

Wahl, Jack E.; Tschoegl, Adrian E.

Working Paper

Incentives for information acquisition

Diskussionsbeiträge - Serie C, No. 8

Provided in Cooperation with:

Department of Economics, University of Konstanz

Suggested Citation: Wahl, Jack E.; Tschoegl, Adrian E. (1985) : Incentives for information acquisition, Diskussionsbeiträge - Serie C, No. 8, Universität Konstanz, Fakultät für Wirtschaftswissenschaften und Statistik, Konstanz

This Version is available at:

<https://hdl.handle.net/10419/92537>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

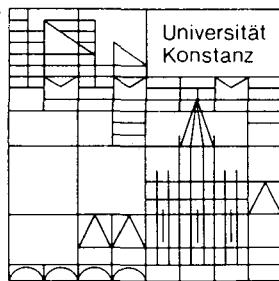
Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



**Fakultät für
Wirtschaftswissenschaften
und Statistik**

Jack Wahl und
Adrian E. Tschoegl

**Incentives
for Information Acquisition**

Diskussionsbeiträge

INCENTIVES FOR INFORMATION ACQUISITION

Jack Wahl und Adrian E. Tschoegl

November 1985

Serie C - Nr. 8

Ag 449/86
Weltwirtschaft
Kiel

Serie A: Volkswirtschaftliche Beiträge
Serie B: Finanzwissenschaftliche Arbeitspapiere
Serie C: Betriebswirtschaftliche Beiträge

Professor Adrian E. Tschoegl, School of Business Administration
The University of Michigan, Ann Arbo

We would like to thank Clifford A. Ball, Günter Franke,
David M. Kreps, John G. Riley, Walter N. Torous, Hal R. Varian,
and various participants at the 1985 Meeting of the European
Finance Association for their helpful comments.

Incentives for Information Acquisition

Abstract

This paper examines necessary conditions for a demand for new information to exist. In this one-period model, investors are homogeneous, have logarithmic utility, and must decide on information acquisition before trading starts, and without knowing what other investors will do. We examine the decision problems under scenarios defined by whether information is costless or costly to acquire, whether aggregate consumption is endogenous or exogeneously given, and whether investors can or cannot costlessly cooperate. In all cases a demand for new information exists as all investors decide to acquire provided the price is below endogeneously determined bounds.

Incentives for Information Acquisition

1.0 Introduction

In equilibrium, information does not "fall from Heaven." Unless investors have an incentive to acquire new information no one will produce it, and therefore, security prices cannot reveal it.

The following example may clarify the information problem we are modeling. A brokerage firm does not ask its customers, "Do you wish to buy the information that we expect Firm X's next quarter earnings will be up 10%?" Instead, it asks, "Will you pay us to research Firm X's future earnings?"

We develop a one-period model which enables us to examine an investor's incentive for acquiring new information. That is, we are interested in necessary conditions for a demand for costly information to exist. Our investor is one of many, all of whom are homogeneous in their endowments and have logarithmic utility functions. Homogeneous endowments leads to clear cut results. Logarithmic utility simplifies the mathematics without loss of generality with respect to the hyperbolic absolute risk aversion (HARA) class of utility functions.

Before trading in the securities market, each investor must first decide whether or not to buy new information from the information market. The investor knows that all other investors face the same decision, but does not know what the others will do. We require this assumption in order to have a decision problem and to derive a simultaneous equilibrium in both the capital and the information market. The capital market equilibrium that investors take as their reference point before deciding on information acquisition is a partial one--it will represent the full equilibrium if and only if no investor acquires new information.

Investors ultimately choose between two rational expectations equilibria: the benchmark, in which prices will reveal only prior (i.e., existing) information if all choose not to acquire, and an alternative in which prices will reveal information which does not yet exist, and which will not come into existence unless some investors decide to acquire it. They do so after comparing these equilibria with some notional others which involve heterogeneous expectations.

It is the introduction of potentially heterogeneous beliefs which drives our results. We examine the effect on the decision to acquire new information under scenarios defined by whether information is costless or costly to acquire, whether aggregate consumption is endogenous or exogenously given, and whether investors can or cannot costlessly cooperate. Cooperation between the informed and the uninformed investors means that they are costlessly able to enter into binding contracts with respect to the financial consequences of the information acquisition decision. In a market without cooperation all investors are assumed to act anonymously.

Under our scenarios, all investors acquire the new information, provided the price the information market charges is below endogeneously determined positive upper bounds. Thus there is a demand for costly information. One possibly unexpected result is that if cooperation is impossible and aggregate consumption is exogenously given, all investors will acquire costly information, even though each would be better off if no investor would acquire it.

The investors are aware that if they all refuse to purchase information and sit on their endowments, they will wind up at the Pareto efficient point at which they begin. If we remove the assumption that investors must go to market, then in this case they would not go, and there would be no demand for information. However, we can replace the assumption with conditions on the

investors' initial endowments. These have the effect of ensuring that all investors will have identical portfolios in equilibrium, but face utility gains from trading securities. Our results then again go through.

While all investors would be better off if none bought information, as long as the information price is low enough the utility gains from trade equal or exceed the utility loss from acquisition. In this situation of fixed aggregate consumption, no cooperation, and costly information, information acquisition represents a purchase of insurance against being exploited by insiders and/or a chance to exploit outsiders while engaging in portfolio adjustment.

When we introduce production, the bounds on the information price change, but the investors' decisions to acquire remain unchanged. In this case we use the gains from production rather than those from trade to bring investors to market. Furthermore, regardless of the exogeneity or endogeneity of aggregate consumption, all investors have an incentive to cooperate when that is possible because doing so enables them to win against the information seller, or nature.

The problem of the information acquisition decision and the related question of the value of information are well-known in decision theory. The main underlying assumption in Savage [1972] and Hirshleifer and Riley [1979] is that information acquisition does not affect securities' prices. Costless information acquisition together with unchanging market prices for securities imply an opportunity set unaffected by information acquisition. Then it cannot be disadvantageous to an investor to acquire information. Although possessing new information, the investor still has the option of realizing his prior investment-consumption decision. This result holds whatever other investors decide to do.

Hirshleifer [1971] and Hakansson, Kunkel, and Ohlson [1982] have developed models which include price changes. These changes can destroy an investor's incentive to acquire information. The effect on incentives depends on the effect of new information on all investors' prior beliefs. With homogeneous prior and posterior beliefs, identical endowments, and given aggregate claims, for example, information acquisition does not change the distribution of claims. When one constrains all investors to act identically, no investor has an incentive to acquire the information [Wahl 1983].

The rational expectations literature also addresses the issue of information acquisition. In Hellwig [1982], information acquisition precedes trading. Thus, uninformed investors can infer new information from prices, but only with a lag. However, in our model, each investor must decide whether or not to acquire information before trading begins. This approach enables us to determine conditions for simultaneous equilibrium in the capital and the information market. Hence, we avoid the Grossman and Stiglitz [1980] paradox. In their paper, the investor's alternatives are to acquire information, or to observe prices which reveal it. In our approach, prices do not reveal new information unless investors first decide to acquire it and, therefore, it is produced.

