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Monopoly pricing under a Medicaid-style most-favored-customer clause and its welfare implication

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THE CENTER FOR THE STUDY
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Working Paper #0100

Monopoly Pricing under a Medicaid-Style
Most-Favored-Customer Clause and Its Welfare
Implication

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Current version: October 2, 2009

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Abstract

To control Medicaid's increasing expenditure on reimbursement of outpatient prescription drugs, the Omnibus Budget Reconciliation Act of 1990 included a rebate program that featured a most favored customer (MFC) clause. This clause guarantees that Medicaid gets a fixed rebate on each unit of purchase by Medicaid consumers. The rebate is calculated as the difference between the minimum price and the average manufacturer price (minimum price provisioning or MPP) or a proportion of the average manufacturer price (average price provisioning or APP). We characterize the optimal pricing strategy of a third-degree price discriminating monopolist in response to the imposition of MPP or APP rules. Under MPP, the minimum price gross of rebate always increases whereas prices gross of rebate in at least some of the markets always decrease. In contrast, under APP, these prices may move in the same direction in all markets, with all increasing in some circumstances and all decreasing in others. We also examine the effects of such provisions on social welfare and provide some useful sufficient conditions for directional changes in social welfare. For example, imposing MPP increases social welfare if it results in higher aggregate demand. Beyond the Medicaid setting, minimum price policies are relevant in a number of applications, including external referencing in drug pricing, price protection in long-term trading contracts and shifts of consumers between markets. We analyze a modified version of our Medicaid MPP model suitable for such settings.

1 Introduction

Medicaid is a U.S. government program to pay for health-care services for some low-income families and individuals. It is funded jointly by the federal and state governments. Growing concern over the rapid increase in Medicaid's spending for outpatient prescription drugs led to the enactment of the Medicaid rebate program in 1990. This rebate program, established by the Omnibus Budget Reconciliation Act (OBRA) of 1990, requires drug manufacturers to offer rebates to Medicaid based on the discounts offered to other large purchasers. This is a form of "most favored customer" (MFC) clause. In particular, Medicaid collects a fixed rebate on each unit of purchase by Medicaid customers. The unit rebate is calculated as the difference between the minimum price and the quantity-weighted average price (minimum price provisioning or MPP), or a fraction of the quantity-weighted average

price (average price provisioning or APP), whichever is higher. As Medicaid consumers constitute a significant fraction of the whole market, the Medicaid rebate program provides drug manufacturers with a strategic incentive to alter their price distribution in the market. This article studies the optimal response of a monopolist to the imposition of these types of minimum price and average price MFC clauses. More specifically, we are interested in examining their effect on pricing when the monopolist practices third degree price discrimination across markets. We also examine how these rebates affect the social welfare.

Drug manufacturers often practice price discrimination to increase profits. For single-source products (e.g., branded drugs with patent protection), suppliers enjoy a high degree of market power. Manufacturers can categorize various purchasers according to their price sensitivity, and charge each group a distinct price. This leads to a high level of price dispersion in the market. In 1991, before the rebate rule went into effect, nearly one-third of the single source drugs had a best price discount of at least 50 percent (i.e., the lowest price charged for the drug was less than half of the highest price charged) (see Congressional Budget Office [4, pp. xi]).

As of 2002, Medicaid constituted approximately 18.5% of the prescription drug market (Duggan and Scott Morton [10])¹. As such a large purchaser, Medicaid might be expected to obtain relatively good prices. However, Medicaid was unable to do this as well as other large purchasers, in part because it reimbursed individual pharmacies and hospitals rather than purchasing in bulk from the manufacturer. To secure better prices for Medicaid patients, OBRA 90 included a voluntary program in which pharmaceutical manufacturers could enroll their product to have access to all state Medicaid formularies². In return, drug manufacturers are required to pay rebates to state and federal Medicaid programs. The rebate rule has a fairly complex structure (and has been modified somewhat over time). As of 2006, manufacturers of branded products are required to offer a rebate to Medicaid at the greater of 15.1% of the Average Manufacturer Price (AMP) or the difference between the AMP and the “best” price (see Hearne [11]). The best price is simply the minimum price at which the product is sold to any purchaser, including hospitals and HMOs³. Generic products are not subject to the best price provision. For generic products, the rebate amounts to 11% of the AMP (see Hearne [11])⁴.

The Medicaid rebate program was introduced to reduce Medicaid’s expenditures on outpatient

¹As of 2006, those individuals over 65 who would have previously been covered by the Medicaid prescription drug program are now covered under Medicare’s Part D prescription drug program. The same is true for some younger disabled individuals. Thus Medicaid’s market share has shrunk somewhat. More recent numbers suggest a market share closer to 15% (Jacobson, Panangala and Hearne [13]).

²When this program was introduced, nearly all branded and generic drug manufacturers did enroll (Scott Morton [29]).

³The Veterans Administration (VA) and Department of Defense (DoD), being large purchasers, enjoy substantial discounts off the wholesale price. When the rebate program was originally enacted, these prices were included in the calculation of the best price. However, in 1992, Congress amended OBRA to exclude prices paid by VA, DoD and some other public purchasers from the calculation of best price.

⁴Average price provisioning is also subject to another restriction in terms of the inflation rate. If AMP rises faster than the inflation rate, an additional rebate, which is equal to the difference between the current AMP and the base year AMP increased by the consumer price index (CPI), is imposed. For a detailed discussion of the Medicaid rebate program, see the Congressional Budget Office report [4]. Duggan and Scott Morton [10] point out that since price increases for any treatment are limited by CPI inflation, if the optimal price for a drug increases faster, there is an incentive to instead introduce and sell a new version of the same drug with a different dosage amount or type (e.g., liquid, capsule, tablet) that would have an unrestricted base price. They find evidence consistent with this behavior by drug manufacturers.

prescription drugs. Although this program seems to have succeeded in lowering Medicaid’s inflation-adjusted drug expenditures (Congressional Budget Office [4]), its overall effects are not obvious. Pharmaceutical manufacturers should react to the rebate rule, thus potentially changing their price distribution across markets. What is the nature of this optimal price response? The savings to Medicaid, if any, would not generally be the same as those calculated without taking into account the change in optimal pricing strategy. Non-Medicaid purchasers are also affected by the rebate rule. For example, Duggan and Scott Morton [10] estimate that for the top 200 drug treatments, the average price of a non-Medicaid prescription would have been 13.3 percent lower in 2002 if the Medicaid MFC clause had not been in effect. The rebate rule also affects drug manufacturers’ profit adversely. It is important to examine the aggregate welfare effects of this cost-saving mechanism. Given the changes that take place as a consequence of the rebate rule, what happens to social welfare?

We analyze a model where a monopolist optimally determines her pricing strategy, subject to MPP or APP clauses. We examine these two types of MFC clauses separately. Medicaid consumers do not pay for their drugs directly (though in some states they do have small co-payments (Hearne [11])). Thus, their price sensitivity may be less compared to non-Medicaid purchasers’ price sensitivity. On the other hand, Medicaid customers’ purchases are influenced by physicians and others (including those running state drug formularies) as well as possible co-payments and thus it may not necessarily be completely inelastic⁵. We therefore assume that Medicaid participant’s demand is a weighted combination of two components: (i) an elastic component, which is the same as non-Medicaid consumers’ demand, and (ii) an inelastic component. The weight of the inelastic component is a parameter in our framework. In examining the impact of the rebate rule on social welfare, we use Marshallian welfare, the sum of consumers’ and producers’ surplus, as our measure of social welfare.

What do we find? A quick preview of some of our results follows. Our analysis of MPP is done with two markets. Under MPP, the minimum price charged always rises compared to the no regulation case. In contrast, the maximum price will (weakly) fall. The maximum price will remain unchanged if Medicaid demand is as elastic as non-Medicaid demand. The welfare effect of MPP may be good or bad. A useful sufficient condition for MPP to be welfare improving is that MPP raise aggregate quantity.

Under APP, prices in all markets move in the same direction if either Medicaid demand is sufficiently inelastic or Medicaid participant’s demand is almost as elastic as non-Medicaid consumers’ demand. When Medicaid participant’s demand is almost as elastic as non-Medicaid consumers’ demand, prices increase. In contrast, when Medicaid participants’ demand is sufficiently inelastic, prices in all markets fall. As with MPP, the welfare effect of imposing APP is ambiguous in general. When prices in all markets fall, both welfare and aggregate quantity increase, while if all prices increase this is welfare and quantity decreasing.

Though the motivation for this paper mainly comes from the MFC clauses that are featured in the Medicaid reimbursement policy, a broad class of contractual problems features similar MFC clauses, especially in the form of MPP. Such clauses are used in contractual agreements in different industries (e.g. external referencing policy in drug pricing in the context of Europe (see Heuer, Mejer and Neuhaus [12], Garcia Mariñoso, Jelovac and Olivella [16]), agreements between health

⁵Note that our work also applies to contracts in natural gas or international trade where the use of a most favored customer clause is common. In those applications, it is more natural to assume that the most favored customer’s demand is also price sensitive.

care providers and health practitioners (see Martin [17]), and most favored nation clauses in legal settlements (see Spier [31], [32]).⁶ Modeling applications of MPP in a more general context requires a modified formulation of MPP compared to the one used in the context of Medicaid. This is because of the following. Though Medicaid collects a rebate from the sale price on each unit of purchase by a Medicaid-covered consumer, the amount of rebate is not known at the time of purchase. The rebate is calculated only later, once the total Medicaid purchases, as well as the relevant minimum and quantity-weighted average prices are known. Furthermore, the rebate is paid by the manufacturer directly to Medicaid, and is essentially invisible to consumers. Therefore, the effective demand by Medicaid customer is likely to be based on the pre-rebate market prices. In other applications of MPP, MFCs are often aware of the minimum price at the time of purchase or more directly involved in the rebate process. To facilitate this wider application of MPP, we also analyze a rebate-responsive version of MPP in which MFCs demand is directly affected by the price net of rebate (i.e., the minimum price). We find that our results on the effect of MPP on pricing and welfare in the context of Medicaid also hold true qualitatively with this alternative version of MPP.

Understanding the effects of these regulations is not simply of interest for evaluation of Medicaid policy, but is also important as a guide to future regulation. For example, recently there has been debate about the appropriate regulatory regime to govern drug purchases and reimbursement under Medicare, the US government program of health insurance for the elderly (Jacobson, Panangala and Hearne [13]).

1.1 Related Literature

The literature related to the Medicaid rebate program and its rebate rules has been primarily empirical. The only theoretical models of monopoly behavior under these rules that we know of are in the brief theory sections of Scott Morton [29] and Congressional Budget Office [4]. The seminal Scott Morton [29] is closely related to and an important motivation for our analysis. We borrow the third degree price discrimination structure and the possibility that Medicaid consumers' demand may be less elastic than other consumers' demand from her model, but there are a number of key differences in our formulation and treatment of the problem. First, we do not limit our analysis to the case of linear market demand curves – we allow general downward sloping, continuously differentiable demands. Nor do we limit ourselves to polar cases in terms of elasticity for Medicaid consumers' demand – we have a continuous parameter indexing elasticity. Second, we analyze how these MFC clauses could affect social welfare, an aspect not studied in Scott Morton ([29], [30]) or Congressional Budget Office [4]. Third, we find conditions under which non-MFC prices in all markets increase when an APP rule is imposed and also conditions under which these prices decrease in all markets as a result of APP. Finally, in our formulation of MPP and APP, we assume that the effective demand of Medicaid consumers is based on the pre-rebate prices, motivated by the fact that the amount of the rebate is neither determined at the time of purchase nor does the ultimate rebate involve any party except the manufacturer and Medicaid. In contrast, Scott Morton assumes that the demand of Medicaid consumers is a function of the post-rebate price. This is most related to our alternative MPP formulation, in which we also assume MFC demand depends on post-rebate prices. In comparing this formulation to Scott Morton, in addition to the first two differences pointed out above, we note that we provide a full characterization

⁶For further discussion, see Section 6.1.2.

of the solution, and, even in the special case of linear demand, this solution only coincides with that in Scott Morton [29] under additional and restrictive assumptions.

