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APPENDIX TO "COMPLEMENTS AND SUBSTITUTES IN SEQUENTIAL AUCTIONS: THE CASE OF WATER AUCTIONS"*

(For Online Publication)

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Abstract

We use data on sequential water auctions to estimate demand when units are complements or substitutes. A sequential English auction model determines the estimating structural equations. When units are complements, one bidder wins all units by paying a high price for the first unit, thus deterring others from bidding on subsequent units. When units are substitutes, different bidders win the units with positive probability, paying prices similar in magnitude, even when the same bidder wins all units. We recover individual demand consistent with this stark pattern of outcomes and confirm it is not collusive, but consistent with non-cooperative behavior. Demand estimates are biased if one ignores these features.

JEL CODES: D44, C13, L10, L40

KEYWORDS: Auctions, Structural Demand Estimation, Market Structure, Competition, Collusion

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Appendix

This is the Web Appendix for "Complements and Substitutes in Sequential Auctions: The Case of Water Auctions" by Javier Donna and Jose-Antonio Espin-Sanchez.

A Technical Proofs and Extensions

A.1 Two Units

Proposition. 1 When $\rho > 0$, the unique pure strategy symmetric sequential equilibrium is:

- First auction:
 - Participation: Bidder i always participate in the first auction, i.e., $y_i^1 = 1$
 - Bidding Strategy. We have two cases:

(a) If
$$(1 + \rho) v_i \equiv \tilde{v}_i \leq c$$
, then $b_i^1(v_i) = b^1(v_i) = v_i$

(b) If
$$(1 + \rho) v_i \equiv \tilde{v}_i > c$$
, then $b_i^1(v_i) = b^1(v_i) = (2 + \rho) v_i - c$

- Second auction:
 - Participation. Here we have two cases:
 - (a) If $(1 + \rho) v_i \equiv \tilde{v}_i \leq c$, then bidder does not participate in the second auction
 - (b) If $(1 + \rho) v_i \equiv \tilde{v}_i > c$, then bidder participates in the second auction if, and only if, he won the first auction, i.e. $y_i^2 = 1$ iff $x_i^1 = 1$
 - Bidding Strategy: If bidder i participates in the second auction (y_i² = 1), he will continue bidding until the price reaches his own valuation, i.e. b²(v_i) = [x_i¹(1 + ρ) + (1 x_i¹)] v_i

Proof. Note that the first case will only happen when the participation cost c is relatively high. In this case, the cost of participating in the second auction is so high that bidder idoes not want to participate in the second auction, even if he won the first unit. Hence, bidder i will behave as in the single unit case and $b_i^1(v_i) = b^1(v_i) = v_i$ is a weakly dominant strategy. Note that because this is an English auction, if only bidder *i* is in the first case and all remaining bidders are in the second case, $b_i^1(v_i) = b^1(v_i) = (2 + \rho)v_i - c$ is still a dominant strategy for bidder *i* because he will pay $p_1 = v_j < (2 + \rho)v_j - c$, where v_j is the maximum valuation of the remaining bidders. This inequality is a direct consequence of bidder *i* being in the first case and $v_i > v_j$.

For the second case, the first step of the proof consists of proving that in any revealing (strictly increasing) equilibrium, i.e. an equilibrium in which it becomes common knowledge after the first auction who is the bidder with the highest valuation, only the winner (the bidder with the highest type) will enter the second auction, and pay the cost c.

Since both a direct mechanism and the sequential auction will provide the same utility to the winner, and both will give the two objects to the bidder with the highest valuation, the total utility for the winner should be $W_i \equiv (2 + \rho) v_i - (2 + \rho) v_j = (2 + \rho) (v_i - v_j)$ in both cases, where j is the bidder with the second highest valuation. In the direct mechanism we can assume that there is a cost of communication to allocate the second unit, which should be paid by every bidder who wants to win the second unit.¹

The second step is to show that the winner will pay $(2 + \rho) v_j - c$ in the first auction. This payment, together with the utility the winner obtains from both goods $(2 + \rho) v_i$ and the cost of entering the second auction, c, will give him the same utility as in the direct mechanism: $(2 + \rho) v_i - ((2 + \rho) v_j - c) - c = (2 + \rho) v_i - (2 + \rho) v_j \equiv W_i$. The utility for the winner in the second auction is $(1 + \rho) v_i - c$, since the equilibrium price in the second auction is zero, and the utility for the loser is zero. Hence, the total value of winning the auction is: $v_i + (1 + \rho) v_i - c = (2 + \rho) v_i - c$.

So far, we have shown that in any revealing equilibrium only the winner will enter the second auction. The intuition for this result is that due to the cost of participating in the second auction it is a necessary condition for bidder *i* to participate in the auction, that his winning probability is strictly positive. Hence, only the winner will enter the second auction. His utility in the second auction is $(1 + \rho) v_i - c$. This fact, together with his utility in the first auction v_i and the revenue equivalence theorem show that he should bid $b_i^1 = (2 + \rho) v_i - c$

¹Note that the participation cost prevents a straightforward application of the revelation principle here. See Stegeman (1996) for details.

and pay $p^1 = (2 + \rho) v_j - c$. Hence, this is a revealing equilibrium. We have shown that, given the payoffs in the second auction, there is only one possible payoff and one possible bid for every player in the first auction. Hence, this is also the unique symmetric equilibrium in pure strategies.

A.1.1 Equilibrium when $-1 < \rho < 0$

Assumption A1:

$$\overline{v} \ge \frac{2c - \rho W}{2 + \rho}$$

Assumption A2:

$$\{(1+\rho)W - E[v_l|v_l > \overline{v}]\}F[(1+\rho)W|v_l > \overline{v}] < c$$

Assumption A2 ensures that the winner of the first auction never enters the second auction. When assumption A1 holds but assumption A2 does not, the winner in the first auction may enter the second auction, even when both bidders have valuations above \bar{v} . The proof is analogous to the one below.

Theorem. 7 Under assumptions A1 & A2, the unique symmetric equilibrium satisfies the following properties:

- (a) First Auction:
 - All bidders participate in the first auction $y_i^1 = 1$ for all *i*.
 - The bidding strategy for bidder i is

$$b_i^1(v_i) = b^1(v_i) = \begin{cases} v_i & \text{if } \tilde{v}_i \le c \\ (2+\rho)v_i - c & \text{if } c < \tilde{v}_i < v_i \le \overline{v} \\ (2+\rho)\overline{v} - c & \text{if } v_i \ge \overline{v} \end{cases}$$

(b) Second Auction:

• Case 1: No pooling in the first auction (at most one bidder has valuation above \overline{v}). Bidder i's participation function $(y_{i-np}^2(v_i, x_i^1))$ is

$$y_{i-np}^2(v_i,0) = 0$$

$$y_i^2(v_{i-np}, 1) = \begin{cases} 0 & \text{if } \tilde{v}_i \le c \\ 1 & \tilde{v}_i > c \end{cases}$$

Case 2: Pooling in the first auction (both bidders above v). Bidder i's participation function (y²_{i-p}(v_i, x¹_i)) is:

$$y_{i-p}^2(v_i, 0) = 1$$

$$y_{i-p}^2(v_i, 1) = 0$$

• If bidder i participates in the second auction, his bidding strategy is:

$$b^{2}(v_{i}, x_{i}^{1}) = \begin{cases} v_{i} & \text{if } x_{i}^{1} = 0\\ \\ \tilde{v}_{i} & \text{if } x_{i}^{1} = 1 \end{cases}$$

Proof. The proof will proceed by backwards induction. Without loss of generality, assume bidder 1 is the bidder with the highest valuation, $v_1 > v_2$. Notice that when $c \to 0$ the probability that we are in the pooling region goes to one. This is because $c \to 0$ implies $\overline{v} \to 0$. When the number of bidders increases this probability also increases.

2-(c): From the bidders' perspective, once the participation decision has been made, the final auction is a standard English auction. As a result, it is a dominant strategy of this stage game for each bidder to bid their valuations.

2-(b)-Case-2: This is the case after pooling in the first auction, that is $v_1, v_2 \ge \overline{v}$. We will write down the relevant expected utilities from entering and argue that any deviation (in pure strategies) results in negative expected utility. In this case, since the winner of the auction is

determined by a random probability lottery, we will refer to the winner and loser valuations as v_w and v_l , respectively. This is simply to emphasize that in this pooling region, the entry decision only depends on the outcome of the first auction, not the relative valuations. First, consider the loser of the first auction. The strategy states that they enter the second auction. The only possible deviation in pure strategies is not to enter the second auction, but given the winner's strategy (not to enter), we get:

$$EU[y_{i-p}^{2}(v_{l},0) = 1|y_{i-p}^{2}(v_{i},1) = 0] = v_{l} - c \ge \overline{v} - c$$

$$\ge P\left(v_{w} < \frac{\overline{v} + c}{1+\rho} \middle| v_{w} > \overline{v}\right) \cdot \overline{v} - c = 0 = EU[y_{i-p}^{2}(v_{l},0) = 0|y_{i-p}^{2}(v_{i},1) = 0]$$

Hence, there is no profitable deviation. Now consider the winner of the first auction. By following the strategy $y_{i-p}^2(\cdot, \cdot)$, this bidder will not enter the auction and get an expected utility of zero. Now, the only possible pure strategy deviation is to enter the auction, which gives him an expected utility of:

$$P(v_{l} < (1+\rho)v_{w}|v_{l} > \overline{v}) \cdot [(1+\rho)v_{w} - E[v_{l}|v_{l} > \overline{v}] - c] - [1 - P(v_{l} < (1+\rho)v_{w}|v_{l} > \overline{v})] \cdot c$$

$$= P(v_{l} < (1+\rho)v_{w}|v_{l} > \overline{v}) \cdot [v_{w}(1+\rho) - E[v_{l}|v_{l} > \overline{v}] - c]$$

$$\leq P(v_{l} < (1+\rho)v_{w}|v_{l} > \overline{v}) \cdot [W(1+\rho) - E[v_{l}|v_{l} > \overline{v}] - c]$$
(by assumption 2) ≤ 0

Thus, by entering, the winner will now get negative utility and, therefore, he has no profitable deviation. Note that this result depends exclusively on A2. If A2 does not hold, but A1 does, then there will be an equilibrium in which the winner also enters the second auction if his valuation is high enough.

2-(a)-Case-1: Now consider the participation decision when no pooling occurred in the first auction, that is: $v_2 \leq \overline{v} < v_1$. Consider bidder 1. His strategy is simply a threshold strategy. As long as his valuation for the second good is larger than the cost of entry, he will enter. Since bidder 2 will not enter, bidder 1 will win the object and pay a price of 0. Therefore, assuming bidder 2 follows $y_{i-np}^2(\cdot, 0)$, bidder 1's strategy is optimal. We must

show that given this strategy by bidder 1, the strategy of bidder 2 is optimal. The threshold of bidder 2, \overline{v} , is the solution in terms of v_2 to:

$$P\left((1+\rho)v_1 < v_2 + c | v_1 > v_2\right) \cdot v_2 = c$$

That is, since bidder 2's valuation will become public after the first auction when there is no pooling, bidder 1 will enter regardless of whether his valuation is above $v_2 + c$. We have to show that this threshold is still binding when bidder 2 knows that bidder 1 will enter with any valuation above c. In this case, $v_2 < \overline{v}$, the only deviation to consider is that bidder 2 enters after losing for some valuation less than \overline{v} . First, note that, if $v_2 < c$, by entering, bidder 2 will get a lower utility, regardless of bidder 1's entry choice. There are two cases to analyze:

(a) Suppose $v_2(1 + \rho) < c$. In this case, it is possible that $(1 + \rho)v_1 < c$, *i.e.*, that bidder 1 will not enter. Then, the expected utility from entering is:

$$P\left((1+\rho)v_{1} < c|v_{1} > v_{2}\right) \cdot (v_{2}-c) + P\left(c < (1+\rho)v_{1} < v_{2}|v_{1} > v_{2}\right) \cdot (v_{2}-c) + P\left((1+\rho)v_{1} > v_{2}|v_{1} > v_{2}\right) \cdot (-c) = P\left((1+\rho)v_{1} < v_{2}|v_{1} > v_{2}\right) \cdot v_{2} - c \\ \leq P\left((1+\rho)v_{1} < v_{2}+c|v_{1} > v_{2}\right) \cdot v_{2} - c < 0, \quad \forall v_{2} < \overline{v}$$

Since by not entering, bidder 2 can ensure 0 expected utility, this deviation is not profitable.

(b) Suppose $v_2(1 + \rho) \ge c$. When this is true, bidder 1 will always enter since $v_1 \ge v_2$. Then, by deviating and entering, bidder 2 gets:

$$P(\tilde{v}_1 < v_2 | v_1 > v_2) \cdot (v_2 - c) + P(\tilde{v}_1 \ge v_2 | v_1 > v_2) \cdot (-c)$$

= $P(\tilde{v}_1 < v_2 | v_1 > v_2) \cdot v_2 - c$
 $\le P(\tilde{v}_1 < v_2 + c | v_1 > v_2) - c \le 0, \quad \forall v_2 < \overline{v}$

Again, note that not entering gives 0 expected utility.

Thus, there is no profitable deviation from the second auction participation strategy.

1-(b): We need to show that if bidders are following the bidding strategy for the first auction as described in Theorem 7, they have no profitable deviation. We will consider each part of the function separately and show that any deviation results in lower utility for the bidders.

 $\tilde{\mathbf{v}}_{\mathbf{i}} \leq \mathbf{c}$: Consider an upwards deviation, $\tilde{b}^1(v_i) = v_i + \varepsilon$, for $\varepsilon > 0$. This will not affect bidder 1. He will win with probability 1 and still pay v_2 . In the case where $v_1 = v_2$, any upward deviation will still result in a utility of 0. Finally, consider bidder 2. If he follows $\tilde{b}^1(\cdot)$, he will now win with positive probability and get utility:

$$U_2 = v_2 - v_1 < 0$$

Thus, no bidder will gain from deviating from the prescribed strategy upwards. Now, consider a downwards deviation. If bidder 2 deviates, he will lose with probability one, and he gets utility of 0. The same is true for the case where $v_1 = v_2$: either deviation still results in utility of 0. However, if bidder 1 plays $\tilde{b}(v_1) = v_1 - \varepsilon$, with positive probability, he will lose when he is the highest type, thus getting 0 utility when previously he was getting $v_1 - v_2 > 0$. Thus, downward deviations are also not profitable. Therefore, when $\tilde{v}_i < c$, the strategy in Theorem 7 has no profitable deviations.

