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Working Paper

Studying international spillovers in a New Keynesian continuous time framework with financial markets

MAGKS Joint Discussion Paper Series in Economics, No. 42-2013

Provided in Cooperation with:

Faculty of Business Administration and Economics, University of Marburg

Suggested Citation: Hayo, Bernd; Niehof, Britta (2013) : Studying international spillovers in a New Keynesian continuous time framework with financial markets, MAGKS Joint Discussion Paper Series in Economics, No. 42-2013, Philipps-University Marburg, Faculty of Business Administration and Economics, Marburg

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MAGKS



**Joint Discussion Paper
Series in Economics**

by the Universities of
Aachen · Gießen · Göttingen
Kassel · Marburg · Siegen

ISSN 1867-3678

No. 42-2013

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This paper can be downloaded from
http://www.uni-marburg.de/fb02/makro/forschung/magkspapers/index_html%28magks%29

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Studying International Spillovers in a New Keynesian Continuous Time Framework with Financial Markets *

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Preliminary Draft. This version September 10, 2013

Abstract

In light of the recent financial and real economic crisis, it seems clear that macroeconomists need to better account for the influence of financial markets. This paper explores the consequences of treating the interaction between different financial markets, monetary policy, and the real economy seriously by developing a fully dynamic theoretical model. Starting from a standard New Keynesian framework, we reformulate and extend the model by means of stochastic differential equations so as to analyse spillover effects and steady-state properties. We solve the model for theoretically derived parameters, distinguishing between (almost) closed, equally sized, and differently sized economies. Applying Bayesian estimation methods, we estimate model parameters for Canada and the United States. Using Lyapunov techniques, we find evidence of instability in the US and Canadian financial systems.

Keywords: New Keynesian Model, Philipps Curve, Taylor Rule, Stochastic Differential Equations.

JEL Classification Numbers: C02, C63, E44, E47, E52 F41

* Thanks to Florian Neumeier, Matthias Uhl Jerg Gutmann, Michael Bergman and participants of ICMAIF 2013

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1 Introduction

The recent economic crisis emphasises the relevance of financial markets for macroeconomic stability. This should not come as a surprise: financial markets played an important role in previous major crises, too, such as the Great Depression and the Asian financial crisis. Arguably, these crises were associated with financial market turmoil and each required a different stabilisation policy to avoid further (financial) contagion. The Asian crisis was caused by speculative attacks on the Thai Baht and the Great Depression was aggravated by a major stock market crash. The ongoing European sovereign debt crisis was triggered by speculative attacks on government bonds following an increase in public debt, which was partly caused by government intervention to support the banking sector following the recent financial crisis.

The current crisis suggests that globalisation has resulted in much greater international economic interconnectedness than was the case in the past. Thus, economic policy-makers need to not only be aware of financial market developments in their own economy, but also of those occurring abroad. In addition, they have to keep in mind differences in the size and structure of financial markets so as to be able to design appropriate policy. In a world still largely dominated by national policy-making, this is a considerable hurdle in adequately responding to macroeconomic and financial crises.

In light of this situation, we believe a better understanding of the role different financial markets play in a macroeconomy is crucial. Furthermore, observing not only financial market actions but also financial market reactions to different policies is helpful in designing appropriate (monetary) policy. Put differently, policy-makers' responses to shocks could likely be improved by the availability of a thorough theoretical framework. However, common macroeconomic approaches used by central banks, both analytically as well as empirically, tend to downplay the role of the financial system in monetary policy, and rarely, if ever, take into account different domestic and foreign financial markets.

Thus, we argue that academic research can help guide central banks' efforts by placing greater emphasis on financial markets and moving beyond common macroeconomic models. Therefore, the purpose of this paper is to explore the consequences of treating seriously the interaction between financial markets, monetary policy, and the real economy in a globalised world by developing a fully dynamic theoretical modelling framework. We are particularly interested in the relationship between financial markets and monetary policy, which can be characterised by a substantial degree of simultaneity.

In mainstream macroeconomic research, the New Keynesian (NK) model Clarida, Gali, and Gertler (1999), Romer (2000), Woodford (1999), and Blinder (1997) is a frequent starting point for analysing monetary policy. We follow this strand of literature and adopt the three-equation NK model as our baseline model. Two of the equations, aggregate demand and aggregate supply, describe the structure of the economy. The third equation specifies monetary policy reactions in the form of a Taylor rule and reflects policy-makers' main targets. A general overview of the Taylor rule is provided by Asso, Kahn, and Leeson (2007). The NK model is a mix of traditional Keynesian and new classical approaches. It is largely consistent with important features of real business-cycle models, but also includes monopolistic competition and nominal rigidities. Arguably, including the Taylor rule as the monetary policy reaction function helps link theory and practice (Gali 2008). Empirical tests of the conventional NK model show some support for this class of models, but the evidence is not overwhelmingly strong (Negro et al. 2007; Gali, Smets, and Wouters 2011; Dennis 2004; Schorfheide 2011; Christiano and Eichenbaum 2005; Rotemberg and Woodford 1998; Smets and Wouters 2003; Peersman and Straub 2006; Lubik and Smets 2005; Ireland 2001).

As our main interest lies in the interaction between monetary policy and financial markets, we need to extend the NK model. Our starting point is that monetary policy reacts to financial markets and financial markets react to monetary policy and that this relationship is characterised by a notable degree of simultaneity. This is not a new idea; other studies take such simultaneity into account. However, most of this work is empirical in nature, e.g., (Rigobon and Sack 2003; Rigobon 2003; Bjornland and Leitemo 2009). Rigobon (2003) and Rigobon and Sack (2003) observe the effects of bonds and stocks on monetary policy and vice versa by applying a novel approach (identification through heteroscedasticity) to circumvent simultaneity issues. Bjornland and Leitemo (2009) adopt a VAR approach for studying how financial markets affect monetary policy. Technically, they deal with the simultaneity issue by imposing *a priori* short-run and long-run restrictions. Both studies find evidence of monetary policy reaction to financial market developments.

Less formally, Hildebrand (2006) argues that financial markets are the link between monetary policy and the real economy and are an important part of the transmission mechanism for monetary policy. Moreover, he argues that financial markets reflect expectations about future inflation and output and therefore are also affected by monetary policy. Christiano et al. (2008) put forward a more formal empirical approach. In considering the problem of boom-bust cycles in the economy, they find evidence that it

might be more expedient for monetary policy-makers to target credit growth instead of inflation. Faia and Monacelli (2007) find evidence that monetary policy should respond to asset price hikes. Reflecting these considerations, Belke and Klose (2010) estimate Taylor rules for the European Central Bank (ECB) and the Federal Reserve (Fed) and include asset prices as additional monetary policy targets. They find statistically significant evidence that short-term interest rates respond to financial market developments, but the estimated quantitative economic effects appear to be small.

The studies just discussed focus on stock or bond markets. However, another important financial market is the one for foreign exchange. Clarida, Gali, and Gertler (2002) incorporate the exchange rate in a NK two-country model, where domestic and foreign households share the same preferences. Under quite restrictive assumptions, they find that purchasing power parity (PPP) holds and that the consumption real exchange rate is constant. Gali and Monacelli (2005) extend this approach by applying Calvo sticky pricing and analysing the policy effects of either a Taylor rule or an exchange rate peg. Engels (2009) extend this open economy model by incorporating local-currency pricing and allowing for differences in domestic and foreign household preferences. Including the exchange rate in the monetary policy analysis not only takes into account a large and important financial market; the exchange rate itself could also be viewed as a policy objective. For example, Leitemo and Söderström (2005) include exchange rate uncertainty in a NK model and analyse different monetary policy rules. They find evidence that an interest rate reaction function in the form of a Taylor rule works particularly well. Similarly, Wang and Wu (2012) report that in their analysis of a group of exchange rate models for 10 OECD countries, the Taylor rule performs best empirically as a monetary policy rule. Taylor (2001) generally discusses the role of the exchange rate in monetary policy rules. Reflecting these empirical and theoretical findings, we model the policy reaction function as a Taylor rule.

