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## Rollover risk as market discipline: A two-sided inefficiency

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# Rollover Risk as Market Discipline: A Two-Sided Inefficiency

Thomas M. Eisenbach

Staff Report No. 597  
February 2013



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## **Rollover Risk as Market Discipline: A Two-Sided Inefficiency**

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JEL classification: G01, G21, G24, G32

### **Abstract**

Why does the market discipline that banks face seem too weak during good times and too strong during bad times? This paper shows that using rollover risk as a disciplining device is effective only if all banks face purely idiosyncratic risk. However, if banks' assets are correlated, a two-sided inefficiency arises: Good aggregate states have banks taking excessive risks, while bad aggregate states suffer from fire sales. The driving force behind this inefficiency is an amplifying feedback loop between asset liquidation values and market discipline. This feedback loop operates in both good and bad aggregate states, but with opposite effects.

Key words: rollover risk, market discipline, bank runs, fire sales, global games

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# 1 Introduction

The use of short-term debt by banks and the resulting rollover risk were prominent features of the financial crisis of 2007–2009. Besides providing liquidity services, the maturity mismatch of banks’ balance sheets can be viewed as playing a disciplining role to address the bankers’ incentive problems (Calomiris and Kahn, 1991; Diamond and Rajan, 2001). Historically, this role was associated with the depositors of commercial banks but in today’s more market-based system of financial intermediation the role can be extended to banks’ (and shadow banks’) creditors in wholesale funding markets (Adrian and Shin, 2010).

The experience leading up to and during the crisis, however, calls into question the effectiveness of short-term debt as a disciplining device: On the one hand, the increasing reliance on short-term debt in the years before the crisis went hand-in-hand with exceedingly risky activities on and off financial institutions’ balance sheets (Admati, DeMarzo, Hellwig, and Pfleiderer, 2010). On the other hand, the run on short-term funding at the heart of the recent crisis was indiscriminate and effectively delivered a “collective punishment,” shutting down the issuers of securities backed not only by real estate loans but also by entirely unrelated assets such as student loans (Gorton and Metrick, 2012). As Carey, Kashyap, Rajan, and Stulz (2012) point out:

“Market discipline” is a commonly suggested method of promoting stability and efficiency. Many studies find evidence that it pushes prices and quantities in the “right” direction in the cross section. [...] Casual observation suggests that market discipline is “too weak” during credit booms and asset price bubbles, and “too strong” after crashes. True? If so, why? Is there a role for policy action? What are the implications for supervision and regulation?

This paper addresses these questions in a general equilibrium model of banks choosing their reliance on short-term debt. The maturity mismatch between assets and liabilities generates rollover risk, which I model using global game techniques. Bankers use the rollover risk as a disciplining device since they face a basic risk-shifting problem. The model shows that this form of market discipline can only be effective and achieve the first-best allocation if banks face purely idiosyncratic risk. When banks face aggregate risk from correlated assets, however, a two-sided inefficiency arises: Good aggregate

states have banks taking excessive risks in projects with negative net present value. Bad aggregate states suffer from fire sales as projects with positive net present value are liquidated.

The driving force behind this inefficiency are amplifying feedback loops between asset liquidation values and market discipline. With correlation in banks' assets, good aggregate states imply good news about the average bank's assets, increasing bank stability. Creditors become complacent, which weakens market discipline. Since not many banks are forced to liquidate assets, liquidation values are inflated. This increases bank stability further, feeding back into even weaker market discipline. In contrast, bad aggregate states imply bad news about the average bank's assets, reducing bank stability and unnerving creditors. Market discipline is strengthened, forcing many banks to liquidate and depressing asset liquidation values. This reduces bank stability further, feeding back into even stronger market discipline. The result of these feedback loops is inefficiently weak market discipline – with inflated asset values and excessive risk taking – in good states and inefficiently strong market discipline – with depressed asset values and excessive liquidation – in bad states.

The model has several implications for regulation and policy interventions. In choosing how much to rely on short-term debt, banks already trade off the two inefficiencies; the resulting allocation is constrained efficient. Any policy to reduce reliance on short-term debt, while decreasing the fire-sale inefficiency of downturns would at the same time increase the risk-taking inefficiency of booms – the overall effect on welfare would be negative. There is, however, scope for welfare improvement by adding an extra element of state contingency to the market discipline. Ideally, banks' exposure to rollover risk should be tailored to each aggregate state. This can be achieved to a degree, for example by adding to the capital structure a layer of contingent-convertible debt with a trigger based on the aggregate state. Alternatively, the state contingency can originate in central bank interventions with broadly targeted support of liquidation values during times of stress. This relaxes the trade-off banks face between the fire-sale inefficiency and the risk-taking inefficiency, improving overall welfare. Finally, regulation can try to address the correlation in banks' assets that is at the heart of the inefficiency. More diversification of risks across banks would result in less volatility in liquidation values and less amplification, thereby reducing the inefficiency.

**Related Literature:** The events of the recent crisis have generated a large body of literature.<sup>1</sup> The realization of rollover risk as the dry-up of short-term funding is well documented for the asset-backed commercial paper market (Covitz, Liang, and Suarez, 2012; Kacperczyk and Schnabl, 2010) and the market for repurchase agreements (Copeland, Martin, and Walker, 2011; Gorton and Metrick, 2012). This has inspired theoretical work on the mechanisms underlying rollover risk in market-based funding, highlighting the fragility of the collateral assets' debt capacity (Acharya, Gale, and Yorulmazer, 2011) or separating the contributions of liquidity concerns and solvency concerns (Morris and Shin, 2010). The main difference in my paper is that it takes an ex-ante perspective in a general equilibrium setting and highlights inefficient risk taking in good states as the mirror image of inefficient fire sales in bad states.

The role of short-term debt as a disciplining device has been discussed in a literature going back to Calomiris and Kahn (1991).<sup>2</sup> This literature commonly features a beneficial disciplining effect that comes at the cost of inefficient liquidation. The choice of maturity structure then has to trade off the two effects. My paper differs, first, in that liquidation is not per se inefficient and, second, in the distinction between two sources of risk. In particular, my paper has an efficient benchmark outcome if only idiosyncratic risk is present. The novel inefficiency arises because of the inability of the disciplining mechanism to deal with two sources of risk.<sup>3</sup>

My paper is also related to Shleifer and Vishny (1992) who study the interaction of debt as a disciplining device with endogenous liquidation values. In their model, disciplining is only necessary in the good state and liquidation always happens at a (potentially inefficient) discount in the bad state. The focus of their paper is how equilibrium liquidation values limit debt capacity. In my model, assets are always sold to outsiders but not necessarily at a discount. More importantly, the incentive problem in my model is present in all aggregate states. Therefore, the optimal maturity structure has to trade off the two inefficiencies of too much liquidation in one state and too little liquidation in another state.

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<sup>1</sup>For overviews of the events see, e.g. Brunnermeier (2009) and Gorton (2008).

<sup>2</sup>See, e.g. Rajan (1992), Leland and Toft (1996) and Diamond and Rajan (2001). For a recent approach with interesting dynamic effects see Cheng and Milbradt (2012). The literature on control rights has similar themes, e.g. Aghion and Bolton (1989).

<sup>3</sup>Another recent paper on maturity structure choice is Brunnermeier and Oehmke (2011). Their model does not have a disciplining problem and the optimal maturity structure is a corner solution of either all short-term or all long-term debt.

Related from a technical point of view are several papers that also use global game techniques to analyze the coordination problem among creditors, notably [Morris and Shin \(2004\)](#), [Rochet and Vives \(2004\)](#) and [Goldstein and Pauzner \(2005\)](#).<sup>4</sup> In my paper, the global game is not as much front and center but rather used as a convenient modeling device. Under weak assumptions, the global game has a unique equilibrium and this equilibrium has continuous comparative statics. This allows studying an ex-ante stage where the maturity structure is chosen optimally, taking into account the effect on the global-game equilibrium at a later stage. Finally, since the global game itself is restricted to a single time period, I avoid the complications in dynamic global games pointed out by [Angeletos, Hellwig, and Pavan \(2007\)](#).

