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The Mixed Economy Case

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# The Price-Wage Inflationary Spiral: The Mixed Economy Case

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THE PRICE-WAGE INFLATIONARY SPIRAL:  
THE MIXED ECONOMY CASE

*(Abstract)*

In the paper we have addressed the problem of price-wage relationship modelling in case of the mixed economy. The empirical investigation was based on the Polish annual data for the period of centrally planned system (1964-1989) and on quarterly data for the period of transition towards the market economy (1990.1-1993.3). The traditional approach proved to be rather inappropriate because of the variables nonstationarity. We tried to identify the long-run behaviour applying the two-step Engle-Granger's or alternatively, Johansen's maximum likelihood (ML) procedures. The ML estimator provided better estimates of cointegration vectors and, which is even more important, allowed to find as many as three of them.

The main conclusion which can be drawn from the empirical findings is that three variables: price index, average wages and labour productivity form a multi-dimensional equilibrium space. This property of the described phenomena should be seriously taken into account while building macroeconomic models explaining the behaviour of the Polish economy.

On the other hand, the results confirm that nominal wages are determined to large extent by prices and labour productivity while prices depend on wage growth and prices of imported goods. This is in accordance with the findings based on more traditional approaches. Nevertheless, it should be emphasised that the adjustment processes are of intertemporal rather than simultaneous character which was not adequately reflected in so far constructed models.

The existence of three cointegration vectors being troublesome because of the interpretation problems is not unusual. However, if it is not a result of misspecification and/or small sample bias, proves that much remains to be learned about the price-wage mechanisms functioning in the economies having the mixed character.

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## I. Introduction\*

Inflation seems to be one of the most carefully observed and analysed economic phenomena. This is due both to the crucial role of price changes in economic development and also because of social sensitivity to the problems brought by inflation. A slow rate of inflation is generally viewed as "normal", but rapid price increases are typically associated with external shocks or a wrong policy conducted by the government, in either case requiring appropriate corrective action. Because of its importance, the rate of inflation is widely considered as a key indicator for an economy.

The literature devoted almost exclusively to the problem of inflation consists of very many monographs and uncountable number of articles (compare for example items cited by W.Frisch [1983], P.A.Rowlatt [1992] ). It was also the subject of various econometric studies. In the East European countries, by comparison, for a long time, inflation was treated by scientists as the marginal problem. The reason has been the political: the possibility of existence of inflation in the socialist economies was questioned. The official propaganda claimed that a stabile growth could be achieved without the increase of cost of living which was said to be characteristic only for the capitalistic system.

In Poland, the decade of '60's almost confirmed the above thesis: the annual growth rates of consumer goods price index was below 2%, while the

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All remaining errors and omissions are mine alone.

increase of excess demand in some markets in these years (mostly for textiles, clothes and durables) was called "a periodical difficulty".

In the '70's a considerable increase of inflation in Poland was observed and since then the economists have developed the interest to study inflation processes. Interestingly, many of them still believed, that the government fully controlled wages in the socialised sector and prices of the goods. Thus the inflation was typically explained as the result of inefficiency in the planning process. Also most of the model-builders denied the possibility of explaining the prices (and wages) and treated them as exogenous (cf. W.Maciejewski *et al.* [1973]).

A real breakthrough took place in the '80's when both open and hidden (suppressed) inflation were carefully studied by theorists and econometricians (cf. W.Charemza, M.Gronicki [1983], [1985], G.W.Kołodko [1987], L.Podkaminer [1982], [1987], R.Porter *et al.* [1982], A.Welfe [1984], [1985], [1989], W.Welfe [1981], [1993], [1985], J.Winiecki [1986]).

So far little attempt was made to model the price-wage nexus which, on the contrary, has very important policy implications. In this paper we try to model price-wage relationship with the special emphasis on the long-run dynamic behaviour. The empirical investigation is based on annual data for the period which corresponds to the centrally planned system (1962-1989) and on quarterly figures for the period of transition (1990.1-1992.3).<sup>1</sup>

## II. The historical perspective

The beginning of '90's has brought in Poland, almost simultaneously, a breakthrough in three spheres: political, economic, and psychological. Political, because after over 40 years, for the first time since II World War, a non-communist government took power. Economic, because on January 1,

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1990, a vast reform, called "the stabilisation programme" or "the Balcerowicz plan", was initiated, which was treated as the birth of a new, free-market era. And psychological, because of the total opening to the West, the introduction of democratic freedom on inexperienced before scale, and privatisation of the state-owned sector which before constituted over 90% of the industry.

This popular view, presented above, created mostly by politicians and shared by the mass media is only partly true. Therefore, we suggest - restricting our attention to the economy - to visualise all the changes as the long-lasting process, rather than an over-night act. The acceptance of this proposition has significant consequences for econometric modelling. Firstly, so far constructed econometric models, although obviously modified, can still be used for the simulation studies or, at least, as a good reference point. Secondly, new models can be based on the annual data covering the period starting in the '70's, and not exclusively on quarterly (monthly) figures since 1990, which in many cases are incomplete and unreliable (for wider discussion see: M.M.Kaźmierska, S.Narel [1992]).

There are a few good reasons to support the hypothesis about the long-run tendency toward the market system. However, to collect the arguments we have to explain briefly what were the characteristic features of the old, "centrally planned system" in Poland, how it has been changing in the past, and what is so far the outcome of the stabilisation plan.

In the most restrictive form "central planning" was the synonym of "command system" which meant the existence of obligatory plans set by central bodies (e.g. ministers) to be fulfilled by socialised enterprises. Interesting enough, it prevailed in Poland in this form at most to the mid '70's. In the '80's special "agreements" between the government and the producers guaranteeing access to scarce raw materials started to play an important role. The difference was quite significant because the terms of these agreements could be negotiated with the officials. At the end of the period

even this practice was limited to very specific products, e.g. military goods, medicines etc.

Because of the existence of non-clearing prices, many goods including intermediate products, raw materials and especially energy had to be subsidised. As the result, excess demand was typical and therefore allocation had to be done not by the market but by the administrative bodies. However, since the '80's after serious attempts to eliminate subsidies paid from the state budget even this system of allocation was active only in case of a very few, centrally balanced strategic materials.