Our paper makes several contributions to the literature. First, we include exchange rates as well as stock prices in the monetary policy equation. Second, we explicitly address the issue of simultaneity between monetary policy and financial markets by incorporating three financial markets (i.e., markets for foreign exchange, bonds, and stocks) as an additional set of equations in our baseline model. Third, we undertake formal mathematical analysis of the extended model in a continuous-time framework. This requires transforming the NK model in stochastic differential equations, using numerical algorithms to derive stable solutions, and studying the evolution of various variables over time. Our reading of the literature is that this combination of applying Taylor rules

to financial markets and extending the NK model with three dynamic financial market equations is unique. Fourth, we estimate model parameters using Bayesian estimation techniques for the United States and Canada.

Our approach is somewhat similar to papers by Asada et al. (2006), Chen et al. (2006a), and Chen et al. (2006b). These authors transform the Keynesian AS/AD model into a disequilibrium model with a wage-price spiral and include two Phillips curves, one targeting wages, the other targeting prices. The model is transformed into five differential equations - explaining real wages, real money balances, investment climate, labour intensity, and inflationary climate - and its dynamics are extensively analysed. Malikane and Semmler (2008b) extend this framework by including the exchange rate and Malikane and Semmler (2008a) consider asset prices. However, none of these studies includes both asset markets and the exchange rate, particularly not in a framework controlling for the simultaneity between monetary policy and financial markets.

In the next section, we develop the theoretical model. Section 2 contains a brief description and analysis of the baseline model. In Section 3, we study this model further by making various assumptions about size and degree of openness of the simulated economies. Section 4 concludes.

2 The Baseline Model

We base our model on the New-Keynesian three equation model of Ball (1998) (respectively Clarida, Gali, and Gertler (1999), Gali and Monacelli (2005), Allsopp and Vines (2000), Blanchard (2007), and Blanchard (2008)). For the sake of brevity, we omit the full derivation of each equation and refer the interested reader to Woodford (2005) and Gali and Gertler (2007).

2.1 The Phillips Curve

Aggregate supply is represented by the NK Phillips curve (NKPC), as it is done by, for example, Blanchard (2007) and Blanchard (2008). The NKPC evolves from a Calvo pricing equation, which mirrors price rigidities by allowing only a fraction of firms to implement price changes at one point in time. Several extensions and variations of the NKPC are proposed in the literature. For example, Roberts (1995) applies an expectation-augmented Phillips curve and Woodford (2005) studies the NKPC by including firm-specific capital. A general overview of recent work in this area is provided

by turnovsky. However, since our focus is on the integration of financial markets, we employ the original NKPC to keep the model tractable. Aggregate demand is modelled by a (dynamic) investment/saving (IS) curve. Alternative open-economy NKPCs are developed by Bofinger, Mayer, and Wollmershäuser (2005), Gali and Monacelli (2005), and Mihailov, Rumler, and Scharler (2011)

We follow Ball (1998) and specify aggregate supply as the result of a firm's incentive to maximise profits in a monopolistic competition setting and assume a linear production technology $Y_t = A_t N_t$, where A_t denotes productivity and N_t denotes labour. Reflecting Calvo (1983), a forward-looking aggregate supply equation can be derived by maximising a firm's profit. We assume that real marginal costs are proportional to the output gap (Woodford 2005; Ball 1998). To incorporate non-rational behaviour, we follow Woodford (2005) and Ball (1998) and allow a fraction of firms to follow a 'rule of thumb' instead of adjusting prices in a forward-looking manner. Furthermore, domestic shocks are subject to a random innovation η (2). In line with Jensen (2002) and Ramón and Vázquez (2006), we incorporate the disturbance as a standard AR(1) process, where ξ_t denotes white noise with zero mean and constant variance (see Equation 2).

In an open economy, the general price level consists of the prices of domestic and foreign goods. Thus, the price of imported goods depends on the real exchange rate e_t (see also Equation 4), which is defined in terms of the domestic price level as:

$$e_t = s_t + \pi_t^* - \pi_t$$

where s_t denotes the nominal exchange rate and π (π^*) is the domestic (foreign) rate of inflation. The exchange rate is subject to the uncovered interest rate parity, and we also account for the Fisher effect, we can transform the exchange rate into Equation 4. Moreover, since foreign and domestic goods are included in the exchange rate, they are not directly expressed in the open-economy Phillips curve (in line with Ball (1998) and Leitemo and Söderström (2005)). Equation (2.1) gives the complete open-economy New Keynesian Phillips curve, where y is the output gap and α_i s are weighing parameters

$$\pi_t = \pi_{t-1} + \alpha_y y_{t-1} - \alpha_e (e_{t-1} - e_{t-2}) + \eta_t \quad (1)$$

$$\eta_t = \alpha_\eta \eta_{t-1} + \xi_t^\epsilon \quad (2)$$

2.2 The Investment and Savings Curve

In constructing the aggregate demand equation, we follow Ball (1998) and adapt a simple open-economy investment and savings (IS) curve. In line with Clarida, Gali, and Gertler (1999) and Allsopp and Vines (2000), we also account for foreign output. We assume a small open economy, the inhabitants of which are representative households seeking to maximise their inter-temporal utility function. A typical utility function employed in the extant literature consists of present and lagged consumption and number of hours worked. Defining N_t as number of labour hours, C_t as a consumption index (consisting of fractions of domestic and foreign goods $C_{H,t}$, $C_{F,t}$) and $C_{i,t}$ as an index of the quantity of goods imported from country i and consumed in country j , we consider the following CES utility function

$$U(C_t, C_{t-1}, N_t) = \Psi \frac{(C_t - C_{t-1})^{1-\alpha}}{1-\alpha} - \frac{N_t^{1+\beta}}{1+\beta}$$

D_t is the nominal payoff in period t and $Q_{t,t+1}$ the stochastic discount factor for a one-period ahead nominal payoff. Domestic households are subject to the following budget constraint:

$$P_t C_t + E(Q_{t,t+1} D_{t+1}) \leq D_t + W_t N_t$$

Solving for the first order conditions yields the Euler equation

$$C_t = \gamma R_t E_t \left[\left(\frac{C_{t+1} - h C_t}{C_t - h C_{t-1}} \right) \left(\frac{P_t}{P_{t-1}} \right) \right]$$

After log-linearisation, ignoring investment and government spending, we obtain

$$\begin{aligned} c_t &= a_1 c_{t-1} + a_2 E_t c_{t+1} - a_3 (i_t - E_t \pi_{t+1} - \bar{r}) \\ y_t &= a_4 c_t + a_5 y_t^* - a_6 y_t + a_7 (e_t + p_t^* - p_t) + u_t \end{aligned}$$

where i is the nominal interest rate, y is the output gap, e the exchange rate and r the natural real interest rate. Combining the two equations and renaming the parameters, we obtain the IS equation.

$$y_t = \lambda_y y_{t-1} - \lambda_i (i_{t-1} - \pi_{t-1}) - \lambda_{y^*} y_{t-1}^* - \lambda_e e_{t-1} + \epsilon_t \quad (3)$$

As in Equation (2), ϵ_t follows an autoregressive process (AR(1)), with a mean-zero innovation with standard deviation σ_ϵ

$$\epsilon_t = \lambda_\epsilon \epsilon_{t-1} + \xi_t^\epsilon$$

2.3 The Exchange Rate

Focusing on the short run, we allow exchange rate adjustment to incorporate the uncovered interest parity condition. However, the long-run-oriented PPP may not hold in this setup. In line with Ball (1998), McCallum (1994) and Batini and Nelson (2000), the exchange rate is a function of the nominal interest rate and inflation. To allow for an explicit analysis of exchange rate bubbles, we follow Batini and Nelson (2000) and add another state variable φ to reflect the potential bursting of a bubble (see Equation (4)). This implies that there is an explosive time series parameter such that a closed solution cannot be computed. A detailed description of the additional variable is provided by Batini and Nelson (2000)

$$e_t = \theta_e e_{t-1} + \theta_i(i_t - \pi_t) - \theta_{i^*}(i_t^* - \pi_t^*) + \psi_t(+\varphi_t) \quad (4)$$

Similar to Equation (2), we define the error term ψ_t to follow an AR(1) process. Note that we set the bubble equal to zero in the following baseline analysis. However, inclusion of a bubble variable makes the model flexible enough to encompass financial market crisis scenarios.

