Matsushima, Noriaki; Miyaoka, Akira

Working Paper

Who benefits from resale-below-cost laws?

ISER Discussion Paper, No. 875

Provided in Cooperation with:
The Institute of Social and Economic Research (ISER), Osaka University

Suggested Citation: Matsushima, Noriaki; Miyaoka, Akira (2013) : Who benefits from resale-below-cost laws?, ISER Discussion Paper, No. 875, Osaka University, Institute of Social and Economic Research (ISER), Osaka

This Version is available at:
hdl:10419/92842

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.
If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
WHO BENEFITS FROM RESALE-BELOW-COST LAWS?

Noriaki Matsushima
Akira Miyaoka

June 2013

The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan
Who benefits from resale-below-cost laws?*

Noriaki Matsushima†
Institute of Social and Economic Research, Osaka University

Akira Miyaoka‡
Graduate School of Economics, Osaka University

June 9, 2013

Abstract

We investigate the effect of banning resale-below-cost offers. There are two retailers with heterogeneous bargaining positions in relation to a monopolistic manufacturer. Each retailer sells two goods: one procured from the monopolistic manufacturer and the other, from a competitive fringe. In equilibrium, banning resale-below-cost offers can decrease the retailers’ prices. The ban can benefit the weak retailer in terms of bargaining position and increase the total consumer surplus, although it harms the dominant retailer and the monopolistic manufacturer. Contrary to the basic scenario, when the weak retailer is horizontally separated, the ban benefits the monopolistic manufacturer.

JEL Classification: L13, L41, M38, D21
Keywords: Loss-leader Pricing, Channel Power, Vertical Relations

*We especially thank Atsushi Kajii, Tadashi Sekiguchi, and the seminar participants at University of Nagasaki for helpful discussions and comments. The authors gratefully acknowledge financial support from the Japan Society for the Promotion of Science. The usual disclaimer applies.

†Noriaki Matsushima. Address: 6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan. E-mail: nmatsush@iser.osaka-u.ac.jp.

‡Akira Miyaoka (Corresponding Author). Address: 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, Japan. E-mail: jge013ma@mail2.econ.osaka-u.ac.jp.
1 Introduction

Retailers use price promotions as both an offensive mechanism to attract competitors’ customers and a defensive strategy to retain current customers (Gupta, 1988; Inman and McAlister, 1993; Raghbir et al., 2004; Srinivasan et al., 2004; Talukdar et al., 2010). As reported in Talukdar et al. (2010, p.336), according to the Promotion Marketing Association, in 2004, US retailers across all product categories spent about $429 billion in such promotions; a more recent estimate by ACNielsen (2007) suggests that promotional sales account for as much as 36% of total grocery sales.

One particularly popular price promotion used by retailers is the “loss-leader (or below-cost) pricing,” which refers to setting retail prices for the selected items at or below retailers’ respective marginal costs (Walters and MacKenzie, 1988). Price promotions through loss-leader pricing draw some profitable shoppers who would otherwise shop at competitors’ stores. Those promotions also draw unprofitable shoppers who only buy promoted items, as confirmed by recent empirical research (Gauri et al., 2008; Talukdar et al., 2010).  

While loss-leader pricing is an important promotion strategy in the retail sector, several countries have adopted resale-below-cost (RBC) laws, which prevent retailers from setting resale prices below purchase prices. In Germany, for instance, RBC laws forbid retailers from setting resale prices below purchase prices for extended periods and allow for private enforcement in the case of a defendant with a dominant position. Similarly, in the US, several states have sales-below-cost motor fuel laws that typically outlaw the selling of motor fuel (gasoline) at retail prices below cost.

---

1The effectiveness of loss-leader promotions is empirically unclear. Walters and Rinne (1986) show that certain portfolios of products promoted as loss leaders have a greater impact on store traffic, store sales, and deal sales than other product portfolios, with no significant impact on retailer profits. Walters and MacKenzie (1988) also find a significant impact of loss leaders on store traffic and store sales, but only two (out of eight) of their categories had significant effects on store profits—one positive and one negative. Recent empirical research does not support the effectiveness of loss-leader promotions either (Srinivasan et al., 2004; Ailawadi et al., 2009).

2The OECD Roundtable on Predatory Foreclosure identified that Ireland, France, and Germany have stringent RBC laws (OECD, 2007). Besides these nations, in the EU, RBC laws exist in Belgium, Hungary, Italy, Luxembourg, Portugal, Spain, and Greece. In the US, although a federal RBC law has not been adopted, many states have RBC laws either applying to all retail products or to specific goods such as gasoline or dairy products (Allain and Chambolle, 2011).

3Walmart’s troubles in Germany are blamed on regulations forbidding Walmart to institute its usual loss-leader strategy for common grocery products such as milk and eggs. The German Supreme Court held that Germany’s RBC prohibition applies regardless of any harm to competition from the RBC prices and that being a large firm relative to small and medium size competitors is sufficient to show that it has “superior” market power (OECD, 2007).
However, in spite of the prevalence of RBC laws, these regulations have been the subject of a heated debate in the area of competition policy. For example, OECD (2007) argues that RBC laws are likely to protect inefficient competitors and harm consumers. In fact, the existing empirical literature on sales-below-cost laws in gasoline retailing has mixed results. Although most studies show that these laws are associated with higher gasoline prices (e.g., Anderson and Johnson, 1999), several recent empirical studies show that the laws actually lower average gasoline prices (Skidmore et al., 2005; Carranza et al., 2009).

By considering the practical importance of loss-leader promotions, we need to understand how such promotions influence firms’ pricing strategies, and how banning RBC influences consumer welfare and firms’ profitability including that of retailers and manufacturers. Therefore, we investigate a linear city (Hotelling) model with two asymmetric retailers, each carrying two products, and with heterogenous consumer groups. One of the products is produced by a monopolistic manufacturer and the other, by a competitive fringe whose wholesale price is set at its marginal cost. Retailers are asymmetric in terms of bargaining positions over the monopolistic manufacturer. The “dominant” retailer can procure the good at the wholesale price equalized to the manufacturer’s marginal cost, but the “weak” retailer must procure at the wholesale price offered by the manufacturer in a take-it-or-leave-it manner. Consumer groups are heterogenous in terms of the number of products they need and per distance transportation costs. The first heterogeneity means that consumers in one group need two products, although those in the other group need only one of the two products. These heterogeneities generate below-cost prices as in DeGraba (2006) and Azar (2010).

In this basic setting, we first show that when it is legally permitted, below-cost pricing can appear as an equilibrium pricing strategy of both retailers and that the weak retailer, which must

---

4 The debate over whether to adopt or overturn sales-below-cost restrictions in gasoline retailing is ongoing (Eckert, 2013).
5 We briefly discuss the relationship between their results and ours later in this section.
6 This assumption reflects the dispute in Germany mentioned in the previous paragraph. That is, we capture a situation in which large retailers have strong bargaining positions, unlike small and medium size retailers. This assumption also captures the US gasoline market in the previous paragraph. That is, we capture a situation in which large retailers are integrated with gasoline wholesalers, which leads to lower internal transfer prices, although small and medium size retailers are independent from those gasoline wholesalers.
procure one of the goods at a higher wholesale price, is more likely to adopt that strategy. We next investigate how a ban on below-cost pricing affects the firms’ behaviors and show that the ban can decrease the retailers’ prices in equilibrium. We also show that the ban always improves the total consumer surplus and can benefit the weak retailer, although it always harms the dominant retailer and the monopolistic manufacturer.

Our results using the basic model are related to the empirical findings in the studies by Skidmore et al. (2005) and Carranza et al. (2009), which are based on theoretical predictions in which below-cost laws can encourage market entry and lead to reduced retail prices. Skidmore et al. (2005) show that these laws in the US retail gasoline market lower average gasoline prices, in part by increasing the number of gasoline outlets. Carranza et al. (2009) show that below-cost regulation imposed in the retail gasoline market in Québec led to more competition, lower prices for consumers, and lower productivity. Our study theoretically shows that a weak retailer benefits from the ban on below-cost pricing, which implies that the ban improves the survival rate of weak retailers. Our study result is related to the empirical findings in the two studies and might also imply that such bans worsen the efficiency in retail markets because the weak retailer’s marginal cost that includes its wholesale price are higher. Our study also shows that such a ban can decrease retail prices even though the number of retailers is exogenously fixed. Therefore, we believe that our study provides another explanation to the empirical findings in the two studies mentioned above, which is useful when setting competition policy.

Further, we extend our model to a setting in which the weak retailer is horizontally separated, thus eliminating the possibility that the separated retailers employ below-cost pricing. This extended model captures a case in which a big-box retailer (the dominant retailer in our model) competes with a shopping mall organized by independent retailers (the separated weak retailers in our model). We show that the ban increases the price of the loss-leader product over most of the parameter range, although the change in the price of the product competing with the loss-leader product depends on the exogenous parameters. We also show that the ban improves the total consumer surplus and benefits the monopolistic manufacturer and the weak retailer trading with it.

The outcomes derived from their theories are similar, although the theories in the two studies differ from each other.
although the ban can harm the other weak retailer and the dominant retailer. That is, the effect of the ban on the profitability of the manufacturer also depends on the retailer-level organizational structure.

Our results have a managerial implication for manufacturers in the context of vertical channels. In our study, the (weak) retailer trading with the monopolistic manufacturer benefits from a below-cost law, which implies that the law stabilizes the weak retailer as a distribution channel because the benefit from the law improves the financial stability of the weak retailer. This stability improvement, in itself, benefits the monopolistic manufacturer as well as the weak retailer. However, the relationship between imposing the law and the profitability of the monopolistic manufacturer depends on whether its product is used as a loss-leader product; in other words, it depends on whether its product is used as a tool to increase store traffic in its trading retailer. Our results imply that when sales managers in manufacturing companies choose retailers, they need to consider not only the regulatory environment in the market but also the pricing strategies of retailers.

Among the theoretical studies that explain the mechanisms of loss-leader promotions (Hess and Gerstner, 1987; Bliss, 1988; Bagwell and Ramey, 1994; Lal and Matutes, 1994; DeGraba, 2006; Azar, 2010), Innes and Hamilton (2009) show the market condition in which loss leadership can appear in the context of vertical restraints. The main concern of their study is to investigate how resale price maintenance (RPM) imposed by a monopolistic manufacturer on two competing symmetric retailers affects the equilibrium prices of those retailers trading two products including that of the manufacturer and the welfare property in equilibrium. They do not consider the effect of banning below-cost pricing and retailers’ asymmetry. Several recent studies discuss the effect of banning below-cost pricing. Chen and Rey (2012) analyze the effect of banning below-cost pricing under asymmetric retail competition with general demand structures, although they ignore the interactions between retailers and their suppliers; von Schlippenbach (2008) constructs a downstream monopoly model in which an upstream supplier produces a core product and competitive fringe suppliers produce complements to the core product. The effects of RBC laws in her model are

---

8 Rey and Vergé (2010) investigate how RPM can be used by duopoly manufacturers to control the retail pricing of duopoly retailers. They show that combining two-part tariffs with RPM can eliminate both interbrand and intrabrand competition and generate industry-wide monopoly pricing.
similar to that in our model, although her model cannot investigate asymmetric effects of RBC laws on heterogenous retailers. Allain and Chambolle (2011) investigate the effect of RBC laws in the context of the vertical restraints discussed in Innes and Hamilton (2009).

Our study is also in line with game-theoretical analyses on marketing channels (Jeuland and Shugan, 1983; McGuire and Staelin, 1983; Coughlan, 1985; Moorthy, 1988; Choi, 1991; Chu and Desai, 1995; Lal and Narasimhan, 1996; Lee and Staelin, 1997; Purohit, 1997; Kim and Staelin, 1999). We incorporate power asymmetry between retailers as in Geylani et al. (2007). Existing marketing literature also considers power within the channel—for instance, Butaney and Wortzel (1988), Messinger and Narasimhan (1995), Bloom and Perry (2001), Banks et al. (2002), and Iyer and Villas-Boas (2003). Raju and Zhang (2005) and Dukes et al. (2006) also investigate asymmetric channel relationships in retail competition. Raju and Zhang (2005) investigate how a manufacturer can best coordinate a channel in the presence of a dominant retailer. Dukes et al. (2006) focus on characterizing the impact of asymmetric retailing costs on wholesale bargaining and on profit distribution.