Within the framework of "central planning" the government tried to control fully the prices. Indeed, in early years, i.e. in the '60's prices of almost everything, including consumption and non-consumption goods, services, raw materials, investment goods and capital were set by special agencies. Their role, however, was gradually decreasing and socialised enterprises were gaining more and more freedom in choosing prices for their products. Consequently, the market forces started to play an important role, which was widely recognised and accepted by politicians. The clear sign of this process was the introduction in the mid '80's of three different kind of prices called: "government", "controlled" and "negotiable". The last one, being in fact free-market, were the most widely used.

Also very closely monitored by the government were wages and personal incomes. The evolution of the system was in this case similar to prices and the weakening role of the central bodies over the years was visible. The direct control in the form of upper limits imposed on growth rates of wages was substituted by fiscal instruments consequently introduced since the late '70's.

The next aspect of "central planning" was the preparation of annual plans believed to be balanced and treated as obligatory documents for the government (and enterprises). They were abolished in 1990 and replaced by



"development programs" which unfortunately proved to be inconsistent. The latter resemble somehow the idea of five-year plans of the past having the same "strategic" character.

Centralisation meant also that the managers of the enterprises were nominated by the central administration. These were usually political decisions, especially in the case of big and important firms. This practice with regard to the state owned enterprises did not change much after 1990.

One of the fundamental tenets of the old system was the "social" ownership of capital. Nevertheless, it should be particularly stressed that over 90% of land was always in hands of private farmers in Poland. There existed also a lot of small, privately owned factories, stores, small trade companies, restaurants etc., however, prior to 1990 they comprised typically less than 15% of non-agricultural GNP.

It is, however, quite remarkable that the evolution of the "centrally planned" system towards a market system started in the '70's was vigorously continued in the '80's, characterised by a series of attempts to implement economic reforms. Although they did not reach most of the targets, these attempts succeeded in substantially broadening the area of freely negotiated prices and wages, brought the departure from central allocation of goods and increased the importance of financial instruments.

On the other hand, the above mentioned stabilisation plan, introduced in 1990, has been successful in eliminating excess demand in practically all markets within a very short period after its initiation (actually in 3-4 months). In the same time the inflation rate declined from 60-80% (typical for 1989/90) to less than 5% on a monthly basis. All of this was accompanied by the introduction of internal convertibility of Polish zloty (keeping the US \$ / zloty exchange rate fixed for more than one year), abolishing the rationing of hard currencies and opening of the economy what brought about a growing competition from imports. Thus, as the result, the system changed from a

rather typical supply-driven, shortage economy to one that was increasingly demand determined. However, deregulation by means of commercialisation and privatisation of state-owned, large firms tends to be quite difficult, and after four years of extensive attempts, still only about half of GNP in Poland is produced by the private sector. The large enterprises in many cases still keep monopolistic or oligopolistic positions and try to maximise wage-bills rather than profits.

Very restrictive deflationary fiscal and monetary policies implemented under the stabilisation plan also resulted in a deep recession characterised by high unemployment rates (over 15% at the end of 1994). The consequences will probably influence the economic development during many years ahead. Clearly, the financial instruments started to play an important role both in actual functioning of the economy and the way it is directed: the number of government actions with the typical use of the exchange rate, taxes, duties etc. is constantly rising.

The above historical analysis allows us to conclude that the economic system in Poland for many years has had a "mixed" character and will very probably keep it in the so called "transition period" which may last as long as to the end of the century. During the process of transformation the unconstrained functioning of the market adjustment mechanisms might be still limited and may differ from the outcomes typically observed in other economies. Thus, the components of the old economic structure will co-exist with the elements of the new system. This puts the econometric modelling of the Polish economy and especially the inflationary mechanisms in an interesting perspective as a very challenging but full of difficulties task.

In the next two sections we will analyse the classical macromodel of price-wage nexus, while in part V the long-run behaviour will be examined within the framework of cointegration.

### III. The basic model

The traditional macromodel of price-wage relationship (cf. J.Tobin [1972]) can be written as

$$\dot{p}_t = \beta_1^p + \beta_2^p \dot{w}_t + \beta_3^p \dot{z}_t + \beta_4^p \dot{m}_t + \xi_t^p \quad (1)$$

$$\dot{w}_t = \beta_1^w + \beta_2^w \dot{p}_t + \beta_3^w \dot{z}_t + \beta_4^w U_t + \xi_t^w \quad (2)$$

where

$P$  - price index of consumer goods (cost of living index),

$W$  - average wages, current prices,

$Z$  - labour productivity, constant prices,

$M$  - prices of imported goods,

$U$  - rate of unemployment,

$\xi_t^p, \xi_t^w$  - disturbance terms,

$t=1, \dots, T$ .

In above notation lower-case letters with a dot designate the growth rates.

The first equation is the result of the acceptance of the cost-push theory of inflation and follows the hypothesis that the prices adopt to the changes in (unit) costs of production enlarged by a mark-up which is assumed to be stable over time. The second equation generalises the concept of the Phillips curve. Also the analysis of the institutional features of the labour market indicate that wages are determined through a collective bargaining process (cf. A.Welfe [1991]). The outcome is the result of the workers' pressure (exerted directly on the factory level or by trade unions) and employer resistance (similar specification was suggested for a developing country by A.Downes *et al.* [1990]).

As shown by W.Franz, R.J.Gordon [1993] the functions (1) and (2) can be derived from the following equation

$$a(L)\Delta p_t = b(L)\Delta w_t + g(L)u_t + d(L)v_{(k)t} + \xi_t, \quad (3)$$

where

$a(L), b(L), g(L), d(L)$  - polynomials in the lag operator  $L$ ,

$\mathbf{v}_{(k)t}$  - vector of other relevant variables,  $k=1, \dots, K$ ,

$\Delta$  - denotes the first difference,

$\xi_t$  - disturbance term,  $\xi_t: \text{IID}(0, \sigma^2)$ ,

lower case letters designate the logarithms.

It leads to the conclusion that there is no such thing as a separate wage or price equation - both of them are the rotation of the structure given by (3). The identification can be done by introducing different sets of variables included in  $\mathbf{v}_{(k)t}$  - vector (which is our approach) or imposing the restrictions on the contemporaneous coefficient on  $w_t$  in price equation and/or on  $p_t$  in wage equation.

#### IV. Further modifications and preliminary empirical results

Firstly, it should be particularly emphasised that the specification of the wage function in nominal rather than in constant terms (cf. S.J.Nickell [1984]) allows to estimate the price elasticity of average wages. This value might be of special interest as, in Poland, it was different than one during the period which covers the sample. Therefore, the impact of prices cannot be eliminated by deflating the nominal wages and this factor had to be explicitly introduced to the equation (for a formal approach to the wage indexation see A.J.Spivak *et al.* [1987]).