The welfare aspect of our work has close connections with the literature on the welfare effects of third degree price discrimination by a monopolist. The effect of price discrimination on social welfare was first studied by Robinson [25]. Schmalensee [27] reexamined the problem, and provided a sufficient condition for welfare to decrease under uniform pricing as compared to third degree price discrimination. He shows that uniform pricing can lead to a decrease in welfare only if it leads to an decrease in aggregate demand. As stated above, we show a similar result for MPP – imposition of MPP can lead to a decrease in welfare only if it leads to a decrease in aggregate demand. Varian [36] extends Schmalensee’s results and proves additional results in a setting where demand in any market can depend on prices in other markets and marginal cost is constant or increasing.⁷ Varian’s [36] techniques prove useful in our welfare analysis of MPP. Concerning the welfare effects of APP, the closest work is Armstrong and Vickers [1, case 2] which analyzes the welfare effect of a price regulation somewhat related to APP. They use a convexity property of the consumer surplus function to establish that consumer surplus decreases when moving from a given uniform price across all markets to price discrimination with a constraint that quantity weighted average price is at most the given uniform price. Moreover, when the negative effect of price discrimination on consumer surplus is sufficiently small, they show the increase in producer’s surplus dominates the loss in consumers’ surplus, and therefore, aggregate welfare increases if the producer is allowed to price discriminate to a small extent. Unfortunately, the benefits of this convexity property of the consumer surplus function are largely limited to circumstances where one of the benchmark pricing schemes is uniform. As neither unconstrained prices nor prices under APP are generally uniform, we are not able to benefit from Armstrong and Vickers [1, case 2]’s techniques.

The empirical work on the Medicaid rebate program includes two United States General Accounting Office (GAO) studies ([34], [35]), a Congressional Budget Office report [4], Scott Morton ([29], [30]) and Duggan and Scott Morton [10]. All of these papers find some evidence of post-rebate rule increases in drug prices for non-Medicaid buyers. GAO [34] studied how Veterans Affairs (VA) and Department of Defense (DoD) prescription drug prices had changed, while GAO [35] examined drug prices to health maintenance organizations (HMOs) and hospitals. In both cases price increases were observed, but the GAO could not determine whether the price growth was attributable to the rebate rules. The Congressional Budget Office [4] report concluded that although the rebate rule lowered Medicaid expenditure, it increased the prices paid by some purchasers in the private sector. Scott Morton [29] finds that the price of branded products facing generic competition rose. For generic drugs, the increase in price is higher as markets become more concentrated. Scott Morton [30] finds that products with higher ex-ante price dispersion show a greater increase in price when the rebate rule is in effect, consistent with the theory. Duggan and Scott Morton [10], as mentioned above, estimate that the average price of a non-Medicaid prescription would have been 13.3 percent lower in 2002 if the Medicaid MFC clause had not been in effect. They also find an increase in new drug introductions for the purpose of raising prices in reaction to a provision in the OBRA 90 legislation that ties increases in existing drug prices to inflation.

Rules like MPP have been studied in a number of other contexts. The impact of similar most

⁷Schwartz [28] extends Varian’s analysis to the case of decreasing marginal cost functions. Cowan [7] and Cowan and Vickers [8] obtain additional welfare results by imposing functional form restrictions on demand.

favored customer clauses in oligopoly settings has been studied extensively in the theoretical literature. Most of the research explores the situation where the sellers strategically exploit the clause to soften price competition. See for example Besanko and Lyon [2], Cooper [5], Cooper and Fries [6], Neilson and Winter [19], [20], [21], Png [23], Png and Hershleifer [24], and Salop [26]. Spier [31] studies uses of MFC-type clauses in settlement of litigation. The use of MPP with long term contracts has been studied by Butz [3] in the context of durable goods monopoly. Butz analyzes how MPP can be used to facilitate commitment not to reduce price in the future, and thereby sustain the monopoly price for the product. In his analysis, MPP is used as a strategic device by the monopolist in its intertemporal game with consumers to change consumer demand by changing beliefs about future prices. Thus even in the monopoly context, the emphasis has been on strategic effects. Our analysis differs substantially from those mentioned in this paragraph because our focus is on the unilateral/own-price effects of such clauses rather than the strategic effects operating through competitor or consumer reaction. In particular, none of our pricing or welfare results may be derived from this literature.

Our analysis of the alternative, rebate-responsive MPP is related to the literature on the theory of pricing with external referencing (ER) and with parallel imports (PI). Applications of ER and PI are common in the context of drug pricing in Europe and in North America (see Garcia Mariñoso, Jelovac and Olivella [16], Pecorino [22] and Jelovac and Bordoy [15]). In ER, a product's price in one market (call it the target price) is required to be below a function of the price of the same product in another market (call it the reference price). An example would be one country requiring that a drug be no more expensive than in a neighboring country. PI refers to allowing the importation of a product that may also be produced domestically. Like ER, PI leads to a link between the target price and the reference price, but with PI this link is indirect. If the home country imports, then the home price is effectively bounded by the foreign price plus the cost of importing. Garcia Mariñoso, Jelovac and Olivella [16] studies the pricing problem with ER in the context of drug pricing regulation in Europe. Pecorino [22] and Jelovac and Bordoy [15] study a similar pricing problem with PI in the context of drug importation. Among their findings is that the reference price may increase in the presence of ER or PI. In the context of rebate-responsive MPP, considering the minimum price as the reference price and the MFC price as the target price, we show a similar result.

This paper is organized as follows. In section 2, we describe the general model and specify the monopolist's objective function under the two rules. In section 4, we solve the optimization problem under MPP and examine its welfare implications. Section 5 carries out a similar investigation for the APP rule. In Section 6.1, we present the analysis of the rebate-responsive form of MPP and include a discussion of some non-Medicaid applications. Section 7 concludes. Proofs not included in the main text are collected in an Appendix.

2 The Model

Consider a monopolist selling a single good in n different markets, indexed by i . We assume that the monopolist cannot discriminate between consumers within a market, but it can prevent arbitrage by consumers across markets. The presence of a MFC provision divides consumers in each market into two categories: MFCs and non-MFCs. If all consumers in market i were non-MFCs, the demand function in market i would be given by a downward sloping, non-negative, continuously differentiable demand curve, $q_i(p_i)$, for the product, where p_i is the price charged in market i . In the context of Medicaid,

MFCs' price sensitivity may be different from non-MFCs' price sensitivity, as Medicaid consumers do not pay for their drugs directly. Their purchases, however, are influenced by physicians and others (including those running state drug formularies) as well as possible co-payments. To incorporate various possibilities, we describe MFC demand as follows: If all consumer in market i were MFCs, the demand function in market i would be given by $(1 - \beta) q_i(p_i) + \beta z_i$, for constants $z_i > 0$ and $\beta \in [0, 1]$, where p_i is the price charged in market i . The constant β measures how inelastic MFC demand is, compared to non-MFC demand. For simplicity, we assume that the fraction of MFCs in each market is the same and we denote this fraction by $\gamma \in [0, 1]$. Therefore, total demand in market i is given by

$$\begin{aligned} & (1 - \gamma) q_i(p_i) + \gamma((1 - \beta) q_i(p_i) + \beta z_i) \\ = & (1 - \beta\gamma) q_i(p_i) + \beta\gamma z_i. \end{aligned} \quad (2.1)$$

Again for simplicity, we consider a linear cost function $C(q) = cq$. We also assume there are gains from trade in all markets, i.e., $q_i(c) > 0$. Without any MFC provision, the monopolist's total profit can be written as

$$\Pi^{no-rebate} = \sum_{i=1}^n (p_i - c) ((1 - \beta\gamma) q_i(p_i) + \beta\gamma z_i). \quad (2.2)$$

Within this model of third-degree price discrimination, we analyze the consequences of MFC clauses. In particular, the MFC clauses related to Medicaid each involve a rebate, which we denote by r , on each unit purchased for a Medicaid-covered consumer. For practical reasons, the rebate amount is calculated only retrospectively, once the Medicaid purchases are known, and is paid directly from the manufacturer to Medicaid.⁸ Thus, the rebate amount is essentially invisible to consumers at the time of purchase. We will assume, therefore, that demand from MFCs is unaffected by the rebate amount.⁹ To avoid confusion between prices gross and net of rebate, we refer to the (gross of rebate) prices, p_i , as *market prices* while the (net of rebate) prices that Medicaid pays for each unit purchased by MFCs in market i , $p_i - r$, are referred to as *post-rebate prices*. With an MFC clause in effect, the monopolist will take the rebate into account and chooses market prices to maximize

$$\Pi^{rebate} = \sum_{i=1}^n (p_i - c) ((1 - \beta\gamma) q_i(p_i) + \beta\gamma z_i) - r\gamma \sum_{i=1}^n ((1 - \beta) q_i(p_i) + \beta z_i). \quad (2.3)$$

There are two different rules that Medicaid uses to calculate the per unit rebate: (i) Minimum price provision (MPP), and (ii) Average price provision (APP). We study them separately.

Under MPP, Medicaid claims the difference between p_q , the quantity weighted average market price, and $p_{\min} \equiv \min(p_1, \dots, p_n)$, the minimum price charged in any market:

$$r = p_q - p_{\min}. \quad (2.4)$$

Under APP, Medicaid claims a fraction $\alpha \in [0, 1]$ of the quantity weighted average market price.¹⁰

⁸ See e.g., Congressional Budget Office report [4] and Scott Morton ([29], [30]).

⁹ While we think this is the most appropriate model for the Medicaid application, in a later section, we also consider a general version of the minimum price provision rule where MFCs' demand is affected by the post-rebate price (i.e., the price ultimately paid). The latter model may be more relevant for other applications including those discussed in Section 6.1.2.

¹⁰ As of 2006, when discounting average price, Medicaid uses $\alpha = 0.151$ for branded drugs and $\alpha = 0.11$ for generic drugs (Hearne [11]).

So,

$$r = \alpha p_q \text{ where } p_q = \frac{\sum_{i=1}^n p_i ((1 - \beta\gamma) q_i(p_i) + \beta\gamma z_i)}{\sum_{i=1}^n ((1 - \beta\gamma) q_i(p_i) + \beta\gamma z_i)}. \quad (2.5)$$

We will assume throughout our analysis that all n markets are served whether or not the MFC provisions are imposed. To this end we impose the following:

Assumption 1 *The demand functions q_i are positive for every market i at the monopolist's optimal market prices for the problem without an MFC provision, the problem with MPP and the problem with APP.*

We also assume that demand in each market is such that profit in that market (assuming no MFC clause) is a strictly concave function of price in that market whenever demand is positive. This assumption ensures that the unique solution of the monopolist's profit maximization problem without an MFC provision may be found by solving the first-order conditions. Formally, the following is assumed for the remainder of the paper:

Assumption 2 *For each market i , $(p_i - c)q_i(p_i)$ is strictly concave in p_i whenever $q_i(p_i) > 0$.*

Additional assumptions will be needed to support the first-order approach under MPP and APP. We defer discussion of those to the sections on MPP and APP respectively.

In addition to the pricing implications of the MFC clauses, we are interested in the social welfare effects. To measure these, we use the classical Marshallian welfare criterion, consumers' surplus plus producers' surplus.¹¹ Since we allow for the possibility that MFC consumers' demand may have an inelastic component, z_i , consumer surplus for these consumers would technically be infinite, rendering Marshallian welfare insensitive to changes in market prices. This can easily be remedied by assuming demand is zero when market prices become high enough. More specifically, assume there is a non-binding finite upper bound on market prices, \bar{M} , such that demand in all markets is zero at prices above \bar{M} . This is equivalent to saying that the inelastic component of demand isn't really perfectly inelastic, but rather is inelastic until price hits \bar{M} , and zero thereafter. For any price vector $P = (p_1, \dots, p_n)$ and associated demand $x(P) = (x_1(p_1), \dots, x_n(p_n))$, the Marshallian welfare measure, $W(P)$, will thus be given by

$$W(P) \equiv \sum_{i=1}^n \left[(p_i - c) x_i(p_i) + \int_{p_i}^{\bar{M}} x_i(v) dv \right]. \quad (2.6)$$

As we will be interested in the *changes* in welfare brought about by the various MFC clauses and not absolute welfare levels, our results will not depend in any way on the precise magnitude of the bound \bar{M} .

