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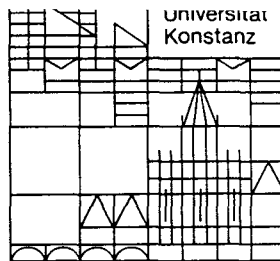
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**Fakultät für
Wirtschaftswissenschaften
und Statistik**

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**Basic Conditions
for Costless Signalling
in Financial Markets¹⁾**

Diskussionsbeiträge

Basic Conditions for
Costless Signalling in Financial Markets ¹⁾

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BASIC CONDITIONS FOR COSTLESS

SIGNALLING IN FINANCIAL MARKETS

ABSTRACT

A costless, fully revealing signalling equilibrium is derived from two easily understandable conditions. The outsider-protection condition states that the outsiders relate the price which they offer to pay for a security inversely to the supply of this security which they interpret as a quality signal. Thereby they attempt to protect themselves against adverse selection. The no-arbitrage condition requires that the exchange rate for two securities must be the same in both primary and secondary markets. These conditions have strong implications for the valuation of securities and optimal insider policies. Therefore a costless signalling equilibrium is obtained.

There is little reason to believe that information is distributed evenly across economic agents. Usually some agents are better informed than others. Managers of a firm, e.g., receive information on the firm's profitability earlier than outside investors. This creates a danger of their being exploited by the insiders through changes in the firm's financing policy or through trade of securities in the securities market. Whenever the firm's securities are mispriced, the insider can sell overpriced and buy underpriced securities in the secondary securities market. Similarly, a change in the firm's financing policy means a substitution of potentially overpriced against potentially underpriced securities, i.e. a trade in the primary securities market. Thereby wealth may be transferred from one group of investors to another.

The information asymmetry is removed if the outsiders observe the insiders' decisions and if the insiders' information can be inferred from these decisions. If, for example, insiders can trade securities anonymously in the secondary market, then outsiders cannot observe their decisions, hence the information asymmetry is not removed and the insiders can earn excess returns. But if the insiders manage a firm, then the outsiders can possibly observe the firm's investment and financing decisions and thereby infer the inside information. As a consequence, the information asymmetry and insider profits would disappear.

Two concepts of signalling inside information have been suggested, the first being the costly signalling equilibrium as dis-

cussed by Spence <11>, Leland and Pyle <7>, Ross <10>, Riley <9> and Talmor <12>, the second being the costless signalling equilibrium as proposed by Bhattacharya <2> and further developed by Brennan and Kraus <3>. Signalling is costly if the production of the signal consumes resources or if the signal is associated with a loss in welfare, generated by deviations from allocation and / or distribution of claims in perfect markets. Both effects are required to be absent in costless signalling models. Costless signalling equilibria produce first best-solutions while costly signalling equilibria produce second best-solutions. This paper deals with costless signalling equilibria.

The basic idea of a signalling equilibrium in financial markets is that the insider-manager of a firm signals his inside knowledge by his choice of the firm's investment and financing policy. The signalling equilibrium is fully revealing if the signals permit the outsiders to infer the insider's knowledge unambiguously. Then the outsiders are able to derive the inside values (= "true" values) of all securities so that the insider cannot exploit the outsiders by choice of the financing policy or by trade in the secondary market. The signalling equilibrium requires, in addition, that neither the insider prefers a change of the firm's policy nor anybody benefits from additional trade in the secondary market.

Bhattacharya and Brennan/Kraus derive the properties of a costless signalling equilibrium in two steps : (1) The insider-manager of a firm acts in the interest of the old securityhol-

ders. Therefore he minimizes the "true" wealth of the new outsider-investors to whom he sells securities to raise money.

(2) Conditions on the outsiders' behavior required by a fully revealing signalling equilibrium are derived. The intuition behind the second step is not easily understandable. Therefore the purpose of this paper is to derive a costless signalling equilibrium in financial markets from easily understandable conditions.

The first condition is the outsider-protection condition. It states that the price which the outsiders offer to pay for a security is inversely related to the supply of this security. The supply is perceived by the outsiders as a quality signal. A larger supply is interpreted as a signal of lower quality so that the outsiders lower their offer price. This behavior represents a simple means of self-protection. The insider would like to sell a large number of pieces of an overpriced security to the outsiders. The outsiders, knowing this, attempt to protect themselves against paying too high prices by inversely relating their price offer to the supply of the security. The outsider-protection condition has nothing to do with attitudes towards risk although risk aversion of outsiders may produce a similar result. The second condition for a costless signalling equilibrium is a no-arbitrage condition. It states that the marginal exchange rates for any two securities must be the same in both the primary and the secondary markets.

The paper shows that a costless signalling equilibrium in financial markets can be derived essentially from the outsider-protection and the no-arbitrage condition. In addition these conditions will be shown to imply valuation functions which exhibit constant negative elasticity with respect to the signal. This leads to a simple rule for determining the optimal financing policy. Pure equity financing is optimal only for a firm with the lowest possible 'true' total value.

The paper is organized as follows. The first section derives the costless signalling equilibrium from the two conditions, the second section presents an example where firms issue common stock, debt and subordinated debt. The third section discusses some generalizations of the model.

I Derivation of the Costless Signalling Equilibrium

1 The Economic Setting

Assume a firm needs currently (at date 0) the amount of money I for some purpose. The old securityholders (= old investors) do not supply the money, therefore new investors provide the money in exchange of risky securities, issued by the firm ²⁾. For simplicity of exposition all investors are assumed to be risk-neutral. In addition, the risk-free interest rate is assumed to be zero. ³⁾ Hence the market value of every security equals its

expected next date-market value including dividends etc. 4) The securities market is assumed to be perfect and competitive, apart from the information asymmetry and the assumption that the old investors do not supply part of the needed money.

Suppose the firm's investment policy is given. Then its next date-random market value is given. Therefore a state of nature at the next date can be defined by the size of the market value. The probability distribution of the market value depends on a vector of parameters $P \equiv (p_1, \dots, p_j, \dots, p_J)$ which is unknown to the old and the new investors. They are outsiders. 5) Those vectors which are regarded by the new investors as possibly 'true', are the elements of the set π . The new investors derive the probability distribution of the firm's market value from the parameter vector \bar{P} . \bar{P} could be a convex combination of all $P \in \pi$, for instance. In any case, $\bar{P} \in \pi$. The firm's manager has inside information on the parameter vector, his vector is denoted by P^* which is also an element of π . Define

v_0, v_1 = the firm's total market value at date 0 resp. date 1,

$f(v_1; P)$ = the probability density function of v_1 , derived from the parameter vector P ,

$F(\quad)$ = the cumulative probability density function.