Secondly, in the classical analysis it is implicitly assumed that the productivity measured on macro level never falls, although the productivity of individual agents may occasionally decline. However, in case of the Polish economy negative values of  $z$  were observed in years 1979-1982 and 1990. On the other hand, no adjustment of wages due to the productivity decreases took place. Therefore we assumed, that the elasticity of wages with respect to labour productivity was equal to zero during this periods. This proposition leads to the following modification of equation (2):

$$\dot{w}_t = \beta_1'' + \beta_2'' \dot{p}_t + \beta_3'' I_{zt}(\dot{z}_t > 0) \dot{z}_t + \beta_4'' U_t + \xi_t'' \quad (4)$$

where  $I_{zt}(\dot{z}_t > 0)$  stands for dummy variable having ones in the periods when  $\dot{z}_t > 0$ .

Thirdly, during the period of the central planning full employment was considered to be one of the fundamental economic targets and also achievements of the socialist system (cf. J.Kornai [1980] p.235). Therefore, the government initiating new investment projects always tried to create enough jobs. Unfortunately, over the years, this policy produced excess demand for labour force in the entire economy which was particularly high in some industrial regions. This resulted in the continuous pressure on wages increase which could be modelled within the framework of the Phillips curve concept extended to the area of "negative" rates of unemployment (cf. A.Welfe [1991]). However, the data available in the official statistics for the period from 1962 to 1989 is limited only to two figures: the number of registered unemployed and number of vacancies, the latter being typically considerably greater. The disaggregation into the industry branches, sectors or regions is not possible. Also the quality of these data is very poor because of frequently changed regulations concerning the registration procedures and the practice of enterprises avoiding the need of reporting vacancies. What concerns quarterly figures, they are imperfect as well. Additionally, being the cost of economic transformation, the unemployment rate raised from 0 to over 10 percent between January 1990 and December 1992. It is quite understandable that this tremendous increase could not have an impact on the wages as it would be expected in more stabile economy. All these reasons accounted for unsatisfactory preliminary empirical results which caused the omission in the wage equation of the variable characterising the situation in the labour market.

Concluding, the estimated form of price and wage functions was the following

$$\dot{p}_t = \beta_1^p + \beta_2^p \dot{w}_t + \beta_3^p \dot{z}_t + \beta_4^p \dot{m}_t + \xi_t^p, \quad (5)$$

$$\dot{w}_t = \beta_1^w + \beta_2^w \dot{p}_t + \beta_3^w I_{zt}(\dot{z}_t > 0) \dot{z}_t + \xi_t^w \quad (6)$$

or alternatively specified in first differences of logarithms (cf. O.J. Blanchard, S. Fisher [1989] pp. 542-4)

$$\Delta p_t = \beta_1^p + \beta_2^p \Delta w_t + \beta_3^p \Delta z_t + \beta_4^p \Delta m_t + \xi_t^p \quad (5a)$$

$$\Delta w_t = \beta_1^w + \beta_2^w \Delta p_t + \beta_3^w I_{zt}(\dot{z}_t > 0) \Delta z_t + \xi_t^w. \quad (6a)$$

The results reported in tables 1a and 1b call for some additional comments.

Estimated elasticities with respect to wages and prices in case of annual data are within the range of (0.70, 0.99), while for quarterly data they are unexpectedly high in the wage equation and low in the price one. This result is obviously difficult to appreciate and points out a serious asymmetry in adjustment processes.

The elasticities with respect to labour productivity are also surprisingly high in all equations based on annual data, especially if compared with results for quarterly figures. Among others, this implies that real wages rise at a higher rate than the productivity of labour which is not possible if the long run is considered.

The constant terms are close to zero which is fortunate because it means that no continuous drift is present in the equations. Ignoring the fact, that for annual data the estimates are statistically insignificant we keep the constants in place to avoid a possible bias.

Autocorrelation has not been detected; (adjusted) determination coefficients are not very high especially for equations estimated on quarterly data; all estimates are rather similar for growth rates and the first differences of logarithms. The latter, however, keep the additiveness property which might be important in case of rapid increases of variables (cf. figure 1) and is congruent with the long-run hypothesis tested below. Therefore in the further analysis we used exclusively the logarithms.

## V. Cointegration analysis and the identification of the long-run structure

Equations (5) and (6) are entirely in the growth rates. Consequently, the term imposing the long-run behaviour of the system is neglected. However, if we believe that the price-wage relationship is dynamically stable (i.e. represents the equilibrium relationship), then prices and wages are cointegrated, provided they are integrated of order one. The series are thus expected to move so that they do not drift apart over time. The same kind of equilibrium relationship between wages and labour productivity can be postulated.

The estimation of the cointegrated vector allows for the separation of the long-run information contained in the data from the complex dynamics (cf. C.W.J.Granger [1981], R.F.Engle, C.W.J.Granger [1987]).

In order to verify the properties of the price-wage nexus we proceeded in two steps.

In the first step the time series were tested for the presence of a unit root. There are many tests which can be used (cf. the bibliography given in F.X.Diebold, M.Nerlove [1990]), however, because of the limited sample sizes they are not very powerful. Therefore one should be careful with any conclusions.

The application of the Dickey-Fuller (DF) test to annual data resulted in autocorrelation of the residuals. Therefore we used the Augmented Dickey-Fuller (ADF) test for the model with and without trend. It is worthwhile to stress that the introduction of the trend did not change the outcome substantially. In case of all variables the calculated values of the ADF statistic were lower than the critical ones (cf. J.G.MacKinnon [1991]) which indicates that the average wages, cost of living index, labour productivity and import price index are difference stationary,  $I(1)$ .

Then we went on to test for the existence of the cointegrating vector(s) and to estimate it.

At the beginning we considered a standard error correction model (ECM), both for the price and wage equation

$$\Delta p_t = \alpha_1^p + \alpha_2^p (p_{t-1} - \varphi_1^p w_{t-1}) + \alpha_3^p \Delta w_t + \alpha_4^p \Delta z_t + \alpha_5^p \Delta m_t + \alpha_6^p z_{t-1} + \alpha_7^p m_{t-1} + \xi_t^p, \quad (7)$$

$$\Delta w_t = \alpha_1^w + \alpha_2^w (w_{t-1} - \varphi_1^w p_{t-1}) + \alpha_3^w \Delta p_t + \alpha_4^w \Delta z_t + \alpha_6^w z_{t-1} + \xi_t^w. \quad (8)$$

The term in brackets is the cointegrating relationship which may be estimated freely or better, by the two-step procedure (cf. R.F.Engle, C.W.J.Granger [1987]). In this case the parameters  $\varphi_1^p$  and  $\varphi_1^w$  should be replaced by  $\hat{\varphi}_1^p$  and  $\hat{\varphi}_1^w$  being the OLS estimates taken from a prior levels (possibly with the trend) regressions. Then, the parameters of equations (7) and (8) are estimated by OLS.