In the following, Section 2 lays out the model and discusses the important features. Section 3 considers the situation of an individual bank, deriving the endogenous rollover risk in Subsection 3.1 and comparing the case without aggregate risk in Subsection 3.2 to the case with aggregate risk in Subsection 3.3. Section 4 analyzes the general equilibrium with many banks and highlights the amplification leading to the two-sided inefficiency. Finally, Section 5 concludes with a discussion of policy implications.

## 2 Model

Time is discrete and there are three periods  $t = 0, 1, 2$ . There is a continuum of banks  $i \in [0, 1]$ , each with the opportunity to invest in a project, and a continuum of investors  $j \in [0, 1]$ . All agents are risk neutral with a discount rate of zero.

**Project:** Bank  $i$ 's project requires an investment of 1 in the initial period  $t = 0$  and has a random payoff in the final period  $t = 2$  given by  $X > 1$  with probability  $\theta_i$  and 0 otherwise. In the interim period  $t = 1$ , the project can still be abandoned and any fraction of its assets can be sold off to alternative uses at a liquidation value of  $\ell < 1$ . At the time of investment in  $t = 0$ , there is uncertainty about both the project's expected payoff  $\theta_i X$  as well as the liquidation value  $\ell$ , which is not resolved until additional

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<sup>4</sup>The global game approach originates with [Carlsson and van Damme \(1993b,a\)](#). [Kurlat \(2010\)](#) studies the trade-off between disciplining and inefficient liquidation using a global game setting. In a related model not using a global game setup, [He and Xiong \(2012\)](#) study the inter-temporal coordination problem among creditors with different maturity dates and derive very similar comparative statics.

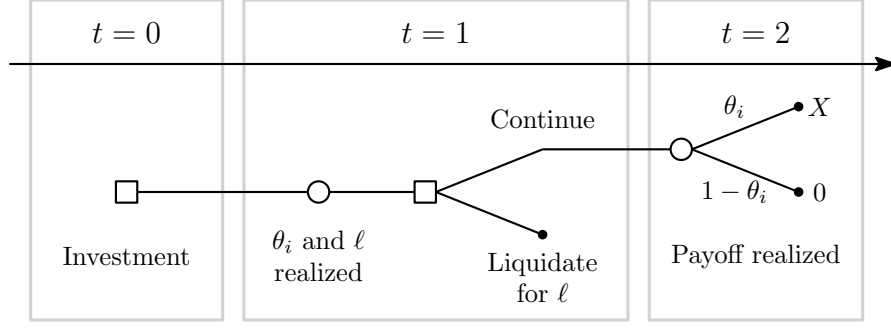


Figure 1: Project time-line for bank  $i$

information becomes available in the interim period  $t = 1$ . The structure of bank  $i$ 's project and its time-line is illustrated in Figure 1.

Importantly, in  $t = 1$  the liquidation value  $\ell$  is not directly linked to the expected payoff  $\theta_i X$  of bank  $i$ 's project. It helps to think of the project as a loan to a borrower against collateral. Over time, the bank learns more about its borrower's repayment probability  $\theta_i$  and can foreclose the loan and sell the collateral. While the value of the collateral depreciates over the course of the two-period loan – first from 1 to  $\ell$ , then from  $\ell$  to 0 – it is not directly linked to the idiosyncratic repayment probability of the borrower. This has the important implication that liquidation is not inherently inefficient. Efficiency requires that a project be abandoned and that its assets be liquidated whenever the expected payoff  $\theta_i X$  turns out to be less than the liquidation value  $\ell$  and vice versa:

$$\begin{aligned}\theta_i X \leq \ell &\Rightarrow \text{abandon} \\ \theta_i X > \ell &\Rightarrow \text{continue}\end{aligned}$$

**Incentive Problem:** A bank financed at least partially with debt faces a basic incentive problem when it comes to continuing or liquidating its project, similar to the risk-shifting problem of [Jensen and Meckling \(1976\)](#). Suppose that in the initial period  $t = 0$  a bank has  $\eta \in [0, 1]$  of equity and raises  $1 - \eta$  in some form of debt. Denote by  $D_t$  the face value of this debt at  $t = 1, 2$ . After learning about  $\theta_i$  and  $\ell$  in the interim period  $t = 1$ , the bank wants to continue its project whenever the expected equity



payoff from continuing is greater than the equity payoff from liquidating:

$$\begin{aligned} \theta_i (X - (1 - \eta) D_2) &> \max \{0, \ell - (1 - \eta) D_1\} \\ \Leftrightarrow \quad \theta_i &> \begin{cases} 0 & \text{for } 1 - \eta \leq \frac{\ell}{D_1} \\ \frac{\ell - (1 - \eta) D_1}{X - (1 - \eta) D_2} & \text{for } 1 - \eta > \frac{\ell}{D_1} \end{cases} \end{aligned}$$

Unless the bank is fully equity financed ( $\eta = 1$ ), its decision doesn't correspond to the efficient one of continuing if and only if  $\theta_i > \ell/X$ . In particular, as long as  $D_1 X > D_2 \ell$ , i.e.  $X$  sufficiently larger than  $\ell$ , the bank wants to take excessive risks in the interim period by continuing projects with negative net present value. Since this incentive problem is present for any  $\eta < 1$ , I consider the cleanest case and assume that banks have no initial equity. This assumption abstracts from the choice of leverage to focus purely on the choice of maturity structure.

**Uncertainty:** There are two aggregate states  $s \in \{H, L\}$  in the interim period  $t = 1$ , with probabilities  $p$  and  $1 - p$  for the high and the low state, respectively. Conditional on the aggregate state  $s$ , the banks' success probabilities  $\{\theta_i\}$  are i.i.d. with cumulative distribution function  $F_s$  on  $[0, 1]$ . The difference between the high state and the low state is that the distribution  $F_H$  strictly dominates the distribution  $F_L$  in terms of first-order stochastic dominance:

$$F_H(\theta) < F_L(\theta) \quad \text{for all } \theta \in (0, 1)$$

This means that higher success probabilities are more likely in state  $H$  than in state  $L$  and therefore that banks' projects are positively correlated. Both the aggregate state  $s$  and each individual bank's success probability  $\theta_i$  are realized at the beginning of  $t = 1$ , before the continuation decision about the project, but after the investment decision in  $t = 0$ .

**Liquidation Value:** The liquidation value for the banks' assets is determined endogenously from a downward-sloping aggregate demand for liquidated assets. If assets are liquidated, they are reallocated to an alternative use with decreasing marginal productivity. For a total mass  $\phi \in [0, 1]$  of assets sold off by all banks, this implies a liquidation value  $\ell(\phi)$  given by a continuous and strictly decreasing function  $\ell : [0, 1] \rightarrow [0, 1]$

which corresponds to the assets' marginal product in the alternative use. Due to the exogenous correlation in the banks'  $\theta_i$ s the model generates fluctuations in equilibrium asset sales  $\phi$  across the aggregate states  $H$  and  $L$ . This implies volatility in the endogenous liquidation value with two different values  $\ell_H = \ell(\phi_H)$  and  $\ell_L = \ell(\phi_L)$  in the two states.

We can think of the assets literally being reallocated to a less productive sector as in [Kiyotaki and Moore \(1997\)](#) or [Lorenzoni \(2008\)](#). This interpretation is in line with the evidence of [Sandleris and Wright \(2011\)](#) who show that a large part of the decrease in productivity in financial crises can be attributed to misallocated resources. Alternatively, the reallocation can be interpreted as a move within the financial sector as documented by [He, Khang, and Krishnamurthy \(2010\)](#). In this case, the reallocation can have real effects by influencing risk premia ([He and Krishnamurthy, 2012a,b](#)) or hurdle rates for new investment ([Stein, 2012](#)).<sup>5</sup>

**Financing:** Each bank has to raise the entire investment amount of 1 through loans from competitive investors in  $t = 0$ . A bank can choose any combination of long-term debt and short-term debt to finance its project.<sup>6</sup> Bank  $i$ 's long-term debt matures in the final period  $t = 2$  at a face value of  $B_i$ . Short term debt has to be rolled over in the interim period  $t = 1$  at a face value of  $R_i$  and – if rolled over – matures at a face value of  $R_i^2$  in the final period  $t = 2$ . Instead of rolling over in  $t = 1$ , a short-term creditor has the right to demand payment of  $R_i$ . This creates the possibility of the bank becoming illiquid in  $t = 1$  since it may face more withdrawals from short-term creditors than it can satisfy even by liquidating the entire project.<sup>7</sup>

Denoting by  $\alpha_i \in [0, 1]$  the fraction of bank  $i$ 's project financed by short-term debt, the bank's choice of debt maturity structure in  $t = 0$  is denoted by the combination of short-term and long-term debt  $(\alpha_i, 1 - \alpha_i)$ . The interest rates  $B_i$  and  $R_i$  are determined

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<sup>5</sup>For evidence on the reduced supply of bank lending to the real sector during the financial crisis of 2007–09 see, e.g. [Ivashina and Scharfstein \(2010\)](#), [Bord and Santos \(2011\)](#) or [Adrian, Colla, and Shin \(2012\)](#).