Section 2 establishes the model, how investors evaluate new information, and the definition of a stable acquisition decision. Section 3 examines the implications of the model for acquisition decisions under the scenarios just outlined above. Section 4 is a summary and conclusion.

2.0 A model of information acquisition

We begin by establishing the economic setting. Next, we determine how investors will assess the value of new information. We close the section with a

definition of a condition under which all investors will choose to acquire new information before the securities market opens.

2.1 Assumptions, definitions, and foundations

We start with the following assumptions:

A.1. The capital market is competitive, complete, and perfect up to a nonnegative information price.

We assume a complete capital market to exclude any allocational inefficiency.

A.2. The exogeneous information market offers information at the same price to every investor. The information seller does not trade in the capital market and no investor acts as an information seller. The information seller knows the investors' demand functions and is risk averse to such a degree that he will not produce information unless he can recover his marginal cost of production for sure. Each investor has to decide whether or not to purchase information before the capital market opens.

A.3. The state space is countable and exogeneously given.

A.4. Every investor is a von Neumann-Morgenstern utility maximizer who optimizes the investment-consumption decision at time \emptyset . The state-independent utility function, U , is time-additive logarithmic in consumption with constant proportional risk aversion (CPRA):

$$U = \rho_{\emptyset} \ln C_{\emptyset} + \rho_1 \ln C_1, \quad (1)$$

$\rho_t \equiv$ parameter of the utility function at time t ($t = \emptyset, 1$);

$\rho_{\emptyset} + \rho_1 \equiv 1$, without loss of generality),

$C_t \equiv$ consumption at time t ($t = \emptyset, 1$).

The expected utility of consumption is:

$$E[\bar{U}] = \rho_{\emptyset} \ln C_{\emptyset}(\emptyset) + \rho_1 \sum_s f(s) \ln C_1(s), \quad (2)$$

$E[\cdot]$ \equiv expectation operator and the tilde denotes state-dependence,

(\emptyset) \equiv state at time \emptyset , which occurs with probability 1,

Σ_s \equiv denotes the sum over the s ($s = 1, 2, \dots, S$) mutually exclusive states at time 1,

(s) \equiv state s at time 1 ($s = 1, 2, \dots, S$),

$f(s)$ \equiv the investor's prior probability belief of the occurrence of state s ($s = 1, 2, \dots, S$).

We use logarithmic utility for the following reasons. First, while mathematically relatively tractable, the results derived below on the basis of the assumption hold for all HARA class utility functions. Second, the available empirical evidence points towards constant proportional risk aversion (CPRA) over wide ranges in wealth (Friend and Blume 1975). While the evidence suggests a coefficient of relative risk aversion greater than that for logarithmic utility, our assumption keeps us within the CPRA class. Finally, Rubinstein (1977) has demonstrated the power of the assumption of logarithmic utility for financial economics.

A.5. Investors are identical in their initial endowments and in the parameters ρ_t of their utility functions ($t = \emptyset, 1$).

Let i ($i = 1, 2, \dots, I$) denote the investor index, I the number of investors in the market, and \bar{C} endowment in consumption claims. Then:

$$\bar{C}_{\emptyset i}(\emptyset) = \bar{C}_{\emptyset 1}(\emptyset) = (1/I) \Sigma_i \bar{C}_{\emptyset i}(\emptyset) \equiv (1/I) C_{\emptyset m}(\emptyset) \quad (i = 1, 2, \dots, I), \quad (3)$$

$$\bar{C}_{1i}(s) = \bar{C}_{11}(s) = (1/I) \Sigma_i \bar{C}_{1i}(s) \equiv (1/I) C_{1m}(s) \quad (i = 1, 2, \dots, I; s = 1, 2, \dots, S)$$

Every investor therefore shares $(1/I)$ of the aggregate initial endowment in consumption claims, $C_{\emptyset m}(\emptyset)$ and $C_{1m}(s)$. Maximizing $E[\tilde{U}_i]$ with respect to $C_{\emptyset i}(\emptyset)$ and $C_{1i}(s)$ subject to the initial endowments $\bar{C}_{\emptyset i}(\emptyset)$ and $\bar{C}_{1i}(s)$ ($s = 1, 2, \dots, S$), gives the optimal investment-consumption decision.

Market equilibrium requires (a) an optimal investment-consumption decision by every investor, and (b) fulfillment of the clearing condition in every

state, i.e., $\sum_i C_{ti}(\cdot) = C_{tm}(\cdot)$ ($t=\emptyset, 1$). Henceforth, we omit the time index t and the other indexes when they are superfluous for understanding.

Differentiating between exogenous aggregate consumption (\equiv without production), and endogenous aggregate consumption (\equiv with production), will lead to interesting results. Therefore assume first:

A.6a. The aggregate consumption in every state, i.e., $C_m(\emptyset)$ and $C_m(s)$ ($s = 1, 2, \dots, S$), is exogenously given.

Lemma 1: Under assumptions A.1. to A.6a, the equilibrium sharing rule in consumption claims is:

$$C_i(\emptyset) = (1/I) C_m(\emptyset), C_i(s) = (1/I) \frac{f_i(s)}{f_m(s)} C_m(s) \quad (4)$$

$$(i = 1, 2, \dots, I; s = 1, 2, \dots, S)$$

and $f_m(s) \equiv \frac{1}{I} \sum_i f_i(s)$ ($s = 1, 2, \dots, S$).

We will call $f_m(\cdot)$ the probability belief of the market, or the market forecast as it exhibits all the properties of a probability measure.

Proof: a). Eq. (3) shows that every investor possesses $(1/I)$ of the corresponding aggregate consumption claims. Identical investors have no reason to change this endowment which meets the market clearing conditions.

b). Suppose that at least two investors differ in their probability assessment of at least two states of nature. Under assumptions A.1 to A.5, the Lagrangean procedure gives investor i 's demand function for contingent consumption claims in state s :

$$C_i(s) = p_1 W(\emptyset) f_i(s)/\pi(s) \quad (s = 1, 2, \dots, S)$$

where $W(\emptyset) \equiv$ initial wealth of every investor, and $\pi(s) \equiv$ contingent price of a \$1 payoff if and only if state s occurs, and $\pi(\emptyset) \equiv 1$. Aggregating the demand functions and inserting the result in investor i 's demand function gives Eq. (4). ***

The aggregate consumption in every state is endogenous if the market as a whole can optimize consumption intertemporally. Therefore, to close the with-production model we make the following assumption:

A.6b. The risk-free marginal productivity of capital, $R_F > -1$, is exogenously given by technology.

This implies that only risk-free production is variable. Risky production remains unchanged. Assumption A.6b implies the following market clearing conditions:

$$\begin{aligned} \sum_i C_i(\emptyset) &= C'_m(\emptyset) = (1-d)C_m(\emptyset) \quad (d < 1), \\ \sum_i C_i(s) &= C'_m(s) = [1+\beta(s)d]C_m(s) \quad (s = 1, 2, \dots, S), \end{aligned} \quad (5)$$

$C'_m(\cdot) \equiv$ optimal aggregate consumption in the corresponding state,

$d \equiv$ coefficient of production change,

$\beta(s) \equiv (1+R_F)C_m(\emptyset)/C_m(s) \quad (s = 1, 2, \dots, S)$.