A two-step estimator is consistent and the estimates converge to the true values even faster than standard econometric estimates. Moreover this procedure entirely based on OLS is relatively simple. However, it requires an arbitrary normalisation and suffers from the small sample bias which might be very large (cf. A. Banerjee *et al.* [1993]). Also a substantial decrease of power of the unit root tests applied to the regression residuals may be expected (a summary of pitfalls is given by S.G.Hall [1989]). Unfortunately, the empirical results presented in tables 2a and 2b confirmed the presence of these disadvantages: the error correction term has the wrong sign in case of wage function for annual data. It is also insignificant in all equations while the non-stationarity is not present in the residuals (was tested separately). Different versions of equations (7) and (8) with additional lags of the variables and/or the error correction term including labour productivity did not provide better estimates of cointegration vectors.



Consequently, we turned to the maximum likelihood procedure proposed by S.Johansen [1988], [1992]. The departure point in this approach is the vector autoregressive model (VAR) given by

$$\mathbf{x}_{(k)t} = \pi + \sum_{s=1}^S \mathbf{x}_{(k)t-s} \Pi_s + \xi_{(k)t} \quad (9)$$

where

$\mathbf{x}_{(k)t} = [x_{1t} x_{2t} \dots x_{kt}]$  is a vector of K variables,  $k=1, \dots, K$

$\xi_{(k)t}$  is a K dimensional vector,  $\xi_{(k)t} : \text{IID}(\mathbf{0}, \Omega)$ ,  $t=1, \dots, T$ .

The above equation can be rewritten as

$$\begin{aligned} \Delta \mathbf{x}_{(k)t} &= \pi + \mathbf{x}_{(k)t-S} \Pi + \Delta \mathbf{x}_{(k)t-1} \Gamma_1 + \dots + \Delta \mathbf{x}_{(k)t-S+1} \Gamma_{S-1} + \xi_{(k)t} = \\ &= \pi + \mathbf{x}_{(k)t-S} \Pi + \sum_{s=1}^{S-1} \Delta \mathbf{x}_{(k)t-s} \Gamma_s + \xi_{(k)t}, \end{aligned} \quad (10)$$

were

$\Pi$  and  $\Gamma_j$  are  $K \times K$  matrixes,

$$\Pi = -(\mathbf{I} - \Pi_1 - \dots - \Pi_S) = -\mathbf{I} + \sum_{s=1}^S \Pi_s,$$

$$\Gamma_j = -(\mathbf{I} - \Pi_1 - \dots - \Pi_j) = -\mathbf{I} + \sum_{s=1}^j \Pi_s, \quad j=1, \dots, S-1.$$

The matrix  $\Pi$  is of special importance as it contains the information about the long-run behaviour of the variables included in the model. Thus the hypothesis about cointegration reduces to the hypothesis that  $\text{rank}(\Pi)=R$ , where  $R$  is the number of cointegration vectors between the variables  $\mathbf{x}_k$ . There are two possible extremes: the matrix  $\Pi$  is the null matrix or it has the full rank ( $R=K$ ). In the first case (10) corresponds to the standard differenced model, while in the second it means that the process is stationary. The most interesting, however, is the event when  $0 < \text{rank}(\Pi) < K$  which implies the presence of two  $K \times R$  matrices:  $A$  containing the weights and  $B$  containing the cointegration vectors, such that

$$\Pi = B A^T, \quad (11)$$

were

$$B = [\beta_{(k)1} \beta_{(k)2} \dots \beta_{(k)R}],$$

$$A = [\alpha_{(k)1} \alpha_{(k)2} \dots \alpha_{(k)R}].$$

From the above it follows that the columns of  $B$  correspond to the  $R$  cointegration vectors while rows of  $A$  to the weights.

It should be particularly stressed that the Johansen procedure enables to identify all cointegration vectors contrary to the Engle-Granger technique assuming that the cointegration vector is unique.

The cointegration vectors, have the important property that a linear combination  $\mathbf{x}_{(k)t} \beta_{(k)r}$  ( $r=1, \dots, R$ ) is stationary even though  $\mathbf{x}_{(k)t}$  is not (Granger's Representation Theorem, cf. R.F.Engle, C.W.J.Granger [1987]). In this case (10) can be interpreted as the ECM.

The hypothesis of at most  $R$  cointegration vectors was verified with the maximal eigenvalue,  $\lambda_{\max}$ , and trace (likelihood ratio test),  $LR$ , statistics (cf. S.Johansen, K.Juselius [1990]). Both statistics are equal if  $R=K-1$ .

Then the cointegration vectors were estimated by the maximum likelihood method (cf. S.Johansen [1988], S.G.Hall [1989], S.Johansen, K.Juselius [1990]). The  $\mathbf{x}_{(k)t}$  vector contained all the variables appearing in the price-wage model:  $\mathbf{x}_{(k)t} = [p_t, w_t, z_t, m_t]$  and also alternatively the trend. The number of lags depended on the data set used.

The empirical results will be presented firstly for the annual data and secondly for the quarterly figures.

#### Sample 1962-1989 (annual data)

Estimating the unrestricted cointegration space, due to the limited sample, we could allow for a maximum of 4 lags only. The results proved to be not sensitive to the lag length, however, the best properties of residuals (no autocorrelation and homoscedastic, see table 3) were found for 2 lags (cf. equation (9)).

The alternative of the trend inclusion into cointegration vectors was important: the values of the  $LR$  statistic were significant at all commonly

accepted levels in case of all lag structures (for 2 lags:  $LR=0.0006$  with marginal significance level 0.980). Therefore, we restricted the trend to be included in the  $\mathbf{x}_{(k)t}$  vector rather than allow it to appear in the equation specified for levels of the variables (cf. equation (9)).