<sup>6</sup>I rule out other forms of financing but there are several different ways to justify debt financing endogenously, see [Innes \(1990\)](#), [DeMarzo and Duffie \(1999\)](#), [Dang, Gorton, and Holmström \(2012\)](#) or [Geanakoplos \(2010\)](#).

<sup>7</sup>The assumption that the short-term interest rate  $R_i$  does not adjust in the interim period isolates the rollover decision as the key margin of adjustment. This is consistent with the evidence of [Copeland, Martin, and Walker \(2011\)](#) who document in the tri-party repo market that lenders simply refused to roll over funding to troubled banks rather than adjusting interest rates.

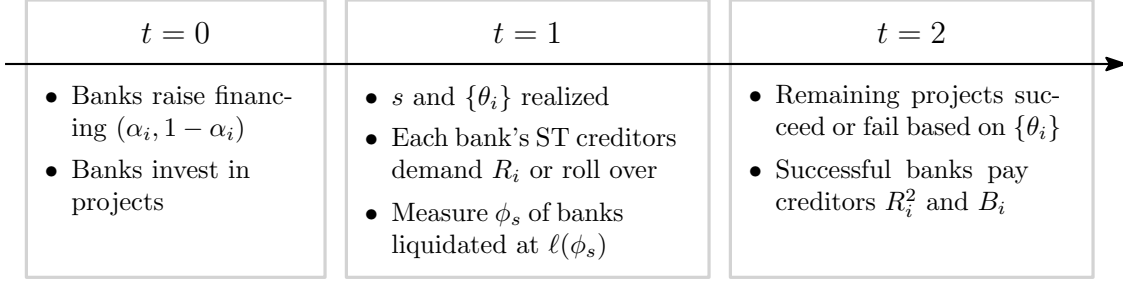


Figure 2: Time-line for the whole economy

endogenously, taking into account both the idiosyncratic and aggregate risk, as well as the the rollover risk arising from the bank's maturity structure.

Figure 2 illustrates the timeline of the whole economy. Since no decisions are made in the final period  $t = 2$ , the first step in solving the model is to analyze the rollover decision of short-term creditors in the interim period  $t = 1$  for given maturity structures  $(\alpha_i, 1 - \alpha_i)$ . The second step is to derive the optimal choice of maturity structure in the initial period  $t = 0$ , taking into account the resulting outcomes in periods  $t = 1, 2$ .

### 3 Individual Bank

I first consider the situation of an individual bank, taking the behavior of other banks and the resulting liquidation value  $\ell$  as given. To reduce notational clutter I drop the bank index  $i$  for now.

#### 3.1 Endogenous Rollover Risk

To solve the model, the first step is to analyze the rollover decision of a bank's short-term creditors in the interim period  $t = 1$ . Denoting the fraction of short-term creditors who withdraw their loans by  $\lambda$ , the bank has to liquidate enough of the project to raise  $\alpha\lambda R$  for repayment. Since the most the bank can raise is by liquidating the entire project for  $\ell$  it will be illiquid whenever  $\lambda > \frac{\ell}{\alpha R}$ .<sup>8</sup>

First, consider the case where the bank remains *liquid*. In this case, short-term creditors who roll over will be repaid  $R^2$  if the project is successful in  $t = 2$ . Given

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<sup>8</sup>A necessary condition for the potential of illiquidity is that  $\alpha R > \ell$ , i.e. a large enough maturity mismatch. Throughout the paper I assume this to be satisfied.

	liquid	illiquid
roll over	$\theta R^2$	0
withdraw	$R$	$\ell$

Figure 3: Payoffs of short-term creditors

the project's success probability  $\theta$  this implies an expected payoff of  $\theta R^2$  from rolling over. Short-term creditors who withdraw simply receive  $R$  in  $t = 1$ . Next, consider the case where the bank becomes *illiquid*. In this case short-term creditors who roll over receive nothing while those who withdraw receive the proceeds of liquidation  $\ell$ . Figure 3 summarizes the payoffs of short-term creditors.

These stylized payoffs create the classic coordination problem at the heart of panic-based bank runs first analyzed by Bryant (1980) and Diamond and Dybvig (1983). With perfect information about the fundamentals  $\theta$  and  $\ell$  and as long as they are not too bad, i.e.  $\theta > 1/R$ , there are multiple equilibria: If an individual creditor expects all other creditors to roll over and the bank to remain liquid, it is individually rational to roll over as well since  $\theta R^2 > R$ . Everyone rolling over and the bank remaining liquid is therefore an equilibrium. At the same time, if an individual creditor expects all other creditors to withdraw and the bank to become illiquid, it is individually rational to withdraw as well since  $\ell > 0$ . Everyone withdrawing and the bank becoming illiquid is therefore also an equilibrium.

From a modeling perspective this indeterminacy is somewhat of a mixed blessing, often resulting in the assumption that a run only happens when it is the only equilibrium (Allen and Gale, 1998; Diamond and Rajan, 2000). For the payoffs in Figure 3 this corresponds to the case of very bad fundamentals ( $\theta < 1/R$ ) where withdrawing is a dominant strategy and the multiplicity disappears with only the run equilibrium remaining. However, many elements of financial regulation and emergency policy measures are rooted in the belief that panic-based runs are a real possibility. Goldstein (2010) discusses the empirical evidence and points out that a clean distinction between fundamentals and panic is impossible since the worse the fundamentals, the more likely panic-based runs are.

In this paper, I therefore use the global game approach to resolve the bank-run multiplicity in a way most similar to Goldstein and Pauzner (2005). This has two

key advantages: First, it delivers a unique perfect Bayesian equilibrium outcome for the creditor game played in  $t = 1$  that is based entirely on the realization of the fundamentals  $\theta$  and  $\ell$ . Second, the implied ex-ante rollover risk is well-defined and responds continuously to the key choice variable in  $t = 0$ , the bank's maturity structure  $\alpha$ .

Instead of perfect and symmetric information, each short-term creditor has to make the roll-over decision based on slightly noisy and idiosyncratic information. Creditor  $j$  receives a noisy signal  $\tilde{\theta}_j = \theta + \nu_j$  about the true success probability  $\theta$ , where the signal noise terms  $\{\nu_j\}$  are independent and identically distributed uniformly on the interval  $[-\varepsilon, \varepsilon]$  for some arbitrarily small  $\varepsilon > 0$ . In contrast, the resolution of the aggregate state determining the liquidation value  $\ell$  is perfectly observed by everyone and becomes common knowledge.