 $\mathbf{c} < \mathbf{\tilde{v}_i} < \mathbf{v} < \mathbf{\overline{v}}$: Agents bid above their valuation for the first object, because if they win, they will also get the second object for free. Thus, the effective valuation of the first object is $(2 + \rho)v_i - c$. Consider an upwards deviation, $\tilde{b}^1(v_i) = (2 + \rho)v_i + c + \varepsilon$. Bidder 1 will still win with probability one and pay the same price. By bidding $\tilde{b}^1(v_2)$, bidder 2 would win the first auction with positive probability. His utility would be:

$$v_2 - [(2+\rho)v_1 - c] + (1+\rho)v_2 - c = (2+\rho)(v_2 - v_1) < 0$$

Thus, an upwards deviation makes no bidder better off, and some strictly worse off.

Now, consider a downwards deviation, $\tilde{b}^1(v_i) = (2 + \rho)v_i + c - \varepsilon$. Bidder 2 will still lose with probability 1. Now, bidder 1 may lose, and thus obtains utility of 0 from deviating. Playing the prescribed strategy would give him the following utility:

$$v_1 - b(v_2) + (1+\rho)v_1 - c \ge v_1 - (2+\rho)v_2 - c + (1+\rho)v_1 - c = (2+\rho) \cdot (v_1 - v_2) > 0$$

Thus, no downwards deviation is profitable.

 $\tilde{\mathbf{v}}_i > \overline{\mathbf{v}}$: The first step is to show that no jump in bidding occurs at \overline{v} . Then, we will show that agents will pool for all values between \overline{v} and W. The difference between this and the previous case is that the loser of the first auction now gets positive utility from entering the second auction. Consider the case where $v_i = \overline{v} + \varepsilon$. We now show that such agents will bid $(2 + \rho)\overline{v} - c$. Assuming ties are broken with an equal probability, the utility to agent *i* bidding $b^1(v_i)$ is:

$$U_{i} = \frac{1}{2} \{ v_{i} - [(2+\rho)\overline{v} - c] \} + \frac{1}{2} [v_{i} - c]$$

$$= v_{i} - \frac{2+\rho}{2}\overline{v}$$

$$= (v_{i} - \overline{v}) - \frac{\rho}{2}\overline{v}$$

$$> 0 \qquad (A.1)$$

This is true for all $\varepsilon > 0$. Now we show that deviating from this results in lower utility. First, consider an upward deviation, $\tilde{b}^1(v_i) = (2 + \rho)\overline{v} - c + \varepsilon$. This will give:

$$U_{i} = v_{i} - [(2 + \rho)\overline{v} - c] + 0$$
$$= (v_{i} - \overline{v}) - (1 + \rho)\overline{v} + c$$
$$\xrightarrow{\varepsilon \to 0} c - (1 + \rho)\overline{v}$$
$$< 0$$

by assumption. Thus, bidders will not bid strictly higher at \overline{v} .

Now consider a downward deviation, say for bidder *i*, with $v_i > \overline{v}$, $\tilde{b}^1(v_i) = (2+\rho)\overline{v} - c - \varepsilon$.

There are two possibilities, either $v_{-i} \geq \overline{v}$ or $v_{-i} \leq \overline{v}$. Such a deviation, will mean that bidder *i* never reaches the pooling region, and that there exists a set of v_{-i} 's such that $b^1(v_{-i}) > \tilde{b}^1(v_i)$. The following is the payoff from playing the candidate equilibrium strategy:

$$\frac{1}{2}P(v_{-i} > \overline{v}) \cdot [v_i - b(v_{-i}) + \tilde{v}_i - c] + P(v_{-i} \le \overline{v}) \cdot [v_i - b(v_{-i}) + \tilde{v}_1 - c] \\
= \frac{1}{2}P(v_{-i} > \overline{v}) \cdot [v_i - (2 + \rho)\overline{v} + \tilde{v}_i] + [1 - P(v_{-i} > \overline{v})] \cdot [v_i - (2 + \rho)\overline{v} + \tilde{v}_1] \\
= (2 + \rho) \left\{ \frac{1}{2}P(v_{-i} > \overline{v}) \cdot (v_i - \overline{v}) + [1 - P(v_{-i} > \overline{v})] \cdot (v_i - v_{-i}) \right\} \quad (A.2)$$

Note that both terms are positive. Define \hat{v}_{-i} so that $b^1(\hat{v}_{-i}) = \tilde{b}^1(v_i)$. Now, the payoff from deviating to $\tilde{b}^1(\cdot)$, is:

$$P(v_{-i} \ge \overline{v}) \cdot 0 + P(v_{-i} \in [\hat{v}_{-i}, \overline{v}]) \cdot 0 + P(v_{-i} < \hat{v}_{-i}) \cdot [v_i - (2 + \rho) + c + \tilde{v}_i - c]$$

= $P(v_{-i} < \hat{v}_{-i}) \cdot (2 + \rho) \cdot (v_i - v_{-i})$
 $\le P(v_{-i} < \hat{v}_{-i}) \cdot (2 + \rho) \cdot (v_i - v_{-i}) + P(v_{-i} \in [\hat{v}_{-i}, \overline{v}]) \cdot (2 + \rho) \cdot (v_i - v_{-i})$

This equals the second term of (A.2) and, since the first term of (A.2) is also positive, the payoff from deviating is strictly less than the payoff from following the strategy.

It remains to show that all agents will pool. An agent with valuation W will not deviate down for the same reason as above. We only need to show that there is no incentive to deviate upwards. From (A.1) we know that any agent with valuation above \overline{v} has positive expected utility by following the prescribed bidding strategy. If type W bids $\tilde{b}^1(W) = (2 + \rho)\overline{v} - c + \varepsilon$, the expected utility is:²

$$P(v_2 \le \overline{v}) \cdot (2+\rho) \cdot (W-v_2) + P(v_2 > \overline{v}) \cdot [W - (2+\rho)\overline{v} + c]$$

 $^{^{2}}$ From the equilibrium, the participation decision of type W will be not to enter the second auction if he wins the first auction.

If, instead, the bidder follows the original strategy, he will get payoff of:

$$P(v_2 \le \overline{v}) \cdot (2+\rho) \cdot (W-v_2) + \frac{1}{2}P(v_2 > \overline{v}) \cdot (2+\rho) \cdot (W-\overline{v})$$

The first terms of these expressions are the same. To complete the proof, we need to show that:

$$\frac{(2+\rho)}{2}(W-\overline{v}) > W - (2+\rho)\overline{v} + c$$

Or, equivalently:

$$\overline{v} > \frac{2c}{2+\rho} - \frac{\rho}{2+\rho}W,$$

which is exactly A1.

Proposition. 2 When $\alpha < \beta$, the probability that a bidder different from the winner enters the last auction is decreasing in the participation cost, c. Moreover, this probability goes to 1 when c goes to zero, i.e:

$$\lim_{c \to 0} \left\{ \Pr\left(y_i^2 = 1 \mid x_j^1 = 1, i \neq j\right) \right\} = 1$$

Proof. It is sufficient to show that one bidder, the bidder with the second highest valuation, will enter with probability approaching 1. If the equilibrium in the first auction is fully revealing, it will become common knowledge that $v_1 \sim \tilde{F}_1(v_1) \equiv F_1(v_1 | v_1 > v_2)$, where bidder 1 is the winner of the first auction and bidder 2 in the second highest bidder in the first auction. The expected utility of entering the second auction for bidder 2 is:³

$$v_2 Pr[v_2 > (1+\rho)v_1] - c = v_2 \tilde{F}_1\left(\frac{v_2}{1+\rho}\right) - c$$
 (A.3)

Bidder 2 will enter the second auction if, and only if, his expected utility is positive, that is: $Pr(y_2^2 = 1 | x_1^1 = 1) = Pr\left(v_2\tilde{F}_1\left(\frac{v_2}{1+\rho}\right) - c > 0\right)$. Notice that c only appears in the right hand side; the probability is decreasing in c. The first term in the left hand side is always

³Note that in the second auction, the winner (in the first auction) now has complete information. Hence a necessary condition for equilibrium is that if bidder 2 wins the second unit, bidder 1 does not enter the second auction. Thus, when bidder 2 wins the second auction, he pays a price of zero.

strictly positive, by construction. This implies that there exist some value for c such that this term is smaller. Formally, we have:

$$\lim_{c \to 0} \left\{ \Pr\left(y_i^2 = 1 \mid x_j^1 = 1, i \neq j\right) \right\} \ge \lim_{c \to 0} \left\{ \Pr\left(y_1^2 = 1 \mid x_2^1 = 1\right) \right\} = \lim_{c \to 0} \left\{ \Pr\left(v_2 \tilde{F}_1\left(\frac{v_2}{1+\rho}\right) - c > 0\right) \right\} = 1$$

Notice that we are considering only the cases where the first auction is fully revealing. If the first auction is only partially revealing (equilibrium is pooling or semi-pooling), then the analysis is less restrictive.⁴ In a pooling bidding region, bidders with valuation within the interval bid the same amount, and ties are broken randomly. Hence, with positive probability it could be the case that the bidder with the highest valuation losses the lottery and we have $v_2 > v_1$. In this case, bidder 2 has even greater incentives to enter the second auction. \Box

Proposition. 3 When goods are substitutes, $-1 < \rho < 0$, it is a dominant strategy for all bidders to bid their valuations for the unit in the last auction, conditional on entering the auction.

Proof. Conditional on entering the last auction, there are no informational or dynamic concerns, and thus all bidders behave as in a standard (single-unit) second price auction. In this type of auction, it is a well known result that it is a dominant strategy to bid one's own valuation. The details of the proof are in Krishna (2010). \Box

Proposition. 4 When $\rho = c = 0$ it is weakly dominant strategy for all bidders to enter all auctions and bid their valuations in every auction, that is, $y_i^k = 1$ and $b_i^k = v_i$, $\forall i, k$.

Proof. The situation here is similar to the single-unit case. The difference is the information that each bidder has in every stage. Nevertheless, this does not affect the result. Note, first, that it is a weakly dominant strategy for every bidder to enter every auction because there are no entry costs and the gains are always zero or positive. Next, note that, conditional on entering, it is a weakly dominant strategy for every bidder to bid their own valuations (for that the unit) in each auction.

⁴When $c \to 0$, the equilibrium is semi-pooling with probability 1. However, at the limit when c = 0, the equilibrium is not pooling, as shown by Black and De Meza (1992).

A.2 Four Units

We now extend the results from Section 2.2 in the paper (K = 2) for the case where four units are sold (K = 4). The marginal utilities in this case are: $MU(1) = (1 - \alpha) v_i$, $MU(2) = (1 - \beta) v_i$, $MU(3) = (1 - 2\beta) v_i$, and $MU(4) = (1 - 3\beta) v_i$. In the remainder of this section we consider the participation cost, c, to be arbitrarily small.

A.2.1 Pure Complements

We say that goods are pure complements if the marginal utility for the first unit is below the marginal utility for the remaining units. Since marginal utility is decreasing after the second unit, this is analogous to say that the marginal utility of the first unit is below the marginal utility of the last unit, $MU(1) \leq MU(4)$, or, equivalently, $3\beta \leq \alpha$.

Proposition. 8 When goods are pure complements, $\beta \leq \frac{\alpha}{3}$, the pure strategy symmetric sequential equilibrium is:

- First auction:
 - Participation: bidder i will always participate in the first auction, i.e. $y_i^1 = 1$
 - Bidding Strategy: $b_i^1(v_i) = b^1(v_i) = [(1 \alpha) + (1 \beta) + (1 2\beta) + (1 3\beta)]v_i 3c = [4 \alpha 6\beta]v_i 3c$
- Second, third, and fourth auctions:
 - Participation: bidder i participates in all the remaining auctions if, and only if, he won the first auction, i.e. $y_i^2 = y_i^3 = y_i^4 = 1$ iff $x_i^1 = 1$
 - Bidding Strategy: If bidder i participates in the second, third or fourth auction $(y_i^k = 1 \text{ for } k = 2, 3, 4)$, he will continue bidding until the price reaches its own valuation, $b_i^k(v_i, X_i^k) = \left[1 - \alpha_1 \{X_i^k = 0\} - \beta X_i^k\right] v_i$, with $X_i^k = \sum_{j=1}^{j=k} x_i^k$

Proof. Following the same argument as in the proof of Proposition 1, the first step of this proof consists of proving that in any revealing (strictly increasing) equilibrium, that is, an equilibrium in which it becomes common knowledge after the first round who is the bidder

with the highest valuation, only the winner (the bidder with the highest type) will enter the remaining auctions, and pay the cost c.

Since both a direct mechanism and the sequential auction will give the same utility to the winner, and both will give the four objects to the bidder with the highest valuation, the total utility for the winner should be $W_i \equiv (4 - \alpha - 6\beta) v_i - (4 - \alpha - 6\beta) v_j =$ $(4 - \alpha - 6\beta) (v_i - v_j)$ in both cases, where j is the bidder with the second highest valuation. For the case of a direct mechanism, we can assume that there is a cost of communication for each of the auctions, which should be paid by every bidder that wants to win the object.

The second step is to show that the winner will pay $(4 - \alpha - 6\beta) v_j - 3c$ in the first auction. This payment, together with the utility the winner gets from both goods, $(4 - \alpha - 6\beta) v_i$, and the cost of entering the remaining auctions, 3c, will give him the same utility as in the direct mechanism: $(4 - \alpha - 6\beta) v_i - ((4 - \alpha - 6\beta) v_j - 3c) - 3c = (4 - \alpha - 6\beta) v_i - (4 - \alpha - 6\beta) v_j \equiv W_i$. The utility for the winner in the second auction is $(1 - \beta) v_i - c$, since the equilibrium price in the second auction is zero, and the utility for the loser is zero. Similarly, the utility for the loser in the third auction is $(1 - 2\beta) v_i - c$, and the utility in the fourth auction is $(1 - 3\beta) v_i - c$. Thus the total value of winning the auction is $(4 - \alpha - 6\beta) v_i - 3c$.

This four unit auction is equivalent to the following single unit auction. Let us define $z_i = (4 - \alpha - 6\beta) v_i - 3c$ and consider a single object auction in which the valuation for the good for bidder *i* is z_i . Thus, we return to the standard single unit auction, and $b_i = z_i = (4 - \alpha - 6\beta) v_i - 3c$ is a weakly dominant strategy.

