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Incentives to Motivate

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Incentives to Motivate

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Incentives to Motivate

Abstract

We present a model in which a motivator can take costly actions - or what we call motivational effort - in order to reduce the effort costs of a worker, and analyze the optimal combination of motivational effort and monetary incentives. We distinguish two cases. First, the firm owner chooses the intensity of motivation and bears the motivational costs. Second, another agent of the firm chooses the motivational actions and incurs the associated costs. In the latter case, the firm must not only incentivize the worker to work hard, but also the motivator to motivate the worker. We characterize and discuss the conditions under which monetary incentives and motivational effort are substitutes or complements, and show that motivational effort may exceed the efficient level.

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Keywords: incentives to motivate.

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“Leadership is based on a spiritual quality — the power to inspire, the power to inspire others to follow.”

Vincent T. Lombardi

1 Introduction

The legendary football coach Vincent Thomas Lombardi was celebrated for his ability to motivate and inspire his players. Even though he achieved an amazing record of victories in a game where tactics and strategy matter, he is not so famous for his tactical skills. Lombardi is legendary for his coaching philosophy and motivational skills. He emphasized hard work and dedication, and players were wholeheartedly devoted to him.

Anyone who follows sports has a sense that it is not only the coach’s knowledge of the game that matters, but also his or her ability to motivate and inspire the players with words and actions. This also applies to work life in general. Leaders continuously emphasize the importance of motivation in terms of "energizing people" or "challenging them to take those actions that will realize results" (Filson, 2004). If one googles "leadership and motivation" one finds an endless list of managerial words of wisdom such as "Great leaders motivate through inspiration", or "Leadership is motivation, the leader is a motivator".1

From an economist’s point of view, this looks more like a technological approach to motivation than an incentive approach. Indeed, economic theories of motivation have primarily focused on incentives, and have not considered motivation to be a kind of technology that helps workers perform better. But when a coach motivates her players or a leader motivates her workers, she may trigger the workers’ effort without increasing their monetary incentives to exert effort. This is emphasized by two contemporary leadership theories in the field of organizational behavior: charismatic leadership and transformational leadership. According to these closely related theories, leaders inspire followers through their words, ideas, and behavior.2 Several studies find a positive relationship between charismatic/transformational leadership,

1The quotes are from CEO Dov Seidman and leadership consultant James Chapman, respectively.
2Max Weber introduced the term “charismatic leadership” in his famous theory of authority (originally published posthumously in 1922). Robert House (1977) further developed Weber’s concept in articulating a theory of charismatic leadership. Bernard Bass (1985) introduced the term “transformational leadership”, contrasting it with transactional leadership: while transactional leaders emphasize rewards in exchange for satisfying performance, transformational leaders inspire their followers by articulating visions and challenging goals. Charismatic and transformational leadership are now often used synonymously.
high performance, and job satisfaction, see Wang (2011) and Robbins and Judge (2013) for recent surveys and overviews.

But even though leadership has an impact on firm performance, a certain leadership style typically does not change a firm’s production technology, i.e., how inputs and particularly work effort transform into output. The channel through which charismatic/transformational leadership affects productivity appears to be its effect on employees’ (dis)utility of work. Effective leadership makes employees like their job better and, as a consequence, they work harder and perform in ways that benefit the organization. As Robbins and Judge (2013, p. 415) put it, "People working for charismatic leaders are motivated to exert extra effort and, because they like and respect their leader, express greater satisfaction." In a similar spirit, Harter et al. (2010) conclude, "Improving employee work perceptions can improve business competitiveness while positively impacting the well-being of employees."

Therefore, a natural way to model charismatic/transformational leadership in a principal-agent framework is to say that the leader or motivator reduces workers’ effort costs. In this paper we make this plausible assumption. We assume that a motivator can take costly actions - or what we call motivational effort - to reduce the effort costs of a worker and analyze the optimal combination of motivational effort and monetary incentives. We distinguish two situations. First, the firm owner chooses the intensity of motivation and bears the motivational costs. Second, another agent of the firm chooses the motivational actions and incurs the associated costs. In the latter case, the firm must then not only incentivize the worker to work hard, but also the motivator to motivate the worker.

Our model allows for a broader interpretation of motivational actions than what is typically emphasized in the literature on charismatic and transformational leadership. We are interested in any action that the leader can take in order to reduce the effort costs of the worker, and we sometimes refer to this as "motivational leadership". The costs of motivational effort can also take many forms and our model allows for various interpretations. For example, the firm can invest in developing its managers’ leadership qualities. Studies suggest that charismatic leaders are not only born but can also be made. Barling et al. (1996) conduct a field experiment with Canadian bank managers and find that branches whose managers underwent transformational leadership training performed better than branches whose managers did not receive such training. With appropriate forms of training, managers can also learn, e.g., how

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3The term "motivational leadership" is often used by consultants. "Charismatic leadership" and in particular "transformational leadership" are narrower academic terms.
to better evaluate critical situations or improve their interpersonal skills. Large firms like BHP Billiton, Nokia, and Adobe hire personal coaches for their top executives to improve their leadership skills (Robbins and Judge, 2013, p. 430). According to the Harvard Business Review, US companies are spending more than $1.5 billion a year on coaching. Renton (2009) reports that about 40% of Britain’s CEOs undergo coaching, as well as an increasing number of senior managers.

Motivational effort costs are also inflicted through communication, attention, or goal setting. Specifying goals that are in line with a worker’s ambitions for personal development requires time-consuming and thus costly communication. Giving sound feedback and appraisals requires careful evaluation of employee performance. And importantly, motivational actions are often at the discretion of the worker’s immediate superior, who is not the residual claimant of the production process but has to bear the costs of motivation. The firm owner then has to incentivize the motivator to motivate the worker, which is also costly.

The main insights of our analysis are as follows:

First, we show that higher-powered monetary incentives to the worker can reduce or enhance his responsiveness to motivational effort. The first case implies that incentives make motivational effort less effective and thus reflects a "hidden cost of reward". Because our motivational effort can be interpreted as an attempt to increase the worker’s intrinsic motivation, this result is related to the well-known crowding-out argument for intrinsic motivation (Lepper and Green, 1978). Monetary incentives, however, can also complement and enhance the effect of motivational effort. We thus also identify a potential "hidden benefit of reward".

Second, we analyze the optimal motivation-incentive mix from the firm’s point of view. Under unlimited liability of the worker, the firm provides first-best incentives that are equal to the marginal productivity of effort. The worker’s bonus is then independent of motivation, because the latter only affects the worker’s disutility of effort. If the worker is subject to limited liability, however, the firm can use incentives and motivation as substitutes as well as complements. The latter case can occur if the worker’s motivation responsiveness increases with the strength of incentives. For example, this is possible when a harder working employee interacts with his superior more frequently and is therefore easier to inspire by charismatic leadership. We show that, in such a situation, motivational effort may even exceed the efficient level and occur in the second-best solution even when it is first-best not optimal to motivate. The reason is that motivational effort can reduce the worker’s rent for each fixed effort level. In this respect, we provide a rather intuitive rationale for motivational effort.
Third, we find that a negative equilibrium relationship may occur between the motivator’s bonus and her effort level. If the worker’s effort becomes more productive (for exogenous reasons), the motivator’s effort level will increase. Cet. par. it may in fact exceed the first-best level of motivation. The firm may then mitigate the motivational effort by lowering the motivator’s incentives to motivate.

Finally, we identify a notable conflict of interest between motivator and worker. When the worker’s rent is decreasing in motivational effort, he clearly prefers a higher bonus rather than more non-monetary motivation. But for a given level of worker effort, the lower-powered the worker’s monetary incentives, the higher often is the motivator’s bonus. Under limited liability, a low bonus to the worker may thus imply a higher rent to the motivator. Consequently, low-powered monetary incentives to the worker may be in the motivator’s interest. Interestingly, we often see negative assessments of monetary incentives in the leadership and coaching literature.4

The rest of the paper is organized as follows. In Section 2 we discuss related literature. In Section 3 we present the basic model and characterize the first-best solution. In Section 4 we analyze the trade-off between motivational effort and monetary incentives in a setting where the firm owner is the motivator. We derive the optimal contract with limited and unlimited liability. The case where the firm needs to hire a motivator to induce motivation is analyzed in Section 5. Section 6 concludes.

2 Relationship to the Literature

In his celebrated book "The Modern Firm", John Roberts (2004) states that "Management (…) is vitally important, but it is not enough. Leadership is needed too (...). Leaders offer direction and then motivate others to believe and to follow." After the black box of the firm was opened in the 1970’s, management has been intensively studied. But leadership has almost been ignored by economists, even though it is a significant subject in the less formal literature on organizational behavior (see e.g. House and Aditya, 1997, for an overview). Recently, however, a small economics literature on leadership has emerged, focusing on the leader as one who has followers because of superior skills or information, see Hermalin (1998, 2007), Komai et al. (2007), Komai and Stegman (2010) and Lazear (2012). But the motivational

4 See for instance the best seller "Drive - the surprising truth about what motivates us", by Daniel H. Pink (2009).
part of leadership has been scantly treated in this literature. Notable exceptions are Van den Steen (2005), who shows how a manager with strong beliefs about the right course of action can attract employees with similar beliefs, and the works by Rotemberg and Saloner (1993, 1994, 2000), who consider the effect of visions and leadership style on employees’ motivation to generate proposals for innovation adoption.