2.4 The Financial Markets

As far as we know, including a complete financial market framework in the NK setup is novel. To facilitate our approach, we initially follow Bekaert, Cho, and Moreno (2010) and model assets in discrete time. We then develop a consumption-based pricing kernel out of the IS equation and derive the asset price afterward in continuous time. To the best of our knowledge, we are the first researchers to employ a continuous-time framework. We believe that this framework is helpful, especially since it allows applying common modelling tools from the field of finance, e.g., Brownian motion processes, to macroeconomic models.

Affine term structure models require linear state variable dynamics and a linear pricing kernel process with conditionally normal shocks (Duffie and Kan 1996). To ensure that the equations derived above belong to the affine class, we need to prove that the shocks

are distributed normally. In Equations (3), (4) and (2.1), we specify all shocks as AR(1) processes (in line with much of the extant econometric and theoretical literature). For example, the supply shock is $\epsilon_t = \lambda_\epsilon \epsilon_{t-1} + \xi_t$ and ξ_t is a Gaussian process. Then ϵ_t is normally distributed if λ_ϵ is close to 1 (Hamilton 1994). Moreover, we assume $\xi_t \sim N(0, 1)$ and, hence, the unconditional distribution (that is, without further knowledge of prior values) is $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. Combining the shocks leads to a normal distribution $\zeta \sim N(0, \Sigma)$ (in matrix notation).

According to asset pricing theory (Cochrane 2001), the first order necessary condition of the pricing kernel B_{t+1} implies

$$E_t(B_{t+1}R_{t+1}) = 1$$

where R_{t+1} is the risk-free rate t . Reformulating yields $R_{t+1} = \frac{P_{n-1,t+1}}{P_{n,t}}$, where $P_{n,t}$ is the price of an asset with n periods at time t and R_t is the payoff (which is normalised to 1). If $B_{t+1} > 0$, the resulting returns satisfy the no-arbitrage condition (Harrison and Kreps 1979). Taking the logarithm of the pricing kernel leads to a conditionally linear process such that

$$b_{t+1} = -i_t - \frac{1}{2}\Lambda_t'\Sigma\Lambda_t - \Lambda_t'\zeta_{t+1}$$

where

$$\begin{aligned}\Lambda_t &= \Lambda_0 + \Lambda_1 x_t = \Lambda \\ &= (1 \ \sigma \ 0 \ 0 \ 0)C - (0 \ (\sigma + \eta) \ 0 \ 0 \ 0)\end{aligned}$$

σ and η are parameters derived from the IS equation ($\frac{1}{\sigma+\eta} = \lambda_i$). As Σ is time independent, we obtain a Gaussian system. Such a system is studied by Dai and Singleton (2000). The model implies an affine term structure and the kernel implies that the IS equation still fits the homoscedastic class (this means that there may be a risk premium, but it does not vary over time).¹

Following Bekaert, Cho, and Moreno (2010) the pricing kernel depends on the inter temporal consumption marginal rate of substitution. Prices are affine functions of the state variables. Asset pricing theory (Cochrane 2001) suggests that the price of an asset/ a security is $p_{n,t} = E_t(b_{t+1} + p_{n-1,t+1}) + 0.5V_t(m_{t+1}, p_{n-1,t+1})$. Bekaert, Cho, and Moreno

¹ Note that a Gaussian prices of risk model and a square root model could also be incorporated, these would allow for time varying premia.

(2010) show that this can be rewritten as an affine equation of the asset/ bond price, where p is (a vector of) assets and β_k are (vectors of) weighing parameters.

$$p_{n,t} = a_n + b_n x_t$$

As the parameters depend on the model, Bekaert, Cho, and Moreno (2010) show that there is no explicit way of reformulating these. Instead, we use our knowledge about the dimensions of the matrices, normalised by maturity n and payoff R and rewrite the price as

$$p_t = \beta + \beta_p p_{t-1} + \beta_i (i_{t-1} - \pi_{t-1}) + \beta_y y_t + \beta_e e_t + \xi_t \quad (5)$$

Note that p might be a vector that includes different financial market instruments. Here, we model bond and stock market equations for each country.

We augment the standard representation of stocks and bonds (see Merton (1969)), reflecting our assumptions of simultaneity and highly interacted markets, by incorporating the monetary policy rate, the foreign bond yield, and the log-linearised domestic stock price into the bond yield equation. Similarly, we add the bond yield, the domestic monetary policy rate, and the foreign stock price to the stock price equation. Since we consider stock prices to represent an important link between the real and the monetary economy, we add the output gap to the stock market equation. Regarding the price equations for stocks and bonds, we account for the future stream of dividend payments by adding the inflation rate positively and the (nominal) interest rate negatively (such that the real interest rate has a negative effect). This approach accounts for the Fisher effect and stock prices rise with increasing inflation. Including the output gap in the stock market is in line with Cooper and Priestley (2009) and Vivian and Wohar (2013). A general approach to accounting for macroeconomic factors in stock returns is provided by Pesaran and Timmermann (1995). We make no assumptions about how bond and stock markets influence each other. When bond yields are inside a small interest rate band (of realistic size), the correlation between bonds yields and stock yields (and equity prices) is mainly positive. However, when yields are outside this band, the correlation is mainly negative.

2.5 The Monetary Policy Rule

If we were studying a closed economy, the optimal monetary policy rule would be a weighted sum of output variance and inflation variance (Taylor 1993). However, in an open economy, the domestic economy interacts with the rest of the world through trade in goods and financial assets. Both aspects are represented by sectors in the model. We also allow for influences of the exchange rate ball and the financial system (Lubik and Smets 2005). To compute the (optimal) Taylor rule, we substitute Equation (4) in Equation (3).

$$y_{t+1} = \lambda_y y_t - \lambda_{y^*} y_t^* - \left(\frac{\lambda_i}{\theta_i} + \lambda_e \right) e_t + \frac{\lambda_i \theta_e}{\theta_i} e_{t-1} - \frac{\lambda_i \theta_{i^*}}{\theta_i} (i_t^* - \pi_t^*) + \frac{\lambda_i}{\theta_i} \psi_t + \epsilon_{t+1} \quad (6)$$

$$\pi_{t+1} = \pi_t + \alpha_y y_t - \alpha_e (e_t - e_{t-1}) + \eta_{t+1} \quad (7)$$

$$p_t = \beta + \beta_p p_{t-1} + \beta_y y_t + \left(\beta_e + \frac{\beta_i}{\theta_i} \right) e_t + \frac{\beta_i \theta_{i^*}}{\theta_i} (i_t^* - \pi_t^*) + \frac{\beta_i}{\theta_i} \psi_t + \xi_t \quad (8)$$

