

Endres, Alfred

**Working Paper**

## Second hand markets and the private supply of excludable public goods

Diskussionsbeiträge - Serie B, No. 7

**Provided in Cooperation with:**

Department of Economics, University of Konstanz

*Suggested Citation:* Endres, Alfred (1979) : Second hand markets and the private supply of excludable public goods, Diskussionsbeiträge - Serie B, No. 7, Universität Konstanz, Fakultät für Wirtschaftswissenschaften und Statistik, Konstanz

This Version is available at:

<https://hdl.handle.net/10419/92533>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

FAKULTÄT FÜR  
WIRTSCHAFTSWISSENSCHAFTEN UND STATISTIK  
UNIVERSITÄT KONSTANZ

---

Second Hand Markets and the Private Supply  
of Excludable Public Goods

Alfred | Endres

Serie B - Nr. 7

DISKUSSIONSBEITRÄGE

D-7750 Konstanz  
Postfach 7733

Serie B:

Finanzwissenschaftliche Arbeitspapiere

Nr. 7

Second Hand Markets and the Private Supply  
of Excludable Public Goods

Alfred | Endres

Serie B - Nr. 7

10. September 1979

AG 5449 79 K  
Weltwirtschaft  
Kiel

Erscheint demnächst in Public Finance

C 90375

Serie A: Volkswirtschaftliche Beiträge

Serie B: Finanzwissenschaftliche Arbeitspapiere

Serie C: Betriebswirtschaftliche Beiträge (in Vorbereitung)

Second Hand Markets and the Private Supply of Excludable Public Goods\*

=====

by

Alfred Endres\*\*

I. Introduction

In the traditional theory of public goods it is claimed that competitive markets severely underprovide public goods. The rationale for this is that non-rivalry in consumption and the impossibility of exclusion are regarded as constituting the properties of public goods. Thus, the total quantity of a pure public good is consumed by all individuals. The utility of each consumer does not depend on how many individuals consume the good. And, it is impossible (or too costly) to exclude consumers who do not pay. Under these conditions, for each potential consumer there is an incentive to veil his preferences since if other consumers finance the provision of the public good, he can get it for nothing (a "free rider"). Therefore, it is argued, either no private markets for public goods will be established or at most, the willingness to pay, revealed in private markets will be too low to induce the optimal supply.

Recently, this consensus has been questioned.

It has been pointed out that for certain public goods exclusion costs may be negligible. The standard example for such an "excludable public good" is the screening of scrambled TV-programs for which decoders are sold<sup>1)</sup>. Non-excludability is overcome, but non-rivalry is preserved. The TV-program can be received by all consumers without consumption rivalry, but seeing the program is useful only for those who paid for a decoder. Several authors have demonstrated that costless exclusion avoids the free rider problem and therefore improves the possibility to provide public goods privately.

It is shown below, however, that the private supply of excludable public goods may provoke the establishment of "second hand markets" which under certain conditions undermine the exclusion system. Alternatively, under different conditions second

---

\* I am indebted to Holger Bonus, Heinz Holländer, J. Thomas Romans, Konrad Stahl and an anonymous referee for their helpful comments on an earlier draft of this paper.

\*\* The author is Visiting Assistant Professor of Economics at the State University of New York at Buffalo, U.S.A., on the leave from the University of Konstanz, W. Germany.

1) See, e.g. P.A. Samuelson [5] or H. Demsetz [1] and [2].  
There are a lot of other examples, especially in the field of information and communication.

hand markets can improve the ability of private markets to provide public goods. The consequences of second hand markets on the possibility of supplying public goods privately are analyzed in the framework of W.H. Oakland's model<sup>2)</sup>.

## II. The Oakland Model

Consider a perfectly competitive economy with  $N$  consumers, one public good  $X$  and one private good  $Y$ . The quantities of the goods are  $x$  and  $y$ .  $X$  is produced with constant marginal costs  $c$  and with no fixed cost. Consumption of  $X$  is completely non-rival and exclusion is costless. Each consumer is indifferent among the units of the public good and consumes each unit only once<sup>3)</sup>.

Oakland assumes that the firms have no knowledge of the preferences of specific individuals. Therefore, a firm cannot command different prices from different individuals for the same unit of the public good.

In long run equilibrium the profits in a perfectly competitive  $X$ -industry will be zero. Therefore, the price a consumer has to pay for a unit which is consumed by  $n$  individuals is

$$p = \frac{c}{n}.$$

The revenue a firm receives for such a unit is  $n \cdot p = c$ . Lower prices would induce losses. Higher prices would be undercut by competitors.

