

Ewerhart, Christian; Quartieri, Federico

Working Paper

Unique equilibrium in contests with incomplete information

Working Paper, No. 133

Provided in Cooperation with:

Department of Economics, University of Zurich

Suggested Citation: Ewerhart, Christian; Quartieri, Federico (2013) : Unique equilibrium in contests with incomplete information, Working Paper, No. 133, University of Zurich, Department of Economics, Zurich, <https://doi.org/10.5167/uzh-85621>

This Version is available at:

<https://hdl.handle.net/10419/92389>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.



**University of
Zurich** ^{UZH}

University of Zurich
Department of Economics

Working Paper Series

ISSN 1664-7041 (print)
ISSN 1664-705X (online)

Working Paper No. 133

Unique Equilibrium in Contests with Incomplete Information

Christian Ewerhart and Federico Quartieri

November 2013

Unique Equilibrium in Contests with Incomplete Information*

Christian Ewerhart[†] Federico Quartieri[‡]

November 2013

Abstract. Szidarovszky and Okuguchi (*Games and Economic Behavior*, 1997) have provided useful conditions for the existence of a unique pure-strategy Nash equilibrium in rent-seeking games of complete information. In this paper, we generalize their results to contests with incomplete information. Two assumptions are imposed on the information structure. First, the players' valuations of winning are assumed to be multiplicatively separable (which includes the polar cases of private values and pure common value). Second, it is assumed that a player is never certain to be the only one with a positive budget. It is also shown that, unless the budgets of all players are zero in all states, at least two players realize a positive expected net rent.

Keywords Contests · Equilibrium existence and uniqueness · Incomplete information · Rent dissipation

JEL Classification D72 · C72

*This paper has been drafted while the second-named author was visiting the Department of Economics at the University of Zurich in spring 2013. Financial support from the Swiss National Science Foundation (research grant IZK0Z1-148239/1) is gratefully acknowledged.

[†](Corresponding) University of Zurich, Winterthurerstrasse 30, CH-8006 Zurich, Switzerland; e-mail: christian.ewerhart@econ.uzh.ch

[‡]IULM University, Via Carlo Bo 1, 20143 Milan, Italy; e-mail: Federico.Quartieri@iulm.it

1 Preliminaries

1.1 Introduction

Over the course of the past several decades, contest theory has emerged through the study of substantial economic problems in areas such as political campaigning, lobbying, R&D, legal dispute, promotion tournaments, marketing, and sports.¹ Based upon game-theoretic methods, the literature has sought to characterize the extent of rent dissipation in equilibrium, i.e., the share of the contested rent that is spent by the competing parties in an attempt to win the contest. Of much value for any such applied analysis, however, is typically also the assurance that an equilibrium solution exists and is unique.²

The literature has considered two main classes of models, the perfectly discriminating contest (or all-pay auction) and the imperfectly discriminating contest. This paper deals with imperfectly discriminating contests, for which Tullock (1980)'s rent-seeking game is an important example. In the case of complete (or public) information, the problem of equilibrium existence and uniqueness has been studied quite thoroughly.³ For rent-seeking games with

¹See Corchón (2007) for an introduction to the theory of contests.

²Indeed, the existence of a unique pure-strategy Nash equilibrium is generally a useful property for modeling frameworks, for reasons such as analytical convenience, predictive power, comparative statics, and global stability.

³Pérez-Castrillo and Verdier (1992) offer a complete characterization of the symmetric pure-strategy Nash equilibrium in the Tullock contest. Szidarovszky and Okuguchi (1997) show that a unique equilibrium exists in rent-seeking games with logit technologies of strictly decreasing returns. Nti (1999) derives a necessary and sufficient condition for the existence of a unique pure-strategy Nash equilibrium in the Tullock contest with asymmetric valuations. Cornes and Hartley (2005) study the pure-strategy equilibrium in asymmetric contests with general technologies. Yamazaki (2008, 2009) reduces further the smoothness assumptions and allows for risk aversion and financial constraints.