The state variables correspond to the expressions on the right side of the equations, modelling a policy maker who chooses the current exchange rate. Transforming these equations yields

$$\lambda_y y_t + \frac{\lambda_i \theta_e}{\theta_i} e_{t-1} + \frac{\lambda_i}{\theta_i} \psi_t - \lambda_{y^*} y_t^* - \frac{\lambda_i \theta_{\pi^*}}{\theta_i} (i_t^* - \pi_t^*) \quad (9)$$

$$\pi_t + \alpha_y y_t + \alpha_e e_{t-1} \quad (10)$$

$$\beta + \frac{\beta_i \theta_{\pi^*}}{\theta_i} (i_t^* - \pi_t^*) + \beta_p p_{t-1} + \frac{\beta_i \theta_e}{\theta_i} e_{t-1} + \beta_y y_t - \frac{\beta_i}{\theta_i} \psi_t \quad (11)$$

Since the model is linear-quadratic, the optimal rule is linear in the three state variables

$$e_t = m \left(\lambda_y y_t + \frac{\lambda_i \theta_e}{\theta_i} e_{t-1} + \frac{\lambda_i}{\theta_i} \psi_t - \lambda_{y^*} y_t^* - \frac{\lambda_i \theta_{\pi^*}}{\theta_i} (i_t^* - \pi_t^*) \right) + n (\pi_t + \alpha_y y_t + \alpha_e e_{t-1}) + \left(\beta + \frac{\beta_i \theta_{\pi^*}}{\theta_i} (i_t^* - \pi_t^*) + \beta_p p_{t-1} + \frac{\beta_i \theta_e}{\theta_i} e_{t-1} + \beta_y y_t - \frac{\beta_i}{\theta_i} \psi_t \right) \quad (12)$$

Substituting ψ_{t-1} by Equation (4) and collecting terms leads to a policy rule targeting the nominal interest rate and exchange rate

$$(1 - \omega_e) e_t + \omega_i i_t = \beta + \beta_p p_{t-1} + \omega_e e_{t-1} + \omega_\pi \pi_t + \omega_y y_t + \omega_{y^*} y_t^* \quad (13)$$

Thus, our Taylor rule accounts for domestic and foreign output, the exchange rate, inflation and the financial market. Note that, δ the state variable reflecting the potential bursting of a bubble from Equation (4), is a combination of the corresponding parameters of our prior equations. However, for ease of presentation, we renamed the fractions in Equation (13). This open-economy Taylor rule allows us to thoroughly study spillover effects between financial markets and monetary policy as well as their transmission to both domestic and foreign real economies. By taking into account these effects, our main open-economy approach is similar to that of Svensson (2000) and Lubik and Schorfheide (2007), and Leitemo and Söderström (2005). Moreover, by including the financial sector, consisting of various markets, we account for a direct relationship between monetary policy and financial markets. Econometric analyses to support this assumption are provided by Rigobon (2003) and Rigobon and Sack (2003).

Our model in discrete time, as derived above, can be summarised as

$$\begin{aligned} y_t &= \lambda_y y_{t-1} - \lambda_i (i_{t-1} - \pi_{t-1}) - \lambda_{y^*} y_{t-1}^* - \lambda_e e_{t-1} + \epsilon_t \\ \pi_t &= \pi_{t-1} + \alpha_y y_{t-1} - \alpha_e (e_{t-1} - e_{t-2}) + \eta_t \\ e_t &= \theta_e e_{t-1} + \theta_i (i_t - \pi_t) - \theta_{i^*} (i_t^* - \pi_t^*) + \psi_t (+\varphi_t) \\ p_t &= \beta + \beta_p p_{t-1} + \beta_i (i_{t-1} - \pi_{t-1}) + \beta_y y_t + \beta_e e_t + \xi_t \\ i_t &= \frac{1}{\omega_i} ((1 - \omega_e) e_t + \beta + \beta_p p_{t-1} + \omega_e e_{t-1} + \omega_\pi \pi_t + \omega_y y_t + \omega_{y^*} y_t^*) \end{aligned} \quad (14)$$

2.6 Switching to Continuous Time

Reflecting work by Asada et al. (2006), Chen et al. (2006a), Chen et al. (2006b), Malikane and Semmler (2008b), and Malikane and Semmler (2008a), we switch to a continuous time framework by taking first differences of the equations and using stochastic differential equations to model the shocks as Brownian motions.

The main concept that allows us to switch from discrete to continuous time is time-scale calculus. Time-scale calculus is a formal unification of the theory of difference equations with that of differential equations. A general overview is provided by Agarwal et al. (2002); the theoretical basis for applying this concept to stochastic differential

equations is given by Sanyal (2008). Taking first differences and changing to differential equations combines the parameters in a new and different way. For reasons of simplicity, instead of using the combined terms, we continue to refer to the parameters by the same names they had previously. Prior parameters can be recovered by solving a system of linear equations.

In line with the relevant finance literature (Barndorff-Nielsen and Shephard 2001; Malkiel and Fama 1970; Fama 1965), we model stock prices as geometric Brownian motion processes and bond yields as Ornstein-Uhlenbeck processes. Since disturbances are specified as AR(1) processes, we can interpret the main macroeconomic variables as standard Brownian motions. Specifically,

$$\begin{aligned}
dy &= (\lambda_y y - \lambda_i i + \lambda_\pi \pi - \lambda_{y^*} y^* - \lambda_e e)dt + \sigma_y dW_t^y \\
d\pi &= (\alpha_\pi \pi + \alpha_y y - \alpha_e e)dt + \sigma_\pi dW_t^\pi \\
de &= (\theta_e e + \theta_i i - \theta_\pi \pi - \theta_{i^*} i^* + \theta_{\pi^*} \pi^*)dt + \sigma_e dW_t^e \\
dp &= (\omega_p p + \omega_e e + \omega_y y + \omega_i i - \omega_\pi \pi + \beta)dt + \sigma_p dW_t^p \\
di &= (\gamma_i i + \gamma_\pi \pi + \gamma_y y - \gamma_{y^*} y^* + \gamma_e e + \gamma_p p)dt
\end{aligned} \tag{15}$$

Note that p can be specified as a vector that includes various assets, each of which can be included linearly and also priced differently.

In general, stability is an important aspect of differential equations. Loosely speaking, stable solutions are those that vary only very little after small changes in the initial values. In economic terms, this means that if an economy drifts away from its steady state, it will return or, alternatively, it will not move very far away. To analyse the stability of Model (15), we rely on Lyapunov techniques. For a thorough discussion, see (Khasminskii 2012). We apply the following Lyapunov function

$$V(x) = \|x\|_2^2 = \left(\sqrt{\sum |x_i|^2} \right)^2 \tag{16}$$

where $\|\cdot\|_2$ denotes the Euclidean norm. Since the zero solution is only locally stable, there is no global stable rest point. However, even though there are only parameter-dependent partial solutions, these are 'almost' stable. In the following section, we analyse stability for each set of parameters we derive.

Note that we concentrate our dynamic analysis on short-run adjustments. Within this time frame, there is no guarantee that the variables will actually return to their starting

values. Thus, although the system is stable in a Lyapunov sense, we find that within our window of analysis, some variables continue in a clear trend and show no tendency to return to their long-term equilibrium.

3 Baseline Scenario

To analyse different policies and scenarios, we delineate various stylised economies both in the context of deriving a stable solution as well as in the context of obtaining empirical evidence. To the best of our knowledge, the literature contains no empirical estimations for such a highly interacted model.