Thus, different units of the public good have different prices, depending on the number of consumers. Since consumption is non-rival and consumers are indifferent between the units of the public good, the greater the number of consumers of a specific unit, the better the situation is for each of them. The prices of the units can vary from the lowest price  $c/N$  (price of a unit consumed by all consumers) to the highest price  $c$  (price of a unit consumed by just one individual).

Consider the price  $c/N$ . Suppose the quantity demanded by the consumer with the lowest demand is  $x_1$  at this price. All other individuals want to consume more than  $x_1$  at  $c/N$ . Perfect competition would secure that the quantity  $x_1$  is provided at the price of  $c/N$  to all consumers.

---

<sup>2)</sup> See W.H. Oakland [4] and the ingenious interpretation given by J.G. Head [3].

<sup>3)</sup> In the TV-example this means, that different programs have the same value for the consumers and that no one wants to see a program twice.

The consumers who want to buy more than  $x_1$  have to pay more than  $c/N$  for the units beyond  $x_1$  because the production costs of these units are no longer divided among  $N$  consumers but among, say,  $N-1$  consumers. Suppose that the individual with the second lowest demand consumes  $(x_2 - x_1)$  additional units at the price of  $c/(N-1)$ . Competitive firms will supply this additional quantity at a price of  $c/(N-1)$  for all consumers who wish to buy at this price.

This system of step-wise price setting continues until a quantity  $x_m$  is reached at which point the (marginal) willingness to pay by the  $N-m$  remaining consumers is just sufficient to cover the price of  $c/(N-m)$ , whereas it is impossible to cover the price of the next unit  $x_{m+1}$ .

To facilitate a graphical illustration the additional assumption is made to Oakland's model that the  $N$  consumers consist of two internally homogeneous groups. Let the groups be  $I = \{i/i \in (1, \dots, r)\}$  and  $II = \{j/j \in (r + 1, \dots, N)\}$ . For each level of  $X$ , the willingness to pay of a member of group I is assumed to be higher than of a member of group II.

Now, Oakland's model can be illustrated as follows:

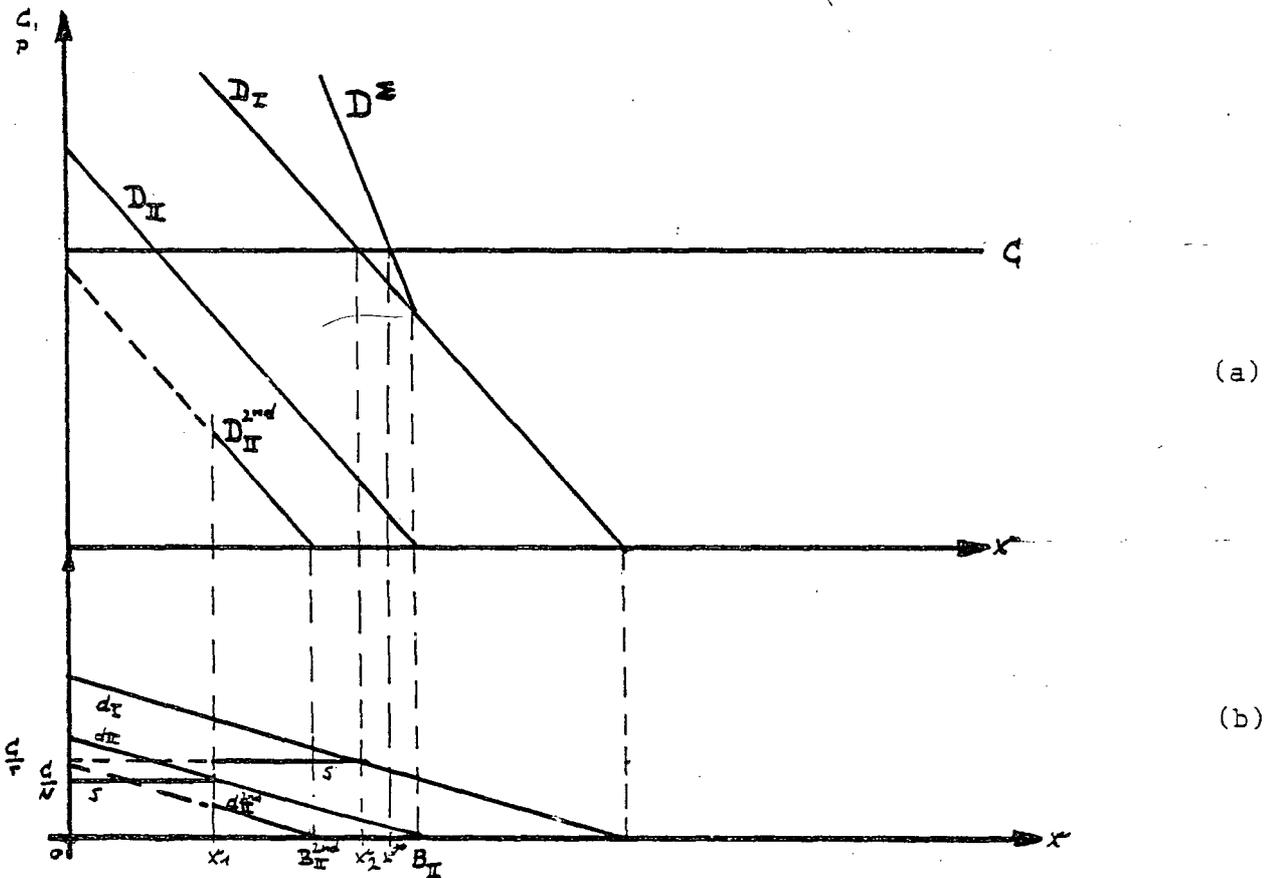


Fig. 1<sup>4)</sup>

<sup>4)</sup> See Head's Fig. VI on p. 17. [3].

Fig. 1a shows the situation of the groups as a whole. Fig. 1b refers to representative members of the groups.  $d_I(d_{II})$  is the demand curve of a member of group I (II).  $D_I(D_{II})$  is the demand function of group I (II), generated from vertical aggregation of the individual demand curves.  $D^\Sigma$  is the total demand curve for all  $N$  consumers.  $c$  is the marginal (average) production cost.  $S$  is Oaklands' step function, yielding the prices for different numbers of consumers.

Up to  $x_1$  units of  $X$ , all members of both groups consume the good. Production costs are financed by all consumers, i.e. the price of a unit is  $c/N$ .  $x_1$  is the equilibrium consumption for each member of group II, i.e.  $c/N = d_{II}(x_1)$  holds. From  $x_1$  on, the members of group II drop out. Since units beyond  $x_1$  have to be financed by the dues of the remaining  $r$  members of group I, price rises to  $c/r$  for all additional units. The members of group I demand  $(x_2 - x_1)$  additional units at that price. At  $x_2$ ,  $c/r = d_I(x)$  or (since  $D_I/r = d_I$ )  $c = D_I(x)$  holds.  $x_2$  is Oakland's equilibrium output. Units beyond  $x_2$  cannot be financed<sup>5)</sup>

The results of the Oakland model are:

- In the competitive equilibrium,  $x_2$  units of the public good are produced.  $(N - r)$  consumers are excluded from  $(x_2 - x_1)$  units of the good.
- The pareto optimal quantity is  $x^* (> x_2)$ , where the sum of all persons marginal willingness to pay equals marginal cost. Due to non-rivalry in consumption, pareto optimal intensity of use is achieved when no consumer is excluded from the consumption of any unit produced.

Thus, competitive supply of public goods generally leads to underprovision and under-utilization, even though exclusion is costless<sup>6)</sup>

---

5) In this example, the willingness to pay of the weakest demanders (members of group II) is sufficient to buy positive quantities of  $X$ . Of course, this is not necessarily so. The case where the members of group II have demands  $d_{II} < c/N$ , for all  $x$ , cannot be excluded. In this case, each member of group I has to pay a price of  $c/r$  for all units. The members of group II do not appear in the market. Note, however, that this does not change the equilibrium quantity  $x_2$  in Fig. 1, given no income effects.

This case is ignored for the present. Below, however, we shall have to deal with a situation, where each consumer is tempted to act as if he belongs to a group II for which  $d_{II} < c/N$  holds.

6) Head [3] p. 17 examines a special case, where the result is optimal. For simplicity, this and other special cases considered by Head are not discussed here.

### III. Second Hand Markets

Oakland claims that his result cannot be changed by arbitrage among consumers. All individuals who consume the  $(x_2 - x_1)$  "expensive" units also consume the  $x_1$  "cheap" units. Since no one wants to consume a unit twice, "individuals who buy at low prices cannot sell to those buying the higher-priced tapes because the latter are already viewing the low-priced tapes. Thus, arbitrage among tapes of different prices is not possible." <sup>7)</sup>

This argument, however, is not convincing. Since  $X$  is a public good, there are incentives for "arbitrage in the opposite direction" from the consumers of the expensive units to those who only consume the cheap units.