36 incomplete information, however, such comprehensive results have not been
37 available, in any case not regarding the uniqueness of the equilibrium.⁴ This
38 is unfortunate because private information in contests is a realistic assump-
39 tion,⁵ and known to impact on the strategic incentives of players in a variety
40 of interesting ways.⁶ Moreover, studying contests with asymmetric informa-
41 tion typically presupposes some knowledge of the corresponding equilibrium
42 set. Thus, there is a gap in the literature that concerns general conditions for
43 the uniqueness of the equilibrium in contests with incomplete information.
44 The purpose of the present paper is it to fill this gap.⁷

45 In this paper, we consider rent-seeking games in which any uncertainty
46 about the primitives of the contest is conveniently captured in a single (po-
47 tentially multi-dimensional) state variable. Specifically, the state of the world
48 pins down the effective number of players, the scale economies and any poten-
49 tial bias of the contest technology, players' valuations of winning, their cost
50 functions, and any budget constraints. Each contestant then receives a pri-
51 vate signal that is not observable to any other contestant. Thus, depending
52 on the nature of the signal, a contestant may be either completely informed,
53 completely uninformed, or possess some arbitrary partial information about
54 the state of the world.

⁴The literature on contests with incomplete information is surveyed in Subsection 1.2.

⁵For example, in a patent race, it is often hard to tell for one market participant how advanced her opponents are with their research. Other examples are readily available.

⁶For example, as Hurley and Shogren (1998a) have pointed out, asymmetric information turns the expenditure of a player into an uncertain investment, which may weaken her incentives. This, in turn, induces the other player to either increase or decrease her expenditures, depending on whether she is underdog or favorite in that situation.

⁷In fact, since the literature does not provide existence results sufficiently general to cover our asymmetric setting with a state-dependent success function and budget constraints, we will offer a self-contained discussion of existence as well.

55 Our results directly generalize the well-known conditions of Szidarovszky
56 and Okuguchi (1997) to contests of incomplete information. Two assump-
57 tions are imposed on the information structure of the contest. First, to ensure
58 uniqueness, it is assumed that players' valuations are multiplicatively separa-
59 ble, i.e., that they can be written as a product of a private-value component
60 and a public-value component. This assumption covers, in particular, the
61 polar cases of private values and pure common value, as well as mixed set-
62 tings. Second, to ensure existence, it is assumed that a player is never certain
63 to be the only one with a positive budget. Under these two assumptions on
64 the information structure, we prove that there exists a unique pure-strategy
65 Nash equilibrium. We also show that, unless all players have zero budgets in
66 all states of the world, at least two players will earn a positive expected net
67 rent.

68 The rest of the paper is structured as follows. The remainder of the
69 present section discussed some related literature. Section 2 describes the
70 set-up. Section 3 presents our main result, the uniqueness theorem. The
71 corresponding existence result is derived in Section 4. Section 5 considers
72 the extent of rent dissipation in contests with incomplete information. Im-
73 plications for specific settings are outlined in Section 6. Section 7 concludes.
74 Technical lemmas have been relegated to an Appendix.

75 **1.2 Related literature**

76 This section briefly reviews some papers on contests with incomplete infor-
77 mation, where we focus on the existence and uniqueness of the pure-strategy

78 equilibrium. For a more complete account of the individual contributions,
79 the reader is referred to the original works.

80 In a seminal paper, Hurley and Shogren (1998a) consider a model with
81 one-sided asymmetric information and private valuations. Assuming that the
82 informed player is never discouraged from competing in the contest, they find
83 a unique equilibrium. Hurley and Shogren (1998b) use the index approach to
84 show that there is at most one interior equilibrium in any two-player lottery
85 contest with private valuations and with two types for one player and three
86 for the other, where types may be correlated. Malueg and Yates (2004) study
87 the unique symmetric equilibrium in a symmetric two-player lottery contest,
88 where each player may possess one of two equally likely valuations. Sui (2009)
89 extends that model by introducing the possibility of resale. Münster (2009)
90 finds a unique equilibrium in a two-player lottery contest with independent
91 valuations that are either positive or zero. Schoonbeek and Winkel (2006)
92 characterize the unique equilibrium in an n -player contest with potential in-
93 activity, where one player has private information about her valuation and all
94 other players are identical. More recently, Einy et al. (2013) prove existence
95 for general (state-independent) success functions and unbounded budgets,
96 and also derive a uniqueness result for a two-player common-value Tullock
97 contest with identical linear costs in which players' information partitions
98 are nested.

