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Money market funds intermediation, bank instability, and contagion

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Staff Reports

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Staff Report No. 599
February 2013



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Abstract

In recent years, U.S. banks have increasingly relied on deposits from financial intermediaries, especially money market funds (MMFs), which collect funds from large institutional investors and lend them to banks. In this paper, we show that intermediation through MMFs allows investors to limit their exposure to a given bank (i.e., reap gains from diversification). However, since MMFs are themselves subject to runs from their own investors, a banking system intermediated through MMFs is more unstable than one in which investors interact directly with banks. A mechanism through which instability can arise in an MMF-intermediated financial system is the release of private information on bank assets, which is aggregated by MMFs and could lead them to withdraw en masse from a bank. In addition, we show that MMF intermediation can also be a channel of contagion among banking institutions.

Key words: Money market funds, bank runs

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1 Introduction

In recent years, large global banks have increasingly relied on deposits from financial intermediaries, especially money market funds (MMFs). MMFs collect funds from institutional and wholesale investors and lend them to banks.

Bank deposits of institutional and wholesale investors are not fully covered by deposit insurance. As a result, they need to limit their exposure to a single banking institution and diversify their portfolio of deposits. Intermediation by institutions such as MMFs allows large investors to reap gains from diversification, while saving on bank-monitoring costs.

In the U.S., MMFs have become a very popular financial instrument, comprising 21 percent of all mutual fund assets. Their assets under management grew from approximately \$2 trillion in 2005 to \$3 trillion at the end of 2008 and contracted only mildly during the financial crisis (to \$2.6 trillion in 2012).¹ MMFs are key providers of short-term funding, especially to the financial sector. As Table 1 shows, in 2012 they were among the largest investors in some asset classes, financing 43 percent of financial commercial paper and 29 percent of certificates of deposit.

Nonfinancial CP	Financial CP	Asset-backed commercial paper (ABCP)	Certificates of Deposit	Repurchase Agreements
43%	43%	38%	29%	33%
75bn	207bn	117bn	434bn	591bn
As a percentage of outstanding assets. June 2012. Source: McCabe et al. (2012).				

In the U.S., MMFs offer demandable deposits (shares) redeemable at par, that is, with fixed net asset value (NAV). When the NAV (i.e., the value of the asset per share) falls below \$0.995 ("breaks the buck"), the MMF is forced by SEC regulation to re-price all its shares. Hence even small losses can start a run since investors have an incentive to redeem their shares before the MMF breaks the buck. In September 2008, the Reserve Primary Fund broke the buck, causing a stampede of withdrawals across the sector. To stem the panic, the Federal Reserve provided a large amount of liquidity through emergency facilities and the Treasury Department guaranteed MMF assets.

¹For a description of the MMF industry, see McCabe et al. (2012).

A banking system intermediated through MMFs can be more unstable than one in which investors interact directly with banks. Since MMFs are themselves subject to run-like redemptions from their own investors, they may react to them by running the banks in which they have deposited, hence amplifying the impact of the initial redemptions. The instability of a financial system in which banks finance themselves through intermediaries such as MMFs was one of the driving forces behind the recent reform effort of the MMF industry by the SEC and the FSOC.²

In this paper, we study an economy à la Diamond and Dybvig (1983) (DD hereafter) with two banks, whose long-term investments have stochastic and (perfectly) negatively correlated returns. Depositing in the two banks allows agents to reduce their risk through diversification. We consider two market structures: direct finance, where investors deposit directly into the banks, and MMF intermediation, where the relationship between investors and banks is intermediated through MMFs.³

In the model, bank bankruptcy arises when a fraction of investors unexpectedly withdraw their funds either from a bank or from an MMF. Investors withdraw either because they receive a liquidity shock, or because they receive a perfectly informative (negative) signal on the return of the investment of one of the two banks.

Under direct finance, unexpected withdrawals cause bank bankruptcy only if the amount withdrawn is large enough to force the bank into liquidation. In contrast, with MMF intermediation, when a fraction of investors unexpectedly redeem from the MMF, their actions represent a (noisy) signal on the state of the world for the MMF. If this signal is strong enough, the MMF will run the bank, withdrawing all its funds and causing bankruptcy even if the fraction of the unexpected redemptions was small enough that bankruptcy would not have occurred under direct finance. The instability of MMF intermediation stems from the fact that the negative information content of an unexpected redemption from an intermediary such as an MMF amplifies the effect of the redemptions themselves. Because of this, an economy intermediated by MMFs is generically more unstable than a direct-finance structure.

The amplification mechanism is possible given that MMFs are subject to run-like redemptions because they offer investors demandable liabilities in order to satisfy their liquidity needs. When an MMF expe-

²See, for instance, Dudley (2012) and Geithner (2012).

³MMFs lend to banks mostly through unsecured commercial paper and other short-term investments (see Table 1). Nevertheless, our model captures the essential economic feature of short-term debt rollover through MMFs' decision to either keep or withdraw the money from the banks.

riences large unexpected redemptions, it runs the bank to protect all its investors, and not just those initiating the redemptions. Because of the bank's fixed promise, the MMF, receiving negative information on the bank's assets, obtains a higher payoff for its investors if it runs than if it does not.

Note that since the withdrawals of funds from the investors may be due to liquidity as opposed to informative reasons, bank bankruptcy may cause inefficient liquidation and a reduction in welfare.

Our model also generates contagion across banks as the diversification opportunities offered by the MMF may turn into a source of fragility when an MMF pulls all its funds away from a bank. In particular, if the other bank in the economy is not viable on its own given the contract that it offers, the whole banking system collapses. This occurs when banks' investments are (sufficiently) risky on their own, but (sufficiently) safe when taken together. In this case, the optimal contracts, which exploit the diversification opportunities offered by the banking system as a whole, may be viable only if investors supply funds to both banks. In a nutshell, we identify a new source of contagion, stemming from the loss of diversification that the liquidation of one bank entails for the investors of the other. This channel is theoretically different from the interbank diversification of Allen and Gale (2000) because it relies on the increase in riskiness of one bank due to the collapse of the other, rather than on a direct loss of funds because of interbank deposits. It is somewhat similar to the work of Lagunoff and Schreft (1999 and 2001).

Section 2 describes our model and characterizes the economic function of MMF intermediation. Section 3 studies the effect of an unexpected withdrawal of funds from the financial system. Section 4 shows that an MMF-intermediated financial system is more fragile than one with direct finance. Section 5 concludes. The proofs are in the appendix.

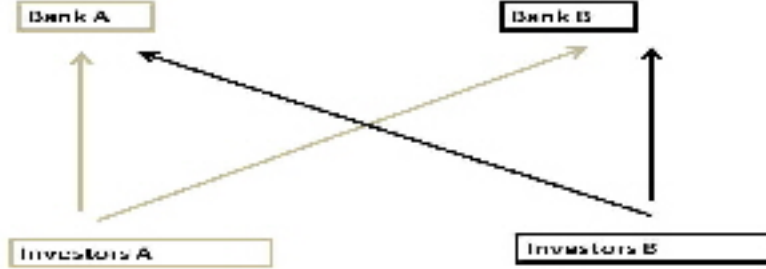
2 The Model

2.1 Technology and Preferences

We describe our economy first with direct finance and then with MMF intermediation. There are two regions, A and B . In each region, there is a continuum of (wholesale or institutional) investors of mass M , which can be interpreted as uninsured wholesale investors,⁴ for a total popula-

⁴Agents supplying funds to banks are normally referred to as depositors, who deposit or withdraw their funds. In contrast, agents supplying funds to MMFs are normally referred to as investors, who purchase or redeem shares of the MMF. In order not to saddle the reader, from now on we will use the term "investors," who

Figure 1:



tion $2M$. Each investor is endowed with \$1 of cash. In each of the two regions there is one bank, Bank *A* and Bank *B*. The structure of the economy is depicted in Figure 1.

There are three dates, 0, 1, and 2, and a unique good that can be consumed, stored, or invested. Everyone in the economy can use storage, which returns one unit of the good at date $t + 1$, for each unit invested at date t , $t = 0, 1$. In contrast, the investment technology is available only to banks. We consider an economy where the returns of the investments of two banks can be either high or low and are perfectly negatively correlated. We assume this in order to maximize the gains from diversification and simplify the model. The returns of the two banks per unit invested are as follows:

	Return Bank <i>A</i>	Return Bank <i>B</i>
Probability 1/2	R^H	R^L
Probability 1/2	R^L	R^H

with $R^H > 1 > R^L$. Since the two states of the world are equally likely, the net present value of a unit of investment is the same for the two banks. As a result, it is optimal to supply an equal amount of funds to both banks at date 0. Investment can also be liquidated at date 1, in which case it returns $0 \leq r \leq R^L$ per unit invested.⁵

may "supply" or "withdraw" their funds from the MMF or from the bank in reference to both direct finance and the MMF-intermediated economy.