Without common knowledge about  $\theta$ , standard global-game techniques yield a *unique* equilibrium for the creditors' coordination game.<sup>9</sup> The equilibrium is symmetric in switching strategies around a signal threshold  $\hat{\theta}$  such that each creditor rolls over for all signals above the threshold and withdraws for all signals below. The equilibrium switching point  $\hat{\theta}$  is determined by the fact that for a creditor *exactly at* the switching point the expected payoff from rolling over has to equal the expected payoff from withdrawing. Given the payoffs in Figure 3, the indifference condition for  $\tilde{\theta}_j = \hat{\theta}$  is:

$$\underbrace{\Pr[\text{liquid} \mid \hat{\theta}] \cdot \hat{\theta} R^2}_{\text{rolling over}} = \underbrace{\Pr[\text{liquid} \mid \hat{\theta}] \cdot R + \Pr[\text{illiquid} \mid \hat{\theta}] \cdot \ell}_{\text{withdrawing}} \quad (1)$$

The main uncertainty faced by an individual creditor is about the fraction  $\lambda$  of other creditors who withdraw since this determines if the bank remains liquid or becomes illiquid. However, if we take the limit as the signal noise  $\varepsilon$  goes to zero, the distribution of  $\lambda$  conditional on being at the switching point  $\hat{\theta}$  conveniently becomes uniform on  $[0, 1]$ . Combined with the fact that the bank remains liquid if and only if  $\lambda \leq \frac{\ell}{\alpha R}$  this means that the indifference condition (1) simplifies to:

$$\frac{\ell}{\alpha R} \cdot \hat{\theta} R^2 = \frac{\ell}{\alpha R} \cdot R + \left(1 - \frac{\ell}{\alpha R}\right) \cdot \ell$$

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<sup>9</sup>For ease of exposition I only sketch the derivation in the main text. See the appendix for the technical details of the global game equilibrium.

Solving for  $\hat{\theta}$  yields the equilibrium switching point:

$$\hat{\theta} = \frac{(1 + \alpha) R - \ell}{R^2} \quad (2)$$

**Proposition 1.** *For  $\varepsilon \rightarrow 0$ , the unique perfect Bayesian equilibrium among short-term creditors is in switching strategies around the threshold  $\hat{\theta}$  as defined by (2). For realizations of  $\theta$  above  $\hat{\theta}$ , all short-term debt is rolled over and the bank remains liquid. For realizations of  $\theta$  below  $\hat{\theta}$ , all short-term debt is withdrawn and the bank becomes illiquid.*

*Proof.* See the appendix. □

The simple structure of the equilibrium highlights three important characteristics of a bank's ex-ante rollover risk, that is *before* the uncertainty about  $\theta$  and  $\ell$  is resolved. This rollover risk is the probability that the bank will suffer a run in the interim period and is given by:

$$\Pr \left[ \theta < \frac{(1 + \alpha) R - \ell}{R^2} \right] \quad (3)$$

First, the rollover risk depends on the fraction of short-term debt  $\alpha$  – both directly as well as indirectly through the endogenous  $R$ . The direct effect is positive: Having a balance sheet that relies more heavily on short-term debt makes the bank more vulnerable to runs since it increases the total amount of withdrawals the bank may face. As will become clear in Lemma 1 below, the overall effect of  $\alpha$  remains positive when taking into account also the effect on  $R$ . By choosing its debt maturity structure, the bank can therefore directly influence its rollover risk.

Second, once the maturity structure is in place, whether the bank suffers a run or not depends on both sources of risk, idiosyncratic and aggregate. Since both  $\theta$  and  $\ell$  in expression (3) are random variables, a run can be triggered by bad news about the project's expected payoff (low  $\theta$ ), or by bad news about the liquidation value (low  $\ell$ ). When deciding whether to roll over, creditors worry about a low  $\theta$  because it means they are less likely to be repaid in  $t = 2$ , should the bank remain liquid. In addition, they worry about a low  $\ell$  because it means the bank can withstand less withdrawals and is more likely to become illiquid in  $t = 1$ . The worry about  $\theta$  is about future insolvency while the worry about  $\ell$  is about current illiquidity.

Third, the two sources of risk interact in determining the bank's rollover risk. In particular, the bank is more vulnerable to idiosyncratic risk for a low realization of the liquidation value. The destabilizing effect of a low liquidation value means that the bank suffers runs for idiosyncratic news that would have left it unharmed had the liquidation value been higher. If the liquidation value fluctuates with the aggregate state, a bank will be more vulnerable to runs in the low aggregate state than in the high aggregate state, for any given ex-ante maturity structure. This effect will play a crucial role in the inefficiency result of this paper.

### 3.2 Efficiency without Aggregate Risk

The second step in the backwards induction is to derive the bank's choice of maturity structure in the initial period  $t = 0$ . To establish the efficiency benchmark, I start with the case of no aggregate risk, that is the distribution of success probabilities is the same across states,  $F_H = F_L =: F$  and the liquidation value is constant,  $\ell_H = \ell_L =: \ell$ .

In the initial period  $t = 0$ , short-term and long-term creditors as well as the bank anticipate what will happen in the following periods. This means that the face values of short-term debt and long-term debt,  $R$  and  $B$  respectively, have to guarantee that investors break even. The bank, when choosing its debt maturity structure  $(\alpha, 1 - \alpha)$ , takes into account the effect of  $\alpha$  on the face values  $R$  and  $B$ , as well as on the rollover risk from the global-game equilibrium in  $t = 1$ .

Given the equilibrium threshold  $\hat{\theta}$  as defined by (2), the break-even constraints for the bank's creditors take a simple form:

$$\begin{aligned} \text{Short-term creditors: } & F(\hat{\theta}) \ell + \int_{\hat{\theta}}^1 \theta R^2 dF(\theta) = 1 \\ \text{Long-term creditors: } & F(\hat{\theta}) \ell + \int_{\hat{\theta}}^1 \theta B dF(\theta) = 1 \end{aligned} \tag{4}$$

For realizations of  $\theta$  below  $\hat{\theta}$ , all short-term creditors refuse to roll over and there is a run on the bank in  $t = 1$ . In this case – which happens with probability  $F(\hat{\theta})$  – the bank has to liquidate all its assets and each creditor receives an equal share of the liquidation proceeds  $\ell$ . For realizations of  $\theta$  above  $\hat{\theta}$ , all short-term creditors roll over and the bank continues to operate the project. In this case, the creditors receive the face value of their loan – the compounded short-term  $R^2$  and the long-term  $B$  – but

only if the project is successful in  $t = 2$  which happens with probability  $\theta$ .

The ex-ante expected payoff of the bank can be derived in a similar way. For realizations  $\theta \leq \hat{\theta}$  there is a run by short-term creditors in the interim period and the bank's payoff is zero. For realizations  $\theta > \hat{\theta}$  there is no run in  $t = 1$  and with probability  $\theta$  the project is successful in  $t = 2$ . In this case the bank receives the project's cash flow  $X$  and has to repay its liabilities  $\alpha R^2 + (1 - \alpha) B$ . The bank's expected payoff therefore is:

$$\int_{\hat{\theta}}^1 \theta (X - \alpha R^2 - (1 - \alpha) B) dF(\theta)$$

Substituting in the values for  $R$  and  $B$  required by the break-even constraints, the bank's payoff becomes:

$$F(\hat{\theta}) \ell + \int_{\hat{\theta}}^1 \theta X dF(\theta) - 1 \quad (5)$$

Due to the rational expectations and the competitive creditors, the bank receives the entire economic surplus of its investment opportunity, given the rollover-risk threshold  $\hat{\theta}$ . The first term in (5) is the economic value realized in the states where the project is liquidated; the second term is the expected economic value realized in the states where the project is continued; the third term is the initial cost of investment. Since it receives the entire economic surplus, the bank fully internalizes the effect of its maturity structure choice on the efficiency of the rollover outcome.

Before analyzing the bank's maturity structure choice, one complication remains: The critical value  $\hat{\theta}$  derived from the rollover equilibrium in  $t = 1$  depends on the short-term interest rate  $R$ . This interest rate in turn is set in  $t = 1$  by the break-even condition which anticipates the rollover threshold  $\hat{\theta}$ . Therefore equations (2) and (4) jointly determine  $\hat{\theta}$  and  $R$  for a given  $\alpha$ . Our variable of interest is the rollover threshold  $\hat{\theta}$  and how it depends on the ex-ante choice of  $\alpha$ , taking into account the endogeneity of  $R$ .