Table 4 gives the test statistics for various values of  $R$  versus  $R+1$  ( $LR$ ) and versus the general alternative ( $\lambda_{\max}$ ) for the model with 2 lags. Both statistics are significant for  $R=0$ ,  $R \leq 1$  and  $R \leq 2$ , hence there is no evidence in the data for more than three cointegration relations between the variables in the model.

The stationary eigenvalues  $\hat{\beta}_{(k)r}$  ( $r=1, \dots, 4$ ) are presented in table 5a.

They allow to identify the following cointegration vectors:

$$C_1 = \mathbf{x}_{(k)t} \hat{\beta}_{(k)1} = p_t - 1.6024 w_t + 1.0393 z_t + 0.8040 m_t + 1.7738 \quad (12)$$

$$C_2 = \mathbf{x}_{(k)t} \hat{\beta}_{(k)2} = p_t - 0.7514 w_t + 0.8143 z_t - 0.3113 m_t - 1.1315 \quad (13)$$

$$C_3 = \mathbf{x}_{(k)t} \hat{\beta}_{(k)3} = p_t - 0.8654 w_t + 0.8494 z_t - 0.3582 m_t - 0.9898. \quad (14)$$

The terms  $C_1$ ,  $C_2$  and  $C_3$  enter each equation of the system with weights given in the second half of table 5a. The weights can be interpreted as measures of the average speed of adjustment towards the equilibrium.

By rearranging the terms so as to write the equations in the ECM form (cf. equation (10)) we have the following

$$\begin{aligned} \Delta p_t = & 1.1900(p_{t-2} - 1.6024 w_{t-2} + 1.0393 z_{t-2} + 0.8040 m_{t-2} + 1.7738) + \\ & + 1.3010(p_{t-2} - 0.7514 w_{t-2} + 0.8143 z_{t-2} - 0.3113 m_{t-2} - 1.1315) + \\ & + 0.0157(p_{t-2} - 0.8654 w_{t-2} + 0.8494 z_{t-2} - 0.3582 m_{t-2} - 0.9898) + \\ & - 0.1359 \Delta p_{t-1} + 1.2675 \Delta w_{t-1} - 2.0251 \Delta z_{t-1} + 0.2492 \Delta m_{t-1} \end{aligned} \quad (15)$$

$$\begin{aligned} \Delta w_t = & -0.0958(w_{t-2} - 0.6241 p_{t-2} - 0.6486 z_{t-2} - 0.5017 m_{t-2} - 1.1070) + \\ & - 0.5357(w_{t-2} - 1.3308 p_{t-2} - 1.0837 z_{t-2} + 0.4143 m_{t-2} + 1.5059) + \\ & - 0.0067(w_{t-2} - 1.1555 p_{t-2} - 0.9815 z_{t-2} + 0.4139 m_{t-2} + 1.1437) + \\ & + 0.3796 \Delta p_{t-1} + 0.5701 \Delta w_{t-1} - 1.2830 \Delta z_{t-1} + 0.6112 \Delta m_{t-1} \end{aligned} \quad (16)$$

$$\begin{aligned}
\Delta z_t = & -0.5224(z_{t-2} + 0.9622 p_{t-2} - 1.5418 w_{t-2} + 0.7736 m_{t-2} + 1.7067) + \\
& -0.2821(z_{t-2} + 1.2280 p_{t-2} - 0.9228 w_{t-2} - 0.3823 m_{t-2} - 1.3895) + \\
& + 0.1248(z_{t-2} + 1.1773 p_{t-2} - 1.0188 w_{t-2} - 0.4217 m_{t-2} - 1.1653) + \\
& + 0.3254 \Delta p_{t-1} - 0.0911 \Delta w_{t-1} + 0.7405 \Delta z_{t-1} - 0.1260 \Delta m_{t-1}
\end{aligned}
\tag{17}$$

If more than one cointegration vectors exist, as it is in our case, they span the cointegration space. Any vector being their linear combination lies in this space and is also a cointegrating vector. Therefore, the economic interpretation is not trivial. In order to overcome these problems some additional restrictions may be imposed (cf. S.Johansen, K.Juselius [1992]), but a fully satisfying procedure has not been proposed yet. However, one can try to give the economic interpretation to each of the above three equations treating them as (estimated) dynamic relationships.

In the price equation all the weights are positive while the influence of wages and labour productivity included in the error correction terms is of the expected character. It means that this equation does not exhibit any tendency towards reducing inflation, on the contrary, it embodies a self accelerating mechanism of the high speed, taking into account the values of weights.

The wage equation does not cause any difficulties and can be interpreted as a classical error correction model.

In the labour productivity equation the first two weights have the correct sign, but the last one has not, however, it is of the smallest absolute value. Furthermore, the comparison of the parameters associated with variables included in the error correction terms leads to the conclusion that this unwelcome effect would be offset by the first two terms.

The estimates of the combined effects of the three identified cointegration vectors are included in the long-run impact matrix  $\hat{\Pi}$  ( $\hat{\Pi} = \hat{B}\hat{A}^T$ ) which is presented in table 6. The relevant relationships after normalisation with respect to prices, wages and labour productivity can then be written as follows

$$p_t = 1.1561 w_t - 0.9213 z_t - 0.2179 m_t - 0.2486 \quad (18)$$

$$w_t = 1.0152 p_t + 0.8894 z_t - 0.0070 m_t - 0.3412 \quad (19)$$

$$z_t = -1.0080 p_t + 1.4378 w_t - 0.6094 m_t - 1.2218 . \quad (20)$$

According to the above estimates, in the long run, the influence of the import prices on wages is negligible and on inflation is also very small. In the last case one normally expects a different relationship, but the negative coefficient may express the tendency to substitute more expensive imported goods by cheaper domestic products. It seems also that the sensitivity of labour productivity to wages is extraordinary high, which could be a feature of the Polish economy. The estimates of other parameters are satisfactory.

The system of equations (18)-(20) can be reduced to the following price-wage relationship

$$p_t = 0.9666 w_t - 0.8487 z_t + 0.2835 \quad (21)$$

$$w_t = 1.0101 p_t + 0.8863 z_t - 0.3219. \quad (22)$$

It should be emphasised that the above equation set is one of many possible representations of the model (18)-(20), however, the most interesting to us. Both elasticities, of prices with respect to wages, and wages with respect to prices, are very close to one which is usually postulated. Also the elasticities with respect to labour productivity belonging to the range (0.8, 0.9) follow closely the macroeconomic equilibrium condition.