If a member  $i$  of group I permits a member  $j$  of group II to consume a unit  $i$  has bought,  $i$ 's utility will not decline. The price  $j$  pays to  $i$  for the consumption right, e.g. an "entrance fee" in the TV-case (or a fee for taping the cable etc.), lowers the net price (market price minus entrance fee)  $i$  pays for the unit. Since  $j$  consumes this unit second hand, such consumption rights may be called "Second hand markets for excludable public goods". The effects of these markets on equilibrium output and equilibrium intensity of use in a competitive public goods producing industry are discussed below <sup>8)</sup>. It will turn out that the interrelation in demand between 1st hand and 2nd hand consumption plays a crucial role in the analysis. Perfect substitution on the one hand and independence on the other hand will be examined as two extreme cases of interrelation.

#### 1. Perfect substitution

For the time being it is assumed that all consumers regard 1st hand and 2nd hand consumption of the public good as identical <sup>9)</sup>. If an individual wants to consume a certain amount of  $X$  at a given price it is irrelevant to him how this quantity is composed of 1st and 2nd hand consumption.

Consider the case, where the members of group II are not willing to demand a unit of the public good at a price of  $c/N$ , i.e.  $d_{II} \leq c/N$  holds for all  $x$  (see fn 6). In this case

---

<sup>7)</sup> Oakland [4], p. 936. See also p. 934. Head [3], p. 15 fn. 43 rules out the resale of consumption rights by assumption.

<sup>8)</sup> In addition to private TV, private lending libraries could create incentives for such 2nd hand markets. Another example is car pooling in a world where private firms supply streets and charge the users. Moreover, the model discussed below can help to understand the obstacles competitive markets for industrial know how have to cope with, even if exclusion of non-buyers is costless.

<sup>9)</sup> 1st and 2nd hand consumption are perfect substitutes such that the indifference curves of all consumers have the slope -1.

the  $r$  members of group I have to finance the supply of the good by themselves. Their willingness to pay is assumed to generate an equilibrium consumption of  $x_2$ . At  $x_2$ ,  $d_I(x) = c/r$  holds. Fig 2 illustrates this situation.