**Lemma 1.** *Equations (2) and (4) implicitly define the interim rollover threshold  $\hat{\theta}$  as a function of the ex-ante maturity structure  $\alpha$ . The mapping  $\hat{\theta}(\alpha)$  is one-to-one and satisfies  $d\hat{\theta}/d\alpha > 0$ .*

*Proof.* See the appendix. □

This lemma establishes the direct link between  $\hat{\theta}$  and  $\alpha$ . In choosing its maturity structure  $\alpha$ , the bank effectively chooses a rollover-risk threshold  $\hat{\theta}(\alpha)$ ; the more short-



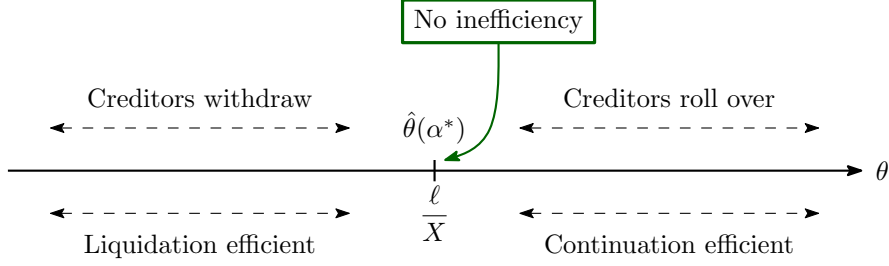


Figure 4: Implemented and efficient rollover risk without aggregate risk

term debt the bank takes on in  $t = 0$ , the higher is the rollover risk it faces in  $t = 1$ . The following proposition characterizes the optimal choice of the bank maximizing its expected payoff (5) subject to the link between maturity structure and rollover risk.

**Proposition 2.** *Without aggregate risk, the bank chooses an optimal maturity structure  $\alpha^*$  that implements the efficient liquidation policy:*

$$\hat{\theta}(\alpha^*) = \frac{\ell}{X}$$

*Proof.* See the appendix. □

The bank uses short-term debt as a disciplining device to implement a liquidation threshold  $\hat{\theta}$  maximizing its payoff. Since the payoff corresponds to the project's full economic surplus, the bank's objective is the same as a social planner's. In the case without aggregate risk, subjecting itself to the market discipline of rollover risk allows the bank to overcome its incentive problem and achieve the first-best policy. Depending on the project's expected payoff after observing  $\theta$ , the first-best policy requires either to continue with the project or to abandon it and put the liquidated assets to alternative use. Continuation is efficient whenever the project's expected payoff is greater than the liquidation value,  $\theta X > \ell$ , and liquidation is efficient whenever  $\theta X < \ell$ . Figure 4 illustrates how the bank uses market discipline to implement the first-best policy. Creditors roll over – allowing the project to continue – for  $\theta > \ell/X$  and withdraw – forcing the project to be liquidated – for  $\theta < \ell/X$ , exactly as required for efficiency. However, this efficiency breaks down in the case with aggregate risk discussed next.

### 3.3 Inefficiency with Aggregate Risk

I now analyze the model with aggregate risk. With probability  $p$  the state is high,  $s = H$ , which means that the success probability is drawn from the distribution  $F_H$  and that the liquidation value is  $\ell_H$ . With probability  $1 - p$  the state is low,  $s = L$ , with distribution  $F_L$  and liquidation value  $\ell_L$ . State  $H$  is the “good” state since  $F_H$  first-order stochastically dominates  $F_L$  and since  $\ell_H > \ell_L$ .

The uncertainty in liquidation values has two main implications for the bank. The first implication is that the first-best policy whether to continue or liquidate the project is affected by the realization of  $\ell$ . For the low liquidation value  $\ell_L$  the project should only be continued if  $\theta X > \ell_L$ , while for the high liquidation value  $\ell_H$  the condition is  $\theta X > \ell_H$ . There are now two cutoffs for the project’s expected payoff: the bar for  $\theta X$  to justify continuing is higher in state  $H$  than in state  $L$ . This means that for realizations of the project’s success probability  $\theta$  in the interval  $[\frac{\ell_L}{X}, \frac{\ell_H}{X}]$ , efficiency calls for liquidation if the assets have a high liquidation value and for continuation if the assets have a low liquidation value.

The second implication of aggregate risk is that the creditor coordination game is different depending on the aggregate state. There are now two equilibrium switching points,  $\hat{\theta}_H$  and  $\hat{\theta}_L$ , one for each realization of  $\ell$ :

$$\hat{\theta}_H = \frac{(1 + \alpha) R - \ell_H}{R^2} \quad \text{and} \quad \hat{\theta}_L = \frac{(1 + \alpha) R - \ell_L}{R^2}$$

If the liquidation value is high, each creditor is less concerned about the other creditors withdrawing their loans and therefore more willing to roll over than when the liquidation value is low. Therefore, the bank will be more stable and less likely to suffer a run by its short-term creditors if the liquidation value is high, which is reflected in the rollover-risk threshold being lower:

$$\hat{\theta}_H < \hat{\theta}_L$$

As in the case without aggregate risk, the bank receives the entire economic surplus of its project, given the liquidation resulting from its maturity structure:

$$p \left( F_H(\hat{\theta}_H) \ell_H + \int_{\hat{\theta}_H}^1 \theta X dF_H(\theta) \right) + (1 - p) \left( F_L(\hat{\theta}_L) \ell_L + \int_{\hat{\theta}_L}^1 \theta X dF_L(\theta) \right) - 1$$

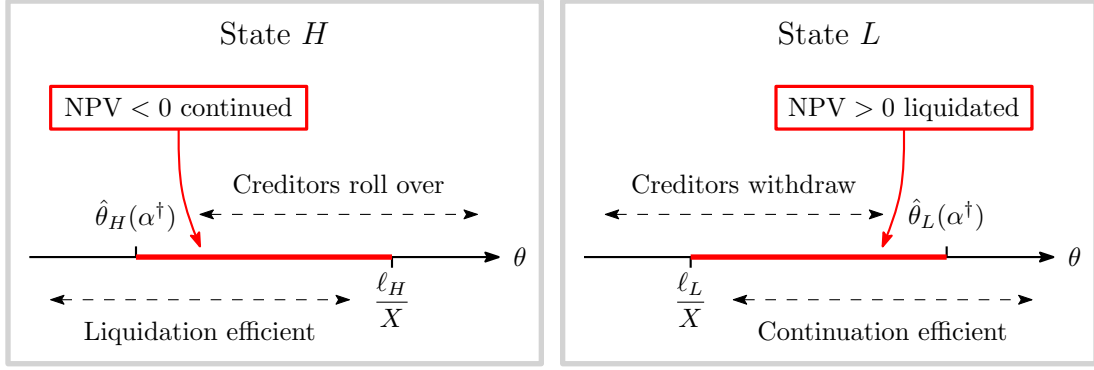


Figure 5: Two-sided inefficiency with aggregate risk

The bank again chooses a maturity structure  $\alpha$  to maximize its expected payoff, now taking into account the effect it has on the two rollover thresholds  $\hat{\theta}_H(\alpha)$  and  $\hat{\theta}_L(\alpha)$ .

**Proposition 3.** *With aggregate risk the bank chooses an optimal maturity structure  $\alpha^\dagger$  resulting in a two-sided inefficiency:*

$$\hat{\theta}_H(\alpha^\dagger) < \frac{\ell_H}{X} \quad \text{and} \quad \hat{\theta}_L(\alpha^\dagger) > \frac{\ell_L}{X}$$

For  $s = H$ , negative-NPV projects are continued whenever  $\theta \in (\hat{\theta}_H(\alpha^\dagger), \frac{\ell_H}{X})$  while for  $s = L$ , positive-NPV projects are liquidated whenever  $\theta \in (\frac{\ell_L}{X}, \hat{\theta}_L(\alpha^\dagger))$ .

*Proof.* See the appendix. □

The key effect of aggregate risk is that it drives a wedge between the efficient liquidation policy and any achievable liquidation policy. The effectiveness of using the maturity structure to eliminate the incentive problem and to implement an efficient liquidation policy is undermined when aggregate risk is added to the bank's idiosyncratic risk. It is important to note that there are efficiency losses for both realizations of the liquidation value, as illustrated in Figure 5. In state  $H$ , when the liquidation value is high, excessively risky projects that should be liquidated because they have negative net present value are continued. In state  $L$  on the other hand, when the liquidation value is low, valuable projects that should be continued because they have positive net present value are liquidated at fire-sale prices.

The two-sided inefficiency comes from the ambivalent role played by the liquidation value of the bank's assets. A high liquidation value makes the bank less vulnerable to

runs but at the same time, the high liquidation value raises the bar in terms of alternate uses for the bank's assets which worsens the incentive problem. Exactly the opposite happens in bad aggregate states where the liquidation value is low. This means that the market-discipline effect of short-term debt is weak in the states where it is needed more and is strong in the states where it is needed less.