On the other hand, the existence of three cointegration vectors in case of four variables means that there is only one factor 'causing' the long-run dynamics. A natural candidate would be the import prices, however, the response from the domestic prices side is not the usual one (cf. equation (15) and (18)). In addition, it is difficult to postulate that average wages and inflation were driven in Poland (in the long run) by import prices. Thus, from the economic point of view, it is much more interesting to concentrate on the system given by equations (21) and (22) which allows to conclude that at a

given labour productivity,  $z_t$ , the price and wage rates are uniquely determined.

Sample 1990.1-1993.3 (quarterly data)

Because of a very limited sample, we could use 1 lag only. Fortunately, the residuals were not heteroscedastic or autocorrelated (cf. table 3). Similarly to the annual data we restricted the trend to be included in the  $\mathbf{x}_{(k)t}$  vector.

Table 4 gives the values of the  $LR$  and  $\lambda_{\max}$  statistics. Both of them are significant for  $R=0$ ,  $R \leq 1$  and  $R \leq 2$ , which means that not more than three cointegration relations between the variables are present in the model.

The stationary eigenvalues  $\hat{\beta}_{(k)r}$  ( $r=1, \dots, 4$ ) and the appropriate weights are presented in table 5b. The signs of all three weights are 'correct' in the productivity equation but 'incorrect' in the price equation. In case of the wage function the signs are mixed. This situation causes obvious interpretation problems.

The combined effects of the three identified cointegration vectors given by the matrix  $\hat{\Pi}$  (cf. table 6) can be reduced to the following two equations:

$$p_t = 1.2841 w_t - 0.1743 z_t - 8.7629 \quad (23)$$

$$w_t = 0.7762 p_t + 0.1535 z_t + 7.7608. \quad (24)$$

The elasticities with respect to wages and prices differ from unity substantially. It seems also that the elasticities with respect to labour productivity are much too small. The other representation of the cointegration vectors, namely eliminating labour productivity instead of import prices leads to the conclusion that the long-run impact of the latter is negligible.

The results based on the quarterly data may be biased because of the very limited number of observations. However, they may be treated as the first approximation for the period of transition.

## VI. Conclusions

In the paper we have addressed the problem of price-wage relationship modelling in the case of Poland. The traditional approach proved to be rather inappropriate because of the variables nonstationarity. We tried to identify the long-run behaviour applying the two-step Engle-Granger's or alternatively, Johansen's maximum likelihood (ML) procedures. The ML estimator provided better estimates of cointegration vectors and, which is even more important, allowed to find as many as three of them.

The main conclusion which can be drawn from the above analysis is that three variables: price index, average wages and labour productivity form a multi-dimensional equilibrium space. This property of the described phenomena should be seriously taken into account while building macroeconometric models explaining the behaviour of the Polish economy.

On the other hand, the results confirm that nominal wages are determined to a large extent by prices and labour productivity while prices depend on wage growth and prices of imported goods. This is in accordance with the findings based on more traditional approaches (cf. W.Welfe [1985], A.Welfe [1991]). Nevertheless, it should be emphasised that the adjustment processes are of intertemporal rather than simultaneous character which was not adequately reflected in so far constructed models.

The existence of many cointegration vectors being troublesome because of the interpretation problems is not unusual (cf. S.Johansen, K.Juselius [1990], A. D. Hall *et al.* [1992], M.Lubrano *et al.* [1993]). However, if it is not a result of misspecification and/or small sample bias, it proves that much remains to be learned about the price-wage mechanisms functioning in the economies having the mixed character.

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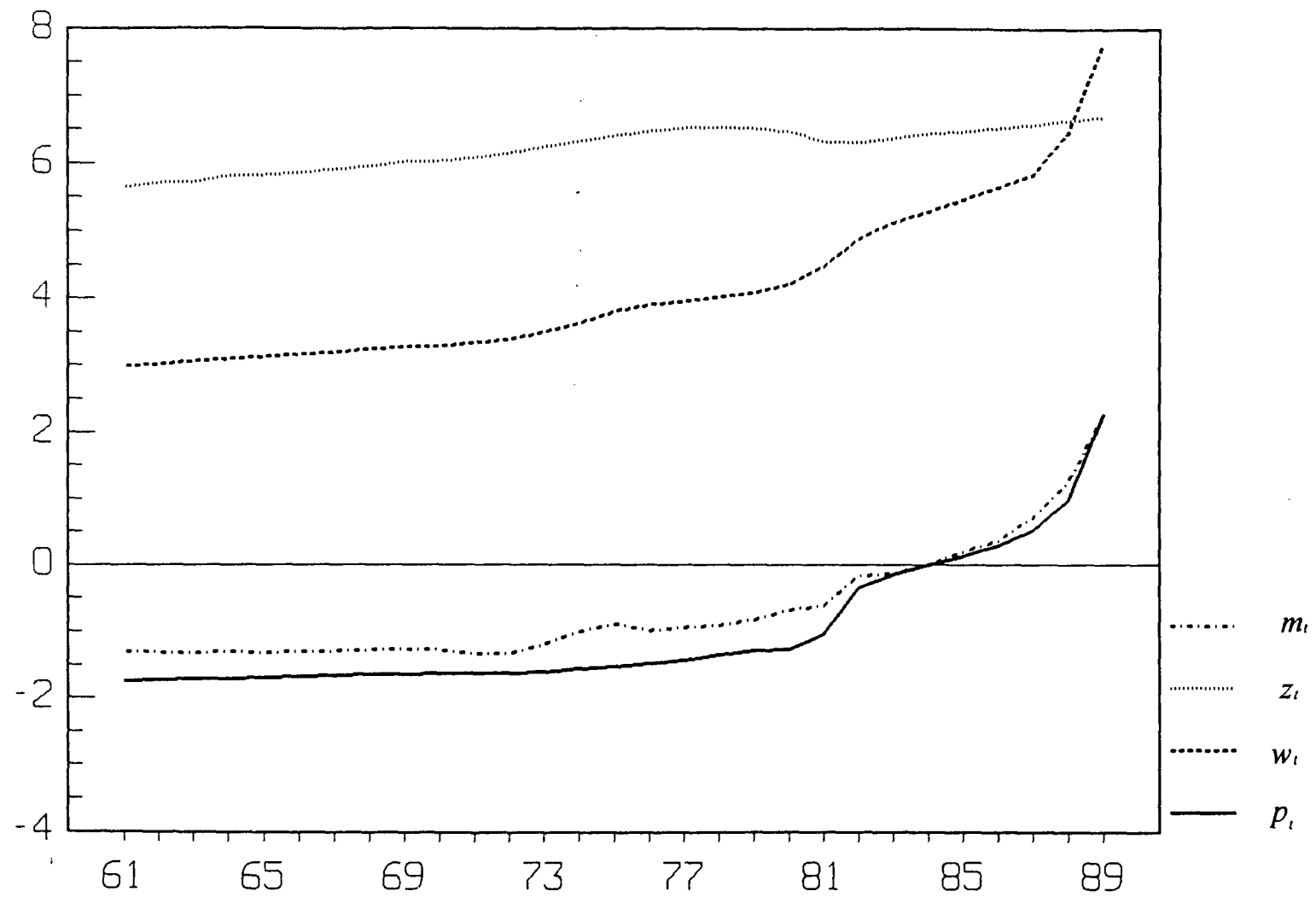