One may interpret $C_m(\cdot)$ as a preliminary production plan for the economy which can be chosen arbitrarily as long as $C_m(\cdot) > 0$. The following implication of (5) clarifies the meaning of d :

$$C'_m(s) - C_m(s) = C_m(\emptyset)d(1+R_F). \quad (6)$$

That is, a positive (negative) d means that the preliminary aggregate consumption in state \emptyset is reduced by $C_m(\emptyset)d$ and that the preliminary aggregate consumption in all states s is augmented (reduced) by the same amount, times one plus the risk-free rate. The optimal d is given by

$$\text{Lemma 2: } \rho_\emptyset(1-d)^{-1} = \rho_1 \sum_s f_m(s) \frac{\beta(s)}{1+\beta(s)d}. \quad (7)$$

Proof: $\text{Max}_d Z \equiv \rho_\emptyset \ln C'_m(\emptyset) + \rho_1 \sum_s f_m(s) \ln C'_m(s)$ subject to Eq. (5)

[Wahl 1983]. ***

Lemma 3: Under assumptions A.1 to A.5, and A.6b, the equilibrium sharing rule in consumption claims is:

$$C_i(\emptyset) = \frac{1}{I} (1-d)C_m(\emptyset),$$

$$C_i(s) = \frac{1}{I} \frac{f_i(s)}{f_m(s)} [1+\beta(s)d]C_m(s) \quad (i = 1,2,\dots,I; s = 1,2,\dots,S). \quad (8)$$

Proof: Lemma 3 follows immediately from Lemma 1 and the market clearing conditions (5). ***

As before, let $\pi(s)$ denote the price of a \$1 contingent payoff in state s , and $\pi(\emptyset) \equiv 1$.

Lemma 4: Under assumptions A.1 to A.5, and A.6b, the equilibrium state-contingent price is:

$$\pi(s) = f_m(s) \frac{\rho_1 (1-d)C_m(\emptyset)}{\rho_\emptyset [1+\beta(s)d]C_m(s)} \quad (s = 1,2,\dots,S). \quad (9)$$

Proof: An equilibrium must exhibit

$$\begin{aligned} \frac{\pi(s)}{\pi(\emptyset)} &= \frac{\partial E[\tilde{U}]}{\partial C(s)} \frac{\partial C(\emptyset)}{\partial E[\tilde{U}]} \\ &= \frac{\rho_1 f(s)C(s)^{-1}}{\rho_\emptyset C(\emptyset)^{-1}} \quad \text{for every investor. Substi-} \end{aligned}$$

tuting in (8) gives (9). ***

With exogenously given aggregate consumption, i.e., $d = 0$, and homogeneous beliefs, i.e., $f_m(s) = f_i(s)$ for all i , Lemma 4 implies the well-known result:

$$\pi(s) = f(s) \frac{\rho_1 C_m(\emptyset)}{\rho_\emptyset C_m(s)} \quad (s = 1,2,\dots,S).$$

To obtain "improved" probability beliefs every investor may buy information in the information market. Let e denote a specific information event and E be the countable set of mutually exclusive information events ($e = 1,2,\dots,E$).

Buying information means that the investor pays a price $p \geq 0$ for a draw from this set. That is, the investor does not know for sure what the content of the "news" he receives will be.

A.7. Every investor knows the likelihood of observing information event e if state s were to occur, $f(e|s)$ ($e = 1, 2, \dots, E; s = 1, 2, \dots, S$).

Using A.4 and A.7 and Bayes' rule, it follows that every investor knows his posterior probability belief, $f(s|e) \propto f(e|s)f(s)$ for all e and s .

New information will in general change state-contingent prices in equilibrium. This implies that the investor's initial wealth $W(\emptyset)$ will depend on the information event that occurs, denoted by $W(\emptyset|e)$ ($e = 1, 2, \dots, E$). The information price p causes a utility loss, K , because acquiring the information changes the investor's wealth to $W(\emptyset|e) - p > 0$.

Lemma 5: $K = -\sum_e f(e) \ln(1 - \frac{p}{W(\emptyset|e)}) \geq 0.$ (10)

Proof: Under assumptions A.1 to A.4 and A.7, $\lambda(e) = 1/W(\emptyset|e)$, where $\lambda(e)$ denotes the Lagrangean multiplier if event e occurs, and the conditional marginal utility loss by information acquisition is $p\lambda(e) = p/W(\emptyset|e)$. The latter expression is the linear approximation of $-\ln[1-p/W(\emptyset|e)]$, if $0 \leq p < W(\emptyset|e)$ for all e . Ex ante, one must take the expectation to adjust for all possible information events, whereby $f(e) = \sum_s f(e|s)f(s)$. ***

2.2. The evaluation of new information

We follow the tradition in decision theory and evaluate information acquisition by the corresponding ex ante change in expected utility, Δ .

Proposition 1: Under assumptions A.1 to A.5, A.6b and A.7,

$$\Delta = F + P - K, \tag{11}$$

where $F \equiv \rho_1 \sum_e f(e) \sum_s f(s|e) \ln \frac{f(s|e)/f_m(s|e)}{f(s)/f_m(s)}$,

which we will call the forecast effect,

$$P \equiv \rho_{\emptyset} \sum_e f(e) \ln \frac{1-d(e)}{1-d} + \rho_1 \sum_e f(e) \sum_s f(s|e) \frac{1+\beta(s)d(e)}{1+\beta(s)d},$$

which we will call the production effect, and

$$K \equiv - \sum_e f(e) \ln \left(1 - \frac{P}{\rho_{\emptyset}^{-1} [1-d(e)] \bar{C}(\emptyset)} \right) \geq 0,$$

which we will call the information cost.

Proof: a). First note that initial endowments and therefore (preliminary) aggregate consumption does not depend on information acquisition.

b). Also note that in the new-information equilibrium the prices of state-contingent claims, $\pi(s|e)$, are independent of the information price p for all s and e [Wahl 1983, pp. 133 and 157]. This is a consequence of the fact that with logarithmic utility, the information expense does not affect the marginal rate of substitution of optimal aggregate consumption between all pairs of states. Then $W(\emptyset|e) = C(\emptyset|e) + \sum_s \pi(s|e)C(s|e)$

$$\begin{aligned} &= (\rho_{\emptyset} I)^{-1} [1-d(e)] C_m(\emptyset) & (12) \\ &= \rho_{\emptyset}^{-1} [1-d(e)] \bar{C}(\emptyset) \end{aligned}$$

by using Eqs. (3), (8) and (9), properly adjusted for information event e .

c). Using Eqs. (2), (8), (10) and (12), it follows ex ante that

$$\begin{aligned} \Delta &\equiv \rho_{\emptyset} \sum_e f(e) \ln \frac{C(\emptyset|e)}{C(\emptyset)} + \rho_1 \sum_e f(e) \sum_s f(s|e) \ln \frac{C(s|e)}{C(s)} \\ &\quad + \sum_e f(e) \ln \left(1 - \frac{P}{W(\emptyset|e)} \right) \\ &= \rho_{\emptyset} \sum_e f(e) \ln \frac{1-d(e)}{1-d} \\ &\quad + \rho_1 \sum_e f(e) \sum_s f(s|e) \ln \left(\frac{f(s|e)/f_m(s|e)}{f(s)/f_m(s)} \cdot \frac{1+\beta(s)d(e)}{1+\beta(s)d} \right) \\ &\quad + \sum_e f(e) \ln \left(1 - \frac{P}{\rho_{\emptyset}^{-1} [1-d(e)] \bar{C}(\emptyset)} \right). \quad *** \end{aligned}$$

Proposition 1 is the key to this article. Different assumptions will specify the sign of Δ by specifying the signs of the production and forecast effects. The information cost is nonnegative, and zero if and only if the information price is zero.