99 A number of papers allow for continuous type distributions. Fey (2008)
100 studies the problem of the existence of a pure-strategy Nash equilibrium
101 in the symmetric two-player lottery contest with uniformly distributed, pri-
102 vately known marginal costs. Ryvkin (2010) examines existence in symmet-

103 ric n -player contests with likewise independently distributed private mar-
104 ginal costs, yet allowing for a wider class of contest success functions and
105 for more general probability densities functions. Wärneryd (2003, 2012) and
106 Rentschler (2009) find a unique equilibrium in common-value contests be-
107 tween players that are either privately informed or completely uninformed.
108 Based on a contraction argument, Wasser (2013a) finds a sufficient condition
109 for uniqueness for a modified lottery contest with heterogeneous continu-
110 ous distributions of marginal costs. Wasser (2013b) proves the existence of a
111 monotone equilibrium in continuous contests with interdependent valuations,
112 yet does not discuss uniqueness.

113 Models of population uncertainty have been studied by several authors.
114 In these models, each player may enter the contest with some exogenous
115 independent probability. Depending on the set-up, the potential number of
116 contestants is either finite (Münster, 2006; Lim and Matros, 2009; Fu et al.,
117 2011) or unbounded (Myerson and Wärneryd, 2006). Münster (2006) proves
118 uniqueness of the pure-strategy equilibrium in this setting.

119 Clark (1997) characterizes the pure-strategy equilibrium in a lottery con-
120 test in which players are uncertain about the success function. Such uncer-
121 tainty may result, for example, when players do not know the exact pref-
122 erences of the contest administrator, or if players have private information
123 about their “talent.” Epstein and Mealem (2013) study a lottery contest
124 with uncertainty about abilities in which one player has perfect information,
125 while the other player knows only her own ability. They characterize the
126 equilibrium for two types, and also explore the case of more than two types.⁸

⁸Baik and Shogren (1995) have considered a model with uncertainty about players’

127 As this overview shows, general results regarding the uniqueness of the
 128 pure-strategy equilibrium in contests of incomplete information have not been
 129 available so far.

130 **2 Set-up**

131 We consider a contest between $n \geq 2$ players. All uncertainty about the
 132 primitives of the contest is summarized in a state variable ω , which is drawn
 133 ex ante from a finite state space Ω .⁹ Player i 's ($i = 1, \dots, n$) expenditure
 134 $x_i \geq 0$ in the contest causes the production of $f_{i,\omega}(x_i) \geq 0$ "lottery tickets"
 135 in state ω . Consequently, player i 's probability of winning the contest is
 136 given by

$$137 \quad p_{i,\omega}(x_1, \dots, x_n) = \frac{f_{i,\omega}(x_i)}{\sum_{j=1}^n f_{j,\omega}(x_j)}, \quad (1)$$

138 provided the denominator in (1) does not vanish. Otherwise, i.e., if the
 139 denominator vanishes, the probability $p_{i,\omega}$ is set equal to some arbitrary
 140 value $p_{i,\omega}(0, \dots, 0) \in [0, 1)$ such that $\sum_{i=1}^n p_{i,\omega}(0, \dots, 0) \leq 1$ for all ω . This
 141 clearly includes the two common cases where either $p_{i,\omega}(0, \dots, 0) = \frac{1}{n}$ for any
 142 i and ω , or $p_{i,\omega}(0, \dots, 0) = 0$ for any i and ω .