'*' always indicates the manager's expectations. For notational simplicity the manager's probability beliefs and value estimates will be called 'true' beliefs and 'true' values although this does not mean that the manager has perfect information on P . It

only means that the manager has superior information. The probability density functions are assumed to be differentiable and integrable with respect to v_1 and differentiable with respect to every parameter p_j .

With limited liability, the firm's market value is ⁶⁾

$$v_0 = v_0(\bar{P}) = \int_0^{+\infty} v_1 f(v_1; \bar{P}) dv_1,$$

the insider-manager's estimate v_0^* is derived from $f(v_1; P^*)$.

As the outsiders do not know the "true" parameters, they can try to infer them from the firm's financing policy. Since the amount of money I , needed by the firm, is given, the outsiders can infer the J "true" parameters only if the manager has at least J degrees of freedom in choosing the financing policy. This policy is modeled here in a rather simple manner which can be generalized as will be shown in section III. There exist $(J+1)$ different types of securities indexed $i = 1, 2, \dots, J+1$. v_{0i} and v_{1i} denote the date 0 - resp. the date 1 - market value of all securities of type i . The distribution of the total market value v_1 across security types is assumed to be given and independent of the inside information, i.e. the state-contingent payoffs v_{1i} are given and independent of the inside information ($i = 1, \dots, J+1$). These payoffs are known to everybody, but the insider-manager has inside information on the probability density func-

tion of v_{1i} ($i = 1, \dots, J+1$) which is derived from the probability density function of v_1 . By definition, $v_1 = \sum_i v_{1i}$. With limited liability, $v_{0i} > 0$ for every security type i .

The manager's decision problem then is to choose for each type of security i the fraction α_i which the new investors receive for paying the amount I ; $\alpha_i \geq 0$.

As a consequence, the old investors receive the fraction $(1-\alpha_i)$. How can the manager achieve this distribution of type- i securities across old and new investors assuming $\alpha_i \in [0, 1]$? Holding the payoff v_{1i} of all type- i securities constant, the manager can issue additional type- i securities and distribute these across old and new investors such that the new investors hold the fraction α_i of all type- i securities. This technique fails only if the old investors already hold type- i securities and $\alpha_i = 1$. In this case the manager has to collect the type- i securities from the old investors in exchange for other securities.

The manager is assumed to maximize the "true" wealth of the old investors, which is the same as minimizing the "true" wealth of the new investors. Let $\alpha \equiv (\alpha_1, \dots, \alpha_{J+1})$ denote the firm's financing policy. Then the manager faces the following decision problem.

$$\text{Min}_{\alpha} \quad \sum_i \alpha_i v_{0i} \quad * \quad (1)$$

s.t.

$$\sum_i \alpha_i v_{0i} = I \quad \text{and} \quad \alpha_i \geq 0 \quad \forall i.$$

The constraint states that in a competitive capital market the market value of the securities which the new investors get must equal the money they pay. A change of the financing policy means that the new investors receive a smaller fraction of some payoffs (securities) in turn for a larger fraction of other payoffs (securities).

2 The Outsider - Protection - Condition

Whether a signalling equilibrium exists or not depends on the behavior of the new investors. Suppose, first, that the new investors do not consider the financing policy as a signal. Then v_{0i} ($i=1, \dots, J+1$) is independent of the financing policy. The optimal solution to the manager's minimization problem would be to sell as many overpriced securities to the new investors as possible.⁸⁾ This would imply adverse selection and force a loss on the new investors if there exists a sufficient number of overpriced securities. The new investors would recognize this and refuse to buy any securities. The market would break down (Akerlof <1>).

A viable behavioral alternative could be that the new investors regard the financing policy α as a quality signal such that their parameter vector \bar{P} is a function of α ; $\bar{P} = \bar{P}(\alpha)$. As a consequence, the market value of type- i securities depends on α ; $v_{0i} = v_{0i}(\bar{P}(\alpha))$; $\forall i$. More precisely, the price which the

new investors are ready to pay for such a security depends on the financing policy α . This offer price equals the market price if the manager chooses financing policy α .

The function $v_{0i} = v_{0i}(\bar{P}(\alpha))$ might be complicated if α is a vector with many components. In order to retain operationality, consider a very simple class of functions

$$v_{0i} = v_{0i}(B_i) \quad ; \quad \forall i,$$

with $B_i \equiv \alpha_i v_{0i} / I$. B_i is the fraction of the amount of money I , to be raised through selling securities of type i . Therefore B_i can be interpreted as the relative monetary supply of type- i securities by the firm. The constraint $\sum_i \alpha_i v_{0i} = I$ translates to $\sum_i B_i = 1$. The firm's financing policy can be defined equivalently by α and by $B \equiv (B_1, \dots, B_{J+1})$.

$v_{0i} = v_{0i}(B_i)$ means that the new investors regard the relative monetary supply B_i as a signal of the quality of security type i . Therefore the probability density $f(v_{1i}; \bar{P})$ depends on B_i only; $f(v_{1i}; \bar{P}) = \hat{f}(v_{1i}; B_i)$.

Suppose, first, that v_{0i} strictly increases in B_i . This means that the new investors offer a higher price for a security of type i if the manager raises the monetary supply of that security. This reinforces the adverse selection. If, for a given financing policy, type- i securities are overpriced, then the manager has an incentive to sell more of these securities to the new investors. If he does it, then the new investors raise the price of this security so that overpricing is reinforced creating an even

stronger incentive for the manager to sell more securities. Thus v_{0i} cannot be increasing in B_i if adverse selection is to be avoided.

Therefore suppose that v_{0i} is inversely related to B_i . If, for a given financing policy, type- i securities are overpriced, then the manager might want to sell more of these securities. Therefore he raises B_i , the monetary supply of these securities. The new investors react by reducing their offer price. If the reduction is sufficiently strong, then overpricing disappears. Possibly the security becomes underpriced. Therefore the manager will not choose $B_i = 1$. Similarly, he will not choose $B_i = 0$ if this signal implies overpricing of type- i securities because $B_i = 0$ rules out any benefits from overpricing. Thus an inverse relation between v_{0i} and B_i possibly protects the new investors against losses from adverse selection. ⁹⁾ This relation is the essence of the outsider-protection condition. In addition, the upper and the lower limit of v_{0i} have to be chosen such that every possible "true" value of v_{0i} is encompassed. More precisely, the upper limit \bar{v}_{0i} is defined by

$$\bar{v}_{0i} = \sup_{P \in \pi} v_{0i}(P),$$

the lower limit \underline{v}_{0i} is defined by

$$\underline{v}_{0i} = \inf_{P \in \pi} v_{0i}(P).$$

Outsider - Protection Condition (OPC) : The new investors regard an increase in the relative monetary supply of type- i secu-

rities, B_i , as a signal of lower quality of these securities and therefore reduce their price offer v_{0i} . For $B_i \rightarrow 0$, $v_{0i}(B_i) \rightarrow \bar{v}_{0i}$, and for $B_i \rightarrow 1$, $v_{0i}(B_i) \rightarrow \underline{v}_{0i}$.