The rest of this paper is organized as follows. In the following section, we set up the model. In Section 3, we derive the equilibrium pricing of each firm when below-cost pricing is feasible and when a ban on below-cost pricing is introduced. In Section 4, we compare the equilibrium with and without the ban and explore the effect of banning below-cost pricing. In Section 5, we analyze the extended model where the weak retailer is horizontally separated. Finally, in Section 6, we present concluding remarks. Proofs of all propositions are provided in Appendix A.9

2 The Model

There are two downstream retailers, two upstream manufacturers, and two goods. While good 1 is supplied to both retailers by a monopolistic manufacturer, good 2 is supplied to both retailers by a competitive fringe. The marginal production cost of each good is constant and denoted by $c_1$ and $c_2$, respectively. For simplicity, the retailers’ marginal costs of retailing are assumed to be zero.

---

9The Mathematica files used to obtain the results in this paper are available from the authors upon request.
In the downstream market, we adopt a standard Hotelling framework (Hotelling, 1929), with two retailers located at the endpoints of the unit interval. Each retailer purchases both goods 1 and 2 from upstream manufacturers and sells them to consumers. In addition, as in Geylani et al. (2007), we assume that the two retailers are asymmetric with respect to the monopolistic manufacturer’s control over its wholesale prices. One retailer is a “dominant” retailer (denoted by \( D \)), and therefore, the manufacturer must set the wholesale price to this retailer at its marginal cost level, that is, \( w_{1D} = c_1 \). However, the other retailer is a “weak” retailer (denoted by \( W \)), and the manufacturer can offer a take-it-or-leave-it wholesale price \( w_{1W} \). As for good 2, the competitive fringe sets the wholesale prices to both retailers at its marginal cost level, that is, \( w_{2D} = w_{2W} = c_2 \). We assume that retailer \( D \) (\( W \)) is located at the left (right) end of the unit interval. The price set by retailer \( i = D, W \) for good \( k = 1, 2 \) is given as \( p_{ki} \). Figure 1 illustrates the industrial structure of our model explained here.

[Figure 1 about here.]

In the downstream market, there are two types of consumers: type 2 and \( B \). Type 2 consumers purchase only good 2 and have a mass of \( \lambda > 0 \). Type \( B \) consumers purchase both goods, 1 and 2, and have a mass that is normalized to 1. Each type of consumers is assumed to be uniformly distributed on the unit interval.\(^{10}\) In order to travel to either retailer, consumers must incur a linear transportation cost for the distance traveled. While the cost per unit of distance for type 2 consumers is \( t \), that for type \( B \) consumers is \( \tau \). The transportation cost of a type 2 consumer located at \( x \) is \( tx \) if he/she purchases good(s) from retailer \( D \) or \( t(1 - x) \) if he/she purchases good(s) from retailer \( W \). Similarly, the transportation cost of a type \( B \) consumer located at \( x \) is \( \tau x \) if he/she purchases good(s) from retailer \( D \) or \( \tau(1 - x) \) if he/she purchases good(s) from retailer \( W \).

Type 2 consumers purchase one unit of good 2 from one of the retailers. Similarly, type \( B \) consumers purchase one unit of both goods from one of the retailers. We assume that type \( B \) consumers do not split purchases between different retailers.\(^{11}\) Therefore, each consumer purchases

\(^{10}\)We do not consider “type 1” consumers who purchase only good 1. If the population of type 1 consumers is smaller than that of the other two types, adding type 1 consumers to the model does not alter our main results qualitatively.

\(^{11}\)This is because the cost of standing in a checkout line and searching for goods is relatively larger than the traveling...
from the retailer for which the sum of the retail prices and the transportation cost is lower. The sum of those costs incurred by a type 2 consumer is given as

\[ \begin{align*}
&\begin{cases} 
  tx + p_{2D}, & \text{if buying from retailer } D, \\
  t(1-x) + p_{2W}, & \text{if buying from retailer } W.
\end{cases}
\end{align*} \tag{1} \]

The sum of those costs incurred by a type B consumer is given as

\[ \begin{align*}
&\begin{cases} 
  \tau x + p_{1D} + p_{2D}, & \text{if buying from retailer } D, \\
  \tau(1-x) + p_{1W} + p_{2W}, & \text{if buying from retailer } W.
\end{cases}
\end{align*} \tag{2} \]

The game runs as follows: First, the monopolistic manufacturer sets the wholesale price, \( w_{1W} \), for retailer \( W \). Note that the rest of the wholesale prices are set at the manufacturers’ marginal cost levels. Second, given the wholesale prices, the retailers set the retail prices, \( p_{ki} \) \((k = 1, 2, i = D, W)\), in the downstream market. Finally, following his/her preference, each consumer purchases good(s) from either retailer.

### 3 Equilibrium

In this section, we derive the equilibrium pricing of the monopolistic manufacturer and the retailers.

In Section 3.1, we first explore the case where downstream retailers can set their retail prices freely. Then, in Section 3.2, we analyze the case when the ban on below-cost pricing is introduced.

#### 3.1 When below-cost pricing is feasible

In this subsection, we derive the equilibrium wholesale and retail prices when downstream retailers can adopt below-cost pricing. Because each type of consumers purchases one good or two goods from the retailer that minimizes the total cost, there is a consumer who is indifferent between the two retailers. Let \( \bar{x}_2 \in [0, 1] \) represent the location of the indifferent consumer of type 2. From (1), \( \bar{x}_2 \) is given as follows:

\[ p_{2D} + t\bar{x}_2 = p_{2W} + t(1-\bar{x}_2) \quad \Leftrightarrow \quad \bar{x}_2 = \frac{1}{2} + \frac{p_{2W} - p_{2D}}{2t}. \tag{3} \]
Similarly, for type $B$ consumers, from (2), the location of the indifferent consumer $\bar{x}_B \in [0, 1]$ is given by

$$\bar{x}_B = \frac{1}{2} + \frac{p_{1W} + p_{2W} - p_{1D} - p_{2D}}{2\tau}. \quad (4)$$

Therefore, given the wholesale prices, the dominant and weak retailers simultaneously choose their retail prices to maximize the following profits, respectively:

$$\Pi_D = (p_{1D} + p_{2D} - w_{1D} - w_{2D})\bar{x}_B + (p_{2D} - w_{2D})\lambda\bar{x}_2, \quad (5)$$

$$\Pi_W = (p_{1W} + p_{2W} - w_{1W} - w_{2W})(1 - \bar{x}_B) + (p_{2W} - w_{2W})\lambda(1 - \bar{x}_2). \quad (6)$$

From (5), we have the following first-order conditions for the dominant retailer:

$$\frac{\partial \Pi_D}{\partial p_{1D}} = \left( \frac{1}{2} + \frac{p_{1W} + p_{2W} - p_{1D} - p_{2D}}{2\tau} \right) - \frac{p_{1D} + p_{2D} - w_{1D} - w_{2D}}{2\tau} = 0, \quad (7)$$

$$\frac{\partial \Pi_D}{\partial p_{2D}} = \left( \frac{1}{2} + \frac{p_{1W} + p_{2W} - p_{1D} - p_{2D}}{2\tau} \right) - \frac{p_{1D} + p_{2D} - w_{1D} - w_{2D}}{2\tau} + \lambda\left( \frac{1}{2} + \frac{p_{2W} - p_{2D}}{2t} \right) - \frac{\lambda(p_{2D} - w_{2D})}{2t} = 0. \quad (8)$$

From (6), the first-order conditions for the weak retailer are the same as (7) and (8) with the subscripts $D$ and $W$ reversed. Then, by solving these four first-order conditions, we have the following retail prices for the given wholesale prices:

$$p_{1D} = \frac{2w_{1D} + w_{1W}}{3} + \tau - t; \quad p_{2D} = w_{2D} + t; \quad (9)$$

$$p_{1W} = \frac{w_{1D} + 2w_{1W}}{3} + \tau - t; \quad p_{2W} = w_{2W} + t. \quad (10)$$

The upstream manufacturers decide their wholesale prices anticipating the retailers’ pricing given by (9) and (10). The competitive fringe prices good 2 at its marginal cost, that is, $w_{2D} = w_{2W} = c_2$. However, while the monopolistic manufacturer sets the wholesale price of good 1 to the dominant retailer at the marginal cost, that is, $w_{1D} = c_1$, it offers a take-it-or-leave-it wholesale price $w_{1W}$ to the weak retailer in order to maximize the following profit:

$$\Pi_1 = (w_{1W} - c_1)(1 - \bar{x}_B). \quad (11)$$
Let $m_{ki}$ denote the price-cost margin of good $k$ for retailer $i$, that is, $m_{ki} = p_{ki} - w_{ki}$. In addition, we denote with an asterisk ($*$) the equilibrium values when the ban on below-cost pricing is not imposed. Then, we obtain the following results that show that below-cost pricing can appear as an equilibrium strategy of downstream retailers.

**Proposition 1.** When the ban on below-cost pricing is not imposed, both retailers’ price-cost margins for good 2 are always positive, that is, $m^*_2i > 0$ for $i = D, W$. By contrast, the price-cost margins of good 1 can be negative when the value of $\tau/t$ is sufficiently small. More precisely,

(a) for $0 < \tau/t < 2/3$, both retailers’ price-cost margins of good 1 are negative, that is, $m^*_1i < 0$ for $i = D, W$,

(b) for $2/3 < \tau/t < 2$, only the weak retailer’s price-cost margin of good 1 is negative, that is, $m^*_{1D} > 0$ and $m^*_{1W} < 0$, and

(c) for $\tau/t > 2$, both retailers’ price-cost margins of good 1 are positive, that is, $m^*_1i > 0$ for $i = D, W$.

Figure 2(a) summarizes the results of Proposition 1. These results suggest that the retailers may set the price of good 1 below the wholesale price in equilibrium when below-cost pricing is feasible. This is due to the existence of type $B$ consumers who purchase both goods at either retailer. Since type $B$ consumers care about the sum of both goods’ prices, the retailers can attract them by lowering either the price of good 1 or the price of good 2, as in the models of DeGraba (2006) and Azar (2010). Lowering the price of good 1 does not affect the profits from type 2 consumers, although lowering the price of good 2 reduces those profits. Therefore, it is more attractive for the retailers to lower the price of good 1. In particular, when $\tau/t$ is sufficiently small ($\tau/t < 2$), type 2 consumers are so price inelastic that the retailers can charge a high markup on good 2, which is too high from the perspective of type $B$ consumers. Therefore, it is optimal for the retailers to make the price-cost margin on good 1 negative to attract type $B$ consumers, while charging a high markup on good 2.
In addition, in equilibrium, the monopolistic manufacturer sets the wholesale price for the weak retailer higher than for the dominant retailer (that is, \( w_{1W}^* > w_{1D}^* = c_1 \)). Therefore, the weak retailer is more likely to price good 1 below cost than the dominant one.\(^{12}\) In fact, some survey evidence suggests that small stores are just as likely as large stores to find it advantageous to employ below-cost pricing as a marketing tool in order to generate store traffic (OECD, 2007).

3.2 When below-cost pricing is banned

Next, we explore the equilibrium pricing of the upstream monopolist and downstream retailers when below-cost pricing is banned. When a ban on below-cost pricing is introduced, each retailer cannot price its goods below the wholesale prices at which it procures the goods. Therefore, given the wholesale prices, the dominant and the weak retailers, respectively, choose their retail prices in order to maximize (5) and (6) under the constraints that \( p_{kD} \geq w_{kD} \) and \( p_{kW} \geq w_{kW} \) for \( k = 1, 2 \).

Anticipating the retail pricing, the upstream manufacturers set their wholesale prices as in the previous subsection. Since good 2 is supplied competitively, its wholesale prices for both retailers are set at the marginal cost level. However, for good 1, the upstream monopolist chooses its wholesale price for the weak retailer to maximize its own profit given by (11), while setting the wholesale price for the dominant retailer at its marginal cost.

