

A Service of

ZBW

Leibniz-Informationszentrum Wirtschaft Leibniz Information Centre for Economics

Schweizer, Urs

### Working Paper Legal Damages at Uncertain Causation

SFB/TR 15 Discussion Paper, No. 160

#### **Provided in Cooperation with:**

Free University of Berlin, Humboldt University of Berlin, University of Bonn, University of Mannheim, University of Munich, Collaborative Research Center Transregio 15: Governance and the Efficiency of Economic Systems

*Suggested Citation:* Schweizer, Urs (2006) : Legal Damages at Uncertain Causation, SFB/TR 15 Discussion Paper, No. 160, Sonderforschungsbereich/Transregio 15 - Governance and the Efficiency of Economic Systems (GESY), München, https://doi.org/10.5282/ubm/epub.13391

This Version is available at: https://hdl.handle.net/10419/93762

#### Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

#### Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



# WWW.ECONSTOR.EU



Discussion Paper No. 160

## Legal Damages at Uncertain Causation

Urs Schweizer\*

#### August 2006

\*Urs Schweizer, Department of Economics, University of Bonn, Adenauerallee 24, 53113 Bonn, Germany. schweizer@uni-bonn.de

Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

 $Sonderforschungsbereich/Transregio~15\cdot~www.gesy.uni-mannheim.de$ 

Universität Mannheim · Freie Universität Berlin · Humboldt-Universität zu Berlin · Ludwig-Maximilians-Universität München Rheinische Friedrich-Wilhelms-Universität Bonn · Zentrum für Europäische Wirtschaftsforschung Mannheim

Speaker: Prof. Konrad Stahl, Ph.D. · Department of Economics · University of Mannheim · D-68131 Mannheim, Phone: +49(0621)1812786 · Fax: +49(0621)1812785

## Legal Damages at Uncertain Causation

Urs Schweizer,\*<sup>†</sup>University of Bonn

August 17, 2006

#### Abstract

The legal notion of damages requires to compare the actual value of the creditor's assets with the hypothetical value that would have prevailed if the debtor had met his obligation. Moreover, values and causation may be uncertain. If nature's contribution is modelled as a random move then the interaction between debtor and nature can be described in normal form which, in turn, allows to capture causality and legal damages in a consistent way. In practice, such random moves of nature are rarely observable. Yet, statistical inference may reveal sufficient information to test for causation and to estimate legal damages on average over observable events as the present paper will establish.

JEL classification: K13, K12, D62

Keywords: estimating legal damages, liability for torts, liability for breach of contracts

<sup>\*</sup>address: Urs Schweizer, Department of Economics, University of Bonn, Adenauerallee 24, 53113 Bonn, Germany, schweizer@uni-bonn.de

<sup>&</sup>lt;sup>†</sup>The author thanks Gerd Muehlheusser, Andreas Roider, Patrick Schmitz and Alexander Stremitzer for helpful comments on a previous version of this paper. He also has benefitted from extensive discussions with Gerhard Wagner and other colleagues from the Law Department in Bonn. Support through SFB/TR 15 is gratefully acknowledged.

#### 1 Introduction

Think of medical practitioner A who was negligent in treating several of his patients. As a consequence, 7 out of 10 were suffering from bodily harm, each equivalent to L = 100 in money terms. Under careful treatment, 4 out of 10 would nonetheless suffer the same losses but 6 out of 10 would have been cured. It is assumed that these statistical data are known from relevant test series of similar cases. What quantum of damages (if any) should be awarded to patients of the negligent practitioner A?

Or think of professional coach A who did not cover the latest revision of competition law, which, however, turned out to be relevant for the exam. Among his students, 6 out of 10 failed the exam. Students of another coach who met his obligation performed better. In fact, only 3 out of 10 failed the exam while 7 out of 10 did pass. Students who pass the exam earn, on average,  $L^P = 20$  below some maximum income level whereas students who failed earn, on average,  $L^F = 70$  below the same maximum. What damages does the negligent coach A owe to his student B, who failed the exam?

The common features of these first two cases are as follows. Party A takes a decision that affects the probability of an accident and is ruled, beyond dispute, to violate A's obligation. Yet, the accident would also have occurred with positive, though lower probability if A had met his obligation. Therefore it remains uncertain whether it was A's negligence that has actually caused the harm or just bad luck.

In a third case, there are two candidates a = 1, 2 that may have caused harm of fixed size L = 100 to victim B. If both candidates had met their obligations the accident would still have occurred with probability  $\varepsilon^{oo} =$ 2/10 whereas if both have neglected their obligations then the probability of an accident would be  $\varepsilon^{nn} = 7/10$ . If just one of the two candidates had neglected his obligation then the probabilities of an accident are  $\varepsilon^{on} = 3/10$ and  $\varepsilon^{no} = 5/10$ , respectively. What damages (if any) would each of the two candidates owe to B if it is known beyond dispute that both have neglected their obligations?

To determine legal damages, by definition, the actual situation under A's negligent decision must be compared with the purely hypothetical situation that would have resulted if A had met his obligation. Since the accident is claimed to occur with positive probability but lower than one, nature is implicitly interpreted as rolling dices. For doctrinal reasons, it proves useful to express the interaction between party A and nature in normal form.<sup>1</sup> According to this view, nature's contribution is modelled as a random move, drawn independently of A's decision. The accident technology then determines whether an accident actually occurs or not as a function of A's decision and the random move of nature.

Suppose the accident has occurred and A's decision was negligent. If the random move of nature were observable then the hypothetical question could easily be addressed. In fact, just plug the non-negligent decision and the same actual move of nature into the accident technology. If the accident still occurs A's negligence did not cause it. If, however, the accident would have been avoided, then A's negligence has actually caused it and A would owe legal damages equivalent to the full harm suffered by party B.

Of course, such moves of nature are rarely observable. Observable may just be the event that an accident has occurred. The present paper propagates to still award true legal damages but on average over the observed event only. Such an approach nicely fits legal doctrine, the only drawback being that the interaction between nature and party A should be known in normal form.

As it turns out, the statistical data presented for the above three examples are not sufficient to determine legal damages on average over the observed event. For the first example, however, just one more piece of information is needed: The probability, with which an accident would occur if A has met his obligation but would have been avoided if A had neglected it. Under these circumstances, party B would enjoy a windfall gain caused by A's negligence that, based on common legal practice, party B could keep for free.