We do not explicitly consider at least two characteristics of the actual Medicaid rebate policy. First, participation in the Medicaid rebate program on the part of drug manufacturers is voluntary in the following sense: a manufacturer could choose not to enroll drugs in the rebate program in exchange for giving up coverage for them under Medicaid, effectively eliminating sales to Medicaid-covered consumers. This could be modeled by including a participation constraint (i.e., that profits under the rebate program should be at least as high as profits without rebates when no Medicaid consumers are served). In practice, it appears that this constraint is not binding. Nearly all branded

¹¹ See Schmalensee [27] and Varian [36] for discussions on the legitimacy of this measure.

and generic drug manufacturers enrolled when the rebate program was introduced (Scott Morton [29]). Furthermore, in our model, it can be shown that this participation constraint is trivially satisfied when Medicaid demand is almost as elastic as non-Medicaid demand (i.e., β is close to 0). For higher values of β , the constraint can be shown to be satisfied under a restriction on the range of relative values of the inelastic component of Medicaid demand as it varies across markets (i.e., the range of ratios of the z_i 's).

Second, the actual Medicaid rebate (at least for branded drugs) is calculated as the rebate from APP, or the rebate from MPP, whichever is higher. We analyze the two rebate forms separately. It is clear that these separate analyses can still give much insight into the combined problem. If at the optimal solution to the combined problem, only one of the two clauses, but not both, is binding, the solution will be exactly either the solution to the MPP problem or the solution to the APP problem and our analysis may be directly applied. If at the optimal solution, however, both clauses are simultaneously binding, then the solution to the combined problem may differ from the optimal solutions obtained through our separate analyses. Ideally we would have liked to analyze this case as well, however it appears to us to be quite intractable. Furthermore, we have been unable to locate evidence that would suggest the dual-binding case occurs in practice.

3 The Benchmark Case: No MFC Provision

As a point of comparison, it is useful to begin our analysis by looking at the profit maximization problem for the monopolist when there is no MFC clause. Without one, the monopolist receives revenue p_i for each unit sold in market i , irrespective of the split between MFCs and non-MFCs within the market. The monopolist therefore chooses prices to maximize profits $\Pi^{no-rebate}$ as defined in (2.2). We call this the unconstrained problem.

Let p_i^m denote the optimal monopoly price in market i . Given our assumptions, p_i^m is the unique solution to the equation

$$(1 - \beta\gamma)(p - c)q'_i(p) + (1 - \beta\gamma)q_i(p) + \beta\gamma z_i = 0. \quad (3.1)$$

With no MFC clause, the social welfare is therefore given as

$$\sum_{i=1}^n \left[(p_i^m - c) \left((1 - \beta\gamma)q_i(p_i^m) + \beta\gamma z_i \right) + \int_{p_i^m}^{\bar{M}} \left((1 - \beta\gamma)q_i(v) + \beta\gamma z_i \right) dv \right].$$

As it is convenient, without loss of generality, we henceforth assume $p_1^m < p_2^m < \dots < p_n^m$.¹² It will also be helpful to denote the uniform monopoly price (i.e., the profit maximizing price under the constraint that the same price must be charged in each market) by p^u , the unique solution to

$$\sum_{i=1}^n [(p - c)(1 - \beta\gamma)q'_i(p) + (1 - \beta\gamma)q_i(p) + \beta\gamma z_i] = 0.$$

¹²If this is strictly violated, simply reindex the markets so that their numbering agrees with the monopoly price induced order. In cases where there is equality in monopoly prices across markets, a similar analysis can be carried out by first combining these markets into one. To see this, let us consider a situation where $p_1^m < \dots < p_k^m = p_{k+1}^m < \dots < p_n^m$. If we define a market indexed by k' by combining market k and market $k+1$ such that $q_{k'} = q_k + q_{k+1}$ and $z_{k'} = z_k + z_{k+1}$, then $p_{k'}^m$ remains the same as p_k^m and p_{k+1}^m . This returns us to a situation where strict inequality is maintained among the optimal individual monopoly prices in each of these markets.

4 Minimum Price Provision

We now examine the MPP problem. Under MPP, combining (2.3) and (2.4), the monopolist chooses market prices to maximize

$$\sum_{i=1}^n (p_i - c) ((1 - \beta\gamma) q_i(p_i) + \beta\gamma z_i) - \gamma (p_q - p_{\min}) \sum_{i=1}^n ((1 - \beta) q_i(p_i) + \beta z_i). \quad (4.1)$$

Let \hat{p}_i denote the market price charged in market i in the solution to this problem. For simplicity and tractability of our results, we consider the two market case ($n = 2$). We also assume the following strengthening of Assumption 2:

Assumption 3 *(4.1) is strictly concave in (p_1, p_2) whenever $(q_1(p_1), q_2(p_2)) \gg 0$.*

Just as Assumption 2 ensured that first-order conditions determined the unique solution to the monopolist's unconstrained problem, Assumption 3 does the same for the MPP problem. To see that this strengthens Assumption 2, notice that when $p_1 = p_2$, (4.1) reduces to $\sum_{i=1}^2 (p_i - c) ((1 - \beta\gamma) q_i(p_i) + \beta\gamma z_i)$, and thus Assumption 3 implies the strict concavity of $(p_i - c) q_i(p_i)$.

Our next result shows that the optimal price in market 1 will remain (weakly) below the optimal price in market 2 after MPP is imposed. The key to this is showing that the monopolist would always prefer to charge a uniform price compared to a situation of charging a high price in the first market and a low price in the second market.

Lemma 1 *Suppose Assumptions 1 and 3 hold. Then $\hat{p}_1 \leq \hat{p}_2$.*

With the aid of this lemma, we can describe the effect of MPP on prices.

Proposition 1 *Suppose Assumptions 1 and 3 hold. If MPP is imposed, the minimum market price increases and the maximum market price decreases compared to the unconstrained case (i.e., $\hat{p}_1 \geq p_1^m$ and $\hat{p}_2 \leq p_2^m$). When some but not all of the consumers are MFCs ($\gamma \in (0, 1)$), these changes are strict. Further, the monopolist will charge a uniform price if and only if*

$$(p^u - c) (1 - \beta\gamma) q_2'(p^u) + ((1 - \beta\gamma) q_2(p^u) + \beta\gamma z_2) \left(1 - \gamma \frac{\sum_{i=1}^2 ((1 - \beta) q_i(p^u) + \beta z_i)}{\sum_{i=1}^2 ((1 - \beta\gamma) q_i(p^u) + \beta\gamma z_i)} \right) \leq 0. \quad (4.2)$$

In such a scenario, the optimal uniform price will be the uniform monopoly price, p^u .

The first part of the above proposition shows that MPP raises the minimum market price but lowers the maximum market price. The basic intuition for this is that, under MPP, the monopolist pays a rebate based on the difference between the minimum market price and the quantity weighted average market price. The monopolist therefore, all else equal, prefers to set prices so that the minimum market price is close to the quantity weighted average market price. This force pushes the minimum market price up and the maximum market price down. At an extreme, the two prices coincide and the monopolist prefers to charge a uniform price. When this happens, the rebate equals zero. Therefore, the only equal market price that can be optimal for the monopolist to charge is the uniform monopoly price, p^u . The second part of the above proposition provides a necessary and sufficient condition under which the monopolist prefers to charge a uniform price. The condition has a simple interpretation

– it says that, starting from uniform monopoly prices (p^u, p^u) , a marginal increase in p_2 reduces the monopolist's profit (net of rebate) from sales in market 2. Alternatively, one could write a similar condition examining a marginal decrease in p_1 . Part of the proof of the proposition shows that it is enough to look at the change in only one of the markets. When (4.2) fails, market prices may be found by replacing p_{\min} with p_1 in (4.1) and setting the partial derivatives with respect to p_1 and p_2 equal to zero. This ensures that \hat{p}_1 and \hat{p}_2 exactly balance the marginal gain in profit due to reduction in rebate with the marginal loss in profit because of deviation from the unconstrained monopoly prices.

4.1 Welfare analysis of MPP

In considering the welfare effects of MPP, it is important to note that any rebates collected are pure transfers from the monopolist to Medicaid. Therefore, the rebate amount does not enter into the measure of welfare directly. Any welfare effect of such a policy will operate only through the change in market prices due to the introduction of MPP.¹³

We start by describing a general result from Varian [36] about change in welfare. We apply the result in our setting to derive bounds on the change in welfare resulting from the imposition of MPP.

Consider an m - good economy for any finite $m > 0$. Let $\mathbf{x}(\mathbf{P}) = (\mathbf{x}_1(p_1), \dots, \mathbf{x}_m(p_m)) \in \mathbb{R}_+^m$ denote the vector of demands associated with price vector $\mathbf{P} = (p_1, \dots, p_m) \in \mathbb{R}_+^m$. Assume that unit cost of production is constant and equal for each good and let $\mathbf{c} = (c, \dots, c) \in \mathbb{R}_+^m$ denote the vector of production costs. The Marshallian welfare measure, as before, is defined as the sum of consumers' surplus and producers' surplus.¹⁴

When changing from a price vector $\mathbf{P}^0 \in \mathbb{R}_+^m$ to a price vector $\mathbf{P}^1 \in \mathbb{R}_+^m$, let $\Delta \mathbf{x} \in \mathbb{R}^m$ and $\Delta W \in \mathbb{R}$ denote the vector of changes in demand and the change in welfare respectively (i.e., $\Delta \mathbf{x} = \mathbf{x}(\mathbf{P}^1) - \mathbf{x}(\mathbf{P}^0)$ and $\Delta W = W(\mathbf{P}^1) - W(\mathbf{P}^0)$).

Fact 1 (Varian [36]) *The change in welfare, ΔW , satisfies the following bounds:*

$$(\mathbf{P}^0 - \mathbf{c}) \cdot \Delta \mathbf{x} \geq \Delta W \geq (\mathbf{P}^1 - \mathbf{c}) \cdot \Delta \mathbf{x}$$

Proof. See the proof of Fact 2 in Varian [36]. ■

The next result uses Fact 1 and the solutions to the monopoly pricing problems with and without MPP to obtain bounds on the change in welfare resulting from a MPP policy.

Proposition 2 *Suppose Assumptions 1 and 3 hold. The change in welfare, when moving from the unconstrained problem to a MPP policy, satisfies the following lower bound:*

$$\Delta W \geq (\hat{p}_1 - c) \Delta Q$$

where ΔQ denotes the corresponding change in aggregate demand. Furthermore, if $q_i(p)$ is concave in $p \geq 0$ for $i = 1, 2$, then the change in welfare satisfies the following upper bound:

$$(\hat{p}_1 - c) \Delta Q - \Delta \pi + (1 - \beta\gamma) (\hat{p}_2 - \hat{p}_1) (q_2(\hat{p}_2) - q_2(p_2^m)) \geq \Delta W$$

¹³If, as is sometimes assumed in the optimal regulation and public finance literatures, there is a social cost of transfers through the government, due to, for example, inefficient taxation, the rebates could have a direct welfare effect as well.

¹⁴In our formulation of social welfare, we consider a finite upper bound in prices, given by \bar{M} , such that demand becomes zero at prices above \bar{M} . In Varian ([36]), there is no finite upper bound in prices ($\bar{M} = \infty$) as he did not explicitly consider demand with an inelastic portion. However, it can be shown easily that all the results on welfare bounds in Varian ([36]) go through with finite \bar{M} .

where $\Delta\pi$ denotes the corresponding change in the monopolist's profit (excluding the rebate from the profit calculation).

The bounds in Proposition 2 use knowledge of only the realized change in aggregate demand, ΔQ , the minimum price, \hat{p}_1 , the manufacturing cost, c , and the loss in profits to the monopolist due to the MPP rule, $\Delta\pi$, to bound the change in welfare. As the monopolist can always do best when pricing is unrestricted, $\Delta\pi$ is never positive under MPP. Moreover, $(1 - \beta\gamma)(\hat{p}_2 - \hat{p}_1)(q_2(\hat{p}_2) - q_2(p_2^m))$ is always positive as $\hat{p}_1 \leq \hat{p}_2 \leq p_2^m$. Thus the bounds are always possible to satisfy. Is interesting to note that even if costs (and thus profits), for example, are unobserved, the lower bound implies that welfare always increases when imposing MPP leads to an increase in aggregate demand. Similarly, if aggregate demand is decreased by MPP, welfare can decrease by no more than the decrease in aggregate demand valued at the minimum price. Under concavity of the demand functions, the upper bound implies that a large enough decrease in aggregate demand ($\Delta Q < [\Delta\pi - (1 - \beta\gamma)(\hat{p}_2 - \hat{p}_1)(q_2(\hat{p}_2) - q_2(p_2^m))] / (\hat{p}_1 - c)$) generates a decrease in welfare.