143 State-contingent production functions arise in a natural way when players
 144 are uncertain about their abilities, as in Clark (1997) or Epstein and Mealem
 145 (2013). These possibilities are obviously covered by our set-up, but cannot
 146 be represented easily in a set-up in which only costs or valuations are state

relative ability before, yet their set-up may not be a game-theoretic one, according to Bolle (1996).

⁹The case of infinite state spaces leads to some technicalities that we prefer not to address in this paper.

147 dependent.

148 Assume that for all i and ω ,

149 (A) the function $f_{i,\omega}$ is twice differentiable, strictly increasing and con-
150 cave, with $f_{i,\omega}(0) = 0$.

151 This assumption captures the case of weakly decreasing returns of the
152 technology that transforms expenditures into “lottery tickets.”

153 Player i 's net rent in state ω is given by

$$154 \quad \Pi_{i,\omega}(x_1, \dots, x_n) = p_{i,\omega}(x_1, \dots, x_n)v_i(\omega) - c_{i,\omega}(x_i), \quad (2)$$

155 where $v_i(\omega) > 0$ denotes player i 's valuation of winning in state ω , and
156 $c_{i,\omega}(x_i)$ is player i 's cost of bidding x_i in state ω . We will alternatively refer
157 to $\Pi_{i,\omega}(x_1, \dots, x_n)$ as player i 's payoff in state ω . Assume that for any i and
158 ω ,

159 (B) the function $c_{i,\omega}$ is twice differentiable, strictly increasing and convex,
160 with $c_{i,\omega}(0) = 0$.

161 Each player i observes a signal θ_i (her “type”) that is taken from a corre-
162 sponding signal space Θ_i . The signal θ_i is the value $\theta_i = t_i(\omega)$ of a mapping
163 $t_i : \Omega \rightarrow \Theta_i$. Thus, θ_i may be thought of as the realization of the “random
164 variable” t_i . Signals are private information of the respective contestants,
165 i.e., player i does not observe the signal $\theta_j = t_j(\omega)$ of any other player $j \neq i$.
166 Note, however, that some primitives of the contest that are directly relevant
167 for player i (specifically, her production function, her valuation of winning,

168 and her cost function) may in general be dependent on, or correlated with,
 169 other players' signals that are not directly observable to her. For convenience,
 170 we write $\mathcal{P}_i(\theta_i) = \{\omega \in \Omega | t_i(\omega) = \theta_i\}$ for type θ_i 's possibility set, i.e., for
 171 the set of states in which player i observes the signal θ_i . Thus, $\omega \in \mathcal{P}_i(\theta_i)$ is
 172 equivalent to $t_i(\omega) = \theta_i$.

173 Let $x_i^{\max}(\theta_i) \geq 0$ denote the budget of type θ_i . If $x_i^{\max}(\theta_i) = 0$, then type
 174 θ_i is forced to remain inactive. Note that the assumption of a finite budget
 175 for all types is without loss of generality because no type θ_i would optimally
 176 choose an expenditure causing interim expected costs in excess of her highest
 177 possible valuation (which is bounded as a consequence of the finiteness of the
 178 state space).¹⁰

179 By a bid function for player i , we mean a mapping $\beta_i : \Theta_i \rightarrow \mathbb{R}_+$ such
 180 that $\beta_i(\theta_i) \in [0, x_i^{\max}(\theta_i)]$. We denote the set of player i 's bid functions by
 181 B_i . For a profile of bid functions $\beta_{-i} = \{\beta_j\}_{j \neq i} \in B_{-i} = \prod_{j \neq i} B_j$, denote
 182 by $\beta_{-i}(t_{-i}(\omega)) = \{\beta_j(t_j(\omega))\}_{j \neq i}$ the corresponding profile of bids resulting in
 183 state ω , where $t_{-i}(\omega) = \{t_j(\omega)\}_{j \neq i}$. Similarly, for any $\beta = \{\beta_i\}_{i=1}^n \in B =$
 184 $\prod_{i=1}^n B_i$, we shall write $\beta(t(\omega)) = \{\beta_i(t_i(\omega))\}_{i=1}^n$ for the corresponding profile
 185 of bids resulting in state ω , where $t(\omega) = \{t_i(\omega)\}_{i=1}^n$.