⁵The assumption is natural if we interpret the banks as investing in financial assets at different maturities, such as a loan, that it tries to sell in an unmodeled market. Because of asymmetric information, market participants may not want to purchase the loans at a price such that $r > R^L$.

In each of the two regions, investors are subject to preference shocks: with probability π investors must consume at date 1 (“impatient” investors), and with probability $1 - \pi$ they must consume at date 2 (“patient” investors). The realization of the shock to their preferences at date 1 is private information. For simplicity’s sake, we assume that investors have logarithmic utility function, so that their expected utility is

$$\pi \log (c_1) + (1 - \pi) \log (c_2),$$

where c_1 and c_2 denote date-1 and date-2 consumption, respectively. From the law of large numbers, a fraction π of agents consume at date 1 and a fraction $(1 - \pi)$ at date 2.

2.2 The Optimal Contract with Direct Finance

We first determine the optimal contract under direct finance. As is standard in this literature, banks are subject to a zero-profit condition and, under direct finance, choose the contracts to maximize the expected utility of investors.

To simplify notation, we express all quantity variables per dollar supplied to the banking system. In particular, we denote by i the total investment per dollar by the two banks. Moreover, we assume that

$$\frac{R^H + R^L}{2} > 1, \tag{1}$$

that is, the expected net present value of each bank’s investment is positive. This condition, as we show in the appendix, guarantees that the optimal level of investment i is positive, since the risk of banks’ long-term technologies can be completely diversified away.

The optimal contract and the optimal investment level are: $c_1 = 1$, $c_2^H = R^H$, $c_2^L = R^L$, and $i = 1 - \pi$, where c_2^H and c_2^L represent date-2 consumption if the bank has a high and a low return, respectively.⁶ The optimal contract implies that the banks store enough funds to satisfy withdrawals from impatient investors only and invest all the remaining funds in the long-term technology. Note that since banks have perfect negatively correlated returns, under the optimal contract, investors will deposit an equal amount in each bank, which, in turn, will invest a fraction $(1 - \pi)$ in its long-run technology, thus allowing patient investors a deterministic return $\frac{c_2^H + c_2^L}{2} = \frac{R^H + R^L}{2}$.

The diversification opportunities that arise from investing in both banks may turn into a source of fragility if a bank is not viable on its

⁶See the appendix for the derivation.

own given the contracts that it offers. Formally, this will occur if patient investors do not want to withdraw funds from one bank and wait in the other versus withdrawing from both banks. That is, if:

$$\log(0.5c_1 + 0.5c_1) > 0.5 \log(0.5c_1 + 0.5c_2^H) + 0.5 \log(0.5c_1 + 0.5c_2^L). \quad (2)$$

Given the optimal contract, condition (2) becomes:

$$\log(1) = 0 > 0.5 \log(0.5 + 0.5R^H) + 0.5 \log(0.5 + 0.5R^L),$$

which is satisfied as long as

$$R^H < \frac{3 - R^L}{1 + R^L}. \quad (3)$$

In the rest of the paper, we assume that condition (3) holds, that is, that banks are not viable on their own. Intuitively, condition (3) requires that R^H cannot be greater than (or equal to) $\frac{3 - R^L}{1 + R^L}$ because, otherwise, each bank would be so profitable that the contract it offers can stand on its own.

The condition (1) for an interior solution for i and the condition (3) that contracts are not viable separately establish a range for R^H :

$$2 - R^L < R^H < \frac{3 - R^L}{1 + R^L},$$

which is always feasible for any R^L .⁷

2.3 MMF Intermediation

The structure of the economy with MMF intermediation is similar to the one under direct finance. However, in each region A and B , there is one MMF—MMF A and MMF B —that channels the funds of its region to the two banks. Each MMF maximizes the expected utility of its investors by investing in banks' deposits (recall that only banks can invest in the long-term technology) and/or into the storage technology. The structure of the economy with MMF intermediation is depicted in Figure 2.

The risk-diversification problem does not change when we introduce MMFs in the economy. As a result, under the optimal contract, the

⁷The equality:

$$2 - R^L = \frac{3 - R^L}{1 + R^L},$$

has two equal roots $R^L = 1$, and it is always satisfied for any $R^L > 0$.

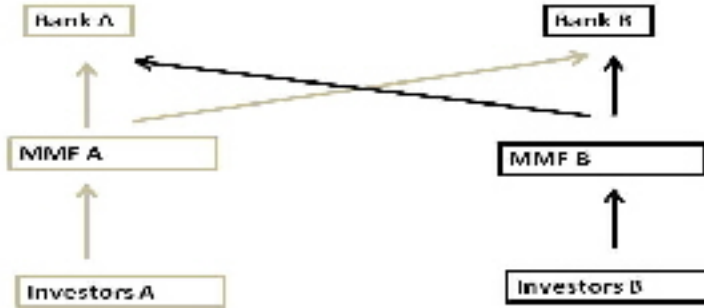


Figure 2: The economy with MMF intermediation.

final consumption for early and late investors must be the same as with direct finance. It is easy to show that this can be accomplished as long as the contracts that the two banks offer to the MMFs are the same as those offered to the wholesale investors with direct finance. Analogously, the contracts that the MMFs offer to their investors must simply aggregate the payouts from the two banks: the contract per dollar invested that each MMF offers is $c_1^{MMF} = 1$, and $c_2^{MMF} = \frac{R^H + R^L}{2}$. That is, the MMFs will offer their investors claims redeemable at par at date 1. Finally, MMFs must share all the funds they collect from their investors equally between the two banks.

In order to understand the role of MMF intermediation in the economy, let us consider the case in which banks must be monitored/screened; otherwise, their return is zero at date 2. The need to monitor the banks could arise from the fact that the opacity of bank loans allows them to underreport the return to the long-term technology and/or offers scope for moral hazard to bank managers. It is well known that since monitoring has a fixed-cost dimension, the duplication of monitoring costs that direct finance entails may be reduced when funds are intermediated through a delegated monitor (Diamond 1984). That is, MMF intermediation allow investors to save on monitoring costs and, as a result, increase their level of consumption and welfare, while at the same time letting them enjoy the gains from diversification.⁸

⁸This is true as long as the cost of monitoring the MMF is lower than that of monitoring banks, which is generally the case since MMFs invest in fixed-income securities

In the next section we show that MMF intermediation also makes the banking system more fragile.

3 An Unexpected Withdrawal of Funds

3.1 Information Arrival and Aggregation

The fragility of an MMF-intermediated system can be captured by considering the effect of an unexpected withdrawal of funds in the economy with direct finance and in that with MMF intermediation.

Let us assume that at date 1 some patient investors unexpectedly withdraw their funds. They do so either because they have received a liquidity shock, i.e., some previously patient investors become impatient and must consume at date 1, or because they have received a perfectly informative signal that the return of the investment of the bank in their region is R_L . This withdrawal, which is unexpected and hence is in excess of the liquidity available at date 1, has a different impact on the stability of the system under direct finance and under MMF-intermediated finance.

Under direct finance, the fraction of funds withdrawn in excess of π may be sufficiently low so as not to push the bank into insolvency and therefore not to alter the equilibrium. However, when investors unexpectedly redeem from the MMF, their actions represent a noisy signal on the state of the world for the MMF to interpret. From the size of the unexpected redemptions in excess of π , the MMF will update its prior belief on the return of the long-term investment of the bank in its own region, and it may run that bank by pulling all its funds away, thereby pushing it into bankruptcy.

More formally, we assume that, with zero probability, a positive measure of patient investors q from region A withdraw their funds at date 1 from Bank A or, under MMF intermediation, from the MMF in region A .⁹ This assumption is in the spirit of Allen and Gale (2000), who consider the realization of an additional state of nature that was assigned a probability zero at date 0.¹⁰

(see Table 1). We did not incorporate the monitoring cost into our the model since it unnecessarily complicates the notation; a model that includes monitoring costs is available on request from the authors. Note also that diversification can be achieved through the interbank market; it is easy to show, however, that MMF intermediation generally saves on monitoring costs also with respect to interbank finance.

⁹Of course, since everything is symmetrical, nothing would change if the unexpected withdrawal of funds occurred in region B .