As every new investor knows the function $v_{0i}(B_i)$, this function is also known to the manager. It is straightforward to show that the OPC is not sufficient for achieving a signalling equilibrium. A necessary condition for a signalling equilibrium is that $v_{0k} / v_{0h} = v_{0k}^* / v_{0h}^*$ for every pair of security types (k,h), i.e. the new investors' price relative must equal the insider-manager's price relative. If this condition is not satisfied, then $\bar{P} \neq P^*$ so that the asymmetry of information continues to exist.

Consider the manager's minimization problem (1) ignoring the nonnegativity constraint. If λ denotes the Lagrange multiplier, then necessary conditions for a minimum are

$$v_{0i}^* = \lambda \left[v_{0i} + \alpha_i \frac{dv_{0i}}{dB_i} \frac{dB_i}{d\alpha_i} \right]; \forall i. \quad (2)$$

Dividing the type-k condition by the type-h condition shows that $v_{0k}^* / v_{0h}^* \neq v_{0k} / v_{0h}$ because of the second term on the r.h.s. of equation (2). Hence the OPC does not imply a signalling equilibrium.

To put it differently, the manager's marginal exchange rate for type-h and type-k securities is v_{0k}^* / v_{0h}^* . The new investors' marginal exchange rate can be derived from the total differential

of the constraint $\sum_i \alpha_i v_{0i} = I$,

$$\sum_i \left[v_{0i} d\alpha_i + \alpha_i \frac{dv_{0i}}{dB_i} \frac{dB_i}{d\alpha_i} d\alpha_i \right] = 0 \quad (3)$$

If $d\alpha_i = 0$ for every i except k and h , then equation (3) yields

$$-\frac{d\alpha_h}{d\alpha_k} = \frac{v_{0k}}{v_{0h}} + \frac{1}{v_{0h}} \left[\alpha_k \frac{dv_{0k}}{dB_k} \frac{dB_k}{d\alpha_k} d\alpha_k + \alpha_h \frac{dv_{0h}}{dB_h} \frac{dB_h}{d\alpha_h} d\alpha_h \right] \quad (4)$$

Hence the new investors' marginal exchange rate equals v_{0k} / v_{0h} plus a term which will be labeled the marginal signalling effect. The marginal signalling effect exists because the new investors adjust their offer prices v_{0k} and v_{0h} to the signals B_k and B_h which change in response to α_k and α_h .

Figure 1 illustrates the preceding result for $J = 1$, i.e. there exist only two securities so that $B_2 = 1 - B_1$. Therefore, by the OPC, v_{01} / v_{02} is strictly decreasing in B_1 , whereas v_{01}^* / v_{02}^* is a constant. The manager chooses a financing policy such that $v_{01}^* / v_{02}^* = -d\alpha_2 / d\alpha_1$. Suppose the marginal signalling effect is positive. Then $v_{01}^* / v_{02}^* > v_{01} / v_{02}$. Therefore the manager chooses $\underset{v}{B}_1$ which exceeds B_1^* .

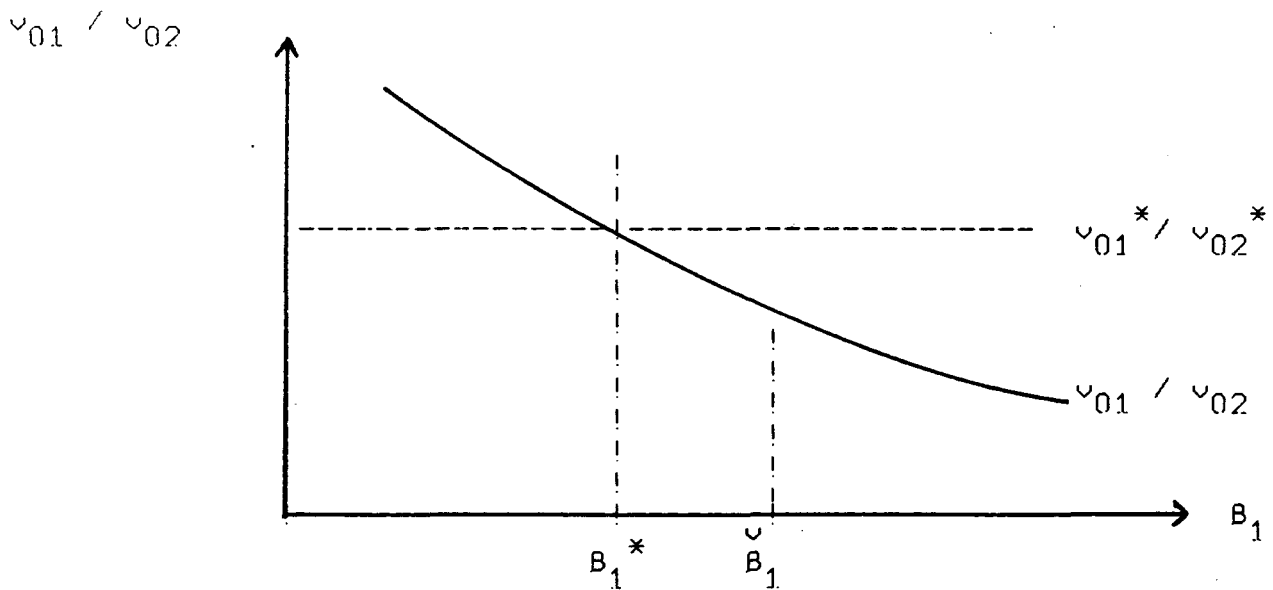


Figure 1 : A firm issues two types of securities. The new investors' price relative is strictly decreasing in the signal B_1 , the manager's price relative is a constant. If the marginal signalling effect is positive, the manager chooses the financing policy B_1 . If it is zero, he chooses B_1^* .

The preceding analysis reveals that a signalling equilibrium with $v_{0k}^*/v_{0h}^* = v_{0k}/v_{0h}$ can be expected to exist only if the marginal signalling effect is zero. This will be assured by a no-arbitrage condition.