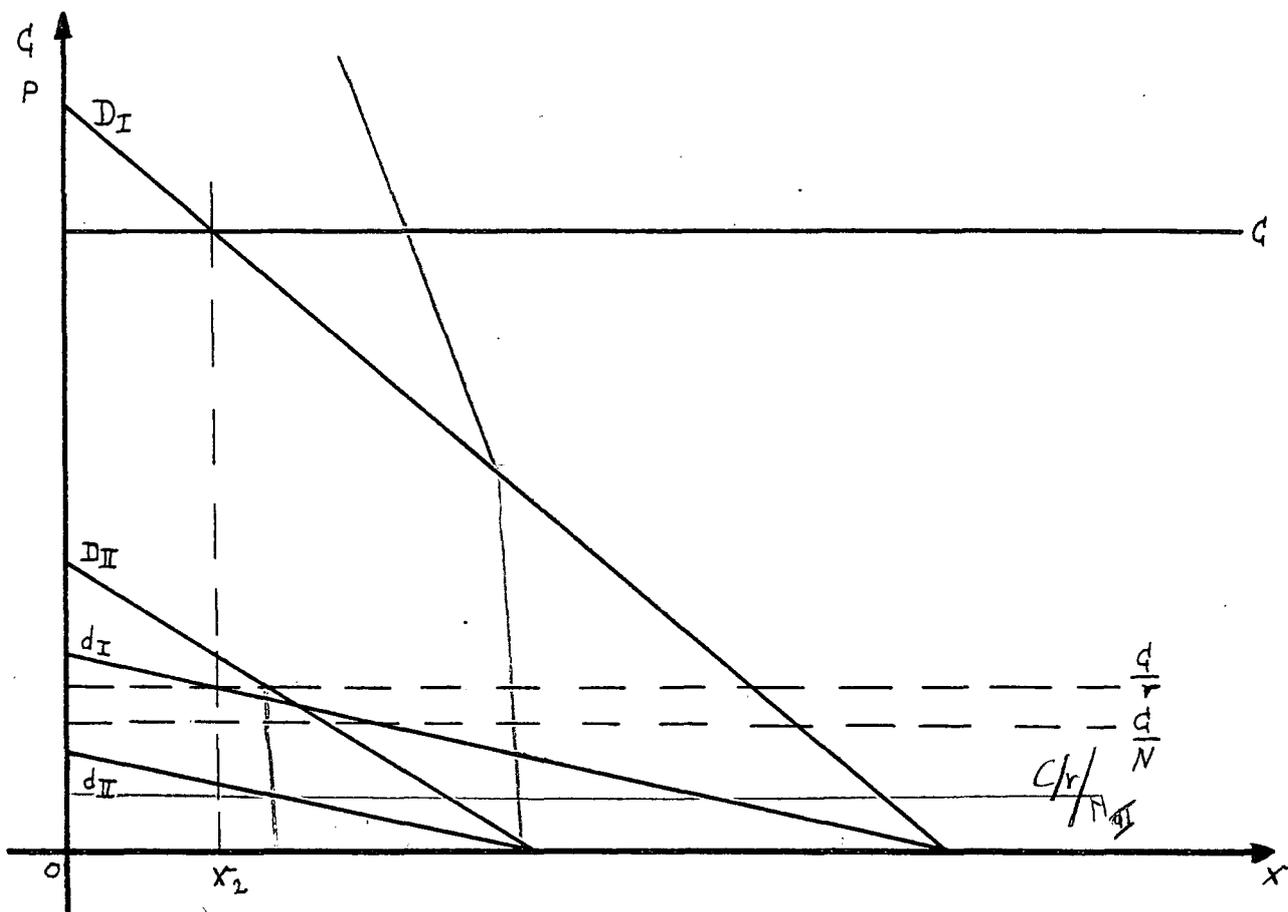


Fig. 2

$d_I$  ( $d_{II}$ ) is the demand curve for a member of group I (II).  $D_I$  ( $D_{II}$ ) is the respective group demand curve.

Although the willingness to pay of the group II members fails to cover the price for 1st hand consumption, it is still positive. There is an incentive for the members of group I to supply the units they bought on the 1st hand market to a 2nd hand market for the members of group II. It is assumed that the 2nd hand market is also perfectly competitive. The utility of each member  $i$  of group I remains the same if a member of group II consumes  $i$ 's units of the public good (non-rival consumption), assuming no transaction or other costs. Competition among the 2nd hand market suppliers of group I will lead to zero price on the 2nd hand market (since marginal costs are zero). But there is a revealed preference problem. Since 1st and 2nd hand use are perfect substitutes for all consumers, no member of group I will demand a unit of the public good

in the 1st hand market at a positive price, if 2nd hand consumption is free. For each member of group I it is rational to veil his preferences and wait until others demand the public good for a positive price in the 1st hand market.

Each member of group I is tempted to act as if he belongs to group II. Therefore, we cannot expect that a private market for the first hand use (and by this, for 2nd hand use) will be established, even if exclusion is costless.<sup>10)</sup>

Thus, given perfect substitution, the existence of 2nd hand markets leads to the result that Oakland's equilibrium quantity  $x_2$  is a "pseudo equilibrium" in Samuelson's sense.

With the undermining of the 1st hand market by "hopeful" 2nd hand demanders the well known free rider problem rises again. Costless exclusion does not avoid this problem, and underproduction is at the level of zero.

## 2. Independence and priority

Now we consider the effects of 2nd hand markets under the assumption that the demands for 1st and 2nd hand consumption are independent. Cross elasticity of demand is zero.

This implies, however, that a consumer may demand a certain unit in both markets. The simplest way to avoid this contradiction to Oakland's assumption that no one consumes the same unit twice, is to supplement the independency assumption by the assumption of priority of 1st hand consumption. Priority means that consumers demand those units in the 2nd hand market for which the 1st hand price is above (or equals) their willingness to pay<sup>11)</sup>. The consequences of this lexicographic relation between the two means of consumption can be seen in Fig. 1. The members of group I do not buy in the 2nd hand market at all since their willingness to pay is above (or equals) price for all  $x_2$  units produced. For graphical simplicity, therefore, no 2nd hand demand curves were drawn for these individuals. The members of group II effectively demand only the units beyond their 1st hand equilibrium quantity  $x_1$  in the 2nd hand market. In Fig. 1,  $d_{II}^{2nd}(x)$  is the 2nd hand demand curve of a member of group II and  $D_{II}^{2nd}(x)$  is the aggregated group demand curve. Following the concept of priority, it is assumed that the willingness to pay for 2nd hand use is lower than for 1st hand use<sup>12)</sup>. Under these conditions a market may be established, where the members of group I supply the units of the range  $(x_1, x_2)$  and the members of group II demand them.