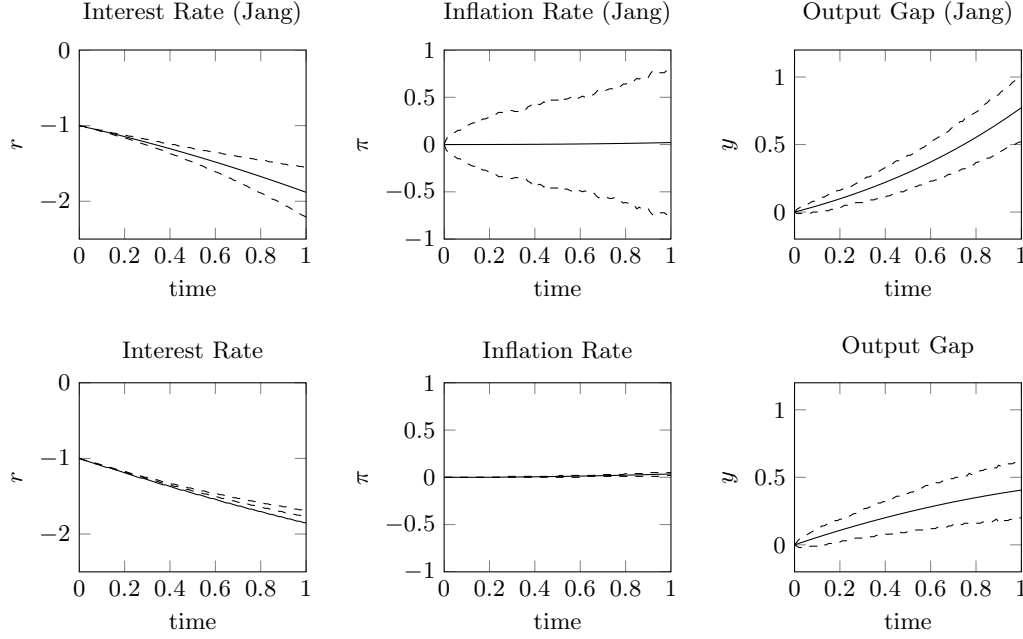
3.1 Core Features of the NK Model

To put our empirical approach into perspective, we compare the main characteristics of the well-established NK model by Clarida, Gali, and Gertler (1999) with the dynamics generated by our model. Specifically, we estimate the continuous-time three-equation model of a closed economy (including the closed-economy IS curve, Philipps curve, and a Taylor rule) via Bayesian techniques and approximate trajectories by applying the Euler-Maruyama scheme (Saito and Mitsui 2001). Our estimations are based on 500,000 replications. (A thorough description of our Bayesian approach and the data is provided in section (3.3).) We then compare our estimates to prior studies. For the practical implementation, we use, first, values from Jang (2012) and, second, our empirical point estimates. Solving the SDEs for both sets of parameters, we generate the trajectories shown in 1. Both approaches arrive at similar solutions and are in accordance with Clarida, Gali, and Gertler (1999): if monetary policy is on a downward path, output increases above potential. However, since this is a short-term model, inflation does not react notably. We show later that due to the influence of the exchange rate, the open economy is characterised by a faster transmission of monetary policy impulses to inflation.

3.2 Economic Analysis of the NK Model with Two Financial Markets

For ease of interpretation, we generate a stable scenario describing stylised versions of closed and open economies. First, we extend the conventional model of an (almost) closed economic environment by including financial markets. Specifically, we set the

Figure 1: Comparison of the Three-Equation NK Model



model parameters so that the influence of the exchange rate is almost zero. Second, we fix the parameters in such a way that we obtain two equally sized open economies. Moreover, we ensure that the financial sector and the real sector of each economy are integrated with each other. Third, we consider the case of a big and a small open economy by allowing for an asymmetric impact of foreign shocks. In this case, we use Bayesian estimation techniques to specify a model based on data from the United States and Canada.

As noted above, the model does not have a unique steady state (stable solution); therefore, we rely on parameter calibration to analyse key characteristics of our continuous-time NK model. As pointed out in Gardiner (2009), calibration of the parameters is an arbitrary decision, and using different parameters can result in different stable solutions. We partially address this criticism by applying economically reasonable parameters. Our approach is as follows. First, we derive a theoretically motivated steady state. Second, we choose parameters such that the ratios of important macroeconomic variables fit standard economic theory and/or empirical stylised facts and provide a stable model solution. Third, we apply initial values that are different from our steady-state values and solve for the corresponding SDEs. We thus can analyse those iterations of the model that (i) provide established trajectories, (ii) destabilise the system, or (iii) contradict mainstream theory.

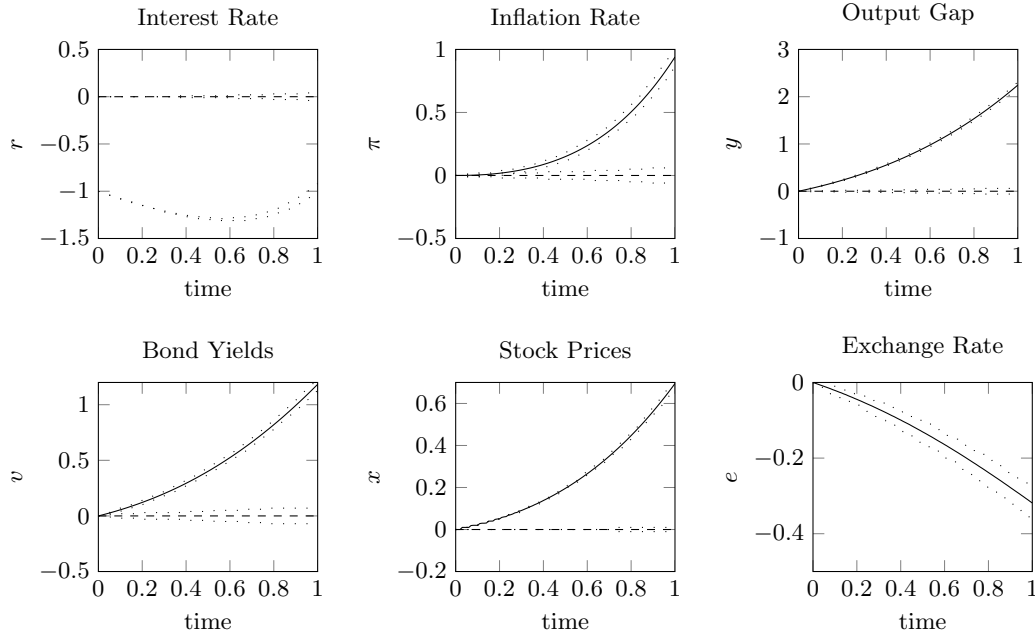
In accordance with the idea behind PPP, the steady-state nominal exchange rate takes the value 1. Furthermore, in line with the inter-temporal budget constraint, we restrict the long-run trade balance to be zero. Additionally, we assume a stable monetary environment and choose small values for the inflation rate and short-term interest rate, which is the monetary policy instrument. To implement the *ex post* Fisher effect, we compute bond yields as the sum of inflation rate and real interest rate. For the sake of simplicity, we set stock prices to unity.

Note that our economy now has two locally stable solutions, the zero solution as well as the economically derived solution used to calibrate the system. For ease of interpretation, we apply the zero solution and then shock the system. There are various ways to fix the initial values. We commence the analysis with a monetary policy shock in the form of a decrease in the interest rate. Alternatively, we could study a 'turbulent times' situation characterised by a given current account disequilibrium.

Model parameters are chosen in line with standard New Keynesian theory and detailed parameter information is given in Table (1). All scenarios start with the same initial values and, by construction, share the same rest point, which makes them easy to compare. We change as few parameters as possible, e.g., in the case of an (almost) closed economy, we greatly diminish the influence of the exchange rate and exclude all international interaction terms. In line with Taylor-rule-related econometric research, we give the largest weight to the interest rate, whereas the parameters associated with the exchange rate or stock and bond markets, which commonly are excluded, receive relatively small weights. We arrange the remaining parameters in a similar way. All analyses of the different stylised economies start with an expansionary monetary policy in Country A. Discovering the interaction between monetary policy and financial markets is the main purpose of our simulations. Note that all scenarios are stable in the Lyapunov sense.