## 4 General Equilibrium and Amplification

After focusing on the situation of an individual bank that takes liquidation values as given, I now derive the general equilibrium with a unit measure of banks where liquidation values are determined endogenously. Specifically, the liquidation value depends on the total mass  $\phi \in [0, 1]$  of assets sold off by all banks and is given by  $\ell(\phi)$  with  $\ell'(\phi) < 0$ .

**Without Aggregate Risk:** It is instructive to start with the case of no aggregate risk. The two equations that jointly define the critical value  $\hat{\theta}$  as a function of the maturity structure  $\alpha$ , the indifference condition (2) and the break-even constraint (4), both depend on the liquidation value  $\ell$  which is now a function of aggregate asset sales  $\phi$ . Writing this relationship as  $\hat{\theta}(\alpha, \phi)$  makes clear the dependence of the implemented rollover risk on both the individual bank's  $\alpha$  as well as the aggregate  $\phi$ . A competitive bank's optimization as characterized in Proposition 2 takes the value of  $\phi$  as given, resulting in the maturity structure  $\alpha^*(\phi)$  and the implemented threshold  $\hat{\theta}(\alpha^*(\phi), \phi)$ .

All banks are identical ex ante, so the competitive equilibrium is symmetric with  $\alpha_i^* = \alpha_j^*$  for all banks  $i, j$ . Given that there is a unit measure of banks and that the success probabilities  $\{\theta_i\}$  are i.i.d., the aggregate mass  $\phi$  of assets sold in  $t = 1$  is equal to the fraction of banks with realizations  $\theta_i \leq \hat{\theta}(\alpha^*(\phi), \phi)$  who experience a run by their short-term creditors and have to liquidate their assets. The competitive equilibrium value  $\phi^{\text{CE}}$  is therefore given by a fixed point:

$$\phi^{\text{CE}} = F\left(\hat{\theta}(\alpha^*(\phi^{\text{CE}}), \phi^{\text{CE}})\right)$$

**With Aggregate Risk:** The case with aggregate risk is only slightly more complicated. There is now a value of  $\phi$  for each aggregate state denoted by the vector

$\Phi = [\phi_H, \phi_L]$ . Both critical values  $\hat{\theta}_H$  and  $\hat{\theta}_L$  depend on both values of  $\phi$ , as well as the choice of  $\alpha$ :  $\hat{\theta}_H(\alpha, \Phi)$  and  $\hat{\theta}_L(\alpha, \Phi)$ . Given the optimization of an individual bank characterized in Proposition 3 the competitive equilibrium is again given by a fixed point:

$$\Phi^{\text{CE}} = \left[ F_H\left(\hat{\theta}_H(\alpha^\dagger(\Phi^{\text{CE}}), \Phi^{\text{CE}})\right), F_L\left(\hat{\theta}_L(\alpha^\dagger(\Phi^{\text{CE}}), \Phi^{\text{CE}})\right) \right] \quad (6)$$

To highlight the amplification effect the two-sided inefficiency has, the competitive equilibrium allocation has to be compared to the first-best allocation. Using the efficient liquidation thresholds, the first-best allocation has asset sales  $\Phi^{\text{FB}}$  implicitly defined by:

$$\Phi^{\text{FB}} = \left[ F_H\left(\frac{\ell(\phi_H^{\text{FB}})}{X}\right), F_L\left(\frac{\ell(\phi_L^{\text{FB}})}{X}\right) \right] \quad (7)$$

By first order stochastic dominance,  $F_H(\theta) < F_L(\theta)$  for any  $\theta \in (0, 1)$  so the first-best allocation satisfies  $\phi_H^{\text{FB}} < \phi_L^{\text{FB}}$  and therefore  $\ell(\phi_H^{\text{FB}}) > \ell(\phi_L^{\text{FB}})$ . This means that even in the first-best allocation there is volatility in liquidation values across aggregate states. However, the competitive equilibrium amplifies this volatility through an inefficiency feedback.

**Proposition 4.** *In the competitive equilibrium with aggregate risk and  $\phi_H^{\text{CE}} < \phi_L^{\text{CE}}$  the two-sided inefficiency creates a feedback effect by amplifying the volatility in liquidation values:*

$$\ell(\phi_H^{\text{CE}}) > \ell(\phi_H^{\text{FB}}) \quad \text{and} \quad \ell(\phi_L^{\text{CE}}) < \ell(\phi_L^{\text{FB}})$$

*Proof.* See the appendix. □

The two-sided inefficiency originates in the fact that the liquidation values vary across aggregate states which is true even in the first-best allocation. Then the inefficiency drives a wedge between the optimal and the implementable policy which is self-reinforcing as illustrated in Figure 6. In state  $H$  the initial good news that the aggregate distribution of projects is  $F_H$  increases average bank stability. This makes short-term creditors relatively placid and weakens market discipline. Fewer banks are forced to liquidate and liquidation values are inflated. The high liquidation values in turn feed back into increased bank stability, further weakening market discipline and so on. The result of this feedback in the good state is the prevalence of excessively risky projects. The opposite happens in state  $L$ : Bad news about projects reduce average

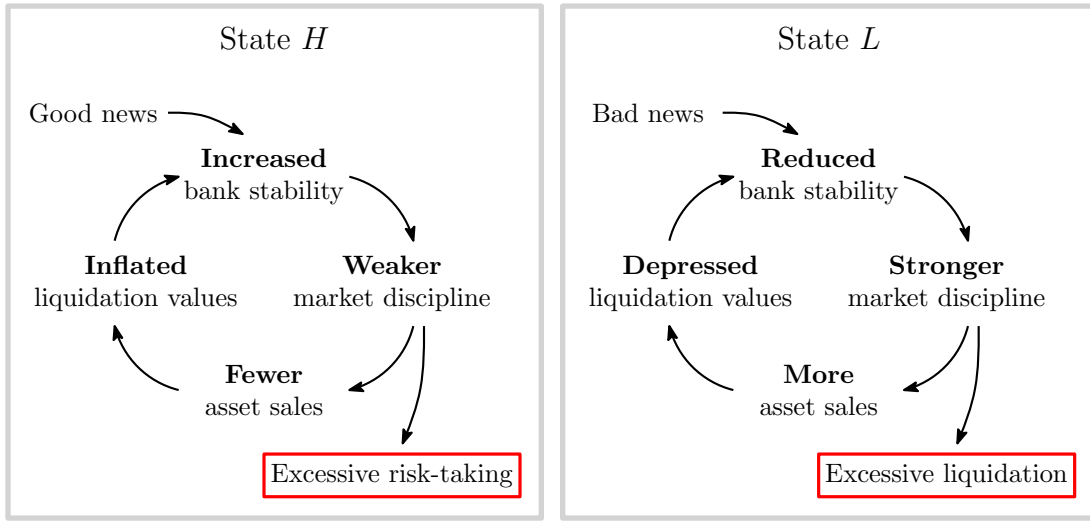


Figure 6: Amplification in both aggregate states

bank stability; short-term creditors become nervous, strengthening market discipline; more banks are forced to fire-sell their assets which depresses liquidation values; finally, fire-sale conditions in asset markets feed back into reduced bank stability which further tightens market discipline and so on. The result of this feedback loop in the bad state is excessive liquidation of good projects.

## 5 Concluding Discussion

This paper provides a theoretical foundation for the stylized fact that the market discipline exerted by banks' short-term debt seems too weak during good times and too strong during bad times. The key mechanism in the model is that a bank's rollover risk is decreasing in the liquidation value of its assets. This is a very basic comparative static with a strong intuition: When creditors decide whether to roll over their loans in a situation where illiquidity is a concern, their decision will depend on how vulnerable the bank is. The main factor determining the bank's vulnerability is how many withdrawals it can satisfy by liquidating assets before it runs out of funds. Therefore higher liquidation values means less jittery creditors which means lower rollover risk. If liquidation values vary across aggregate states, so will the rollover risk a bank faces.

