frac{\partial N_O}{\partial N} > 0, \tag{36}$$

$$\begin{aligned}\frac{\partial^2 \pi_k}{\partial f_k \partial N} = &\left[\frac{\partial V}{\partial \tilde{F}} + \frac{\partial^2 V}{\partial \tilde{F}^2}\tilde{F}\epsilon_{\tilde{F} f_k} - C_a\right]\frac{\partial \tilde{F}}{\partial N_O}\frac{\partial N_O}{\partial N} + \left(\frac{\partial V}{\partial \tilde{F}}\tilde{F} - V + \frac{\partial L}{\partial s_k}\right)\frac{\partial \epsilon_{\tilde{F} f_k}}{\partial N_O}\frac{\partial N_O}{\partial N} \\ &+ \left[\frac{\partial \tilde{F}}{\partial N_O}\frac{p_k}{\tilde{F}} - \frac{\partial p_k}{\partial N_O}\right]\frac{\partial N_O}{\partial N}\frac{\partial^2 L}{\partial {s_k}^2}\frac{f_k}{\tilde{F}}\left(1 - \epsilon_{\tilde{F} f_k}\right) > 0 \,.\end{aligned} \tag{37}$$

Note that equation (36) has the same structure as equations (26, (27)) while equation (37) has the same structure as equations (28, (29)). This shows that Corollary 1 holds if Proposition 1 holds.

**Proof of Proposition 2** To determine the effects of an increase in technological opportunity $O$ we investigate the following cross-partial derivatives:

$$\frac{\partial^2 \tilde{\pi}_i}{\partial o_i \partial O} \qquad \text{and} \qquad \frac{\partial^2 \tilde{\pi}_i}{\partial \tilde{f}_i \partial O} \quad . \tag{38}$$

If the game set out above is smooth supermodular, it follows from equations (26) and (28) that both cross-derivatives here are negative. To see this note that $o_j$ and $O$ only enter this

model as a ratio: an increase in $O$ is equivalent to a reduction in $o_j$.[31] Equations (26) and (28) are both positive if the game $G$ is smooth supermodular. Their signs are determined by the derivatives $\frac{\partial \tilde{F}}{\partial o_j} > 0$ and $\frac{\partial p_i}{\partial o_j} < 0$. The derivatives $\frac{\partial \tilde{F}}{\partial O} < 0$ and $\frac{\partial p_i}{\partial O} > 0$ have exactly opposite signs, reversing the signs of the cross-partial derivatives above.

**Proof of Proposition 3** On one hand $F$ enters our model through the ratio $\phi_i$. Therefore, an increase in $F$ is the same as a reduction in $f_j$, indicating that greater complexity should reduce patenting efforts if Proposition 1 holds. The argument is analogous to that made in the case of Proposition 2. On the other hand $\tilde{F}$ also increases directly in $F$:

$$\frac{\partial^2 \pi_k}{\partial o_k \partial F} = \frac{f_k}{\tilde{F}} \left( \frac{\partial V}{\partial \tilde{F}} \tilde{F} - V + \frac{\partial L}{\partial s_k} \right) \left[ \frac{p_i}{\tilde{F}} \frac{\partial \tilde{F}}{\partial F} - \epsilon_{\tilde{F} f_k} \frac{\partial p_k}{\partial F} \right] \tag{39}$$

$$\frac{\partial^2 \pi_k}{\partial f_k \partial F} = \left[ \left( \frac{\partial V}{\partial \tilde{F}} \tilde{F} - V + \frac{\partial L}{\partial s_k} \right) \frac{(1 - 2\epsilon_{\tilde{F} f_k})}{\tilde{F}} + \frac{\partial^2 V}{\partial \tilde{F}^2} \tilde{F} \epsilon_{\tilde{F} f_k} \right] \frac{\partial \tilde{F}}{\partial F} + \left[ \frac{\partial \tilde{F}}{\partial F} \frac{p_k}{\tilde{F}} - \frac{\partial p_k}{\partial F} \right] \frac{\partial^2 L}{\partial {s_k}^2} \frac{f_k}{\tilde{F}} \left(1 - \epsilon_{\tilde{F} f_k}\right) \tag{40}$$

Here the terms in round brackets in Equations (39) and (40) are positive if the game is smooth supermodular. These positive terms will determine the sign of both conditions if the elasticity of covered facets $(\epsilon_{\tilde{F} f_k})$ goes to zero. As noted in the proof of Lemma 3 this arises as more firms compete within a technological opportunity.