5 Average Price Provision

We now analyze the APP problem. We allow for n markets here as, for APP, this additional generality comes at no cost and may be helpful in applications. Under APP, the monopolist chooses prices to maximize

$$\sum_{i=1}^n (p_i - c) ((1 - \beta\gamma) q_i(p_i) + \beta\gamma z_i) - \alpha\gamma p_q \sum_{i=1}^n ((1 - \beta) q_i(p_i) + \beta z_i). \quad (5.1)$$

Let \hat{p}_i denote the optimal market price in market i after APP is imposed. Then $\hat{p} = (\hat{p}_1, \hat{p}_2, \dots, \hat{p}_n)$ solves the first-order conditions:

$$\begin{aligned} & \frac{d}{dp_i} \left[\sum_{i=1}^n (p_i - c) ((1 - \beta\gamma) q_i(p_i) + \beta\gamma z_i) \right] \\ & - \alpha\gamma \frac{d}{dp_i} [p_q \sum_{i=1}^n ((1 - \beta) q_i(p_i) + \beta z_i)] = 0 \quad , \text{ for } i = 1, 2, \dots, n. \end{aligned} \quad (5.2)$$

This says that optimal market prices under APP equate the marginal gain in profit due to reduction in total rebate paid with the marginal loss in profit due to deviation from the unconstrained monopoly prices. As in our analysis of MPP, we strengthen Assumption 2 to ensure that the first-order conditions determine a unique global optimum. To this end, assume the following:

Assumption 4 (5.1) is globally concave in (p_1, p_2, \dots, p_n) whenever $(q_1(p_1), \dots, q_n(p_n)) \gg 0$.

How do the market prices under APP compare to those in the unconstrained problem? Observe that at the solution of the unconstrained problem, (p_1^m, \dots, p_n^m) , the first term in (5.2) is equal to zero for all markets i . Hence, the left hand side of the first order condition for the APP problem, computed at (p_1^m, \dots, p_n^m) , reduces to

$$-\alpha\gamma \frac{d}{dp_i} [p_q \sum_{i=1}^n ((1 - \beta) q_i(p_i) + \beta z_i)] \big|_{p=(p_1^m, \dots, p_n^m)}. \quad (5.3)$$

Therefore, the sign of this expression is the key to understanding whether imposing APP will raise or lower market prices. The firm's only motive for moving prices away from the unconstrained monopoly

level is to reduce the rebates it has to pay. If, at unconstrained market prices, raising prices increases the total rebate under the APP rule, (5.3) is negative, implying that APP will result in lower market prices. Similarly, if raising prices decreases the total rebate, (5.3) is positive, implying that APP will result in higher market prices.

In general, either case is possible. However, the inelasticity parameter, β , is very helpful in determining which case is relevant. We show that when the inelasticity parameter β takes extreme values one can unambiguously compare the APP market prices, \hat{p} , to the unconstrained prices. When the price responsiveness of Medicaid covered consumers is similar to that of other consumers (i.e., β low enough), APP will increase market prices. If, instead, Medicaid covered consumers are much less price responsive (i.e., β high enough), APP will decrease market prices. The following proposition formalizes this claim:

Proposition 3 *Suppose Assumptions 1 and 4 hold. There exist $\underline{\beta}$ and $\bar{\beta}$, $0 < \underline{\beta} \leq \bar{\beta} < 1$, such that (i) for $\beta < \underline{\beta}$, all market prices strictly increase under APP compared to the unconstrained case, and (ii) for $\beta > \bar{\beta}$, all market prices strictly decrease under APP compared to the unconstrained case.*

What is the intuition for the role of β in determining these effects? Under APP, the total rebate paid is a fraction of the quantity weighted average market price times the total MFC demand for the product. The monopolist, all else equal, prefers to reduce the rebate it pays. Starting at the solution of the unconstrained problem, an increase in market prices generates two effects on the total rebate. First, the quantity weighted average price increases. Second, total MFC demand for the product falls. When β takes values close to zero, i.e., when MFCs' demand is almost as elastic as non-MFCs' demand for the product, the demand reduction effect dominates and so by increasing prices from the unconstrained monopoly level, the firm can reduce the rebate it pays. On the other hand, when β takes values close to one, i.e., when MFC demand is almost inelastic, there is little demand reduction and the effect on quantity weighted average price dominates, leading the monopolist to reduce the rebate by decreasing prices. Note that the boundaries of these regions, $\underline{\beta}$ and $\bar{\beta}$, will vary with the fraction of MFCs, γ , the firm's cost, c , and the demand functions q_i . The determination of these boundaries is described in the proof. For intermediate values of β , prices in different markets may move in different directions as a result of APP.

5.1 Welfare Analysis of APP

The change in social welfare engendered by APP depends on how it causes market prices to move. From Proposition 3, we know that for $\beta < \underline{\beta}$, all market prices increase, and move further away from the competitive price (which is $p_i = c$ for all i). As a result, aggregate quantity falls and social welfare decreases. Conversely, for $\beta > \bar{\beta}$, all market prices decrease and move toward the competitive price. As a result, aggregate quantity rises and social welfare increases. This argument proves the following result:

Proposition 4 *Suppose Assumptions 1 and 4 hold. For $\beta < \underline{\beta}$, social welfare decreases when APP is imposed and (ii) for $\beta > \bar{\beta}$, social welfare increases when APP is imposed.*

Thus, under APP, at least in the cases where all market prices move in the same direction, whether the policy is welfare improving is easy to detect by looking to see if market prices fall.

6 Extensions and Applications

6.1 Rebate-responsive Minimum Price Provision

In our analysis, we have assumed that demand from Medicaid consumers (to the extent that it is price-sensitive) is based on pre-rebate market prices. This appears reasonable in the Medicaid context, not least because the rebates are essentially invisible to consumers. However, as mentioned in the Introduction, clauses similar to MPP appear in other contexts as well. Often, the analogue to rebates in these applications are more immediate and visible than under Medicaid. Thus, to expand the scope of application, and also as a robustness check on our Medicaid analysis, this section presents and analyzes a variation of MPP where MFC demand responds to post-rebate prices.

Under rebate-responsive MPP (RMPP), MFCs in market i receive a rebate r , which is the difference between the market price, p_i , and the minimum price, p_{\min} . The post-rebate price for MFCs in market i , is therefore given by

$$p_i - r = p_i - (p_i - p_{\min}) = p_{\min}.$$

Unlike what we assumed earlier, MFCs demand depends on the post-rebate price, $p_i - r$. With RMPP, the monopolist therefore chooses market prices, p_i , to maximize

$$(1 - \gamma) \sum_{i=1}^n (p_i - c) q_i(p_i) + \gamma \sum_{i=1}^n ((p_i - r) - c) ((1 - \beta) q_i(p_i - r) + \beta z_i) \quad (6.1)$$

$$= (1 - \gamma) \sum_{i=1}^n (p_i - c) q_i(p_i) + \gamma \sum_{i=1}^n (p_{\min} - c) ((1 - \beta) q_i(p_{\min}) + \beta z_i). \quad (6.2)$$

We define \tilde{p}_i as the optimal monopoly price if facing only the non-MFCs in market i . Without loss of generality and because it will prove convenient, we order the markets so that $\tilde{p}_1 < \tilde{p}_2 < \dots < \tilde{p}_n$. Note that this ordering of the markets is on the basis of the optimal monopoly prices when facing the non-MFC consumers only, and that this is different from the way we ordered markets in the previous sections. Here, \tilde{p}_i solves

$$(p - c)q'_i(p) + q_i(p) = 0.$$

The following condition is useful in characterizing the optimal solution under RMPP:

$$\beta\gamma \sum_{i=1}^n z_i + (1 - \beta\gamma) \sum_{i=1}^n [(\tilde{p}_n - c)q'_i(\tilde{p}_n) + q_i(\tilde{p}_n)] \geq 0. \quad (\text{Condition U})$$

If the same price is being charged in all markets, the left-hand side of Condition U is the derivative of the profit function with respect to price evaluated at a price of \tilde{p}_n . Therefore, given strict concavity, Condition U is necessary and sufficient for the optimal uniform price to be above \tilde{p}_n . Note that, by definition, $[(\tilde{p}_n - c)q'_n(\tilde{p}_n) + q_n(\tilde{p}_n)]$ is zero, whereas, by concavity, $[(\tilde{p}_n - c)q'_i(\tilde{p}_n) + q_i(\tilde{p}_n)]$ is negative for any other i . The next result describes the optimal solution under RMPP and shows that Condition U determines whether this solution involves uniform pricing.

Proposition 5 *Suppose Assumptions 1 and 2 hold. If Condition U is violated, the solution of the profit maximization problem under RMPP is of the form*

$$(\underbrace{\hat{p}, \dots, \hat{p}}_{k \text{ times}}, \tilde{p}_{k+1}, \dots, \tilde{p}_n)$$

where $\hat{p} \in [\tilde{p}_k, \tilde{p}_{k+1})$ and $k \in \{1, 2, \dots, n-1\}$. If Condition U holds, the solution of the profit maximization problem under RMPP will be of the form $(\underbrace{\hat{p}, \dots, \hat{p}}_{n \text{ times}})$ (i.e., uniform pricing) where $\hat{p} \geq \tilde{p}_n$.

Several comments are in order. First, note that Assumption 2 is enough to guarantee strict concavity of the profit function in the minimum market price and the validity of the first-order approach, unlike in our earlier MPP analysis. The reason for this is that, since the minimum price (post-rebate price) rather than the market price affects MFC demand, the expressions involving prices in the objective function (6.2) are either linear in price (the inelastic part) or in the form of a standard profit function addressed by Assumption 2. Second, under RMPP, the minimum price may be charged in more than one market, even though distinct prices would be charged in each market in the absence of RMPP.¹⁵ Third, n markets with RMPP turns out to be no more messy or difficult than the two market case, which was less true of MPP and led to our choice to present the two market case in that analysis. Finally, as was true for MPP, market prices may decrease in some of the markets under RMPP compared to the unconstrained case. In those markets where the minimum price is not charged under RMPP, the monopolist will optimally charge the monopoly price as if demand in that market came only from non-MFCs. In those markets, before MPP is imposed, the optimal market price was higher (with equality if $\beta = 0$) than the optimal monopoly price based on only the non-MFC section (this is because of the fact that if demand in a market is composed of both elastic and inelastic demands, then the optimal monopoly price for the combined market is higher than the optimal monopoly price for the elastic demand section only). Therefore, these prices decrease under MPP. However, as with MPP, we will now show that prices cannot fall in all markets under RMPP. In particular, the minimum market price charged under RMPP will always be higher than the minimum market price under unconstrained price discrimination. Formally:

Proposition 6 *Suppose Assumptions 1 and 2 hold. The minimum market price increases under RMPP, compared to the unconstrained case. In those markets where the minimum price is not charged, market prices decrease under RMPP, compared to the unconstrained case.*

6.1.1 Welfare Analysis

As before, we apply Fact 1 to derive bounds on the change in welfare resulting from the imposition of RMPP. The following proposition uses Fact 1 and the solutions to the monopoly pricing problems in the inelastic demand framework with and without RMPP to obtain bounds on the change in welfare.