Closer to our approach, however, is the important work on identity by Akerlof and Kranton (2000, 2005). They assume that effort costs are a function of identity, and that the firm can take actions that affect the workers’ identity. In particular they differentiate between insiders and outsiders, where only insiders identify with the firm/employer’s values. Like them, we assume that the firm can affect the workers’ effort costs, but we do not allow for discrete preference changes, and the trade-offs and interpretations we present are more in line with standard principal-agent terminology (discussing, e.g., marginal effects on motivation responsiveness). In this respect our paper is more related to Dur et al. (2010), who analyze a situation where the agent’s marginal costs of effort are decreasing and the worker’s well-being is increasing in the attention paid by the principal. In contrast to them, we allow for a more general effort cost function, which is crucial for deriving our main results. As another difference, Dur et al. focus on a commitment problem on the side of the principal, which is not an issue in our setup. And most importantly, neither Dur et al. nor Akerlof and Kranton study how the motivator should be incentivized. Moreover, Akerlof and Kranton do not consider limited liability and the rent extraction aspect, which is important in our paper.

When the firm needs to incentivize both a worker and a motivator, it faces a team incentive problem. Our paper is thus related to Itoh (1991a) who analyzes the incentives for workers to help each other, and in particular Itoh (1991b) who analyzes a situation where workers can socialize with each other and thereby affect each other’s utility functions. Dur and Sol (2010) also study social interaction between workers and how it is affected by the financial incentive systems. But the literature on team incentives does not relate to the kind of motivational effort we discuss. In contrast to the standard team literature, we analyze a team incentive problem where the agents have very different roles: The team consists of an agent - the worker - who is essential for production, and another agent - the motivator - who can help the worker but cannot produce anything without him.

Finally, our paper is related to a recent literature on perks and benefits, in particular Oyer (2008) and Marino and Zábojník (2008). Oyer studies how firm and worker characteristics may affect the trade-off between salaries and benefits, and models a situation where workplace
benefits such as entertainment options and errand services lower the workers’ effort costs. Benefit in his model could be reinterpreted as motivational effort, but Oyer does not consider the trade-off between benefits and monetary incentive provision, as he analyzes a full information model with no moral hazard problem. Marino and Zábojnik study the trade-off between work-related perks and incentive provision. In their model, perks improve the worker’s effort productivity, and give the worker a certain benefit in combination with effort. With respect to the latter aspect, our model is more general, which gives rise to different results concerning the interaction of perks and monetary incentives. In contrast to Marino and Zábojnik, we show that perks and monetary incentives can be substitutes as well as complements.

It can be instructive to position our approach within a simple taxonomy of motivation, see Table 1. The workers’ utility from being motivated can be realized ex post or ex ante, and it can be monetary or non-monetary.

<table>
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<th>Ex post</th>
<th>Monetary</th>
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<td>Standard principal-agent models</td>
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Table 1

The standard principal-agent approach is based on monetary rewards given ex post the worker’s effort, such as bonuses. But economists have increasingly recognized the importance of non-material incentives, such as the intrinsic pleasure of doing a good job (see Bénabou and Tirole, 2003; Besley and Ghatak, 2005), or the social esteem or respect that follows from good performance (see Ellingsen and Johannesson, 2008). Like the standard principal-agent models, the worker’s utility from motivation is also here realized ex post. In contrast, the gift-exchange literature and its emphasis on reciprocal preferences has shown both theoretically and experimentally that workers can be motivated by ex ante material rewards. A worker that receives a higher fixed wage responds by exerting higher effort (see Falk and Fehr, 2000, for an overview).

Finally, the huge literature on organizational behavior and motivational leadership focuses to a large extent on ex ante non-material realization of motivational utility. The immediate
payoff from being motivated by a leader is a reduction in non-material costs of exerting a given effort level. This effort cost reduction can of course then materialize in ex post rewards from higher effort. The novelty of our approach is to formalize motivational effort/motivational leadership, and to combine it with a standard principal-agent model with ex post material rewards.

3 The Model

We consider a model where a worker produces an output $q$ for a firm. The output can be either high or low, i.e., $q \in \{q_L, q_H\}$ with $q_L < q_H$. The probability of producing high output $q_H$ is given by the worker’s effort level $e \in [0, 1]$, i.e., $\Pr[q = q_H|e] = e$. The worker’s effort is non-observable, whereas output is observable and verifiable. The firm pays the worker a non-contingent fixed wage $s$ and a bonus $b$ if output is high.

In addition, the worker can be motivated by motivational effort $a \geq 0$. We assume that, if the worker is exposed to motivational effort, he enjoys working more and also finds it less troublesome to increase his effort. Hence, the worker’s private effort costs $C(e, a)$ are affected by the level of motivation that he experiences.\(^5\) The function $C(e, a)$ is strictly increasing and strictly convex in $e$, i.e., $C_e(e, a) > 0$ and $C_{ee}(e, a) > 0$ for $e > 0$ and all $a$. Motivation reduces both the worker’s absolute and marginal effort costs for all positive effort levels, i.e., $C_a(e, a) < 0$ and $C_{ea}(e, a) < 0$ for all $e > 0$ and all $a$. For $e = 0$, however, we assume that the worker’s absolute and marginal effort costs are zero (i.e., $C(0, a) = C_e(0, a) = 0$ for all $a$), and thus cannot be further reduced by motivation, i.e., $C_a(0, a) = C_{ea}(0, a) = 0$ for all $a$.\(^6\)

The costs of motivation are denoted by $K(a)$. They are strictly increasing and convex in the level of motivation, $K_a > 0$, $K_{aa} \geq 0$ for all $a > 0$. Zero motivational effort, $a = 0$, corresponds to a situation without motivation and, therefore, $K(0) = 0$. However, the marginal motivational costs at zero may be positive, i.e., $K_a(0) \geq 0$, which will imply that motivation may be too costly to implement.\(^7\) Both motivational effort and motivational costs

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\(^5\)Even though monetary incentives are a source of motivation, we mainly reserve the term “motivation” when talking about motivational effort. We also use “motivational effort” and “non-monetary motivation” synonymously.

\(^6\)As an alternative modelling approach, work effort could be costless up to a certain threshold, and motivation shifts the worker’s cost function to the right. Then, the maximum costless effort level increases and the marginal costs for each costly effort level strictly decrease. This would lead to similar results as those we will present here.

\(^7\)The reason could be that the motivator has high opportunity costs or, when the firm wants to hire a
are non-contractible. We first consider a situation where the firm chooses $a$ itself and bears the motivational costs. We can think of this as the firm owner being the motivator, or as motivation being delegated to a third party whose motivational actions are not subject to an incentive problem.\footnote{In the latter case, the firm owner could hire a particularly charismatic CEO, who naturally motivates other top executives just by interacting with them. However, the firm must offer higher compensation to a manager with extraordinary leadership qualities than to a less gifted manager because the latter has less attractive outside options on the labor market. The compensation differential then reflects the firm’s motivational cost. As another example, the firm can invest in developing its managers’ leadership qualities.}

We denote the sum of work effort costs and motivational costs by $\Gamma(e, a) := C(e, a) + K(a)$ and define $H$ as the Hessian of $\Gamma(e, a)$. To ensure strict convexity of the total cost function $\Gamma(e, a)$, we assume that $H$ is positive definite, i.e., $\det H = C_{ee}(C_{aa} + K_{aa}) - C_{ea}^2 > 0$ for all $e$ and $a$.

The worker has a reservation utility of zero and is risk neutral. He may, however, be protected by limited liability. In this case, payments to the worker must be non-negative. We will analyze the firm’s contracting problem in the case of both unlimited and limited liability of the worker.

Timing is as follows. First, the firm offers the worker a contract $(s, b)$ and announces to exert motivational effort $a$. The worker can accept or reject the contract offer. If he accepts the contract, he enters the firm and the firm chooses the motivational effort $\hat{a}$ at cost $K(\hat{a})$. The worker observes $\hat{a}$ and can decide whether to stay with the firm or quit.\footnote{Under unlimited liability, this interim participation decision will ensure that the firm can induce the first-best solution. Thus, allowing the worker to quit after observing the motivational level is in the interest of the firm. It serves as a self-commitment device. Under limited liability (or if the motivator is an agent of the firm), the interim participation decision will not be relevant for the results. In contrast to our model, Dur et al. (2010) assume that the worker cannot quit after observing the principal’s action.} If the worker stays with the firm, he exerts effort $e$ at cost $C(e, \hat{a})$. Finally, $q$ is realized and the firm pays the worker.