Figure 1a. Logarithms of price index ( $p_t$ ), average wages ( $w_t$ ), labour productivity ( $z_t$ ) and price index of imported goods ( $m_t$ ). Sample 1962-1989

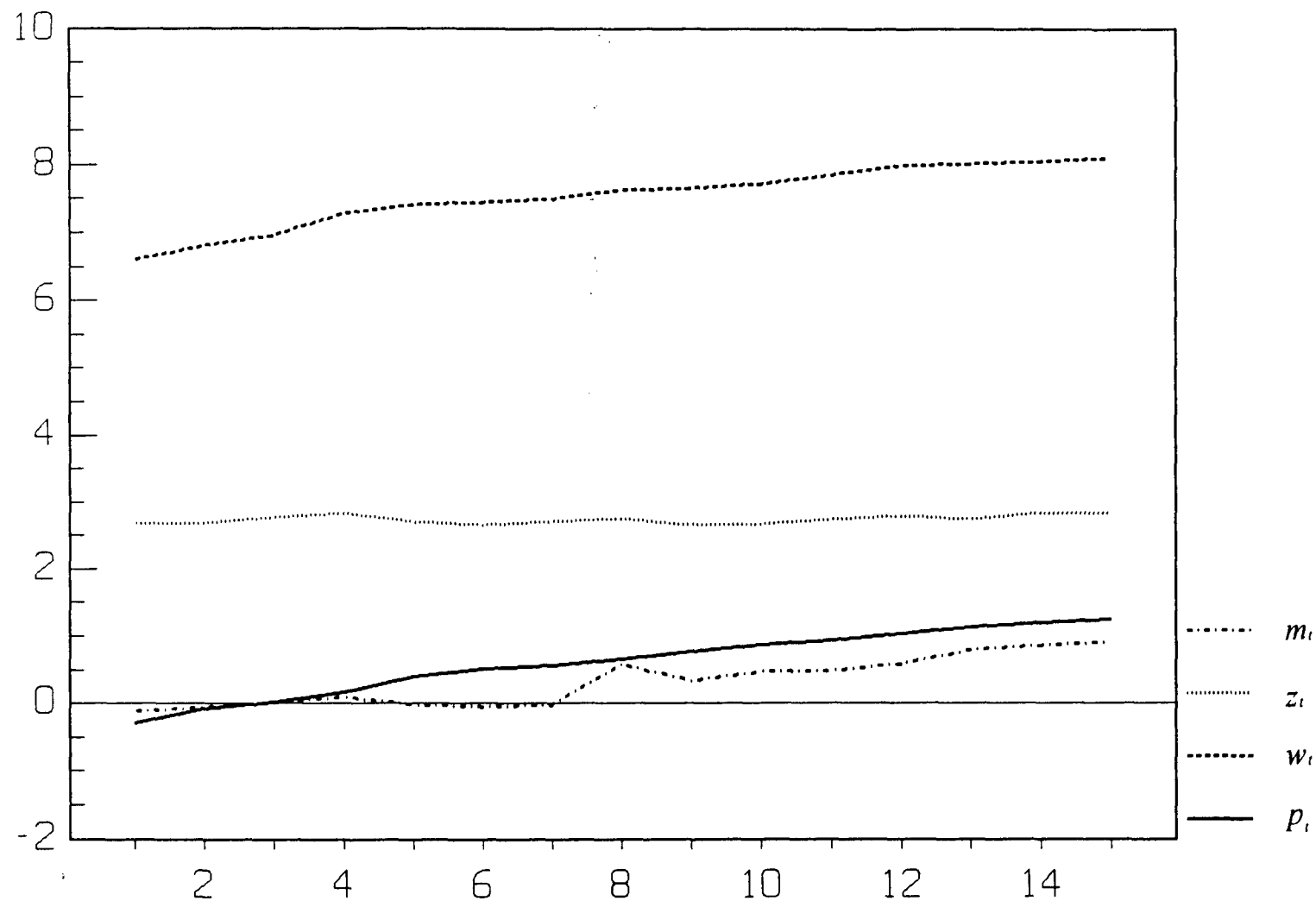


Figure 1b. Logarithms of price index ( $p_t$ ), average wages ( $w_t$ ), labour productivity ( $z_t$ ) and price index of imported goods ( $m_t$ ). Sample 1990.1-1993.3

Table 1a. Estimation results of the price and wage equations (sample 1962-1989)

Variable	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	D-W	$\overline{R^2}$
			growth rates			
Prices	0.059 (1.46)	0.700 (4.55)	-1.786 (-2.49)	0.391 (1.76)	1.71	0.957
Wages	-0.054 (-1.25)	0.995 (23.37)	1.839 (2.41)	-	1.95	0.953
			first differences of logarithms			
Prices	0.032 (1.19)	0.804 (5.24)	-1.258 (-2.64)	0.244 (1.38)	1.78	0.938
Wages	-0.018 (-0.67)	0.936 (19.63)	1.211 (2.57)	-	1.94	0.934

Table 1b. Estimation results of the price and wage equations (sample 1990.1-1993.3)

Variable	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	D-W	$\overline{R^2}$
			growth rates			
Prices	0.072 (5.04)	0.469 (4.88)	-0.639 (-4.20)	-0.002 (-0.06)	2.22	0.708
Wages	-0.074 (-1.90)	1.500 (5.11)	1.099 (4.29)	-	2.41	0.686
			first differences of logarithms			
Prices	0.064 (5.31)	0.486 (5.32)	-0.560 (-4.36)	0.001 (0.00)	2.09	0.744
Wages	-0.071 (-2.12)	1.520 (5.58)	0.952 (4.67)	-	2.27	0.723

Table 2a. Estimation results of the ECM models (sample 1962-1989)

Variable	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\alpha}_4$	$\hat{\alpha}_5$	$\hat{\alpha}_6$	$\hat{\alpha}_7$	D-W	$\overline{R^2}$
Prices	-0.165 (-0.32)	-0.036 (-0.39)	0.755 (4.22)	-0.002 (-0.17)	0.340 (1.54)	-0.012 (-1.38)	-0.023 (-0.52)	1.82	0.923
Wages	-0.207 (-0.67)	0.035 (0.59)	0.909 (13.94)	0.009 (1.05)	-	0.013 (1.85)	-	1.92	0.922

Note:

The values of  $\hat{\phi}_1^p$  and  $\hat{\phi}_1^w$  were equal 0.964895 and 0.921911 respectively.