As shown in Proposition 1, even though the ban on below-cost pricing is inactive, the retailers never set the prices below their wholesale prices when \( \tau/t \geq 2 \). Therefore, when \( \tau/t \geq 2 \), the ban is not binding, and it affects neither the retail nor wholesale pricing in equilibrium. On the other hand, when \( \tau/t < 2 \), the retailers have incentives to adopt below-cost pricing. Therefore, the retailers’ and manufacturers’ equilibrium prices will alter by introducing the ban on below-cost pricing.

Let variables with a tilde (~) denote equilibrium values when below-cost pricing is banned. Then, the following proposition summarizes the equilibrium retail pricing under a ban on below-cost pricing.

**Proposition 2.** When the ban on below-cost pricing is imposed, both retailers’ price-cost margins

\(^{12}\)In Section 5, we consider an extended model where the weak retailer is horizontally separated. In that setting, only the dominant retailer can adopt below-cost pricing in equilibrium, as in Chen and Rey (2012). See Section 5 for details.
of good 2 are always positive, that is, $\bar{m}_{2i} > 0$ for $i = D, W$. On the other hand, the price-cost margins of good 1 can be zero when $\tau/t$ is sufficiently small. More precisely,

(a) for $0 < \tau/t \leq T$, both retailers’ price-cost margins of good 1 are zero, that is, $\bar{m}_{1i} = 0$ for $i = D, W$,

(b) for $T < \tau/t \leq 2$, only the weak retailer’s price-cost margin of good 1 is zero, that is, $\bar{m}_{1D} > 0$ and $\bar{m}_{1W} = 0$, and

(c) for $\tau/t > 2$, both retailers’ price-cost margins of good 1 are positive, that is, $\bar{m}_{1i} > 0$ for $i = D, W$,

where the exact expression for $T$ is derived in the Appendix.

Figure 2(b) illustrates the results of Proposition 2. Proposition 1 shows that retailer $W$’s price-cost margin of good 1 is smaller than that of retailer $D$. The pricing of retailer $W$ is more likely to bind the constraint of the ban.

4 The Effects of Banning Below-cost Pricing

In the previous section, we derived the equilibrium price with and without the ban on below-cost pricing. In this section, by comparing the results in the previous section, we analyze the effects of banning below-cost pricing.

First, we explore the effects of the ban on the wholesale and the retail prices.

Proposition 3. The effects of banning below-cost pricing on wholesale and retail prices are summarized as follows:

(a) The monopolistic manufacturer’s wholesale price always decreases.

(b) Both retailers’ prices of good 1 decrease when $\tau/t$ is sufficiently large.

(c) Both retailers’ prices of good 2 never increase. In particular, they decrease except when $\tau/t$ is too large.
When the ban on below-cost pricing is not imposed, as explained in Proposition 1, the weak retailer tries to attract consumers away from the dominant retailer by setting the price of good 1 at a level below the wholesale price. When the ban is imposed, the weak retailer must set the price of good 1 at least equal to the wholesale price. Anticipating this higher retail price set by the weak retailer, the monopolistic manufacturer sets a lower wholesale price to mitigate the demand loss of good 1 distributed by the weak retailer.

Figure 3(a) summarizes the effect of banning below-cost pricing on the retail prices. Because the products sold by each retailer are complements, an increase in the price of one product is more likely to decrease the price of the other one. As shown in Figure 3(a), however, when \( \tau/t \) is large (\( T_p < \tau/t < \bar{T} \)), both \( p_{1i} \) and \( p_{2i} \) simultaneously decrease by banning below-cost pricing. We explain the mechanism below.

As for the weak retailer’s pricing of good 1, banning below-cost pricing has two opposite effects. While it can lower the price through the reduction of the wholesale price as explained above, it can also raise the price by preventing the weak retailer from having a negative price-cost margin on good 1. This trade-off depends on the pricing for good 2. When the ban is imposed, the price-cost margin of good 1 increases from a negative level to zero, which encourages the weak retailer to attract consumers of type \( B \) through a decrease in \( p_{2W} \). This incentive to decrease \( p_{2W} \) is relatively weaker as \( \tau/t \) is larger, because an increase in the price-cost margin on good 1 through the ban is relatively smaller.

---

13 The exact expressions for \( T_p \) and \( \bar{T} \) in the figure are derived in the Appendix.
14 A similar result is obtained by von Schlippenbach (2008). In her model, the downstream monopolist supplies the monopolistic manufacturer’s product and complementary products procured from competitive suppliers. Although competitive suppliers’ products are complements to the monopolistic supplier’s product, a decrease in the price of the monopolistic manufacturer’s product causes decreases in the prices of those complementary products.
15 For \( \bar{T} < \tau/t < 2 \), the mechanism behind a decrease in \( p_{1i} \) through the ban is the same as explained below. However, the reason the ban does not affect \( p_{2i} \) is as follows. In this range, it is optimal for the monopolistic manufacturer to set its wholesale price such that the constraint of the ban for the weak retailer’s price of good 2 is just binding. Therefore, given the wholesale price set optimally, the constraint of the ban is not effective, and the equilibrium retailers’ responses are given by (9) and (10).
16 This can be also confirmed from the Kuhn-Tucker conditions derived in the Appendix. From (29) and (30), the best response price of good 2 for retailer \( W \) can be derived as \( p_{2W} = (p_{2D} + t + c_2)/2 - \mu_W/A \), where \( \mu_W \) is the Lagrange multiplier of retailer \( W \)’s maximization problem. The Lagrange multiplier \( \mu_W \) is decreasing in \( \tau/t \) and it is equal to zero.

12
price significantly in order to maintain the demand for good 1 distributed by the weak retailer. Therefore, when \( \frac{\tau}{t} \) is large, the effect of the reduction of the wholesale price dominates the other one, and then, the ban decreases the weak retailer’s prices of both goods. In addition, because of strategic complementarity between both retailers’ pricing, the dominant retailer’s prices of both goods also decrease through the ban.

Next, we consider the effects of banning below-cost pricing on the profits of the monopolistic manufacturer and downstream retailers.

**Proposition 4.** The effects of banning below-cost pricing on firms’ profits are summarized as follows:

(a) The monopolistic manufacturer’s profit always decreases.

(b) The dominant retailer’s profit always decreases.

(c) The weak retailer’s profit increases when \( \frac{\tau}{t} \) is sufficiently large.

These results are illustrated in Figure 3(b).\(^{17}\) The ban prevents the weak retailer from setting the price of good 1 at a below-cost level, which makes it difficult for the monopolistic manufacturer to set a higher wholesale price. This reduces the profit of the monopolistic manufacturer. The ban lowers the prices of good 2, which lowers the profits of the retailers from type 2 consumers. The lower \( \frac{\tau}{t} \) is, the stronger is the negative effect, because the relative profitability of the market for type 2 consumers becomes larger. Because of the negative effect, the ban always reduces the profit of the dominant retailer. For the weak retailer, however, the negative effect does not always dominate the positive effect of the decrease in the wholesale price set by the monopolistic manufacturer. When \( \frac{\tau}{t} \) is large, the positive effect can dominate the negative one.

Finally, we analyze how introducing the ban affects consumer surplus. In this study, since we assume that all consumers are served in equilibrium, consumer surplus for type 2 and type B without a ban on below-cost pricing. Therefore, the ban shifts the best response curve downward less significantly when \( \frac{\tau}{t} \) is large.

\(^{17}\)In the Appendix, we explain how to derive \( T_H \) in the figure.
consumers are, respectively, given by

\[ CS_2 = -\lambda \left( \int_0^{\bar{x}_2} (p_{2D} + tm)dm + \int_{\bar{x}_2}^{1} (p_{2W} + t(1 - m))dm \right) , \]

(12)

\[ CS_B = - \int_0^{\bar{x}_B} (p_{1D} + p_{2D} + \tau m)dm - \int_{\bar{x}_B}^{1} (p_{1W} + p_{2W} + \tau (1 - m))dm. \]

(13)

The total consumer surplus is given by \( CS_T = CS_2 + CS_B \). Then, we have the following results on the effects of banning below-cost pricing on consumer surplus.

**Proposition 5.** The effects of banning below-cost pricing on consumer surplus are summarized as follows:

(a) **Banning below-cost pricing never harms type 2 consumers. In particular, it benefits them except when \( \tau/t \) is too large.**

(b) **Banning below-cost pricing benefits type B consumers when \( \tau/t \) is sufficiently large.**

(c) **Banning below-cost pricing always increases the total consumer surplus.**

These results are also summarized in Figure 3(c). As shown in Proposition 3, when the ban is imposed, the prices of good 2 set by both retailers do not increase and, in particular, decrease when \( \tau/t \) is not large enough. Therefore, the ban at least does not harm type 2 consumers, who purchase only good 2, and can benefit them through a decrease in the price of good 2. By contrast, type B consumers, who buy both goods, can be hurt by the ban, which directly eliminates the below-cost prices of good 1. When \( \tau \) is relatively smaller than \( t \) (\( \tau/t \) is smaller than 1), attracting type B consumers is relatively easy for each retailer because those consumers are relatively price elastic than type 2 consumers. To do so, controlling the price of good 1 is relatively useful for each retailer to attract type B consumers. The ban restricts the retailer’s pricing for good 1, which leads to increases in those prices. As a result, the restriction mitigates competition between the retailers to attract type B consumers, which increases the total payment of those consumers.
5 Extension: A Big-box Retailer vs. a Shopping Mall

In the basic model, we assumed that each retailer sells both goods, and this assumption enables both retailers to engage in below-cost pricing. How does the equilibrium property change if we assume that the weak retailer is separated? In this section, we extend the basic model and investigate how the structural change of the retail sector affects the effect of banning below-cost pricing.

5.1 The extended model

We consider the case where the dominant retailer competes with two weak retailers, $W_1$ and $W_2$, who carry good 1 and 2, respectively (see Figure 4). We interpret the group of the separated weak retailers as a shopping mall, which competes with a big-box retailer.\(^\text{18}\) In this setting, since each weak retailer carries only one brand, it cannot set below-cost prices, which always lead to negative profits. However, the dominant retailer sells both brands and can engage in below-cost pricing, as in the basic model. As shown below, this structural change of the retail sector drastically alters the effect of banning below-cost pricing.

\[\text{[Figure 4 about here.]}\]

The profits of the separated weak retailers are, respectively, given as follows:

\[
\Pi_{W_1} = (p_{1W} - w_{1W})(1 - \bar{x}_B),
\]

\[
\Pi_{W_2} = (p_{2W} - w_{2W})(1 - \bar{x}_B + \lambda(1 - \bar{x}_2)).
\]

Note that each weak retailer does not internalize the effect of its own pricing on the other weak retailer through the behavior of type $B$ consumers. The profits of the dominant retailer and the monopolistic manufacturer are the same as in the basic model and represented by (5) and (11), respectively. The timing structure of the game is also the same as in the basic model.

\(^{18}\)The case discussed here is also related to a case in which one manufacturer sets up a store-within-a-store, and the retailer buys the other’s product at a wholesale price and sells it at a marked-up retail price (Jerath and Zhang, 2010). The separated retailer trading with a competitive fringe is related to a manufacturer within a store and that trading with the monopolistic manufacturer is related to the retailer buying the other manufacturer’s product at a wholesale price.
As mentioned above, in this extended model, only the dominant retailer can set below-cost prices in equilibrium. Figure 5 illustrates the equilibrium price-cost margin of good 1 set by the dominant retailer with and without the ban on below-cost pricing. When below-cost pricing is legally permitted, the dominant retailer sets the price of good 1 below the wholesale price if $\tau/t$ is sufficiently small. The intuition behind this result is the same as in the basic model. By contrast, when the ban is imposed, it is binding in the parameter range where the dominant retailer engaged in below-cost pricing without the ban. In the relevant range of parameters, the dominant retailer sets the price of good 1 equal to the wholesale price, and the ban affects the prices, profits, and consumer surplus in equilibrium, as summarized in Figure 6. We examine each of these effects in detail below.

![Figure 5 about here.]

![Figure 6 about here.]