We have proven that in any revealing equilibrium only the winner will enter the remaining auctions. His utilities in the second, third, and fourth auctions are $(1 - \beta) v_i - c$, $(1 - 2\beta) v_i - c$ and $(1 - \beta) v_i - c$, respectively. This fact, his utility in the first auction being $(1 - \alpha)v_i$, and the revenue equivalence theorem show that he will bid $b_i^1 = (4 - \alpha - 6\beta) v_i - c$ and pay $p^1 = (4 - \alpha - 6\beta) v_j - c$. Hence, this is a revealing equilibrium. Therefore, we have shown that, given the payoffs in the second, third, and fourth auctions, there is only one possible payoff and one possible bid for every player in the first auction. Hence, this is also the unique symmetric equilibrium in pure strategies. In this case, the strategies and the outcomes are also analog to the single-unit case. The equilibrium is fully revealing because all second, third, and fourth units yield greater marginal utility than the first unit.

Corollary. 5 When goods are pure complements, $\beta \leq \frac{\alpha}{3}$, the marginal utility of the winner for all four auctions satisfies:

$$[4 - \alpha - 6\beta] v_{N:N} - 3c \ge \sum_{k=1}^{4} p^k = [4 - \alpha - 6\beta] v_{N-1:N} - 3c$$
(A.4)

A.2.2 Non pure complements

The full characterization of the equilibrium for the case where $\beta > \frac{\alpha}{3}$ is outside the scope of this paper. Nevertheless, we will extend the results from propositions 2 and 3 to compute the equilibrium price equations that we use in the structural estimation in Section 5 in the paper.

Proposition. 9 When $\frac{\alpha}{K-1} < \beta$, the probability that a bidder different from the winner enters the last auction is decreasing in the participation cost, c. Moreover, this probability goes to 1 when c goes to zero, that is:

$$\lim_{c \to 0} \left\{ \Pr\left(y_i^K = 1 \mid x_j^1 = 1, i \neq j\right) \right\} = 1$$

Proof. The argument of the proof is the same as in Proposition 3.

Proposition. 10 When goods are non pure complements, $\beta > \frac{\alpha}{3}$, it is a dominant strategy for all bidders to bid their valuations in the last auction (for that unit), conditional on entering the auction.

Proof. The argument for the proof is the same as in Proposition 3. \Box

In this case, there is no equilibrium in weakly dominant strategies. However, in any Nash equilibrium, the bidding strategy in the last auction will always be to bid one's own valuation, conditional on entering the last auction. Since this would be the last stage of the game, it is a weakly dominant strategy to bid one's own valuation.

For the case of non pure complements, there are more possible configurations for the winner and the second highest bidder in the last auction when four units are sold. We summarize them in the following corollary.

Corollary. 6 When goods are non pure complements, $\beta > \frac{\alpha}{3}$, the marginal utility of the winner in the last auction, depending on how many units he won, satisfies:

If he won all four units:

$$(1 - 3\beta)v_{N:N} - c \ge p^4 = (1 - \alpha)v_{N-1:N} - c \tag{A.5}$$

If he won three units, two out of the first three, and the last one:

$$(1 - 2\beta)v_{N:N} - c \ge p^4 = (1 - \beta)v_{N-1:N} - c \tag{A.6}$$

Several remarks concerning the behavior in these cases are worth discussing. First, note that in both cases, whoever wins three or four units will necessarily be the bidder with the highest type. When the same bidder buys all four units, we can only claim equality on the price paid for the last sequential unit (equation A.5). In this case, we know that all the remaining bidders are bidding in the last auction according to $MU_i(1) = (1 - \alpha) v_i$. This is because none of them bought any of the first three units. In the case of equation A.6, however, we do not know if the second highest bidder for the last unit has already bought one unit, and hence he is bidding according to $MU_{N-1}(2) = (1 - \beta) v_{N-1:N}$; or he has not bought any units, and is thus bidding according to $MU_{N-2}(1) = (1 - \alpha) v_{N-2:N}$. We assume that the winner in the first 3 auctions is also the runner-up in the last auction.⁵ Hence, we have $p^4 = (1 - \beta)v_{N-1:N} - c$.

In the cases where the winner of the last unit (K = 4) has bought less than 3 units, we are unable to say whether he is the highest or the second highest type without imposing further structure. A similar argument applies to the case where the winner of the last unit only buys one good. We can only tell whether the last winner is bidding according to $MU_i(1) = (1 - \alpha)v_i$ or $MU_i(k)$, depending on his previous purchases. Nonetheless, if we

⁵ This assumption rests in the fact that, when the number of bidders in the auction grows, the probability that the winner of the first three auctions will be the runner up in the last auction goes to 1.

do not know his ranking on the valuations, we cannot use this information to infer price behavior.

Proposition. 11 When $\rho = c^k = 0$, $\forall k$ and K = 4, it is a weakly dominant strategy for all bidders to enter all auctions and bid their valuations in every auction, that is, $y_i^k = 1$ and $b_i^k = v_i$, $\forall i, k$.

Proof. The argument of the proof is the same as that of Proposition 4. \Box

As in the two units case, this is the unique equilibrium in weakly dominant strategies, but not the unique Nash equilibrium.⁶ Note, however, that the bidder with the highest valuation will win all units, and will pay a price equal to the valuation of the bidder with the second highest valuation.

⁶We can construct any such equilibrium by using parameters $\tau_k \forall k$, such that, $\tau_1 + \tau_2 + \tau_3 = 4$, and define the strategies for all players as $y_i^k = 1$, $b_i^1 = \tau_1 v_i$, $b_i^2 = \tau_2 v_i$, $b_i^3 = \tau_3 v_i$, and $b_i^4 = (4 - \tau_1 - \tau_2 - \tau_3) v_i$, $\forall i, k$. This is, indeed, an equilibrium. Also, for $\tau_k \neq 1 \forall k$, these strategies are not weakly dominant. A corollary to proposition 11 is that, when $\tau_k \neq 1 \forall k$, this is an equilibrium in weakly dominant strategies. In Subsection 5.4 in the paper we use this result when we compare our model estimates to Haile and Tamer (2003).

B End-Digit Preferences and Regime Determination

When goods are pure complements, very low prices—or, according to the auctioneer who ran the auctions, symbolic prices (Botía, Francisco, personal interview, Murcia, June 17, 2013)⁷ are paid, by the winner of the first unit, for the second, third, and fourth units (Figure 2 in Section 3 in the paper). The predicted price pattern by our model for each each regime (pure complements and non pure complements) provides us with a straightforward empirical method to determine them. We show, in this section, that end-digit preference for these prices is consistent with this empirical prediction, without specifying further assumptions on the model's primitives. When goods are pure complements a key prediction from proposition 8 (see Subsection A.2) is that the same bidder will win all units, pay his valuation for the whole bundle in the first auction, and pay a price of zero for the second, third, and fourth units. The baseline model implicitly assumes that the seller will not require a reserve price for the second, third, and fourth objects. We do not observe zero prices (for second, third, and four units in the data), but very low prices (relative to the first price): they are symbolic prices. Although there is no reserve price in the actual auctions, we interpret the minimum price as a general agreement to bid a symbolic price in subsequent auctions. A common effect in our data is that farmers bid certain preferred end-digits prices substantially more often than others. This provides us with valuable information to determine both regimes.

Studies of digit distribution go back to Benford (1938) who documented that in large data sets, leading digits are not distributed evenly (1 is the most common and 9 the rarest), and proposed a distribution for first digits of numbers in naturally occurring data. Abrantes-Metz, Villas-Boas and Judge (2011) use Benford's second digit reference distribution to track the daily London Interbank Offered Rate (Libor) from 2005 to 2008 and find that in two periods, Libor rates depart significantly from the expected Benford reference distribution; collusion or rate manipulation appear as likely outcomes to this behavior, the authors suggest. Rauch et al. (2011) use a Benford test to investigate the quality of macroeconomic data relevant to the deficit criteria reported to Eurostat by the European Union member states; they find that the data reported by Greece shows the greatest deviation from Benford's law among all

⁷A summary is available online in the Web Appendix at http://www.jdonna.org/water-auctions-web.

euro states.

In regards to end-digit preferences, Kandel, Sarig and Wohl (2001) use Israeli IPO auctions to present evidence that investors have end-digit preference for round numbers (prices that end with 0 or 5), and that prices that end with 0 are used more often than those ending with 5. To our knowledge, we are the first to use end-digit preferences for inference in the empirical auctions literature. End-digit preferences and systematic age misreporting are important and broadly studied issues in demography, particularly in survey and census data when respondents inaccurately report ages or dates of birth (Myers 1940; Das Gupta 1975; Coale and Li 1991; and Siegel, Shryock and Swanson 2003). The concern in these cases is, typically, heaping on particular ages such as those ending in 0 or 5. Craven and Baten (2009) use some of these techniques to investigate the phenomenon of age heaping, and to test the hypothesis that an unequal distribution of human capital reduces welfare growth. Baker (1992) focuses on digit preferences in CPS unemployment duration data, where he raises the question of what can be said without making any specific assumptions concerning the true nature of end-digit preference in the CPS, and shows that employment duration is sensitive to the choice of a corrective for end-digit preference. Finally, end-digit preference has also been studied in the medical literature pertaining to individuals reporting body weight and height, blood pressure, and cigarette consumption (Bopp and Faeh 2008). Our set up is different from all these studies as we do not a priori impose a uniform (nor a particular) distribution on end-digit preference, nor do we have a compelling argument to.

Table A1 shows in column 2 the frequency distribution by the last digit of price for firstunit prices.⁸ We observe strong preferences for 0 and 1, and somewhat weaker preferences for 5. In 32.1% of the cases we see a multiple of 10, in 32.2% we see a price ending in 1, while 8.4% report a multiple of 5. The frequencies also show some preference for 2 and 6, but not a marked one. After taking into account these effects, we find that 48% of the first-unit price observations are inconsistent with a uniform distribution in each digit.⁹ That is, we would need to reclassify 48% of the cases to obtain a uniform distribution by digit. This is clearly

 $^{^{8}}$ We obtain similar patterns if we restrict the sample by month or schedule (day-time or night-time) or both.

⁹This number corresponds to the value of the Whipple's concentration index (Siegel, Shryock and Swanson 2003). In the absence of digit preference one would expect 10% in each terminal digit.

not the case for our underlying distribution. We interpret these results as strong end-digit preference for 0 and weak end-digit preference for 5.

	All u	$_{\rm nits}$	First	Unit	Non First Unit		Second Unit		Third Unit		Fourth Unit	
	(1)	(2)	(3)	(4)	(5)	(6))
Last Digit	Freq	%	Freq	%	Freq	%	Freq	%	Freq	%	Freq	%
0	2,905	49.9	468	32.1	2,437	55.8	809	55.5	811	55.7	817	56.1
1	$1,\!541$	26.4	469	32.2	1,072	24.5	370	25.4	369	25.3	333	22.9
2	201	3.5	116	8.0	85	1.9	32	2.2	24	1.7	29	2.0
3	138	2.4	59	4.0	79	1.8	29	2.0	23	1.6	27	1.9
4	68	1.2	40	2.7	28	0.6	10	0.7	9	0.6	9	0.6
5	531	9.1	123	8.4	408	9.3	114	7.8	144	9.9	150	10.3
6	218	3.7	91	6.2	127	2.9	43	3.0	37	2.5	47	3.2
7	71	1.2	33	2.3	38	0.9	19	1.3	12	0.8	7	0.5
8	61	1.0	24	1.6	37	0.9	8	0.6	12	0.8	17	1.2
9	48	0.8	24	1.6	24	0.6	12	0.8	5	0.3	7	0.5
Total	5,828	100	1,457	100	4,371	100	1,457	100	1,457	100	$1,\!457$	100

 Table A1: End-Price Preferences

Sample restricted to 4CU auctions (that is, instances when the same farmer won all four units in a 4-unit auction). *Last Digit* refers to the end-digit winning price. Non integer winning prices are excluded.

Strong preference for digit 1 is not, in general, an indication of a preference for this digit *per se* but, instead, a sign of competition. According to our model, first-unit prices are always competitive (in the sense that all N bidders will enter the auction when no information has yet been revealed), regardless of the regime. Nevertheless, second to fourth-unit prices are not competitive in the pure complements regime (competition in this regime takes part in the first unit where they bid for the whole bundle, and then pay a symbolic price for the second to fourth units since it is optimal for the remaining N - 1 bidders not to enter in these sequential auctions). Hence, end-digit preference for 1 in second to fourth-unit prices, as a sign of competition, are indicative of a non pure complements regime. Alternatively, end-digit preference for 0 for second to fourth-unit prices are indicative of a pure complement regime. Moreover, in the pure complement regime the model predicts that all second, third, and fourth consecutive prices will simultaneously behave in this fashion. Column 3 in Table A1 display the frequency distribution by the last digit of price for the second to fourth units.

This behavior provides us with a natural lower bound for the pure complements regime, namely, second, third, and fourth unit prices within the same four-unit auction show a strong end-digit preference for 0. We use this behavior, along with the model, to identify the two regimes.¹⁰

Figure A1 displays the histogram of the percentage change of first price against the median of second to fourth price, by regime.¹¹ It can be seen in the figure that end-digit preference behavior (as defined above) also captures, in general, the other empirical prediction from the model, namely, that prices are competitive in the non pure complements regime but exhibit the deterrence effect in the pure complements one (the way to see this in the figure is that percentage change from the first to the second, third or fourth prices is high when goods are pure complements). This is remarkable as the end-digit preference behavior used to identify the regime is unrelated *a priori* to this second empirical prediction. This provides further evidence in favor of our model.¹² Regime identification is done by using the strongest version of the empirical prediction to identify the pure complements case, that is, the case in the left panel in Figure A1.

 $^{^{10}}$ We could also use weaker or stronger definition of end-digit or round-number preferences to obtain different bounds for the empirical distributions of prices in each regime. We could, for example, assume that in the pure complements regime second, third, and fourth unit prices within the same four-unit auction show simultaneously a strong end-digit preference for 0 or 5. Our results are robust to include end-digit preference for 5 as well.

Note that the strongest version of the empirical prediction is be that all second, third, and fourth prices display an end-digit preference for 0 in any given four-unit auction for the same individual. A weaker version would be that two out of the three (among second to fourth) prices show an end-digit preference for 0. The weakest version is that just one of these three prices exhibit an end-digit preference for 0. The last (weakest) specification only provides us an upper bound for pure complements regime identification since, as shown in Table A1, the underlying distribution displays an end-digit preference for 0 even in the non pure complements regime.