¹⁰More recently, Gennaioli et. al (2012) argued that investors may not take into consideration certain highly improbable risks, such as the probability that the share price of a money market mutual fund may fall below 1.

As a result of the shock q , the total amount of withdrawals at date 1 from region A 's investors is $\pi + (1 - \pi)q$. Note that, since we assumed that condition (3) holds, that is, that Bank B 's contract is not viable on its own, agents receiving negative information about Bank A will also withdraw from Bank B in the direct-finance case.¹¹

The probability that the unexpected withdrawal is informative is increasing in q , the observed level of unexpected withdrawal itself:

$$Pr(\{\text{Shock due to informational reason}\}) = f(q), \quad f'(q) > 0$$

that is, the higher the fraction of withdrawals in excess of π , the more likely it is that it happens for information reasons. Note that in order to keep the algebra simple, from now on we will consider the case

$$f(q) = q$$

that is, the probabilities that the unexpected withdrawal is informative or that it is due to a preference shock are q and $(1 - q)$, respectively, where $(1 - \pi)q$ is the observed level of unexpected withdrawal itself.¹² All the results we present, however, hold for any increasing function $f(q)$.

Under MMF intermediation, MMF A sees the unexpected withdrawal of funds by its investors and interprets this as (imperfect) bad news on the return of the assets of Bank A . In particular, after observing the unexpected withdrawal $(1 - \pi)q$, the MMF A updates the joint probabilities that Bank A and Bank B have a high or a low return in the following manner:

$$\begin{aligned} Pr(R_A^H, R_B^H|q) &= 0(q) + 0(1 - q) = 0, \\ Pr(R_A^H, R_B^L|q) &= 0(q) + 0.5(1 - q) = 0.5(1 - q), \\ Pr(R_A^L, R_B^L|q) &= 0.5(q) + 0(1 - q) = 0.5q, \\ Pr(R_A^L, R_B^H|q) &= 0.5(q) + 0.5(1 - q) = 0.5. \end{aligned}$$

Note that when the shock is informative, the release of information is not about the state of nature; rather, it is about the return of one of the two banks (Bank A).¹³ In other words, the probability of the return

¹¹Note that negatively informed patient investors will always find it convenient to withdraw because $c_1 > c_2^L$ (as we explain below, even if Bank A goes into bankruptcy, informed patient investors will always be able to recoup c_1).

¹²In an addendum available on request from the authors, we show that $f(q) = q$ can be derived from a simple informational structure.

¹³In other words, the zero-probability event consists of a change in the returns to the long-run technologies in the two states of nature, which become (R_A^L, R_B^L) and (R_A^L, R_B^H) .

of Bank B being high or low is not affected by the arrival of negative information on Bank A . In fact, after observing $(1 - \pi)q$, the conditional probability of Bank B being good or bad is not affected. That is,

$$Pr(R_B^L|q) = Pr(R_B^H|q) = 0.5.$$

Nevertheless, since the contract that Bank B offers is not viable on its own, because of condition (3), the destruction of diversification opportunity stemming from the release of information on Bank A may also send Bank B into bankruptcy, an issue that we will analyze below. That is, the bankruptcy of Bank B occurs not because of any information on Bank B 's return, but as a result of the bankruptcy of Bank A .

Note that the assumption that investors in region A can only be informed on Bank A (and that the MMF in region A knows as much) is useful to highlight the mechanism through which contagion due to loss of diversification opportunities occurs, that is, in clarifying that it is a separate channel from informational contagion. Should we assume that, with a given probability, informed withdrawers in region A receive a signal on Bank B , the mechanism of MMF-induced financial fragility would not change, but contagion would occur for a mix of informational and loss-of-diversification reasons. Finally, note that the release of information on Bank A makes the contract offered by Bank B undesirable. This is reminiscent of the point made by Hirshleifer (1971) on information having negative welfare impact because it can impede insurance.

Finally, the fact that unexpected withdrawal is an imperfect signal on the bank long-term investment generates “confounding” as in Chari and Jagannathan (1988). Confounding is desirable in our model because otherwise any realization of q , however small, would generate the collapse of Bank A and also the collapse of Bank B because of assumption (3).

3.2 Bankruptcy

The excess withdrawal of funds may cause bank bankruptcy in our economy. To study its impact on the banking system, we need to make some assumptions on how the banks' assets are split in case of bankruptcy. In particular we assume that:

- Banks abide by the sequential service constraint when facing withdrawals at date 1, both under direct finance and in the MMF-intermediated economy.
- Patient investors withdrawing their funds early do so at the beginning of the queue. This captures the notion that since they are potentially informed about bank asset returns, they may be able to jump ahead of the line.¹⁴ The assumption reflects the fact that institutional

¹⁴Note that liquidity withdrawers will also try to jump ahead of the queue as they

investors are prone to run in a crisis.

- Analogously and for the same reasons, if one MMF makes unexpected withdrawals, it is first in the queue with respect to the other MMF.¹⁵

We now contrast the effect of the unexpected withdrawal of funds $(1 - \pi)q$ in an economy with direct finance and one with MMF intermediation.

3.2.1 Bankruptcy with Direct Finance

In the case of direct finance, the unexpected withdrawal of funds from Bank *A* will push it into bankruptcy if the proportion q of patient investors who withdraw their funds early is such that:

$$\left(\pi + \frac{(1 - \pi)}{2}q\right)c_1 > 1 - i + ri. \quad (4)$$

That is, bankruptcy will occur when the bank's date-1 liabilities, per unit deposited in the bank, (LHS of 4) exceed its date-1 assets (RHS). This condition, given the optimal contract described above, becomes:

$$\pi + \frac{(1 - \pi)}{2}q > \pi + r(1 - \pi).$$

Hence, the bank goes bankrupt if and only if $q > 2r$. Note that since $q \in (0, 1)$, a necessary condition for bankruptcy to occur under direct finance is

$$r < \frac{1}{2}. \quad (5)$$

From now on, however, we concentrate on realizations of q such that the bank does not go bankrupt with direct finance, and we show that the same realizations of q may instead trigger bankruptcy under MMF intermediation. That is, we assume:

$$q \leq 2r. \quad (6)$$

Let us make two observations. First, since we assumed that the proportion $\frac{(1-\pi)q}{2}$ of patient investors who unexpectedly withdraw at date 1 are early in the queue, they will be able to withdraw $c_1 = 1$ as long as

are aware that the bank/MMF may not be able to serve latecomers if there are excess withdrawals at time 1.

¹⁵These assumptions allow us to characterize the equilibrium in the economy in the simplest possible way. As will be clear, however, the fragility of an MMF-intermediated economy does not stem from the particular bankruptcy assumption that we adopted, but from the ability of MMFs to aggregate information among their investors.

$$\frac{(1 - \pi)q}{2} \leq \pi + r(1 - \pi), \quad (7)$$

where the LHS of (7) is the amount of funds withdrawn at date 1 by the patient investors, and the RHS are the bank's assets at date 1.¹⁶ Obviously, as long as the level of withdrawal is such that the bank is not pushed into bankruptcy, that is, as long as $q \leq 2r$, the bank will always be able to pay $c_1 = 1$ to the patient investors withdrawing their funds early. As a result, as mentioned above, an informed patient investor knowing that the return of Bank A 's assets is low will find it optimal to withdraw since $c_1 > c_2^L$.

Second, if bankruptcy occurs, impatient investors (from both Banks A and B) will not necessarily get c_1 since there are not enough resources in the bank, even after liquidating all the long-term assets. Moreover, even if bankruptcy does not occur, patient investors will not receive the optimal contract at date 2 since some (or all) of the funds have been liquidated.

As a final remark, observe that since $r \leq R^L$, then Bank A will never liquidate all its long-term assets when it observes an excess withdrawal of funds; it will only liquidate whatever is needed to repay the proportion of patient investors who withdraw their funds early.

3.2.2 Bankruptcy with MMF Intermediation

Even if both the MMFs and the banks issue the same claims demandable at par at date 1, upon observing unexpected redemptions MMF A behaves differently from the bank in the direct-finance case. In fact, the MMF can withdraw its funds from Bank A at the contract $c_1 = 1$, while in direct finance, when Bank A liquidates early to meet the unexpected withdrawal $(1 - \pi)q$, it does so at $r < 1$. Therefore, if after observing the unexpected redemptions $(1 - \pi)q$, MMF A believes that Bank A 's return is low with high enough probability, it will withdraw all its funds, and not only what is needed to meet the unexpected redemptions. This amplification mechanism makes the MMF-intermediated structure more unstable than direct finance.