**The effect of the ban on prices** The ban increases $p_{1D}$ for most of the cases where the ban is binding for the dominant retailer in equilibrium. Obviously, this is because the ban prevents the dominant retailer from setting the price of good 1 below the wholesale price, $c_1$. The constraint on $p_{1D}$ has a direct negative impact on $p_{2D}$.

Next, we consider the effect of the ban on the monopolistic manufacturer’s wholesale pricing. We call the weak retailer trading with the monopolistic manufacturer and the other weak retailer “the trading weak retailer” and “the outside weak retailer”, respectively. From (11), the monopolistic

---

19Proofs of all results in this section are provided in Appendix B.

20According to Figure 5, the ban is binding even in the parameter range where the dominant retailer does not engage in below-cost pricing without the ban (i.e., $T^c < \tau/t < \hat{T}^c$). In this range, under the ban, the monopolistic manufacturer sets a lower wholesale price in order to induce the weak retailer to decrease $p_{1W}$. Since the dominant retailer cannot reduce $p_{1D}$ below $c_1$ in response to the decrease in $p_{1W}$ owing to the ban, the monopolistic manufacturer can significantly attract the demand for its product through the weak retailer.

21In Figure 6, we do not illustrate the effects on the monopolistic manufacturer’s profit and total consumer surplus, because the direction of those effects is independent of the parameter values, as explained below.

22Note that for $\tau/t$ around the threshold value of $\tau/t$ in which the ban is binding for the dominant retailer in equilibrium, $p_{1D}$ decreases by the ban. The reason for this phenomenon is explained in footnote 20.
manufacturer’s first-order condition is given as
\[
\frac{\partial \Pi_1}{\partial w_{1W}} = (1 - \bar{x}_B) + (w_{1W} - c_1) \frac{\partial (1 - \bar{x}_B)}{\partial w_{1W}}.
\] (16)

The ban on below-cost pricing affects both terms in the right-hand side of this condition.

First, the ban works as a commitment of the dominant retailer to set \( p_{1D} = c_1 \), which implies that it becomes the first-mover in the price competition with the trading weak retailer. This also implies that the trading weak retailer has the second-mover advantage, which increases the demand from type \( B \) consumers for the weak retailers, \( 1 - x_B \). This induces the monopolistic manufacturer to increase its wholesale price for the trading weak retailer.

Second, the price commitment has another effect on the competition between the dominant retailer and the trading weak retailer. Under the ban, the dominant retailer cannot reduce its price of good 1 in response to a decrease in the trading weak retailer’s price. Therefore, the ban changes the strategic interaction between them from strategic complements to strategic independence, which means that a decrease in the trading weak retailer’s price attracts the demand for it more than the case without the ban. In other words, the elasticity of demand for this retailer, which is correlated to \( \frac{\partial (1 - \bar{x}_B)}{\partial w_{1W}} \), becomes higher. This effect gives the monopolistic manufacturer an incentive to decrease its wholesale price.\(^{23}\)

The relative scale of these two effects depends on \( \tau/t \). When \( \tau/t \) is small, the first effect is large enough to dominate the second effect. However, when \( \tau/t \) is large, while the second effect remains effective, the first effect becomes sufficiently small because an increase in \( p_{1D} \) through the ban is very small. Therefore, as illustrated in Figure 6(a), the ban increases (decreases) \( w_{1W} \) when \( \tau/t \) is small (large).

**The effect of the ban on profits**  The ban always benefits the monopolistic manufacturer and the weak retailer trading with it because the dominant retailer becomes less aggressive in the pricing

\(^{23}\)To be precise, there is another effect of the ban that counteracts the price-commitment effect explained in this paragraph. Under the ban, since the dominant retailer cannot adjust \( p_{1D} \) flexibly, it responds to a decrease in \( p_{1W} \) by decreasing \( p_{2D} \) more aggressively than the case without the ban. This effect makes the demand for the trading weak retailer less elastic and gives the monopolistic manufacturer an incentive to increase \( w_{1W} \). However, since this effect is relatively weaker than the price-commitment effect explained in this paragraph, the overall effect of the ban on the demand elasticity tends to reduce \( w_{1W} \).
of $p_{1D}$. The other weak retailer also benefits from the ban if and only if its retail price increases in equilibrium.

As regards the dominant retailer’s profit, Figure 6(c) shows that the effect of the ban is non-monotonic in $\tau/t$. Behind this result, there are three effects of the ban on the dominant retailer’s profit. First, the ban restricts the scope of the dominant retailer to control demand for the goods through a change in $p_{1D}$ because it cannot set $p_{1D}$ at a below-cost level. This loss of flexibility hurts the dominant retailer, and this effect is stronger as $\tau/t$ is smaller because the below-cost pricing is more valuable.

Second, as explained above, the ban affects the wholesale price of good 1 for the weak retailer. In particular, when $\tau/t$ is large enough, the ban reduces the wholesale price, which gives the weak retailers a competitive advantage vis-a-vis the dominant retailer.

Third, since the ban works as a commitment of the dominant retailer to set $p_{1D} = c_1$, this can mitigate the price competition and benefit the dominant retailer. The strength of this effect is non-monotonic in $\tau/t$. When $\tau/t$ is nearly equal to the threshold value, this effect does not work because $p_{1D} = c_1$ is nearly equal to the equilibrium price in the case without the ban. On the other hand, when the value of $\tau/t$ is small enough, this effect does not work, because a small $\tau/t$ implies high elasticity of type $B$ consumers’ demand. Therefore, this effect is strong for an intermediate range of $\tau/t$.

These three effects produce the result in Figure 6(c). When $\tau/t$ is sufficiently small or large, since the first or second effects become dominant, the ban hurts the dominant retailer. However, when $\tau/t$ falls within an intermediate range, the third effect dominates the other effects and the ban benefits the dominant retailer.

**The effect of the ban on consumer surplus** The ban always increases the total consumer surplus, which seems similar to the result in the basic model. The main factor behind the welfare property is decreases in the prices of good 2.
5.2 Discussion

When compared to that of the basic model, the results in this section imply that the effects of banning below-cost pricing on the prices and the profits of each firm crucially depend on the downstream market structure.\textsuperscript{24} As discussed in the Introduction, the existing empirical literature has mixed results on the effects of (banning) below-cost pricing on the prices and retailers’ profits. In addition, there are also mixed empirical or anecdotal evidences on whether (banning) below-cost pricing benefits or hurts the upstream manufacturers. On one hand, Srinivasan et al. (2004) show empirical findings that price promotions have a predominantly positive impact on manufacturer revenues. On the other hand, it is also observed that manufacturers sometimes object to loss-leader pricing on the ground that competition between retailers can bring pressure on the manufacturers to lower the wholesale prices.\textsuperscript{25} Our findings could be one possible explanation for these seemingly contradictory observations.

6 Conclusion

This study analyzed below-cost pricing by retailers and the effects of resale-below-cost laws that prohibit retailers from pricing their goods below wholesale prices. The key feature of our model is the power asymmetry between retailers. One retailer (weak retailer) does not have bargaining power over a monopolistic manufacturer; the other retailer (dominant retailer) has full bargaining power over manufacturers, including the monopolistic one.

Under a basic model in which each retailer sells two products including that of the monopolistic manufacturer, we showed that below-cost pricing could appear as an equilibrium pricing strategy of both retailers and that the weak retailer is more likely to adopt that strategy. We also showed that the ban could decrease the prices of the retailers in equilibrium. This result is related to empirical

\textsuperscript{24}In the basic model of previous sections, we assumed that resale-below-cost laws are equally imposed on both dominant and weak retailers. If the laws are imposed only on the dominant retailer because of its strong market power, the effect of the ban on the monopolistic manufacturer is similar to that in the model of this section. That is, the ban benefits it.

\textsuperscript{25}For example, recently in Germany, the German Farmer Association (Deutscher Bauernverband) and the German Association of Brand Manufacturers (Deutscher Markenverband) complained against below-cost prices by large retailers (von Schlippenbach, 2008). In addition, Marvel (1994) also reports that manufacturers often object to loss-leader status because it makes retailers reluctant to order inventories in the presence of demand uncertainty.
findings (that is, in Skidmore et al., 2005 and Carranza et al., 2009) that are based on theoretical predictions in which below-cost laws can encourage market entry leading to reduced retail prices. Our study also showed that the ban could decrease retail prices even though the number of retailers is exogenously fixed. We therefore believe that our study provides another explanation to the empirical findings mentioned above.

In addition, we analyzed an extended model in which the weak retailer is horizontally separated. This extended model captures a case in which a big-box retailer competes with a shopping mall organized by independent retailers. We found that when a ban on below-cost pricing is binding, the ban benefits the monopolistic manufacturer and the weak retailer trading with it, although the ban harms the other weak retailer and the dominant retailer. The result implies that the effect of the ban on the manufacturer’s profitability crucially depends on the retailer’s pricing strategy.

In this study, we employed the Hotelling framework with two multi-product retailers and with heterogenous consumer groups, as in DeGraba (2006) and Azar (2010). However, some previous studies use other demand structures to analyze loss-leader pricing. We leave it for future work to analyze the retailers’ pricing behavior and the effects of banning loss-leader pricing under such demand structures.

Appendix A: Proofs

Proof of Proposition 1

Substituting wholesale prices (other than \( w_{1W} \)) and retailers’ responses given by (9) and (10) into the manufacturer’s profits (11), we have

\[
\Pi_1 = (w_{1W} - c_1) \left( \frac{1}{2} - \frac{w_{1W} - c_1}{6\tau} \right). \tag{17}
\]

Then, the monopolistic manufacturer optimally sets the wholesale price at

\[
w_{1W}^* = c_1 + \frac{3\tau}{2}. \tag{18}
\]

\[\text{For instance, Innes and Hamilton (2009) use the Hotelling-type spatial competition model combined with the demand functions derived from a quasi-linear utility function. Allain and Chambolle (2011) use a linear demand framework developed by Dobson and Waterson (1996), which captures both interbrand and intrabrand competition.}\]
From (18), the equilibrium retail prices are obtained as follows:

\[ p^*_1D = c_1 + \frac{3\tau}{2} - t; \quad p^*_1W = c_1 + 2\tau - t; \quad \text{and} \quad p^*_2D = p^*_2W = c_2 + t. \] (19)

For good 2, it is easy to see that \( m^*_2i = p^*_2i - c_2 = t \) and that both retailers’ price-cost margin is always positive. On the other hand, for good 1, each retailer’s price-cost margin becomes

\[ m^*_1D = p^*_1D - c_1 = \frac{3\tau}{2} - t \gtrless 0 \Leftrightarrow \frac{\tau}{t} \gtrless \frac{2}{3}; \] (20)

\[ m^*_1W = p^*_1W - w^*_1W = \frac{\tau}{2} - t \gtrless 0 \Leftrightarrow \frac{\tau}{t} \gtrless 2. \] (21)

Therefore, we obtain the results in Proposition 1. \[ \blacksquare \]

**Proof of Proposition 2**

In the following proof, we first derive an equilibrium under a ban, temporarily ignoring the constraints for good 2 (\( p^*_2i \geq w^*_2i \) for \( i = D, W \)), and later we check that those constraints are certainly not binding in that equilibrium. Then, the Lagrange functions of each retailer’s profit maximization problem are as follows:

\[ L_D = (p^*_1D + p^*_2D - c_1 - c_2)\bar{x}_B + (p^*_2D - c_2)\lambda \bar{x}_2 + \mu_D(p^*_1D - c_1), \] (22)

\[ L_W = (p^*_1W + p^*_2W - w^*_1W - c_2)(1 - \bar{x}_B) + (p^*_2W - c_2)\lambda (1 - \bar{x}_2) + \mu_W(p^*_1W - w^*_1W), \] (23)

where \( \bar{x}_2 \) and \( \bar{x}_B \) are respectively given by (3) and (4), and \( \mu_D \) and \( \mu_W \) are Lagrange multipliers.