¹¹The figure looks similar if we use the second, or the third, or the fourth, or the average of second to fourth prices.

¹²Using a modified version of this assumption that differentiates end-digit preference for prices ending in 0 that exhibit more frequency (e.g. prices like 100 are more frequent than 150) yields almost identical results.

Figure A1: Within Price Distribution by Regime: First Price vs. Median Second to Fourth prices



Both figures display, for instances when the same farmer won all four units in a 4-unit auction, the normalized percentage change, Δ , of the first winning price against the median of the second to fourth winning prices: $\Delta = \frac{p_1 - m}{(p_1 + m)/2}$, with $m = median (p_2, p_3, p_4)$. Note that $\Delta < 2$ and that $\Delta \rightarrow 2$ if, and only if, $p_1 \rightarrow \infty$, or $m \rightarrow 0$, or both; that is, when the percentage change goes to infinity. In the figure on the left, Lower Bound is computed by assuming that all second, third, and fourth unit prices paid by the (same) farmer within the same four-unit auction display end-price preference for 0. In the figure on the right, Upper Bound is computed by assuming that only one among second, third, or fourth unit prices paid by the (same) farmer within the same four-unit auction display end-price preference for 0.

A final robustness check further shows that the approach in this section consistently identifies both regimes in terms of our model. Columns 1 and 2 (first unit) in Table A2 display, by regime, the frequency distribution in terms of end-digit prices for first-unit, among each four-unit auction. As emphasized above, both regimes should exhibit competition for first-unit prices according to our model. This competition is captured by the same distribution among ending digits in both regimes. This is what we observe in columns 1 and 2 (first unit). Columns 3 and 4 in Table A2 (fourth unit) show that, as predicted by the model, fourth-unit prices for non pure complements are also competitive: 29.5% of preference for 0 vs. 39.7% for 1. (Note that, in column 3—Pure Complements—, the percentage of observations with last digit 0 is 100% by construction.)

		First Unit				Fourth Unit			
	Pure (Compl	Not P	ure Compl	Pure	Compl	Not P	Not Pure Compl	
	(1	1)		(2)		(3)		(4)	
Last Digit	Freq	%	Freq	%	Freq	%	Freq	%	
0	220	38.3	236	29.0	575	100	240	29.5	
1	185	32.2	264	32.5	0	0	323	39.7	
2	42	7.3	69	8.5	0	0	24	2.9	
3	17	3.0	41	5.0	0	0	26	3.2	
4	16	2.8	22	2.7	0	0	6	0.7	
5	45	7.8	76	9.4	0	0	125	15.4	
6	34	5.9	50	6.2	0	0	43	5.3	
7	8	1.4	22	2.7	0	0	6	0.7	
8	6	1.0	14	1.7	0	0	13	1.6	
9	2	0.4	19	2.3	0	0	7	0.9	
Total	575	100	813	100	575	100	813	100	

Table A2: End-Price Preferences by Regime

Sample restricted to 4CU auctions. *Last Digit* refers to the end-digit winning price. Non integer winning prices are excluded. Regime is determined by assuming all second, third, and four unit prices paid by the same winner within the same four-unit auction display end-price preference for 0.

C Data Description

In this section we provide a detailed description of the auction allocating system and the data. We then introduce the empirical regularities that support the contention that the size of sunk costs relative to decreasing marginal returns provide the basis for the existence of two regimes. Finally, we justify the modeling assumptions made in Section 2 from the paper within our empirical context.

C.1 Data Summary Overview

We combine data from different sources for our analysis. To get a sense of the industry context during the period under analysis, we present a brief description for the region's demographics, agriculture production and weather.¹³ Murcia's population share in Spain was around 3% during the period. As a municipality, Mula comprised 2% of Murcia in 1954, ranking Mula 20th in terms of population. The three main citrus fruits produced in the area are apricot, lemon, and peach trees. Murcia's share of these crops was 50% (2.3 million), 44% (1.5 million), and 42% (4.3 million), respectively, in terms of Spain's total production of these fruits for the year 1962. *Regadio* land in Murcia constitutes 4% (70,000 ha.) of Spain's.

Auction data, the primary source of data for this study, is obtained from the historical archive of Mula. We complement auctions data with daily rainfall data for Mula and monthly price indexes for Spain, which we obtain from the *Agencia Estatal de Metereología*, AEMET (which is the National Meteorological Agency), and the *Instituto Nacional de Estadística de España*, INE (which is the National Statistics Institute of Spain), respectively.¹⁴

Mediterranean climate rainfall occurs mainly in spring and autumn. Peak water requirements for the products cultivated in the region are reached in spring and summer, between

 $^{^{13}}$ These descriptive statistics Population Agricultural Cenare obtained from and sus from the National *Statistics* Institute of Spain (INE) (available online at http://www.ine.es/inebaseweb/treeNavigation.do?tn=201299&tns=199923#199923).

¹⁴In terms of purchasing power, one peseta from 1950 is approximately equivalent to 0.43 U.S. dollars from 2013. Note that $\frac{43.06}{166.39} \times 1.18 \times 1.40 = 0.43$ where 43.06 is the purchasing power of one peseta from 1950 in 1999 (obtained from *Servicios de estudios BBVA*, available online at http://www.elmundo.es/anuario/2001/sec/eco/391.htm), 166.39 is the exchange rate of pesetas per euro in January 1999 (obtained from the European Central Bank), 1.18 is the exchange rate of U.S. dollars per euro in January 1999 in 2013 (obtained from the European Central Bank), and 1.40 is the purchasing power of one dollar from 1999 in 2013 (obtained from the U.S. Bureau of Labor Statistics, available online at http://data.bls.gov/cgi-bin/cpicalc.pl?cost1=1&year1=1999&year2=2013).



Figure A2: Rain and Frequency Distribution of 4CU Over the Sample Period

The figure displays for each month: i) the number of auctions where the same farmer wins all four consecutive units, and ii) total rain using a Nadaraya-Watson kernel regression (of 'total rain' on 'month of the year') with an Epanechnikov kernel with bandwidth selected by cross validation.

April and August. During this period more frequent irrigation is advisable because citrus trees are more sensitive (in terms of quality of production) to water deficits.

Soaring demand is reflected by the frequency of auctions where the same farmer buys all four consecutive units (4CU), which reaches its peak during these months, as it is depicted in Figure 6 in the paper. The frequency of 4CU is not homogenous over time, but is related to seasonal rainfall, as can be seen in Figure A2. Weather is important for our analysis as it is a determinant of seasonality. The coastal strip of southeast Spain is the most arid region of all continental Europe due to the *foehn* effect, and because of its location to the west of the mountain chain *Sistema Penibético*, which includes the *Mulhacen* (the second highest mountain in Europe).¹⁵ Although annual average rainfall is 320 mm, rainfall frequency distribution is skewed, making the majority of years dryer than this annual average. Aridity during the summer is especially acute, and autumn is the only relatively humid season. The

¹⁵A *foehn* wind "results from the ascent of moist air up the windward slopes; as this air climbs, it expands and cools until it becomes saturated with water vapour, after which it cools more slowly because its moisture is condensing as rain or snow, releasing latent heat. By the time it reaches the peaks and stops climbing, the air is quite dry. The ridges of the mountains are usually obscured by a bank of clouds known as a foehn wall, which marks the upper limit of precipitation on the windward slopes. As the air makes its leeward descent, it is compressed and warms rapidly all the way downslope because there is little water left to evaporate and absorb heat; thus, the air is warmer and drier when it reaches the foot of the leeward slope than when it begins its windward ascent" (Encyclopedia-Britannica).

number of days when torrential rain occurs is not particularly high, but when such rain occurs it is substantial.¹⁶ Potential evaporation is four or five times higher than rainfall and the number of arid months vary from 7 to 11 in our sample.¹⁷ These arid conditions found in southeast Spain are related to the circular air movement in the occidental Mediterranean area and to the Atlantic-origin storms. As explained in Subsection 3.2 in the paper, we further augment our data with individual characteristics of the farmers' land, which we obtain from the 1954/55 agricultural census.

C.2 Empirical Regularities

In our theoretical model, it is important to be able to identify cases in the four-unit auctions where the same farmer won four or three sequential units, so as to avoid imposing further structure on the dynamic strategic considerations of the bidders, which are outside the scope of this paper.¹⁸ Thus, following the theoretical model from Section 2 in the paper, we have selected for our estimation auctions where a single person wins all units or where the last winner also won two out of the first three units (for a total of three units). This represents 54% of the total number of water units sold in our sample as displayed in Table A3. The table exhibits the frequency distribution of units sold by number of units bought by the same farmer. Overall, 42% (5, 880/13, 992) of the units were sold in 4CU.

 $^{^{16}}$ As an example, on October 10th 1943, 681 mm of rain water were measured in Mula, more than twice the yearly average for our sample.

¹⁷ Drought definition has been studied by several scientific studies. In our analysis, we define a drought as an indicator that equals one when average monthly rain during the specific year is below a consensus threshold defined in the literature in terms of the historic annual average (following Gil Olcina 1994 we use a threshold of 40% in Figure A6). For further details, see for example a study from Valiente 2001 where the author presents diverse methodological issues analyzing drought definitions.

¹⁸A complete characterization of the equilibrium when goods are non pure complements would require further structure on the primitives of the model as the equilibrium depends on the believes about other player's types, and the strategies, that each bidder have. We analyze these dynamics effects in Donna and Espin-Sanchez (2013).

	Number of Units	Frequency	Total
1CU	3,530	0.20	3,530
$2\mathrm{CU}$	2,866	0.17	6,396
$3\mathrm{CU}$	1,716	0.10	8,112
$4\mathrm{CU}$	$5,\!880$	0.34	13,992
No bids	3,203	0.19	$17,\!195$
Total	17,195	1.00	

Table A3: Frequency Distribution of Units Bought by the Same Farmer

The table displays the frequency distribution of units in the auctions disaggregated by the units bought sequentially by the same farmer: (i) 1CU refers to the case where the same farmer buys only one unit; (ii) 2CU refers to the case where the same farmer buys two sequential units; (iii) 3CU refers to the case where the same farmer buys three sequential units; finally, (iv) 4CU refers to the case where the same farmer buys all four consecutive units. There are no observations where the same farmer buys more than four consecutive units. No bids refers to cases where auctions were run but where no one bid for the last units (see, for example, the last four units in Figure 4 in the paper).

Selected summary statistics for the main variables are provided in Table A4. Importantly for our analysis, we only observe the transaction price (winning bid) and the identity of the winner (name). Winning prices range from \$0.05 to \$4,830, with the mean being \$271.6. As expected, winning prices and frequency distribution of 4CU are strongly correlated with past rain (Figure A2). Interestingly, in the sample used in the structural estimation, the counterintuitive positive correlation between average daily winning prices and daily rainfall recovers its "correct" sign (statistically significant at 1% level) once we condition on seasonality. The endogeneity issue arises because both demand (due to the nature of the trees) and supply (rainfall) are high during spring, generating an artificial positive correlation between the variables that is, ultimately, caused by seasonality (we further discuss seasonality below).

Variable	Mean	SD	Min	Max	Obs
Rain (mm/m^2)	8.53	46.33	0	980.00	3,834
Price (pesetas)	271.61	374	.05	4,830	$13,\!872$
Land Extension (hectares)	5.54	32.24	.25	900	819
Selling Price (pesetas)	15.07	222.52	.02	5,700	964
Kg sold	5,569.70	10,003.76	0	110,000	$1,\!000$
Number of Trees	161.49	493.45	1	$12,\!300$	946

Table A4: Summary Statistics of Selected Variables

Summary Statistics of Selected Variables. SD stands for Standard Deviation, Obs stands for Total Number of Observations.

There is substantial price variation, both within and across four-unit auctions. Table A5 exhibits the distribution of winning prices by both the number of consecutive units bought by the same individual (1CU, 2CU, 3CU, or 4CU) and by sequential auction (1st, 2nd, 3rd, or 4th). Interestingly, the greater variation that we observe for 4CU (with respect to non-4CU) has a well defined pattern. While mean prices for the first auction in 4CU are considerably higher than for non-4CU (Table A5, Panel 2: 677.6 for 4CU against 211.1 for 1CU, 305 for 2CU, or 410 for 3CU), mean prices for fourth auctions in 4CU are the smallest one (Table A5, Panel 5: 210.1 for 4CU against 233.4 for 1CU, 239.6 for 2CU, or 311.6 for 3CU). Median and maximum prices display similar patterns.

Table A5: Distribution of Winning Prices: by Number of CU and Sequential Auction

Panel 1: Price distribution by number of consecutive bids: All Auctions									
	Median	Mean	SD	Max	Min	Obs			
1CU	101	218.2	327.9	3000	0.05	3530			
$2\mathrm{CU}$	123	256.7	364.6	2700	0.05	2866			
$3\mathrm{CU}$	190	320.0	415.5	4050	0.05	1716			
$4\mathrm{CU}$	182	339.9	470.2	4830	0.05	5880			

Panel 1: Price distribution by number of consecutive bids: All Auctions

Panel 2: Price distribution by number of consecutive bids: First Auction

	Median	Mean	SD	Max	Min	Obs
1CU	100	211.1	304.1	2921	0.05	977
$2\mathrm{CU}$	150	305.0	427.8	2700	0.05	673
$3\mathrm{CU}$	220.5	410.0	512.5	4050	0.05	382
$4\mathrm{CU}$	451	677.6	689.5	4830	0.05	1470

Panel 3: Price distribution by number of consecutive bids: Second Auction

	Median	Mean	SD	Max	Min	Obs
1CU	93.25	219.8	373.0	3000	0.10	624
$2\mathrm{CU}$	103.5	230.2	328.0	2685	0.05	867
$3\mathrm{CU}$	181	294.9	364.7	2850	0.05	539
$4\mathrm{CU}$	101	242.7	309.3	2605	0.05	1470

Panel 4: Price distribution by number of consecutive bids: Third Auction

	Median	Mean	SD	Max	Min	Obs
1CU	94	200.8	312.3	2357	0.10	715
$2\mathrm{CU}$	126.5	256.5	353.9	2601	0.10	778
$3\mathrm{CU}$	151.5	285.55	379.0	2801	0.05	536
$4\mathrm{CU}$	100	229.2	294.2	2701	0.05	1470

Panel 5: Price distribution by number of consecutive bids: Fourth Auction

	Median	Mean	SD	Max	Min	Obs
1CU	114.5	233.4	330.2	2601	0.05	1214
$2\mathrm{CU}$	113.5	239.6	344.8	2601	0.10	548
$3\mathrm{CU}$	167	311.6	411.6	2630	0.05	259
$4\mathrm{CU}$	100	210.1	272.6	2935	0.05	1470

Panel 6: Price distribution for 4CU

Auction	Median	Mean	SD	Max	Min	Obs
1st to 4th	182	339.9	470.2	4830	0.05	5880
1st	451	677.6	689.5	4830	0.05	1470
2nd	101	242.7	309.3	2605	0.05	1470
3rd	100	229.2	294.2	2701	0.05	1470
4th	100	210.1	272.6	2935	0.05	1470
1st and 2nd	253	460.2	576.9	4830	0.05	2940
2nd and 3rd	101.0	235.9	301.9	3001	0.05	2940
3rd and 4th	100.0	219.7	283.7	2935	0.05	2940
1st to 3rd	200.0	383.2	512.4	4830	0.05	4410
2nd to 4th	100.0	227.3	292.7	3001	0.05	4410

The table displays the Distribution of Winning Prices. Panels 1 to 5 presents the Distribution of Prices disaggregated by cases where the same farmer buys one, two, three, or four consecutive units (1CU, 2CU, 3CU, or 4CU, respectively). Panel 1 presents the Distribution of Prices for All Auctions (that is, First, Second, Third, and Fourth Auctions). Panel 2 presents the Distribution of Prices for First Auctions. Panel 3 presents the Distribution of Prices for Second Auctions. Panel 4 presents the Distribution of Prices for Third Auctions. Panel 5 presents the Distribution of Prices for Fourth Auctions. Finally, Panel 6 presents Distribution of Prices just for 4CU (that is, for the subsample of 5880 auctions where the same farmer won all four consecutive units). Note that the first line in Panel 6 (1st to 4th) displays the same information as the last line in Panel 1 (4CU). The second line in Panel 6 (1st) displays the same information as the last line in Panel 6 (2nd) displays the same information as the last line in Panel 6 (2nd) displays the same information as the last line in Panel 6 (2nd) displays the same information as the last line in Panel 6 (2nd) displays the same information as the last line in Panel 6 (2nd) displays the same information as the last line in Panel 6 (2nd).