The first scenario in Figure (2) illustrates the dynamic adjustment in two closed economies. It enriches the three-equation New Keynesian model by taking into account financial markets. A good example of a big and relatively closed economy is the United States, which, however, is embedded in a globalised world. In the following figures, a solid line represents Country A and a dashed line Country B. Country A's central bank engages in expansionary monetary policy by decreasing the short-term interest rate. This raises both its output gap as well as its inflation rate. The latter effect is rather sluggish and, therefore, we find lower domestic real interest rates throughout our observation window, further driving the output gap and inflation. We also observe spillovers from

Figure 2: Trajectories for the Closed Economy

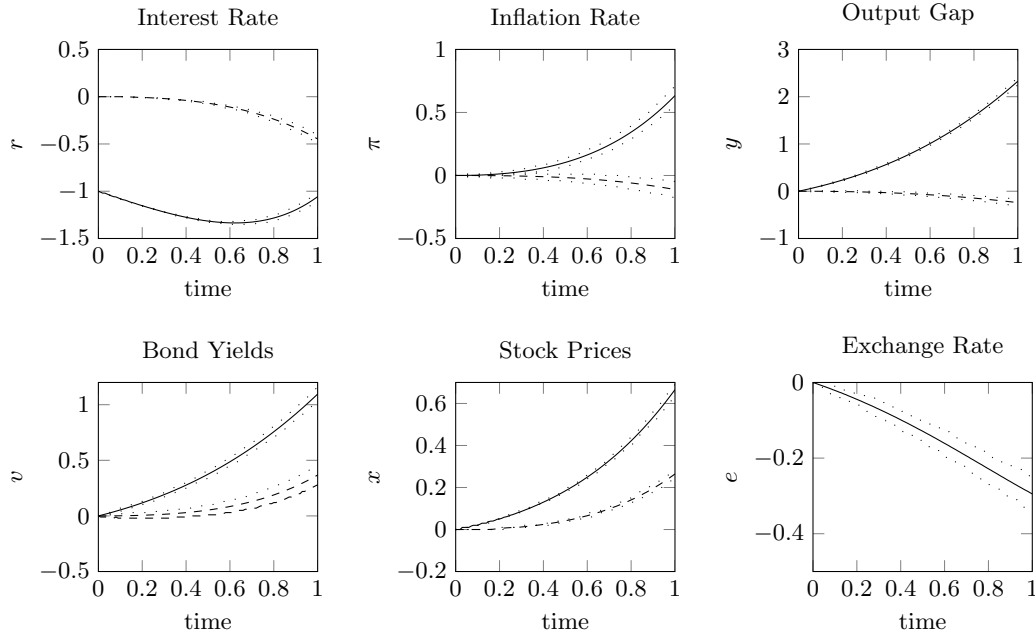


Solid lines show trajectories for Country A, dashed lines for Country B

monetary policy to financial markets. Following the increase in liquidity, bond yields as well as stock prices increase over time, reflecting the booming real economy and the rising inflation rate. Thus, the model illustrates that the central bank's provision of excessive liquidity can generate stock market booms and high bond yields. The drop in real interest rates leads to a continuous exchange rate depreciation, but this has little impact on other variables due to the closed-economy nature of our example. This is also why there is almost no reaction in Country B.

The second scenario (see Figure 3) reflects two open economies under the assumption of equally-sized countries. A real-world example is France and the United Kingdom. Suppose again that Country A's central bank engages in expansionary monetary policy by decreasing the short-term interest rate. On the one hand, Country A's economy shows a qualitative adjustment similar to that which occurred in the closed-economy case. Country B, on the other hand, now experiences dynamic change, too. First, the macroeconomic adjustment is now downward, with both output gap and inflation rate declining. This reflects the loss in competitiveness brought about by the appreciation of Country B's currency. The reverse is found for the other financial markets. Here, financial market integration lets Country B's stock and bond market response reflect

Figure 3: Trajectories for the Open Economy



Solid lines show trajectories for Country A, dashed lines for Country B

Country A's, albeit more sluggishly and less strongly. Thus, we find that in the case of two big and open economies, expansionary monetary policy in one country can bring about a real recession in another country, while at the same time fuelling a stock market boom and causing an increase in bond yields.

3.3 Employing Empirically Estimated Parameters: The United States and Canada

Our second approach to analysing economic behaviour is based on using econometric methods to determine model parameters. Thus, by employing macroeconomic data, we obtain estimates of each equation. As we are working with stochastic differential equations, we apply Bayesian estimation techniques, specifically, Markov Chain Monte Carlo (MCMC) methods (approximate Bayesian computation, see Beaumont, Zhang, and Balding (2002)). Two inputs are crucial to obtaining plausible results through MCMC estimations: first, the choice of priors and, second, the choice of initial values. Our choice of prior distributions for New Keynesian models is similar to decisions by, among others Smets and Wouters (2007), Negro et al. (2007) or Lindé (2005). We follow Kimmel (2007) or Jones (2003) and choose normal distributions for financial instruments.

An overview of the priors is given in the first four columns of Table (1). We run 500,000 simulations to obtain our results, with an average acceptance ratio of about 50%.

Data are obtained from the Bureau of Economic Analysis, the Federal Reserve St. Louis, the US Bureau of Labor Statistics, and Statistics Canada. We use Canada and the United States as benchmark countries, which allows us to obtain estimates for two highly integrated countries, one of which is a big and the other a small economy. We employ quarterly data for 34 years, from 1978:Q1 to 2012:Q4, resulting in 152 observations. The output gap is obtained as the transitory component after applying the HP filter to logged quarterly GDP. The monetary policy interest rate is the quarterly average of the effective Federal Funds rate and the Bank Rate in case of the United States and Canada, respectively. The inflation series is constructed as $400(CPI_t/CPI_{t-1} - 1)$. Regarding financial variables, for the United States (Canada) we employ the S& P 500 (TSX) to capture stock prices. Yields on the average of three-year T-Bills, in the case of the United States, and five-year T-Bills, in the case of Canada, are used as bond returns.

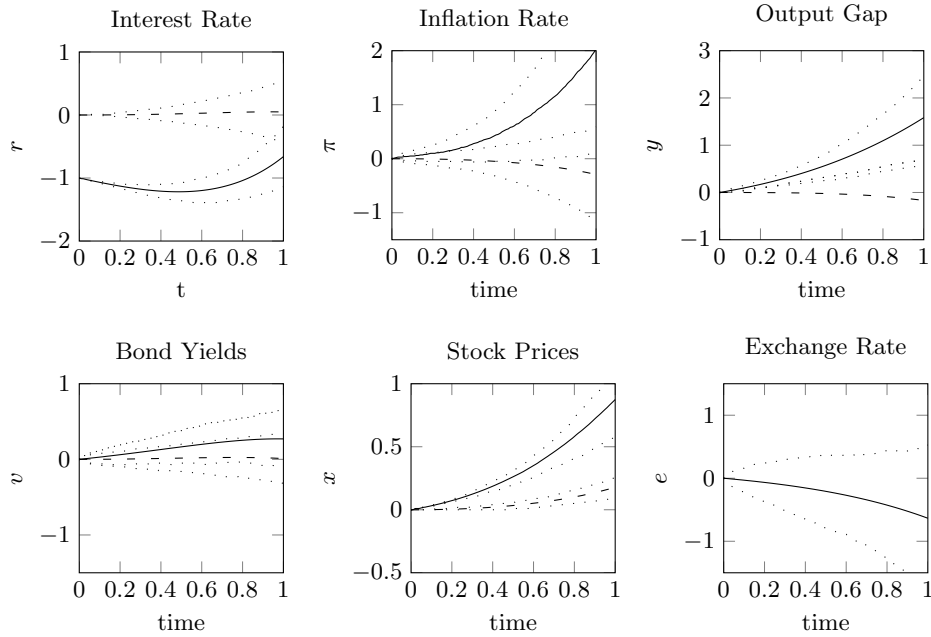
Table 1: Priors and Posteriors for the extended NK model (USA & Canada)