It will be shown, at the one extreme where the probability of windfall gains is equal to the probability of an accident under non-negligent behavior, legal damages on average over the observed event are equivalent to the full loss suffered by B, well in line with the traditional negligence rule pioneered by Brown (1973). At the other extreme where windfall gains can be ruled out entirely, legal damages on average over the observed event turn out to be

<sup>&</sup>lt;sup>1</sup>The insight that interaction in general can be expressed in normal form is ascribed to John von Neumann as Myerson (1999) has pointed out.

equivalent to Shavell's (1985) proportionality rule. Particularly interesting seems to be the intermediate case where the probabilities of accidents are pure in the sense that all patients are of the same type: If a patient recovers in spite of negligent treatment then he is still believed to suffer with probability 4 out of 10 in the hypothetical situation where the practitioner had met his obligation. For the given statistical data, under pure uncertainty, legal damages on average over the observable event are equal to 60 percent of the victim's actual loss as will be shown later in this paper.<sup>2</sup>

The second example turns out to be even more puzzling. Suppose the student did pass the exam in spite of the coach's negligent behavior. Then legal damages on average over the observed event that the student has passed the exam may still happen to be positive. On intuitive grounds, one is tempted to argue that no accident has occurred and, hence, no damages are due. The puzzle resolves if the following consideration is taken into account.

The coach was negligent but the student did pass the exam. Thereafter, this student was less successful at the labor market where he ended up with a low paying job. Compare this with the hypothetical situation where the coach had met his obligation but where, this time, the student failed the exam. At the labor market, however, this student may be more lucky, ending up in fact with a higher paying job. If such a hypothetical situation is believed to occur with positive probability then, indeed, the negligent coach may owe a positive quantum of damages even to students who passed the exam. If such damages were denied, incentives could well be distorted.

So far, losses were assumed to be of fixed size. Yet, the actual as well as the hypothetical value of assets affected by an accident may vary to a larger degree. If they do the following conceptual issue arises. Suppose party A has neglected his obligation and the actual value of the affected assets is observable. The actual value must be compared with the hypothetical one if A had met his obligation. The hypothetical value may now be higher or lower than the actual value. If it is lower then windfall gains from negligent behavior are involved and the question arises: While taking averages over observable events, should B keep such hypothetical windfall gains for free or should they be offset against B's losses over the observed event?

<sup>&</sup>lt;sup>2</sup>Gerhard Wagner and other legal scholars have pointed out to me that this rule is used in France, where it is referred to as "perte d'une chance".

Kahan (1989) argues that party A owes damages equal to the difference between the value of the affected assets if A had met versus A's actually having neglected his obligation. It will be shown that Kahan's rule coincides with average legal damages if windfall gains are offset against losses over the observed event.

From a practical perspective, calculating legal damages on average over observable events turns out to be simpler if windfall gains are offset against losses. From a doctrinal perspective, it may be objectionable to treat windfall gains differently just because they happen not to be observable. From the economic perspective, it does not really matter. In fact, as long as the victim is (at least) fully compensated and meeting the obligation is socially preferred to neglecting it – the Hand Formula – then party A has the incentive to meet his obligation under both schemes. While leaving windfall gains for free may lead to overcompensation of B, party A can easily escape liability by meeting his obligation.

The paper also addresses the case of uncertain causation in the presence of multiple injurers. To express the interaction between all these injurers and nature in normal form, the accident technology must be specified as a function of the profile of individual decisions and the random move of nature. For simplicity, cost complementarities between injurers are ruled out. The accident technology under consideration, however, allows for externalities between individual decisions of the fully general type.

Suppose each of the injurer has the obligation to decide in a way such that the sum of total costs and expected losses attains a minimum if all injurers are meeting their obligations (extended Hand Formula). Suppose further that the victim is fully compensated (in ex ante terms at least) for expected losses from deviations. Then, as will be shown, details of sharing damages owed to B among negligent injurers are of no importance with respect to incentives because each injurer can unilaterally escape liability by meeting his obligation. In fact, not even collusion, let alone non-cooperative behavior, will allow the group of potential injurers to gain from neglecting some or all of their obligations if such damages are implemented.

The paper is organized as follows. Section 2 deals with cases of fixed harm size. The notion of legal damages on average over observable events is defined explicitly and it is shown that such damages provide efficient incentives. The interaction in normal form is constructed from the statistical data and from hypotheses on potential chains of causation. The first example of the introduction will be revisited in this section as well.

While harm size is still kept fixed, section 3 adds observable signals, which the quantum of damages must be based on. Shavell (1985) has examined a model of this type. It is shown that Shavell's proportionality rule allows for an interpretation as legal damages on average over observable events. The second example of the introduction is also studied in detail, the observable signal being whether the student has passed or failed the exam.

In section 4, accidents are assumed to cause losses of variable size. The issue of windfall gains being offset or not against losses on average over observable events is discussed. The efficiency of incentives is shown to prevail for both versions of the damage rule. Section 5 establishes the efficiency result if several injurers may have caused the accident and revisits the third example of the introduction. Section 6 concludes.

#### 2 Binary asset values

Party A is facing a decision  $r \in R$  at costs c(r) that affects the value of party B's assets in an uncertain way. For the present section, it is assumed that assets are attaining just two values from the range  $\rho_B = \{b^0 = -L, b^1 = 0\}$ . I have the following interpretation in mind. If there is an accident then B suffers from harm of fixed size L whereas, if there is none, the value of B's assets is not affected at all. Moreover, let us assume that it is known from test series of similar cases that the relative frequency of an accident amounts to  $0 \leq \varepsilon(r) \leq 1$  provided that A has taken decision  $r \in R$ .

If this test series contains sufficiently many independent draws,  $\varepsilon(r)$  can also be referred to as the probability of an accident. Whether nature is actually rolling dices or not may remain a matter of philosophical dispute. Yet, probabilities have proven to be a most useful device of modelling and describing uncertain events.

The economic analysis of tort law refers to the setting at hand as the accident model. The model serves to investigate incentives for precaution arising from negligence rules. Suppose it were A's obligation to decide  $r^o \in R$ 

but, instead, A has actually decided  $r^n \neq r^{o.3}$  By such negligent behavior, A has saved  $c(r^o) - c(r^n)$  in terms of private costs but, at the same time, has raised the probability of an accident by  $\varepsilon(r^n) - \varepsilon(r^o) > 0$ . The Hand Formula

$$0 < c(r^{o}) - c(r^{n}) < [\varepsilon(r^{n}) - \varepsilon(r^{o})] \cdot L$$

is assumed to be fulfilled and, hence, it is socially desirable that A meets his obligation.

If an accident occurs, A may be held liable. The legal question behind injurer A's liability concerns the hypothetical situation that would have resulted if, ceteris paribus, A had met his obligation. According to common legal doctrine, party A is held liable only if, by meeting the obligation, the accident would have been avoided. Put differently, party A's negligence must have caused the accident for A to owe any damages.