Proposition 7 *Suppose Assumptions 1 and 2 hold. The change in welfare, when moving from no RMPP to an RMPP policy, satisfies the following lower bound:*

$$\Delta W \geq (\hat{p} - c) \Delta Q$$

where ΔQ denotes the corresponding change in aggregate demand. Furthermore, if $q_i(p)$ is concave in $p \geq 0$ for all $i = 1, 2, \dots, n$, then the change in welfare satisfies the following upper bound:

$$(\hat{p} - c) \Delta Q - \Delta \pi + B_1 + B_2 \geq \Delta W$$

¹⁵This point is germane to the relation with Scott Morton's [29] analysis of minimum price provision. Her model corresponds to RMPP assuming linear demand and $\beta = 0$ or $\beta = 1$. Comparing our result under those assumptions to her solution, we see that Scott Morton [29] must be implicitly assuming that the minimum price is charged in only one market (i.e., $k = 1$ in our proposition).

where

$$B_1 = \begin{cases} (1 - \gamma) \sum_{i=k+1}^n (\tilde{p}_i - \hat{p}) (q_i(\tilde{p}_i) - q_i(p_i^m)) & \text{if Condition } U \text{ is violated and the minimum market price is charged in } k \text{ markets with } k < n \\ 0 & \text{if Condition } U \text{ holds} \end{cases},$$

and

$$B_2 = \begin{cases} \frac{\beta\gamma(1-\gamma)}{1-\beta\gamma} \sum_{i=k+1}^n (\hat{p} - \tilde{p}_i) z_i & \text{if Condition } U \text{ is violated and the minimum market price is charged in } k \text{ markets with } k < n \\ 0 & \text{if Condition } U \text{ holds} \end{cases},$$

and $\Delta\pi$ denotes the change in the monopolist's profit.

The lower bound in Proposition 7 use knowledge of only the realized change in aggregate demand, the minimum price and the cost to bound the change in welfare. As was true with MPP, it is important to note that even if cost, for example, is unobserved, the lower bound implies that welfare always increases when imposing RMPP results in an increase in aggregate demand.

6.1.2 Applications

Here we sketch a few applications of RMPP.

- a) Many European countries have adopted an external reference (ER) pricing scheme in regulating pharmaceutical prices. Under ER, the regulating country sets up a price cap based on prices in a selected group of countries. For example, the Netherlands and Switzerland introduced ER to regulate pharmaceutical prices in 1996. In the Netherlands, for a drug to be included in the list of reimbursed pharmaceuticals by the national health insurance, its price should not exceed the average price of the drug in Germany, France, UK and Belgium. Similarly, Switzerland averages the prices charged in Germany, Denmark, the Netherlands and UK. As of 2007, most of the European countries (excluding Denmark, Germany, Sweden and UK) have incorporated some forms of ER in their drug pricing regulations, but there is large variation in the design of the rules. First, the choice of reference countries differ. Second, some countries (for example, the Netherlands) uses the average of prices charged in the reference countries, while others (for example, Greece) use the minimum of the prices charged in the reference countries as a reference price.¹⁶ These rules were primarily introduced to import the low prices from the reference markets to the regulated market. However, the effectiveness of these rules in achieving this objective is significantly constrained by the drug manufacturers' response through changing the price distribution across markets. Our RMPP framework is related to applications of ER. In RMPP, the MFC price is capped by the minimum price among prices charged in n markets. Like ER, RMPP creates a cross market effect among the individual market prices. As we find with RMPP, prices in some of the reference markets may increase. Similar predictions are documented by Garcia Mariñoso, Jelovac and Olivella [16], who study the effect of ER in

¹⁶ For a detailed discussion on application of reference pricing in the context of OECD countries, see Heuer, Mejer and Neuhaus [12], Jacobzone [14] and Garcia Mariñoso, Jelovac and Olivella [16]. These rules are also frequently changed. For example, Sweden discontinued the practice of ER in 2002.

the European drug pricing context. Understanding the effects of these regulations is important for both the evaluation of the current policy as well as for providing normative suggestions for future regulation. Unfortunately, there are not many empirical studies determining the effect of ER in isolation. An important exception is Heuer, Mejer and Neuhaus [12] who find empirical evidence that drug manufacturers strategically respond to ER by delaying the launch of new drugs in low-price countries.

- b) Long term trading contracts with price protection: This type of contract is often present in markets where market power is on the side of the buyer. Applications include natural gas contracts (see e.g. Crocker and Lyon [9]) and other utility contracts. Sellers often sign contractual agreements with large buyers (or buyers with large sellers) to provide the buyers (or sellers) with price protection over an extended time period. We can accommodate this problem in our set up in the following way. Consider this as an n period problem, where demand may change from period to period. A section of buyers, treated as most favored customers, will be paying the minimum price that prevails over the n periods. However, the seller is allowed to charge different prices in different periods to other customers. As long as it is not possible to substitute demand in one period for demand in another, we can treat these n different periods as n different markets with distinct demand curves. If the section of most favored customers remains a fixed fraction of the total consumers in every market, this formulation will directly fit our model.
- c) Exogenous shift of consumers between markets: Consider the example of an electronics goods manufacturer who sells her product in different locations through retailers. Retailers differ in their bargaining power, depending on the size and elasticity of their individual markets. Assuming a high level of search cost, this would typically result in high dispersion in retail prices. Now consider an exogenous mechanism that can reduce the search cost for a section of consumers. For example, with the growth of web based transactions, almost every retailer now maintains a web site that allows online purchase of electronics goods. Not everybody can easily access or feels comfortable using that market, but for those who do, search cost is reduced to a large extent. Assuming that the fraction of consumers who may exercise the online purchasing option remains relatively constant across different markets, this implies that a section of consumers from every market now pay the minimum price (ignoring differences in retailer service provision and return policies).

What is important from a theoretical perspective is that the external referencing to calculate a price cap or the long term trading contractual agreements or exogenous shifts in location of consumers create a cross-market effect among the individual market prices in the monopolist's objective function. In each market, a fraction of the consumers is now paying a price that is connected to the prices charged in other markets. The RMPP model precisely deals with the situation where this cross-market connection is induced through minimum price protection.

7 Summary

Our analyses in sections 4 and 5 show how the MPP and APP rebate rules affect a monopolist's optimal pricing strategy as well as social welfare under third-degree price discrimination. In the context of

MPP, we present our analysis with two markets. The minimum market price charged always rises compared to the no regulation case. In contrast, prices in markets where the minimum is not charged will fall. The welfare effect of MPP may be good or bad. A useful sufficient condition for MPP to be welfare improving is that MPP raise aggregate quantity. We also analyze a rebate-responsive version, RMPP, where MFCs demand are affected by the post-rebate (i.e., minimum) price. We find that RMPP has effects on prices and social welfare similar to MPP and suggest a number of applications beyond the Medicaid context.

Under APP, we find that all market prices move in the same direction in two different scenarios: when MFCs demand is sufficiently inelastic or when MFCs' demand is sufficiently similar to non-MFCs demand. When MFCs' demand is sufficiently inelastic, all market prices decrease, resulting in an increase in aggregate quantity and social welfare. In contrast, when MFCs' demand is close to non-MFCs' demand, all market prices increase, resulting in a decrease in aggregate quantity and social welfare.

The analysis of these policies is surprisingly intricate, even in a relatively simple setting such as ours. This suggests that great care is needed when implementing such MFC rules and that making provisions for data collection to support follow-up empirical work measuring the pricing and demand response has high potential value in avoiding mistakes or helping fine-tune the policy. Some theoretical issues that we have not addressed here, such as incorporating demand uncertainty, second-degree price discrimination and the effect on dynamic R&D incentives for the manufacturer are interesting topics for future work to explore.

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8 Appendix

Proof of Lemma 1. Suppose, if possible, $\hat{p}_1 > \hat{p}_2$. There are three possible cases to consider. First, let us suppose that $\hat{p}_1 \geq p^u \geq \hat{p}_2$. Compare this with charging $p_1 = p^u = p_2$. The rebate will be

weakly lower in the latter case. Profits in the second market will be weakly higher, as price is moving closer to p_2^m (by Assumption 3). Similarly, in the first market profits will be weakly higher, as price is moving closer to p_1^m from above (by Assumption 3). Thus, $p_1 = p^u = p_2$ dominates $\hat{p}_1 \geq p^u \geq \hat{p}_2$. Next suppose $\hat{p}_1 > \hat{p}_2 \geq p^u$. Compare this with $p_1 = \hat{p}_2 = p_2$. Again the rebate is lower in the latter case, profits from the second market are the same, while profits from the first market increase since p_1 is getting closer to p_1^m from above (by Assumption 3). Thus $p_1 = \hat{p}_2 = p_2$ dominates $\hat{p}_1 > \hat{p}_2 \geq p^m$. Finally, suppose $p^u \geq \hat{p}_1 > \hat{p}_2$. By similar arguments this is dominated by $p_1 = \hat{p}_1 = p_2$. ■

Proof of Proposition 1. Given Lemma 1, there are two possibilities to consider: $\hat{p}_1 = \hat{p}_2$ and $\hat{p}_1 < \hat{p}_2$. In the first scenario (i.e., when $\hat{p}_1 = \hat{p}_2$), the optimal solution will be to charge the uniform monopoly price p^u . Since both prices are the same, the effective rebate equals zero, which is the lowest possible effective rebate. Therefore, the solution of (4.1) is also the solution of the maximization problem when the monopolist maximizes profits (the first sum in equation 4.1), under the constraint of uniform pricing.

A remaining question is thus, when is the uniform monopoly price optimal? When it is not, we know that $\hat{p}_1 < \hat{p}_2$. We claim that it is sufficient to prove optimality of the uniform monopoly price (under Assumption 2) by checking whether starting from the uniform monopoly price it does not give a local improvement to either lower p_1 or raise p_2 . When will these moves not give a local improvement? When the partial derivative of (4.1) with respect to p_1 when taken from below and evaluated at uniform monopoly prices is positive and the partial derivative of (4.1) with respect to p_2 when taken from above and evaluated at uniform monopoly prices is negative. Formally, these one-sided partial derivatives are, from below and above respectively:

$$(p - c)(1 - \beta\gamma)q_1'(p) + ((1 - \beta\gamma)q_1(p) + \beta\gamma z_1) - \gamma(p_q(p, p_2) - p)(1 - \beta)q_1'(p) \quad (8.1)$$

$$- \gamma \left(\frac{dp_q(p, p_2)}{dp} - 1 \right) ((1 - \beta)(q_1(p) + q_2(p_2)) + \beta(z_1 + z_2))$$

where

$$\frac{dp_q(p, p_2)}{dp} = \frac{(p - p_q(p, p_2))(1 - \beta\gamma)q_1'(p) + ((1 - \beta\gamma)q_1(p) + \beta\gamma z_1)}{((1 - \beta\gamma)q_1(p) + \beta\gamma z_1) + ((1 - \beta\gamma)q_2(p_2) + \beta\gamma z_2)}$$

and

$$(p - c)(1 - \beta\gamma)q_2'(p) + ((1 - \beta\gamma)q_2(p) + \beta\gamma z_2) - \gamma(p_q(p_1, p) - p_1)(1 - \beta)q_2'(p) \quad (8.2)$$

$$- \gamma \frac{dp_q(p_1, p)}{dp} ((1 - \beta)(q_1(p) + q_2(p_2)) + \beta(z_1 + z_2))$$

where

$$\frac{dp_q(p_1, p)}{dp} = \frac{(p - p_q(p_1, p))(1 - \beta\gamma)q_2'(p) + ((1 - \beta\gamma)q_2(p) + \beta\gamma z_2)}{((1 - \beta\gamma)q_1(p) + \beta\gamma z_1) + ((1 - \beta\gamma)q_2(p_2) + \beta\gamma z_2)}.$$

When calculated at $(p_1 = p^u, p_2 = p^u)$, (8.1) and (8.2) simplify to:

$$(p^u - c)(1 - \beta\gamma)q_1'(p^u) + ((1 - \beta\gamma)q_1(p^u) + \beta\gamma z_1) \quad (8.3)$$

$$+ \gamma((1 - \beta\gamma)q_2(p^u) + \beta\gamma z_2) \frac{\sum_{i=1}^2 ((1 - \beta)q_i(p^u) + \beta z_i)}{\sum_{i=1}^2 ((1 - \beta\gamma)q_i(p^u) + \beta\gamma z_i)}$$

and

$$(p^u - c)(1 - \beta\gamma)q_2'(p^u) + ((1 - \beta\gamma)q_2(p^u) + \beta\gamma z_2) \left(1 - \gamma \frac{\sum_{i=1}^2 ((1 - \beta)q_i(p^u) + \beta z_i)}{\sum_{i=1}^2 ((1 - \beta\gamma)q_i(p^u) + \beta\gamma z_i)} \right) \quad (8.4)$$

respectively.