Note that we have \( w^*_{1D} = c_1 \) and \( w^*_{2D} = w^*_2W = c_2 \) by assumption. Then, we have the following Kuhn-Tucker conditions for the dominant retailer:

\[ \frac{\partial L_D}{\partial p^*_1D} = \left( \frac{1}{2} + \frac{p^*_1W + p^*_2W - p^*_1D - p^*_2D}{2\tau} \right) - \frac{p^*_1D + p^*_2D - c_1 - c_2}{2\tau} + \mu_D = 0, \] (24)

\[ \frac{\partial L_D}{\partial p^*_2D} = \left( \frac{1}{2} + \frac{p^*_1W + p^*_2W - p^*_1D - p^*_2D}{2\tau} \right) - \frac{p^*_1D + p^*_2D - c_1 - c_2}{2\tau} + \lambda \left( \frac{1}{2} + \frac{p^*_2W - p^*_2D}{2\tau} \right) = 0, \] (25)

\[ \frac{\partial L_D}{\partial \mu_D} = p^*_1D - c_1 \geq 0, \] (26)

\[ \mu_D \geq 0, \] (27)

\[ \mu_D(p^*_1D - c_1) = 0. \] (28)
Similarly, we obtain the Kuhn-Tucker conditions for the weak retailer as follows:

\[
\frac{\partial L}{\partial p_{1W}} = \left( \frac{1}{2} + \frac{p_{1D} + p_{2D} - p_{1W} - p_{2W}}{2\tau} \right) - \frac{p_{1W} + p_{2W} - w_{1W} - c_2}{2\tau} + \mu_W = 0, \\
\frac{\partial L}{\partial p_{2W}} = \left( \frac{1}{2} + \frac{p_{1D} + p_{2D} - p_{1W} - p_{2W}}{2\tau} \right) - \frac{p_{1W} + p_{2W} - w_{1W} - c_2}{2\tau} \\
+ \lambda \left( \frac{1}{2} + \frac{p_{2D} - p_{2W}}{2\tau} \right) - \frac{\lambda(p_{2W} - c_2)}{2\tau} = 0, \\
\frac{\partial L}{\partial \mu_W} = p_{1W} - w_{1W} \geq 0, \\
\mu_W \geq 0, \\
\mu_W(p_{1W} - w_{1W}) = 0.
\]

In the analysis below, we deal with the following three cases separately: (i) both (26) and (31) are binding; (ii) only (26) is binding; (iii) only (31) is binding. Note that the case where neither (26) nor (31) are binding corresponds to the analysis without a ban on below-cost pricing in Section 3.1.

**Case I: Both (26) and (31) are binding**

First, we explore the case where both retailers’ prices of good 1 are bound by the wholesale prices in equilibrium. From (24), (25), (29), (30), and binding conditions, we have

\[
p_{1D} = c_1; \quad p_{2D} = c_2 + \frac{t(w_{1W} - c_1 + 3(1 + \lambda)\tau)}{3(t + \lambda\tau)}; \quad \mu_D = \frac{\lambda(t - \tau + c_1 - w_{1W})}{2(t + \lambda\tau)}; \\
p_{1W} = w_{1W}; \quad p_{2W} = c_2 + \frac{t(c_1 - w_{1W} + 3(1 + \lambda)\tau)}{3(t + \lambda\tau)}; \quad \mu_W = \frac{\lambda(t - \tau + w_{1W} - c_1)}{2(t + \lambda\tau)}. \tag{34}\]

Substituting the above equations into (11), the monopolistic manufacturer’s profit becomes

\[
\Pi_I^I = (w_{1W} - c_1) \left[ \frac{1}{2} - \frac{(w_{1W} - c_1)(t + 3\lambda\tau)}{6\tau(t + \lambda\tau)} \right]. \tag{36}\]

The monopolistic manufacturer maximizes this profit under the constraints (27) and (32), which can be rewritten as \(c_1 + \tau - t \leq w_{1W} \leq c_1 + t - \tau\). Note that, in this case, there exists an equilibrium as long as \(t - \tau \geq 0\). Assuming an interior solution, we obtain the optimal wholesale price and the manufacturer’s profit as follows:

\[
\bar{w}_{1W}^I = c_1 + \frac{3\tau(t + \lambda\tau)}{2(t + 3\lambda\tau)}, \tag{37}\]
\[ \tilde{\Pi}_1^I = \frac{3\tau(t + \lambda \tau)}{8(t + 3\lambda \tau)}. \]  

(38)

This is valid as long as \( \tilde{w}_{1W} \leq c_1 + t - \tau \), which implies that

\[ 0 < \frac{\tau}{t} \leq \frac{6\lambda - 5 + \sqrt{36\lambda^2 + 12\lambda + 25}}{18\lambda}. \]  

(39)

Otherwise, we have \( \tilde{w}_{1W} = c_1 + t - \tau \), and the manufacturer’s profit becomes

\[ \tilde{\Pi}_1^I = (t - \tau) \left[ \frac{1}{2} - \frac{(t - \tau)(t + 3\lambda \tau)}{6\tau(t + \lambda \tau)} \right]. \]  

(40)

\textbf{Case II: Only (31) is binding}

Second, we focus on the case where only the weak retailer’s equilibrium price of good 1 is bound by the wholesale price. Note that, since (26) holds with strict inequality, we have \( \mu_D = 0 \) from (28).

Then, in the same way as in the previous case, from (24), (25), (29), (30), and binding conditions, we have

\[ p_{1D} = c_1 + \frac{\tau - t + w_{1W} - c_1}{2}; \quad p_{2D} = c_2 + \frac{\tau(c_1 - w_{1W} + 3(\tau + \lambda \tau))}{6(t + \lambda \tau)}; \quad \mu_D = 0; \]  

(41)

\[ p_{1W} = w_{1W}; \quad p_{2W} = c_2 + \frac{\tau(c_1 - w_{1W} + 3(1 + \lambda)\tau)}{3(t + \lambda \tau)}; \quad \mu_W = \frac{\lambda(3(t - \tau) + w_{1W} - c_1)}{4(t + \lambda \tau)}. \]  

(42)

Substituting these equations into (11), the profit of the upstream monopolist can be rewritten as follows:

\[ \Pi_{1}^{II} = (w_{1W} - c_1) \left[ \frac{1}{2} - \frac{(w_{1W} - c_1)(2t + 3\lambda \tau) + 3\lambda \tau(t - \tau)}{12\tau(t + \lambda \tau)} \right]. \]  

(43)

The monopolistic manufacturer maximizes this profit under the constraints (26) with strict inequality and (32), which can be rewritten as \( w_{1W} \geq c_1 + \max\{t - \tau, 3(\tau - t)\} \). Assuming an interior solution, the optimal wholesale price and the manufacturer’s profit are

\[ \tilde{w}_{1W}^{II} = c_1 + \frac{3\tau(t(2 - \lambda) + 3\lambda \tau)}{2(2t + 3\lambda \tau)}, \]  

(44)

\[ \tilde{\Pi}_1^{II} = \frac{3\tau(t(2 - \lambda) + 3\lambda \tau)^2}{16(t + \lambda \tau)(2t + 3\lambda \tau)}. \]  

(45)

This is valid as long as \( \tilde{w}_{1W}^{II} \geq c_1 + \max\{t - \tau, 3(\tau - t)\} \), which implies that

\[ \frac{9\lambda - 10 + \sqrt{81\lambda^2 + 60\lambda + 100}}{30\lambda} < \frac{\tau}{t} \leq \frac{5\lambda - 2 + \sqrt{25\lambda^2 + 28\lambda + 4}}{6\lambda}. \]  

(46)
For $\tau/t > (5\lambda - 2 + \sqrt{25\lambda^2 + 28\lambda + 4})/6\lambda$, since $\Pi_{II}^{II}$ is decreasing in $w_{1W} \geq c_1 + 3(\tau - t)$, the optimal wholesale price and the associated profit are

$$\tilde{w}_{1W}^{II} = c_1 + 3(\tau - t), \quad (47)$$

$$\tilde{\Pi}_{II}^{II} = \frac{3(t - \tau)}{6}. \quad (48)$$

For $(3\lambda - 4 + \sqrt{9\lambda^2 + 16})/12\lambda \leq \tau/t \leq (9\lambda - 10 + \sqrt{81\lambda^2 + 60\lambda + 100})/30\lambda$, since $\Pi_{I}^{II}$ is decreasing in $w_{1W} \geq c_1 + 3(\tau - t)$, we have $\tilde{w}_{1W}^{II} = c_1 + 3(\tau - t)$ and

$$\tilde{\Pi}_{I}^{II} = (t - \tau) \left[ \frac{1}{2} - \frac{(t - \tau)(t + 3\lambda\tau)}{6(t + \lambda\tau)} \right]. \quad (49)$$

Finally, for $0 < \tau/t < (3\lambda - 4 + \sqrt{9\lambda^2 + 16})/12\lambda$, the monopolist cannot earn positive profit for any $w_{1W} \geq c_1 + t - \tau$.

**Case III: Only (26) is binding**

Finally, we turn to the case where only the dominant retailer’s equilibrium price of good 1 is bound by the wholesale price. Note that, since (31) holds with strict inequality, we have $\mu_{W} = 0$ from (33).

Then, from (24), (25), (29), (30), and binding conditions, we have

$$p_{1D} = c_1; \quad p_{2D} = c_2 + \frac{\Gamma(w_{1W} - c_1 + 3(1 + \lambda)\tau)}{3(t + \lambda\tau)}; \quad \mu_{D} = \frac{\lambda(3(t - \tau) + c_1 - w_{1W})}{4(t + \lambda\tau)}; \quad (50)$$

$$p_{1W} = w_{1W} + \frac{t - \tau + c_1 - w_{1W}}{2}; \quad p_{2W} = c_2 + \frac{\Gamma(w_{1W} - c_1 + 3(t + \tau + 2\lambda\tau))}{6(t + \lambda\tau)}; \quad \mu_{W} = 0. \quad (51)$$

Substituting the above equations into (27) and (31) and combining them lead to

$$w_{1W} - c_1 < \tau - t \leq -\frac{w_{1W} - c_1}{3}. \quad (52)$$

However, in equilibrium, since the upstream monopolist must set its wholesale price such that $w_{1W} \geq c_1$, this inequality cannot hold. Therefore, this case can never happen in equilibrium.

Now, we find the optimal wholesale prices for given $\tau/t$ by comparing the optimal manufacturer’s profit derived above in each case.
(i) \(0 < \tau/t < (3\lambda - 4 + \sqrt{9\lambda^2 + 16})/12\lambda\)

In this case, the upstream monopolist can earn positive profit only when it chooses the whole-sale price at which both retailers’ prices of good 1 are bound. Therefore, the equilibrium wholesale price and monopolist’s profit are given by (37) and (38).

(ii) \((3\lambda - 4 + \sqrt{9\lambda^2 + 16})/12\lambda \leq \tau/t \leq (9\lambda - 10 + \sqrt{81\lambda^2 + 60\lambda + 100})/30\lambda\)

In this case, a comparison between (38) and (49) leads to

\[
\tilde{\Pi}_I^f - \tilde{\Pi}_I^{ff} = \frac{(2t^2 + (6\lambda - 5)t\tau - 9\lambda\tau^2)^2}{24\tau(t + \lambda\tau)(t + 3\lambda\tau)} > 0.
\]

Therefore, the equilibrium wholesale price and monopolist’s profit are given by (37) and (38).

(iii) \((9\lambda - 10 + \sqrt{81\lambda^2 + 60\lambda + 100})/30\lambda < \tau/t \leq (6\lambda - 5 + \sqrt{36\lambda^2 + 12\lambda + 25})/18\lambda\)

In this case, interior solutions (37) and (44) are both valid. Then, by comparing (38) and (45), we have

\[
\tilde{\Pi}_I^f - \tilde{\Pi}_I^{ff} \equiv 0 \text{ if and only if } \tau/t \leq T,
\]

where

\[
T = \frac{135\lambda^2 + 90\lambda + 211 + (18\lambda - 29)X^{1/3} + X^{2/3}}{63\lambda X^{1/3}},
\]

and \(X = -63 \sqrt{3(1 + \lambda)} \sqrt{-162\lambda^4 + 486\lambda^3 + 153\lambda^2 + 504\lambda - 25 + 729\lambda^3 + 4698\lambda^2 + 2889\lambda + 3016}\). In addition, it can be checked that \(T\) satisfies

\[
\frac{9\lambda - 10 + \sqrt{81\lambda^2 + 60\lambda + 100}}{30\lambda} < T < \frac{6\lambda - 5 + \sqrt{36\lambda^2 + 12\lambda + 25}}{18\lambda}
\]

for all \(\lambda > 0\). Therefore, the equilibrium wholesale price and monopolist’s profit are given by (37) and (38) if \((9\lambda - 10 + \sqrt{81\lambda^2 + 60\lambda + 100})/30\lambda < \tau/t \leq T\), and by (44) and (45) if \(T < \tau/t \leq (6\lambda - 5 + \sqrt{36\lambda^2 + 12\lambda + 25})/18\lambda\).