The probability of an accident may remain positive even if A meets his obligation, i.e.,  $\varepsilon(r^o) > 0$ , in which case causation is uncertain. To settle the issue of liability from a theoretical perspective, it proves useful to visualize nature's contribution in what game theorists call the normal form of interaction. In normal form, nature is simultaneously "choosing" from a set  $\omega \in \Omega$ of alternative moves of nature – the outcome space – as party A is choosing from his set  $r \in R$  of strategies. While A is assumed to behave strategically, nature is assumed to be governed by an exogenous probability measure  $\pi$ : For any subset (event)  $\Omega'$  of the outcome space  $\Omega$ ,  $\pi(\Omega')$  denotes the probability, with which the event  $\Omega'$  occurs. The accident model in normal form combines this probability measure with a function  $e: R \times \Omega \to \{0, 1\}$ , referred to as the accident technology. By construction, this function attains the value  $e(r, \omega) = 1$  if and only if an accident is resulting from the interaction.

To begin with, suppose the actual move of nature  $\omega$  were observable. Then legal doctrine would rule A liable for the full loss L if the accident has actually occurred under A's negligent behavior but would have been avoided if A had met his obligation. For short, the quantum of damages then amounts to

$$D(r^{n},\omega) = \max\left[e(r^{n},\omega) - e(r^{o},\omega), 0\right] \cdot L.$$

 $<sup>^{3}</sup>$ Instead of a tort relationship, the obligation may also arise from a contractual relationship.

The same rule may alternatively be expressed in terms of events. Let  $\Omega_{ij} = \{\omega \in \Omega : e(r^n, \omega) = i \land e(r^o, \omega) = j\}$  denote events leading to the partition  $\Omega = \Omega_{00} \cup \Omega_{01} \cup \Omega_{10} \cup \Omega_{11}$  of the outcome space. Given the negligent decision  $r^n$ , an accident occurs in the event  $\Omega_1 = \Omega_{10} \cup \Omega_{11}$  whereas no accident occurs in the event  $\Omega_0 = \Omega_{00} \cup \Omega_{01}$ . While A owes damages of L in the event  $\Omega_{10}$  he would escape liability in the event  $\Omega_{11}$ . In the event  $\Omega_{01}$ , B enjoys windfall gains due to A's negligence. To be consistent with the statistical data of the model,  $\pi(\Omega_1) = \varepsilon(r^n)$  and

$$0 \le \pi(\Omega_{01}) \le \pi(\Omega_{01} \cup \Omega_{11}) = \varepsilon(r^o)$$

must hold.

Unfortunately, different versions of the accident model in normal form may be consistent with the same statistical data but, nevertheless, may lead to different judgment as will be shown later in this section.

The move of nature itself need not be observable but the fact that an accident has occurred may still be. In this case, it is the event  $\Omega_1$  that can be observed. What level of damages should be awarded in such an event? The obvious solution consists of still awarding true legal damages but on average over the observed event only. More precisely, if the event  $\Omega' \subset \Omega$  is observed, average legal damages amount to

$$d(r^n, \Omega') = E[D(r^n, \omega) \mid \Omega'],$$

i.e. the expected value of true legal damages conditional on the observed event. Accordingly, in the event  $\Omega_1$ , average legal damages amount to

$$d(r^n, \Omega_1) = E[D(r^n, \omega) \mid \Omega_1] = \frac{\pi(\Omega_{10})}{\pi(\Omega_1)} \cdot L$$

while, in the event  $\Omega$  corresponding to no information, average legal damages amount to

$$d(r^n, \Omega) = \pi(\Omega_{10}) \cdot L = \pi(\Omega_1) \cdot d(r^n, \Omega_1).$$

Of course, no damages are due if A has met his obligation, i.e.  $d(r^o, \Omega') = 0$ for any observable event  $\Omega'$ . The following proposition can be established.

**Proposition 1** Legal damages on average over the event that an accident has occurred amount to

$$d(r^n, \Omega_1) = \frac{\varepsilon(r^n) - \varepsilon(r^o) + \pi(\Omega_{01})}{\varepsilon(r^n)} \cdot L$$

and can be calculated from the statistical data if, in addition, the probability  $\pi(\Omega_{01})$  of windfall gains is known. Moreover, in expected terms, party B is at least as well off as if A had met his obligation<sup>4</sup> and, hence, would owe no damages to B, i.e.

$$\pi(\Omega_1) \cdot [d(r^n, \Omega_1) - L] \ge -\pi(\Omega_{01} \cup \Omega_{11}) \cdot L$$

whereas party A has the incentive to meet his obligation as

$$c(r^n) + \pi(\Omega_1) \cdot d(r^n, \Omega_1) > c(r^o)$$

holds.

**Proof.** If  $\omega \in \Omega_0$  then no accident occurs even though A has neglected his obligation and, hence, no damages are due. Consider the partition  $\Omega = \Omega_1 \cup \Omega_{00} \cup \Omega_{01}$  of the outcome space. It then follows from Bayes' rule and from consistency with the statistical data that

$$\begin{aligned} [\varepsilon(r^n) - \varepsilon(r^o)] \cdot L &= E\left[e(r^n, \omega) - e(r^o, \omega)\right] \cdot L = \\ &= \pi(\Omega_1) \cdot E\left[e(r^n, \omega) - e(r^o, \omega) \mid \Omega_1\right] \cdot L - \pi(\Omega_{01}) \cdot L = \\ &= \pi(\Omega_1) \cdot d(r, \Omega_1) - \pi(\Omega_{01}) \cdot L \end{aligned}$$

from which the first claim follows immediately.

It then follows from the first claim that

$$\pi(\Omega_1) \cdot [d(r^n, \Omega_1) - L] = \pi(\Omega_{10}) \cdot L - \pi(\Omega_1) \cdot L = -\pi(\Omega_{11}) \cdot L \ge -\pi(\Omega_{11} \cup \Omega_{01}) \cdot L$$

such that the second claim is established, which jointly with the Hand Rule implies

$$c(r^{n}) + \pi(\Omega_{1}) \cdot d(r^{n}, \Omega_{1}) >$$

$$c(r^{o}) - [\varepsilon(r^{n}) - \varepsilon(r^{o})] \cdot L + \pi(\Omega_{1}) \cdot d(r^{n}, \Omega_{1}) \geq$$

$$c(r^{o}) - \varepsilon(r^{n}) - \varepsilon(r^{o}) \cdot L + \pi(\Omega_{1}) \cdot L - \pi(\Omega_{01} \cup \Omega_{11}) \cdot L = c(r^{o})$$

and the third claim is established as well.  $\blacksquare$ 

In order to calculate legal damages on average over the event  $\Omega_1$  that an accident has occurred, the probability  $\pi(\Omega_{01})$  of windfall gains must be

 $<sup>^{4}</sup>$ This corresponds to the saddle point property that Schweizer (2005a) has identified as the driving force behind efficient incentives.

known. Consider, first, the case of pure uncertainty where nature is rolling dices with independent probabilities  $\varepsilon(r^n)$  and  $\varepsilon(r^o)$  as indicated by tree (a) of Figure 1. Under pure uncertainty, the probability of windfall gains amounts to  $\pi(\Omega_{01}) = [1 - \varepsilon(r^n)] \cdot \varepsilon(r^o)$  such that legal damages on average over the event amount to

$$d(r^n, \Omega_1) = [1 - \varepsilon(r^o)] \cdot L$$

as follows from the first claim of Proposition 1. Notice,  $1 - \varepsilon(r^o)$  is the probability with which the accident would have been avoided if A had met his obligation. It is this loss of chance (in French, "perte d'une chance") that determines the percentage of the harm, which B can recover.