Figure A3: Winning Prices: by Number of Consecutive Units Bought by the Same Farmer and by Unit



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Figure A3 presents price variation by number of consecutive auctions won by the same individual (left panel) and by sequential auction (right panel).

Figure A3 shows that the stark pattern of outcomes from Figures 1 and 2 (Section 3 in the paper) is consistent across the whole sample. On the one hand, in the top panel of Figure A3 we can see that price dispersion—as well as mean and median prices—are higher when the same farmer wins all four consecutive units (in the top panel, last vertical box labeled 4).

On the other hand, in the bottom panel, where we further disaggregate each box from the top panel by unit (first unit, second unit, third unit, and fourth unit), we can see that the above mentioned higher price dispersion labeled with 4 in this graph—as well as the higher mean and median—is generated by the greater variation in prices for first units and not by prices in second, third, and fourth units.

This particular pattern in prices is caused by the above mentioned deterrence effect whereby farmers exhibit different behavior based on seasonality and rainfall (that is, residual demand for water). During high demand and low rainfall months (summer, for example) the same farmer buys all four sequential units paying a high price for the first unit (with respect to the median or average price conditional on rain) and very low prices for the remaining units. In months where demand is not high (due to farming seasonalities) or where rainfall is high, winning prices for all units are similar in magnitude, regardless of whether the same farmer wins all sequential units (4CU) or different farmers win subsequent units (1CU, 2CU or 3CU).

Aggregate prices over time display consistent trends with the ones found in the empirical literature on sequential auctions. Figure A4 shows that, on average, per unit prices decline by sequential unit (with the first unit of each day considerably higher than second to fourth units, for the reason explained in the previous paragraphs), and by day of the week (prices decline from Monday to Friday). Figure A4 also shows that per unit prices are slightly higher during the day than during the night.



The figure displays the distribution of winning prices by: i) Unit (First Unit in Blue, Second Unit in Red, Third Unit in Green, and Fourth Unit in Orange); Weekday (Mo=Monday, Tu=Tuesday, We=Wednesday, Th=Thursday, and Fr=Friday); and Schedule (Day=Day-Time and Night=Night-Time). Thus, the figure displays the distribution of prices of each of the 40 units auctioned per week for the whole sample (disaggregated by Unit, Weekday, and Schedule). Each vertical box (unit) displays the maximum price (upper adjacent value), 75th percentile (upper hinge), median (black circle marker), 25th percentile (lower hinge), and minimum price (lower adjacent value).

High water requirements for citrus during summer cause prices to soar during those months (Figure A5). As expected, prices are also higher during droughts after conditioning on seasons (Figure A6).¹⁹

¹⁹See drought definition in footnote 17.





The figure displays the distribution of winning prices by season. Each vertical box displays the maximum price (upper adjacent value), 75th percentile (upper hinge), median (black circle marker), 25th percentile (lower hinge), and minimum price (lower adjacent value).





The figure displays the distribution of winning prices by: i) Season and Drought Indicator. Each vertical box displays the maximum price (upper adjacent value), 75th percentile (upper hinge), median, 25th percentile (lower hinge), and minimum price (lower adjacent value). See drought definition in footnote 17. The numbers below each box correspond to the percentage (in terms of the whole sample) of observations in each box (that is, al these numbers sum up to 100%).

To get a more precise understanding of these patterns and the factors affecting winning prices, Table A6 shows that these correlations are robust after conditioning on past rain, unit, weekday, schedule, week-of-the-year, month, and individual fixed effects. Specifically, the table displays the results obtained by regressing daily unit prices on a seven-day-rain moving average (Rain MA7), the rain on the day bought in the auction and the mentioned fixed effects. The estimated coefficients on Rain MA7 have the expected sign and they are statistically significant at 1% level. From column 1, a 10 millimeter increase in average rain in the previous week is associated with a decrease of 40.5 pesetas in the equilibrium price paid in the auction. The regression in column 2 adds unit, weekday, and schedule fixed effects. The estimated coefficient on *Rain MA7* increases in magnitude and also has the expected sign. It can also be seen from this regression that, as noticed in previous figures, price declines within day and across units (both for day-time and night-time auctions) and across schedules (price is on average 110 pesetas lower for night-time auction than for day-time auctions). As regards weekdays average prices, the estimated coefficients show that equilibrium prices decline monotonically within the week (Figure A4). Columns 3 and 4 add, respectively, month seasonal dummies and individual fixed effects (we have 537 different individuals in our sample) to the specification in column 2. The estimated coefficient on Rain MA7 in column 3, though smaller, has again the expected sign and it is statistically different from zero. Similar qualitative results are obtained in column 4; however, the estimated coefficient on Rain $MA\gamma$ has increased. Note that the goodness of the fit in this last regression of the table is 36%, indicating that average (or *ex-ante*) prices are explained by observables such as rain in the previous week and time of the allocation relatively well (favoring, thus, the idea of common knowledge for the parameters within four-unit auctions). Although not reported, we performed as a robustness exercise an analogue analysis using average daily prices within schedule; we obtained similar results.

Variables	(1)	(2)	(3)	(4)
Rain MA7	-4.0543***	-4.1117***	-2.9911^{***}	-3.1741***
	(0.6742)	(0.6894)	(0.5580)	(0.6227)
Rain Day Bought	-0.2346	-0.1853	0.0519	0.1779
	(0.1434)	(0.1416)	(0.1558)	(0.1531)
Unit 2 Day		-167.9547***	-167.8286***	-180.6172***
		(19.4659)	(19.4542)	(21.8896)
Unit 3 Day		-173.0328***	-172.9066***	-188.0507***
		(19.9287)	(19.9165)	(22.7544)
Unit 4 Day		-176.5446^{***}	-176.5451^{***}	-190.8276***
		(20.4404)	(20.4387)	(23.0968)
Unit 2 Night		-237.5795***	-237.8597***	-249.3275***
		(24.9031)	(24.9367)	(27.3493)
Unit 3 Night		-243.3244***	-243.5220***	-257.6533***
		(25.4507)	(25.4838)	(28.3077)
Unit 4 Night		-254.8376***	-255.1867***	-266.4109***
		(25.7254)	(25.7817)	(28.9070)
Tuesday		26.0232^{***}	32.1906^{***}	10.0596
		(7.4927)	(8.2359)	(12.4758)
Wednesday		-34.5756***	-29.3838**	-31.7270**
		(10.6269)	(11.9714)	(15.4702)
Thursday		-59.6530***	-55.4057^{***}	-42.9164***
		(10.7704)	(12.3518)	(15.0439)
Friday		-94.9538***	-95.5421^{***}	-76.3654^{***}
		(12.7055)	(14.4995)	(17.2939)
Night		-110.0908***	-111.0406***	-102.6780***
		(11.3642)	(10.9544)	(13.4322)
Unit FE	No	Yes	Yes	Yes
Weekday FE	No	Yes	Yes	Yes
Schedule FE	No	Yes	Yes	Yes
Month FE	No	No	Yes	Yes
Individual FE	No	No	No	Yes
R^2	0.016	0.083	0.230	0.359
Observations	$13,\!801$	$13,\!801$	$13,\!801$	$13,\!801$

Table A6: Correlation Between Winning Prices and Covariates

All columns are OLS regressions. Dependent variable is the winning price in each auction (one *cuarta*). Robust standard errors in parentheses. FE stands for *Fixed Effects. Individual FE* refers to a set of dummy variables identifying different winners (names) in our sample. We obtain similar results including *Week FE* (a set of dummy variables identifying 52 or 53 weeks of the corresponding year). *** p<0.01, ** p<0.05, * p<0.1. Sample restricted to auctions with positive bids during the period January 1954 to August 1966.

A nice feature of our data is that we are able to observe the identity of the winner. This is crucial to our analysis since it allows us to identify auctions where the same farmer buys all units (4CU). Complementary data from the agricultural census, where we also observe the identity of the land owners, allows us to match these characteristics to each auction's winner. Aside from the variation in these characteristics, which is important to justify our conditionalindependence assumption across auctions, these data allow us to confirm specialized bidding behavior from certain outliers who own a great amount of land and, therefore, bid and win more often. This is depicted in Figure A7. We also use these characteristics in Subsection 5.5 in the paper to analyze an alternative hypothesis to our model (bid rigging).

Figure A7: Winning Farmers and Winning Prices



The figure displays the distribution of Bids per Person (number of auction that the farmer won in the whole sample) and the Mean Price (in the auctions that the farmer won). Each point correspond to a different bidder. In total, there are 537 different points (farmers) who won auctions (see Table 2 in the paper). Outliers are in red with labels.

C.3 Fitting the Model to the Data

Now we show how the described data fit the model rationalizing its assumptions in our empirical context. Validating the model requires that we justify the following assumptions in our empirical setting: (i) conditional on auction-specific covariates, farmers have independent and private valuations (CIPV), (ii) the relevant unit of analysis for individual bidders' demand are four-unit auctions, (iii) conditional on auction-specific covariates accounts for possible dynamic strategic behavior relevant to our analysis, (iv) sunk cost and decreasing marginal returns are important features in this market and not a mechanical result, and (v) participation costs affect bidders in the market.

As emphasized in the theoretical model, we assume that bidders draw new independent and private valuations at every four-unit auction. The conditional-independence assumption allows us to model the equilibrium as Bayesian equilibrium. For the estimation we assume that farmers have independent and private valuations at each four-unit auction, conditional on observed covariates. The first justification for CIPV is that each bidding farmer (who may or may not be a water-owner) has his own land extension, and his own mixture of trees and crops. This eliminates a pure common value scenario. Second, the products being sold are units of water. Assuming that farmers have private (from other farmers) information about the characteristic of this product is not in line with the homogeneous nature of water units. Finally, the conditional-independence assumption is the most sensible in our context, given the varying nature of farming products and soil conditions across farmers. To understand why, recall that the sellers in the water market are a holding formed by the water owners and the buyers are farmers that own fertile land. Around 500 different farmers are observed to win auctions in our sample. Not all of these farmers show up at every auction nor decide to participate if they are present. Farming products cultivated in the area are mainly fruit and citrus trees (lemon, orange, peach, mandarin, and apricot), and vegetables (tomato, lettuce, and onion). The amount of water required by the trees depends on the time of the year and type of crop (citrus trees should not be irrigated daily). Moreover, and given that we condition on seasonality, water requirements vary across products. For example, water needs for grapefruit and lemons are about 20% higher than those for oranges, while

Timing offer planting	Month								
rinning arter planting	Dec Feb.	Mar Apr.	May - Jun.	Jul Sep.	Oct Nov.				
0 - 1 month			2 to 3 days						
2 - 3 months			3 to 5 days						
4 months to 1 year	14 days	7 to 10 days	5 to 7 days	2 to 5 days	5 to 10 days				
1 to 2 years	14 to 21 days	10 to 14 days	7 to 10 days	7 to 10 days	10 to 14 days				
3 years or older	21 to 30 days	14 to 21 days	14 days	10 to 14 days	14 to 21 days				

Table A7: Irrigation Requirements for Citrus Trees

Obtained from Table 2 in Wright (2000), modified from Chott and Bradley (1997).

water requirements for mandarins are about 10% less. Ground conditions (which also vary across areas where different farmers have their land) also affect water necessity. Table A7 displays appropriate intervals for watering citrus. The variations across farmers generated by these factors provide support for the fact that the conditional-independence assumption seems satisfied, given that each day the market is quite specific and since we work with data for four-consecutive auctions as a unit of analysis (sequential auctions).²⁰

The most comprehensive independent unit of analysis that could be considered are weekly auctions, encompassing all 40 units sold per week. This would be the relevant definition to answer questions related to demand fluctuations generated by supply shocks (such as no-auctions due to drought or excessive rain) on an aggregate level. Alternatively, the narrowest possible unit observed (bought) is a *cuarta* (1 of the 40 weekly units). As we explain below in this subsection, the presence of sunk costs and decreasing marginal returns indicate that *cuartas* within a day-schedule are not conditional-independent. Moreover, they are not the relevant unit of analysis to investigate individual farmers' demand nor the price pattern described above.