### 3.1 First-Best Work Effort and Motivational Effort

The first-best work effort $e^{FB}$ and the first-best motivational effort $a^{FB}$ serve as a benchmark. They maximize the social surplus, i.e.,

$$
(e^{FB}, a^{FB}) = \arg \max_{e \in [0, 1], a \geq 0} q_L + e \cdot \Delta q - \Gamma(e, a), \quad (1)
$$

\[\text{particularly charismatic manager, search costs are large.}\]
where $\Delta q := q_H - q_L$. The assumption $C(0, a) = C_e(0, a) = 0$ implies that the efficient work effort is strictly positive, i.e., $e^{FB} > 0$. Whether the worker should be motivated ($a^{FB} > 0$) or not ($a^{FB} = 0$) depends on how work effort and motivational effort interact in the total cost function $\Gamma(e, a)$. A sufficient condition to obtain $a^{FB} > 0$ is that, for each positive work effort, total costs are initially decreasing in $a$, i.e., $\Gamma_a(e, 0) < 0$ for all $e > 0$. This is the case if, e.g., $K_a(0) = 0$. By contrast, a sufficient condition for $a^{FB} = 0$ is that an infinitesimal amount of motivation always increases total costs, i.e., $\Gamma_a(e, 0) \geq 0$ for all $e$. Thus, even though motivation always reduces the worker’s effort costs, it is not necessarily efficient to induce motivation. If problem (1) has an interior solution, i.e., $a^{FB} > 0$ and $e^{FB} < 1$, first-best effort levels are characterized by the first-order conditions

$$\Gamma_e(e^{FB}, a^{FB}) = C_e(e^{FB}, a^{FB}) = \Delta q; \tag{2}$$
$$\Gamma_a(e^{FB}, a^{FB}) = C_a(e^{FB}, a^{FB}) + K_a(a^{FB}) = 0. \tag{3}$$

4 Monetary Incentives versus Motivational Effort

We now proceed to the contracting game where the firm’s objective is to implement the profit-maximizing combination of work effort and motivational actions. We solve the game by backward induction and thus first analyze the worker’s effort choice.

4.1 The Worker’s Optimal Effort Choice

The worker chooses his effort given the contract $(s, b)$ and motivation $a$. The optimal effort $e(a, b)$ maximizes the worker’s expected net payment, i.e.,

$$e(a, b) = \arg \max_{\bar{e} \in [0, 1]} s + \bar{e}b - C(\bar{e}, a). \tag{4}$$

The corresponding first-order condition yields the incentive constraint,

$$b = C_e(e, a), \tag{IC}$$
Equation (IC) implicitly defines $e(a, b)$ and describes the bonus that the firm has to offer to induce effort level $e$ given motivation $a$.\footnote{It is easy to see that the first-order condition holds at the worker’s optimal effort choice even if the firm wishes to induce the minimum or maximum effort, $e = 0$ or $e = 1$, respectively. To make the worker choose $e = 0$, the firm optimally sets $a = b = 0$. If the firm wants the worker to exert $e = 1$, it is not optimal to choose $a$ and $b$ such that the worker’s expected net payment is still increasing at $e = 1$, i.e., it cannot be the case that $b - C_e(1, a) > 0$.} From implicit differentiation of (IC) we obtain the worker’s incentive responsiveness $e_b$ and his "motivation responsiveness" $e_a$, where

$$e_b = \frac{1}{C_{ee}} > 0 \text{ and } e_a = -\frac{C_{ea}}{C_{ee}} > 0. \tag{5}$$

Accordingly, the worker exerts more effort the higher his bonus and the higher the motivational effort. The latter observation follows from our assumption $C_{ea} < 0$ and is in line with the empirical studies indicating that motivational leadership increases productivity, which we referred to in the introduction. The higher the incentive responsiveness (the lower $C_{ee}$), the higher is also the motivation responsiveness.

Next, we are interested in how the worker’s motivation responsiveness changes when incentives increase, which is reflected by $e_{ab}$. From (5) we obtain

$$e_{ab} = -\frac{C_{eae} + C_{eee}e_a}{C_{ee}^2}. \tag{6}$$

Intuitively, with a higher bonus, the worker increases his effort for each given level of motivation, which changes the impact of motivation on his marginal effort costs (reflected by $C_{eae}$) and the difficulty of raising effort further (reflected by $C_{eee}$). Both effects jointly determine the sign of $e_{ab}$. Since $C_{ee}(e, a)$ denotes the worker’s marginal bonus, the third derivatives $C_{eae} = C_{eea}$ and $C_{eee}$ also indicate how the marginal bonus changes with higher motivation and higher effort, respectively. It seems reasonable to assume that $C_{eee} \geq 0$, i.e., to elicit marginally higher effort, the firm has to increase the bonus more strongly the harder the worker works.

However, motivation can affect the marginal bonus in different ways, making the sign of $e_{ab}$ ambiguous. If $C_{eae} < 0$ and the firm increases motivational effort, it can achieve a marginal increase in work effort by a smaller bonus increase. If, in addition, $C_{eee}$ is small, we obtain $e_{ab} > 0$, i.e., the worker’s motivation responsiveness is increasing in the bonus. Such a case occurs for example if a harder working agent interacts with his motivator (e.g.,...
superior) more frequently and is therefore more responsive to motivational effort. Alternatively, a worker who is more occupied with his job could also be more eager for his motivator’s attention or feedback, which then also has a stronger effect on the worker’s job satisfaction and, consequently, marginal disutility of effort. By contrast, if $C_{eae} > 0$, we are in a situation where $e_{ab} < 0$, i.e., motivation responsiveness is decreasing in monetary incentives. This case occurs if, after a bonus increase, the agent works at an intensity that makes it extremely difficult to further raise effort. Or, from a certain point on, the agent’s opportunities to affect the realization of output are strongly limited (recall that effort is measured as the probability of high output in our model). Consequently, the agent is less responsive to motivational effort. Note that, because $C_{ea}(0; a) = 0$ and $C_{ea}(e; a) < 0$ for $e > 0$, $C_{eae}$ must initially be negative. Thus, $C_{eae} > 0$ can indeed occur only if the worker’s effort already is sufficiently high. Finally, the worker’s motivation responsiveness could be independent of monetary incentives, i.e., $e_{ab} = 0$.

The following proposition summarizes the main results of this subsection.

**Proposition 1** The worker’s motivation responsiveness may be increasing, decreasing, or independent of his monetary incentives, i.e., the sign of $e_{ab}$ is ambiguous.

Our result on the ambiguity of $e_{ab}$ is related to the current discussion on the effect of monetary incentives on intrinsic motivation. On the one hand, there is the well-known crowding out argument saying that higher-powered monetary incentives crowd out intrinsic motivation, also termed as the "hidden cost of reward" by Lepper and Greene (1978). Agency theory provides several versions of the argument: Monetary rewards may change the worker’s preferences (Frey, 1997), undermine incentives for social esteem (Benabou and Tirole, 2006, and Ellingsen and Johannesson, 2008), or affect workers’ perceptions of their tasks or own abilities (Benabou and Tirole, 2003). We show that this hidden cost of reward can be captured by the sign of $e_{ab}$. Monetary incentives may make an employer’s attempts to increase intrinsic motivation or, more generally, utility from work, less fruitful ($e_{ab} < 0$). This conclusion also

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It is straightforward to specify specific cost functions for the different situations. First consider a cost function of the type $C(e; a) = c(e)g(a)$. Our initial assumptions on $C$ imply that $c_e, c_{ee} > 0$ and $g_a < 0$. We thus obtain $C_{eae} < 0$. Consequently, we have $e_{ab} > 0$ whenever $c_{ee} = 0$, which is the case for a quadratic function $c(e)$. In contrast, we obtain $C_{eae} > 0$ and $e_{ab} < 0$, e.g., for the cost function $C(e, a) = \frac{e^2}{2(e+1/e^2)}$ whenever $te^2 > a$. Finally, an example for $C_{eae} = 0$ and $e_{ab} = 0$ is $C(e, a) = c^2 + e(1 - a)$ with $a \leq 1$.

In our model, motivational effort $a$ can be interpreted as measures aimed at increasing the worker’s intrinsic motivation. Even though $a$ is not the level of intrinsic motivation, the assumption that higher $a$ lowers effort costs may reflect a mapping between $a$ and intrinsic motivation.
has a natural counterpart. Since $e_{ab} = e_{ba}$, it also says that if a worker is highly motivated by non-monetary motivational effort, he may respond less to monetary incentives.

On the other hand, if the worker’s cost function is such that $e_{ab} > 0$, we have a "hidden benefit of reward" that has not been addressed in the economics literature so far. Monetary incentives then complement and enhance the effect of motivational effort and vice versa. Several organizational behavior papers find that more incentive pay leads to higher levels of intrinsic motivation for salespeople, see Babakus et al. (1996), Baldauf et al. (2002), Miao and Evans (2007), and DelVecchio and Wagner (2011). A hidden benefit of rewards is also demonstrated in the literature on goal setting. One way to exert motivational effort is to formulate goals, either for each employee, for groups of employees, or for the whole firm. Empirical evidence (see Locke and Latham, 2002) suggests that demanding but achievable goals have a motivating effect on workers, and may thus potentially reduce effort costs. Locke and Latham (1984) show that goal-setting works even better when it is accompanied by financial incentives. This can be captured by $C_{eae} < 0$, meaning the impact of motivation on marginal effort costs is more pronounced when the worker exerts more effort, e.g., due to monetary incentives. As a consequence, $e_{ab} > 0$ becomes more likely.