The first example of the introduction may serve as an illustration. Under careful treatment, 6 patients out of 10 are cured while 4 out of 10 would still suffer from harm equivalent to L = 100. If these relative frequencies are interpreted as probabilities in the usual way then  $\varepsilon^o = 4/10$  and  $1 - \varepsilon^o = 6/10$ such that, under pure uncertainty, legal damages on average over the observed event amount to  $d(r^n, \Omega_1) = 60$  as claimed in the introduction.

Consider, second, tree (b) of Figure 1. Here, uncertainty is type-contingent in the following sense. Nature is rolling a dice which determines whether the case is of type t = N (with probability  $\varepsilon(r^o)$ ) or of type t = A (with probability  $1 - \varepsilon(r^o)$ ). Types react in different ways to negligent treatment. In fact, if the case is of type t = N, then the accident occurs whether A is meeting his obligation or not. If, however, the case is of type t = A then the accident is avoided if A has met his obligation whereas the accident occurs with probability  $\varepsilon_A^n$  if A has neglected it. To ensure consistency with the statistical data,

$$\varepsilon_A^n = \frac{\varepsilon(r^n) - \varepsilon(r^o)}{1 - \varepsilon(r^n)}$$

must be imposed. Notice, under this interpretation, windfall gains will never occur and legal damages on average over the observed event amount to

$$d(r^n, \Omega_1) = \frac{\varepsilon(r^n) - \varepsilon(r^o)}{\varepsilon(r^n)} \cdot L$$

as follows from the first claim of Proposition 1. Such damages are in line with Shavell's (1985) proportionality rule.<sup>5</sup>

[Figure 1 here, approximately]

 $<sup>^{5}</sup>$ For the general case studied by Shavell, the reader is referred to section 3 below.

For the statistical data of the example, legal damages on average over the observed event amount to

$$d(r^n, \Omega_1) = \frac{7/10 - 4/10}{7/10} \cdot 100 \approx 42,86$$

if windfall gains can be ruled out entirely.

At the other extreme, finally, where all accidents that occur if A has met his obligation are of the windfall type, i.e.  $\pi(\Omega_{01}) = \varepsilon(r^o)$ , legal damages on average over the observed event amount to  $d(r^n, \Omega_1) = L$ , well in line with the traditional negligence rule as pioneered by Brown (1973).

The present analysis uncovers legal damages on average over the observed event as the unifying doctrine behind all these rules. It is the probability of windfall gains caused by A's neglecting his obligation that makes the difference.

The general extensive form of the accident model consists of three stages. At stage 0, nature is choosing the type  $t \in T$  of the case from a (finite) set of alternatives. Type t is chosen with probability  $\mu_t$  where  $\sum_{t \in T} \mu_t = 1$ . At stage 1, party A decides between neglecting  $(r^n)$  and meeting his obligation  $(r^o)$ . At stage 2, suppose the case is of type t. Then the accident causing harm to B of fixed size L occurs with probability  $\varepsilon_t^n$  if A has neglected his obligation and with probability  $\varepsilon_t^o$  if A has met it. The situation is referred to as one of pure uncertainty provided that a single type exists whereas, if there are several types, uncertainty is called type-contingent.

The general extensive form of the accident model is consistent with the statistical data if

$$\sum_{t\in T} \mu_t \cdot \varepsilon_t^n = \varepsilon(r^n) \text{ and } \sum_{t\in T} \mu_t \cdot \varepsilon_t^o = \varepsilon(r^o)$$

both hold. The probability of windfall gains amounts to

$$\pi(\Omega_{01}) = \sum_{t \in T} \mu_t \cdot (1 - \varepsilon_t^n) \cdot \varepsilon_t^o$$

and vanishes only under the following condition. For all types t, either the accident is avoided for sure if A has met his obligation, i.e.  $\varepsilon_t^o = 0$  or, if it still occurs with positive probability, then the accident must occur for sure if A neglects his obligation, i.e.  $\varepsilon_t^n = 1$ . As soon as at least one type violates this condition, i.e.  $\varepsilon_t^o > 0$  and  $\varepsilon_t^n < 1$  for some type t then windfall gains occur with positive probability.

### **3** Observable signals

In the previous section, it was assumed to be observable whether an accident has occurred or not. The analysis is now extended to include more general events. To this end, signals are introduced as functions  $Q: R \times \Omega \to \rho_Q$  that map any combination of party A's decision and random move of nature into a range  $\rho_Q$  of observable signals. Suppose, by deciding  $r^n \neq r^o$ , party A has neglected his obligation. If signal  $q \in \rho_Q$  shows up then it is the event

$$\Omega^q = \{ \omega \in \Omega : Q(r^n, \omega) = q \}$$

that can be observed. Legal damages on average over this event amount to

$$d(r^n, \Omega^q) = \frac{\pi(\Omega^q \cap \Omega_{10})}{\pi(\Omega^q)} \cdot L$$

whereas if, in addition, it is observed that an accident has actually occurred then legal damages on average over this event amount to

$$d(r^n, \Omega^q \cap \Omega_1) = \frac{\pi(\Omega^q \cap \Omega_{10})}{\pi(\Omega^q \cap \Omega_1)} \cdot L.$$

Notice the events  $\Omega_{ij}$  are defined as in the previous section. In particular, it holds that  $\Omega_{10} \subset \Omega_1 = \Omega_{10} \cup \Omega_{11}$  and, hence,  $d(r^n, \Omega^q) \leq d(r^n, \Omega^q \cap \Omega_1)$ .

If legal damages are granted on average over such events, it follows from the Hand Formula and Proposition 1 in combination with Bayes' rule that

$$\sum_{q \in \rho_Q} \pi(\Omega^q) \cdot d(r^n, \Omega^q) - \pi(\Omega_1) \cdot L \ge -\pi(\Omega_{11} \cup \Omega_{01}) \cdot L$$

and

$$c(r^n) + \sum_{q \in \rho_Q} \pi(\Omega^q) \cdot d(r^n, \Omega^q) > c(r^o)$$

must both hold. The term on the right of the first inequality is party B's expected loss which B must bear if A has met his obligation and, for that reason, escapes liability. The term on the left is B's net position if A has neglected his obligation and owes legal damages on average over the observed events to B. The first inequality then establishes that party B is at least as well off as if A had met his obligation. The second inequality shows that, as a consequence of the Hand Formula, party A still has the incentive to meet his obligation.