**Proof of Proposition 4** To see that this is true consider the first and second order derivatives of the payoff function with respect to technological opportunities invested in:

$$\frac{\partial \pi}{\partial o_k} = (V - L - C_a) p_k - \frac{\partial C_c}{\partial o_k} = 0 \qquad \frac{\partial^2 \tilde{\pi}}{\partial {o_k}^2} = -\frac{\partial^2 C_c}{\partial {o_k}^2} \quad . \tag{41}$$

If we assume that costs of coordinating technological opportunities are strictly convex: $\frac{\partial^2 C_c}{\partial {o_k}^2} > 0$, then Proposition 4 can be proved with the help of the implicit function theorem:

$$\frac{\partial o_k}{\partial O} = -\frac{\partial^2 \tilde{\pi}}{\partial o_k \partial O} \Big/ \frac{\partial^2 \tilde{\pi}}{\partial {o_k}^2} > 0 \quad , \tag{42}$$

where $\frac{\partial^2 \tilde{\pi}}{\partial o_k \partial O} = (V - L - C_a) \frac{\partial p}{\partial O} > 0$.

**Proof of Corrolary 2** To see this is true note that $\frac{\partial^2 \tilde{\pi}}{\partial o_k \partial N} = (V - L - C_a) \frac{\partial p_k}{\partial N_O} \frac{\partial N_O}{\partial N} < 0$. Then:

$$\frac{\partial o_k}{\partial N} = -\frac{\partial^2 \tilde{\pi}}{\partial o_k \partial N} \Big/ \frac{\partial^2 \tilde{\pi}}{\partial {o_k}^2} < 0 \quad . \tag{43}$$

[31] Compare the discussion of the expected number of rivals investing in the same technological opportunity ($N_O$) in Appendix A.1.

# C Results from OLS and Fixed Effects Regressions

Table 7: Patent Applications Estimates using OLS and Fixed Effects

| | OLS models | | | Fixed effects models | | |
|---|---|---|---|---|---|---|
| **Variable** | OLS 1 | OLS 2 | OLS 3 | FE 1 | FE 2 | FE 3 |
| log Patentcount$_{t-1}$ | 0.586*** | 0.562*** | 0.560*** | 0.170*** | 0.155*** | 0.154*** |
| | (0.002) | (0.002) | (0.002) | (0.002) | (0.002) | (0.002) |
| log Patentcount$_{t-1}$× Triples | | 0.001*** | 0.001*** | | 0.001*** | 0.001*** |
| | | (0.000) | (0.000) | | (0.000) | (0.000) |
| Non Patent References (NPR) | 0.057*** | 0.067*** | 0.058*** | 0.004 | 0.012 | -0.011 |
| | (0.002) | (0.002) | (0.003) | (0.007) | (0.008) | (0.009) |
| NPR × Triples | | -0.001*** | -0.001*** | | 0.000* | 0.001** |
| | | (0.000) | (0.000) | | (0.000) | (0.000) |
| NPR × Triples × Large | | | -0.000*** | | | -0.000* |
| | | | (0.000) | | | (0.000) |
| NPR × Large | | | 0.022*** | | | 0.036*** |
| | | | (0.003) | | | (0.006) |
| Fragmentation | 0.569*** | 0.546*** | 0.544*** | 0.456*** | 0.432*** | 0.431*** |
| | (0.004) | (0.004) | (0.004) | (0.004) | (0.004) | (0.004) |
| Fragmentation × Triples | | 0.001*** | 0.001*** | | 0.001*** | 0.001*** |
| | | (0.000) | (0.000) | | (0.000) | (0.000) |
| Triples | 0.000*** | 0.001*** | 0.001*** | 0.001*** | -0.001** | -0.001** |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| Areas | 0.018*** | 0.018*** | 0.018*** | 0.082*** | 0.082*** | 0.082*** |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| Large | 0.183*** | 0.189*** | 0.174*** | 0.259*** | 0.259*** | 0.226*** |
| | (0.004) | (0.004) | (0.005) | (0.005) | (0.005) | (0.008) |
| Year dummies | YES | YES | YES | YES | YES | YES |
| Primary area dummies | YES | YES | YES | YES | YES | YES |
| Constant | 0.106*** | 0.105*** | 0.116*** | 0.031* | 0.041** | 0.066*** |
| | (0.011) | (0.011) | (0.011) | (0.015) | (0.015) | (0.016) |
| R-squared | 0.686 | 0.688 | 0.688 | 0.323 | 0.324 | 0.324 |
| N | 173448 | 173448 | 173448 | 173448 | 173448 | 173448 |