(iv) \((6\lambda - 5 + \sqrt{36\lambda^2 + 12\lambda + 25})/18\lambda < \tau/t \leq 1\)

By comparing (40) with (45), we have

\[
\tilde{\Pi}_I^f - \tilde{\Pi}_I^{ff} = -\frac{(4t^2 + (9\lambda - 10)t\tau - 15\lambda\tau^2)^2}{48\tau(t + \lambda\tau)(2t + 3\lambda\tau)} < 0.
\]

Therefore, the equilibrium wholesale price and monopolist’s profit are given by (44) and (45).
(v) $1 < \tau/t \leq 2$

In this case, the monopolist can obtain positive profit only when it chooses the wholesale price at which only weak retailers’ price of good 1 is bound. Therefore, as explained in Case II, the equilibrium wholesale price and monopolist’s profit are respectively given by (44) and (45) if $1 < \tau/t \leq T$, and by (47) and (48) if $T < \tau/t \leq 2$, where $T$ is given by

$$T = \frac{5\lambda - 2 + \sqrt{25\lambda^2 + 28\lambda + 4}}{6\lambda}. \quad (58)$$

Then, the equilibrium under a ban on below-cost pricing can be summarized as follows. If $0 < \tau/t \leq T$, both retailers’ prices of good 1 are bound (that is, $\tilde{m}_{1i} = 0$ for $i = D, W$), and the equilibrium wholesale and retail prices are calculated as follows:

$$\tilde{w}_{1W} = c_1 + \frac{3\tau(t + \lambda r)}{2(t + 3\lambda r)}; \quad (59)$$

$$\tilde{p}_{1D} = c_1; \quad \tilde{p}_{2D} = c_2 - \frac{\tau(3 + 2\lambda) t + \lambda(7 + 6\lambda) r}{2(t + \lambda r)(t + 3\lambda r)}; \quad (60)$$

$$\tilde{p}_{1W} = c_1 + \frac{3\tau(t + \lambda r)}{2(t + 3\lambda r)}; \quad \tilde{p}_{2W} = c_2 + \frac{\tau(1 + 2\lambda) t + \lambda(5 + 6\lambda) r}{2(t + \lambda r)(t + 3\lambda r)}. \quad (61)$$

If $T < \tau/t \leq \bar{T}$, only the weak retailer’s price of good 1 is bound (that is, $\tilde{m}_{1D} > 0$ and $\tilde{m}_{1W} = 0$), and the equilibrium wholesale and retail prices are obtained as follows:

$$\tilde{w}_{1W} = c_1 + \frac{3\tau(t(2 - \lambda) + 3\lambda r)}{2(2t + 3\lambda r)}; \quad (62)$$

$$\tilde{p}_{1D} = c_1 + \frac{-4t^2 + (10 - 9\lambda)tr + 15\lambda r^2}{4(2t + 3\lambda r)}; \quad \tilde{p}_{2D} = c_2 + \frac{4t^2 + (2 + 15\lambda)tr + 3\lambda(1 + 4\lambda) r^2}{4(t + \lambda r)(2t + 3\lambda r)}; \quad (63)$$

$$\tilde{p}_{1W} = c_1 + \frac{3\tau(t(2 - \lambda + 3\lambda r)}{2(2t + 3\lambda r)}; \quad \tilde{p}_{2W} = c_2 + \frac{\tau(2 + 5\lambda) t + \lambda(1 + 2\lambda) r}{2(t + \lambda r)(2t + 3\lambda r)}. \quad (64)$$

Finally, if $\bar{T} < \tau/t \leq 2$, only the weak retailer’s price of good 1 is bound (that is, $\tilde{m}_{1D} > 0$ and $\tilde{m}_{1W} = 0$), and the equilibrium wholesale and retail prices are given as follows:

$$\tilde{w}_{1W} = c_1 + 3(\tau - t); \quad (65)$$

$$\tilde{p}_{1D} = c_1 + 2(\tau - t); \quad \tilde{p}_{2D} = c_2 + t; \quad (66)$$

$$\tilde{p}_{1W} = c_1 + 3(\tau - t); \quad \tilde{p}_{2W} = c_2 + t. \quad (67)$$

It is easy to see that the prices of good 2 set by both retailers are always above their wholesale prices, that is, $\tilde{m}_{2i} > 0$ for $i = D, W$. This completes the proof of Proposition 2. \[\blacksquare\]
Proof of Proposition 3

In the following proofs, we define \( \Delta w_{1W} \equiv \tilde{w}_{1W} - w_{1W}^* \) and \( \Delta p_{ki} \equiv \tilde{p}_{ki} - p_{ki}^* \) for \( k = 1, 2, i = D, W \).

(a) From Propositions 1 and 2, we have

\[
\Delta w_{1W} = \begin{cases} 
-\frac{3\lambda \tau^2}{t + 3\lambda \tau} & \text{if } 0 < \frac{\tau}{t} \leq T, \\
-\frac{6\lambda \tau^2}{2(2t + 3\lambda \tau)} & \text{if } T < \frac{\tau}{t} \leq \overline{T}, \\
-\frac{3(2t - \tau)}{2} & \text{if } \overline{T} < \frac{\tau}{t} < 2.
\end{cases}
\]  

(68)

It is easy to see that we have \( \Delta w_{1W} < 0 \) for \( 0 < \tau/t < 2 \).

(b) For the price of good 1 set by the dominant retailer, we have

\[
\Delta p_{1D} = \begin{cases} 
\frac{2t - 3\tau}{2} & \text{if } 0 < \frac{\tau}{t} \leq T, \\
\frac{4t^2 + (6\lambda - 2)t\tau - 3\lambda \tau^2}{4(2t + 3\lambda \tau)} & \text{if } T < \frac{\tau}{t} \leq \overline{T}, \\
\frac{2t - \tau}{2} & \text{if } \overline{T} < \frac{\tau}{t} < 2.
\end{cases}
\]  

(69)

As shown in Figure 3(a), it can be verified that \( \Delta p_{1D} > 0 \) for \( 0 < \tau/t < T_p \) and \( \Delta p_{1D} < 0 \) for \( T_p < \tau/t < 2 \), where

\[
T_p = \frac{3\lambda - 2 + \sqrt{9\lambda^2 + 36\lambda + 4}}{6\lambda}.
\]  

(70)

Similarly, for the price of good 1 set by the weak retailer, we have

\[
\Delta p_{1W} = \begin{cases} 
\frac{2t^2 + (6\lambda - 1)t\tau - 9\lambda \tau^2}{2(t + 3\lambda \tau)} & \text{if } 0 < \frac{\tau}{t} \leq T, \\
\frac{4t^2 + (3\lambda - 2)t\tau - 3\lambda \tau^2}{2(2t + 3\lambda \tau)} & \text{if } T < \frac{\tau}{t} \leq \overline{T}, \\
-(2t - \tau) & \text{if } \overline{T} < \frac{\tau}{t} < 2.
\end{cases}
\]  

(71)

In the same way as for the dominant retailer’s price of good 1, we can show that we have \( \Delta p_{1W} > 0 \) for \( 0 < \tau/t < T_p \) and \( \Delta p_{1W} < 0 \) for \( T_p < \tau/t < 2 \).
(c) For the price of good 2 set by the dominant retailer, we have

\[
\Delta p_{2D} = \begin{cases} 
-\frac{t(2t^2 + 3(2\lambda - 1)t\tau - 7\lambda t^2)}{2(t + \lambda t)(t + 3\lambda t)} & \text{if } 0 < \frac{\tau}{t} \leq T, \\
-\frac{t(4t^2 + (5\lambda - 2)t\tau - 3\lambda t^2)}{4(t + \lambda t)(2t + 3\lambda t)} & \text{if } \frac{T}{t} < \frac{\tau}{t} \leq \overline{T}, \\
0 & \text{if } \overline{T} < \frac{\tau}{t} < 2.
\end{cases}
\]

As shown in Figure 3(a), we can show that \( \Delta p_{2D} < 0 \) for \( 0 < \tau/t \leq T \).

Similarly, for the weak retailer’s price of good 2, we have

\[
\Delta p_{2W} = \begin{cases} 
-\frac{t(2t^2 + (6\lambda - 1)t\tau - 5\lambda t^2)}{2(t + \lambda t)(t + 3\lambda t)} & \text{if } 0 < \frac{\tau}{t} \leq T, \\
-\frac{t(4t^2 + (5\lambda - 2)t\tau - 3\lambda t^2)}{2(t + \lambda t)(2t + 3\lambda t)} & \text{if } \frac{T}{t} < \frac{\tau}{t} \leq \overline{T}, \\
0 & \text{if } \overline{T} < \frac{\tau}{t} < 2.
\end{cases}
\]

In the same way as above, it can be verified that \( \Delta p_{2W} < 0 \) for \( 0 < \tau/t \leq \overline{T} \). ■

Proof of Proposition 4

In the following proofs, we define \( \Delta \Pi_s \equiv \tilde{\Pi}_s - \Pi^*_s \) for \( s = 1, D, W \).

(a) When below-cost pricing is feasible, the monopolistic manufacturer’s equilibrium profit is

\[
\Pi^*_1 = \frac{3\tau}{8}.
\]

On the other hand, when a ban on below-cost pricing is introduced, the monopolistic manufacturer’s equilibrium profit is given by

\[
\tilde{\Pi}_1 = \begin{cases} 
\frac{3\tau(t + \lambda t)}{8(t + 3\lambda t)} & \text{if } 0 < \frac{\tau}{t} \leq T, \\
\frac{3\tau(t(2 - \lambda) + 3\lambda t)^2}{16(t + \lambda t)(2t + 3\lambda t)} & \text{if } \frac{T}{t} < \frac{\tau}{t} \leq \overline{T}, \\
\frac{3\tau(\tau - t)}{2\tau} & \text{if } \overline{T} < \frac{\tau}{t} < 2.
\end{cases}
\]
Then, the effect of banning below-cost pricing on the monopolistic manufacturer’s profit can be calculated as follows:

\[
\Delta \Pi_1 = \begin{cases} 
- \frac{3\lambda \tau^2}{4(t + 3\lambda \tau)} & \text{if } 0 < \frac{\tau}{t} \leq T, \\
- \frac{3\lambda \tau((4 - \lambda)^2 - 2(1 - 3\lambda \mu \tau - 3\lambda \tau^2))}{16(t + \lambda \tau)(2t + 3\lambda \tau)} & \text{if } T < \frac{\tau}{t} \leq \bar{T}, \\
- \frac{3(2t - \tau)^2}{8\tau} & \text{if } \bar{T} < \frac{\tau}{t} < 2.
\end{cases}
\] (76)

For \( 0 < \tau/t \leq T \) and \( T < \tau/t < \bar{T} \), it is easy to see that we have \( \Delta \Pi_1 < 0 \). For \( \bar{T} < \tau/t < 2 \), although the expression is a bit complicated, we can also show that \( \Delta \Pi_1 < 0 \).