3 The No - Arbitrage Condition

So far only the primary market has been considered, i.e. the choice of the firm's financing policy. Now assume that the firm's securities are also traded in a secondary market. This market is anonymous which means that an investor does not know whether the

other investors trading in the secondary market are insiders or outsiders. This assumption is reasonable only if insider trading is not legally prohibited and if insiders are not obliged to announce trades immediately.

Assumption : The firm's securities are traded in an anonymous secondary market, i.e. the characteristics of the trading investors are not revealed.

As the secondary market is anonymous, no signal can be derived from activities in this market.¹⁰⁾ Hence, given the financing policy of the firm, the marginal exchange rate between type-k and type-h securities equals v_{0k} / v_{0h} in the secondary market. It deviates from the marginal exchange rate in the primary market if the marginal signalling effect is not zero. The existence of two different marginal exchange rates usually creates profitable arbitrage opportunities.

Suppose, for instance, that the marginal exchange rate in the primary market is $-d\alpha_h / d\alpha_k = 1$ and $v_{0k} / v_{0h} = .8$ in the secondary market. Moreover, assume that the manager owns the fraction γ of all securities issued previously by the firm. Then the manager can change the financing policy by raising an additional dollar through selling additional type-k securities to the new investors and raising one dollar less by selling less type-h securities. This means that the old investors sell type-k securities against type-h securities in the primary market to the new investors. At the same time the manager buys type-k securities

for $.8 \gamma$ dollars and sells type-h securities for γ dollars in the secondary market. Hence the manager's stake in the firm remains unchanged, but he earns an arbitrage profit of $.2 \gamma$ dollars.

Even if the manager does not own any securities but acts in the interest of the old investors, he could create an arbitrage profit for them by changing the firm's financing policy and informing them secretly about their appropriate reaction in the secondary market.

Another possibility for earning arbitrage profits would be to announce some financing policy first; then the old investors trade in the secondary market. Finally the manager announces a new change in financing policy which offsets the old investors' trade in the secondary market and earns them an arbitrage profit.

Generally speaking, the existence of two different marginal exchange rates creates for the new investors the danger of arbitrage losses, generated by a change in the firm's financing policy and trade in the secondary market. Hence the new investors can buy securities safely only if both marginal exchange rates are the same. This motivates the no-arbitrage condition.

No - Arbitrage Condition (NAC) : The marginal exchange rates for any pair of securities are required to be the same in the primary and in the secondary market.

4 Implications of the Outsider - Protection and the No - Arbitrage - Condition

The NAC constrains the marginal signalling effects to be zero. This has quite strong implications as revealed by proposition 1. Define the elasticity of the market value v_{0j} with respect to B_j by $\epsilon_j(B_j)$, i.e. $\epsilon_j(B_j) = (dv_{0j} / dB_j) (B_j / v_{0j})$.

Proposition 1 : Suppose that the outsider-protection condition (OPC) holds. Then the no-arbitrage condition (NAC) implies $\alpha_j > 0$ and $B_j \in (0,1) \forall j$ and

- - $\epsilon_1(B_1) = \epsilon_2(1-B_1) < 0 \quad \forall B_1 \in (0,1)$ if only two types of securities exist,

- - $\epsilon_j(B_j) = c < 0$ for every type of security if more than two types of securities exist.

Proof : The NAC requires that the marginal signalling effect be zero for every pair of securities (k,h) . Hence the bracketed term in equation (4) must be zero so that

$$\alpha_k \frac{dv_{0k}}{dB_k} \frac{dB_k}{d\alpha_k} d\alpha_k = - \alpha_h \frac{dv_{0h}}{dB_h} \frac{dB_h}{d\alpha_h} d\alpha_h \quad \forall (k,h) \text{ with } k \neq h. \quad (5)$$

From $\sum_i B_i = 1$ follows

$$\sum_i \frac{dB_i}{d\alpha_i} d\alpha_i = 0.$$

Hence
$$\frac{dB_k}{d\alpha_k} d\alpha_k = - \frac{dB_h}{d\alpha_h} d\alpha_h \quad (6)$$

if $d\alpha_i = 0$ except for $i = k$ and $i = h$. Insert (6) in (5) and obtain

$$\alpha_k \frac{dv_{0k}}{dB_k} = \alpha_h \frac{dv_{0h}}{dB_h} \quad \forall (k,h) \text{ with } k \neq h. \quad (7)$$

This equation rules out $\alpha_k = 0$ since this would imply $\alpha_h = 0 \forall h$. Moreover, all α 's must be of the same sign so that they are required to be positive. Hence $B_i > 0 \forall i$. Then $\sum_i B_i = 1$ implies $B_i \in (0,1) \forall i$. Equation (7) can be rewritten as

$$\frac{\alpha_k v_{0k}}{I} \frac{1}{v_{0k}} \frac{dv_{0k}}{dB_k} = \frac{\alpha_h v_{0h}}{I} \frac{1}{v_{0h}} \frac{dv_{0h}}{dB_h}$$

or
$$\epsilon_k(B_k) = \epsilon_h(B_h) ; \forall (k,h) \text{ with } k \neq h. \quad (8)$$

This equation must hold for any financing policy. If only two types of securities exist, then $B_1 = 1 - B_2$.

Hence
$$\epsilon_1(B_1) = \epsilon_2(1-B_1) \quad \forall B_1 \in (0,1).$$

If more than two types of securities exist, then equation (8) must hold for any $(B_k, B_h) \in \{(0,1)\}^2$. This can be true only if ϵ_k and ϵ_h equal some constant c .

As $dv_{0i} / dB_i < 0$ is required by the OPC, $\epsilon_i < 0$ ■

By proposition 1, the outsider-protection and the no-arbitrage conditions require the valuation functions $v_{0j}(B_j)$ to have constant negative elasticity of magnitude c if more than two security types exist. Hence

$$v_{0j}(B_i) = B_i^c v_{0j} \quad ; \quad c < 0 \quad ; \quad \forall i .$$

If only two types of securities exist, then the requirement is weaker. For any financing policy, both valuation functions must exhibit the same elasticity.

It should be noted that the new investors necessarily receive a positive fraction of every type of security. Otherwise the marginal signalling effects could not be zero everywhere. The next step derives the optimal financing policy under the OPC and the NAC and shows that an exchange equilibrium obtains.