2.3 A definition of a stable information acquisition decision

We define the informed investor as one who trades using posterior beliefs after having acquired information. The uninformed investor is one who does not acquire information and therefore trades using prior beliefs. The ex ante change in an investor's expected utility from acquiring information depends on whether other investors do or do not decide to buy information. There will also be an ex ante change in the expected utility of an uninformed investor if some other investor should buy information.

Let $\Delta_{a|\bar{a}}$ denote the ex ante change in expected utility of each of those investors (insiders) who acquire information, subject to the condition that some or all others do not. Similarly, $\Delta_{\bar{a}|a}$ is the ex ante change in the expected utility of individual outsiders, those who do not acquire information, given that there are investors who do. Thus informed investors have made the decision "a", and uninformed investors the decision " \bar{a} ". Note that all insiders are informed, but informed investors are not necessarily insiders. When all investors acquire (do not acquire) there are no insiders (outsiders).

Definition: A stable acquisition decision is defined by the following vector ranking:

$$\begin{pmatrix} \Delta_{a|\bar{a}} \\ \Delta_{\bar{a}|a} \end{pmatrix} \succeq \begin{pmatrix} \Delta_{\bar{a}|\bar{a}} \\ \Delta_{\bar{a}|a} \end{pmatrix} \quad (13)$$

in which " \succeq " means that at least one inequality holds. Note that $\Delta_{\bar{a}|\bar{a}} \equiv 0$ because when no investor acquires information there is no change from the benchmark of the market equilibrium under prior probability beliefs.

The definition simply means that the decision to acquire dominates the decision not to acquire. Therefore the decision "a" is stable [see Cornell and Roll 1981, p. 202, and the references cited therein].

Each individual for whom the ranking in (13) holds will surely acquire because he will always be better off no matter what the other investors do. That is, each investor's decision is independent of the magnitude of the probability that others acquire. As we shall demonstrate, the standard assumption set we have used implies a stable acquisition decision for all investors.

3.0 The information acquisition decision

In order to simplify the paper, we make the following common assumption:

A.8. All investors share the same information at time \emptyset , and all are identical in their information processing abilities.

Assumption A.8 yields two implications. First, it gives the traditional assumption of homogeneous prior beliefs, i.e., $f_i(s) = f_m(s)$ for all s and i . Every individual's forecast is identical with the market forecast before any individual acquires new information. Second, A.8 implies that for homogeneous posterior beliefs to obtain, i.e., $f_i(s|e) = f_m(s|e)$ for all i , s , and e , it is necessary and sufficient that all individuals acquire the information. If only some investors are informed, A.8 implies

Lemma 6: Let $\alpha \equiv I_a/I$ with I_a being the number of acquiring investors [$\alpha \in (0,1)$]. Then the probabilities fit one of the following inequalities:

$$f(s|e) > f_m(s|e) > f(s)$$

or

(14)

$$f(s|e) < f_m(s|e) < f(s) \quad (s = 1,2,\dots,S; e = 1,2,\dots,E).$$

Proof: Eq. (4) shows $f_m(s|e) \equiv \frac{1}{I} \sum_i f_i(s|e) = \alpha f(s|e) + (1-\alpha)f(s)$ under A.8. But with $\alpha \in (0,1)$, $f_m(s|e)$ must be between $f(s|e)$ and $f(s)$ for all s and e . ***

While Lemma 6 requires the absence of any price signal to outsiders, the fact that the acquisition decision precedes trading guarantees this absence. The Lemma also shows that if not all investors are informed, the informed will never push the market prices of the state-contingent claims to the levels they would reach if all investors were informed. Those levels are unreachable because investors are risk-averse and limit their portfolio changes.

Finally, we introduce an assumption often implicit in other models. We make it explicit because doing so leads to interesting results, especially when we remove it.

A.9. All investors must go to the securities market and enter their reservation prices based on their information.

This assumption is irrelevant in all but one of our cases, but in that one it is crucial. However, noticing where it binds and the other assumptions on investors' endowments with which one can replace it provides some insights.

In the sections below we discuss the implications of the model under several situations given by 1) the presence or absence of production, 2) the presence or absence of cooperation, and 3) the presence of costless ($p=0$) or costly ($p>0$) information.

3.1 Without production and without cooperation

We begin by considering the situation when new information is costless.

Proposition 2: Assume A.1 to A.5, A.6a, A.7, A.8, and $p = 0$.

With exogenous aggregate consumption and costless information acquisition, the ex ante change in an investor's expected utility is

- a) positive (negative) for insiders (outsiders), i.e.,

$$\Delta_{a|\bar{a}} > 0 \quad (\Delta_{\bar{a}|a} < 0),$$

- b) greater for being one of the informed when all are informed (homogeneous posterior beliefs) than for being an outsider, i.e., $\Delta_{a|a} > \Delta_{\bar{a}|a}$.

Proof: No production and costless information imply $P = 0$ because $d = d(e) = 0$ for all e , and $K = 0$ because $p = 0$. Then using Eq. (11) with assumption A.8, it follows that

$$a.1) \Delta_a|\bar{a} = F_{a|\bar{a}} \equiv \rho_1 \sum_e f(e) \sum_s f(s|e) \ln \frac{f(s|e)}{f_m(s|e)} > 0$$

by Jensen's inequality. $\Delta_a|\bar{a}$ is strictly positive because of Eq. (14).

$$a.2) \Delta_{\bar{a}}|a = F_{\bar{a}|a} \equiv \rho_1 \sum_e f(e) \sum_s f(s|e) \ln \frac{f(s)}{f_m(s|e)}$$

$$= -\rho_1 \sum_e f(e) \{ \sum_s [f(s|e) - f_m(s|e)] \ln \frac{f_m(s|e)}{f(s)} + \sum_s f_m(s|e) \ln \frac{f_m(s|e)}{f(s)} \} < 0$$

because the first summand in the brackets is positive by Eq. (14), and the second is also positive by Jensen's inequality;

b) $\Delta_a|a = F_{a|a} \equiv \rho_1 \ln 1 = 0$ because $f_i(s|e) = f_m(s|e)$ for all i, e, s . *** With Proposition 2, Eq. (13) holds. This immediately results in

Theorem 1: Under the assumptions of Proposition 2, the decision to acquire new information is stable. Every investor has an incentive to acquire new information.