Complementarity between monetary incentives and motivational effort has also been emphasized in the leadership literature, contrasting charismatic-transformational leadership with transactional leadership. While transformational leaders inspire their followers by offering "a purpose that transcends short-term goals and focuses on higher order intrinsic needs" (Judge and Piccolo, 2004, p. 755), transactional leaders emphasize the exchange of resources such as (monetary) rewards or praise in return for satisfying performance. Recent work by organizational psychologists suggests that both leadership styles coexist, complement, and reinforce each other (see Gürerk et al., 2009, p. 594, and further references therein). In our model, we can interpret monetary incentives as a form of transactional leadership, whereas our motivational effort may correspond to transformational actions. The complementarity of the two leadership styles is then reflected in our model by $e_{ab} > 0$.

In the end, whether incentives make motivation more or less effective (and vice versa) is an empirical question, and most likely context specific. The main insight from Proposition 1 is that the worker’s response to a combination of non-monetary motivation and monetary incentives can be captured in a methodological simple way by considering a general function $C(e, a)$ that maps effort and motivation to the (dis)utility from work.
4.2 The Firm’s Contracting Problem

4.2.1 Optimal Contracting Under Unlimited Liability

We first solve the firm’s contracting problem under unlimited liability, i.e., when there are no exogenously imposed lower bounds on the worker’s wage. The solution proceeds in two steps: In the first step, we solve the firm’s first-stage optimization problem, assuming that the firm can commit to the motivational effort \( a \) that it announces. In the second step, we show that, under the previously derived contract, the firm will indeed choose the motivational level announced at the first stage, i.e., \( \hat{a} = a \).

The firm’s first-stage optimization problem is:

\[
\begin{align*}
\max_{e,a,b,s} & \quad q_L + e\Delta q - (eb + s) - K(a) \\
\text{s.t.} & \quad s + eb - C(e,a) \geq 0, \quad \text{(PC)} \\
& \quad b = C_e(e,a) \quad \text{(IC)}
\end{align*}
\]

Accordingly, the firm maximizes expected output net of wage costs and motivational costs, taking into account the worker’s participation constraint (PC) and incentive constraint (IC). It is readily verified that the optimal solution \((e^{UL}, a^{UL}, b^{UL}, s^{UL})\) comprises first-best effort and motivation, \( e^{UL} = e^{FB} \) and \( a^{UL} = a^{FB} \), which implies that \( b^{UL} = C_e(e^{FB}, a^{FB}) = \Delta q \). The optimal fixed payment \( s^{UL} \) makes the constraint (PC) just binding. It remains to verify that the firm indeed finds it optimal to exert \( \hat{a} = a^{FB} \) after the worker has signed the contract. At this stage, the firm faces the following optimization problem:

\[
\begin{align*}
\max_{\hat{a}} & \quad q_L + e(\hat{a}, b^{UL})(\Delta q - b^{UL}) - s^{UL} - K(\hat{a}) \\
\text{s.t.} & \quad s^{UL} + e(\hat{a}, b^{UL})b^{UL} - C(e, \hat{a}) \geq 0
\end{align*}
\]

Since the contract \((s^{UL}, b^{UL})\) is designed such that (9) is binding for \( \hat{a} = a^{FB} \), the firm can ensure that the worker does not quit only by implementing \( \hat{a} \geq a^{FB} \). Consequently, to minimize motivational costs, the firm indeed exerts first-best motivational effort.\(^\text{13}\)

\(^{13}\)Note that, if the worker is not allowed or able to leave the firm after he observes the level of motivation, the firm would not invest in motivation at all given that the bonus is \( b^{UL} = \Delta q \). Such a situation is analyzed by Dur et al. (2010).
**Proposition 2** Under unlimited liability of the worker, the firm implements the first-best solution \((e^{FB}, a^{FB})\). The optimal bonus is \(b^{UL} = \Delta q\).

The worker chooses first-best effort when his monetary incentives make him internalize the impact of his effort on output. Therefore, the bonus \(b^{UL}\) equals the marginal productivity of work effort, \(\Delta q\). Monetary incentives are thus independent of the specific "motivation technology", i.e., how motivation affects the worker’s effort cost and motivational costs. The reason is that the motivation technology has no direct impact on the productivity of work effort.

### 4.2.2 Optimal Contracting Under Limited Liability

We now analyze the firm’s contracting problem under limited liability, assuming that the firm cannot extract payments from the worker, i.e., \(s, s + b \geq 0\). The central questions we want to answer in this section are: How does motivation affect the firm’s wage costs under limited liability? Will there be too much or too little motivational effort in the second-best solution compared to the first-best solution? What is the optimal interaction of motivation and monetary incentives?

Because the worker’s liability limit and his reservation utility are zero, the limited liability constraint \(s \geq 0\) is binding and the worker earns a rent. The following lemma shows how the rent varies with motivation.

**Lemma 1** Under limited liability, the worker earns a strictly positive rent \(R(e, a) = ec_e(e, a) - C(e, a)\) for all \(e > 0\). This rent is decreasing in motivation \(a\) if and only if \(C_{ee} < 0\).

The proof is given in the Appendix.

As explained in Section 4.1, \(C_{ee} < 0\) means that the worker’s marginal bonus \(C_{ee}(e, a)\) is decreasing in motivation. In other words, when the worker experiences more motivation, he responds more strongly to a bonus increase. As a consequence, the rent he earns for exerting a given effort level decreases in motivation. Whether more motivation makes a higher bonus more or less effective \((C_{ee} < 0\) or \(C_{ee} > 0\), respectively), should depend on the specific situation. If, however, the former holds, motivation has an additional benefit for the firm: under limited liability, motivation does not only decrease the worker’s effort costs but also his

\[\text{Limited liability may arise from wealth constraints or from laws that impose lower bounds on wages, i.e., minimum wage legislation.}\]
rent. As the next proposition shows, this additional benefit may make the firm invest more heavily in motivation than is efficient. We denote the optimal work and motivational effort under limited liability by $e^{LL}$ and $a^{LL}$, respectively.

**Proposition 3** It is possible that the firm motivates under limited liability even though motivation is not efficient, i.e., $a^{LL} > 0$ and $a^{FB} = 0$. Furthermore, $a^{LL}$ is strictly larger than the level of motivation that minimizes total costs $\Gamma(e^{LL}, a)$ if and only if $C_{eae} < 0$.

The proof is given in the Appendix.

Building on our discussion in Section 4.1, the firm invests too much in motivation if the worker is more responsive to, e.g., charismatic leadership, feedback, or attention the harder he works. By contrast, in a situation where the worker finds it especially hard to increase effort or to affect the probability of high output, the firm motivates too little. It is worthwhile to note that, even if the worker’s rent is decreasing in motivation for each fixed effort level (i.e., $C_{eae} < 0$), this does not mean that the worker does not benefit from motivation. When the firm motivates the worker, it typically also pays a (weakly) higher bonus\(^{15}\) and induces a higher effort level than without motivation. The reason is that motivation makes monetary incentives more effective and thus less costly to the firm. The worker’s rent thus increases because he has lower effort costs and obtains a higher bonus.

We now analyze whether the firm employs incentives and motivation as complements or substitutes in its optimal contract. To do so, we assume that motivational costs have the specific form $K(a) = \gamma k(a)$, with $\gamma > 0$. Our purpose is to analyze how a decrease in the parameter $\gamma$, reflecting that motivation becomes less costly to the firm, affects the optimal level of motivation and the worker’s bonus. Motivation gets less costly, e.g., when the motivator is less occupied with other tasks and thus his opportunity costs of time fall, or when costs of leadership training decrease, or when technological or organizational changes make it easier to implement more attractive job characteristics such as more task variety, flexible working hours, or work from home.

**Proposition 4** Assume that $K(a) = \gamma k(a)$. We then have $da^{LL}/d\gamma < 0$ and

$$\text{sign} \left( \frac{db^{LL}}{d\gamma} \right) = -\text{sign} \left( e_{ba}[\Delta q - b^{LL}] - e_a \right).$$

\(^{15}\)For example, if $C(e, a) = \frac{ce^2}{2(1+\alpha)}$ and $K(a) = \frac{t}{2}a^2 + ta$, $t > 0$, as in the proof of Proposition 3, the bonus is $b = \Delta q/2$ both with and without motivation.
Thus, as motivation becomes less costly, the firm exerts more motivational effort. Higher motivation is accompanied by lower monetary incentives if the worker's incentive responsiveness is decreasing in motivation \((e_{ba} \leq 0)\). Otherwise, however, higher motivation may entail stronger monetary incentives.

The proof is given in the Appendix.

Thus, motivation and monetary incentives are substitutes whenever \(e_{ba} < 0\). However, if \(e_{ba} > 0\), motivation and incentives may also be complements.\(^{16}\) Intuitively, increasing motivation has two effects on the optimal bonus: First, the worker's incentive responsiveness \(e_b\) changes, making a bonus increase more or less effective \((e_{ba} \leq 0)\).\(^{17}\) Second, the worker's effort increases \((e_a > 0)\). Consequently, any bonus has to be paid more often, which favors a smaller bonus. Thus, the overall effect on \(b^{LL}\) is ambiguous.