Proposition 2 : Suppose the OPC and the NAC hold.

a) If there exist more than two security types, then the manager's optimal financing policy is given by

$$B_i^+ = \phi_i / \sum_k \phi_k \quad \forall i \quad (9)$$

with $\phi_i \equiv \left[v_{0i}^* / v_{0i} \right]^{1/c}$.

b) If there exist two security types, then the optimal policy B_1^+ is derived from

$$v_{01}(B_1) = \frac{v_{01}^*}{v_{02}^*} v_{02}^{(1-B_1)} . \quad (10)$$

Proof : a) Suppose $J > 1$. Consider the minimization problem

$$\text{Min}_{\alpha} \sum_i \alpha_i v_{0i}^* , \text{ s.t. } \sum_i B_i \geq 1 .$$

Necessary conditions for a minimum are

$$v_{0i}^* = \lambda \frac{dB_i}{d\alpha_i} ; \forall i , \text{ with } \lambda > 0 . \quad (11)$$

Differentiate B_i with respect to α_i and obtain

$$\frac{dB_i}{d\alpha_i} = \frac{v_{0i}(B_i)}{1-i} + \frac{\alpha_i}{1} \frac{dv_{0i}}{dB_i} \frac{dB_i}{d\alpha_i} = \frac{v_{0i}(B_i)}{1-i} \left[1 - \epsilon_i(B_i) \right]^{-1} .$$

By proposition 1 , $\epsilon_i(B_i) = c$. Hence equation (11) yields

$$v_{0i}^* / v_{0h}^* = v_{0i}(B_i^+) / v_{0h}(B_h^+) = (B_i^+)^c v_{0i} / (B_h^+)^c v_{0h} .$$

This equation can be rewritten as

$$\begin{aligned} B_i^+ &= B_h^+ \left[v_{0i}^* / v_{0i} \right]^{1/c} \left[v_{0h}^* / v_{0h} \right]^{-1/c} \\ &= B_h^+ \phi_i / \phi_h \quad \forall (i,h) . \end{aligned}$$

Sum the last equation over $i = 1, \dots, J+1$ and obtain

$$1 = B_h^+ \sum_i \phi_i / \phi_h \quad (12)$$

which is the desired result. This solution is a unique minimum. This follows because the set of feasible solutions is convex¹¹⁾ and the problem $\text{Max } \sum_i \alpha_i v_{0i}^*$, s.t. $\sum_i B_i \geq 1$, would have no finite optimal solution.

From equation (12) follows $B_h^+ \in (0,1)$ so that $\alpha_h^+ > 0$; $\forall h$. Therefore this solution is also an optimal solution of the manager's minimization problem (1).

b) Suppose $J = 1$. Then $\epsilon_1(B_1) = \epsilon_2(1-B_1)$ [proposition 1]. Thus equation (11) implies $v_{01}^* / v_{02}^* = v_{01}(B_1^+) / v_{02}(1-B_1^+)$. As v_{01} / v_{02} is strictly decreasing in B_1 , this solution is a unique minimum ■

Proposition 2 tells the manager how to derive the optimal financing policy. As this policy implies $v_{0i}^* / v_{0h}^* = v_{0i}(B_i^+) / v_{0h}(B_h^+)$ for every pair of securities, a corollary follows.

Corollary : Suppose the OPC and the NAC hold. Then the optimal financing policy leads to an exchange equilibrium.

5 The Costless Signalling Equilibrium

An exchange equilibrium is not necessarily a signalling equilibrium because the exchange equilibrium assures only the equality of the manager's and the outsiders' marginal exchange rates, but not the equality of their price levels. It is possible, for

instance, that in an exchange equilibrium all securities are overpriced at a rate of 10 percent, relative to the insider's values. This is inconsistent with a signalling equilibrium. Such a problem cannot exist if the exchange equilibrium is fully revealing, i.e. if the new investors can infer the manager's parameter vector P^* unambiguously from the chosen financing policy. Knowing P^* enables the new investors to derive all prices on their own. The equilibrium is fully revealing if there exists a one-to-one correspondence between π and B , with B being the set of all possibly optimal financing policies B^+ . A one-to-one correspondence exists if each $P^* \in \pi$ implies a unique $B^+ \in B$ and if each $B^+ \in B$ implies a unique $P^* \in \pi$. This correspondence restricts the nature of the unknown parameters and /or the nature of the security payoffs. ¹²⁾ An example with two unknown parameters will be presented in the next section. The preceding results are summarized by proposition 3.

Proposition 3 : Assume that the outsider-protection and the no-arbitrage conditions hold. In addition assume that there exists a one-to-one correspondence between the set π of unknown parameter vectors and the set B of optimal financing policies. Then the exchange equilibrium, characterized by propositions 1 and 2, is a costless, fully revealing signalling equilibrium.

A related characterization of a costless, fully revealing signalling equilibrium can be derived easily from the preceding analysis. Suppose two firms of types 1 and 2 with different parameter vectors P_1^* and P_2^* need the same amount of money I . The manager

minimizes the 'true' value of the new investors' securities. The minimal value equals the amount I in a fully revealing equilibrium. Hence the 'true' value of these securities must be smaller for firm 1 than for firm 2 at B^+ , the optimal financing policy of firm 1. Similarly, the 'true' value of these securities must be smaller for firm 2 than for firm 1 at B^{++} , the optimal financing policy of firm 2. This is illustrated in figure 2, assuming $B^+ < B^{++}$.

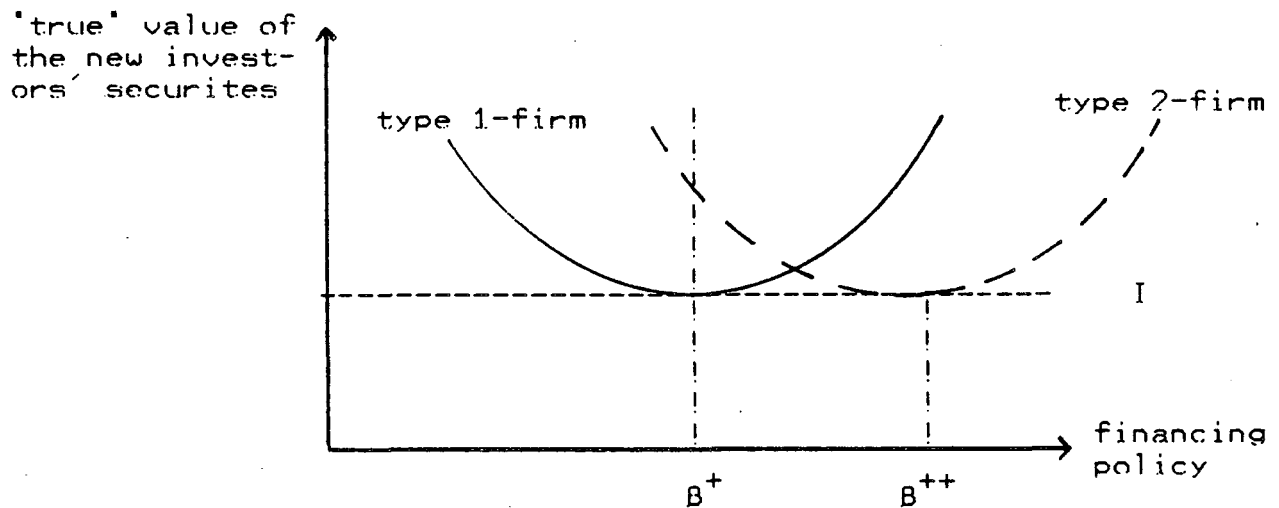


Figure 2 : The 'true' values of the new investors' securities are minimal at the optimal financing policies.