To illuminate Theorem 1, let us examine the decisions of investors j and k , both having to decide independently whether or not to acquire new information. The signs of the ex ante changes in expected utilities are shown in the following table. For instance, (+,-) represents the pair of the signs of $\Delta_a|\bar{a}$ for j and $\Delta_{\bar{a}}|a$ for k .

Table 1

j \ k	a	\bar{a}
a	(0,0)	(+,-)
\bar{a}	(-,+)	(0,0)

Clearly the decision to acquire dominates that of not acquiring information and all will acquire. Note that if all acquire, none will have an advantage. Thus, if it were not for the possibility that some investors might not acquire, the result would not be that all would acquire. Instead, all investors would be indifferent between acquiring and not acquiring. However, the consequences for any investor j in the event that some investor k might not acquire information when j does, produces the requisite incentive. This result is independent of the probability, from the viewpoint of investor j , that any other investors may or may not acquire.

Suppose now that the price of new information is strictly positive ($p > 0$), because the information is costly to produce.

Let p^0 and p^* be break-even prices of new information such that $\Delta_{a|\bar{a}}(p=p^0) = 0$ and $\Delta_{a|a}(p=p^*) = \Delta_{\bar{a}|a}(p=p^*)$. Then we have

Lemma 7: Assume the assumption set of Proposition 2. Then

- a) $\Delta_{a|\bar{a}} \geq 0$ if and only if $p \leq p^0 \equiv \rho_{\emptyset}^{-1} \bar{C}(\emptyset) (1 - e^{-F_{a|\bar{a}}})$;
 b) $\Delta_{a|a} \geq \Delta_{\bar{a}|a}$ if and only if $p \leq p^* \equiv \rho_{\emptyset}^{-1} \bar{C}(\emptyset) (1 - e^{F_{\bar{a}|a}})$. (15)

Proof: The results follow from Eq. (11) taking into account that without production $d = d(e) = 0$ for all e and $\Delta_{a|\bar{a}} = F_{a|\bar{a}} - K(p=p^0)$ for a) above, and $\Delta_{a|a} = -K(p=p^*) = F_{\bar{a}|a} = \Delta_{\bar{a}|a}(p=p^*)$ for b). ***

Lemma 8: Let $c > 0$ be the marginal cost of producing the information. Then with A.2, it follows:

$$0 < c \leq pI_a \text{ implies } p \leq \text{Min}\{p^0, p^* | p^0 \neq p^*\}. \quad (16)$$

Proof: Suppose that either $p^0 < p^*$ and $p > p^0$ or that $p^* < p^0$ and $p > p^*$. In both cases the price p may result in no investors acquiring information. If $p \geq \text{Max}\{p^0, p^* | p^0 \neq p^*\}$, then clearly no investor will acquire information;

if p is between p^0 and p^* , investors will not know what to do ex ante and so it is possible that none will acquire. Therefore, because the information seller faces a risk that he may not recover his production cost, he will not produce the information. This contradicts $p > 0$. ***

We are now ready to examine the effect of costly information on the investor's acquisition decision.

Proposition 3: Assume A.1 to A.5, A.6a, and A.7 to A.9. With exogenously given aggregate consumption and a positive information price, the ex ante change in an investor's expected utility is

- a) nonnegative for insiders, i.e., $\Delta_a | \bar{a} \geq 0$,
- b) not smaller for being one of the informed when all are informed than for being an outsider, i.e., $\Delta_a | a \geq \Delta_{\bar{a}} | a$.

Proof: Combining Lemmata 7 and 8 [Eqs. (15) and (16)], one can deduce that if $p \leq \text{Min}\{p^0, p^* | p^0 \neq p^*\}$, this implies that $\{p \leq p^0 < p^*\}$ or $\{p \leq p^* < p^0\}$.

This in turn implies that either

- $\{\Delta_a | \bar{a} \geq 0 \text{ and } \Delta_a | a > \Delta_{\bar{a}} | a\}$, or
- $\{\Delta_a | \bar{a} > 0 \text{ and } \Delta_a | a \geq \Delta_{\bar{a}} | a\}$. ***

With Proposition 3, Eq. (13) holds. This immediately results in

Theorem 2: Under the assumptions of Proposition 3, the decision to acquire new information is stable.

Theorem 2 shows that utility maximizing investors have an incentive to acquire new information, even when each of them will bear a utility loss ex ante. Note that $\Delta_a | a = -K < 0$.

The problem facing each investor is how to minimize his losses. The underlying assumption set and Lemma 8 imply that for the information market to exist with certainty it is necessary for the information seller to set a price such that for every investor it is less expensive to lose wealth to the information seller than to insiders because of the decision to remain an

outsider. One can infer that in equilibrium information would not be produced for which the production cost would result in an information cost exceeding the expected utility loss to an investor from being an outsider or the gain from being an insider, whichever is less.

This result depends crucially on Assumption 9, that investors must go to the securities market. If we remove that assumption, then no investor would buy new information and all investors would refuse to go market and simply stay with their endowments. This points out the importance for equilibrium models of realizing that when one assumes that markets exist, one is assuming that participants have some reason to go to market, or at least not to avoid it. Thus a necessary condition for the existence of the information market is the existence of a securities market.

Though removing Assumption 9, we can provide for the existence of a securities market by imposing the following conditions on investors' endowments.

A.5'. Investors can be separated into two groups such that all investors in the same group have identical endowments. These endowments are deterministic but intertemporally different, and meet the two sets of conditions:

$$\rho_0 \bar{q}_{j0} + \rho_1 \bar{q}_{j1} = \rho_0 \bar{q}_{k0} + \rho_1 \bar{q}_{k1} = \frac{1}{I}, \text{ and}$$

$$\frac{\rho_0}{\bar{q}_{j0}} \frac{\rho_1}{\bar{q}_{j1}} = \frac{\rho_0}{\bar{q}_{k0}} \frac{\rho_1}{\bar{q}_{k1}}$$

where $\bar{q}_{jt} \in (0,1)$ and $\bar{q}_{kt} \in (0,1)$ are group representative investor j's and k's endowments as fractions of state contingent aggregate consumption for time t ($t = 0,1$).

The first set of conditions gives investors an incentive to trade, and means that in equilibrium all investors will hold identical fractions of aggregate consumption. Thus we ensure that all investors will have an incentive to go to the securities market and will not differ in wealth.

The second condition results in the price equivalent of the utility of trade in securities being the same for all investors. The two conditions together assure that all investors will make the same decision when faced with the same information price.

Note that one of the conditions in the first set in A.5' is redundant because the weighted sum of representative endowments is 1, the weights being the number of investors in each group. Furthermore, the economically meaningful solutions of the equations restrict the region of the parameters ρ_t .