The setting including observable signals allows to capture the model examined by Shavell (1985) as well as the case of the negligent coach from the introduction of the present paper. To capture Shavell's model, consider the range  $\rho_Q = \{0, A, N, L\}$  of potential signals with the following interpretation in mind. If signal q = 0 shows up no accident has occurred whereas an accident has occurred if any of the other three signals is observed. If q = A then, by assumption, the accident is known to be caused by party A while it is caused by nature (the natural agent in Shavell's words) if signal q = N occurs. If, however, signal q = L is observed the accident remains of ambiguous origin.

Implicitly at least, Shavell rules out windfall gains, i.e.  $\pi(\Omega_{01}) = 0$ . Moreover, in my notation, the accident is said to be caused by A and by nature in the events  $\Omega_{10}$  and  $\Omega_{11}$ , respectively. Shavell denotes the probabilities of these events by  $\pi(\Omega_{10}) = p$  and  $\pi(\Omega_{11}) = n$ . As a final piece of notation, Shavell introduces the probabilities

$$\alpha = \text{prob} \{ \Omega^L \cap \Omega_{10} \mid \Omega_{10} \} \text{ and } \beta = \text{prob} \{ \Omega^L \cap \Omega_{11} \mid \Omega_{11} \}$$

which denote the conditional probabilities of an accident caused by party A and nature, respectively, appearing to be of ambiguous origin.<sup>6</sup>

Given these statistical data, it follows that the probabilities of an accident caused by party A and nature appears to be of ambiguous origin amount to  $\pi\{\Omega^L \cap \Omega_{10}\} = \alpha \cdot p$  and  $\pi\{\Omega^L \cap \Omega_{11}\} = \beta \cdot n$ , respectively and, hence, legal damages on average over the event that an accident of ambiguous origin has occurred amount to

$$d(r^n, \Omega^L) = \frac{\pi \{ \Omega^L \cap \Omega_{10} \}}{\pi \{ \Omega^L \}} \cdot L = \frac{\alpha \cdot p}{\alpha \cdot p + \beta \cdot n} \cdot L,$$

which is equivalent to Shavell's proportionality rule. If, however, party A is observed as the origin of the accident then legal damages on average over this event of size L are granted, i.e.  $d(r^n, \Omega^A) = L$  whereas A does not owe any damages if the accident is known to be caused by the natural agent, i.e.  $d(r^n, \Omega^N) = 0$ . In this sense, Shavell's rule can be interpreted quite generally as legal damages on average over observable events.

<sup>&</sup>lt;sup>6</sup>Notice the game tree (b) in Figure 1 corresponds th Shavell's model for the parameters  $\alpha = \beta = 1$ .

To examine the case of the negligent coach in the present framework, the range  $\rho_Q = \{P, F\}$  of observable signals must be considered with the interpretation that signals q = P and q = F occur if student B has passed and failed the exam, respectively. If it is just observed whether party B has passed or failed, legal damages on average over these events amount to

$$d(r^n, \Omega^P) = \frac{\pi(\Omega^P \cap \Omega_{10})}{\pi(\Omega^P)} \cdot L \text{ and } d(r^n, \Omega^F) = \frac{\pi(\Omega^F \cap \Omega_{10})}{\pi(\Omega^F)} \cdot L_{2}$$

respectively. If, in addition, it is observed that party B has actually suffered a loss, legal damages amount to

$$d(r^n, \Omega^P \cap \Omega_1) = \frac{\pi(\Omega^P \cap \Omega_{10})}{\pi(\Omega^P \cap \Omega_1)} \cdot L \text{ and } d(r^n, \Omega^F \cap \Omega_1) = \frac{\pi(\Omega^F \cap \Omega_{10})}{\pi(\Omega^F \cap \Omega_1)} \cdot L,$$

respectively. Notice, legal damages on average over the event that party B has passed the exam are positive if, and only if the probability  $\pi(\Omega^P \cap \Omega_{10})$  that the student passes the exam, still suffers from the loss of income but would suffer from no such loss if the coach had met his obligation is positive. If damages were denied whenever the student has passed the exam in spite of the coach's negligent teaching, incentives may be distorted as the following example of pure uncertainty illustrates.

Let  $L^P = \eta^P \cdot L < L^F = \eta^F \cdot L$  denote the average loss of income of students who have passed and failed the exam, respectively, as compared to the maximum income level. Under pure uncertainty, these averages are assumed to depend just on the observed signal though not on the quality of coach A's teaching. The probabilities  $\gamma^o > \gamma^n$  that a student fails the exam, however, depend on whether the coach has met his obligation or not, i.e. whether he has decided  $r^o$  or  $r^n$ . Notice, under pure uncertainty, all students are assumed to be of the same capabilities such that passing or failing the exam is entirely due to the teaching quality and nature's trembles.

The probabilities of passing and failing the exam and, at the same time, suffering from a loss amount to

$$\pi(\Omega^P \cap \Omega_1) = (1 - \gamma^n) \cdot \eta^P$$
 and  $\pi(\Omega^F \cap \Omega_1) = \gamma^n \cdot \eta^F$ 

if the coach has neglected his obligation. The probabilities that these losses occur and are caused by A's negligence amount to

$$\pi(\Omega^P \cap \Omega_{10}) = (1 - \gamma^n) \cdot \eta^P \cdot \left[1 - \gamma^o \cdot \eta^F - (1 - \gamma^o) \cdot \eta^P\right]$$

and

$$\pi(\Omega^F \cap \Omega_{10}) = \gamma^n \cdot \eta^F \cdot \left[1 - \gamma^o \cdot \eta^F - (1 - \gamma^o) \cdot \eta^P\right].$$

It follows that, under pure uncertainty, legal damages on average over the event that an accident has occurred and the student fails or passes the exam are identical, i.e.

$$d(r^n, \Omega^P \cap \Omega_1) = d(r^n, \Omega^F \cap \Omega_1) = \left[1 - \gamma^o \cdot \eta^F - (1 - \gamma^o) \cdot \eta^P\right] \cdot L$$

and, hence, are both positive if students enjoying careful coaching can expect to avoid losses with positive probability.

For illustration, take the statistical data presented in the introduction  $(\gamma^n = 6/10, \gamma^o = 3/10, L = 100, L^P = 20 \text{ and } L^F = 70)$ . Then legal damages on average over the observed event amount to

$$d(r^n, \Omega^P \cap \Omega_1) = d(r^n, \Omega^F \cap \Omega_1) = 100 - \frac{3}{10} \cdot 70 - \frac{7}{10} \cdot 20 = 65$$

and are positive indeed.

If damages were denied whenever the student has passed the exam in spite of negligent coaching, the student would remain at least as well off as if the coach had met his obligation and, hence, would have avoided liability for sure if the inequality

$$\pi(\Omega^F \cap \Omega_{10}) - \pi(\Omega_{10} \cup \Omega_{11}) \ge -\pi(\Omega_{01} \cup \Omega_{11})$$

or, equivalently, the inequality

$$\pi(\Omega_{01}) \ge \pi(\Omega^P \cap \Omega_{10})$$

were met. In other words, to still ensure compensation of the student who is denied recovery whenever he passes the exam, the probability of windfall gains caused by negligent coaching would have to be sufficiently high. Otherwise incentives of the coach may be distorted downwards even if the Hand Formula is met.