Note that we can interpret a more broadly as any actions or investments the firm can undertake to lower the worker's effort costs. One possible interpretation are perks or work place benefits. Oyer (2008) convincingly argues that perks and benefits such as free meals, free parking, electronic equipment, or the provision of "concierge services" can lower employees' effort costs. Marino and Zábojnik (2008) incorporate perks in an otherwise standard principal-agent model. They show, among other things, that the firm can use perks to reduce the worker's monetary incentives. Unlike us, they focus on a situation in which perks increase the productivity and the utility of a risk-averse agent without affecting his effort costs. They find that offering perks then allows the firm to decrease the agent's bonus and, consequently, his risk premium. Perks and incentives are thus substitutes. By contrast, our model shows that, when perks increase the incentive responsiveness of the worker \((e_{ba} > 0)\), the firm may also employ the two instruments as complements. The specific functional forms in Marino and Zábojnik (2008) do not allow for such an effect. Analogous to our argumentation in Section 4.1, \(e_{ba} = e_{ab} > 0\) is more likely to occur when higher effort (due to a higher bonus) enhances the cost-reducing effect of perks on the worker's marginal effort cost even further \((C_{cae} < 0)\). For example, the worker could realize higher benefits from an innovative electronic device when he spends more time using it, thereby learning about additional features that facilitate work.

\(^{16}\)It is also possible that the optimal bonus is independent of motivation. For example, if \(C(e, a) = \frac{e^2}{a(e+a)}\) the optimal bonus is \(b^{LL} = \Delta q/2\).

\(^{17}\)The proof of Proposition 4 shows that \(\Delta q > b^{LL}\).
5 The Motivator as an Agent of the Firm

Although our motivational effort can be interpreted as any kind of production input that may lower the worker’s effort costs, it is also clearly distinguishable from other effort cost reducing inputs such as perks. Unlike perks, the demand for motivational effort can create an incentive problem, namely how to incentivize the motivator. Therefore, we now consider a situation where the motivational actions are not chosen by the firm owner but by another agent of the firm, who also bears the costs of motivation. We can think of the motivator as a leader or someone above the worker in the hierarchy.\textsuperscript{18} The motivator’s effort level is not observable to the firm, so that the firm must contract on the worker’s output to incentivize the motivator. It pays the motivator a bonus $b_M$ if the worker’s output is high. In addition, the motivator receives a non-contingent fixed payment $s_M$. Like the worker, the motivator is risk neutral, has a reservation utility of zero, and may be protected by limited liability.

The timing of the contracting game is now as follows: First, the firm offers the motivator a contract $(s_M, b_M)$ and the worker a contract $(s, b)$. The parties observe each other’s contracts and decide whether to accept or reject. If both parties accept, the motivator chooses her motivational effort $a$ at cost $K(a)$. Afterwards, the worker chooses his effort at cost $C(e, a)$. Next, output is realized and the firm pays the motivator and the worker.

We again solve the model by backward induction. We have already analyzed the last stage of the game where the worker chooses effort (see Section 4.1). We can therefore proceed to analyze how the motivator responds to given contracts $(s, b)$ and $(s_M, b_M)$.

5.1 The Motivator’s Optimal Effort Choice

The motivator chooses her motivational effort given the contracts $(s, b)$, $(s_M, b_M)$ and anticipating the worker’s effort choice $e(a, b)$ as implicitly given by (IC). The motivator’s optimal effort $a(b, b_M)$ is thus determined by

$$a(b, b_M) = \arg \max_{\hat{a}} s_M + e(\hat{a}, b)b_M - K(\hat{a}).$$

\textsuperscript{18}If the motivator performs other tasks besides motivation within the firm, we neglect those tasks and the corresponding compensation schemes in our analysis.
We assume that the motivator’s problem is concave in $a$, i.e.,

$$e_{aa}(a, b) b_M - K_{aa} < 0 \quad \text{for all } a \geq 0. \quad (12)$$

Thus, the optimal motivational effort $a(b, b_M)$ is implicitly defined by

$$e_a(a, b) b_M = K_a(a). \quad \text{(IC-M)}$$

We can observe that the motivator’s responsiveness to her own monetary incentives is always positive, $a_{b_M} > 0$. Furthermore, the worker’s bonus $b$ also affects the motivator’s effort level,

$$a_b = -\frac{e_{ab} b_M}{e_{aa}(a, b) b_M - K_{aa}}. \quad (13)$$

The relationship between the worker’s incentives and the motivator’s effort is ambiguous because $\text{sign}(a_b) = \text{sign}(e_{ab})$ and, by equation (6), $\text{sign}(e_{ab})$ can be positive or negative.

**Proposition 5** The motivator’s effort is increasing in his bonus $b_M$. Moreover, his effort is increasing in the worker’s bonus $b$ if and only if $e_{ab} > 0$, i.e., if the worker’s motivation responsiveness increases in $b$.

When monetary incentives to the worker amplify the effect of motivational effort (i.e., $e_{ab} > 0$ as discussed in Section 4.1), it also increases the motivator’s effort level. In contrast, if there is a hidden cost of reward (i.e., $e_{ab} < 0$), then higher monetary incentives to the worker do not only crowd out the effect of motivational effort. It also crowds out motivational effort. The interaction between non-monetary motivation and incentives thus transmits to the effort-level chosen by the motivator - which is illuminating, but not surprising. By equation (6) and the subsequent discussion, we obtain $e_{ab} < 0$ whenever $C_{eae} > 0$, i.e., when the worker already exerts sufficiently high effort and/or finds it particularly hard to affect output. Our model thus predicts that motivators of such agents should exert less motivational effort when monetary incentives to the worker increase.
5.2 The Firm’s Contracting Problem with a Motivator

5.2.1 Optimal Contracting Under Unlimited Liability

We first analyze the firm’s contracting problem under unlimited liability:

\[
\max_{e,a,b,b_M,s,s_M} q_L + e \Delta q - [e(b + b_M) + s + s_M] \\
\text{s.t.} \quad s + eb - C(e, a) \geq 0, \quad (PC) \\
\quad s_M + eb_M - K(a) \geq 0, \quad (PC-M) \\
\quad (IC),(IC-M).
\]

Accordingly, the firm maximizes expected output net of wage costs. Thereby, it has to take into account the worker’s and motivator’s participation constraint (PC) and (PC-M), respectively, and each party’s optimal effort choice for given bonuses, (IC) and (IC-M), respectively. The firm optimally chooses the fixed wages \(s\) and \(s_M\) such that (PC) and (PC-M) are just binding. Consequently, the firm’s wage costs are equal to the total costs \(\Gamma(e, a)\). The firm therefore induces the worker and the motivator to exert first-best effort levels \((e^{FB}, a^{FB})\). As in the case where the firm motivates the worker itself, the worker’s optimal bonus is \(b^{FB} = \Delta q\) (compare Proposition 2). By (IC-M), if \(a^{FB} > 0\), the motivator’s optimal bonus is given by

\[
b^{FB}_M = \frac{K_a(a^{FB})}{e_a(a^{FB}, \Delta q)}. \quad (15)
\]

The motivator’s bonus is thus determined by the ratio of marginal motivational costs and the agent’s motivation responsiveness \(e_a\) at \(a = a^{FB}\) and \(b = b^{FB} = \Delta q\). Consequently, the motivator’s bonus crucially depends on the characteristics of the worker’s effort cost function \(C(e, a)\).

Using (2) and (3), it can be easily verified that, when the first-best motivational effort is positive and \(\Delta q\) increases, both the worker and the motivator exert more effort \((de^{FB}/d\Delta q, da^{FB}/d\Delta q > 0)\). We now investigate how, in such a situation, the firm optimally adopts the contracts to induce higher effort levels. Obviously, the worker’s bonus \(b^{FB} = \Delta q\) will increase when his effort becomes more valuable to the firm. The effect on the motivator’s bonus, however, is
There are two effects on $b_{FB}^M$. First, the motivator needs to be incentivized to incur higher marginal effort costs, which favors a higher bonus. This is reflected by the first, positive term on the right-hand side of (16). Second, the higher worker bonus and the increased level of motivation changes the worker’s motivation responsiveness ($e_a$) and, thereby, the effectiveness of motivation. This effect is given by the second term on the right-hand side of (16), whose sign is undetermined because both $e_{ab}$ and $e_{aa}$ can be negative or positive.\footnote{From (5) we obtain $e_{aa} = -\frac{(C_{eaa} + C_{eaa_e}e_a)C_{ae} - (C_{eaa} + C_{eaa_e}e_a)C_{ee}}{e^2_{ee}}$.} Consequently, if $e_{ab}$ and/or $e_{aa}$ are positive, implying that the worker responds more strongly to motivation if his bonus and/or motivation increases, the overall effect on $b_{FB}^M$ may be negative. Thus, even though the motivator works harder as $\Delta q$ increases, she may obtain a lower bonus. In such a situation, the motivator increases her effort because she anticipates that the worker will respond more intensely to motivation.