Lemma 8': Let $c > 0$ be the marginal cost of producing the information. Then with A.2 and without A.9, it follows:

$$0 < c \leq pI_a \text{ implies } p \leq \text{Min}\{p^+, \text{Min}(p^0, p^* | p^0 \neq p^*)\}$$

where p^+ is the price equivalent of the utility of trading.

Proof: a) Using the conditions of A.5' and the logarithmic utility function (1), investor i 's utility gain from trade becomes $\ln(\frac{1}{I}) - (\rho_0 \ln \bar{q}_{j0} + \rho_1 \ln \bar{q}_{j1})$ for $\bar{q}_{it} \in (0,1)$. This difference is strictly positive due to the relationship between the arithmetic and geometric means of nonnegative real numbers. Define $p_i^+ = \frac{1}{I} \sqrt[\rho_0]{\bar{q}_{i0}^{\rho_0}} \sqrt[\rho_1]{\bar{q}_{i1}^{\rho_1}}$, and it also follows that $p_i^+ = p_1^+$ for all i .

b) Suppose that $p^+ < p \leq \text{Min}\{p^0, p^* | p^0 \neq p^*\}$.

Then all investors would prefer to refuse to trade, and would not purchase the information. This contradicts $p > 0$. If $p^+ \geq \text{Min}\{p^0, p^* | p^0 \neq p^*\}$ then

Lemma 8 applies. ***

We are now ready for a revised Proposition 3 and Theorem 2.

Proposition 3': Assume A.1 to A.5', A.6a, A.7 and A.8. With exogenously given aggregate consumption, investors having a utility gain from trade, and a positive information price, the ex ante change in an investor's utility is

a) nonnegative for insiders, i.e., $\Delta_a | \bar{a} \geq 0$, and

b) not smaller for being one of the informed when all are informed than for being an outsider, i.e., $\Delta_a|a \geq \Delta_{\bar{a}}|a$.

Proof: $p \leq \text{Min}\{p^+, \text{Min}(p^0, p^* | p^0 \neq p^*)\}$ implies adding and subtracting a positive constant to Δ . Hence the proof of Proposition 3 holds.

With Proposition 3', Eq. 13 holds. This gives us

Theorem 2': Under the assumptions of Proposition 3', the decision to acquire new information is stable.

Theorem 2' shows that utility maximizing investors have an incentive to acquire new information even when all would be better off by not acquiring it if the utility gain from trading is at least as great as the utility loss from acquisition. That is, the investor will acquire new information when this gain from going to market is at least as great as the cost of insuring that he is (not) at an informational (dis)advantage vis-à-vis other investors. In equilibrium then, information would not be produced for which the production cost would result in an information cost exceeding the least of the utility of trading in securities to achieve an optimal portfolio, the expected utility loss to being an outsider, or the gain from being an insider.

We now drop assumption A.5' and restore A.5, i.e., identical initial endowments. For the remainder of the paper the requirement that investors must go to the securities market is unnecessary.

3.2 Without production but with cooperation

In the absence of cooperation, every investor is driven to acquire new information by the desire to avoid the utility loss caused by redistributing consumption claims, even with costly information and gains from trading. With cooperation, investors can avoid the utility loss. That is, they can make agreements before acquiring information to undo any redistribution of consumption claims which would result from trading after acquisition. Because with

cooperation there is no cost (gain) in being an outsider (insider), investors would be unwilling to pay anything for information. Hence, no strictly positive information price is sustainable in equilibrium. A strictly positive information price would result in the decision not to acquire information dominating the decision to acquire it. Therefore, we have

Theorem 3: In the absence of production and with costly information, the information market can only exist if cooperation too is costly.

Information which cannot change aggregate consumption, for instance by affecting its intertemporal allocation, but which serves only to redistribute consumption claims among investors in each state of nature will command a zero price when investors can cooperate costlessly. This is consistent with the position in Hirshleifer [1971]. In the next sections we turn to the question of the impact of the existence of production, i.e., utility gains from intertemporal allocation of aggregate consumption, on the maximum information price and the acquisition decision.

3.3 With production and without cooperation

Suppose $p = 0$, i.e., new information is costless. The following lemma gives the sign of the production effect, P .

Lemma 9: Let $P(\cdot)$ represent both cases, $P_{a|a}$ and $P_{\bar{a}|a}$. Then $P(\cdot) > 0$. (17)

Proof: Define $\hat{P} \equiv \rho_0 \ln(1-d) + \rho_1 \sum_s f(s) \ln[1+\beta(s)d]$, with d being the solution of Eq. (7) (see Lemma 2). Because $\left. \frac{\partial^2 Z}{\partial d^2} \right|_d < 0$, $\hat{P} = Z^{\max}$.

\hat{P} is convex in the prior probability beliefs $f(s)$ ($s = 1, 2, \dots, S$), i.e.,

$$d^2 \hat{P} = \sum_{q=1}^S \sum_{r=1}^S \frac{\partial^2 \hat{P}}{\partial f(q) \partial f(r)} df(q) df(r) > 0.$$

The calculation is simplified by following a two-step procedure. First, calculate $d^2 \hat{P}$ with respect to d and $f(s)$ ($s = 1, 2, \dots, S$), ignoring for the moment the fact that d is a function of the probabilities. Second, calculate the

implicit derivative of Eq. (7), $\frac{d\hat{p}}{dd}$, and insert this in the result of the first step. Then

$$d^2\hat{p} = \rho_1^2 \frac{[\sum_s df(s) \frac{\beta(s)}{1+\beta(s)d}]^2}{\frac{\rho_0}{(1-d)^2} + \rho_1 \sum_s f(s) [\frac{\beta(s)}{1+\beta(s)d}]^2} > 0. \quad (18)$$

Because $\sum_e f(e)f(s|e) = f(s)$ and by using the definition of the market forecast in Lemma 6, $\sum_e f(e)f_m(s|e) = \sum_e f(e)[\alpha f(s|e) + (1-\alpha)f(s)] = f(s)$ ($s = 1, 2, \dots, S$), the convexity of \hat{P} implies

$$P_{a|a} \equiv \sum_e f(e) \hat{P}_{a|a}(e) - \hat{P}_{a|a} > 0, \text{ and}$$

$$P_{\bar{a}|a} \equiv \sum_e f(e) \hat{P}_{\bar{a}|a}(e) - \hat{P}_{\bar{a}|a} > 0. \quad ***$$

This leads to

$$\underline{\text{Lemma 10:}} \quad P_{a|a} > P_{\bar{a}|a} = P_{a|\bar{a}}. \quad (19)$$

Proof: Combine the convexity of \hat{P} in the prior market forecast in Lemma 9 with the relative "spreading" of the posterior market forecast in Lemma 6. Then the inequality follows because $\sum_e f(e)f(s|e) = \sum_e f(e)f_m(s|e) = f(s)$, for all s . The equality follows because the market forecast is the same for insiders and outsiders. ***

Lemma 10 reveals that the greater the number of investors who are informed, the greater is the ex ante utility gain from production for both insiders and outsiders. The individual production effect is greatest with homogeneous posterior beliefs, for then the change in the prices of the state-contingent claims is the greatest for every information event.