Of course, the fact that MMF A withdraws all its funds from Bank A does not necessarily imply that Bank A is bankrupt, which will only happen when:

¹⁶The condition (7) computed for the highest possible level of withdrawal, i.e., for $q = 1$, becomes:

$$(1 - \pi) < 2(\pi + r(1 - \pi)),$$

which is always true for $\pi > \frac{1}{3}$.

$$\left(\pi + \frac{(1 - \pi)}{2}\right)c_1 > 1 - i + ri,$$

which, given the optimal contract, becomes:

$$\begin{aligned} \left(\pi + \frac{(1 - \pi)}{2}\right) &> \pi + r(1 - \pi), \\ \text{or } r &< \frac{1}{2}. \end{aligned}$$

Note that $r < \frac{1}{2}$ is the same as condition (5), which makes bankruptcy possible in direct finance for a high enough realization of q .

Note that bankruptcy is not the result of a sunspot (e.g., a wave of pessimism), but stems from the unexpected withdrawal of funds $(1 - \pi)q$ by patient investors and, in the MMF-intermediated economy, from the information that such an unexpected withdrawal conveys to the MMF.

3.3 The MMF Reaction to an Unexpected Withdrawals of Funds

We now investigate how MMF A will behave after observing an unexpected withdrawal of funds $(1 - \pi)q$. The MMF will be able to offer c_1 to all its investors withdrawing their funds at date 1 as long as:

$$\left(\pi + (1 - \pi)q\right)c_1 \leq 2(\pi + r(1 - \pi)), \quad (8)$$

where the LHS of (8) is the withdrawal of funds from MMF A and the RHS are the combined assets of both banks A and B at date 1. Since $c_1 = 1$ under the optimal contract, (8) becomes

$$(1 - \pi)q \leq \pi + 2r(1 - \pi). \quad (9)$$

Note that since we are only considering realizations of q such that the banking system does not go bankrupt under direct finance¹⁷ (i.e., $q \leq 2r$), condition (9) becomes:

¹⁷In contrast, for $q = 1$, the condition would be:

$$\begin{aligned} 1 - \pi &< \pi + 2r - 2r\pi \\ \pi &> \left(\frac{1 - 2r}{2 + 2r}\right) \end{aligned}$$

Note that, in this case, if the MMF withdraws all its funds early at the rate c_1 from Bank A and B , the banks will never go bankrupt. This is because all their combined assets $2(\pi + r(1 - \pi))$ are equal to or greater than the MMF maximum withdrawal, which is equal to $\pi + (1 - \pi)$.

$$(1 - \pi)2r \leq \pi + 2r(1 - \pi),$$

which, as in the case of the analogous condition with direct finance, is always satisfied. As a result, the MMF is sure that both impatient and patient investors redeeming early receive c_1 . This allows us to study the reaction of the MMF upon observing the unexpected redemptions $(1 - \pi)q$, while disregarding the welfare of the investors redeeming early.

In particular, the MMF must choose the proportion by which it meets the unexpected redemptions $(1 - \pi)q$ by withdrawing funds from Bank A and from Bank B . These proportions, which we denote by λ and $(1 - \lambda)$, respectively, are the results of the MMF re-optimization upon observing the unexpected withdrawal of funds. Since the MMF knows it will be able to pay its investors redeeming early the amount $c_1 = 1$, the proportions λ and $(1 - \lambda)$ are derived only by looking at the welfare of the remaining patient investors (i.e., $(1 - \pi)(1 - q)$). Note that although the excess withdrawal of funds occurs in region A (and with probability q it reflects bad information on Bank A 's long-term investment), in general the MMF will decide to meet the unexpected redemptions by pulling funds from both banks. The reason is that although the expected return on Bank A assets has decreased (whereas that on Bank B assets has not), in general it may not be optimal to meet the unexpected redemptions exclusively from Bank A , as the two banks provide a hedge one against the other. The optimal λ is given by

$$\lambda = \max\left(\frac{\left(\frac{R^H}{R^L} - 1\right)(1 - q) + \frac{r\left(\frac{R^H}{R^L} + 1\right)}{(1 - \pi)} - q}{(2 - q)\left(\frac{R^H}{R^L} - 1\right)}, 1\right).$$

In the interest of space, we do not report the derivation of the optimal level of λ in the main text of the paper, but describe it in the appendix. Note that, as shown in the appendix, if $q = 0$, then $\lambda = \frac{1}{2}$, which means that if the unexpected withdrawal of funds is low enough not to contain any information on Bank A , it will be met by withdrawing funds equally from both banks. Note also that if $q = 1$, then $\lambda = 1$. This means that if the unexpected withdrawal is so high that the MMF knows that the return on Bank A assets is low, it will be met by withdrawing from Bank A only.

Recall that the total amount withdrawn at date 1 by MMF A is $\pi + (1 - \pi)q$. Since the banks have invested in the long-term asset a fraction $i = 1 - \pi$ of each unit deposited, the overall liquidation of the long-term assets per unit deposited in the banking system is $\frac{(1 - \pi)q}{r}$. Given the optimal fraction λ withdrawn by MMF A from Bank A , the withdrawal per unit of deposit from Bank A is $\lambda(\pi + (1 - \pi)q)$ and that

from Bank B is $(1 - \lambda)(\pi + (1 - \pi)q)$. Moreover, the liquidation of Bank A 's assets yields $\lambda \frac{(1 - \pi)q}{r}$; and the liquidation of Bank B 's assets yields $(1 - \lambda) \frac{(1 - \pi)q}{r}$. Because of the liquidation of both banks' assets to meet the unexpected withdrawal of funds, the contracts that the banks can afford to offer at date 2 will change. In particular, the payoffs offered by Bank A are:

$$\widehat{c}_{2,A}^H = \max(R^H(1 - \lambda \frac{(1 - \pi)q}{r}), 0) \quad (10)$$

$$\widehat{c}_{2,A}^L = \max(R^L(1 - \lambda \frac{(1 - \pi)q}{r}), 0), \quad (11)$$

and the payoffs offered by Bank B are:

$$\widehat{c}_{2,B}^H = \max(R^H(1 - (1 - \lambda) \frac{(1 - \pi)q}{r}), 0), \quad (12)$$

$$\widehat{c}_{2,B}^L = \max(R^L(1 - (1 - \lambda) \frac{(1 - \pi)q}{r}), 0). \quad (13)$$

4 The Fragility of MMF Intermediation

We now show that there are levels of withdrawals and redemptions such that there is no bank bankruptcy with direct finance, but bankruptcy occurs with MMF intermediation. Obviously, to study the MMF behavior, we must take into account that the unexpected redemptions $(1 - \pi)q$ give the MMF information about the return of the long-term asset in Bank A . In particular, upon observing q , the MMF updates its prior on the return of Banks A and B according to the probabilities described in Section 3.1.

We are looking for a condition on q such that MMF A , after having received unexpected redemption $(1 - \pi)q$, prefers to withdraw all its holdings from Bank A and trigger its liquidation,¹⁸ as opposed to liquidating only the minimum from both banks to satisfy unexpected redemptions and keep the rest in the banks. Recall that the MMF maximizes the expected utility of its investors.

The expected utility of the MMF investors if the MMF decides to withdraw only $(1 - \pi)q$, and not to force Bank A 's liquidation, is:

$$EU^{\text{Non-Liquidation}} = \underbrace{(\pi + (1 - \pi)q) u(c_1)}_0 + (1 - (\pi + (1 - \pi)q)) \quad (14)$$

$$\left[0.5(1 - q)u\left(\frac{\widehat{c}_{2,A}^H + \widehat{c}_{2,B}^L}{2}\right) + 0.5u\left(\frac{\widehat{c}_{2,B}^H + \widehat{c}_{2,A}^L}{2}\right) + 0.5qu\left(\frac{\widehat{c}_{2,B}^L + \widehat{c}_{2,A}^L}{2}\right) \right],$$

¹⁸Since we assumed that $r < \frac{1}{2}$, Bank A will go bankrupt if MMF A withdraws all its assets (see Section 3.2.2).

where $\widehat{c}_{2,i}^H, \widehat{c}_{2,i}^L$ $i = A, B$ are the payouts of Banks A and B at date 2 after MMF A withdraws its funds defined in (10), (11), (12), and (13). Note that since the MMF knows it can pay its investors redeeming at date 1 the amount $c_1 = 1$, the first term drops out from (14).