(b) Without a ban, the dominant retailer’s equilibrium profit is given by

\[
\Pi_D^* = \frac{4t\lambda + 9\tau}{8}.
\] (77)

When below-cost pricing is banned, the profit becomes

\[
\tilde{\Pi}_D = \begin{cases} 
\frac{t\lambda(3t + 2t\lambda + 7\lambda \tau + 6\lambda^2 \tau)^2}{8(t + \lambda \tau)(t + 3\lambda \tau)^2} & \text{if } 0 < \frac{\tau}{t} \leq T, \\
\frac{t\lambda(4t^2 + (2 + 15\lambda)\lambda \tau + 3\lambda(1 + 4\lambda)\tau^2)^2 + \tau(t(6 + \lambda) + 5\lambda \tau)^2}{32(t + \lambda \tau)^2(2t + 3\lambda \tau)^2} & \text{if } T < \frac{\tau}{t} \leq \bar{T}, \\
\frac{t^2 - 4t \tau + t\lambda \tau + 4\tau^2}{2\tau} & \text{if } \bar{T} < \frac{\tau}{t} < 2.
\end{cases}
\] (78)

Although we do not report the full expression of \( \Delta \Pi_D \) here, we can show that we have \( \Delta \Pi_D < 0 \) for \( 0 < \tau/t < 2 \).

(c) Without a ban on below-cost pricing, the weak retailer’s equilibrium profit is

\[
\Pi_W^* = \frac{4t\lambda + \tau}{8}.
\] (79)

When a ban on below-cost pricing is put into force, the equilibrium profit becomes

\[
\tilde{\Pi}_W = \begin{cases} 
\frac{t\lambda(t + 2t\lambda + 5\lambda \tau + 6\lambda^2 \tau)^2}{8(t + \lambda \tau)(t + 3\lambda \tau)^2} & \text{if } 0 < \frac{\tau}{t} \leq T, \\
\frac{t\lambda(2t + 5t\lambda + 3\lambda \tau + 6\lambda^2 \tau)^2}{8(t + \lambda \tau)(2t + 3\lambda \tau)^2} & \text{if } T < \frac{\tau}{t} \leq \bar{T}, \\
\frac{t(t + \lambda \tau)}{2\tau} & \text{if } \bar{T} < \frac{\tau}{t} < 2.
\end{cases}
\] (80)
Because it is a bit messy, we do not provide the full expression of $\Delta \Pi_W$ here. From (79) and (80), as shown in Figure 3(c), we can find a threshold $T_{\Pi}$, which satisfies

$$
\frac{4t\lambda + \tau}{8} = \frac{t\lambda(2t + 5t\lambda + 3\lambda\tau + 6\lambda^2\tau^2)}{8(t + \lambda\tau)(2t + 3\lambda\tau)^2},
$$

and show that $\Delta \Pi_W \preceq 0$ if and only if $\tau/t \preceq T_{\Pi}$. ■

**Proof of Proposition 5**

In the following proofs, we define $\Delta CS_h \equiv \tilde{CS}_h - CS_h^*$ for $h = 2, B, T$.

(a) From Proposition 3, we have $\Delta p_{2i} < 0$ if $0 < \tau/t \leq \bar{T}$ and $\Delta p_{2i} = 0$ if $\bar{T} < \tau/t < 2$ for $i = D, W$. Therefore, it is straightforward to see that we have $\Delta CS_2 \geq 0$ for $0 < \tau/t < 2$ and, in particular, $\Delta CS_2 > 0$ if $0 < \tau/t \leq T$.

(b) For $\bar{T} < \tau/t < 2$, Proposition 3 shows that we have $\Delta p_{1i} < 0$ and $\Delta p_{2i} = 0$ for $i = D, W$. Therefore, it is easy to see that we have $\Delta CS_B > 0$ for this range. For $0 < \tau/t \leq \bar{T}$, when the ban on below-cost pricing is not imposed, the type $B$ consumers’ surplus is

$$
CS_B^* = -(c_1 + c_2) - \frac{31\tau}{16}.
$$

(82)

On the other hand, when the ban is imposed, we have

$$
\tilde{CS}_B = -(c_1 + c_2) - \frac{\tau((16\lambda + 31)t^2 + 12(4\lambda + 7)\lambda\tau + 21\lambda^2\tau^2)}{16(t + \lambda\tau)(t + 3\lambda\tau)},
$$

(83)

if $0 < \tau/t \leq T$, and

$$
\tilde{CS}_B = -(c_1 + c_2) - \frac{\tau((-\lambda^2 + 20\lambda + 124)t^2 + 2(11\lambda + 114)\lambda\tau + 103\lambda^2\tau^2)}{64(t + \lambda\tau)^2},
$$

(84)

if $T < \tau/t \leq \bar{T}$. Using (82)–(84), we have

$$
\Delta CS_B = \begin{cases} 
-\frac{\lambda\tau(2t^2 + (6\lambda - 5)\lambda\tau - 9\lambda^2\tau^2)}{2(t + \lambda\tau)(t + 3\lambda\tau)} & \text{if } 0 < \frac{\tau}{t} \leq T, \\
\frac{\lambda\tau(\tau - t)((20 - \lambda)\lambda\tau + 21\lambda\tau)}{64(t + \lambda\tau)^2} & \text{if } T < \frac{\tau}{t} \leq \bar{T}.
\end{cases}
$$

(85)

As shown in Figure 3(b), we can find a threshold value of $\tau/t$ such that $\Delta CS_B \preceq 0$ if and only if $\tau/t \preceq 1$.
From the results in (a) and (b), it is easy to see that we have $\Delta CS_T > 0$ for $T < \tau/t < 2$. For $0 < \tau/t \leq T$, when below-cost pricing is feasible, type 2 consumers’ surplus is

$$CS_2^* = -\lambda \left[ c_2 + \frac{5t\lambda}{4} \right]$$

(86)

On the other hand, when below-cost pricing is banned, it becomes

$$\overline{CS}_2 = -\lambda \left[ c_2 + t(2t^2 + (4 + 11.\lambda)r^2 + (39.\lambda^2 + 24.\lambda - 1)\lambda\tau^2 + \lambda(45.\lambda^2 + 36.\lambda - 1)\lambda\tau^3)(4(t + \lambda\tau)(t + 3.\lambda\tau)^2)^{-1}\right]$$

(87)

if $0 < \tau/t \leq T$, and

$$\overline{CS}_2 = -\lambda \left[ c_2 + t(112t^4 + 56(15.\lambda + 2)\lambda^2 + (1047.\lambda^2 + 428.\lambda - 4)t^2\lambda^3 + 6.\lambda(340.\lambda^2 + 89.\lambda - 2)\lambda\tau^3 + 9.\lambda^2(80.\lambda^2 + 24.\lambda - 1)t^4)\right]$$

$$\times(64(t + \lambda\tau)^2(2t + 3.\lambda\tau)^2)^{-1}\right]$$

if $T < \tau/t \leq \overline{T}$. Using (85)–(88), we can obtain $\Delta CS_T$ for $0 < \tau/t \leq \overline{T}$. Although we do not report the full expression of $\Delta CS_T$ here for the sake of brevity, we can show that $\Delta CS_T > 0$ for $0 < \tau/t < \overline{T}$.

**Appendix B: Extension**

**B.1 Equilibrium**

When below-cost pricing is feasible

Since the dominant retailer’s profit function is given by (5), which is same as in the basic model, the first-order conditions for the dominant retailer are given by (7) and (8). On the other hand, from (14) and (15), the first-order conditions for the weak retailers are given as follows:

$$\frac{\partial \Pi_{W1}}{\partial p_{1W}} = \left(\frac{1}{2} + \frac{p_{1D} + p_{2D} - p_{1W} - p_{2W}}{2\tau}\right) - \frac{p_{1W} - w_{1W}}{2\tau} = 0, \quad (89)$$

$$\frac{\partial \Pi_{W2}}{\partial p_{2W}} = \left(\frac{1}{2} + \frac{p_{1D} + p_{2D} - p_{1W} - p_{2W}}{2\tau}\right) - \frac{p_{2W} - w_{2W}}{2\tau}$$

$$+ \lambda \left(\frac{1}{2} + \frac{p_{2D} - p_{2W}}{2t}\right) - \frac{\lambda(p_{2W} - w_{2W})}{2t} = 0. \quad (90)$$

31
By solving these four first-order conditions and substituting \( w_{1D} = c_1 \) and \( w_{2D} = w_{2W} = c_2 \), we have the following retail prices for given \( w_{1W} \):

\[
\begin{align*}
p_{1D} &= c_1 + \frac{3(w_{1W} - c_1)(t + \lambda \tau) - 4t^2 + 9\lambda t^2 + t\tau(7 - 6\lambda)}{8t + 9\lambda t}, \\
p_{2D} &= c_2 + \frac{t(4t + 3\lambda + 9\lambda \tau - (w_{1W} - c_1))}{8t + 9\lambda t}, \\
p_{1W} &= c_1 + \frac{6(w_{1W} - c_1)(t + \lambda \tau) + 3\tau(t(2 - \lambda) + 3\lambda \tau)}{8t + 9\lambda t}, \\
p_{2W} &= c_2 + \frac{t(3\tau(2 + 3\lambda) - 2(w_{1W} - c_1))}{8t + 9\lambda t}.
\end{align*}
\]

(91)

The monopolistic manufacturer chooses \( w_{1W} \) to maximize its own profit, taking into consideration the above retailers’ responses. Substituting the above retail prices into the manufacturer’s profits (11), we have

\[
\Pi_1 = (w_{1W} - c_1) \left( \frac{1}{2} - \frac{(2t + 3\lambda \tau)(w_{1W} - c_1) + t\tau(2 + 3\lambda)}{16t\tau + 18\lambda t^2} \right).
\]

(92)

Then, the monopolistic manufacturer optimally sets the wholesale price at

\[
w_{1W}^* = c_1 + \frac{3\tau(2t - t\lambda + 3\lambda \tau)}{4t + 6\lambda t},
\]

(93)

and its equilibrium profit becomes

\[
\Pi_1^* = \frac{9\tau(t(2 - \lambda) + 3\lambda \tau)^2}{8(8t + 9\lambda t)(2t + 3\lambda \tau)}.
\]

(94)

In addition, by substituting (93) into (91) and (92), we obtain the equilibrium retail prices as follows:

\[
\begin{align*}
p_{1D}^* &= c_1 + \frac{-16t^3 + t^2(46 - 57\lambda)\tau + 3(41 - 15\lambda)\lambda t^2 + 81\lambda^2 t^3}{2(2t + 3\lambda \tau)(8t + 9\lambda t)}, \\
p_{2D}^* &= c_2 + \frac{t(16t^2 + 3t(2 + 21\lambda)\lambda + 9\lambda(1 + 6\lambda)\tau^2)}{2(2t + 3\lambda \tau)(8t + 9\lambda t)}, \\
p_{1W}^* &= c_1 + \frac{3\tau(5t^2(2 - \lambda) + 3t(9 - 2\lambda)\lambda t + 18\lambda^2 \tau^2)}{(2t + 3\lambda \tau)(8t + 9\lambda t)}, \\
p_{2W}^* &= c_2 + \frac{3\tau\tau(t(2 + 3\lambda) + 3(1 + 3\lambda)\lambda t)}{(2t + 3\lambda \tau)(8t + 9\lambda t)}.
\end{align*}
\]

(95)

For good 2, it is easy to see that both retailers’ price-cost margin is always positive, \( m_{2i}^* = p_{2i}^* - c_2 > 0 \). On the other hand, for good 1, the dominant retailer’s price-cost margin becomes

\[
m_{1D}^* = p_{1D}^* - c_1 = \frac{-16t^3 + t^2(46 - 57\lambda)\tau + 3(41 - 15\lambda)\lambda t^2 + 81\lambda^2 t^3}{2(2t + 3\lambda \tau)(8t + 9\lambda t)}.
\]

(96)

Let \( T^c \) denote the threshold value of \( \tau/t \) at which we have \( m_{1D}^* = 0 \). As drawn in Figure 5, we have \( m_{1D}^* < 0 \) when \( \tau/t < T^c \).
When below-cost pricing is banned

In the extended model, since each weak retailer carries one good, it cannot set the price of its good below the wholesale price. Therefore, only the dominant retailer can engage in below-cost pricing in the equilibrium. In the following proof, we first derive an equilibrium under a ban, temporarily ignoring the constraints for good 2 \( p_2 \geq w_2 \), and later we check that this constraint is certainly not binding in that equilibrium. Then, the Lagrange function and Kuhn-Tucker conditions for the dominant retailer are given by (22) and (24)–(28), respectively. On the other hand, the first-order conditions for the weak retailers are given by (89) and (90).