Par	Dist	Prior		Posterior (USA)			Posterior (CA)		
		Mean	SD	50%	5%	75%	50%	5%	75%
λ_y	Beta	0.50	0.250	0.58	-0.10	1.09	0.5	-0.36	1.06
λ_i	Gamma	1.00	0.300	0.73	0.17	1.25	0.96	0.37	1.51
λ_{y^*}	Beta	0.50	0.250	0.04	-0.73	0.67	-0.22	-0.73	0.32
λ_e	Gamma	1.00	0.300	0.85	0.10	1.43	0.79	0.28	1.24
α_π	Gamma	1.00	0.300	0.54	-0.24	1.00	0.56	0.24	0.89
α_y	Beta	0.50	0.250	1.08	0.45	1.64	0.63	-0.39	1.21
α_e	Gamma	1.00	0.300	0.98	0.47	1.49	0.93	0.20	1.49
θ_e	Normal	0.50	0.125	0.56	-0.08	1.02			
θ_i	Normal	0.50	0.125	0.29	-0.57	0.83	0.17	-0.6	0.73
θ_π	Normal	0.50	0.125	0.47	-0.21	0.92	0.51	-0.19	0.98
ω_b^b	Normal	0.50	0.125	0.33	-0.45	0.8	0.31	-0.38	0.76
$\omega_{b^*}^b$	Normal	0.50	0.125	0.46	-0.18	0.89	0.4	-0.56	0.92
ω_s^b	Normal	0.50	0.125	0.47	0.09	0.79	0.53	0.15	0.89
ω_s^b	Normal	0.50	0.125	0.68	0.28	1.08	0.36	-0.55	0.86
ω_e^b	Normal	0.50	0.125	0.50	-0.29	0.97	0.57	-0.04	1.00
ω_y^b	Normal	0.50	0.125	0.36	-0.51	0.88	0.68	0.21	1.12
ω_i^b	Normal	0.50	0.125	0.28	-0.51	0.78	0.05	-0.63	0.70

ω_π^b	Normal	0.50	0.125	-0.06	-0.80	0.71	0.32	-0.6	0.86
ω_b^s	Normal	0.50	0.125	0.53	0.15	0.90	0.41	0.13	0.69
$\omega_{b^*}^s$	Normal	0.50	0.125	0.28	-0.59	0.79	-0.05	-0.72	0.69
ω_s^s	Normal	0.50	0.125	0.37	-0.46	0.86	-0.09	-0.73	0.66
ω_s^s	Normal	0.50	0.125	0.21	-0.6	0.71	0.60	0.10	1.04
ω_e^s	Normal	0.50	0.125	0.04	-0.65	0.70	0.20	-0.56	0.70
ω_y^s	Normal	0.50	0.125	0.49	-0.28	0.94	0.15	-0.74	0.88
ω_i^s	Normal	0.50	0.125	0.28	-0.57	0.81	0.76	0.19	1.33
ω_π^s	Normal	0.50	0.125	0.21	-0.69	0.83	0.38	-0.63	1.17
γ_π	Normal	0.50	0.125	0.99	0.43	1.54	0.90	0.35	1.49
γ_i	Gamma	1.00	0.300	1.04	0.45	1.58	0.89	-0.05	1.51
γ_e	Gamma	1.00	0.300	0.79	-0.11	1.40	0.68	-0.38	1.37
γ_b	Gamma	1.00	0.300	1.07	0.46	1.68	0.80	0.12	1.40
γ_{b^*}	Gamma	1.00	0.300	0.76	0.17	1.26	0.97	0.38	1.51
γ_s	Gamma	1.00	0.300	1.14	0.70	1.57	1.06	0.64	1.48
γ_{s^*}	Gamma	1.00	0.300	1.06	0.42	1.64	0.47	0.16	0.72
γ_y	Beta	0.50	0.250	0.22	-0.97	1.12	0.93	0.32	1.44
γ_{y^*}	Beta	0.50	0.250	0.70	0.12	1.24	0.36	-0.30	0.68
σ_y	Beta	0.10	0.200	-0.22	-1.00	0.57	-0.32	-1.11	0.47
σ_π	Beta	0.10	0.200	0.22	-0.58	1.20	-0.39	-1.22	0.43
σ_e	Gamma	0.65	0.200	0.91	0.00	1.85			
σ_b	Gamma	0.65	0.200	0.40	-0.54	1.16	0.35	-0.36	1.05
σ_s	Gamma	0.65	0.200	0.66	-0.76	1.89	0.42	-0.63	1.51

Table 1 shows that, in many cases, prior and posterior values are remarkably close together, especially in the case of the United States. But even there we find some notable deviations and, in two cases, changes in signs (ω_{pi^b}) (σ_y) has a prior value of 0.5 (0.1), but its median posterior is -0.06 (-0.22)). In the case of Canada, the difference between prior and posterior is generally greater and there are five switches in signs ($\lambda_{y^*}, \omega_s^{b^*}, \omega_s^s, \sigma_y, \sigma_\pi$). Overall, we think the model holds up well and is still reasonably specified after estimation. Still, taking means of our variables as equilibrium points, we find that under these parameter values, the solution is unstable in the Lyapunov sense. However, the zero solution, which we use as the starting point for our simulations, is

Figure 4: **Simulations Implementing Bayesian Estimates: Expansionary Monetary Policy in the US**

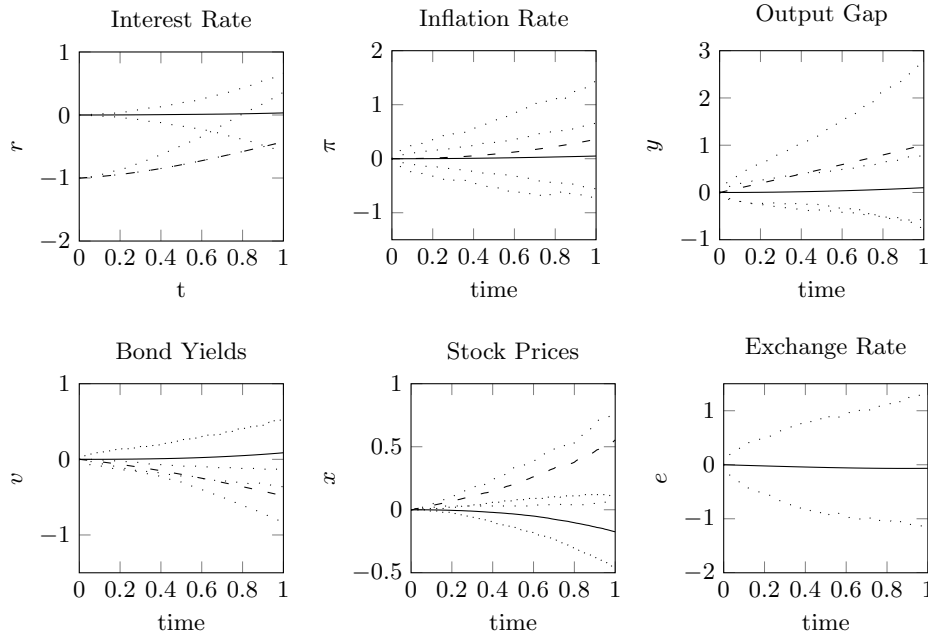


(almost) stable.