### 4 General distributions of asset values

More often than not, the actual value of assets affected by an accident and, a fortiori, their value in the hypothetical situation where the injurer had met his obligation is uncertain. To deal with such cases, the accident model is extended to allow for losses of variable size.

The potential injurer is still facing a decision  $r \in R$  at costs c(r) that affects the value of party B's assets. The uncertain value is from the (finite) range  $\rho_B$ . The distribution of values depends on A's decision. Let f(b,r)denote the probability that B's assets attain the value  $b \in \rho_B$  provided that A has decided  $r \in R$ . These distributions play the role of the statistical data of a case for the present extension of the accident model. Probabilities sum up to one, i.e.  $\sum_{b \in \rho} f(b, r) = 1$  and the expected value of B's assets as a function of A's decision amounts to

$$\beta(r) = \sum_{b \in \rho_B} f(b, r) \cdot b.$$

Suppose it is A's obligation to meet the standard  $r^o \in R$  but A is actually neglecting his obligation and decides  $r^n \neq r^o$  instead. The Hand Formula

$$\beta(r^n) - c(r^n) < \beta(r^o) - c(r^o)$$

is assumed to be met. What damages if any will be due?

Kahan (1989) has considered a version of the accident model that specifies the expected loss L(r) as a deterministic function of precaution. He argues that the injurer's liability for accidents caused by his negligence would be the difference  $L(r^n) - L(r^o)$ . Since losses have the meaning of values with a negative sign, such liability could equivalently be defined as  $\beta(r^o) - \beta(r^n)$ . It will be later shown that Kahan's interpretation captures legal damages if taken on average over the appropriate event.

To deal with the hypothetical value of B's assets, it proves useful again to consider the interaction between nature and party A in normal form. The outcome space, from which nature is choosing at random, is still denoted by  $\Omega$ . In extension of the accident technology, the value of B's assets resulting from A's decision  $r \in R$  and nature's move  $\omega \in \Omega$  amounts to  $B(r, \omega) \in \rho_B$ . The normal form is consistent with the statistical data of the extended model if

$$\pi\{\omega\in\Omega:B(r,\omega)=b\}=f(b,r)$$

holds for all  $b \in \rho_B$  and  $r \in R$ .

If the hypothetical move of nature were observable, the legally correct quantum of liability would be obvious: Victim B could recover that part of harm only, which is caused by A's neglecting his obligation, i.e. the difference  $B(r^o, \omega) - B(r^n, \omega)$ . Occasionally, this difference may happen to be negative, in which case party B would be enjoying a windfall gain caused by A's negligence. According to common legal practice, B may keep such windfall gains for free. As a consequence, the quantum of legal damages would amount to

$$D(r,\omega) = \max[B(r^o,\omega) - B(r^n,\omega), 0]$$

Yet, the move of nature may not be observable. It may only be known, that the actual move of nature must belong to some event  $\Omega' \subset \Omega$ . Average legal damages then amount to the expected value of true legal damages conditional on the observed event, i.e. to

$$d(r, \Omega') = E[D(r, \omega) \mid \Omega'].$$

Notice, with variable loss size, the observable event may contain, at the same time, moves of nature, under which B suffers harm, as well as others, under which he enjoys windfall gains due to A's negligence. For such events, potential windfall gains may be offset against losses over the observed event. Legal damages reflecting this rule are denoted by

$$\Delta(r, \Omega') = E[B(r^o, \omega) - B(r, \omega) \mid \Omega'].$$

More precisely, since even this term may happen to be negative, legal damages amount to

$$\delta(r, \Omega') = \max\left[\Delta(r, \Omega'), 0\right]$$

if windfall gains are offset against losses over the observed event. Comparing the two measures,

$$0 \le \delta(r, \Omega') \le d(r, \Omega')$$

most obviously hold for any decision and event. Notice, while Bayes' rule applies for  $d(r, \Omega')$  and  $\Delta(r, \Omega')$ , the damage measure  $\delta(r, \Omega')$  need not obey this rule any more.

From the incentive perspective, it does not matter, which version of the rule is taken. In fact, consider any partition  $\Omega = \Omega^1 \cup ... \cup \Omega^i \cup ... \cup \Omega^I$  of the outcome space into observable events. Then the following proposition holds.

**Proposition 2** In expected terms, party B is at least as well off as if A had met his obligation and, hence, would not owe any damages to B, i.e.

$$\beta(r^n) + \sum_{i=1}^{I} \pi(\Omega^i) \cdot d(r^n, \Omega^i) \ge \beta(r^n) + \sum_{i=1}^{I} \pi(\Omega^i) \cdot \delta(r^n, \Omega^i) \ge \beta(r^o)$$

whereas party A has the incentive to meet his obligation as

$$c(r^n) + \sum_{i=1}^{I} \pi(\Omega^i) \cdot d(r^n, \Omega^i) \ge c(r^n) + \sum_{i=1}^{I} \pi(\Omega^i) \cdot \delta(r^n, \Omega^i) > c(r^o)$$

holds, no matter whether windfall gains are offset against losses over the observed event or not.

**Proof.** In fact, by definition, it follows from Bayes' rule that

$$\sum_{i=1}^{I} \pi(\Omega^{i}) \cdot \delta(r^{n}, \Omega^{i}) \geq \sum_{i=1}^{I} \pi(\Omega^{i}) \cdot \Delta(r^{n}, \Omega^{i}) = \beta(r^{o}) - \beta(r^{n})$$

must hold, from which the first claim of the proposition easily follows. By making use of the Hand Formula, it then follows that

$$c(r^n) + \sum_{i=1}^{I} \pi(\Omega^i) \cdot \delta(r^n, \Omega^i) \ge c(r^n) + \beta(r^o) - \beta(r^n) > c(r^o)$$

must hold, which establishes the second claim of the proposition.  $\blacksquare$ 

If the data of the extended model arise from test series of cases that differ only by trembles of nature taking place after A has chosen the level of precaution, the situation is again referred to as pure uncertainty. To simplify notation, the statistical data, i.e. the distribution of asset values under the two levels are denoted by  $f^n(b) = f(b, r^n)$  and  $f^o(b) = f(b, r^o)$ , respectively.