**Proposition 6** Assume that the marginal productivity of work effort, $\Delta q$, increases. Then, both the worker’s bonus and the motivator’s effort increase. However, the motivator may receive a lower bonus. This is the case if and only if

$$\frac{K_{aa} e_{aq}^F B e_a}{e_a^2} - \frac{K_a \left(e_{ab} + e_{aa} e_{aq}^F B\right)}{e_a^2} < 0.$$  

(17)

We may thus have a negative equilibrium relationship between the motivator’s effort and the bonus she receives. One way to express the intuition is as follows: If the worker’s responsiveness to monetary incentives and/or motivation increases in the level of motivation, then a higher productivity, cet. par, may lead to an inefficiently high level of motivation ($a > a_{FB}^F B$). The firm will then reduce the motivator’s incentives to motivate.

### 5.2.2 Optimal Contracting Under Limited Liability

In this section we assume that both the motivator and the worker are protected by limited liability. When we analyzed the limited liability case without a motivator (Section 4.2.2), we
found that the firm, under certain conditions, chooses an inefficiently high motivational effort level in order to reduce the worker’s rent. A question now is whether this result continues to hold when the firm hires a motivator. Inducing motivation now entails a rent payment to the motivator and, therefore, becomes more costly to the firm. Our main questions are: Will limited liability make it less likely that the firm induces motivation? Can we still have excessive motivational effort in the second-best solution when the firm must leave a rent to the motivator? And how is the motivator’s and worker’s rent affected by the bonuses they receive?

The firm’s optimization problem now reads as

\[
\begin{align*}
\max_{e, b, b_M} & \quad q_L e + e \Delta q - [e(b + b_M) + s + s_M] \\
\text{s.t.} & \quad s + eb - C(e, a) \geq 0, \quad (PC) \\
& \quad s_M + eb_M - K(a) \geq 0, \quad (PC-M) \\
& \quad (IC), (IC-M), \\
& \quad s, s_M, s + b, s_M + b_M \geq 0.
\end{align*}
\]

The last line ensures that the payments to both the worker and the motivator are always non-negative. As the next proposition shows, even though motivation now entails a rent payment to the motivator, the firm may still induce more motivation than is efficient.

**Proposition 7** When having to incentivize a motivator, it is possible that (i) the firm induces motivation only under unlimited liability, i.e., \(a_F^B > 0\) and \(a_{LL}^M = 0\). However, it is also possible that (ii) motivation occurs only under limited liability, i.e., \(a_F^B = 0\) and \(a_{LL}^M > 0\).

The proof is given in the Appendix. For case (i), it shows that, even if exerting an infinitesimal amount of motivation is costless for the motivator \((K_a(0) = 0)\), the firm may decide against motivation. If the firm could motivate the worker itself, it would do so. However, incentivizing a motivator is too costly because of the rent she earns. As the proof shows, such a case can occur if marginal motivational effort costs are large relative to the impact that motivation has on the worker’s costs. Then, the motivator’s bonus increases more sharply in motivation than the worker’s bonus decreases. However, as case (ii) shows, there may also be situations where the firm hires a motivator even though motivation is inefficient. Then,
motivation has a stronger advantageous effect on the wage paid to the worker than it increases the wage paid to the motivator.

Finally, it is interesting to analyze the relationship between the worker’s and the motivator’s rent. To do so, we consider a situation where the firm wishes to induce a fixed work effort $e$. This work effort can be implemented by all combinations of $a$ and $b$ satisfying the worker’s incentive constraint (IC). The question we want to answer is: How do the worker’s and the motivator’s rent change under the different feasible combinations and, consequently, what combination does each party prefer? Assume that, starting from a certain combination $a = a_1$ and $b = b_1$ that induces $e$, the firm decides to marginally increase motivation. This requires to adjust the bonuses $b_M$ and $b$ such that the motivator is willing to exert more effort, while the worker’s effort level remains constant. The motivator’s initial bonus is

$$b_M = \frac{K_a(e, a_1)}{e_a(a_1, C_e(e, a_1))}. \quad (20)$$

If the firm wishes to increase $a$, holding $e$ constant (by decreasing $b = C_e(e, a_1)$), the motivator’s bonus changes as follows:

$$\frac{\partial b_M}{\partial a} = \frac{K_a e_a - (e_{aa} + e_{ab} C_{ea}) K_a}{e_a^2} = \frac{K_a e_a - e_{aa} K_a - e_{ab} C_{ea} K_a}{e_a^2} \quad (21)$$

The term $K_a e_a - e_{aa} K_a$ is positive because, by the second-order condition for the motivator’s problem, (12), we have $\frac{K_a}{e_a} > b = \frac{K_a}{e_a}$. The sign of the term $e_{ab} C_{ea} K_a$, however, depends on $e_{ab}$. If $e_{ab} \geq 0$, a lower bonus for the worker leads to lower motivation responsiveness, which in turn has a negative effect on the motivator’s incentive to motivate (see Proposition 5). Thus, it is clear that the motivator’s bonus must increase. Consequently, the motivator’s rent,

$$R^M(e, a) = e(a, C_e(e, a)) - \frac{K_a(e, a)}{e_a(a, C_e(e, a))} - K(a), \quad (22)$$

also gets larger. The reason is that, with the higher bonus, the motivator would earn a higher rent than before if she still chose $a = a_1$. However, she prefers to exert higher motivational effort. Consequently, this higher effort must entail an even larger rent. If, however, $e_{ab} < 0$, the worker is more responsive to motivation after a bonus decrease. When this effect dominates in (21), the motivator’s bonus actually decreases in motivation. Because motivational costs increase, the motivator’s rent is also lower. Thus, the motivator prefers a lower bonus for
the worker and more motivation if $e_{ab} \geq 0$, but may favor a higher worker bonus and less motivation if $e_{ab} < 0$.

From the analysis in Section 4.2.2, we can infer the interests of the worker: He prefers a higher bonus and less motivation if and only if his rent $R(e, a)$ is decreasing in motivation, i.e., if $C_{eaa} < 0$. This is always the case if $e_{ab} > 0$. Consequently, we obtain the following result.

**Proposition 8** If $e_{ab} > 0$, there always is a conflict of interest between worker and motivator: To be incentivized to exert a given effort level, the worker prefers stronger monetary incentives and less motivation, whereas the motivator prefers lower monetary incentives and more motivation for the worker. If $e_{ab} < 0$, there is a conflict of interest between the two parties if and only if $K_{aa}e_a - (e_{aa} + e_{ab}C_{eaa})K_a > 0$ and $C_{eaa} < 0$ (the motivator prefers more motivation and the worker less) or $K_{aa}e_a - (e_{aa} + e_{ab}C_{eaa})K_a < 0$ and $C_{eaa} > 0$ (the motivator prefers less motivation and the worker more).

If the worker’s motivation responsiveness is increasing in incentives ($e_{ab} > 0$), motivator and worker would never agree on a motivation-incentive mix: The motivator then always advocates relatively more motivation and the worker less. However, if $e_{ab} < 0$, the interests of the two parties may be aligned. Interestingly, the motivator may then even favor lower motivation and higher incentives for the worker. This is the case when a small bonus makes the worker highly responsive to motivation. A high bonus to the worker then acts as a self-commitment device for the motivator not to increase motivational effort even if her own bonus decreases. The worker does not like a higher bonus when $C_{eaa} > 0$. As argued in Section 4.1, such a case can occur only if the worker already works quite hard. Motivation then lowers his effort costs so strongly that he is willing to accept a lower bonus in return.

6 Discussion

Our model shows that different (conflicting) insights gained in fields such as management, leadership, and organizational psychology can be understood and analyzed within a simple microeconomic framework. As a consequence, we do not offer clear cut empirical predictions. Different results are possible, depending on the parameters. This should, though, illuminate potential empirical strategies. The main challenge for empirical and/or experimental work is to find the technology of motivation, in particular how the worker’s effort costs are affected by non-monetary motivational effort. If one understands this cost function, then one can achieve
a deeper understanding of the interaction between monetary incentives and non-monetary motivation.

But implications can also be drawn without knowing the exact cost function. One implication of our model is that firms have to pay particular attention to the motivation-incentive mix when workers earn rents. Rents occur when employees are protected by limited liability, which may arise from liquidity or wealth constraints, or from laws that require employers to pay a minimum fixed wage. We find that, under a binding wage floor constraint, monetary incentives cannot be chosen independently of motivation. By contrast, if an employee’s participation constraint is binding, the optimal bonus does not change with motivation. If the worker’s reservation utility is high, participation constraints can be binding even when wage floors such as liability limits or minimum wages exist. More able or better qualified workers typically have higher reservation utilities because they can easily find another well-paid job outside their current firm. Our model suggests that, ceteris paribus, such workers tend to obtain stronger monetary incentives than colleagues with less attractive outside options. Moreover, these high-powered incentives do not vary with the firm’s motivational effort (as long as effort costs do not decrease so strongly with higher motivation that the participation constraint is no longer binding). Our model also implies that the firm is more likely to overinvest in motivation for workers who have less attractive outside options, e.g., workers on lower hierarchy levels who obtain minimum wages as fixed compensation.