Figures (4) and (5) show trajectories of the estimated parameters. Solid lines represent the United States, dashed lines Canada. Figure ep(4) shows our model's dynamic response to a expansionary US monetary policy shock after implementing the Bayesian estimates. In the United States, the output gap increases after implementation of expansionary monetary policy. Stock and bond prices increase, the former reflecting the upward path of the real economy, the latter anticipating the increase in inflation. These results are consistent with our model and the underlying economic theory. Comparison with Figure (3) suggests that the dynamic adjustment for the United States is similar to that which occurs in an open economy. Reflecting the different size of the two countries, Canada reacts strongly to US monetary policy. Due to an appreciation of its exchange rate, Canada loses competitiveness and, correspondingly, output decreases slightly. Since the price of imported goods declines, inflation starts declining. Concerning real macroeconomic variables, the estimated spillover effects are smaller than those found in the simulations based on parameter values taken from the literature.

Specifically, the interest rate is basically not affected, which shows the short-term horizon of our model, as Canadian monetary policy has not yet reacted to the destabilisation triggered by the exchange rate. The effect of US monetary policy on Canadian financial

Figure 5: **Simulations Implementing Bayesian Estimates: Expansionary Monetary Policy in Canada**



markets is much more severe than on real variables. In particular, we observe opposite effects than on the real variables. Stock prices react positively to US monetary policy, reflecting the strong connection between the countries and a positive international stock market correlation. In contrast, bond prices in Canada are much less affected and their movement is slowly dominated over time by the working of the Fisher effect.

The outcome of conducting monetary policy in Canada is shown in Figure (5). The drop in short-term interest generates a positive output gap and slightly increases inflation. Stock prices increase, mirroring the boom of the real economy. In this estimation, bond and stock yields are negatively correlated, which is shown by the fact that bond yields decrease even though inflation increases. Regarding international spillover effects, we observe almost none between the real economies and very few in the case of bond yields. As Canada's economy is much smaller than that of the United States, we also observe no notable effect on the exchange rate. The reverse is found for the US stock market, which declines in the aftermath of expansionary Canadian monetary policy. Thus, this time, we observe a negative correlation between the two equity markets. This shows that under empirically estimated parameter values, both positive and negative financial market spillovers of monetary policy actions may occur.

4 Conclusions

In this paper, we extend the well-known, open economy New Keynesian model of Clarida, Gali, and Gertler (1999) and Lubik and Schorfheide (2007) in two important ways. First, we include a well-developed financial sector and, second, we apply stochastic differential equations and move the analysis to a continuous-time framework. We employ classic research from the field of finance and model the financial sector by including the market for foreign exchange, the bond market, and the stock market, both in the domestic as well as in the foreign economy, thereby acknowledging that these markets are driven by different aspects of the economy. For example, bonds are strongly affected by sovereign debt, whereas stock markets are heavily influenced by the real economy. Applying stochastic differential equations allows us to rely on established research starting with Merton and specify the financial markets either as Brownian motion or Ornstein-Uhlenbeck processes. Furthermore, we employ Lyapunov techniques to analyse the stability of the solutions and steady-state properties. Thus, in our analysis, we combine New Keynesian macroeconomic analysis, classic finance research, and standard mathematical procedures.

Our main research quest is to understand the effects of monetary policy in a theoretical framework where we allow for a feedback between financial markets and the real economy. In line with economic theory and empirical evidence, we start with a steady-state solution. We assume balanced trade and consider the influence of PPP as well as of the Fisher equation. In a second step, we simulate the reaction to monetary policy between two (almost) closed economies, two equally sized open economies, and two open economies of different size. We base the model parameters on empirical findings from studies on the Taylor rule, New Keynesian Phillips curve, and the IS curve; our financial equations rely on findings by Merton (1970) and Black and Scholes (1973). However, we extend all equations by accounting for spillover effects from monetary variables to real variables and vice versa. Finally, we solve the model for an initial value, which is common, but deviate from the steady-state point and observe and analyse how the solutions develop.

All simulation scenarios start by with expansionary monetary policy. Following the decrease in short-term interest rates, we study the transmission channel to analyse spillover effects between monetary and real variables, as well as between domestic and foreign markets. Specifically, reflecting market size and financial market structure, we identify differences in the influence of monetary policy. In the case of a closed economy, by con-

struction, there are no spillover effects. In the case of two equally sized, almost closed economies, we identify transmission effects on real variables, such as the output gap, monetary variables, such as the interest rate, and financial markets. Given the rather closed nature of the simulation framework, domestic effects prevail. In the case of two big and open economies, expansionary monetary policy in one country can bring about a real recession in another country, while at the same time fuelling a stock market boom and causing an increase in bond yields. This suggests interesting spillover effects from monetary policy to both the real economy and the financial sector.

Our third simulation combines the theoretical case of unequally sized economies with real-world data from the United States and Canada. Employing quarterly data over the period 1978:Q1: to 2012:Q4, we use point estimates based on Bayesian estimation techniques to derive the model's parameters. The simulation results support our findings from the purely theoretically parameterized model. We find spillover effects from monetary policy if conducted in the United States but only very small effects if the policy is initiated by the Bank of Canada. However, Canadian monetary policy can trigger negative correlations between US and Canadian stock markets. Moreover, US monetary policy appears to have a larger effect on Canada than Canadian monetary policy itself. This finding is consistent with evidence reported by Hayo and Neuenkirch (2012) on how monetary policy communication impacts financial markets in the United States and Canada.

Our study has some interesting policy implications. We find evidence that monetary policy actions spill over to other countries. The impact and size of the effect depend on, first, the linkage between the markets and, second, the structure of the markets. Policy-makers, particularly those of very open and well interacted countries, should take into account that spillovers could have effects that (depending on the degree of interaction) might even be larger than domestic policies. In contrast, we find evidence in support of the conventional wisdom that big countries and relatively closed economies can design and engage in policy primarily based on domestic factors. We also discover evidence that monetary policy in one country can substantially affect financial markets in other countries, even trigger booms and busts. It is possible that, depending on the specific situation, positive or negative financial market correlations can occur. These results suggest that deriving stylised policy conclusions supposedly applicable to each and every case is a practice best avoided. Moreover, the recent financial and economic crisis lends additional and very tangible support to our findings.

There are various fruitful ways of extending our analysis. First, analysing spillover

effects from the demand side (IS curve) to both financial markets and monetary policy might provide new insight. We conducted the analysis by changing monetary policy and observing the transmission mechanism. However, working in the opposite direction might be just as interesting. In particular, it would be interesting to analyse how financial market shocks influence monetary policy setting. The model also permits comparing different monetary policy reaction functions, e.g., distinguished by differences in the influence of financial markets.

In this paper, we did not account for a fiscal policy. Thus, extending the model by including government fiscal policy targets could yield interesting insights, especially since we observe that bond markets react very little to monetary policy. Thus, the hypothesis that bond markets are relatively more driven by fiscal policy could be tested within the context of a differently specified model. Furthermore, the model could be expanded by accounting for an even higher number of financial markets. Inclusion of the Black-Scholes formula or advanced option pricing techniques would allow analysis of an almost complete model of the financial system. Finally, considering the recent financial and sovereign debt crisis, a specific analysis of crises could be interesting. As outlined above, the model is potentially capable of studying the impact of financial market bubbles in the two-economy framework. Thus, including jump-diffusion processes would provide an additional source of financial market volatility and could help explain times of crisis.

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