This reproduces an important result of Bhattacharya and Brennan / Kraus: In a fully revealing costless signalling equilibrium the securities, sold by a firm to new stockholders, are priced on the correct supposition that they have the lowest possible 'true' value. In other words, when the outsiders are offered new securities by a firm, they check for every type of firm what the 'true' value of these securities would be if they were issued by that

type of firm. Then they buy the securities on the supposition that they are issued by that type of firm such that they have the lowest possible 'true' value. This supposition is correct in equilibrium. Therefore it is impossible for the manager to gain anything by cheating, i.e. by signalling a parameter vector other than the 'true' one.

The next proposition shows that the type of firm with the lowest possible 'true' total value sells the same fraction α^+ of all securities to the new investors. Thus for this type of firm there exists no need to issue more than one security type for signalling purposes. This firm could be purely equity-financed.

Proposition 4 : Suppose that the assumptions of proposition 3 hold. In addition, assume $d^2 v_{0i} / dB_i^2 > 0$ for $i = 1, 2$ if only two types of securities exist ($J=1$). Then the manager of a firm with the lowest possible 'true' total value sells the same fraction α^+ of all securities to the new investors.

Proof : The 'true' total value of a firm is $\sum_i v_{0i}(B_i^+)$ in a costless signalling equilibrium. It has to be shown that this value is minimal if $\alpha_1^+ = \dots = \alpha_{J+1}^+$. Consider the problem

$$\text{Min}_B \sum_i v_{0i}(B_i) \text{ , s.t. } \sum_i B_i = 1 \text{ .}$$

Necessary conditions for a minimum are

$$\frac{dv_{0i}}{dB_i} = \lambda \quad ; \quad \forall i \text{ .}$$

Hence $dv_{0i} / dB_i = dv_{01} / dB_1 \quad \forall i$, or

$$\frac{v_{0i}}{B_i} \epsilon_i(B_j) = \frac{v_{01}}{B_1} \epsilon_1(B_1) \quad , \quad \forall i \quad , \quad \text{or}$$

$$\frac{\epsilon_i(B_j)}{\alpha_i} = \frac{\epsilon_1(B_1)}{\alpha_1} \quad , \quad \forall i \quad .$$

By proposition 1 , $\epsilon_i(B_j) = \epsilon_1(B_1)$, so that $\alpha_j = \alpha_1 > 0$, $\forall i$.

Sufficiency for a minimum requires that $d^2 v_{0j} / dB_j^2 > 0 \quad \forall i$.

For $J = 1$ this has been assumed; for $J > 1$, it is implied by constant negative elasticity of $v_{0j}(B_j)$ ■

Proposition 4 shows that firms with high "true" total values choose financing policies which deviate strongly from pure equity financing. Hence the distance between actual financing policy and pure equity financing can be considered a signal of the firm's total value.

II An Example With Two Unknown Parameters

This section presents an example. Assume that the firm's date 1-market value v_1 is normally distributed. The manager knows the expected value μ^* , but the new investors do not. If the manager sells only common stock to the new investors, they would underprice the common stock, because only firms with the lowest possible "true" total value are purely equity financed. Therefore the manager intends to signal μ^* by selling common stock and debt. The market value of debt, however, depends on μ and on σ , the standard deviation of v_1 . Again, σ^* is known to the manager,

but not to the new investors. Therefore the manager sells debt, subordinated debt and common stock so that both, μ and σ , can be signalled. The following notation will be used

- D \equiv date 1-claim of debt holders ,
 U \equiv date 1-claim of subordinated debt holders ,
 v_{D1} \equiv date 1-payoff to debt holders ,
 v_{U1} \equiv date 1-payoff to subordinated debt holders ,
 v_{S1} \equiv date 1-payoff to stockholders .
 $f^*(v_1)$ \equiv the manager's normal density function of v_1 ;
 $F^*(v_1)$ denotes the cumulative normal density function ;
 z \equiv $(v_1 - \mu) / \sigma$, the standardized normal variate ,
 $f(z)$ \equiv the standardized normal density function ; $F(z)$ denotes the cumulative standardized normal density function .

The payoffs are defined as follows :

$$\begin{aligned}
 v_{D1} &= \begin{cases} v_1 & , \text{ if } v_1 < D , \\ D & , \text{ if } D \leq v_1 , \end{cases} \quad 13) \\
 v_{U1} &= \begin{cases} 0 & , \text{ if } v_1 < D , \\ v_1 - D & , \text{ if } D \leq v_1 < D + U , \\ U & , \text{ if } D + U \leq v_1 , \end{cases} \\
 v_{S1} &= \begin{cases} 0 & , \text{ if } v_1 < D + U , \\ v_1 - D - U & , \text{ if } D + U \leq v_1 . \end{cases}
 \end{aligned}$$

The 'true' value of the debt claims is

$$v_{0D}^* = \int_{-\infty}^D v_1 dF^*(v_1) + D [1 - F^*(D)].$$

Defining $z_D^* \equiv (D - \mu^*) / \sigma^*$ and using the statistics of truncated normal distributions (cf. Johnson and Kotz < 6 , p.81 >) yields

$$\begin{aligned} v_{0D}^* &= F(z_D^*) \left[\mu^* - \sigma^* f(z_D^*) / F(z_D^*) \right] + D [1 - F(z_D^*)] \\ &= D - \sigma^* \left[f(z_D^*) + z_D^* F(z_D^*) \right]. \end{aligned} \quad (13)$$

Define $z_{D+U}^* = (D + U - \mu^*) / \sigma^*$ and v_{0U}^* as the 'true' value of subordinated debt. Then

$$v_{0U}^* = D + U - \sigma^* \left[f(z_{D+U}^*) + z_{D+U}^* F(z_{D+U}^*) \right] - v_{0D}^*$$

and the 'true' value of common stock is

$$v_{0S}^* = \mu^* - v_{0D}^* - v_{0U}^*.$$

The manager, knowing the 'true' values and the functions $v_{0i}(B_i)$, derives the optimal financing policy as stated in proposition 2. Then an exchange equilibrium obtains. Whether this is a fully revealing signalling equilibrium, depends on the existence of a one-to-one correspondence between π and B . Therefore conditions for this correspondence will be derived.