In contrast, the expected utility of the MMF investors if the MMF decides to force Bank A into liquidation is:

$$EU^{\text{Liquidation}} = (\pi + (1 - \pi)q) u(\widehat{c}_1) + (1 - (\pi + (1 - \pi)q)) \left[0.5u\left(\frac{c_2^H + \widehat{c}_1}{2}\right) + 0.5u\left(\frac{c_2^L + \widehat{c}_1}{2}\right) \right], \quad (15)$$

where

$$\widehat{c}_1 \equiv \min\left(\frac{\pi + r(1 - \pi)}{\frac{\pi}{2} + \frac{(1 - \pi)}{2}}, 1\right) = \min(2[\pi + r(1 - \pi)], 1) \quad (16)$$

is how much the MMF obtains if it forces Bank A to liquidate all its assets at date 1.¹⁹ Note that if the MMF A forces Bank A into bankruptcy it will not necessarily be able to pay all its investors $c_1 = 1$ and, therefore, generally, $\widehat{c}_1 < c_1$.²⁰

At date 1, MMF A will withdraw from Bank A all its holdings (as opposed to only the unexpected withdrawal $(1 - \pi)q$) if the expected utility of its investors upon total withdrawal from Bank A ($EU^{\text{Liquidation}}$) is greater than the expected utility upon keeping funds in Bank A ($EU^{\text{Non Liquidation}}$).²¹ Thus, we can establish a level of q such that MMF A , after having observed the unexpected withdrawal $(1 - \pi)q$, prefers to withdraw all its holdings from Bank A and trigger its liquidation (if $r < \frac{1}{2}$), as opposed to liquidating only the minimum from both banks to satisfy the unexpected withdrawal of funds.

The following proposition compares the stability of MMF intermediation and direct finance.

Proposition 1: For any values of R^H and R^L satisfying condition (1) and for $\pi \geq \frac{0.5-r}{1-r}$, there is an interval of realization of q for which bankruptcy occurs with MMF intermediation and not with direct finance.

¹⁹Note that $\pi + r(1 - \pi)$ are Bank A 's assets at date 1; moreover, in the denominator $\pi/2$ is the mass of impatient investors that MMF A has to satisfy from Bank A , and $(1 - \pi)/2$ is the mass of patient investors that MMF A has to satisfy from Bank A (remember that we assume MMF A is first in the queue).

²⁰Given that the information event is zero probability, MMF B will never withdraw in excess of π since it has no information on the return of the long-term asset.

²¹For simplicity's sake, we assume in the proofs that the MMF withdraws when the inequality holds weakly.

The proof is in the appendix.

Proposition 1 establishes that a MMF-intermediated system is more fragile than direct finance. This happens because MMFs give investors demandable liabilities in order to satisfy their liquidity needs, which makes MMFs liabilities subject to run-like redemptions. The unexpected early redemptions contain negative information on Bank A 's assets, which may make it optimal for the MMF to run the bank. Note that the MMF decides to run the bank in order to protect all its investors, and not just those unexpectedly withdrawing early. Indeed, given the bank's fixed promise at date 1, the MMF obtains a higher payoff for its investors if it runs than if it does not. Since the unexpected early redemptions may be due to liquidity as opposed to informative reasons, bank bankruptcy under MMF intermediation may cause inefficient liquidation of the long-term investment.

Note that, in this economy, MMF intermediation generates financial fragility even though MMFs only maximize the welfare of their investors. The instability does not arise from any friction (such as agency problems), but simply from the ability of MMFs to aggregate private information and use it to the benefit of its investors.

As mentioned above, bankruptcy in an MMF-intermediated economy occurs because there is a threshold of q , such that any realization of q greater than that leads MMF A to withdraw its funds from Bank A and, as a result, Bank A collapses. In the following proposition we provide an upper bound for such a threshold.

Proposition 2: For any values of R^H and R^L satisfying condition (1) and for $\pi \geq \frac{0.5-r}{1-r}$, let us define by \hat{q} the threshold such that any realization of q greater than \hat{q} leads to bankruptcy under MMF intermediation. We can show that

$$\hat{q} \leq \tilde{q} \equiv \frac{\log \frac{(R^H+1)(R^L+1)}{(R^H+R^L)^2}}{\log \left(\frac{2}{\left(\frac{R^H}{R^L}+1\right)} \right)}.$$

The proof is in the appendix, where we also show that:

$$\frac{\partial \tilde{q}}{\partial R^L} > 0, \quad \frac{\partial \tilde{q}}{\partial R^H} > 0,$$

that is, \tilde{q} increases with both R^L and R^H . When R^L is higher, the negative information conveyed by the excess withdrawal is less important; therefore, a higher level of withdrawal is needed for the MMF to cause the bank's bankruptcy. Similarly, when R^H increases, the higher return

in the high state of the world increases the expected utility from not withdrawing from the bank; as a result, a higher excess withdrawal is needed for the MMF to cause bankruptcy.

4.1 Contagion

The presence of banks in the two regions offers wholesale investors hedging opportunities. However, when a bank is liquidated, this hedging opportunity vanishes, which may create contagion across the banking system.

In particular, if the unexpected withdrawals of funds $(1 - \pi)q$ are large enough that MMF A withdraws all its funds from Bank A , the MMF may also withdraw from Bank B . This happens if the contract offered by Bank B is not viable on its own, given the conditional information that the MMF has on Bank B returns (see Section 3.1) and the fact that the payout from Bank A is now \hat{c}_1 for both early and late withdrawers. This will happen if:

$$\begin{aligned} \log(0.5\hat{c}_1 + 0.5\hat{c}_1) &\geq (\pi + (1 - \pi)q) \log(\hat{c}_1) \\ &+ (1 - (\pi + (1 - \pi)q)) \left[0.5 \log\left(\frac{c_{2,B}^H + \hat{c}_1}{2}\right) + 0.5 \log\left(\frac{c_{2,B}^L + \hat{c}_1}{2}\right) \right]. \end{aligned} \quad (17)$$

When condition (17) is satisfied, the instability of MMF finance is “contagious” because it generates the collapse of the whole banking system. We establish the following result:

Proposition 3: For $\pi \geq \frac{0.5-r}{1-r}$ and given assumption (3), bankruptcy of Bank A triggers bankruptcy of Bank B .

The proof is in the appendix.

With this result, we identify a new source of contagion, stemming from the loss of diversification that the liquidation of one bank entails for the investors of the other. This channel is theoretically different from the interbank diversification of Allen and Gale (2000) because it relies on the increase in riskiness of one bank due to the collapse of the other, rather than on a direct loss of funds because of interbank deposits.

4.2 Fragility and Contagion: an Example

By means of an numerical example we establish, first, that there exists an economy in which on date 1 an unexpected redemption $(1 - \pi)q$ causes bankruptcy under MMF intermediation, while a unexpected withdrawal of the same size does not cause bankruptcy under direct finance. Second, we establish that contagion can happen.

Consider an economy where

$$R^L = 0.25 \text{ and } R^H = 2.$$

Assume that the liquidation value $r = 0.249 < R^L$. Therefore, since $r < 1/2$, by condition (5) bankruptcy is possible under direct finance. Also assume that the fraction of impatient π equals 0.8.

With a logarithmic utility function, the optimal contract offered by the banks is

$$c_1 = 1 \quad c_2^H = 2 \quad c_2^L = 0.25.$$

Moreover, since $R^H < \frac{3-R^L}{(1+R^L)} = 2.2$, by condition (3) the optimal contracts offered by Banks A and B are not viable separately.

Consider a level of excess withdrawal $q = 0.35$. This means that the unexpected withdrawal is $(1 - \pi)q = 0.2(0.35) = 0.07$. Since, $q < 2r = 0.498$, by condition (5) such level of unexpected withdrawal does not cause bankruptcy in an economy with direct finance.

What happens instead with MMF intermediation? Upon observing the unexpected redemption, the MMF A will update upwards the probability that return of Bank A long-term investment yields $R^L = 0.25$.

Given this information, if MMF A decides to withdraw only what is needed to meet the unexpected withdrawal of funds from its investors $(1 - \pi)q$, it would withdraw all the funds from Bank A and nothing from Bank B (i.e., the optimal λ equals 1).

Because of this, upon observing the withdrawal, Bank A would only be able to offer

$$\hat{c}_2^H = R^H \left(1 - \frac{(1 - \pi)q}{r}\right) = 1.44 \quad \text{and} \quad \hat{c}_2^L = R^L \left(1 - \frac{(1 - \pi)q}{r}\right) = 0.18,$$

whereas Bank B would not have to modify its payouts. As a result, from (14) the expected utility of MMF A investors would be -0.10 .