When another agent of the firm motivates the worker, we show that high-productive workers may trigger the motivator’s effort to such an extent that the firm may want to mitigate motivation by lowering the motivator’s bonus. This may create a negative equilibrium relationship between the motivator’s bonus and her effort level. We also identify a potential conflict of interest between motivator and worker. Motivators may have an interest in low-powered incentives (and low rents) to the workers they motivate, because this raises the need for higher bonuses (and thus higher rent) to the motivator. This can contribute to explain why motivators and authors of popular management books so often emphasize the importance of non-monetary motivation, and why leaders often have higher-powered incentives than lower-level employees. The latter can also be explained by the fact that the ability to motivate is a scarce resource. If the motivator cannot herself be motivated by a motivator, she has to be motivated by money. However, if the worker responds more strongly to motivation when his bonus is low, the motivator can also prefer a high bonus for the worker to avoid being driven to provide high motivational effort. Moreover, the worker may prefer motivation to monetary
incentives if he already works quite hard.

The optimal choice between monetary incentives and motivational effort will of course vary. Jobs differ in their characteristics and employees have different preferences for leadership styles and work environments. Thus, motivation has a different impact in different employment relationships. For example, studies show that charismatic leadership is more effective if the subordinate’s task has an ideological component, the work environment is subject to stress and uncertainty, or the firm is small (Robbins and Judge 2013, p. 415 and 419). According to the GLOBE study by House et al. (2004), employees from different cultures have distinct preferences for leadership styles. Our model not only suggests that multinational firms should adapt their motivational effort to individual tastes for leadership. They should also offer different compensation packages when first-best monetary incentives are not feasible, e.g., due to wage floors. Even if motivational effort cannot be perfectly tailored to individual employees, individually adapted incentive pay can mitigate this problem. Companies have already realized that it is important to learn what motivates an individual employee. Large firms such as Kraft Foods or Deutsche Telekom use the so-called "Reiss Motivation Profile" to find out more about their employees’ preferences.

The model captures differences in the effectiveness of motivation and its optimal interaction with monetary incentives by the worker’s effort cost function \( C(e, a) \). We show that, to assess the usefulness of motivation and to determine the optimal motivation-incentive mix, it is essential for a firm to know whether motivational effort becomes more or less effective in increasing work effort with stronger monetary incentives (i.e., whether \( C_{eae} < 0 \) or \( C_{eae} > 0 \) holds, respectively). The main challenge for empirical and/or experimental work is to determine the situation-specific interaction of monetary incentives and motivation. So far, empirical studies have shown that different leadership styles or leadership training have an impact on firm performance. Our model highlights that it is also crucial to determine whether and how this impact depends on the strength of monetary incentives provided to those affected by motivation. If performance responds less to motivational effort under higher-powered incentives, then the firm should use motivation and incentives as substitutes. Otherwise, it may be optimal to use incentive and motivation as complementary devices, and the firm may benefit from implementing excessive motivation.

\[ \text{20 See "Gesucht: Der perfekte Kollege", Die Zeit, 21.06.2012, p. 75. The Reiss Profile was developed and publicised by Prof. Steven Reiss, Emeritus Professor of Psychology and Psychiatry at Ohio State University (USA), see, e.g., Reiss (2002).} \]
7 Conclusion

In this paper we take a technological approach to motivation by modeling "motivational effort" as something that reduces other workers' effort costs. A worker can get motivated by visionary talks, pats on the back, or just mere attention, making effort more enjoyable and less costly.

Our simple framework makes it possible to study important details on the interaction between monetary incentives and non-monetary motivation. We can distinguish between incentive responsiveness and motivation responsiveness, and we can characterize the conditions under which monetary incentives and motivational effort are substitutes or complements in the firm's optimal motivation-incentive mix. In the former case, higher-powered incentives to the worker reduce his responsiveness to motivational effort, which is a version of the well-known crowding out argument or "hidden cost of reward". In the latter case monetary incentives complement and enhance the effect of motivational effort, which is a less known "hidden benefit from reward".

Interestingly, it can be shown that firms may induce motivational effort not just in order to reduce the workers' effort costs, but also to reduce the workers' rent. This may lead to excessive motivation in equilibrium. In the case where the firm hires a motivator, high-productive workers may trigger the motivator's effort to such an extent that the firm may want to mitigate motivation by lowering the motivator's bonus. This may create a negative equilibrium relationship between the motivator's bonus and her effort level. Finally, we identify a potential conflict of interest between motivator and worker. Motivators may have an interest in low-powered incentives to the workers they motivate, because this raises the need for higher bonuses to the motivator.

One could argue that motivational effort is analytically hard to distinguish from many other production inputs since there are many kinds of inputs that may lower the worker's effort costs. Still, we believe our modeling approach and interpretation is worthwhile. First, it clarifies how "technological motivation" in terms of, e.g., inspiration or visions relates to standard incentive models of motivation. Second, it opens for a new incentive problem that is not relevant for other kinds of production inputs, namely how to incentivize the motivator. The model can of course be extended in various ways, to include incomplete (relational) contracting, multitasking, and/or imperfect performance measures. Motivational effort may in fact be an important response to incentive problems when good performance measures are not available.
8 Appendix

Proof of Lemma 1. Consider the firm’s first-stage problem, assuming that it will adhere to the motivational effort announced at stage 1:

$$\max_{e,a,b,s} q_L + e\Delta q - (eb + s) - K(a)$$

s.t.  
$$s + eb - C(e,a) \geq 0,$$  
$$b = C_e(e,a),$$  
$$s, s + b \geq 0.$$  

(23)

By (IC), $b$ is non-negative. Given an arbitrary $b \geq 0$ and $a$, the worker’s expected net payment under his optimal effort choice, $e(a,b)b - C(e(a,b),a)$, is at least zero. The reason is that the worker can always ensure himself a payoff of zero by exerting zero effort. Thus, to satisfy (PC) and (LL), $s = 0$ is optimal. Given $a$, the firm’s wage costs for inducing a fixed effort level $e$ thus are $eb = eC_a(e,a)$. Because $eC_e(e,a) > C(e,a)$ for all $a \geq 0$ and $e > 0$, the worker earns a strictly positive rent of $R(e,a) = eC_e(e,a) - C(e,a)$ for $e > 0$. We have $R_a < 0$ if and only if $C_a$ is concave in $e$, i.e., $C_{aee} = C_{eae} < 0$. It remains to check whether the firm will indeed exert the motivational effort announced at the first stage. The first-stage optimization problem can be simplified to

$$\max_{e,a} q_L + e(\Delta q - C_e(e,a)) - K(a).$$

(24)

We assume that the objective function in (24) is strictly concave\(^\text{21}\) and denote the solution of (24) by $(e^{LL}, a^{LL})$. The bonus is $b^{LL} = C_e(e^{LL}, a^{LL})$. At the stage where the firm chooses motivational effort, the firm solves

$$\max_{\bar{a}} q_L + e(\bar{a}, b^{LL})(\Delta q - b^{LL}) - K(\bar{a})$$

s.t.  
$$e(\bar{a}, b^{LL})b^{LL} - C(e(\bar{a}, b^{LL}), \bar{a}) \geq 0.$$  

(25)

(26)

\(^{21}\)This is the case if the Hessian of $eC_e(e,a) + K(a)$ is positive definite, i.e., $2C_{ee} + eC_{eee} > 0$ and $(2C_{ee} + eC_{elee})(eC_{eee} + K_{aa}) - (C_{ea} + eC_{eeea})^2 > 0$. The conditions are satisfied for the cost functions from footnote 11 when $K_{aa}$ is sufficiently large.
The worker’s interim participation constraint is satisfied for all \( \tilde{a} \). Thus, the firm chooses \( \tilde{a} \) such that

\[
e_a(\tilde{a}, b^{LL})(\Delta q - b^{LL}) - K'(\tilde{a}) = 0.
\]  

(27)

However, the firm’s first-stage optimization problem can also be written as

\[
\max_{a,b} q_L + e(a, b)(\Delta q - b) - K(a),
\]  

(28)

implying that \( e_a(a^{LL}, b^{LL})(\Delta q - b^{LL}) - K'(a^{LL}) = 0 \) and thus \( \tilde{a} = a^{LL} \).

Proof of Proposition 3. The proof is by example. Assume that \( C(e, a) = \frac{ce^2}{2(1+a)} \) and \( K(a) = \frac{k}{2}a^2 + ta, t > 0. \)

A sufficient condition for \( a^{FB} = 0 \) is that an infinitesimal amount of motivation always increases total costs, i.e.,

\[
\Gamma_a(e, 0) \geq 0 \text{ for all } e.
\]  

(29)

Thus, \( a^{FB} = 0 \) if

\[
C_a(e, 0) + K_a(0) = -\frac{ce^2}{2(1+a)^2} + k \cdot 0 + t \geq 0 \text{ for all } e.
\]  

(30)

Since \( e \leq 1 \), this condition is satisfied if \( t \geq \frac{c}{2} \).

Now consider the case of limited liability. A sufficient condition for \( a^{LL} > 0 \) is that the firm’s expected costs decrease in motivation at the effort level \( e_0^{LL} \) that is optimal given that \( a = 0 \), i.e.,

\[
e_0^{LL}C_a(e_0^{LL}, 0) + K_a(0) < 0,
\]  

(31)

where \( e_0^{LL} = \arg \max_{e} e(\Delta q - C_e(e, 0)) \).