Proposition 4: Assume A.1 to A.5, A.6b, A.7 and A.8. With endogenous aggregate consumption as defined in A.6b and costless information acquisition, the ex ante change in an investor's expected utility is

- a) positive for insiders, i.e., $\Delta_{a|\bar{a}} > 0$, and
- b) greater for being one of the informed when all are informed than for being an outsider, i.e., $\Delta_{a|a} > \Delta_{\bar{a}|a}$.

Proof: By using Eq. (11) with $p = 0$, it follows that

- a) $\Delta_{a|\bar{a}} = F_{a|\bar{a}} + P_{a|\bar{a}} > 0$ because $F_{a|\bar{a}} > 0$ from Proposition 2, and $P_{a|\bar{a}} = P_{\bar{a}|a} > 0$ from Lemmata 9 and 10; and
- b) $\Delta_{a|a} - \Delta_{\bar{a}|a} = F_{a|a} + P_{a|a} - (F_{\bar{a}|a} + P_{\bar{a}|a}) > 0$ because $F_{a|a} = 0$, $F_{\bar{a}|a} < 0$ from Proposition 2, and $P_{a|a} - P_{\bar{a}|a} > 0$ from Lemma 10.***

With Proposition 4, Eq. (13) holds. This then gives

Theorem 4: Under the assumptions of Proposition 4, the decision to acquire new information is stable.

With a positive price for new information, the information cost in general will depend on all information events and the market forecast because these influence the coefficient of production change. Looking at the break-even information prices for being an insider or not trading as an outsider, p^0 and p^* respectively, such that $\Delta_{a|\bar{a}}(p=p^0) = 0$ and $\Delta_{a|a}(p=p^*) = \Delta_{\bar{a}|a}(p=p^*)$, Lemma 11 follows from Eq. (11).

Lemma 11: Given the assumption set of Proposition 4, then

- a) $\Delta_{a|\bar{a}} \geq 0$ if and only if $p \leq p^0$ with p^0 being determined by
$$F_{a|\bar{a}} + P_{a|\bar{a}} = -\sum_e f(e) \ln \left(1 - \frac{p^0}{\rho_\emptyset^{-1} [1 - d_{a|\bar{a}}(e)] \bar{C}(\emptyset)} \right);$$
- b) $\Delta_{a|a} \geq \Delta_{\bar{a}|a}$ if and only if $p \leq p^*$, with p^* being determined by
$$-F_{\bar{a}|a} + P_{a|a} - P_{a|\bar{a}} = -\sum_e f(e) \ln \left(1 - \frac{p^*}{\rho_\emptyset^{-1} [1 - d_{a|a}(e)] \bar{C}(\emptyset)} \right).$$

Even with endogenous aggregate consumption, the upper bound for the information price given in Lemma 8 has to hold, for this is a necessary condition for an equilibrium. Then it follows

Proposition 5: Assume A.1 to A.5, A.6b, A.7 and A.8. With endogenous aggregate consumption and a positive information price, the ex ante change in an investor's expected utility is

- a) nonnegative for insiders, i.e., $\Delta_a|_{\bar{a}} \geq 0$,
- b) not smaller for being one of the informed when all are informed than for being an outsider, i.e., $\Delta_a|_a \geq \Delta_{\bar{a}}|_a$.

Proof: Combine Lemmata 11 and 8, and use the logic of the proof of Proposition 3. ***

With Proposition 5 Eq. (13) holds. This yields

Theorem 5: Under the assumptions of Proposition 5 the decision to acquire new information is stable.

From Theorems 2, 2' and 5 it is clear that if investors cannot cooperate, all will acquire the information, whether or not production decisions are possible. The opportunity to optimize market consumption intertemporally may change the price the information market can quote for information, but this does not affect the investors' decision to acquire. The existence of production shifts the bounds on the information price, leaving the decision unchanged.

3.4 With production and cooperation

In Section 3.2 above we showed that investors had an incentive to cooperate if the cost of cooperation was zero because by so doing they could drive down the price the information seller could charge. However, in the absence of production the upper bound on the information price was zero.

So far, cooperation has only involved undoing any redistribution of claims arising from the forecast effect. With endogenous aggregate consumption a further aspect of cooperation arises, namely that investors may agree not to act as free riders. We deal with each of these aspects in turn.

3.4.1 Cooperation as a no redistribution contract only

Let us introduce production and omit (for the moment) the information cost. Outsiders will only cooperate when the cooperation contract requires that the undoing of the redistribution of consumption claims (induced by the forecast effect), take place. Then, if this cooperation contract is possible:

$$\begin{pmatrix} \Delta_{a|\bar{a}} \\ \Delta_{\bar{a}|a} \end{pmatrix} = \begin{pmatrix} P_{a|\bar{a}} \\ P_{\bar{a}|a} \end{pmatrix} \geq \begin{pmatrix} 0 \\ P_{\bar{a}|a} \end{pmatrix} = \begin{pmatrix} 0 \\ \Delta_{\bar{a}|a} \end{pmatrix}. \quad (20)$$

Therefore, every investor has an incentive, before information costs, to acquire the information no matter whether other investors acquire or not, as long as redistribution will be undone.

Now suppose that the information market wishes to charge a price equal to the individual's expected "information gain", i.e., $p = p'$ with p' being the solution of Eq. (21):

$$-\sum_e f(e) \ln \left(1 - \frac{p'}{\rho_{\emptyset}^{-1} [1 - d_{a|a}(e)] \bar{C}(\emptyset)} \right) = P_{a|a}. \quad (21)$$

An information price of p' is impossible. Let us divide the investors into two groups, j and k , and look at a representative investor in each group. Table 2 shows the signs of each investor's ex ante change in expected utility if $p = p' > 0$.

Table 2

	k		
j		a	\bar{a}
	a	(0,0)	(-,+)
	\bar{a}	(+,-)	(0,0)

This implies that nonacquisition of the information dominates acquisition. Thus with a cooperation contract that requires only that redistribution

be undone, $p = p'$ cannot be an equilibrium price because no investor will buy at that price.

Lemma 12: Given the assumption set of Proposition 5 and costless cooperation with respect to the redistribution of claims. Then

a) $\Delta_{a|\bar{a}} \geq 0$ if and only if $p \leq p^0$ being the solution of

$$-\sum_e f(e) \ln \left(1 - \frac{p^0}{\rho_{\emptyset}^{-1} [1 - d_{a|\bar{a}}(e)] \bar{C}(\emptyset)} \right) = P_{a|\bar{a}} ;$$

b) $\Delta_{a|a} \geq \Delta_{\bar{a}|a}$ if and only if $p \leq p^*$, being the solution of

$$-\sum_e f(e) \ln \left(1 - \frac{p^*}{\rho_{\emptyset}^{-1} [1 - d_{a|a}(e)] \bar{C}(\emptyset)} \right) = P_{a|a} - P_{\bar{a}|a} .$$

Theorem 6: Under the assumption set of Proposition 5 and costless cooperation the decision to acquire new information is stable.