10) There may be a different result, if - contrary to the model given above - there exists one strongest demander whose willingness to pay exceeds marginal cost  $c$  for a certain amount of  $x$ . This case, however, would not be consistent with the competitive large number framework, considered above.

11) The assumptions of independence and priority are artificial. They are useful, however, for together with perfect substitution they limit the spectrum of real possibilities. Furthermore, they facilitate the analysis of movements away from the Oakland equilibrium, when cross elasticity of demand is low. Instead of the priority assumption it may be assumed that a comparison of consumer's surplus is the criterion for the consumers' choice between 1st and 2nd hand consumption. The equilibrium attained under this assumption, however, does not differ from the equilibrium under the much simpler priority assumption, in principle. This fact emphasizes the expediency of the priority assumption.

12) The 2nd hand user of a TV-program incurs e.g. transportation costs.

If we again assume perfect competition and zero cost in the 2nd hand market, the supply curve is identical with the abscissa in Fig. 1. To separate the effects of the 2nd hand market on equilibrium quantity and equilibrium intensity of use it is assumed for the time being that the satiation quantity  $B_{II}^{2nd}$  of the 2nd hand demanders is not bigger than the 1st hand equilibrium quantity  $x_2$  (as shown in Fig. 1). Given that, the equilibrium price in the 2nd hand market is zero for all units. At that price the 2nd hand equilibrium consumption for the members of group II is  $(B_{II}^{2nd} - x_1)$ .

This means that the 2nd hand market improves the intensity of use because, contrary to Oakland's result, no one is excluded from the units beyond  $x_1$ . Of course, the situation would be even better, if the members of group II would be admitted to the 1st hand use of all  $x_2$  units produced and not only to the - lower valued - 2nd hand use from  $x_1$  on.

Given  $B_{II}^{2nd} \leq x_2$  the equilibrium quantity of the public good is not affected by the existence of the 2nd hand market.

To consider the effects of 2nd hand markets on the production level, it is now assumed that  $B_{II}^{2nd}$  is bigger than  $x_2$ , as in Fig. 3. In this case it will be shown that 2nd hand markets increase the equilibrium quantity of the public good.

There is an incentive for each member  $i$  of group I to demand units beyond Oaklands equilibrium quantity  $x_2$  in the 1st hand market if the revenue received in the 2nd hand market is not less than the difference between the price  $i$  has to pay on the 1st hand market and  $i$ 's own willingness to pay. (This revenue is the sum of the prices paid by the consumers in the 2nd hand market).

To supply a unit beyond  $x_2$  in the 2nd hand market, each member of group I must first buy it in the 1st hand market at a price of  $c/r$ . The revenue in the 2nd hand market necessary to make the deal worth-while is therefore  $c/r - d_I(x)$ . Since a higher revenue will be prevented by competition among the members of group I, the 2nd hand supply function of each member of group I is  $c/r - d_I(x)$  for all units beyond  $x_2$ . The supply is faced by the 2nd hand demand of the  $(N - r)$  members of group II. If the (identical) members of group I share consumers symmetrically, each supplier has  $(N - r)/r$  customers<sup>13)</sup>. Thus, the price per consumer

is  $\frac{c/r - d_I(x)}{(N - r)/r}$ <sup>14)</sup>. At this price the equilibrium consumption of each group II

13)  $(N - r)/r$  is assumed to be a natural number.

14) This is the efficient way of supplying units beyond  $x_2$ . Especially, it is not possible for a member  $i$  of group I to undercut this price and thereby pulling all consumers to his business:

If  $i$  would be the only one to supply units beyond  $x_2$  in the 2nd hand market he would first have to buy each of those units as the only demander in the 1st hand market at a price of  $c$ . If all  $(N - r)$  members of group II and

member is, say,  $x_2^{2nd}$ . Units beyond  $x_2^{2nd}$  cannot be financed. So  $x_2^{2nd}$  is the equilibrium output.