Recall that the uniform monopoly price is defined by the condition

$$(p^u - c)(1 - \beta\gamma)(q'_1(p^u) + q'_2(p^u)) + \sum_{i=1}^2 ((1 - \beta\gamma)q_i(p^u) + \beta\gamma z_i) = 0.$$

Using this to substitute into (8.3) gives:

$$\begin{aligned} & - (p^u - c)(1 - \beta\gamma)q'_2(p^u) - ((1 - \beta\gamma)q_2(p^u) + \beta\gamma z_1) \\ & + \gamma((1 - \beta\gamma)q_2(p^u) + \beta\gamma z_2) \frac{\sum_{i=1}^2 ((1 - \beta)q_i(p^u) + \beta z_i)}{\sum_{i=1}^2 ((1 - \beta\gamma)q_i(p^u) + \beta\gamma z_i)}, \end{aligned}$$

which is positive (so there is no gain from lowering p_1) exactly when

$$(p^u - c)(1 - \beta\gamma)q'_2(p^u) + ((1 - \beta\gamma)q_2(p^u) + \beta\gamma z_2) \left(1 - \gamma \frac{\sum_{i=1}^2 ((1 - \beta)q_i(p^u) + \beta z_i)}{\sum_{i=1}^2 ((1 - \beta\gamma)q_i(p^u) + \beta\gamma z_i)} \right) \leq 0.$$

The other partial (8.4) is negative (so there is no gain from raising p_2) exactly at the same condition. Thus, whenever (4.2) holds true, the uniform monopoly price is optimal, and otherwise the optimum will have $\hat{p}_1 < p^u < \hat{p}_2$.

We further claim that $\hat{p}_1 \geq p_1^m$ and $\hat{p}_2 \leq p_2^m$. At the uniform monopoly price, this is trivially true. Consider the possibility when $\hat{p}_1 < \hat{p}_2$. Suppose, if possible, $\hat{p}_1 < p_1^m$. By raising p_1 a bit, we raise profits in market 1, while, because \hat{p}_1 is getting closer to \hat{p}_2 the total rebate shrinks (formally as shown above the total rebate shrinks by $\gamma q_2(p_2)$ as \hat{p}_1 increases) thus it cannot be optimal to have $\hat{p}_1 < p_1^m$. To see the other inequality, suppose $\hat{p}_2 > p_2^m$. By lowering \hat{p}_2 a bit we raise profits in market 2. What happens to the rebate? As long as \hat{p}_2 is weakly below the monopoly price for market 2 that would hold if cost were \hat{p}_1 , then lowering \hat{p}_2 lowers the total rebate. Furthermore, that is the relevant region because, assuming concavity, we need to look at only whether it would be optimal to raise \hat{p}_2 above p_2^m starting from \hat{p}_2 . Thus it is never optimal to have $\hat{p}_2 > p_2^m$. ■

Proof of Proposition 2. To apply Fact 1, take

$$\mathbf{x}(\mathbf{P}) = ((1 - \beta\gamma)q_1(p_1) + \beta\gamma z_1, (1 - \beta\gamma)q_2(p_2) + \beta\gamma z_2).$$

From the right-hand-side inequality of Fact 1, we see that

$$\begin{aligned} \Delta W & \geq (1 - \beta\gamma)[(\hat{p}_1 - c)(q_1(\hat{p}_1) - q_1(p_1^m)) + (\hat{p}_2 - c)(q_2(\hat{p}_2) - q_2(p_2^m))] \\ & = (1 - \beta\gamma)(\hat{p}_1 - c) \sum_{i=1}^2 (q_i(\hat{p}_i) - q_i(p_i^m)) + (1 - \beta\gamma)(\hat{p}_2 - \hat{p}_1)(q_2(\hat{p}_2) - q_2(p_2^m)). \end{aligned} \quad (8.5)$$

Notice that the change in aggregate demand, ΔQ , is given by

$$\Delta Q = \sum_{i=1}^2 (1 - \beta\gamma)(q_i(\hat{p}_i) - q_i(p_i^m)) \quad (8.6)$$

The inequality in (8.5) therefore gives us

$$\Delta W \geq (\hat{p}_1 - c) \Delta Q + (1 - \beta\gamma)(\hat{p}_2 - \hat{p}_1)(q_2(\hat{p}_2) - q_2(p_2^m)).$$

By Proposition 1, $\hat{p}_1 \leq \hat{p}_2 \leq p_2^m$ and therefore, we have $(1 - \beta\gamma)(\hat{p}_2 - \hat{p}_1)(q_2(\hat{p}_2) - q_2(p_2^m)) \geq 0$. Hence, we get the following

$$\Delta W \geq (\hat{p}_1 - c) \Delta Q.$$

Next, assume that $q_i(p)$ is concave in $p \geq 0$ for $i = 1, 2$. We, therefore, have

$$q_i(\hat{p}_i) - q_i(p_i^m) \leq (\hat{p}_i - p_i^m) q_i'(p_i^m).$$

Or,

$$(p_i^m - c)(1 - \beta\gamma)(q_i(\hat{p}_i) - q_i(p_i^m)) \leq (\hat{p}_i - p_i^m)(p_i^m - c)(1 - \beta\gamma)q_i'(p_i^m).$$

Since p_i^m maximizes $(p - c)((1 - \beta\gamma)q_i(p) + \beta\gamma z_i)$ by definition, the first-order condition gives

$$(p_i^m - c)(1 - \beta\gamma)q_i'(p_i^m) = -((1 - \beta\gamma)q_i(p_i^m) + \beta\gamma z_i).$$

Therefore, we have

$$(p_i^m - c)(1 - \beta\gamma)(q_i(\hat{p}_i) - q_i(p_i^m)) \leq (p_i^m - \hat{p}_i)((1 - \beta\gamma)q_i(p_i^m) + \beta\gamma z_i). \quad (8.7)$$

From the left-hand-side inequality of Fact 1, we see that

$$\Delta W \leq (1 - \beta\gamma)[(p_1^m - c)(q_1(\hat{p}_1) - q_1(p_1^m)) + (p_2^m - c)(q_2(\hat{p}_2) - q_2(p_2^m))]$$

Combining the above inequality with the inequality in (8.7), we get

$$\Delta W \leq \sum_{i=1}^2 (p_i^m - \hat{p}_i)((1 - \beta\gamma)q_i(p_i^m) + \beta\gamma z_i). \quad (8.8)$$

The change in the monopolist's profit (including rebate) can be written as

$$\begin{aligned} \Delta\pi &= \sum_{i=1}^2 (\hat{p}_i - c)((1 - \beta\gamma)q_i(\hat{p}_i) + \beta\gamma z_i) - \sum_{i=1}^2 (p_i^m - c)((1 - \beta\gamma)q_i(p_i^m) + \beta\gamma z_i) \\ &= \sum_{i=1}^2 \hat{p}_i((1 - \beta\gamma)q_i(\hat{p}_i) + \beta\gamma z_i) - \sum_{i=1}^2 p_i^m((1 - \beta\gamma)q_i(p_i^m) + \beta\gamma z_i) - c\Delta Q. \end{aligned}$$

After adding and subtracting $\Delta\pi$ to the right-hand side expression in (8.8) and rearranging terms, we get

$$\begin{aligned} \Delta W &\leq \sum_{i=1}^2 \hat{p}_i((1 - \beta\gamma)q_i(\hat{p}_i) + \beta\gamma z_i) - \sum_{i=1}^2 \hat{p}_i((1 - \beta\gamma)q_i(p_i^m) + \beta\gamma z_i) - c\Delta Q - \Delta\pi \\ &\leq (1 - \beta\gamma)[\hat{p}_1(q_1(\hat{p}_1) - q_1(p_1^m)) + \hat{p}_2(q_2(\hat{p}_2) - q_2(p_2^m))] - c\Delta Q - \Delta\pi \\ &= (\hat{p}_1 - c)\Delta Q - \Delta\pi + (1 - \beta\gamma)(\hat{p}_2 - \hat{p}_1)(q_2(\hat{p}_2) - q_2(p_2^m)). \end{aligned}$$

■

Proof of Proposition 3. We first calculate the derivative of the quantity weighted average price with respect to individual prices. Define $m_i \equiv \frac{\beta\gamma}{1-\beta\gamma}z_i$.

$$\begin{aligned} \frac{d}{dp_i} p_q &= \frac{d}{dp_i} \frac{\sum_{i=1}^n p_i(q_i(p) + m_i)}{\sum_{i=1}^n (q_i(p) + m_i)} \\ &= \frac{(\sum_{i=1}^n (q_i(p) + m_i))(p_i q_i'(p) + q_i(p) + m_i) - (\sum_{i=1}^n p_i(q_i(p) + m_i))q_i'(p)}{(\sum_{i=1}^n (q_i(p) + m_i))^2} \\ &= \frac{(p_i - p_q)q_i'(p) + q_i(p) + m_i}{\sum_{i=1}^n (q_i(p) + m_i)}. \end{aligned}$$

The first-order conditions of the APP-constrained problem are given by

$$\frac{d}{dp_i} \left\{ \sum_{i=1}^n (p_i - c)((1 - \beta\gamma)q_i(p_i) + \beta\gamma z_i) \right\} \quad (8.9)$$

$$-\alpha\gamma \frac{d}{dp_i} \{p_q \sum_{i=1}^n ((1 - \beta)q_i(p_i) + \beta z_i)\} = 0, \text{ for } i = 1, 2, \dots, n. \quad (8.10)$$

If computed at (p_1^m, \dots, p_n^m) , the left-hand side of the first-order condition is

$$-\alpha\gamma \frac{d}{dp_i} \{p_q \sum_{i=1}^n ((1-\beta) q_i(p_i) + \beta z_i)\} \big|_{p=(p_1^m, \dots, p_n^m)} \quad (8.11)$$

This is because the first part of (8.9), when computed at (p_1^m, \dots, p_n^m) , is zero as (p_1^m, \dots, p_n^m) is the solution of the unconstrained problem. Further, notice that

$$\begin{aligned} & \frac{d}{dp_i} \{p_q \sum_{i=1}^n ((1-\beta) q_i(p_i) + \beta z_i)\} \\ &= (\sum_{i=1}^n ((1-\beta) q_i(p_i) + \beta z_i)) \frac{dp_q}{dp_i} + p_q (1-\beta) q'_i(p_i) \\ &= (1-\beta) p_q q'_i(p_i) + (\sum_{i=1}^n ((1-\beta) q_i(p_i) + \beta z_i)) \frac{(p_i - p_q) q'_i(p_i) + q_i(p_i) + m_i}{\sum_{i=1}^n (q_i(p_i) + m_i)} \\ &= \frac{1}{\sum_{i=1}^n (q_i(p_i) + m_i)} \left[\frac{p_q q'_i(p_i) \{((1-\beta) \sum_{i=1}^n (q_i(p_i) + m_i) - \sum_{i=1}^n ((1-\beta) q_i(p_i) + \beta z_i))\}}{(p_i q'_i(p_i) + q_i(p_i) + m_i) (\sum_{i=1}^n ((1-\beta) q_i(p_i) + \beta z_i))} \right] \\ &= \frac{1}{\sum_{i=1}^n (q_i(p_i) + m_i)} \left[\frac{p_q q'_i(p_i) \left\{ \left((1-\beta) \sum_{i=1}^n \frac{\beta \gamma z_i}{1-\beta \gamma} \right) - \sum_{i=1}^n \beta z_i \right\}}{(p_i q'_i(p_i) + q_i(p_i) + m_i) (\sum_{i=1}^n ((1-\beta) q_i(p_i) + \beta z_i))} \right] \quad (8.12) \end{aligned}$$