When the dominant retailer’s prices of good 1 is bound by the wholesale price in equilibrium, from (24), (25), (29), (30), and binding conditions, we have

\[
\begin{align*}
p_{1D} &= c_1; \quad p_{2D} = c_2 + \frac{f((w_{1W} - c_1)(t + \lambda \tau) + (t(5 + 4.6\lambda + (7 + 6.6\lambda\tau))\tau)}{2(t + \lambda \tau)(2t + 3.6\lambda \tau)}; \\
p_{1W} &= \frac{3(w_{1W} + \tau)(t + \lambda \tau) + c_1(t + 3.6\lambda \tau)}{2(2t + 3.6\lambda \tau)}; \\
p_{2W} &= c_2 + \frac{f((t(3 + 4.6\lambda + 5 + 6.6\lambda\tau)\tau - (w_{1W} - c_1)(t + \lambda \tau))}{2(t + \lambda \tau)(2t + 3.6\lambda \tau)}; \\
\mu_D &= \frac{\lambda(4t^2 - (7 - 6.6\lambda)\tau - 9.6\lambda^2 - 3(w_{1W} - c_1)(t + \lambda \tau))}{4(t + \lambda \tau)(2t + 3.6\lambda \tau)}.
\end{align*}
\]

Substituting the above equations into (11), the monopolistic manufacturer’s profit becomes

\[
\Pi_1 = (w_{1W} - c_1) \left[ \frac{1}{2} - \frac{(w_{1W} - c_1 + \tau)(t + 3.6\lambda \tau)}{4\tau(2t + 3.6\lambda \tau)} \right].
\]

The monopolistic manufacturer maximizes this profit under the constraint (27), which can be rewritten as

\[
w_{1W} \leq c_1 + \frac{4t^2 - (7 - 6.6\lambda)\tau - 9.6\lambda^2}{3(t + \lambda \tau)}.
\]

Assuming an interior solution, we obtain the optimal wholesale price and the manufacturer’s profit as follows:

\[
\begin{align*}
\tilde{w}_{1W} &= c_1 + \frac{3\tau(t + \lambda \tau)}{2(t + 3.6\lambda \tau)}, \\
\tilde{\Pi}_1 &= \frac{9\tau(t + \lambda \tau)^2}{16(t + 3.6\lambda \tau)(2t + 3.6\lambda \tau)}.
\end{align*}
\]
This is valid as long as $\tilde{w}_{1W}$ satisfies (102). Let $\bar{T}^e$ denote the value of $\tau/t$ at which $\tilde{w}_{1W}$ satisfies (102) with equality. Then, (103) is valid when $\tau/t \leq \bar{T}^e$.

By comparing $T^e$ with $\bar{T}^e$, we can find that $T^e < \bar{T}^e$ for $\lambda > 0$. This implies that for $T^e < \tau/t < \bar{T}^e$, the monopolistic manufacturer chooses $w^*_1$ or $\tilde{w}_{1W}$ depending on relative size of $\Pi^*_1$ and $\tilde{\Pi}_1$.

By computing $\tilde{\Pi}_1 - \Pi^*_1$ directly, we can find a threshold $\hat{T}^e \in (T^e, \bar{T}^e)$ such that we have $\tilde{\Pi}_1 > (<) \Pi^*_1$ when $\tau/t < (>) \hat{T}^e$.

Then, the equilibrium under a ban on below-cost pricing can be summarized as follows. If $0 < \tau/t \leq \hat{T}^e$, the dominant retailer’s price of good 1 is bound (that is, $\tilde{m}_{1D} = 0$), and the equilibrium wholesale and retail prices are calculated as follows:

\[
\tilde{w}_{1W} = c_1 + \frac{3\tau(t + \lambda \tau)}{2(t + 3\lambda \tau)}; \quad (105)
\]

\[
\tilde{p}_{1D} = c_1; \quad \tilde{p}_{2D} = c_2 + \frac{\tau(\lambda^2(13 + 8\lambda) + 2\lambda(25 + 18\lambda)\tau + 9\lambda^2(5 + 4\lambda)^2)}{4(t + 3\lambda \tau)(t + \lambda \tau)(2t + 3\lambda \tau)}; \quad (106)
\]

\[
\tilde{p}_{1W} = c_1 + \frac{3\tau(t + \lambda \tau)(5t + 9\lambda \tau)}{4(t + 3\lambda \tau)(2t + 3\lambda \tau)}; \quad (107)
\]

\[
\tilde{p}_{2W} = c_2 + \frac{\tau(\lambda^2(3 + 8\lambda) + 2\lambda(11 + 18\lambda)\tau + 9\lambda^2(3 + 4\lambda)^2)}{4(t + 3\lambda \tau)(t + \lambda \tau)(2t + 3\lambda \tau)}. \quad (108)
\]

It is easy to see that the price of good 2 set by the dominant retailer is always above their wholesale price, that is, $\tilde{m}_{2D} > 0$. On the other hand, if $\tau/t > \hat{T}^e$, the dominant retailer’s price of good 1 is not bound and the equilibrium under the ban corresponds to that without the ban.

**B.2 Effects of banning below-cost pricing**

Wholesale and retail prices

From the above analysis, we can derive the changes of the wholesale and retail prices through the ban on below-cost pricing as follows:

\[
\Delta w_{1W} = \frac{3\lambda \tau(t^2 - (4 - 3\lambda)\tau - 6\lambda \tau^2)}{2(t + 3\lambda \tau)(2t + 3\lambda \tau)}; \quad (109)
\]

\[
\Delta p_{1D} = \frac{16\lambda^3 + t^2(57\lambda - 46)\tau + 3\lambda(15\lambda - 41)\tau^2 - 81\lambda^2 \tau^3}{2(t + 3\lambda \tau)(8t + 9\lambda \tau)}; \quad (110)
\]

\[
\Delta p_{1W} = \frac{3\lambda \tau(20\lambda^3 + t^2(84\lambda - 71)\tau + 18\lambda(4\lambda - 11)\lambda \tau^2 - 135\lambda^2 \tau^3)}{4(t + 3\lambda \tau)(2t + 3\lambda \tau)(8t + 9\lambda \tau)}; \quad (111)
\]
Although we do not provide the full expression of $\Delta \Pi$. On the other hand, under the ban on below-cost pricing, we have

$$
\Delta \Pi = -\frac{\tau(32t^4 + 2t^3(95t - 46)t + t^2\lambda(348\lambda - 451)t^2 + 18t\lambda^2(11\lambda - 39)t^3 - 351\lambda^3t^4)}{4(t + \lambda t)(t + 3\lambda t)(2t + 3\lambda t)(8t + 9\lambda t)}.
$$

$$
\Delta \Pi = \frac{\tau(20t^3 + t^2(84\lambda - 71)t + 18t\lambda(4\lambda - 11)t^2 - 135\lambda^2t^3)}{4(t + \lambda t)(t + 3\lambda t)(2t + 3\lambda t)(8t + 9\lambda t)}.
$$

Then, we can obtain Figure 6(a) and (b).

**Profits of firms**

For the monopolistic manufacturer, from (94) and (104), we have

$$
\Delta \Pi_1 = \frac{9\tau(t + \lambda t)^2(8t + 9\lambda t) - 2(t(2 - \lambda) + 3\lambda(t + 3\lambda t))^1}{16(t + 3\lambda t)(2t + 3\lambda t)(8t + 9\lambda t)}.
$$

For the relevant parameter range where the ban is binding, it can be found that we have $\Delta \Pi_1 > 0$.

When below-cost pricing is not banned, the equilibrium profits of retailers are obtained as follows:

$$
\Pi_D = (256t^2\lambda + 4t^2(676 + 204\lambda + 513\lambda^2)t + 3t^3\lambda(4588 + 1188\lambda + 1935\lambda^2)t^2
+ 9t^2\lambda^2(2884 + 570\lambda + 765\lambda^2)t^3 + 81t\lambda^3(265 + 30\lambda + 36\lambda^2)t^4
+ 6561\lambda^4t^5)(8(2t + 3\lambda t)(8t + 9\lambda t))^1,
$$

$$
\Pi_W^W = \frac{9\tau(t(2 - \lambda) + 3\lambda t)^2}{8(8t + 9\lambda t)^2},
$$

$$
\Pi_W^W = \frac{9\tau(t + \lambda t)(t(2 + 7\lambda) + 3(1 + \lambda)\lambda t)^2}{2(2t + 3\lambda t)^2(8t + 9\lambda t)^2}.
$$

On the other hand, under the ban on below-cost pricing, we have

$$
\Pi_D = \frac{\tau(t^2(13 + 8\lambda) + 2t\lambda(25 + 18\lambda)t + 9\lambda^2(5 + 4\lambda)t^2)^2}{32(t + \lambda t)(t + 3\lambda t)(2t + 3\lambda t)^2},
$$

$$
\Pi_W^W = \frac{9\tau(t + \lambda t)^2}{32(2t + 3\lambda t)^2},
$$

$$
\Pi_W^W = \frac{\tau(t^2(3 + 8\lambda) + 2t\lambda(11 + 18\lambda)t + 9\lambda^2(3 + 4\lambda)t^2)^2}{32(t + \lambda t)(t + 3\lambda t)(2t + 3\lambda t)^2}.
$$

Although we do not provide the full expression of $\Delta \Pi_D$, $\Delta \Pi_W^W$, and $\Delta \Pi_W^W$, we can obtain Figure 6(c) and (d).
Consumer surplus

When the ban on below-cost pricing is not imposed, type 2 and type B consumers’ surplus are respectively calculated as follows:

\[
CS_2^* = -\lambda \left[ c_2 + t(1792\lambda^4 + 672\lambda^3(2 + 21\lambda)\tau + 9\lambda^2(4039\lambda^2 + 588\lambda - 4)\tau^2 \\
+ 54\lambda(714\lambda^2 + 127\lambda - 2)\tau^3 + 81\lambda^2(180\lambda^2 + 36\lambda - 1)\tau^4 \right] \\
\times (16(2t + 3.\tau)^2(8t + 9.\lambda^2)^2)^{-1},
\]

(117)

\[
CS_B^* = - (c_1 + c_2) - \frac{\tau t^2(2140 + 228\lambda - 9.\lambda^2) + 54t\lambda(86 + 5.\lambda)\tau + 2511\lambda^2\tau^2}{16(8t + 9.\lambda^2)^2}.
\]

On the other hand, when the ban is imposed, we have

\[
\overline{CS}_2 = -\lambda \left[ c_2 + t(16\lambda^5 + 32t^4(2 + 7.\lambda)\tau + t^3(1188\lambda^2 + 576\lambda - 25)\tau^2 \\
+ t^2(2988\lambda^2 + 1872\lambda - 115)\tau^3 + 9t\lambda(396\lambda^2 + 288\lambda - 19)\tau^4 \\
+ 81\lambda^3(20\lambda^2 + 16\lambda - 1)\tau^5)(16(t + \lambda\tau)(t + 3.\lambda\tau)^2(2t + 3.\lambda\tau)^2)^{-1} \right],
\]

(118)

\[
\overline{CS}_B = - (c_1 + c_2) - \frac{\tau t^4(535 + 256.\lambda) + 6t^3\lambda(511 + 256.\lambda)\tau}{16t^3(33 + 16\lambda)\tau^2 + 54t^2\lambda^3(81 + 32.\lambda)\tau^3 + 837t^4\lambda^4\tau^4} \\
\times (64(t + \lambda\tau)(t + 3.\lambda\tau)(2t + 3.\lambda\tau)^2)^{-1}.
\]

Although we do not report the full expression of \(\Delta CS_2 \) and \(\Delta CS_B \), we can obtain Figure 6(e) and 6(f).

References


Monopolistic manufacturer

Competitive fringe

Dominant retailer

Weak retailer

Consumers

Figure 1: Industrial structure of the model
(a) When the ban is not imposed

(b) When the ban is imposed

Figure 2: Equilibrium price-cost margin of good 1
Figure 3: The effect of banning below-cost pricing
Figure 4: Industrial structure of the extended model
Figure 5: Equilibrium price-cost margin of good 1 sold by the dominant retailer
Figure 6: The effect of banning below-cost pricing in the extended model