Proof: From Lemmata 9 and 10 we have that $P_{a|\bar{a}}$ and $(P_{a|a} - P_{\bar{a}|a})$ are positive. This implies a positive p^0 and p^* . Then:

a). If $p = 0$, then with Lemma 12, Eq. (13) must hold.

b). If $p > 0$, then Lemma 8 must hold. Combining Lemmata 8 and 12, Eq. (13) again holds. ***

With production and a no redistribution contract, investors will decide to buy information which is costly to produce. The incentive, however, is the utility gain from production, not the desire to avoid losses to others or to gain from them. While the information market will share the gains from production, it does not exhaust them ($p < p'$). If it were to attempt to do so, every investor would have an incentive to be a free rider on the production gain which results from the acquisition of new information by others. The result would be that none would acquire.

3.4.2 Cooperation as a no redistribution and no free riding contract

The extension of the cooperation contract to no free riding makes possible an information price, and hence a shared cost of information in the interval $\text{Min}\{p^0, p^* | p^0 \neq p^*\} < p < p'$. Because investors agree to undo the distributional consequences of free riding, there is no production gain from being an outsider, nor an information cost in excess of the production gain to being an insider. Thus Table 2 converges to Table 3 below, where clearly acquisition dominates nonacquisition.

Table 3

j \ k	a	\bar{a}
a	(+,+)	(0,0)
\bar{a}	(0,0)	(0,0)

Note, agreeing to undo any redistribution of claims after trade is logically equivalent to agreeing not to free ride before trade.

Lemma 13: Given the assumption set of Proposition 5 and costless cooperation to exclude redistribution of claims and free riding. Then

$$\{\Delta_a | \bar{a} = 0 \text{ and } \Delta_a | a > \Delta_{\bar{a}} | a\} \text{ if and only if } p < p'.$$

Lemma 14: $0 < c \leq pI_a$ implies $p < p'$.

The proofs of Lemmata 13 and 14 are obvious. It follows that Theorem 6 remains unchanged: the decision to acquire is stable. The only change which has occurred is that now in the upper-bound on the information price is p' instead of the lower one from Lemma 8. Therefore, the contractual exclusion of free riding enables the information seller to produce information which would otherwise not be acquired.

4.0 Summary and conclusion

In this paper we have developed a model with an information market, and investors who are homogeneous in their endowments, have logarithmic utility functions, and use Bayes' rule in updating their prior probability beliefs after acquiring new information. We employ a one-period model with information acquisition being possible only before trading in securities occurs.

We examine the implications of the model under a variety of scenarios organized by exogenously or endogenously given aggregate consumption, costless or costly information, and with or without the possibility of cooperation. In all cases, as long as all investors would not be better off refusing to go to the securities market, all will acquire information, provided the price the information market charges is below endogenously determined upper bounds. In general, these bounds are positive. Thus, a necessary condition for the existence of an information market, the presence of a demand for costly information, is satisfied.

Because all investors acquire information and have the same information processing abilities, when they begin with homogeneous prior probability beliefs they also end with homogeneous posterior beliefs. Therefore, when trading in claims opens, while trade may occur for other, unspecified reasons, it will not be a consequence of heterogeneous beliefs.

One of the major implications of the model is that when information is costly and aggregate consumption is given exogeneously, if investors cannot cooperate to undo the redistribution of claims which would result from trading with heterogeneous posterior beliefs, all will acquire the information. This is despite the fact that all would be better off if no one acquired it. In essence, all investors will prefer to lose wealth to the information market when buying new information is less costly in terms of utility loss than being

uninformed and losing in trade with informed investors, or not trading at all. This result is consistent with the existence of brokerage houses and other securities research sellers. These continue to sell their information to investors despite the fact that each purchaser, knowing that a large number of other investors have also purchased the same information, does not expect any informational advantage.

This situation creates an incentive for investors to cooperate. If they can do so, the model implies that they will be unwilling to pay anything for information which could serve only to redistribute consumption claims on the existing, exogeneously given aggregate consumption. We thus prove one of the points Hirshleifer [1971] makes in his example. Our result is also consistent with the evidence in Ball, Torous, and Tschoegl [1985]. These authors argue that the common stockmarket convention of quoting prices in eighths is a way for market participants cooperatively to avoid investing in very precisely determining prices because the expected utility gain of increased precision is zero before information costs. Furthermore, our results suggest the interpretation of mandatory disclosure laws as a form of social cooperation in reducing expenditure on information production (Diamond 1985).

When aggregate consumption is determined endogeneously, the model further implies that even in the presence of costless cooperation, a positive price for new information is possible. Thus the information market could exist, depending on the cost of producing information. However, the maximum price it can charge is such that even though all investors acquire information at that price, the ex ante change in their utility is positive. Thus the information market shares the gains from intertemporal optimization of aggregate consumption, but does not exhaust them.

Future extensions of the model might take several directions. One could further loosen the assumption of homogeneous endowments, or explicitly model the information sellers, and thus the supply of new information. This would further enable one to derive implications for social welfare. Finally, it would be interesting to permit heterogeneous posterior beliefs. All of these possibilities, and others, remain subjects for further research.

Bibliography

- Ball, Clifford A., Walter N. Torous, and Adrian E. Tschoegl. 1985. "The Degree of Price Resolution: The Case of the Gold Market," Journal of Futures Markets 5 (1), pp. 29-43.
- Cornell, Bradford, and Richard Roll. 1981. "Strategies for Pairwise Competitions in Markets and Organizations," Bell Journal of Economics 12, pp. 201-213.
- Diamond, Douglas W. 1985. "Optimal Release of Information by Firms," Journal of Finance (forthcoming).
- Friend, I. and Marshall E. Blume. 1975. "The Demand for Risky Assets." American Economic Review 65, pp. 900-22.
- Hakansson, Nils H., T. Gregory Kunkel, and James A. Ohlson. 1982. "Sufficient and Necessary Conditions for Information to Have Social Value in Pure Exchange," Journal of Finance 37, pp. 1169-1181.
- Hellwig, Martin F. 1982. "Rational Expectations Equilibrium with Conditioning on Past Prices: A Mean-Variance Example," Journal of Economic Theory 26, pp. 274-312.
- Hirshleifer, Jack. 1971. "The Private and Social Value of Information and the Reward to Inventive Activity," American Economic Review 17, pp. 1375-1421.
- _____, and John G. Riley. 1979. "The Analysis of Uncertainty and Information--an Expository Survey," Journal of Economic Literature 17, pp. 1375-1421.
- Grossman, Sanford J., and Joseph E. Stiglitz. 1980. "On the Impossibility of Informationally Efficient Markets," American Economic Review 70, pp. 393-408.
- Rubinstein, Mark. 1977. "The Strong Case for the Generalized Logarithmic Utility Model as the Premier Model of Financial Markets," in H. Levy and M. Sarnat, eds., Financial Decision Making Under Uncertainty (Academic Press: New York), pp. 11-62.
- Savage, Leonard J. 1972. The Foundations of Statistics, (Dover: New York).
- Wahl, Jack. 1983. Informationsbewertung and -effizienz auf dem Kapitalmarkt. (Physica-Verlag: Würzburg-Wien).