Note that when computed at (p_1^m, \dots, p_n^m) , $(p_i q'_i(p_i) + q_i(p_i) + m_i) = c q'_i(p_i)$ (by (3.1)). Also, as $\frac{\beta \gamma (1-\beta)}{1-\beta \gamma} - \beta = -\frac{\beta(1-\gamma)}{1-\beta \gamma}$, we can simplify (8.12), when computed at (p_1^m, \dots, p_n^m) , as

$$\frac{1}{\sum_{i=1}^n (q_i(p) + m_i)} \left[c q'_i(p_i) (\sum_{i=1}^n ((1-\beta) q_i(p_i) + \beta z_i)) - \frac{\beta(1-\gamma)}{1-\beta \gamma} p_q q'_i(p_i) \sum_{i=1}^n z_i \right]. \quad (8.13)$$

Therefore, the first-order derivative of the APP-constrained problem, when computed at (p_1^m, \dots, p_n^m) , can be written (by (8.11)) as

$$\frac{\alpha\gamma}{\sum_{i=1}^n (q_i(p_i) + m_i)} \left[\frac{\beta(1-\gamma)}{1-\beta \gamma} p_q q'_i(p_i) \sum_{i=1}^n z_i - c q'_i(p_i) (\sum_{i=1}^n ((1-\beta) q_i(p_i) + \beta z_i)) \right]. \quad (8.14)$$

Notice that $\frac{\alpha\gamma}{\sum_{i=1}^n (q_i(p) + m_i)} > 0$, $\frac{\beta(1-\gamma)}{1-\beta \gamma} p_q q'_i(p_i) \sum_{i=1}^n z_i \leq 0$ and $c q'_i(p_i) (\sum_{i=1}^n ((1-\beta) q_i(p_i) + \beta z_i)) < 0$ always. Therefore, sign of (8.14) will be determined by relative values of the two terms, $\frac{\beta(1-\gamma)}{1-\beta \gamma} p_q q'_i(p_i) \sum_{i=1}^n z_i$ and $c q'_i(p_i) (\sum_{i=1}^n ((1-\beta) q_i(p_i) + \beta z_i))$. In general, the first-order conditions can take either sign. In order to prove the proposition, we calculate the partial derivatives at two extreme values of β . At $\beta = 0$, when MFC demand is as elastic as non-MFCs, the term in (8.14) can be written as

$$-\alpha\gamma c q'_i(p_i),$$

which is always positive, as $q'_i(p_i) < 0$ for all $i = 1, 2, \dots, n$. On the other hand, at $\beta = 1$, when MFC demand is completely inelastic, the term in (8.14) can be written as

$$\frac{\alpha\gamma(1-\gamma)}{\sum_{i=1}^n ((1-\gamma) q_i(p) + \gamma z_i)} [(p_q - c) q'_i(p_i) \sum_{i=1}^n z_i],$$

which is always negative for all $i = 1, 2, \dots, n$.

Since (8.14) evaluated at $p_i = p_i^m$ (itself a continuous function of β) is continuous in $\beta \in [0, 1]$, and takes a positive value at $\beta = 0$ and a negative value at $\beta = 1$, we can find two numbers $\underline{\beta}_i$ and $\bar{\beta}_i$ such that $0 \leq \underline{\beta}_i \leq \bar{\beta}_i \leq 1$ and (8.14) evaluated at $p_i = p_i^m$ is always negative for $\beta \in [\bar{\beta}_i, 1]$

and always positive for $\beta \in [0, \underline{\beta}]$. Set $\underline{\beta} = \min_i \{\underline{\beta}_i\}$ and $\bar{\beta} = \max_i \{\bar{\beta}_i\}$. In this way, (8.14) fully determines. Therefore, for $\beta \in [0, \underline{\beta}]$, the partial derivative with respect to p_i , computed at (p_1^m, \dots, p_n^m) , is positive for all i . By global concavity, we see that price increases in every market, in comparison to (p_1^m, \dots, p_n^m) , the solution of the unconstrained problem. Similarly, for $\beta \in [\bar{\beta}, 1]$ the partial derivative with respect to p_i , computed at (p_1^m, \dots, p_n^m) , is negative for all i . By global concavity, we see that price decreases in every market, in comparison to (p_1^m, \dots, p_n^m) , the solution of the unconstrained problem. ■

Proof of Proposition 5. Let a solution vector be (p_1^*, \dots, p_n^*) and $J = \{i \in \{1, 2, \dots, n\} \mid p_i^* = \min \{p_1^*, \dots, p_n^*\}\}$.

Claim 1: If $j \notin J$, then $p_j^* = \tilde{p}_j$.

If $j \notin J$, then $p_j^* > \min \{p_1^*, \dots, p_n^*\}$. p_j^* is also the solution of the optimization problem: $\max_p (p - c) q_j(p)$ such that $p \geq \min \{p_1^*, \dots, p_n^*\}$.

If $\tilde{p}_j > \min \{p_1^*, \dots, p_n^*\}$, and as \tilde{p}_j maximizes $(p - c) q_j(p)$ globally, $p_j^* = \tilde{p}_j$.

If $\tilde{p}_j \leq \min \{p_1^*, \dots, p_n^*\}$, then $(p - c) q_j(p)$ being concave in p , is maximized at $p = \min \{p_1^*, \dots, p_n^*\}$ over the range $\{p : p \geq \min \{p_1^*, \dots, p_n^*\}\}$. This implies that $p_j^* = \min \{p_1^*, \dots, p_n^*\}$, or, $j \in J$. which is ruled out.

Claim 2: If $j \in J, l \notin J$, then $j < l$.

If not, let us suppose $\exists l \notin J$ and $j \in J$ such that $j > l$.

Then, Claim 1 suggests $p_l^* = \tilde{p}_l$. Moreover, $\tilde{p}_l > \min \{p_1^*, \dots, p_n^*\}$ since $l \notin J$. As $j > l$, we have $\tilde{p}_j > \tilde{p}_l > \min \{p_1^*, \dots, p_n^*\}$. Therefore, $j \notin J$. Contradiction.

Claim 3: $\min \{p_1^*, \dots, p_n^*\} \in [\tilde{p}_k, \tilde{p}_{k+1})$ for $k = \max J$.

By Claim 1, $p_{k+1}^* = \tilde{p}_{k+1} > \min \{p_1^*, \dots, p_n^*\}$. Suppose $p_k^* < \tilde{p}_k$. Then, the monopolist could strictly increase profits by setting $p_k^* = \tilde{p}_k$. This increases profits from the non-MFC customers in market k , and leaves all other terms in the profit expression unchanged.

Claim 4: $k < n$.

Suppose $k = n$. Then $\min \{p_1^*, \dots, p_n^*\} = p^u$, the uniform monopoly price. Since $\tilde{p}_n > p^u$, this contradicts Claim 3.

Claims 1, 2, 3 and 4 together yield that the solution is of the desired form.

It remains to show that the solution is unique. Suppose $(\tilde{p}_1, \dots, \tilde{p}_n)$ is a different solution. It can differ from (p_1^*, \dots, p_n^*) only in the choice of k and \hat{p} . We now show that there is a unique profit maximizing choice of k and \hat{p} so that the existence of such different solutions is not possible. For any fixed k , it follows from Assumption 2 that there is a unique profit maximizing price which satisfies $\max_p \sum_{i=1}^k (p - c) q_i(p) + \gamma \sum_{i=k+1}^n (p - c) q_i(p)$. Call this $\hat{p}(k)$. Suppose that there exist $k_1 < k_2$ such that $\hat{p}(k_1) \in [\tilde{p}_{k_1}, \tilde{p}_{k_1+1})$ and $\hat{p}(k_2) \in [\tilde{p}_{k_2}, \tilde{p}_{k_2+1})$ as was shown to be required for profit maximization by the first part of this proof. By revealed preference, profits from the first k_1 markets and the MFCs from the remaining markets are strictly higher when charging $\hat{p}(k_1)$ rather than $\hat{p}(k_2)$. Since $\hat{p}(k_2) \in [\tilde{p}_{k_2}, \tilde{p}_{k_2+1})$, profits from the non-MFCs in markets $k_1 + 1, \dots, k_2$ would be higher by charging the monopoly prices in those markets. Combining these facts implies that profits are higher with $k = k_1$ and $\hat{p} = \hat{p}(k_1)$ than with $k = k_2$ and $\hat{p} = \hat{p}(k_2)$. This shows that a profit maximizing solution of the required form must be unique. ■

Proof of Proposition 6. First, we show that the minimum price increases after RMPP is imposed. To study properties of the minimum market price, we construct an alternative optimization problem

and show that its optimal solution coincides with the optimal solution of the original problem (6.1). We then derive properties of the optimal minimum market price by studying the first order condition of this modified problem.

Given Proposition 5, the maximization problem (6.1) may be rewritten as the following problem of maximizing with respect to k and p_{\min} , where only an upper bound on p_{\min} is imposed:

$$\max_{p < \tilde{p}_{k+1}, k \in \{1, 2, \dots, n\}} \sum_{i=1}^k (p - c) ((1 - \beta\gamma)q_i(p) + \beta\gamma z_i) + (1 - \gamma) \sum_{i=k+1}^n (\tilde{p}_i - c)q_i(\tilde{p}_i) \quad (8.15)$$

$$+ \gamma \sum_{i=k+1}^n (p - c) ((1 - \beta)q_i(p) + \beta z_i)$$

where \tilde{p}_{n+1} defined as ∞ .¹⁷

Let \hat{p} and \hat{k} solve (8.15). We now show that the unique solution to the following unconstrained optimization problem is $p = \hat{p}$:

$$\max_p \sum_{i=1}^{\hat{k}} (p - c) ((1 - \beta\gamma)q_i(p) + \beta\gamma z_i) + \gamma \sum_{i=\hat{k}+1}^n (p - c) ((1 - \beta)q_i(p) + \beta z_i). \quad (8.16)$$

By strict concavity, this problem has a unique solution – call it p' . By inspection, if $p' < \tilde{p}_{\hat{k}+1}$ then $p' = \hat{p}$. Otherwise, the monopolist could strictly increase profits by setting $p = p'$ (instead of \hat{p}) in (8.15). Can it be that $p' \geq \tilde{p}_{\hat{k}+1}$ for some $\hat{k} \in \{1, 2, \dots, n-1\}$? Then $\hat{p} < p'$. Since p' optimizes a strictly concave function, any increase in p above \hat{p} , no matter how small, will increase the value of the objective function in (8.16). But some increase is always feasible in problem (8.15), as $\hat{p} < \tilde{p}_{\hat{k}+1}$ and so could be increased at least some amount and still remain the minimum. This would contradict the optimality of \hat{p} in (8.15) and so it cannot be that $p' \geq \tilde{p}_{\hat{k}+1}$. Therefore, $p' < \tilde{p}_{\hat{k}+1}$ and $p' = \hat{p}$.

Therefore, \hat{k} and \hat{p} solve the first order condition (in price) of (8.16):

$$(1 - \beta\gamma) \sum_{i=1}^{\hat{k}} [(\hat{p} - c)q'_i(\hat{p}) + q_i(\hat{p})] + \gamma(1 - \beta) \sum_{i=\hat{k}+1}^n [(\hat{p} - c)q'_i(\hat{p}) + q_i(\hat{p})] + \beta\gamma \sum_{i=1}^n z_i = 0. \quad (8.17)$$

The above condition characterizes the minimum market price under RMPP.