This can be done as follows. The new investors observe the firm's financing policy and therefore know the value relatives $v_{0D}^* : v_{0U}^* : v_{0S}^*$, i.e. they know $\bar{g}_D \equiv v_{0D}^* / \mu^*$ and $\bar{g}_{D+U} = (v_{0D}^* + v_{0U}^*) / \mu^*$. Now consider a (σ, μ) - diagram as illustrated in figure 3. Take a specific value of μ . Then there exists a unique value of σ satisfying $v_{0D} / \mu = \bar{g}_D$ because $\delta v_{0D} / \delta \sigma = -f(z_D) < 0$. Thus it is possible to construct an iso-value-relative curve in the (σ, μ) - diagram such that $v_{0D} / \mu = \bar{g}_D$ holds for every point of this curve.

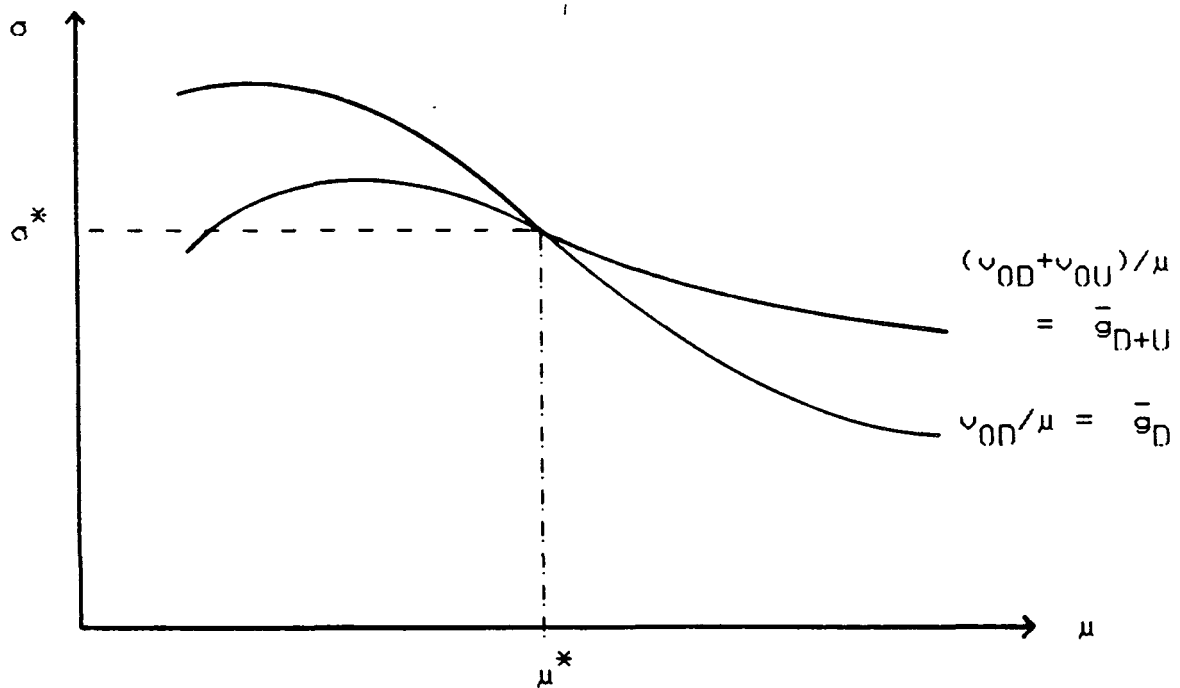


Figure 3 : Iso-Value-relative curves for debt and debt including subordinated debt. If both curves intersect once, then the point of intersection defines the insider's values μ^* and σ^* .

Similarly an iso-value-relative curve can be constructed for $\bar{g}_{D+U} = (v_{0D} + v_{0U}) / \mu$. If both iso-value-relative curves

intersect once, then the new investors can infer μ^* and σ^* from the point of intersection so that the exchange equilibrium is a fully revealing signalling equilibrium.

Proposition 5 : A sufficient condition for a costless, fully revealing signalling equilibrium is that

$$\frac{x}{\mu} (1 - \frac{x}{\mu}) > (\frac{\sigma}{\mu})^2 \quad \text{for } x = D \quad \text{and } x = D+U \quad (14)$$

holds for all vectors $(\sigma, \mu) \in \pi$.

This proposition is proved in the appendix. Condition (14) is sufficient, but not necessary. It constraints the firm's risk (σ/μ) in response to the leverage ratios D/μ and $(D+U)/\mu$. This is perhaps best illustrated by the probability of default $F(z_{D+U}^*)$.

Assume $D+U < \mu$, otherwise the firm would be bankrupt at date 0. Consider those values of μ such that the l.h.s. of condition (14) is smaller for $x = D+U$ than for $x = D$. Then it follows from the definition of z_{D+U}^* and condition (14) $[y = D+U]$

$$z_y = (\frac{y}{\mu} - 1) \frac{\mu}{\sigma} < (\frac{y}{\mu} - 1) \left[\frac{y}{\mu} (1 - \frac{y}{\mu}) \right]^{-1/2} = -\sqrt{\frac{\mu}{y} - 1}.$$

Hence the probability of default, $F(z_y)$, is constrained from above. If, e.g. $(D+U)/\mu = .8$, then $-\sqrt{(\mu/y) - 1} = -.5$ so that the probability of default must not exceed $F(-.5) = 31$ percent.

IV Some Generalizations

The preceding analysis has been simplified by some assumptions which will be discussed briefly now. First, a non-zero risk-free interest rate does not affect the exchange equilibrium as the price relatives are unaffected. Second, risk averse behavior of investors complicates the analysis but does not endanger the existence of a costless, fully revealing signalling equilibrium as long as the capital markets are pareto-efficient. If the latter condition does not hold, then signalling might interfere with optimal allocation of risk so that the signalling equilibrium becomes costly.