Our original question is motivated by the particular price behavior caused by the deterrence effect. This particular behavior is observed within four-unit auctions and, consistent with this empirical observation, is the unit of analysis in the theoretical model (Section 2 in the paper). This is an implication of the way the auction is structured: twelve hours of water (subdivided into four *cuartas* of three hours each) during day-time and twelve hours

²⁰Our justification of the CIPV paradigm is in line with the literature on empirical auctions. For first price descending auctions see, for example, Laffont, Ossard and Vuong (1995) in an application to agricultural products (greenhouse eggplants in Marmande, France) where the number of bidders vary between 11 and 18. For English auctions, Haile and Tamer (2003) apply their limited structure model to U.S. Forest Service timber auctions, where the number of bidders vary from 2 to 12.

of water during night-time, each weekday. The logic behind this structure is related to water requirements in the area. First, water scarcity in the region made water accountability crucial. The standard unit used to measure surface area in Murcia is called *tahulla*. One *tahulla* is, by definition, the surface area which can be irrigated in such a way that water level rises 1-foot high in 1 minute.²¹ Needless to say, depending on soil conditions, the surface area from one-*tahulla* varies from one town to another.²² A four-consecutive units auction—half day, twelve hours of irrigation—is, in that sense, the amount of water that absorbs a regular *parcela* (individual piece of land). Water requirements could and actually do differ (a) across farmers depending on farming trees and land extension, and (b) for the same farmer over time depending on past rainfall.

Second, the irrigation technique used in Mula is flood irrigation. This ancient method of irrigating crops is still used today. The farmer builds small embankments in his *parcela* and water is delivered to the land by the channel system that simply flows over the ground through the crop. Flood irrigation requires a minimum of water delivery that, for a regular *parcela*, is captured by one *tahulla*.

Finally, a supply-side consideration also plays its role. The reason to supply water for 12 hours (during day-time and during night-time) is to guarantee a particular and homogenous quantity for each *cuarta* (which depends on water pressure since water units are defined in hours). Given that the *De La Cierva* dam is continuously filled with water from the river, spreading the supply provision across weekdays ensures the homogeneity of water units.

Our data confirm these three points, validating that the relevant unit of analysis for individual demand within the CIPV paradigm are four-consecutive units: the most frequent quantity purchased by farmers are twelve hour of water (42% of sold units in Table A3).

The way in which the auction system is carried out every week raises the question of the importance of dynamic strategic considerations between four-unit auctions both among days (Monday to Friday for a specific schedule) and between schedules (day-time vs night-time for

 $^{^{21}}$ Although close in magnitude, the traditional Murcian measure of foot is not exactly the same to the foot measure used in the U.K. and the U.S. (Valiente 2001)

 $^{^{22}}$ The surface area of 1-tahulla is 1,118 square meters in Murcia and 1,185 square meters in the old Kingdom of Aragón, except the region of Pías Fundaciones. The tahulla is used in regadío lands since Charles IV (king of Spain from 14 December 1788 until his abdication on 19 March 1808). In secano lands the surface area measure that is used is the fanega and the celemín. For further details see Vera Nicolás (2004).

a specific day). Tables A5 and A6 show that winning prices decline across days (for a given schedule) and at night (for a given day), which is consistent with the literature on empirical sequential auctions. These dynamic strategic considerations are outside the scope of the present investigation, and we abstract from them in our model in Section 2 in the paper.²³ However, is important to note that, even if present, dynamic behavior considerations do not invalidate our model's assumptions. As emphasized above, the conditional-independent units of analysis are four-unit auctions (not day-auctions of eight units or week-auctions of 40 units) which, conditional on covariates, are homogeneous goods. As can be seen from the correlations presented in Table A6, previous patterns are consistent along the whole sample and robust to the inclusion of a whole set of fixed effects and covariates. Moreover, residuals from these reduced-form regressions do not show dynamic patterns. This is not surprising as the principal difference between these four-unit auctions is related to the uncertainty of future rain. As it is explained in the estimation section, we include covariates for schedule, day-ofthe-week, and past rain in our structural estimation that capture technological or strategic effects. Future rain, on the other hand, is also included as a proxy for farmers' beliefs to account for these (possible) strategic behaviors not accounted by previous covariates. In that sense, our estimates should be interpreted as four-unit day-schedule specific auctions, conditional on past rain and seasonality. It seems implausible that after accounting for these observables and unobservables,²⁴ and given that the relevant unit of analysis are fourunit auctions, dynamic behavior would affect our results concerning individual demand.²⁵ Nevertheless, we further investigate the residuals after our estimation in Subsection 5.3 in the paper to analyze specific patterns related to this concern: residuals from the structural estimates in Tables 3 and 4 do not show dynamic patterns and are not correlated with the covariates.

There are two main specific features from the model that need to be justified in our empirical setting. First, the sunk cost (SC) that farmers incur when they buy their first

²³For a broader discussion see Donna and Espin-Sanchez 2013.

 $^{^{24}}$ While farmers use their (reasonable good) predictions in their decisions, we use actual future rainfall in our estimation.

²⁵Once we condition on these covariates, the concern that bidders' outside option would vary according to the day of the week (or schedule) is addressed by redefining the idiosyncratic individual valuation in such a way that the new one be the original valuation net of the outside option. By normalizing the outside option of Friday-night to zero the model's assumptions remain valid.

unit. Water is allocated during the auction and is distributed on the specific day and time of the irrigation accordingly. Water stored in the dam is delivered to the farmer's plot on this date using the channel system. Except the main canal, all channels are dug into the ground (Figure A8). On the day of the irrigation, a guard opens the corresponding gates to allow the water to flow to the appropriate farmer's land. These channels are land-specific in the sense that different areas and lands have their own system of channels which only carry water when the corresponding gates are opened.²⁶ There is a water loss that is incurred because water flows over a dry channel. Engineers have estimated this loss to be between 15% and 40% (20% on average) of the water carried by one *cuarta* when the channel is completely dry (see Vera Nicolás 2004). This is the SC incurred by the bidder for his first unit. The SC is only incurred once, for the first unit, since water losses associated with a wet channel are negligible. In the model, the SC effect is captured by the parameter α , whose interpretation is the percentage of water loss from the first unit because water is flowing through a dry channel (hence, proportional to the valuation of the bidder for the unit of water). One would expect that, conditional on rain, water loss would be constant within season with relatively more importance (higher α) in summer.²⁷

 $^{^{26}}$ A concern is that farmers whose lands lie next to each other may be buying different sequential units for the same auction. In this case, the SC would only be incurred by the first farmer for his first unit but not for the second farmer for his first unit. Fortunately, we have data on the specific location of the farmers that we are able to match to auction winners to analyze these situations, as we do in Subsection 5.5 in the paper.

²⁷We discuss variation of SC across auctions (conditional on covariates) in Section 4 in the paper.

Figure A8: The Channel System in Mula and the Sunk Cost of Initiating the Irrigation





The main canal (left panel) was made of concrete. The individual sub-channels (right panel) were dug into the ground. Thus, in these sub-channels, a water loss is incurred because water flows over a dry sub-channel (some water is absorbed by the ground).

The second feature refers to the decreasing marginal returns (DMR) effect. The classic textbook case for DMR is appropriate for our empirical application: given that the amount of land owned by each farmer is fixed, marginal productivity of subsequent units of water is decreasing. When assessing the relative importance of DMR, the impact in summer would generally be greater than in autumn. More generally, one would expect DMR to be affected by season and rain. When water requirements are high, the slope of the marginal productivity function will be relatively flat, as in the left panel in Figure A9. This is likely to occur in spring and summer. On the other hand, when water requirements are low, the slope of the marginal productivity function will be steeper, as in the right panel in Figure A9. This is likely to happen in autumn or winter. In the model, the DMR effect is captured by the parameter β . In Section 4 we discuss how we model DMR for the estimation to take into account the points addressed in this paragraph.

There are several reasons to believe that farmers face participation costs in this market. The first component are opportunity costs. Farmers who value their time may prefer not to participate in the whole auction session. Auctions were run on Friday evenings during work hours. Attending the auction entailed alternative use of working time for the farmer.



Figure A9: Marginal Returns of Irrigation Water

This is Figure 4 in the paper. Marginal returns of water in summer (left) and autumn (right).

To contextualize this component in our empirical setting, it is worth noting that it is not uncommon to observe the farmer's wife, a son, or other relative substitute for the farmer on certain occasions.²⁸ Another component of participation costs are the various hassle costs associated with active bidding. Actually, only a fraction of the individuals who attended a Friday auction were actively engaged in the bidding for a particular sequential auction of water and not everyone who was present participated in every auction (Botía, interview). As von der Fehr (1994) points out, a reasonable assumption for why only a portion of attendees participate may be that they consider it so unlikely to that they will win at a price below what they will be willing to pay, that they are not willing to bother to engage in bidding. Finally, and related to the previous argument, participation costs may also be interpreted as the cost of preparing the bid and the cost of learning what the item is worth in the specific environment (that is, conditional on the specific covariates of the auction).

Empirical evidence from our data is consistent with the assertion that farmers dislike participation, facing positive entry costs. In particular, we observe multiple weeks per year where auctions are run, farmers show up and buy the first units of water, but where no one bids for the last units. Since there is no reservation price and the minimum bid increment are cents, they could potentially win all the remaining units bidding this amount. On the

 $^{^{28}}$ We are able to identify these cases by matching census land data (with information on the person who registered the property) with auctions data (where the name of the winning bidder on the auction sheet is a relative of the name written on the census card). In other situations it is more evident as the winning name in the auction sheet simply states, *Wife (or son) of* the farmer who owns the land.

contrary, they decide not to bid and leave the auction. For example, on January 22, 1954, units 1 to 16 were sold to seven different farmers but no one bid for units 17 to 20 (Figure A10). In 1954 we observe similar behavior for 14 weeks,²⁹ and this is consistent along the remaining years in our sample: out of the 3203 auctions where no one bid (Table A3), we use those 2423 where some bidders where present (auctions similar to the one in Figure A10). The interpretation is that the utility for all bidders is smaller than the participation cost, conditional on covariates. We use this information in Section E in this Online Appendix to partially identify participation costs.

Figure A10: Auction Sample: Auction where Farmers Are Present and No Bids Are Placed

Auction $\#$	Name	Price	Day	
1	Sebastian Aguilar	48	Мо	
2	Felipe Amaro	42		2. plipe Grien 98 - 3. plipe Grieno 98 - 4. genero 98 - 5. plipe Grieno 98 - 5. plipe Grieno 59 - 5. plipe Grieno 59 - 5. plipe Grieno 59 -
3	Felipe Amaro	48		
4	Diego Guirao	50		
5	Felipe Amaro	54	Tu	7. Cintobal Rouns 51-
6	Antonio Llamas	51		9 - Gistabel future 2.
7	Cristobal Romero	47		11 · l luinuo 1 -
8	Cristobal Romero	50		12 ° d' dur 1- 13 ° fuur Ullage 2 75
9	Cristobal Gutierrez	2	We	14 ° U uurus 1 - 15 ° U Uurus 1 -
10	Cristobal Gutierrez	5		16 · · · · · · · · · · · · · · · · · · ·
11	Cristobal Gutierrez	1		18 :
12	Cristobal Gutierrez	1		20 , Dia
13	Luis Moya	2.75	Th	Noche
14	Luis Moya	1		Voz půbica To al liquido
15	Luis Moya	1		Mula 22 de ferro de 195 Y
16	Luis Moya	1		El Fresidente, A D
17				Jun Joins
18			Fr	- Op-
19			ГІ	
20				

This is Figure 5 in the paper. Sample from original data obtained from the historical archive: Auction where farmers are present and no bids are placed (Winter - January 22, 1954, Day).

²⁹Weeks of January 22, February 5, April 5, May 1, May 8, May 15, May 22, May 29, June 5, June 12, July 3, July 10, November 26, and December 3.

C.4 High Definition Figures

In this Subsection we present four high definition versions of Figure 6 in the paper.



Figure A11: Winning and Estimated Prices

This is a high definition version of Figure 6 in the paper. The figure displays real prices against predicted prices using three different models: (i) our structural model (specification 3 in Table 3 in the paper), (ii) a standard (button) English auction model (specification 5 in Table 3 in the paper), and (iii) a reduced-form model for the sample using as regressors: *Past Rain*, unit (3 dummy variables), weekday (4 dummy variables), schedule (1 dummy variable), month (11 dummy variables), year (12 dummy variables), and individual fixed effects, in addition to a constant (for details about the reduced-form specification see Table A6 in Subsection C.2 in this Online Appendix). The graph shows the mean monthly averages of the prices.







This is a high definition version of Figure 6 in the paper. The figure displays real prices against predicted prices using a standard (button) English auction model (specification 5 in Table 3 in the paper). The graph shows the mean monthly averages of the prices.





as regressors: Past Rain, unit (3 dummy variables), weekday (4 dummy variables), schedule (1 dummy variable), month (11 dummy variables), year (12 dummy variables), and individual fixed effects, in addition to a constant (for details about the reduced-form specification see Table A6 in Subsection C.2 in this Online This is a high definition version of Figure 6 in the paper. The figure displays real prices against predicted prices using a reduced-form model for the sample using Appendix). The graph shows the mean monthly averages of the prices.

D Estimation: Implementation Details

Note from equation 8 (Section 5 in the paper) that, conditional on θ , the log-likelihood function is the sum of three well defined components: a part associated with the winning bids where the same bidder wins all four units and goods are pure complements, a part where the same bidder wins all four units and goods are non pure complements, and a last part where the last winner also bought two out of the first three units, three units in total. To recover the parameters we follow an iterative two stage procedure. In the first stage, conditional on θ , we recover the parameter vector $\gamma \in \mathbb{R}^{19}$ by MLE using the distribution of the $(N-1)^{th}$ order statistic.³⁰ In the second stage, conditional on γ , we recover $\theta \in \mathbb{R}^5$ by generalized method of moments (GMM, Hansen 1982) using the model moment equations.³¹ We then iterate these two stages until convergence.³² Implementing the estimation in two stages results in substantial computational savings (to compute the bootstrapped standard errors) as we express analytically the solution of the gradient of the GMM estimator.^{33 34}

 $^{^{30}}$ For the exponential distribution we obtain this distribution solving numerically the integral in equation 5 (Section 4 in the paper). For the Exponentiated Gamma distribution we are able to obtain a closed-form solution of this expression (see footnote 31 in the paper). This motivated its use originally (see Donna and Espin-Sanchez (2012)).

³¹The log-likelihood is the sum of the three mentioned components. The known function multiplying private valuations differs for each component (equation 7 in Section 4 in the paper). However, because private valuations, v_i , are conditional-independent draws from the same parametric distribution $F(\cdot; \mu|\theta, Z_t, \gamma)$, all the coefficients in the mean valuations equation (equation 9 in Section 5 in the paper) are point identified. This would not be true if the log-likelihood would only have, say, the first component.