Table 2b. Estimation results of the ECM models (sample 1990.1-1993.3)

Variable	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\alpha}_4$	$\hat{\alpha}_5$	$\hat{\alpha}_6$	$\hat{\alpha}_7$	D-W	$\overline{R^2}$
Prices	-0.528 (-0.36)	-0.127 (-0.50)	0.459 (2.97)	-0.497 (-2.30)	-0.006 (-0.12)	-0.106 (-0.40)	-0.007 (-0.12)	1.86	0.692
Wages	0.807 (0.47)	-0.250 (-0.93)	1.381 (4.30)	0.877 (2.92)	-	0.326 (1.27)	-	1.80	0.717

Note:

The values of  $\hat{\varphi}_1^p$  and  $\hat{\varphi}_1^w$  were equal 0.100229 and 0.876861 respectively.

Table 3. The tests for autocorrelation and heteroscedasticity

Equation	Breusch-Godfrey	Jarque-Bera	Breusch-Godfrey	Jarque-Bera
	Annual data		Quarterly data	
Prices	10.02 (0.12)	0.37 (0.83)	1.89 (0.39)	0.17 (0.92)
Wages	8.98 (0.17)	0.68 (0.71)	2.55 (0.28)	0.41 (0.82)
Productivity	0.59 (1.00)	0.76 (0.68)	0.02 (0.99)	0.20 (0.90)
Import prices	9.94 (0.13)	0.65 (0.72)	1.50 (0.47)	0.19 (0.91)

Notes: Marginal significance levels are in parentheses. For the Breusch-Godfrey test 6 lags were used in case of annual data and 2 in case of quarterly data.

Table 4. The test statistics for number of cointegration vectors

Hypothesis	Annual data		Quarterly data	
	max. eigenvalue	trace	max. eigenvalue	trace
$R \leq 3$	3.7737	3.7737	5.3216	5.3216
$R \leq 2$	<b>10.5988</b>	<b>14.3724</b>	<b>16.6852</b>	<b>22.0068</b>
$R \leq 1$	<b>16.3517</b>	<b>30.7241</b>	<b>44.5764</b>	<b>66.5832</b>
$R = 0$	<b>25.2864</b>	<b>56.0105</b>	<b>178.1632</b>	<b>244.7465</b>

Note: The values written in bold are the ones which are significant.

Table 5a. Eigenvectors  $\hat{\beta}_{(k)r}$  and their weights  $\hat{\alpha}_{(k)r}$  (sample 1962-1989)

Variable	$\hat{\beta}_{k1}$ or $\hat{\alpha}_{1r}$	$\hat{\beta}_{k2}$ or $\hat{\alpha}_{2r}$	$\hat{\beta}_{k3}$ or $\hat{\alpha}_{3r}$	$\hat{\beta}_{k4}$ or $\hat{\alpha}_{4r}$
Eigenvectors				
Prices	1.0000	1.0000	1.0000	1.0000
Wages	-1.6024	-0.7514	-0.8654	0.3235
Productivity	1.0393	0.8143	0.8494	-0.8262
Import prices	0.8040	-0.3113	-0.3582	-1.4995
Constant	1.7738	-1.1315	-0.9898	3.7732
Weights				
Prices	1.1900	1.3010	0.0157	0.9034
Wages	0.1535	0.4025	0.0058	1.0512
Productivity	-0.5429	-0.2297	0.1060	-0.5505
Import prices	0.0587	0.0198	0.0574	0.0766
Constant	0.0000	0.0000	0.0000	0.0000

Table 5b. Eigenvectors  $\hat{\beta}_{(k)r}$  and their weights  $\hat{\alpha}_{(k)r}$  (sample 1990.1-1993.3)

Variable	$\hat{\beta}_{k1}$ or $\hat{\alpha}_{1r}$	$\hat{\beta}_{k2}$ or $\hat{\alpha}_{2r}$	$\hat{\beta}_{k3}$ or $\hat{\alpha}_{3r}$	$\hat{\beta}_{k4}$ or $\hat{\alpha}_{4r}$
Eigenvectors				
Prices	1.0000	1.0000	1.0000	1.0000
Wages	-0.8804	-0.8545	0.3499	-1.0809
Productivity	0.8955	0.7167	2.6193	0.2004
Import prices	-0.2587	-0.2562	-1.0107	0.2103
Constant	3.7000	3.9160	-10.5977	6.9521
Weights				
Prices	1.8156	2.2200	2.1092	12.2015
Wages	-2.0395	-1.4512	1.9029	5.3302
Productivity	-0.0122	-0.2151	-0.1307	-0.0050
Import prices	-0.0629	-0.0300	0.2291	-0.5319
Constant	0.0000	0.0000	0.0000	0.0000

Table 6. The matrix  $\hat{\Pi} = \hat{B}\hat{A}^T$  restricted to three cointegration vectors

	$\Delta p$	$\Delta w$	$\Delta z$	$\Delta m$
Annual data				
$p$	2.5067	0.5617	-0.6666	0.1358
$w$	-2.8979	-0.5533	0.9508	-0.1585
$z$	2.3094	0.4921	-0.6613	0.1258
$m$	0.5461	-0.0039	-0.4030	0.0205
Const	0.6231	-0.1888	-0.8080	0.0205
Quarterly data				
$p$	6.1448	-1.5879	-0.3580	0.1361
$w$	-2.7575	3.7015	0.1488	0.1612
$z$	8.7417	2.1178	-0.5073	0.5221
$m$	-3.1704	-1.0238	0.1903	-0.2076
Const	-6.9415	-33.3957	0.4971	-2.7780