Let $j \in \{1, 2, \dots, n\}$ be the market in which the unconstrained monopoly price was lowest. Denoting that monopoly price by p_j^m , it is the unique solution of

$$(1 - \beta\gamma) [(p - c)q'_j(p) + q_j(p)] + \beta\gamma z_j = 0. \quad (8.18)$$

If we can show that $p_j^m \leq \hat{p}$, this will complete the first part of the proof. To see that $p_j^m \leq \hat{p}$, consider the function

$$S(p) \equiv (1 - \beta\gamma) \sum_{i=1}^{\hat{k}} [(p - c)q'_i(p) + q_i(p)] + \gamma(1 - \beta) \sum_{i=\hat{k}+1}^n [(p - c)q'_i(p) + q_i(p)] + \beta\gamma \sum_{i=1}^n z_i.$$

By Assumption 2, $S(p)$ is decreasing in p . Furthermore, for every $i = 1, 2, 3, \dots, n$, $(1 - \beta\gamma) [(p_j^m -$

¹⁷In this scenario, prices in all markets could even be greater than \tilde{p}_n . To accommodate such a possibility, we set the upper limit as infinity (by setting $\tilde{p}_{n+1} = \infty$).

c) $q'_i(p_j^m) + q_i(p_j^m)] + \beta\gamma z_i \geq 0$. Notice that

$$\begin{aligned}
S(p) &= \sum_{i=1}^{\widehat{k}} [(1-\beta\gamma) [(p-c)q'_i(p) + q_i(p)] + \beta\gamma z_i] \\
&\quad + \sum_{i=\widehat{k}+1}^n [\gamma(1-\beta) [(p-c)q'_i(p) + q_i(p)] + \beta\gamma z_i] \\
&\geq \sum_{i=1}^{\widehat{k}} [(1-\beta\gamma) [(p-c)q'_i(p) + q_i(p)] + \beta\gamma z_i] \\
&\quad + \sum_{i=\widehat{k}+1}^n [\gamma(1-\beta) [(p-c)q'_i(p) + q_i(p)] + \beta\gamma \frac{\gamma(1-\beta)}{1-\beta\gamma} z_i] \\
&= \sum_{i=1}^{\widehat{k}} [(1-\beta\gamma) [(p-c)q'_i(p) + q_i(p)] + \beta\gamma z_i] \\
&\quad + \frac{\gamma(1-\beta)}{1-\beta\gamma} \sum_{i=\widehat{k}+1}^n [(1-\beta\gamma) [(p-c)q'_i(p) + q_i(p)] + \beta\gamma z_i].
\end{aligned}$$

Hence, $S(p_j^m) \geq 0$ because term-by-term the final expression is non-negative. From (8.17), we know that $S(\widehat{p}) = 0$. Therefore, we have $p_j^m \leq \widehat{p}$ since $S(p)$ is decreasing in p .

The second part of the proposition, stating that prices decrease under RMPP in those markets where the minimum price is not charged, directly follows from Proposition 5. ■

Proof of Proposition 7. To make it easier to apply Fact 1, we make the following adjustment in our notation: when writing the price vector we will treat the MFC and non-MFC sections of each market as two different markets. The generic price vector is thus $\mathbf{P} = (p_1, \dots, p_{2n}) \in \mathbb{R}_+^{2n}$ where p_i and p_{n+i} denote the prices faced by the non-MFC and the MFC sections of market i respectively. Without MPP, the monopolist's optimal price vector is given by $(p_1^m, \dots, p_n^m, p_1^m, \dots, p_n^m)$. Under RMPP, applying Proposition 5, the optimal price vector is $(\widehat{p}, \dots, \widehat{p}, \widetilde{p}_{k+1}, \dots, \widetilde{p}_n, \widehat{p}, \dots, \widehat{p})$ where k is endogenously determined such that the minimum price is charged in the first k markets, if Condition U is violated or $\left(\underbrace{\widehat{p}, \dots, \widehat{p}}_{2n \text{ times}}\right)$ if Condition U holds. With the split-market representation of prices, note that the corresponding market demands will be

$$\mathbf{x}(\mathbf{P}) = ((1-\gamma)q_1(p_1), \dots, (1-\gamma)q_n(p_n), \gamma\{(1-\beta)q_1(p_{n+1}) + \beta z_1\}, \dots, \gamma\{(1-\beta)q_n(p_{2n}) + \beta z_n\}).$$

Let us first consider the case when Condition U is violated. Applying Fact 1 yields

$$\begin{aligned}
&(1-\beta\gamma) \sum_{i=1}^k (p_i^m - c) (q_i(\widehat{p}) - q_i(p_i^m)) + (1-\gamma) \sum_{i=k+1}^n (p_i^m - c) (q_i(\widetilde{p}_i) - q_i(p_i^m)) \\
&\quad + \gamma(1-\beta) \sum_{i=k+1}^n (p_i^m - c) (q_i(\widehat{p}) - q_i(p_i^m)) \\
&\geq \Delta W \\
&\geq (1-\beta\gamma) \sum_{i=1}^k (\widehat{p} - c) (q_i(\widehat{p}) - q_i(p_i^m)) + (1-\gamma) \sum_{i=k+1}^n (\widetilde{p}_i - c) (q_i(\widetilde{p}_i) - q_i(p_i^m)) \\
&\quad + \gamma(1-\beta) \sum_{i=k+1}^n (\widehat{p} - c) (q_i(\widehat{p}) - q_i(p_i^m)).
\end{aligned}$$

The change in aggregate demand, ΔQ , is given by

$$\Delta Q = (1-\beta\gamma) \sum_{i=1}^k (q_i(\widehat{p}) - q_i(p_i^m)) + (1-\gamma) \sum_{i=k+1}^n (q_i(\widetilde{p}_i) - q_i(p_i^m)) + \gamma(1-\beta) \sum_{i=k+1}^n (q_i(\widehat{p}) - q_i(p_i^m)). \quad (8.20)$$

The right-hand side inequality in (8.19) gives us

$$\begin{aligned}
\Delta W &\geq (1 - \beta\gamma) \sum_{i=1}^k (\hat{p} - c) (q_i(\hat{p}) - q_i(p_i^m)) + (1 - \gamma) \sum_{i=k+1}^n (\tilde{p}_i - \hat{p} + \hat{p} - c) (q_i(\tilde{p}_i) - q_i(p_i^m)) \\
&\quad + \gamma (1 - \beta) \sum_{i=k+1}^n (\hat{p} - c) (q_i(\hat{p}) - q_i(p_i^m)) \\
&= (\hat{p} - c) \Delta Q + (1 - \gamma) \sum_{i=k+1}^n (\tilde{p}_i - \hat{p}) (q_i(\tilde{p}_i) - q_i(p_i^m)) \\
&\geq (\hat{p} - c) \Delta Q \text{ since } \hat{p} \leq \tilde{p}_i \leq p_i^m \text{ for } i = k+1, \dots, n, \text{ by Proposition 6.}
\end{aligned}$$

Next consider the possibility when Condition U holds. The optimal price vector under MPP is

$\left(\underbrace{\hat{p}, \dots, \hat{p}}_{2n \text{ times}} \right)$ and the change in demand, ΔQ , is given by,

$$\Delta Q = (1 - \beta\gamma) \sum_{i=1}^n (q_i(\hat{p}) - q_i(p_i^m)).$$

Applying Fact 1 yields

$$\begin{aligned}
(1 - \beta\gamma) \{ \sum_{i=1}^n (p_i^m - c) (q_i(\hat{p}) - q_i(p_i^m)) \} &\geq \Delta W \\
&\geq (1 - \beta\gamma) \{ \sum_{i=1}^n (\hat{p} - c) (q_i(\hat{p}) - q_i(p_i^m)) \}.
\end{aligned} \tag{8.21}$$

The right-hand side inequality in (8.21) therefore gives us

$$\Delta W \geq (\hat{p} - c) \Delta Q.$$

Next, assume that $q_i(p)$ is concave in $p \geq 0$ for all $i = 1, 2, \dots, n$. We, therefore, have

$$q_i(\hat{p}) - q_i(p_i^m) \leq (\hat{p} - p_i^m) q'_i(p_i^m). \tag{8.22}$$

Or,

$$(p_i^m - c) (q_i(\hat{p}) - q_i(p_i^m)) \leq (p_i^m - c) (\hat{p} - p_i^m) q'_i(p_i^m).$$

Since p_i^m maximizes $(p - c) ((1 - \beta\gamma) q_i(p) + \beta\gamma z_i)$, using the first-order condition we get

$$(p_i^m - c) q'_i(p_i^m) = -q_i(p_i^m) - \frac{\beta\gamma z_i}{(1 - \beta\gamma)}. \tag{8.23}$$

Therefore, we have

$$(p_i^m - c) (q_i(\hat{p}) - q_i(p_i^m)) \leq (p_i^m - \hat{p}) \left(q_i(p_i^m) + \frac{\beta\gamma z_i}{(1 - \beta\gamma)} \right). \tag{8.24}$$

Similarly, we get

$$q_i(\tilde{p}_i) - q_i(p_i^m) \leq (\tilde{p}_i - p_i^m) q'_i(p_i^m),$$

or,

$$(p_i^m - c) (q_i(\tilde{p}_i) - q_i(p_i^m)) \leq (p_i^m - c) (\tilde{p}_i - p_i^m) q'_i(p_i^m). \tag{8.25}$$

Applying (8.23), inequality (8.25) becomes

$$(p_i^m - c) (q_i(\tilde{p}_i) - q_i(p_i^m)) \leq (p_i^m - \tilde{p}_i) \left(q_i(p_i^m) + \frac{\beta\gamma z_i}{(1 - \beta\gamma)} \right). \tag{8.26}$$

Combining (8.24) and (8.26) with the left-hand side inequality in (8.19), we get

$$\begin{aligned} \Delta W \leq & (1 - \beta\gamma) \sum_{i=1}^k (p_i^m - \hat{p}) \left(q_i(p_i^m) + \frac{\beta\gamma z_i}{(1 - \beta\gamma)} \right) + (1 - \gamma) \sum_{i=k+1}^n (p_i^m - \tilde{p}_i) \left(q_i(p_i^m) + \frac{\beta\gamma z_i}{(1 - \beta\gamma)} \right) \\ & + \gamma(1 - \beta) \sum_{i=k+1}^n (p_i^m - \hat{p}) \left(q_i(p_i^m) + \frac{\beta\gamma z_i}{(1 - \beta\gamma)} \right). \end{aligned} \quad (8.27)$$

Notice that the change in the monopolist's profit can be written as

$$\begin{aligned} \Delta \pi = & (1 - \beta\gamma) \sum_{i=1}^k (\hat{p} - c) \left(q_i(\hat{p}) + \frac{\beta\gamma z_i}{(1 - \beta\gamma)} \right) + (1 - \gamma) \sum_{i=k+1}^n (\tilde{p}_i - c) q_i(\tilde{p}_i) \\ & + \gamma(1 - \beta) \sum_{i=k+1}^n (\hat{p} - c) \left(q_i(\hat{p}) + \frac{\beta\gamma z_i}{(1 - \beta\gamma)} \right) - (1 - \beta\gamma) \sum_{i=1}^n (p_i^m - c) \left(q_i(p_i^m) + \frac{\beta\gamma z_i}{(1 - \beta\gamma)} \right). \end{aligned}$$

After adding and subtracting $\Delta \pi$ to the right-hand side expression in (8.27) and rearranging terms, we get

$$\Delta W \leq (\hat{p} - c) \Delta Q - \Delta \pi + (1 - \gamma) \sum_{i=k+1}^n (\tilde{p}_i - \hat{p}) (q_i(\tilde{p}_i) - q_i(p_i^m)) + \frac{\beta\gamma(1 - \gamma)}{1 - \beta\gamma} \sum_{i=k+1}^n (\hat{p} - \tilde{p}_i) z_i. \quad (8.28)$$

Next consider the possibility when Condition U holds. The optimal price vector under MPP is

$\left(\underbrace{\hat{p}, \dots, \hat{p}}_{2n \text{ times}} \right)$, and as shown above, we have

$$(1 - \beta\gamma) \left\{ \sum_{i=1}^n (p_i^m - c) (q_i(\hat{p}) - q_i(p_i^m)) \right\} \geq \Delta W.$$

Combining this inequality with (8.24), we get

$$\Delta W \leq (1 - \beta\gamma) \left\{ \sum_{i=1}^n (p_i^m - \hat{p}) \left(q_i(p_i^m) + \frac{\beta\gamma z_i}{(1 - \beta\gamma)} \right) \right\}. \quad (8.29)$$

Further, if Condition U holds, the change in profit is given by

$$\Delta \pi = (1 - \beta\gamma) \left\{ \sum_{i=1}^n (\hat{p} - c) \left(q_i(\hat{p}) + \frac{\beta\gamma z_i}{(1 - \beta\gamma)} \right) - \sum_{i=1}^n (p_i^m - c) \left(q_i(p_i^m) + \frac{\beta\gamma z_i}{(1 - \beta\gamma)} \right) \right\}.$$

After adding and subtracting $\Delta \pi$ to the right-hand side expression in (8.29) and rearranging terms, we get

$$\Delta W \leq (\hat{p} - c) \Delta Q - \Delta \pi.$$

■