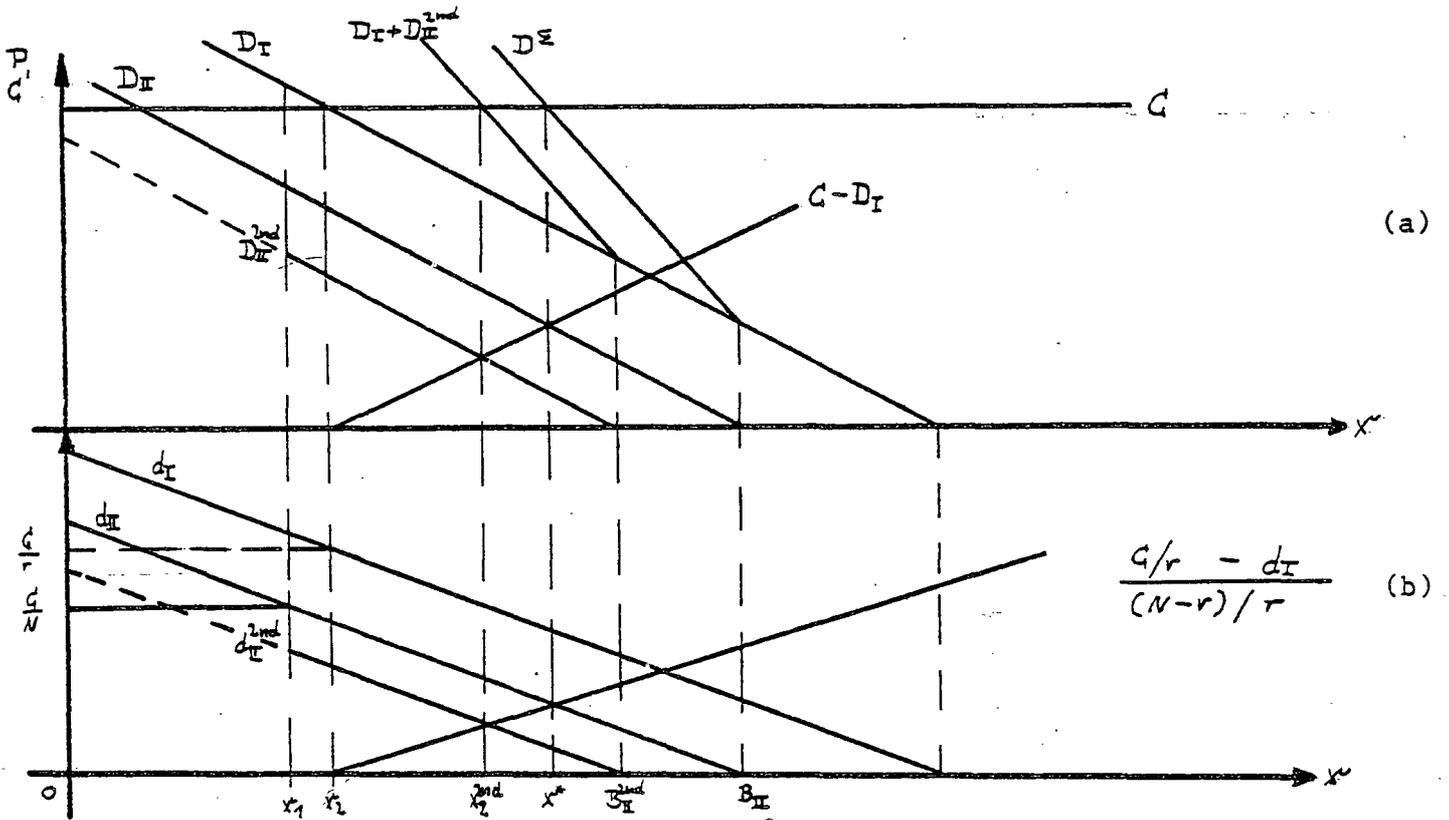


Fig. 3

Fig. 3 illustrates the situation for units beyond  $x_2$ . Fig. 3a refers to groups

as a whole and Fig. 3b to group members. At  $x_2^{2nd}$  the  $\frac{c/r - d_I}{(N-r)/r}$ -curve and the 2nd hand demand curve ( $d_{II}^{2nd}$ ) of each group II member intersect. Since

$D_{II}^{2nd} = (N-r)d_{II}^{2nd}$  and  $c - D_I = (N-r) \frac{c/r - d_I}{(N-r)/r}$  15), the curves  $D_{II}^{2nd}$  and  $C - D_I$  also intersect at  $x_2^{2nd}$ .

the  $(r - 1)$  remaining members of group I would buy the units supplied in the 2nd hand market, the 2nd hand price of a unit would be  $\frac{c - d_I(x)}{N - 1}$  per consumer. Since it can be shown that  $\frac{c - d_I(x)}{N - 1} > \frac{c/r - d_I(x)}{(N-r)/r}$ , there is no possibility for a member of group I to undercut  $\frac{c/r - d_I(x)}{(N-r)/r}$  and thereby destroy the market structure specified above (see appendix).