When considering policy implications, it is important to note that the competitive equilibrium is constrained efficient. Since each bank maximizes the economic surplus

of its investment opportunity it has the same objective function as a social planner who is constrained to choosing a debt-maturity structure. The banks fully internalize the effect of their maturity structure when trading off the inefficiencies in the two aggregate states. Therefore, a policy intervention such as a tax on the use of short-term debt would reduce efficiency. It would lead to an increase of the inefficiency due to excessive risk-taking which would more than outweigh the reduction of the inefficiency due to fire sales. Given the wedge between what is efficient and what is achievable when choosing a debt-maturity ex ante, the model points to a lack of choosing state contingency in the exposure to rollover risk ex post. This issue can be addressed if the debt burden that has to be rolled over is higher in good states than in bad states. Adding a layer of contingent-convertible debt to the capital structure of banks has the desired effect if the conversion trigger is based on an aggregate state variable, e.g. stress in bank funding markets.<sup>10</sup> This model also lends support to central banks' direct interventions in the short-term funding markets. By partially substituting for dried-up lending, this policy effectively supports liquidation values, thereby preventing some of the inefficient liquidation. Interestingly, the state-contingency of such a policy – if anticipated ex ante – may also reduce the risk-taking inefficiency in good states since it relaxes the trade-off banks face between the two inefficiencies ex ante. Finally, the inefficiency mechanism in this paper is provides an argument for reducing correlation in banks' assets. This would reduce the volatility in liquidation values and move the allocation towards the first-best with only idiosyncratic risk. A regulatory charge based on a measure such as CoVaR ([Adrian and Brunnermeier, 2011](#)) would have such an effect.

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<sup>10</sup>The proposal for a “lockbox” of liquidity reserves that is tied to a systemic trigger by [Kashyap, Rajan, and Stein \(2008\)](#) is another interesting mechanism that would introduce state contingency.

## Appendix

**Proof of Proposition 1:** To apply the standard global games results summarized by [Morris and Shin \(2003\)](#) the payoffs have to satisfy certain properties. Using the payoffs of short-term creditors in Figure 3, the payoff difference between withdrawing and rolling over is:

$$\Delta(\lambda, \theta) = \begin{cases} R - \theta R^2 & \text{for } \lambda \leq \frac{\ell}{\alpha R} \\ \ell & \text{for } \lambda > \frac{\ell}{\alpha R} \end{cases}$$

This payoff difference is monotone in  $\theta$  (state monotonicity) and there is a unique  $\theta^*$  that solves  $\int_0^1 \Delta(\lambda, \theta) d\lambda = 0$  (strict Laplacian state monotonicity). In terms of limit dominance, for  $\theta < 1/R$  we have  $\Delta(\lambda, \theta) > 0$  for all  $\lambda$  (lower dominance region). Taking the approach of [Goldstein and Pauzner \(2005\)](#) I assume that for sufficiently high  $\theta$  the bank cannot become illiquid, e.g. because the project matures early and pays off  $X$  for sure. This implies an upper dominance region  $(\bar{\theta}, 1]$  such that for  $\theta > \bar{\theta}$  we have  $\Delta(\lambda, \theta) < 0$  for all  $\lambda$ . The payoff difference  $\Delta(\lambda, \theta)$  is not monotone in  $\lambda$  but it satisfies the following single-crossing property: For each  $\theta$  there exists a  $\lambda^* \in \mathbb{R} \cup \{-\infty, +\infty\}$  such that  $\Delta(\lambda, \theta) < 0$  for all  $\lambda < \lambda^*$  and  $\Delta(\lambda, \theta) > 0$  for all  $\lambda > \lambda^*$ . In addition, the signal about  $\theta$  with uniform noise satisfies the monotone likelihood ratio property. Given all these properties, there is a unique equilibrium and it is in symmetric switching strategies around a critical value  $\hat{\theta}$ .<sup>11</sup>

In equilibrium, a creditor with signal  $\tilde{\theta}_j = \hat{\theta}$  has to be indifferent between rolling over and withdrawing:

$$\Pr[\lambda \leq \frac{\ell}{\alpha} \mid \hat{\theta}] \cdot E[\theta \mid \hat{\theta}] \cdot R^2 = \Pr[\lambda \leq \frac{\ell}{\alpha} \mid \hat{\theta}] \cdot R + \Pr[\lambda > \frac{\ell}{\alpha} \mid \hat{\theta}] \cdot \ell \quad (8)$$

Given the signal structure, for a particular realization  $\theta$  the distribution of signals is uniform on  $[\theta - \varepsilon, \theta + \varepsilon]$  and for a particular signal realization  $\tilde{\theta}$  the conditional

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<sup>11</sup>See Lemma 2.3 and the following discussion in [Morris and Shin \(2003\)](#) as well as Theorem 1 in [Goldstein and Pauzner \(2005\)](#) for details.



distribution of  $\theta$  is

$$f(\theta|\tilde{\theta}) = \begin{cases} \frac{f(\theta)}{F(\tilde{\theta}+\varepsilon)-F(\tilde{\theta}-\varepsilon)} & \text{for } \theta \in [\tilde{\theta}-\varepsilon, \tilde{\theta}+\varepsilon], \\ 0 & \text{otherwise.} \end{cases}$$

With the distribution of  $\theta|\hat{\theta}$  we have an expression for  $E[\theta|\hat{\theta}]$  in condition (8) so it remains to derive the distribution of  $\lambda|\hat{\theta}$ . We can derive the corresponding c.d.f.  $G(\lambda|\hat{\theta})$  as follows: The probability that a fraction less than  $\lambda$  receives a signal less than  $\hat{\theta}$  (and therefore withdraws) equals the probability that  $\theta$  is greater than the cutoff  $\theta'$  defined by

$$\begin{aligned} \frac{\hat{\theta} - (\theta' - \varepsilon)}{2\varepsilon} &= \lambda \\ \Rightarrow \theta' &= \hat{\theta} + \varepsilon - 2\varepsilon\lambda \end{aligned}$$

We therefore have

$$\begin{aligned} G(\lambda|\hat{\theta}) &= 1 - F(\hat{\theta} + \varepsilon - 2\varepsilon\lambda|\hat{\theta}) \\ &= 1 - \int_{\hat{\theta}-\varepsilon}^{\hat{\theta}+\varepsilon-2\varepsilon\lambda} \frac{f(\theta)}{F(\hat{\theta}+\varepsilon)-F(\hat{\theta}-\varepsilon)} d\theta \\ &= \frac{F(\hat{\theta}+\varepsilon) - F(\hat{\theta}+\varepsilon-2\varepsilon\lambda)}{F(\hat{\theta}+\varepsilon) - F(\hat{\theta}-\varepsilon)}. \end{aligned}$$

Finally, we have to derive the limits as the signal noise  $\varepsilon$  goes to zero. First, we

have that  $\lim_{\varepsilon \rightarrow 0} E[\theta|\hat{\theta}] = \hat{\theta}$ . Second, we have that

$$\begin{aligned}
\lim_{\varepsilon \rightarrow 0} G(\lambda|\hat{\theta}) &= \lim_{\varepsilon \rightarrow 0} \frac{F(\hat{\theta} + \varepsilon) - F(\hat{\theta} + \varepsilon - 2\varepsilon\lambda)}{F(\hat{\theta} + \varepsilon) - F(\hat{\theta} - \varepsilon)} \\
&= \lim_{\varepsilon \rightarrow 0} \frac{f(\hat{\theta} + \varepsilon) - f(\hat{\theta} + \varepsilon - 2\varepsilon\lambda)(1 - 2\lambda)}{f(\hat{\theta} + \varepsilon) + f(\hat{\theta} - \varepsilon)} \quad \text{by l'Hôpital's rule} \\
&= \lim_{\varepsilon \rightarrow 0} \frac{f(\hat{\theta} + \varepsilon) - f(\hat{\theta} + \varepsilon - 2\varepsilon\lambda)}{f(\hat{\theta} + \varepsilon) + f(\hat{\theta} - \varepsilon)} + \lim_{\varepsilon \rightarrow 0} \frac{2\lambda f(\hat{\theta} + \varepsilon - 2\varepsilon\lambda)}{f(\hat{\theta} + \varepsilon) + f(\hat{\theta} - \varepsilon)} \\
&= \frac{0}{2f(\hat{\theta})} + \frac{2\lambda f(\hat{\theta})}{2f(\hat{\theta})} \\
&= \lambda
\end{aligned}$$