Here,

\[
e_0^{LL} = \arg \max_{e \in [0,1]} e\Delta q - ce^2.
\]  

(32)

Assuming that this problem has an interior solution, we obtain \( e_0^{LL} = \frac{\Delta q}{2c} \) for \( \Delta q < 2c \). By (31), \( a^{LL} > 0 \) if

\[
-\left( \frac{\Delta q}{2c} \right)^2 \frac{c}{(1 + 0)^2} + t < 0 \iff t < \frac{1}{c} \left( \frac{\Delta q}{2} \right)^2.
\]  

(33)

22In the motivational cost function, we need the \( a^2 \)-term to ensure convexity of the total cost function and the \( ta \)-term to ensure that \( K_a(0) > 0 \) and, hence, \( a^{FB} = 0 \) is possible.
Furthermore, we have
\[ t = \frac{c}{2} < \frac{1}{c} \left( \frac{\Delta q}{2} \right)^2 \iff \sqrt{2}c < \Delta q. \] (34)

It follows that \( a^{FB} = 0 \) and \( a^{LL} > 0 \) for, e.g., \( t = c/2 \) and \( \Delta q \in (\sqrt{2}c, 2c) \).

If \( a^{LL} > 0 \), then \((e^{LL}, a^{LL})\) are characterized by the first-order conditions
\[
\begin{align*}
\Delta q - C_e(e^{LL}, a^{LL}) - e^{LL} C_{ee}(e^{LL}, a^{LL}) &= 0, \quad (35) \\
e^{LL} C_{ea}(e^{LL}, a^{LL}) + K_a(a^{LL}) &= 0. \quad (36)
\end{align*}
\]

Given that the worker exerts effort \( e \), the conditional efficient level of motivation, \( a^*(e) \), minimizes total costs, i.e.,
\[ a^*(e) = \arg \min_a C(e, a) + K(a). \] (37)

If \( a^*(e) > 0 \), then \( C_a(e, a^*) + K_a(a^*) = 0 \). Comparing the latter equation with (36), \( a^{LL} > a^*(e^{LL}) \) if and only if \( e^{LL} C_{ea}(e^{LL}, a^{LL}) < C_a(e^{LL}, a^{LL}) \) or, equivalently, \( C_{aee} < 0 \). ■

**Proof of Proposition 4.** We first rewrite the firm’s problem (24) in terms of \( b \) and \( a \),
\[
\max_{b,a} e(a, b)(\Delta q - b) - \gamma k(a). \] (38)

Using the first-order conditions of (38), it is straightforward to verify that a decrease in \( \gamma \) entails a rise in the intensity of motivation, i.e., \( da^{LL} / d \gamma < 0 \). For the effect on the bonus \( b^{LL} \), we obtain (10). Given the motivational level \( a \), the conditional efficient work effort is
\[ e^*(a) = \arg \max_e e \Delta q - C(e, a), \] (39)

which is equivalent to \( \Delta q - C_e(e^*, a) = 0 \). This implies together with (35) that \( e^{LL} < e^*(a^{LL}) \) and, hence, \( b^{LL} < \Delta q \). ■

**Proof of Proposition 7.** We first characterize the solution to the firm’s problem. From (IC) and (IC-M), we see that \( b \) and \( b_M \) are non-negative. Given arbitrary non-negative bonuses and the optimal effort responses, the worker’s and the motivator’s expected bonus payment net of effort costs, \( eb - C(e, a) \) and \( eb_M - K(a) \), respectively, are at least zero. Thus, to satisfy (PC), (PC-M), and (19), the firm optimally sets \( s = s_M = 0 \). The firm’s problem can thus be
simplified to

\[
\max_{\varepsilon,a,b,M} \varepsilon(\Delta q - b - b_M)
\]
\[
\text{s.t. } b = C_e(e,a), \quad b_M = \frac{K_a(a)}{e_a(a,b)}.
\]

Defining \(\Psi(e,a)\) as the bonus offered to the motivator, \(\Psi(e,a) := \frac{K_a(a)}{e_a(a,C_e(e,a))}\), we can further rewrite the firm’s problem as

\[
\max_{\varepsilon,a} \varepsilon(\Delta q - C_e(e,a) - \Psi(e,a)).
\]

We again assume that the objective function is strictly concave\(^{23}\) and denote the solution to (42) by \((e_{LL}^M, a_{LL}^M)\). The firm still induces positive work effort, \(e_{LL}^M > 0\). When deciding whether the worker should be motivated or not, the firm trades off the benefit of lowering the worker’s expected bonus payment against the costs of motivation. These costs are now equal to the motivator’s expected bonus payment. Because worker and motivator earn a rent when they exert positive effort, the firm’s wage costs always exceed the total costs \(\Gamma(e,a)\). A sufficient condition for \(a_{LL}^M > 0\) is that the firm’s expected costs decrease in \(a\) for each positive effort level:

\[
e(C_{ea}(e,0) + \Psi_a(e,0)) < 0 \text{ for all } e > 0
\]
\[
\Leftrightarrow C_{ea}(e,0) + \Psi_a(e,0) < 0 \text{ for all } e > 0
\]

The second inequality shows that the expected wage costs are decreasing in \(a\) whenever the sum of the bonuses decreases in motivation. More specifically, for \(a_{LL}^M > 0\), it is sufficient that the sum of the bonuses is decreasing in motivation at \(e_0^{LL}\), i.e.,

\[
C_{ea}(e_0^{LL},0) + \Psi_a(e_0^{LL},0) < 0.
\]

An example for case (i) is \(C(e, a) = \frac{e^2}{2(1+ca)}\) and \(K(a) = k \frac{a^2}{T}\). From \(K_a(0) = 0\), we obtain

\(^{23}\)This is the case if the Hessian \(L\) of \(e(C_e + \Psi)\) is positive definite, i.e., \(2C_{ee} + eC_{eee} + \Psi_e + \Psi_{ee} > 0\) and \(\det L > 0\) with

\[
L = \begin{pmatrix}
2C_{ee} + eC_{eee} + \Psi_e + \Psi_{ee} & C_{ea} + eC_{eea} + \Psi_a + \Psi_{ea} & e(C_{eaa} + \Psi_{aa}) \\
C_{ea} + eC_{eea} + \Psi_a + \Psi_{ea} & C_{ea} + eC_{eea} + \Psi_a + \Psi_{ea}
\end{pmatrix}.
\]
Next, we verify that the sum of the bonuses, \( C_e(e, a) + \Psi(e, a) \), is increasing in \( a \) for all \( e \) and, consequently, \( a^M_L = 0 \). We have \( e(a, b) = (1 + ca)b \). Thus, recalling that 
\[
\Psi(e, a) = \frac{K_a(a)}{e_a(a, C_e(e, a))},
\]
the sum of the bonuses is
\[
e \frac{e}{1 + ca} + \frac{ka}{c(1 + ca)} = \frac{e}{1 + ca} + \frac{k}{ce}(1 + ca).
\]
This sum is increasing in \( a \) if
\[
-\frac{ce}{(1 + ca)^2} + \frac{k}{ce}(1 + 2ca) > 0 \iff (1 + 2ca)(1 + ca)^2 > \frac{c^2}{k}e^2.
\]
The last inequality holds for all \( a \geq 0 \) and \( e \in [0, 1] \) if \( k > c^2 \).

As an example for case (ii), consider \( C(e, a) = \frac{c^2}{2(1 + a)} \) and \( K(a) = \frac{k}{2}a^2 + ta \), as in the proof of Proposition 3. We already know that \( a^{FB} = 0 \) if \( t > \frac{c}{2} \). Furthermore, for the limited liability case, we have \( e_{0}^{LL} = \frac{\Delta q}{2c} \) if \( \Delta q < 2c \). Since \( e(a, b) = \frac{1}{c}(1 + a)b \), the motivator’s bonus and marginal bonus is \( \Psi(e, a) = \frac{ka + t}{1 + (a + 1)c} = \frac{(1 + a)(ka + t)}{e} \) and \( \Psi_a = \frac{ka + t + k(1 + a)}{e} = \frac{t + k(1 + 2a)}{e} \), respectively. By (45), we obtain \( a^M_L > 0 \) if
\[
C_{e0}(e_{0}^{LL}, 0) + \Psi_a(e_{0}^{LL}, 0) = -\frac{\Delta q}{2c} \frac{e}{(1 + 0)^2} + \frac{t + k}{2c} < 0
\]
\[\iff t < c \left( \frac{\Delta q}{2c} \right)^2 - k. \tag{49}\]
Furthermore,
\[
t = \frac{c}{2} < c \left( \frac{\Delta q}{2c} \right)^2 - k \iff 2c \left( \frac{1}{2} + \frac{k}{c} \right)^{1/2} < \Delta q \tag{50}\]
It follows that \( a^{FB} = 0 \) and \( a^M_L > 0 \) for, e.g., \( t = c/2 \) and \( \Delta q \in \left( 2c \left( \frac{1}{2} + \frac{k}{c} \right)^{1/2}, 2c \right) \), which is possible if \( \frac{1}{2} + \frac{k}{c} < 1 \). ■

References


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