Third, it has been assumed for every type of security i that the date 1 payoff on all securities of this type, v_{j1} , is exogenously given and hence independent of the financing policy. In reality, the manager might want to redistribute the payoffs according to the chosen financing policy. If he wants to sell many debt claims to the new investors, for example, then he may have to create additional debt claims. Such a policy-dependent redistribution of payoffs among security types invalidates the no-arbitrage condition if an investor cannot offset the effects of a change in the firm's financing policy on his date 1 payoff by trading in the secondary market. This cannot happen, however, if a sufficient number of linearly independent securities is traded in the secondary market. Then a policy-dependent redistribution of payoffs complicates the analysis, but it does not endanger the existence of the signalling equilibrium, provided that the outsiders know

the state-contingent payoffs and therefore can evaluate them properly.

Fourth, the date 1 payoff on all securities of the firm, v_1 , may depend on the financing policy. If the firm issues warrants, for example, then in some states at date 1 the payoff v_1 is affected by the cash inflow from the exercise of the warrants. Again, this complicates the analysis but does not endanger the existence of the signalling equilibrium if the outsiders know the state-contingent payoffs and if the no-arbitrage condition is not invalidated.

Fifth, although the needed amount of money, I , has been assumed to be exogenous, the analysis remains the same if the manager chooses the level of I , provided that the new investors do not regard the level as a signal.

V Conclusion

Bhattacharya and Brennan/Kraus derived conditions for a costless signalling equilibrium which are not easy to understand. This paper shows that a costless signalling equilibrium in financial markets can be derived from an outsider-protection and a no-arbitrage condition. The outsider-protection condition is an important device against adverse selection, the no-arbitrage condition requires that marginal exchange rates be the same in the primary and in the secondary market. Both conditions appear

to be easily understandable. The implications are quite strong. If more than one parameter is unknown, then the current market value of the securities of any type must exhibit constant negative elasticity with respect to the signal. This leads to a simple rule for deriving the optimal financing policy. The implied exchange equilibrium is a costless, fully revealing signalling equilibrium if and only if there exists a one-to-one correspondence between the unknown parameters and the optimal financing policy. This correspondence restricts the nature of the unknown parameters and / or the nature of the security payoffs.

The outsider-protection condition being used is most simple since it relates the market value of a payoff to only one variable of the firm's financing policy. Although this simplicity is analytically desirable, it may be unrealistic. Therefore empirical tests of this condition would be helpful. So far, research on costless signalling equilibrium is still in its infancy.

Appendix : Proof of Proposition 4

The proof proceeds as follows : It will be shown that under condition (14) of proposition 4 the iso-value-relative curve for (D+U) has a larger slope than that for D . Hence the curves can intersect at most once. If no intersection exists, then the outsiders cannot infer μ^* and σ^* . This is possible only if the manager's behavior is inconsistent with the model. Hence the new investors would refuse to buy any securities. Conversely, if the manager behaves according to the model, then an intersection must exist.

Therefore it suffices to show that under condition (14) the iso-value-relative curve for (D+U) has a slope $d\sigma_{D+U} / d\mu$ which is larger than $d\sigma_D / d\mu$. A sufficient condition for this to be true is that

$$\frac{d \left[\frac{d\sigma_x}{d\mu} \right]}{dx} > 0 \quad \forall x \in [D, D+U] .$$

The iso-value-relative curve for x is defined by $v_{0x} = \bar{g}_x \mu$.

Total differentiation yields

$$\frac{\delta v_{0x}}{\delta \mu} d\mu + \frac{\delta v_{0x}}{\delta \sigma} d\sigma = \bar{g}_x d\mu , \text{ so that}$$

$$\frac{d\sigma}{d\mu} = \frac{\bar{g}_x - \delta v_{0x} / \delta \mu}{\delta v_{0x} / \delta \sigma} = \frac{\bar{g}_x - F(z_x)}{-f(z_x) x} .$$

Substituting $\bar{g}_x = v_{0x} / \mu$ in connection with equation (13) yields

$$\frac{d\sigma_x}{d\mu} = \frac{-x \left[1 - F(z_x) \right]}{\mu f(z_x)} + \frac{\sigma}{\mu} .$$

Differentiation with respect to x yields

$$\frac{d \left[\frac{d\sigma_x}{d\mu} \right]}{dx} = \frac{- \left[1 - F(z_x) \right] \left[1 + x z_x / \sigma \right] + (x/\sigma) f(z_x)}{\mu f(z_x)} .$$

This differential is positive if

$$1 + x z_x / \sigma = 1 + \frac{x(x - \mu)}{\sigma^2} < 0 ,$$

or $(\sigma / \mu)^2 < (x / \mu) (1 - x / \mu) .$

If this condition holds for every $x \in [D, D+U]$, then $d\sigma_{D+U} / d\mu > d\sigma_D / d\mu$. As $(x / \mu) (1 - x / \mu)$ attains its minimum either at $x = D$ or at $x = D+U$, condition (14) in proposition 4 is sufficient.

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Footnotes

- 1) Helpful comments by Robert Geske and Nikolaus Läufer are gratefully acknowledged.
- 2) The economic setting resembles that of Myers and Majluf (8). They analyze investment decisions under asymmetric information.
- 3) These assumptions can be removed easily in a pareto-efficient market. See Brennan and Kraus (3, p. 43).
- 4) This might be questionable if adverse selection exists. But the economic setting and the conditions of this paper rule out adverse selection as will be shown.
- 5) The firm's manager is an insider although he may be an investor.
- 6) If the firm's manager belongs to the investors, his trading decisions have no appreciable effect on pricing in a competitive market.
- 7) As the payoffs v_{1i} are given, it is possible that $\alpha_i > 1$ which means that the old investors sell some securities short to the new investors. The manager can adjust the payoffs, however, such that $\alpha_i < 1 \quad \forall i$ as shown in section III.

- 8) The manager might face a risk of being accused of fraud if he sells overpriced securities. Overpricing, however, is difficult to prove since valuation depends heavily on subjective expectations.
- 9) Similarly a specialist relates the bid-ask spread to the size of the transaction in order to protect himself against losses by insider trading. See Copeland and Galai <4>, Glosten and Milgrom <5> .
- 10) This is true at least if all insiders together neither buy nor sell substantial numbers of securities in the secondary market.
- 11) The function $-\sum_i B_i(\alpha_i)$ is separable in $\alpha_1, \alpha_2, \dots, \alpha_{j+1}$ and $-d^2 B_i / d\alpha_i^2 > 0, \forall i$.
- 12) Brennan and Kraus < 3, pp. 40 ff> derive some properties of these restrictions.
- 13) The probability of negative v_1 - values is assumed to be negligible.