So the distribution of  $\lambda$  conditional on being at the switching point becomes uniform as the signal noise goes to zero. Combining everything, we can rewrite condition (8) as

$$\frac{\ell}{\alpha R} \cdot \hat{\theta} R^2 = \frac{\ell}{\alpha R} \cdot R + \left(1 - \frac{\ell}{\alpha R}\right) \cdot \ell$$

This implies the critical value  $\hat{\theta}$  of Proposition 1.  $\square$

**Proof of Lemma 1:** The two key equations are the indifference condition (IC) and the break-even constraint (BC):

$$\hat{\theta} R^2 = (1 + \alpha) R - \ell \tag{IC}$$

$$F(\hat{\theta}) \ell + \int_{\hat{\theta}}^1 \theta R^2 dF(\theta) = 1 \tag{BC}$$

To show that the mapping between  $\alpha$  and  $\hat{\theta}$  is one-to-one, we first need to show that only one  $\alpha$  implements each  $\hat{\theta}$ . This is straightforward since (IC) is linear in  $\alpha$  and (BC) doesn't depend on  $\alpha$  at all.

Next, we need to show that each  $\alpha$  implements only one  $\hat{\theta}$ . Differentiating the left-hand side of (BC) with respect to  $\hat{\theta}$  without substitution of  $R$  we get:

$$2R \left. \frac{dR}{d\hat{\theta}} \right|_{(IC)} \int_{\hat{\theta}}^1 \theta dF(\theta) - f(\hat{\theta}) (\hat{\theta} R^2 - \ell) \tag{9}$$

Implicit differentiation of (IC) yields:

$$\begin{aligned}
\left. \frac{dR}{d\hat{\theta}} \right|_{(IC)} &= - \frac{R^2}{2\hat{\theta}R - (1 + \alpha)} \\
&= - \frac{R^3}{(1 + \alpha)R - 2\ell} \quad \text{using (IC)} \\
&< 0 \quad \text{since } R \geq \alpha R > \ell
\end{aligned} \tag{10}$$

Again using (IC) we get:

$$\begin{aligned}
\hat{\theta}R^2 - \ell &= (1 + \alpha)R - 2\ell \\
&> 0 \quad \text{since } R \geq \alpha R > \ell
\end{aligned} \tag{11}$$

Using (10) and (11) implies that the expression (9) is strictly negative. Therefore each  $\alpha$  implements only one  $\hat{\theta}$  and we can conclude that the mapping between  $\alpha$  and  $\hat{\theta}$  is one-to-one.

Finally we need to show that  $d\hat{\theta}/d\alpha > 0$ . Implicit differentiation of (BC) yields:

$$\frac{d\hat{\theta}}{d\alpha} = - \frac{2R \left. \frac{dR}{d\alpha} \right|_{(IC)} \int_{\hat{\theta}}^1 \theta dF(\theta)}{2R \left. \frac{dR}{d\hat{\theta}} \right|_{(IC)} \int_{\hat{\theta}}^1 \theta dF(\theta) - f(\hat{\theta})(R^2\hat{\theta} - \ell)} \tag{12}$$

The denominator is equal to expression (9) which we have already established is negative. Implicit differentiation of (IC) yields:

$$\begin{aligned}
\left. \frac{dR}{d\alpha} \right|_{(IC)} &= \frac{R}{2\hat{\theta}R - (1 + \alpha)} \\
&> 0 \quad \text{analogous to } \left. \frac{dR}{d\hat{\theta}} \right|_{(IC)} \text{ in (10)}
\end{aligned}$$

Therefore the nominator in (12) is positive and we can conclude that  $d\hat{\theta}/d\alpha > 0$ .  $\square$

**Proof of Proposition 2:** Given Lemma 1 this result is straightforward. The bank solves the following problem:

$$\max_{\alpha} \left\{ F(\hat{\theta}(\alpha)) \ell + \int_{\hat{\theta}(\alpha)}^1 \theta X dF(\theta) - 1 \right\}$$

The first order condition to this problem is:

$$f(\hat{\theta}(\alpha)) \hat{\theta}'(\alpha) (\ell - \hat{\theta}(\alpha) X) = 0$$

With the properties of  $\hat{\theta}(\alpha)$  established in Lemma 1, this implies the efficient rollover threshold  $\hat{\theta}(\alpha^*) = \ell/X$  which is implemented by an optimal maturity structure  $\alpha^* = \hat{\theta}^{-1}(\ell/X)$ .  $\square$

**Proof of Proposition 3:** The bank solves the following problem:

$$\begin{aligned} \max_{\alpha} \left\{ p \left( F_H(\hat{\theta}_H(\alpha)) \ell_H + \int_{\hat{\theta}_H(\alpha)}^1 \theta X dF_H(\theta) \right) \right. \\ \left. + (1-p) \left( F_L(\hat{\theta}_L(\alpha)) \ell_L + \int_{\hat{\theta}_L(\alpha)}^1 \theta X dF_L(\theta) \right) - 1 \right\} \end{aligned}$$

The first order condition to this problem is:

$$\begin{aligned} p f_H(\hat{\theta}_H(\alpha)) \hat{\theta}'_H(\alpha) (\ell_H - \hat{\theta}_H(\alpha) X) \\ + (1-p) f_L(\hat{\theta}_L(\alpha)) \hat{\theta}'_L(\alpha) (\ell_L - \hat{\theta}_L(\alpha) X) = 0 \end{aligned}$$

Since  $\ell_H > \ell_L$  and  $\hat{\theta}_H(\alpha) < \hat{\theta}_L(\alpha)$  the first order condition implies that for the optimal maturity structure  $\alpha^\dagger$  we have:

$$\ell_H - \hat{\theta}_H(\alpha^\dagger) X > 0 \quad \text{and} \quad \ell_L - \hat{\theta}_L(\alpha^\dagger) X < 0$$

In state  $H$ , projects are inefficiently continued for  $\theta \in (\hat{\theta}_H(\alpha^\dagger), \frac{\ell_H}{X})$ , while in state  $L$ , projects are inefficiently liquidated whenever  $\theta \in (\frac{\ell_L}{X}, \hat{\theta}_L(\alpha^\dagger))$ .  $\square$

**Proof of Proposition 4:** Proposition 3 implies that in the competitive equilibrium we have:

$$\begin{aligned} \hat{\theta}_H(\alpha^\dagger(\Phi^{\text{CE}}), \Phi^{\text{CE}}) &< \frac{\ell(\phi_H^{\text{CE}})}{X} \\ \text{and } \hat{\theta}_L(\alpha^\dagger(\Phi^{\text{CE}}), \Phi^{\text{CE}}) &> \frac{\ell(\phi_L^{\text{CE}})}{X} \end{aligned}$$

These inequalities imply that:

$$F_H\left(\hat{\theta}_H(\alpha^\dagger(\Phi^{\text{CE}}), \Phi^{\text{CE}})\right) < F_H\left(\frac{\ell(\phi_H^{\text{CE}})}{X}\right) \quad (13)$$

$$\text{and } F_L\left(\hat{\theta}_L(\alpha^\dagger(\Phi^{\text{CE}}), \Phi^{\text{CE}})\right) > F_L\left(\frac{\ell(\phi_L^{\text{CE}})}{X}\right) \quad (14)$$

Given the implicit definitions of  $\Phi^{\text{CE}}$  in (6) and of  $\Phi^{\text{FB}}$  in (7) as well as the fact that  $F_s(\ell(\phi)/X)$  is decreasing in  $\phi$  for both  $s = H, L$  we can conclude from the inequalities (13) and (14) that  $\phi_H^{\text{CE}} < \phi_H^{\text{FB}}$  and  $\phi_L^{\text{CE}} > \phi_L^{\text{FB}}$ , respectively. These inequalities, in turn, imply that  $\ell(\phi_H^{\text{CE}}) > \ell(\phi_H^{\text{FB}})$  and  $\ell(\phi_L^{\text{CE}}) < \ell(\phi_L^{\text{FB}})$ .  $\square$

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