Suppose the event  $\Omega' = \{\omega \in \Omega : Q(r^n, \omega) = b'\}$  is observed. This means that the actual value b' of B's assets is known but their hypothetical value  $b^o$  if A had met his obligation remains uncertain. Under pure uncertainty, this event occurs with probability f(b') and legal damages on average over the event  $\Omega'$  amount to

$$d(r, \Omega') = \sum_{b^o \in \rho_B} f^o(b^o) \cdot \max[b^o - b', 0]$$

or, if windfall gains are offset against losses, to  $\delta(r, \Omega') = \max[\Delta(r, \Omega'), 0]$ where

$$\Delta(r, \Omega') = \sum_{b^o \in \rho_B} f^o(b^o) \cdot (b^o - b') = \beta(r^o) - b'.$$

As the second version is more easy to grasp, I focus on the rule where windfall gains are offset against losses. In this case, the actual value b' must be compared to the hypothetical expected value of the assets. The difference if positive corresponds to legal damages on average over the observed event provided that windfall gains are offset against losses.

If, however, the actual value of B's assets under the negligent behavior of A remains uncertain as well, then legal damages on average over this event amount to

$$\delta(r^n, \Omega) = \Delta(r^n, \Omega) = \beta(r^o) - \beta(r^n)$$

and are equal to Kahan's (1989) rule. Notice, for this equivalence to be true, the actual value of B's assets must remain uncertain and windfall gains must be offset against losses over the whole outcome space  $\Omega$ .

The following numerical example illustrates the alternative methods. The value of B's assets potentially attains the three levels  $b^0 < b^1 < b^2$  with probabilities  $f^n(b^0) = 1/4$ ,  $f^n(b^1) = 1/2$  and  $f^n(b^2) = 1/4$  if A has neglected his obligation and with probabilities  $f^o(b^0) = 1/6$ ,  $f^o(b^1) = 1/3$  and  $f^o(b^2) = 1/2$  if A had met his obligation. The expected value of B's assets amounts to

$$\beta(r^n) = b^0 + \frac{3}{4} \cdot (b^1 - b^0) + \frac{1}{4} \cdot (b^2 - b^1)$$

and to

$$\beta(r^{o}) = b^{0} + \frac{5}{6} \cdot (b^{1} - b^{0}) + \frac{1}{2} \cdot (b^{2} - b^{1}),$$

respectively. Notice that the actual expected value is lower than the hypothetical expected value because

$$\beta(r^{o}) - \beta(r^{n}) = \frac{1}{12} \cdot (b^{1} - b^{0}) + \frac{1}{4} \cdot (b^{2} - b^{1}) > 0$$

obviously holds.

Suppose the actual value of the affected assets under negligent behavior is  $b' = b^1$ . In this event it follows that average legal damages are

$$d(r^n, \Omega') = \frac{1}{2} \cdot (b^2 - b^1) > \delta(r^n, \Omega') = \frac{1}{2} \cdot (b^2 - b^1) - \frac{1}{6} \cdot (b^1 - b^0)$$

if windfall gains are kept for free or offset against losses over the event, respectively..

Under pure uncertainty, party B will enjoy hypothetical windfall gains with positive probability. A priory theories on possible chains of causation, however, may rule out such windfall gains. If they do, the statistical data cannot be interpreted as pure uncertainty any more. Rather, the test series from which these data are derived must cover situations that differ also by type. If, under negligent behavior, the actual value of B's assets is known, such information allows to update beliefs about the distribution of types. Since the extension to type-contingent uncertainty is straightforward, details are not presented in the paper.

## 5 Multiple injurers

For many practical cases of uncertain causation, more than one potential injurer will be involved. If an accident occurs and several parties have neglected their obligations, two questions must be settled. First, what quantum of damages if any is granted to the victim and, second, how should the negligent injurers share this quantum? As it turns out, even if the accident model is known in normal form and the move of nature were observable, new conceptual issues arise.

To discuss them, the accident model is extended as follows. The class of potential injurers is denoted by a = 1, ..., A. Party a is facing a decision  $r_a \in R_a$  and bears costs  $c_a(r_a)$ . Profiles of decisions are denoted by

$$r = (r_1, \dots, r_a, \dots, r_A) \in R = R_1 \times \dots \times R_a \times \dots \times R_A$$

and total costs by

$$c(r) = \sum_{a=1}^{A} c_a(r_a).$$

For simplicity, losses if they occur are assumed to be of fixed size L. At profile  $r \in R$ , accidents are assumed to occur with probability  $\varepsilon(r)$ . Loss size L and probabilities  $\varepsilon(r)$  are referred to as the statistical data of the case.

Suppose it is party *a*'s obligation to decide  $r_a^o \in R_a$ . The profile  $r^o = (r_1^o, ..., r_A^o) \in R$  of obligations is assumed to satisfy the Hand Formula

$$c(r^{o}) + \varepsilon(r^{o}) \cdot L \le c(r) + \varepsilon(r) \cdot L$$

for any other profile  $r \in R$ .

The actual decision is denoted by profile  $r^n = (r_1^n, ..., r_A^n)$ . At least some though not necessarily all of the A parties are assumed to having neglected their obligations, i.e.  $r^n \neq r^o$ . Let  $A^n = \{a : r^n_a \neq r^o_a\}$  denote the set of negligent parties.

To express the accident model with several injurers in normal form, the accident technology  $e: R \times \Omega \to \{0, 1\}$  is now defined for any combination of a decision profile with a move of nature. Suppose an accident has actually occurred, i.e.  $e(r^n, \omega) = 1$ . Suppose it also would have occurred even if all parties had met their obligations, i.e.  $e(r^o, \omega) = 1$ . Then party B could not recover his loss. Therefore, in what follows, let me assume that the accident would have been avoided if all had met their obligations, i.e.  $e(r^o, \omega) = 0$ . To determine legal damages, the question must be addressed whose party's deviation has caused the accident.

Suppose the move of nature is observable and, for simplicity, just two candidates are involved. If

$$e(r_1^o, r_2^n, \omega) = 0 < e(r_1^n, r_2^o, \omega) = 1$$

holds then party 1 has obviously caused the accident. In fact, if just party 2 had kept his obligation, the accident would still have occurred whereas it would have been avoided even if only party 1 had met his obligation. Similarly, if  $e(r_1^o, r_2^n, \omega) = 1 > e(r_1^n, r_2^o, \omega) = 0$  then it was party 2's deviation, which has caused the accident.

Yet, even with just two candidates, additional combinations may arise. First, if

$$e(r_1^o, r_2^n, \omega) = e(r_1^n, r_2^o, \omega) = 1$$

then the accident were only avoided if both parties had jointly met their obligations and, second, if

$$e(r_1^o, r_2^n, \omega) = e(r_1^n, r_2^o, \omega) = 0$$

then the accident would have been avoided it either of them had met his obligation. For these two combinations, the occurrence of an accident cannot be attributed to a single injurer and, hence, the rule, according to which the two parties should share total damages owed to B, cannot be derived from principles of one-party-causation.