*p<0.05, ** p<0.01, *** p<0.001

# D LED Technology

Light emitting diodes (LED) are based on physical principles that were discovered in the early $20^{th}$ century and were first introduced as a practical electronic component by Holonyak and Bevacqua (1962). LEDs consist of semiconducting material that has been impregnated with impurities to create so-called p-n-junctions that generate the physical characteristic of diodes, i.e., current is flowing only from the anode-side to the cathode-side, but not in the reverse direction. Depending on the materials used to impregnate the chip underlying the diode and the way of applying it to the supporting material, different wavelengths of light are emitted by LEDs. Historically, the first usable LEDs were infrared and red devices based on gallium arsenide.

Since the emergence of the first red LEDs, major research paths in LED-technology can be classified in two broad categories comprising (i) the identification of different materials to produce different colours and (ii) improvement of efficiency and operational parameters. The combination of the results from R&D in these two dimensions led to the gradual improvement of this technology.

The nature of the research conducted within the realm of LEDs is a good example of how we think about technology areas in terms of technology opportunity and patentable facets. First, the different materials that are used to impregnate semiconducting materials can be thought of as separate technological opportunities in the technology area of LEDs. Discovery of novel materials that can be used in the production of LEDs stems from basic research that can be conducted within firms or within universities.

Second, different materials require novel production techniques since efficient impregnation of the semiconducting base of LEDs largely depends on the characteristics of the material used (Yam and Hassan, 2005). Therefore, the emergence of novel materials opens up a certain number of patentable facets. Once a novel material has been discovered, firms have to adapt their production techniques to efficiently manufacture LEDs using that material and they have to invest in opportunity-specific R&D to do so. We model these specific R&D efforts as $C_o$ in our theoretical model. Note that such opportunity specific R&D can also lead to more efficient LEDs over time.

Both novel manufacturing techniques as well as efficiency gains can be protected by patent rights and therefore can be considered as examples of patentable facets. If separate firms engage in R&D activities within opportunities it is likely that more than one firm obtains patents on crucial production steps. This might give rise to situations where firms need to access competitors property rights - which we consider to be a hallmark of a complex technology. In fact, patents are crucial in the LED industry and a high degree of cross-licensing and infringement law-suits among can be observed. A list of relevant deals and disputes can be found on http://www.ledsmagazine.com/features/1/8/21/1.

# E Complex and Discrete Technologies

Table 8: Classification of technology areas according to OST-INPI/FhG-ISI

| Area Code | Description | Classification |
|---|---|---|
| 1 | Electrical machinery, electrical energy | Complex |
| 2 | Audiovisual technology | Complex |
| 3 | Telecommunications | Complex |
| 4 | Information technology | Complex |
| 5 | Semiconductors | Complex |
| 6 | Optics | Complex |
| 7 | Analysis, measurement, control technology | Complex |
| 8 | Medical technology | Complex |
| 9 | Nuclear engineering | Complex |
| 10 | Organic fine chemistry | Discrete |
| 11 | Macromolecular chemistry, polymers | Discrete |
| 12 | Pharmaceuticals, cosmetics | Discrete |
| 13 | Biotechnology | Discrete |
| 14 | Agriculture, food chemistry | Discrete |
| 15 | Chemical and petrol industry, basic mat | Discrete |
| 16 | Chemical engineering | Discrete |
| 17 | Surface technology, coating | Discrete |
| 18 | Materials, metallurgy | Discrete |
| 19 | Materials processing, textiles paper | Discrete |
| 20 | Handling, printing | Discrete |
| 21 | Agricultural and food processing, machin | Discrete |
| 22 | Environmental technology | Complex |
| 23 | Machine tools | Complex |
| 24 | Engines, pumps and turbines | Complex |
| 25 | Thermal processes and apparatus | Complex |
| 26 | Mechanical elements | Complex |
| 27 | Transport | Complex |
| 28 | Space technology, weapons | Complex |
| 29 | Consumer goods and equipments | Complex |
| 30 | Civil engineering, building, mining | Complex |

Description of the 30 technology areas contained in the OST-INPI/FhG-ISI technology nomenclature. We classified the 30 technology areas as complex or discrete attempting to replicate the classification of Cohen et al. (2000).