³²For the initial condition we use a consistent estimate of γ and θ obtained by full maximum likelihood estimation using equation 8 (Section 5 in the paper). See Aguirregabiria and Mira (2002) and Aguirregabiria and Mira (2007) for further details.

³³For details about the moment equations, the empirical analogues, and the gradient that we use in our program see Section F in this Online Appendix.

³⁴We performed a Monte Carlo study to evaluate how well the proposed estimation procedure performs in our setting. Using a smaller subset of parameters (7 instead of 22) and the model constraints (see Section 2 in the paper), we did not register substantial improvement in efficiency of MLE relative to the two-stage estimator. Of course, the point estimates obtained by the two procedures were not statistically different. The Monte Carlo study is available online in the earlier working paper (Donna and Espin-Sanchez 2012).

E Participation Costs: Identification and Estimation

Although, throughout the previous estimation procedure, participation costs, c, have been fixed at an arbitrary small magnitude, we recover them from our data. We use our model and data where auctions were run, no bids observed and farmers were present, along with the structural estimates. Participation costs are identified by the necessary condition for a bidder to bid in the first auction that is given by:

$$(1 - \alpha)v_{N:N} < c$$

More generally, a condition that additionally involves second, third, and fourth marginal utilities for the case where the bidder also enters the individual auctions for two, three or four units should be considered. In these cases, participation costs are also greater than the average marginal utility for second, third, and fourth units. Formally:

$$Max\left\{(1-\alpha), \frac{(2-\alpha-\beta_t)}{2}, \frac{(3-\alpha-2\beta_t)}{3}, \frac{(4-\alpha-3\beta_t)}{4}\right\}v_{N:N} < c$$
(E.1)

Note that, when $\alpha < \beta_t$, the former condition is sufficient, implying the latter. In our econometric specification the structural parameter α is fixed while the parameter β_t varies according to the farmers' expectations of (exogenous) future rain. One would expect to observe auctions without bids when farmers' expectations for rain, as captured by actual future rain, are high (which in the model is represented by a relatively high β_t). Therefore, absence of bids will only occur when $\alpha < \beta_t$, thus, the former identification restriction is sufficient.

Analogously, using the model and the remaining data not used in the structural estimation, we obtain an upper bound using that participation cost are lower than the minimum registered price (conditional on covariates, sunk cost and decreasing marginal returns).

F Moment Conditions, Gradient and Empirical Analogues

1 Moment Conditions

1.1 Moments

To recover the structural parameters, α and β , we use the following moment conditions. To simplify the notation, let $\theta = (\alpha, \beta_0^C, \beta_1^C, \beta_0^S, \beta_1^C)$, $Y_t \equiv (p_t, R_t, v_{N-1:N,t}, \{D_t^j\}_{j \in \{a,b,c\}}, c)$ and $\bar{x}_j = \frac{1}{\sum_{t=1}^T 1[D_{j,t}=1]} \sum_{t=1}^T D_{j,t} x_t$, $j \in \{a, b, c\}$.

$$f_1(Y_t, \theta) \equiv \mathbb{E}\left(D_{a,t} \sum_{k=1}^4 p_t^k - \left[4 - \alpha - 6\left(\beta_0^C + \beta_1^C D_{a,t} R_t\right)\right] D_{a,t} v_{N-1:N,t} + 3c\right) = 0$$

$$f_2(Y_t, \theta) \equiv \mathbb{E} \left(D_{b,t} p_t^4 - (1 - \alpha) D_{b,t} v_{N-1:N,t}^t + c \right) = 0$$

$$f_3(Y_t, \theta) \equiv \mathbb{E}\left(D_{c,t}p_t^4 - \left[1 - \beta_0^S + \beta_1^S D_{c,t} R_t\right] D_{c,t} v_{N-1:N,t} + c\right) = 0$$

$$f_4(Y_t,\theta) \equiv \mathbb{E}\left(\left[D_{a,t}\sum_{k=1}^4 p_t^k - \bar{p}_a\right]^2 - \left[\left(4 - \alpha - 6\beta_0^C\right)\left(D_{a,t}v_{N-1:N,t} - \bar{v}_{N-1:N}\right) - 6\beta_1^C\left(D_{a,t}R_tv_{N-1:N,t} - \bar{R}_a\bar{v}_{N-1:N}\right)^2\right) = 0$$

Or, equivalently:

$$f_1(Y_t,\theta) = \mathbb{E}\left(D_{a,t}\sum_{k=1}^4 p_t^k\right) - \left[4 - \alpha - 6\left(\beta_0^C + \beta_1^C \mathbb{E}\left(\sum_{t=1}^T D_{a,t}R_t\right)\right)\right] \mathbb{E}\left(D_{a,t}v_{N-1:N,t}\right) + 3c = 0$$

$$f_2(Y_t,\theta) = \mathbb{E}\left(D_{b,t}p_t^4\right) - (1-\alpha)\mathbb{E}\left(D_{b,t}v_{N-1:N,t}\right) + c = 0$$

$$f_{3}(Y_{t},\theta) = \mathbb{E}\left(D_{c,t}p_{t}^{4}\right) - \left[1 - \beta_{0}^{C} + \beta_{1}^{C}\mathbb{E}\left(D_{c,t}R_{t}\right)\right]\mathbb{E}\left(D_{c,t}v_{N-1:N,t}\right) + c = 0$$

$$f_{4}(Y_{t},\theta) \equiv \mathbb{V}\left(D_{a,t}\sum_{k=1}^{4} p_{t}^{k}\right) - \left(4 - \alpha - 6\beta_{0}^{C}\right)^{2} \mathbb{V}\left(D_{a,t}v_{N-1:N,t}\right) + 12\left(4 - \alpha - 6\beta_{0}^{C}\right)\beta_{1}^{C}\mathbb{E}\left(D_{a,t}R_{a,t}\right)\mathbb{V}\left(D_{a,t}v_{N-1:N,t}\right) - 36\left(\beta_{1}^{C}\right)^{2}C_{a,t} = 0$$

$$f_5\left(Y_t,\theta\right) \equiv \mathbb{V}\left(D_{b,t}p_{b,t}^4\right) - (1-\alpha)^2 \,\mathbb{V}\left(D_{b,t}v_{N-1:N,t}\right) = 0$$

$$f_{6}(Y_{t},\theta) \equiv \mathbb{V}\left(D_{c,t}p_{c,t}^{4}\right) - \left(1 - \beta_{0}^{S}\right)^{2} \mathbb{V}\left(D_{c,t}v_{N-1:N,t}\right) \\ - 2\left(1 - \beta_{0}^{S}\right)\beta_{1}^{S} \mathbb{E}\left(D_{c,t}R_{c,t}\right) \mathbb{V}\left(D_{c,t}v_{N-1:N,t}\right) + \left(\beta_{1}^{C}\right)^{2}C_{c,t} = 0$$

where $\mathbb{V}\left(\cdot\right)$ denotes variance and:

$$C_{j,t} \equiv \left[\mathbb{E} (D_{j,t}R_t)\right]^2 \mathbb{V} (D_{j,t}v_{N-1:N,t}) + \left[\mathbb{E} (D_{j,t}v_{N-1:N,t})\right]^2 \mathbb{V} (D_{j,t}R_t) + \mathbb{V} (D_{j,t}v_{N-1:N,t}) \mathbb{V} (D_{j,t}R_t)$$

, $j \in \{a, c\}.$

1.2 Moments Empirical Analogues

For the estimation we use the empirical analogues, $\hat{f}_i(Y_t, \theta) = \frac{1}{T} \sum_{t=1}^T f_i(Y_t, \theta)$, i = 1, ..., 6. Specifically:

$$\begin{aligned} \hat{f}_1\left(Y_t,\theta\right) = & \frac{1}{\sum_{t=1}^T 1\left[D_{a,t}=1\right]} \sum_{t=1}^T D_{a,t} \sum_{k=1}^4 p_t^k - \left[4 - \alpha - 6\left(\beta_0^C + \beta_1^C \frac{1}{\sum_{t=1}^T 1\left[D_{a,t}=1\right]} \sum_{t=1}^T D_{a,t} R_t\right)\right] \\ & \frac{1}{\sum_{t=1}^T 1\left[D_{a,t}=1\right]} \sum_{t=1}^T D_{a,t} v_{N-1:N,t} + 3c = 0 \end{aligned}$$

$$\hat{f}_{2}(Y_{t},\theta) = \frac{1}{\sum_{t=1}^{T} \mathbb{1}\left[D_{b,t}=1\right]} \sum_{t=1}^{T} D_{b,t} p_{t}^{4} - (1-\alpha) \frac{1}{\sum_{t=1}^{T} \mathbb{1}\left[D_{b,t}=1\right]} \sum_{t=1}^{T} D_{b,t} v_{N-1:N,t} + c = 0$$

$$\hat{f}_{3}(Y_{t},\theta) = \frac{1}{\sum_{t=1}^{T} \mathbb{1}\left[D_{c,t}=1\right]} \sum_{t=1}^{T} D_{c,t} p_{t}^{4} - \left[1 - \beta_{0}^{C} + \beta_{1}^{C} \frac{1}{\sum_{t=1}^{T} \mathbb{1}\left[D_{c,t}=1\right]} \sum_{t=1}^{T} D_{c,t} R_{t}\right]$$
$$\frac{1}{\sum_{t=1}^{T} \mathbb{1}\left[D_{c,t}=1\right]} \sum_{t=1}^{T} D_{c,t} v_{N-1:N,t} + c = 0$$

$$\begin{split} \hat{f}_{4}\left(Y_{t},\theta\right) = & \frac{1}{\sum_{t=1}^{T} 1\left[D_{a,t}=1\right]} \sum_{t=1}^{T} \left[D_{a,t} \sum_{k=1}^{4} p_{t}^{k} - \frac{1}{\sum_{t=1}^{T} 1\left[D_{a,t}=1\right]} \sum_{t=1}^{T} D_{a,t} \sum_{k=1}^{4} p_{t}^{k} \right]^{2} \\ & - \left(4 - \alpha - 6\beta_{0}^{C}\right)^{2} \frac{1}{\sum_{t=1}^{T} 1\left[D_{a,t}=1\right]} \sum_{t=1}^{T} \left[D_{a,t} v_{N-1:N,t} - \frac{1}{\sum_{t=1}^{T} 1\left[D_{a,t}=1\right]} \sum_{t=1}^{T} D_{a,t} v_{N-1:N,t} \right]^{2} \\ & + 12 \left(4 - \alpha - 6\beta_{0}^{C}\right) \beta_{1}^{C} \frac{1}{\sum_{t=1}^{T} 1\left[D_{a,t}=1\right]} \sum_{t=1}^{T} D_{a,t} R_{t} \frac{1}{\sum_{t=1}^{T} 1} \\ & \left[D_{a,t}=1 \right] \sum_{t=1}^{T} \left[D_{a,t} v_{N-1:N,t} - \frac{1}{\sum_{t=1}^{T} 1\left[D_{a,t}=1\right]} \sum_{t=1}^{T} D_{a,t} v_{N-1:N,t} \right]^{2} - 36 \left(\beta_{1}^{C}\right)^{2} C_{a,t} = 0 \end{split}$$

$$\hat{f}_{5}(Y_{t},\theta) = \frac{1}{\sum_{t=1}^{T} \mathbb{1}[D_{b,t}=1]} \sum_{t=1}^{T} \left[D_{b,t}p_{t}^{4} - \frac{1}{\sum_{t=1}^{T} \mathbb{1}[D_{b,t}=1]} \sum_{t=1}^{T} D_{b,t}p_{t}^{4} \right]^{2} - (1-\alpha)^{2} \frac{1}{\sum_{t=1}^{T} \mathbb{1}[D_{b,t}=1]} \sum_{t=1}^{T} \left(D_{b,t}v_{N-1:N,t} - \frac{1}{\sum_{t=1}^{T} \mathbb{1}[D_{b,t}=1]} \sum_{t=1}^{T} D_{b,t}v_{N-1:N,t} \right)^{2} = 0$$

$$\begin{split} \hat{f}_{6}\left(Y_{t},\theta\right) = & \frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]} \sum_{t=1}^{T} \left[D_{c,t}p_{t}^{4} - \frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]} \sum_{t=1}^{T} D_{c,t}p_{t}^{4} \right]^{2} - \left(1 - \beta_{0}^{S}\right)^{2} \frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]} \\ & \sum_{t=1}^{T} \left[D_{c,t}v_{N-1:N,t} - \frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]} \sum_{t=1}^{T} D_{c,t}v_{N-1:N,t} \right]^{2} - 2\left(1 - \beta_{0}^{S}\right)\beta_{1}^{S} \frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]} \\ & \sum_{t=1}^{T} D_{c,t}R_{t} \frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]} \sum_{t=1}^{T} \left[D_{c,t}v_{N-1:N,t} - \frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]} \sum_{t=1}^{T} D_{c,t}v_{N-1:N,t} \right]^{2} \\ & + \left(\beta_{1}^{C}\right)^{2}C_{c,t} = 0 \end{split}$$

where for $j\epsilon \{a, c\}$:

$$\begin{split} \hat{C}_{j,t} &\equiv \left[\frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T} D_{j,t} R_{t}\right]^{2} \frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T} \left[D_{j,t} v_{N-1:N,t} - \frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T} D_{j,t} v_{N-1:N,t}\right]^{2} \\ &+ \left[\frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T} D_{j,t} v_{N-1:N,t}\right]^{2} \frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T} \left[D_{j,t} R_{t} - \frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T} D_{j,t} R_{t}\right]^{2} \\ &+ \left[\frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T} D_{j,t} v_{N-1:N,t}\right]^{2} \frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T} \left[D_{j,t} R_{t} - \frac{1}{\sum_{t=1}^{T} 1\left[D_{j,t}=1\right]} \sum_{t=1}^{T} D_{j,t} R_{t}\right]^{2} \end{split}$$

2 Gradient

2.1 Gradient

Let $\hat{F}(\theta) = \left[\hat{f}_1(\theta) \cdots \hat{f}_6(\theta)\right]'$, \hat{W} be a positive-definite weighting matrix (computed based on our data), and m' denotes transposition. The GMM estimator is given by:

$$\hat{\theta} = \underset{\theta \in \Theta}{\arg\min} \hat{F}(\theta)' \hat{W} \hat{F}(\theta)$$

where Θ is the (compact) parameter set obtained from our model in Section 3.