What would happen if MMF A decides to pull all its funds from Bank A ? Since π is relatively high (0.8), MMF A would be able to pay $\hat{c}_1 = c_1 = 1$ to all investors withdrawing early. As a result, from (15) the expected utility of MMF A investors would be -0.004 , higher than if MMF A decides not to pull its funds from Bank A .

Finally it is easy to verify that given the decision to pull out from Bank A , MMF A would also find it convenient to withdraw its funds from Bank B .²² That is, we found a level of unexpected withdrawals such that with direct finance there is no bankruptcy, whereas with MMF intermediation both Bank A and Bank B go bankrupt.

²²This follows from the fact that, under the maintained assumptions, the LHS of equation (17) is equal to 0 as

$$\hat{c}_1 \equiv \min(2[\pi + r(1 - \pi)], 1) = 1$$

and the RHS of equation (17) is equal to the expected utility under liquidation, namely equation (15), which is equal to -0.004 .

5 Conclusion

In this paper we show that MMF intermediation allows investors to limit their exposure to a single banking institution and reap the gains from diversification. However, a banking system intermediated through MMFs is more unstable than one in which investors interact directly with banks because MMFs are themselves subject to runs from their own investors. The mechanism through which instability arises is the release of private information on bank assets, which is aggregated by MMFs and lead them to withdraw en masse from a bank. Finally, we show that MMF intermediation is itself a channel of contagion among banking institutions. Our results provide a theoretical underpinning for the idea that an MMF-intermediated financial system can be particularly fragile. This fragility has been the impetus of recent regulatory efforts of the industry by the SEC and FSOC.

Over the recent decades, banks have relied more and more on financial intermediaries, such as money markets funds, to finance their investment. Our results, suggest that this trend, while providing investors with valuable diversification opportunities, may increase the instability of the banking system.

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7 Appendix

7.1 The Optimal Contract with Direct Finance

We derive optimal contract as the solution to the planner problem in an economy with direct finance. Note that although each bank A and B offers potentially different contracts

$$c_1^A, c_2^{A,H}, c_2^{A,L}, c_1^B, c_2^{B,H}, c_2^{B,L}$$

it is trivial to show that, under the optimal contract, bank contracts would be identical and investors would invest an equal amount in each bank. Therefore, to simplify notation, we denote the optimal contract by c_1, c_2^H, c_2^L .

Denote with s storage to date 2 per unit of deposit. The optimal contract is the solution to the following optimization problem:

$$\begin{aligned} & \text{Max } \pi u(c_1) + (1 - \pi) \left[u\left(\frac{c_2^H + c_2^L}{2}\right) \right], \\ & \text{w.r.t. } c_1, c_2^H, c_2^L, i, s \\ & \text{s.t.} \\ & \text{date 1} \quad : \quad \pi c_1 = 1 - i - s, \\ & \text{date 2: } (1 - \pi)c_2^H = iR^H + s, \\ & \text{date 2: } (1 - \pi)c_2^L = iR^L + s, \\ & i + s \leq 1, \quad -i \leq 0, \quad -s \leq 0, \end{aligned}$$

where, recall, the second utility term comes from the fact that by investing $\frac{1}{2}$ in each bank, and since banks have perfect negative correlation, patient investors obtain a deterministic return at date 2.

Substituting the equality constraint

$$\begin{aligned} & \text{Max } \pi u\left(\frac{1 - i - s}{\pi}\right) + (1 - \pi) \left[u\left(\frac{i(R^H + R^L) + 2s}{2(1 - \pi)}\right) \right], \\ & \text{w.r.t. } i, s \\ & \text{s.t} \\ & i + s \leq 1, \quad -i \leq 0, \quad -s \leq 0. \end{aligned}$$

The FONCs are:

$$\begin{aligned} -u'(c_1) + u'\left(\frac{c_2^H + c_2^L}{2}\right) \frac{(R^H + R^L)}{2} - \lambda &= 0 \\ -u'(c_1) + u'\left(\frac{c_2^H + c_2^L}{2}\right) 2 - \mu &= 0. \\ \lambda i = 0, \mu s &= 0, \\ \text{where } \lambda, \mu &\geq 0 \end{aligned}$$

which, with the natural log utility function, becomes

$$\begin{aligned} -\frac{1}{c_1} + \frac{1}{c_2^H + c_2^L} (R^H + R^L) + \lambda &= 0 \\ -\frac{1}{c_1} + \frac{1}{c_2^H + c_2^L} 2 + \mu &= 0. \\ \lambda i = 0, \mu s &= 0, \\ \text{where } \lambda, \mu &\geq 0. \end{aligned}$$

There are three cases:

Case 1), with $s = 0, i > 0$.

Then the multiplier $\lambda = 0$ and the first constraint,

$$-\frac{1}{c_1} + \frac{1}{c_2^H + c_2^L} (R^H + R^L) = 0.$$

The solution to the optimization problem is interior and $i = 1 - \pi$.

The second constraint,

$$-\frac{1}{c_1} + \frac{1}{c_2^H + c_2^L} 2 + \mu = 0,$$

where $\mu > 0$. For the constraint to be satisfied, it must be the case

that

$$-\frac{1}{c_1} + \frac{2}{c_2^H + c_2^L} < 0,$$

that is,

$$-\frac{1}{c_1} + \frac{2}{(R^H + R^L)} < 0,$$

or

$$R^H + R^L > 2,$$

which is the condition (1) for an interior solution.

Case 2), with $s > 0, i = 0$.

Then, the multiplier $\mu = 0$, and the second constraint,

$$-\frac{1}{c_1} + \frac{2}{c_2^H + c_2^L} = 0.$$

For the constraint to be satisfied, it must be the case that

$$-\frac{\pi}{1-s} + \frac{(1-\pi)}{s} = 0, \text{ that is,}$$

$$s = (1 - \pi).$$

The first constraint,

$$-\frac{1}{c_1} + \frac{1}{c_2^H + c_2^L}(R^H + R^L) + \lambda = 0,$$

which since $\lambda \geq 0$ implies

$$-\frac{\pi}{1-s} + \frac{(1-\pi)(R^H + R^L)}{s \cdot 2} \leq 0,$$

that is,

$$R^H + R^L \leq 2,$$

in which case the banks' net present value is smaller than zero, and the optimal contract implies zero investment in the long technology.

Case 3), with $s > 0, i > 0$.

Then, both multipliers $\mu, \lambda = 0$, and the constraints become:

$$-\frac{1}{c_1} + \frac{1}{c_2^H + c_2^L}2 = 0,$$

and

$$-\frac{1}{c_1} + \frac{1}{c_2^H + c_2^L}(R^H + R^L) = 0,$$

which can never be the case unless $R^H + R^L = 2$.

As mentioned in the text, we assumed that condition (1) holds, that is, $R^H + R^L > 2$, which implies $i > 0, s = 0$.

7.2 The Optimal Withdrawal by the MMF

Recall that MMF A chooses how much to withdraw from Banks A and B assuming that it can still obtain c_1 for all its investors redeeming early (that is, $\hat{c}_1 = c_1$). This allows us to disregard the welfare of the investors redeeming early from the MMF A . As a result, the optimal withdrawal of MMF A from the two banks is the result of the following maximization problem:

$$Max_{w.r.t. \tilde{\lambda}} 0.5(1-q)u\left(\frac{c_2^{H,A} + c_2^{L,B}}{2}\right) + 0.5u\left(\frac{c_2^{H,B} + c_2^{L,A}}{2}\right) \quad (18)$$

$$+ 0.5qu\left(\frac{c_2^{L,A} + c_2^{L,B}}{2}\right)$$

s.t.

$$c_2^{H,A} = \max\left(R^H\left(1 - \tilde{\lambda}\frac{(1-\pi)q}{r}\right), 0\right),$$

$$c_2^{L,A} = \max\left(R^L\left(1 - \tilde{\lambda}\frac{(1-\pi)q}{r}\right), 0\right),$$

$$c_2^{H,B} = \max\left(R^H\left(1 - (1-\tilde{\lambda})\frac{(1-\pi)q}{r}\right), 0\right),$$

$$c_2^{L,B} = \max\left(R^L\left(1 - (1-\tilde{\lambda})\frac{(1-\pi)q}{r}\right), 0\right).$$

where $\tilde{\lambda}$, and $(1 - \tilde{\lambda})$ represent the fraction of withdrawal that the *MMF* *A* will do in Bank *A* and *B* respectively, and $c_2^{i,j}$ represents date-2 consumption if the returns are low or high, $i = L, H$, by bank $j = A, B$. Let us analyze the three terms in (18) we need to maximize separately:

Term 1:

$$\begin{aligned} \frac{c_2^{H,A} + c_2^{L,B}}{2} &= \frac{(R^H(1 - \tilde{\lambda}\frac{(1-\pi)q}{r})) + (R^L(1 - (1 - \tilde{\lambda})\frac{(1-\pi)q}{r}))}{2} = \\ &= \frac{R^L}{2} \left(\frac{R^H}{R^L} (1 - \tilde{\lambda}\frac{(1-\pi)q}{r}) + (1 - (1 - \tilde{\lambda})\frac{(1-\pi)q}{r}) \right) = \\ &= \frac{R^L}{2} \left(\frac{R^H}{R^L} + 1 \right) - \frac{R^L}{2} \frac{(1-\pi)q}{r} \left(\tilde{\lambda} \left(\frac{R^H}{R^L} - 1 \right) + 1 \right), \end{aligned}$$

which is decreasing in $\tilde{\lambda}$.