Since I am not aware of legal principles that fully settle the above issue, I rather consider the class of all damage rules which satisfy two basic principles. First, the victim is granted damages that, in expected terms, makes the

victim at least as well off as if all potential injurers had met their obligations. Second, by meeting his obligation, an injurer can unilaterally escape liability and, hence, damages granted to the victim must be shared by those parties that have actually neglected their obligations.

In formal terms, suppose  $\Omega = \Omega^1 \cup ... \cup \Omega^i \cup ... \Omega^I$  is a partition of the outcome space into observable events  $\Omega^i$ . Let  $d_a(r, \Omega^i)$  denote damages owed by party a to party B if the event  $\Omega^i$  is observed and let

$$\delta(r) = \sum_{a=1}^{A} \sum_{i=1}^{I} \pi(\Omega^{i}) \cdot d_{a}(r, \Omega^{i})$$

denote total damages granted to B in expected terms, both at actual decision profile r. Then, as the first principle requires,

$$\delta(r) \ge [\varepsilon(r) - \varepsilon(r^o)] \cdot L$$

must hold for any decision profile  $r \in R$  and, according to the second principle,

$$d_a(r_a^o, r_{-a}, \Omega^i) = 0$$

must hold where  $r_{-a} = (r_1, ..., r_{a-1}, r_{a+1}, ..., r_A)$  denotes the decisions of all parties except a. The following proposition establishes that meeting all obligations is a Nash equilibrium of the game among the potential injurers. Moreover, even if the parties a = 1, ..., A (or at least some of them) would be able to collude they could not improve their joint situation as the following proposition establishes. The proposition holds for any damage rule that obeys the above two basic principles.<sup>7</sup>

**Proposition 3** If the damage rules satisfies the two basic properties then, by meeting all their obligations, the injurers minimize the sum of precaution costs and damages owed to B, i.e.  $c(r) + \delta(r) \ge c(r^{\circ})$  must hold for all decision profiles  $r \in R$ . Moreover, all parties have the incentive to meet their obligations, i.e.  $r^{\circ}$  is a Nash equilibrium of the game induced by any damage rule satisfying the above two properties.

**Proof.** It follows from the Hand Formula and the property that party B is at least as well off as if all injurers had met their obligations that

 $c(r) + \delta(r) \ge c(r^{o}) + [\varepsilon(r) - \varepsilon(r^{o})] \cdot L + \delta(r) \ge c(r^{o})$ 

<sup>&</sup>lt;sup>7</sup>For a systematic discussion of general multilateral obligations, the reader is referred to Schweizer (2005b).

must hold indeed, which establishes the first claim. In particular, it then follows that  $c(r_a, r_{-a}^o) + \delta(r_a, r_{-a}^o) \ge c(r^o)$  and, hence, that

$$c_a(r_a) + \delta(r_a, r_{-a}^o) \ge c_a(r_a^o)$$

must both hold. The last inequality establishes that meeting the own obligation is a best response to all the other injurer meeting theirs and, hence, the profile  $r^o$  is shown to be a Nash equilibrium.

Notice, two obvious versions of the damage rule would both satisfy the required properties. Either party a owes damages

$$d_a(r^n, \Omega^i) = \frac{c_a(r_a^o) - c_a(r_a^n)}{c(r^o) - c(r^n)} \cdot d(r^n, \Omega^i)$$

in the proportion of individual cost savings or damages owed to B are shared equally among negligent injurers, i.e.

$$d_a(r^n, \Omega^i) = \frac{1}{\#A^n} \cdot d(r^n, \Omega^i).$$

In terms of efficiency, both methods of sharing liabilities would provide incentives to meet all obligations and be coalition-proof as follows from the above proposition.

The following numerical example from the introduction may illustrate the findings summarized by Proposition 3. Suppose two parties a = 1, 2 are candidates for having caused harm of fixed size L = 100 to B. If both had met their obligations the accident would still have occurred with probability  $\varepsilon(r^o) = 2/10$  whereas if both have neglected their obligations then the probability of an accident would be  $\varepsilon(r^n) = 7/10$ . If just one of the two candidates had neglected his obligation then the probability of an accident is  $\varepsilon(r_1^o, r_2^n) = 3/10$  and  $\varepsilon(r_1^n, r_2^o) = 5/10$ , respectively. To meet the two basic principles, party 2 owes damages in expected terms  $\delta(r_1^o, r_2^n) \ge 10$  whereas party 1 owes damages in expected terms  $\delta(r_1^n, r_2^o) \ge 30$  to B if just one of them has neglected his obligation. If both of them have neglected their obligations they jointly owe  $\delta(r^n) \ge 50$  to B in expected terms. With respect to incentives, it does not matter how the two candidates have to share damages  $\delta(r^n) \ge 50$ .

#### 6 Concluding remarks

By definition, estimating legal damages requires to compare the actual, possibly uncertain, value of assets with their hypothetical value that would have prevailed if the debtor had met his obligation. If the interaction between debtor and nature were known in normal form and the move of nature could be observed, comparing actual and hypothetical value would be straightforward. If just the normal form were known but the move of nature remains hidden, true legal damages could still be determined though on average over the observed event only. The present paper shows that awarding legal damages on average over observable events provides efficient incentives.

The traditional accident model assumes the probability of an accident to be a function of the debtor's decision and harm, if it occurs, to be of fixed size. In general, different versions of the accident model in normal form may be consistent with the same data of a traditional accident model. Worse, average legal damages may well be different for different versions of the normal form consistent with the same data. Only if the probability of an accident in case the debtor has met his obligation is negligible, average legal damages are equal to the full harm, no matter which normal form. Otherwise, additional hypotheses about potential chains of causation are needed to determine legal damages on average over observable events.

Throughout the paper it was assumed that the debtor knows his obligation at the time of his decision and that courts can verify beyond any doubt whether a debtor has met or neglected his obligation. It is an interesting topic of future research to give up some of these assumptions. It seems particularly worthwhile to examine the case where the debtors' decisions are actions that remain hidden to courts. While there exists a vast literature on the hidden action problem, the question of interest would be whether legal practice can be interpreted as if it were implementing some of the findings from the hidden action literature.

#### 7 References

Brown, J. P. (1973), "Toward an Economic Theory of Liability", Journal of Legal Studies 2, 323 - 349.

- Kahan, M. (1989): "Causation and incentives to take care under the negligence rule", Journal of Legal Studies, 18, 427 - 447.
- Myerson, R. B. (1999), "Nash equilibrium and the history of economic theory", Journal of Economic Literature XXXVII, 1067 - 1082.
- Schweizer, U. (2005 a), "Law and Economics of Obligations," International Review of Law and Economics 25, 209 – 228.
- Schweizer, U. (2005 b), "The pure theory of multilateral obligations", Journal of Institutional and Theoretical Economics (JITE) 161, 239 254...
- Shavell, S. (1985): "Uncertainty over causation and the determination of civil liability", The Journal of Law and Economics 28, 587 609.



## Figure 1: pure vs. type-contingent uncertainty