The gradient is given by:

$$\frac{\partial F\left(\theta\right)}{\partial\theta'} = \begin{bmatrix} \frac{\partial f_{1}(\theta)}{\partial\alpha} & \frac{\partial f_{1}(\theta)}{\partial\beta_{0}^{C}} & \frac{\partial f_{1}(\theta)}{\partial\beta_{1}^{C}} & \frac{\partial f_{1}(\theta)}{\partial\beta_{0}^{S}} & \frac{\partial f_{1}(\theta)}{\partial\beta_{1}^{S}} \\ \vdots & & \vdots \\ \vdots & & & \vdots \\ \frac{\partial f_{6}(\theta)}{\partial\alpha} & \cdots & \cdots & \frac{\partial f_{6}(\theta)}{\partial\beta_{1}^{S}} \end{bmatrix}$$

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Where:

$$\frac{\partial F\left(\theta\right)}{\partial \alpha} = \begin{array}{c} \frac{\frac{\partial f_{1}\left(\theta\right)}{\partial \alpha}}{\frac{\partial f_{2}\left(\theta\right)}{\partial \alpha}} \\ \frac{\partial f_{2}\left(\theta\right)}{\partial \alpha} \\ \frac{\partial f_{3}\left(\theta\right)}{\partial \alpha} \\ \frac{\partial f_{4}\left(\theta\right)}{\partial \alpha} \\ \frac{\partial f_{5}\left(\theta\right)}{\partial \alpha} \\ \frac{\partial f_{5}\left(\theta\right)}{\partial \alpha} \\ \frac{\partial f_{5}\left(\theta\right)}{\partial \alpha} \end{array} = 2 \begin{bmatrix} f_{1}\left(\theta\right) \mathbb{E}\left(D_{a,t}v_{N-1:N,t}\right) \\ 0 \\ 2f_{4}\left(\theta\right) \left[\left(4 - \alpha - 6\beta_{0}^{C}\right) - 6\beta_{1}^{C}\mathbb{E}\left(D_{a,t}R_{t}\right)\right] \mathbb{V}\left(D_{a,t}v_{N-1:N,t}\right) \\ 2f_{5}\left(\theta\right)\left(1 - \alpha\right) \mathbb{V}\left(D_{b,t}v_{N-1:N,t}\right) \\ 0 \end{bmatrix}$$

$$\frac{\partial F\left(\theta\right)}{\partial \beta_{0}^{C}} = \begin{array}{c} \frac{\frac{\partial f_{1}(\theta)}{\partial \beta_{0}^{C}}}{\frac{\partial f_{2}(\theta)}{\partial \beta_{0}^{C}}} \\ \frac{\partial f_{3}(\theta)}{\partial \beta_{0}^{C}} \\ \frac{\frac{\partial f_{3}(\theta)}{\partial \beta_{0}^{C}}}{\frac{\partial f_{4}(\theta)}{\partial \beta_{0}^{C}}} \end{array} = 12 \begin{bmatrix} f_{1}\left(\theta\right) \mathbb{E}\left(D_{a,t}v_{N-1:N,t}\right) \\ 0 \\ 0 \\ 2f_{4}\left(\theta\right) \begin{bmatrix} \left(4 - \alpha - 6\beta_{0}^{C}\right) - 6\beta_{1}^{C}\mathbb{E}\left(D_{a,t}R_{t}\right)\end{bmatrix} \mathbb{V}\left(D_{a,t}v_{N-1:N,t}\right) \\ 0 \\ 0 \\ \frac{\partial f_{6}(\theta)}{\partial \beta_{0}^{C}} \\ 0 \end{bmatrix}$$

$$\frac{\partial F\left(\theta\right)}{\partial \beta_{1}^{C}} = \begin{pmatrix} \frac{\partial f_{1}(\theta)}{\partial \beta_{1}^{C}} \\ \frac{\partial f_{2}(\theta)}{\partial \beta_{1}^{C}} \\ \frac{\partial f_{3}(\theta)}{\partial \beta_{1}^{C}} \\ \frac{\partial f_{4}(\theta)}{\partial \beta_{1}^{C}} \\ \frac{\partial f_{5}(\theta)}{\partial \beta_{1}^{C}} \\ \frac{\partial f_{5}(\theta)}{\partial \beta_{1}^{C}} \\ \frac{\partial f_{6}(\theta)}{\partial \beta_{1}^{C}} \end{pmatrix} = 12 \begin{bmatrix} f_{1}\left(\theta\right) \mathbb{E}\left(D_{a,t}R_{t}\right) \mathbb{E}\left(D_{a,t}v_{N-1:N,t}\right) \\ 0 \\ 2f_{4}\left(\theta\right) \left[\left(4 - \alpha - 6\beta_{0}^{C}\right) \mathbb{E}\left(D_{a,t}R_{t}\right) \mathbb{V}\left(D_{a,t}v_{N-1:N,t}\right) - 6\beta_{1}^{C}C_{a,t}\right] \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{\partial F\left(\theta\right)}{\partial \beta_{0}^{S}} = \begin{array}{c} \frac{\frac{\partial f_{1}\left(\theta\right)}{\partial \beta_{0}^{S}}}{\frac{\partial f_{2}\left(\theta\right)}{\partial \beta_{0}^{S}}} \\ \frac{\partial f_{2}\left(\theta\right)}{\partial \beta_{0}^{S}} \\ \frac{\partial f_{3}\left(\theta\right)}{\partial \beta_{0}^{S}} \\ \frac{\partial f_{4}\left(\theta\right)}{\partial \beta_{0}^{S}} \\ \frac{\partial f_{5}\left(\theta\right)}{\partial \beta_{0}^{S}} \\ \frac{\partial f_{5}\left(\theta\right)}{\partial \beta_{0}^{S}} \\ \frac{\partial f_{6}\left(\theta\right)}{\partial \beta_{0}^{S}} \end{array} = 2 \begin{bmatrix} 0 \\ 0 \\ f_{3}\left(\theta\right) \mathbb{E}\left(D_{c,t}v_{N-1:N,t}\right) \\ 0 \\ 0 \\ 2f\left(\theta\right) \left[\left(1-\beta_{0}^{S}\right)+\beta_{1}^{S}\mathbb{E}\left(D_{c,t}R_{t}\right)\right] \mathbb{V}\left(D_{c,t}v_{N-1:N,t}\right) \end{bmatrix}$$

$$\frac{\partial F\left(\theta\right)}{\partial \beta_{1}^{S}} = \begin{array}{c} \frac{\frac{\partial f_{1}(\theta)}{\partial \beta_{1}^{S}}}{\frac{\partial f_{2}(\theta)}{\partial \beta_{1}^{S}}} \\ \frac{\partial f_{3}(\theta)}{\partial \beta_{1}^{S}} \\ \frac{\partial f_{4}(\theta)}{\partial \beta_{1}^{S}} \\ \frac{\partial f_{5}(\theta)}{\partial \beta_{1}^{S}} \\ \frac{\partial f_{5}(\theta)}{\partial \beta_{1}^{S}} \\ \frac{\partial f_{6}(\theta)}{\partial \beta_{1}^{S}} \end{array} = 2 \begin{bmatrix} 0 \\ 0 \\ f_{3}\left(\theta\right) \mathbb{E}\left(D_{c,t}R_{t}\right) \mathbb{E}\left(D_{c,t}v_{N-1:N,t}\right) \\ 0 \\ 0 \\ 2f_{6}\left(\theta\right) \left[-\left(1-\beta_{0}^{S}\right) \mathbb{E}\left(D_{c,t}R_{t}\right) \mathbb{V}\left(D_{c,t}v_{N-1:N,t}\right) + \beta_{1}^{S}C_{c,t}\right] \end{bmatrix}$$

2.2 Gradient Empirical Analogue

Finally, the empirical analogue we use in the estimation is given by:

$$\frac{\partial \hat{F}(\theta)}{\partial \alpha} = 2 \begin{bmatrix} \hat{f}_{1}(\theta) \frac{1}{\sum_{t=1}^{T} 1[D_{a,t}=1]} \sum_{t=1}^{T} D_{a,t}v_{N-1:N,t} \\ \hat{f}_{2}(\theta) \frac{1}{\sum_{t=1}^{T} 1[D_{b,t}=1]} \sum_{t=1}^{T} D_{b,t}v_{N-1:N,t} \\ 0 \\ \frac{\partial \hat{f}_{4}(\theta)}{\partial \alpha} \\ 2\hat{f}_{5}(\theta) (1-\alpha) \frac{1}{\sum_{t=1}^{T} 1[D_{b,t}=1]} \sum_{t=1}^{T} \left[D_{b,t}v_{N-1:N,t} - \frac{1}{\sum_{t=1}^{T} 1[D_{b,t}=1]} \sum_{t=1}^{T} D_{b,t}v_{N-1:N,t} \right]^{2} \\ 0 \end{bmatrix}$$

where:

$$\begin{split} \frac{\partial \hat{f}_4\left(\theta\right)}{\partial \alpha} =& 2\hat{f}_4\left(\theta\right) \left[\left(4 - \alpha - 6\beta_0^C\right) - 6\beta_1^C \frac{1}{\sum_{t=1}^T 1\left[D_{a,t} = 1\right]} \sum_{t=1}^T D_{a,t} R_t \right] \frac{1}{\sum_{t=1}^T 1\left[D_{a,t} = 1\right]} \\ & \sum_{t=1}^T \left[D_{a,t} v_{N-1:N,t} - \frac{1}{\sum_{t=1}^T 1\left[D_{a,t} = 1\right]} \sum_{t=1}^T D_{a,t} v_{N-1:N,t} \right]^2 \\ & \frac{\partial \hat{F}\left(\theta\right)}{\partial \beta_0^C} = 12 \begin{bmatrix} \hat{f}_1\left(\theta\right) \frac{1}{\sum_{t=1}^T 1\left[D_{a,t} = 1\right]} \sum_{t=1}^T D_{a,t} v_{N-1:N,t} \\ 0 \\ 0 \\ \frac{\partial \hat{f}_4\left(\theta\right)}{\partial \beta_0^C} \\ 0 \\ 0 \end{bmatrix} \end{split}$$

where:

$$\begin{split} \frac{\partial \hat{f}_4\left(\theta\right)}{\partial \beta_0^C} =& 2\hat{f}_4\left(\theta\right) \left[\left(4 - \alpha - 6\beta_0^C\right) - 6\beta_1^C \frac{1}{\sum_{t=1}^T 1\left[D_{a,t} = 1\right]} \sum_{t=1}^T D_{a,t}R_t \right] \frac{1}{\sum_{t=1}^T 1\left[D_{a,t} = 1\right]} \\ & \sum_{t=1}^T \left[D_{a,t}v_{N-1:N,t} - \frac{1}{\sum_{t=1}^T 1\left[D_{a,t} = 1\right]} \sum_{t=1}^T D_{a,t}v_{N-1:N,t} \right]^2 \\ & \frac{\partial \hat{F}\left(\theta\right)}{\partial \beta_1^C} = 12 \begin{bmatrix} \hat{f}_1\left(\theta\right) \frac{1}{\sum_{t=1}^T 1\left[D_{a,t} = 1\right]} \sum_{t=1}^T D_{a,t}R_t \frac{1}{\sum_{t=1}^T 1\left[D_{a,t} = 1\right]} \sum_{t=1}^T D_{a,t}v_{N-1:N,t} \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{bmatrix} \end{split}$$

where:

$$\begin{aligned} \frac{\partial \hat{f}_4\left(\theta\right)}{\partial \beta_1^C} =& 2\hat{f}_4\left(\theta\right) \left[\left(4 - \alpha - 6\beta_0^C\right) \frac{1}{\sum_{t=1}^T 1\left[D_{a,t} = 1\right]} \sum_{t=1}^T D_{a,t} R_t \frac{1}{\sum_{t=1}^T 1\left[D_{a,t} = 1\right]} \right] \\ & \sum_{t=1}^T \left[D_{a,t} v_{N-1:N,t} - \frac{1}{\sum_{t=1}^T 1\left[D_{a,t} = 1\right]} \sum_{t=1}^T D_{a,t} v_{N-1:N,t} \right]^2 - 6\beta_1^C \hat{C}_{a,t} \end{aligned}$$

$$\frac{\partial \hat{F}\left(\theta\right)}{\partial \beta_{0}^{S}} = 2 \begin{bmatrix} 0\\ 0\\ \hat{f}_{3}\left(\theta\right) \frac{1}{\sum_{t=1}^{T} \mathbb{1}\left[D_{c,t}=1\right]} \sum_{t=1}^{T} D_{c,t} v_{N-1:N,t} \\ 0\\ 0\\ 0\\ \frac{\partial \hat{f}_{5}\left(\theta\right)}{\partial \beta_{0}^{S}} \end{bmatrix}$$

where:

$$\begin{split} \frac{\partial \hat{f}_{5}\left(\theta\right)}{\partial \beta_{0}^{S}} =& 2\hat{f}_{6}\left(\theta\right) \left[\left(1-\beta_{0}^{S}\right) + \beta_{1}^{S} \frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]} \sum_{t=1}^{T} D_{c,t}R_{t} \right] \frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]} \\ & \sum_{t=1}^{T} \left[D_{c,t}v_{N-1:N,t} - \frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]} \sum_{t=1}^{T} D_{c,t}v_{N-1:N,t} \right]^{2} \\ & \frac{\partial \hat{F}\left(\theta\right)}{\partial \beta_{1}^{S}} = 2 \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]} \sum_{t=1}^{T} D_{c,t}R_{t} \frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]} \sum_{t=1}^{T} D_{c,t}v_{N-1:N,t} \\ & 0 \\ & 0 \\ & 0 \\ & \frac{\partial \hat{f}_{6}\left(\theta\right)}{\partial \beta_{1}^{S}} \end{bmatrix} \end{split}$$

where:

$$\begin{aligned} \frac{\partial \hat{f}_{6}\left(\theta\right)}{\partial \beta_{1}^{S}} =& 2\hat{f}_{6}\left(\theta\right) \left[-\left(1-\beta_{0}^{S}\right) \frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]} \sum_{t=1}^{T} D_{c,t} R_{t} \frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]} \right] \\ & \sum_{t=1}^{T} \left[D_{c,t} v_{N-1:N,t} - \frac{1}{\sum_{t=1}^{T} 1\left[D_{c,t}=1\right]} \sum_{t=1}^{T} D_{c,t} v_{N-1:N,t} \right]^{2} + \beta_{1}^{S} \hat{C}_{c,t} P_{c,t} P_{c,$$

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