Term 2:

$$\begin{aligned} \frac{c_2^{H,B} + c_2^{L,A}}{2} &= \frac{(R^H(1 - (1 - \tilde{\lambda})\frac{(1-\pi)q}{r})) + (R^L(1 - \tilde{\lambda}\frac{(1-\pi)q}{r}))}{2}, \\ &= \frac{R^L}{2} \left(\frac{R^H}{R^L} (1 - (1 - \tilde{\lambda})\frac{(1-\pi)q}{r}) + (1 - \tilde{\lambda}\frac{(1-\pi)q}{r}) \right), \\ &= \frac{R^L}{2} \left(\frac{R^H}{R^L} + 1 \right) - \frac{R^L}{2} \frac{(1-\pi)q}{r} ((1 - \tilde{\lambda}) \left(\frac{R^H}{R^L} - 1 \right) + 1), \end{aligned}$$

which is increasing in $\tilde{\lambda}$.

Term 3:

$$\begin{aligned} \frac{c_2^{L,B} + c_2^{L,A}}{2} &= \frac{(R^L(1 - (1 - \tilde{\lambda})\frac{(1-\pi)q}{r})) + (R^L(1 - \tilde{\lambda}\frac{(1-\pi)q}{r}))}{2} \\ &= R^L - \frac{R^L}{2} \frac{(1-\pi)q}{r}, \end{aligned}$$

which is independent from $\tilde{\lambda}$. Therefore, the solution to the maximization of (18) is the solution to the maximization of the first two terms. That is,

$$\begin{aligned} &0.5(1 - q)u\left(\frac{R^L}{2}\left(\frac{R^H}{R^L} + 1\right) - \frac{R^L}{2}\frac{(1-\pi)q}{r}\left(\tilde{\lambda}\left(\frac{R^H}{R^L} - 1\right) + 1\right)\right) + \\ &0.5u\left(\frac{R^L}{2}\left(\frac{R^H}{R^L} + 1\right) - \frac{R^L}{2}\frac{(1-\pi)q}{r}\left((1 - \tilde{\lambda})\left(\frac{R^H}{R^L} - 1\right) + 1\right)\right), \end{aligned}$$

or

$$\begin{aligned} &(1 - q)u\left(\frac{R^L}{2}\left(\frac{R^H}{R^L} + 1\right) - \frac{R^L}{2}\frac{(1-\pi)q}{r}\left(\tilde{\lambda}\left(\frac{R^H}{R^L} - 1\right) + 1\right)\right) + \\ &u\left(\frac{R^L}{2}\left(\frac{R^H}{R^L} + 1\right) - \frac{R^L}{2}\frac{(1-\pi)q}{r}\left((1 - \tilde{\lambda})\left(\frac{R^H}{R^L} - 1\right) + 1\right)\right), \end{aligned}$$

which, with a logarithmic utility function, becomes:

$$(1-q) \log\left(\frac{R}{2}\left(\frac{R^H}{R^L} + 1\right) - \frac{R(1-\pi)q}{2r}(\tilde{\lambda}\left(\frac{R^H}{R^L} - 1\right) + 1)\right) + \\ \log\left(\frac{R^L}{2}\left(\frac{R^H}{R^L} + 1\right) - \frac{R^L(1-\pi)q}{2r}((1-\tilde{\lambda})\left(\frac{R^H}{R^L} - 1\right) + 1)\right),$$

which is equivalent to

$$(1-q) \log\left(\left(\frac{R^H}{R^L} + 1\right) - \frac{(1-\pi)q}{r}(\tilde{\lambda}\left(\frac{R^H}{R^L} - 1\right) + 1)\right) + \\ \log\left(\left(\frac{R^H}{R^L} + 1\right) - \frac{(1-\pi)q}{r}((1-\tilde{\lambda})\left(\frac{R^H}{R^L} - 1\right) + 1)\right).$$

The FONC of the maximization problem is:

$$(1-q) \frac{-\frac{(1-\pi)q}{r}\left(\frac{R^H}{R^L} - 1\right)}{\left(\left(\frac{R^H}{R^L} + 1\right) - \frac{(1-\pi)q}{r}(\tilde{\lambda}\left(\frac{R^H}{R^L} - 1\right) + 1)\right)} + \\ \frac{\frac{(1-\pi)q}{r}\left(\frac{R^H}{R^L} - 1\right)}{\left(\left(\frac{R^H}{R^L} + 1\right) - \frac{(1-\pi)q}{r}((1-\tilde{\lambda})\left(\frac{R^H}{R^L} - 1\right) + 1)\right)} = 0,$$

which is equivalent to:

$$(1-q) \left(\left(\left(\frac{R^H}{R^L} + 1 \right) - \frac{(1-\pi)q}{r}((1-\tilde{\lambda})\left(\frac{R^H}{R^L} - 1\right) + 1) \right) \right) + \\ \left(\left(\frac{R^H}{R^L} + 1 \right) - \frac{(1-\pi)q}{r}(\tilde{\lambda}\left(\frac{R^H}{R^L} - 1\right) + 1) \right) = 0.$$

Then:

$$-(1-q) \left(\left(\left(\frac{R^H}{R^L} + 1 \right) - \frac{(1-\pi)q}{r}((1-\tilde{\lambda})\left(\frac{R^H}{R^L} - 1\right) + 1) \right) \right) + \quad (19) \\ \left(\left(\frac{R^H}{R^L} + 1 \right) - \frac{(1-\pi)q}{r}(\tilde{\lambda}\left(\frac{R^H}{R^L} - 1\right) + 1) \right) = 0.$$

Let us denote $W = \frac{(1-\pi)q}{r}$, and observe that $W \leq \frac{(1-\pi)2r}{r} < 2$. To simplify notation, denote $A = \left(\frac{R^H}{R^L} + 1\right)$ and $D = \left(\frac{R^H}{R^L} - 1\right) = A - 2$, so that equation (19) becomes:

$$-(1-q)(A - W((1-\tilde{\lambda})D + 1)) + (A - W(\tilde{\lambda}D + 1)) = 0.$$

That is,

$$\tilde{\lambda} = \frac{WD(1-q) + qA - qW}{(2-q)DW}, \quad (20) \\ = \frac{W\left(\frac{R^H}{R^L} - 1\right)(1-q) + q\left(\frac{R^H}{R^L} + 1\right) - qW}{(2-q)\left(\frac{R^H}{R^L} - 1\right)W}.$$

That is,

$$\tilde{\lambda} = \frac{\left(\frac{R^H}{R^L} - 1\right)(1 - q) + \frac{r\left(\frac{R^H}{R^L} + 1\right)}{(1 - \pi)} - q}{(2 - q)\left(\frac{R^H}{R^L} - 1\right)}.$$

Note that, for simplicity's sake, we solved the maximization problem without imposing the condition that the proportion withdrawn from Bank A must be less than 1, and without explicitly considering the non-negativity of the payoff at time 2 that agents receive from both banks. Thus, the optimal level of withdrawal from Bank A is:

$$\lambda = \min \left(\frac{\left(\frac{R^H}{R^L} - 1\right)(1 - q) + \frac{r\left(\frac{R^H}{R^L} + 1\right)}{(1 - \pi)} - q}{(2 - q)\left(\frac{R^H}{R^L} - 1\right)}, 1 \right).$$

Note that if $q = 0$,

$$\lambda = \frac{W\left(\frac{R^H}{R^L} - 1\right)(1 - q) + q\left(\frac{R^H}{R^L} + 1\right) - qW}{(2 - q)\left(\frac{R^H}{R^L} - 1\right)W} = \frac{W\left(\frac{R^H}{R^L} - 1\right)}{2\left(\frac{R^H}{R^L} - 1\right)W} = \frac{1}{2},$$

which means that if the unexpected redemption is low enough not to contain any information on Bank A , it will be met by withdrawing equally from both banks.