15) Note that  $d_I = D_I/r$ .

It follows that under the assumptions of independence and priority 2nd hand markets improve the results of the Oakland-economy in two respects:

1) The equilibrium output is pushed towards Pareto optimum. Instead of Oakland's equilibrium quantity  $x_2$  a larger quantity  $x_2^{2nd}$  is produced. However, 2nd hand markets still fail to achieve pareto optimal output, where aggregate marginal willingness to pay for 1st hand consumption equals marginal cost ( $x^*$  in Fig. 3).

2) The intensity of use of the produced units is improved by the 2nd hand market. Whereas, in Oakland's model all members of group II are excluded from consuming units beyond  $x_1$  they are now admitted to consume all units produced, in the 2nd hand market.

Pareto optimal utilization, however, would require that all consumers are admitted to the 1st hand consumption of the produced units, since consumption is non-rival.

#### IV. Summary and Conclusion

There are incentives for the buyers of excludable public goods to resell consumption rights for 2nd hand use to non-buyers. The existence of markets for 2nd hand use has important consequences for the possibility of supplying excludable public goods privately:

- If 1st and 2nd hand use are perfect substitutes, the possibility of 2nd hand markets provokes free rider behaviour that prevents the establishment of private markets for public goods. In this case, costless exclusion devices do not solve the free rider problem and therefore do not enable private firms to supply public goods.
- On the other hand, under the assumptions of independence and priority, 2nd hand markets improve the ability of competitive firms to provide public goods. Compared with the results of Oakland's model, equilibrium output and equilibrium intensity of usage are both increased. The heart of the matter in this case is that the 2nd hand markets produce a price discrimination between the two consumer groups, following the willingnesses to pay of their members, even though no individual firm has any knowledge of the preferences of specific individuals. Pareto optimal resource allocation, however, is not achieved, since 2nd hand consumption is assumed to be inferior to 1st hand consumption.

The theory of 2nd hand markets for excludable public goods has important policy implications, especially regarding economic and legal analysis of information and communication technologies as well as copyright issues.

#### References

- [1] H. Demsetz; The Private Supply of Public Goods.  
Journal of Law and Economics, Vol.13, 1970, pp.293-306.
- [2] H. Demsetz; Joint Supply and Price Discrimination.  
Journal of Law and Economics, Vol.16, 1973, pp.389-405.
- [3] J.G. Head; Misleading Analogies in Public Goods Analysis.  
Finanzarchiv, Vol.36, 1977, pp.1-18.

[4] W.H. Oakland; Public Goods, Perfect Competition and Underproduction.  
Journal of Political Economy, Vol.82, 1974, pp.927-939.

[5] P.A. Samuelson; Aspects of Public Expenditure Theories.  
Review of Economics and Statistics, Vol.40, 1958, pp.332-338.

Appendix

To show that

$$\frac{c/r - d_I(x)}{(N-r)/r} < \frac{c - d_I(x)}{N-1} \text{ holds, first, } d_I \text{ is substituted in both expressions:}$$

The price in the 2nd hand market cannot be higher than the willingness to pay of each member of the weaker group II ( $d_{II}^{2nd}$ ). Since  $d_I > d_{II}^{2nd}$ , for all  $x$ , it follows, that  $d_I$  must be higher than the price in the 2nd hand market.

If  $\frac{c/r - d_I}{(N-r)/r}$  should turn out to be the lower of the two prices, this means

$$d_I > \frac{c/r - d_I}{(N-r)/r}$$

$$\Leftrightarrow \boxed{d_I > c/N}$$

If  $\frac{c - d_I}{N-1}$  should turn out to be the lower of the two prices, this means

$$d_I > \frac{c - d_I}{N-1}$$

$$\Leftrightarrow \boxed{d_I > c/N}$$

Thus, whatsoever the lower of the two prices may be,  $\frac{c/r - d_I}{(N-r)/r}$  or  $\frac{c - d_I}{N-1}$ ,

$d_I > c/N$  holds anyway.

Therefore,  $c/N + b$  ( $b > 0$ ) can be substituted for  $d_I$  in the expressions for

both prices. With that, my assertion  $\frac{c/r - d_I}{(N-r)/r} < \frac{c - d_I}{N-1}$  is equivalent to

$$\frac{c/r - c/N - b}{(N-r)/r} < \frac{c - c/N - b}{N-1}$$

$$\Leftrightarrow -\frac{1}{(N-r)/r} < -\frac{1}{N-1} \text{ q.e.d. (since } r > 1\text{).}$$