Note also that if $q = 1$, then

$$\lambda = \min \left(\frac{\frac{R^H}{R^L} + 1 - W}{\left(\frac{R^H}{R^L} - 1\right)W}, 1 \right) = 1,$$

since $\frac{R^H}{R^L} + 1 < \frac{R^H}{R^L}W$ (recall that $W < 2$ and $\frac{R^H}{R^L} > 1$). This means that if the unexpected redemption is so high that the MMF knows that the return on Bank A assets is low, it will be met by withdrawing from Bank A only.

Finally, note that from (20),

$$\begin{aligned} \frac{d}{dq} \frac{WD(1 - q) + qA - qW}{(2 - q)DW} &= \\ \frac{1}{DW(q - 2)^2} (2A - 2W - DW) &= \frac{1}{DW(q - 2)^2} (2A - W(2 + D)) = \\ \frac{1}{DW(q - 2)^2} (2A - WA) &= \frac{A}{DW(q - 2)^2} (2 - W) > 0, \end{aligned}$$

since $W < 2$. That is, the higher the level of unexpected redemptions q , the higher the proportion of funds withdrawn from Bank A as opposed to Bank B (since the probability that Bank A has a low return is higher).

7.3 Proof of Proposition 1

From the assumption $\pi > \frac{0.5-r}{1-r}$, we know from (16) that $\widehat{c}_1 = c_1 = 1$.

The MMF run condition (15) > (14) becomes:

$$\begin{aligned} & (1 - (\pi + (1 - \pi)q)) 0.5 * \\ & \left[(1 - q) \log\left(\frac{\widehat{c}_{2,A}^H + \widehat{c}_{2,B}^L}{2}\right) + \log\left(\frac{\widehat{c}_{2,B}^H + \widehat{c}_{2,A}^L}{2}\right) + q \log\left(\frac{\widehat{c}_{2,B}^L + \widehat{c}_{2,A}^L}{2}\right) \right] \\ & \leq (1 - (\pi + (1 - \pi)q)) 0.5 \left[\log\left(\frac{c_2^H + 1}{2}\right) + \log\left(\frac{c_2^L + 1}{2}\right) \right]. \end{aligned}$$

Since

$$\begin{aligned} \widehat{c}_{2,A}^H, \widehat{c}_{2,B}^H &\leq c_2^H = R^H \\ \widehat{c}_{2,A}^L, \widehat{c}_{2,B}^L &\leq c_2^L = R^L \end{aligned}$$

and $(1 - (\pi + (1 - \pi)q)) \geq 0$ a fortiori it will be that:

$$\begin{aligned} (1 - q) \log\left(\frac{R^H + R^L}{2}\right) + \log\left(\frac{R^H + R^L}{2}\right) + q \log(R^L) &\leq \\ \log\left(\frac{R^H + R^L}{2}\right) + \log\left(\frac{R^L + 1}{2}\right). & \end{aligned} \quad (21)$$

Hence the inequality (21) will hold iff:

$$\begin{aligned} \log(R^H + R^L)^{(2-q)} + \log(2R^L)^q &\leq \log(R^H + 1) + \log(R^L + 1), \\ (R^H + R^L)^{(2-q)}(2R^L)^q &\leq (R^H + 1)(R^L + 1). \end{aligned} \quad (22)$$

Assume that $r = \frac{1}{2} - \eta$ so that direct finance may lead to bankruptcy for q high enough. Consider $q = 1 - 2\eta = 2r$, so that for this realization of q there is no bankruptcy with direct finance (however small η is). Let us now show that bankruptcy will occur for MMF intermediation for η small enough.

Observe that for $q = 1$ the inequality (22) becomes

$$(R^H + R^L)(2R^L) < (R^H + 1)(R^L + 1),$$

which is always satisfied for any values of R^H and R^L , since $R^L < 1$. Thus by continuity, there will a value of η , such that $q = 1 - 2\eta$ has bankruptcy under MMF intermediation, but not under direct finance.

7.4 Proof of Proposition 2

From the proof of Proposition 1 we know that bankruptcy will occur if (22) is satisfied. By algebraic manipulation of (22) we obtain:

$$q \leq \tilde{q} \equiv \frac{\log \frac{(R^H+1)(R^L+1)}{(R^H+R^L)^2}}{\log \left(\frac{2}{\frac{R^H}{R^L}+1} \right)}. \quad (23)$$

Note that both the denominator and the numerator of the RHS of (23) are negative. The denominator is negative since $\frac{2}{\frac{R^H}{R^L}+1} < 1$. It is easy to show that the numerator is also negative. To see that consider that

$$1 < \frac{1}{2} (R^H + R^L),$$

by condition (1). Thus:

$$\frac{1}{2} (R^H + 1) + \frac{1}{2} (R^L + 1) < (R^H + R^L),$$

which, because of the concavity of the log function, yields

$$\log \frac{(R^H + 1)(R^L + 1)}{(R^H + R^L)^2} < 0.$$

This also shows that the RHS of (23) > 0 .

We also want to study the sign of the derivatives of \tilde{q} with respect to R^L and R^H :

$$\frac{d\tilde{q}}{dR^L} = \frac{R^L + R^H + 2}{R^L \left(\ln \frac{\frac{R^H}{R^L} + 1}{2} \right) (R^H + 1) (R^L + 1)} > 0.$$

Furthermore, denote

$$N = \log \frac{(R^H + 1)(R^L + 1)}{(R^H + R^L)^2} \text{ and } D = \log \left(\frac{2R^L}{R^H + R^L} \right).$$

Then

$$\begin{aligned} \frac{\partial N}{\partial R^H} &= \frac{(R^H + R^L)^2}{(R^H + 1)(R^L + 1)} \left[\frac{(R^L + 1)(R^H + R^L)^2 - 2(R^H + R^L)(R^H + 1)(R^L + 1)}{(R^H + R^L)^4} \right] = \\ &= \frac{1}{(R^H + 1)} \left[\frac{R^L - R^H - 2}{R^H + R^L} \right] < 0, \end{aligned}$$

and

$$\frac{\partial D}{\partial R^H} = -\frac{1}{R^H + R^L} < 0.$$

Hence

$$\begin{aligned} \text{sign} \frac{\partial \tilde{q}}{\partial R^H} &= \text{sign} \left(\frac{\partial N}{\partial R^H} D - \frac{\partial D}{\partial R^H} N \right) => \\ \frac{\partial N}{\partial R^H} D - \frac{\partial D}{\partial R^H} N &= \left[\frac{\overbrace{R^L - R^H - 2}^{<0}}{(R^H + 1)(R^H + R^L)} \right] D + \frac{1}{R^H + R^L} N. \end{aligned}$$

Recall that $D < 0$ and $N < 0$ and that $\tilde{q} \equiv \frac{N}{D} < 1$. Hence $|D| > |N|$. Thus sufficient condition for

$$\text{sign} \left(\frac{\partial N}{\partial R^H} D - \frac{\partial D}{\partial R^H} N \right) > 0$$

is that

$$\begin{aligned} \left| \frac{R^L - R^H - 2}{(R^H + 1)(R^H + R^L)} \right| &> \frac{1}{R^H + R^L} \Leftrightarrow \\ \left| \frac{R^L - R^H - 2}{R^H + 1} \right| &> 1 \end{aligned}$$

which is true since

$$\frac{R^L - R^H - 2}{R^H + 1} = -1 - \frac{1 - R^L}{R^H + 1} < -1.$$

7.5 Proof of Proposition 3

Recall that if $\pi > \frac{0.5-r}{1-r}$, then $\hat{c}_1 = 1$. Therefore, the contagion condition (17) becomes

$$0 > (1 - (\pi + (1 - \pi)q)) \left[0.5 \log \left(\frac{c_{2,B}^H + 1}{2} \right) + 0.5 \log \left(\frac{c_{2,B}^L + 1}{2} \right) \right]. \quad (24)$$

We know from condition (3) that

$$0 > \left[0.5 \log \left(\frac{c_{2,B}^H + 1}{2} \right) + 0.5 \log \left(\frac{c_{2,B}^L + 1}{2} \right) \right],$$

which ends the proof.