186 Let $q(\omega)$ denote the ex-ante probability of state ω , where we assume
 187 $q(\omega) > 0$ without loss of generality. The unconditional probability that
 188 signal θ_i realizes is then given by $q_i(\theta_i) = \sum_{\omega \in \mathcal{P}_i(\theta_i)} q(\omega)$. Based on the
 189 private signal θ_i received, player i forms a posterior belief on Ω , given by a
 190 probability $q_i(\omega | \theta_i)$ for each ω . It is assumed that each possible realization
 191 of player i 's signal occurs with positive probability, i.e., $q_i(\theta_i) > 0$ for any i

¹⁰A similar argument has been used by Yamazaki (2008).

192 and any θ_i . Hence, according to Bayes' rule, $q_i(\omega|\theta_i) = q(\omega)/q_i(\theta_i)$ for any
 193 $\omega \in \mathcal{P}_i(\theta_i)$, and $q_i(\omega|\theta_i) = 0$ otherwise. Note that $q_i(\omega|\theta_i) > 0$ is equivalent to
 194 $\omega \in \mathcal{P}_i(\theta_i)$, i.e., the support of type θ_i 's posterior is just $\mathcal{P}_i(\theta_i)$. The interim
 195 expected payoff for type $\theta_i \in \Theta_i$ is given by the conditional expectation

$$196 \quad \bar{\Pi}_i(x_i, \beta_{-i}, \theta_i) = \sum_{\omega \in \mathcal{P}_i(\theta_i)} q_i(\omega|\theta_i) \Pi_i(x_i, \beta_{-i}(t_{-i}(\omega)), \omega). \quad (3)$$

197 A pure-strategy Nash equilibrium is then a profile $\beta^* \in B$ such that

$$198 \quad \bar{\Pi}_i(\beta_i^*(\theta_i), \beta_{-i}^*, \theta_i) \geq \bar{\Pi}_i(x_i, \beta_{-i}^*, \theta_i) \quad (4)$$

199 for any i , θ_i and any $x_i \in [0, x_i^{\max}(\theta_i)]$.

200 **3 Uniqueness**

201 **3.1 Approach to uniqueness**

202 Our approach to uniqueness rests upon J.B. Rosen's (1965) result for con-
 203 cave n -person games with strategy spaces that are convex subsets of some
 204 Euclidean space. Rosen (1965) considers the Jacobian matrix J associated
 205 with players' marginal payoff functions, and requires that $J + J^T$, i.e., the
 206 sum of J and its transpose, be negative definite at all strategy profiles. To
 207 obtain some intuition, consider the pseudogradient associated with the pay-
 208 off functions in an asymmetric two-player lottery contest. I.e., to each pair
 209 of bids $(x_1, x_2) \in \mathbb{R}_+^2 \setminus \{(0, 0)\}$, one attaches a vector whose i 's component
 210 corresponds to player i 's marginal payoff, for $i = 1, 2$. Figure 1 shows the

211 corresponding directional field, in which the length of the pseudogradient at
 212 each point is normalized to one.¹¹ At the unique interior equilibrium β^* , the
 213 pseudogradient vanishes. Suppose there was another interior equilibrium β^{**}
 214 that differs from β^* . Then the inner product between the pseudogradient
 215 and the vector pointing from β^{**} to β^* would have to vanish at both β^* and
 216 β^{**} . But under Rosen's (1965) condition on the Jacobian, this inner product
 217 turns out to be strictly declining as one moves along the straight line from
 218 β^{**} to β^* , which is impossible. The argument works, in fact, equally well for
 219 boundary equilibria. Hence, there is at most one equilibrium.

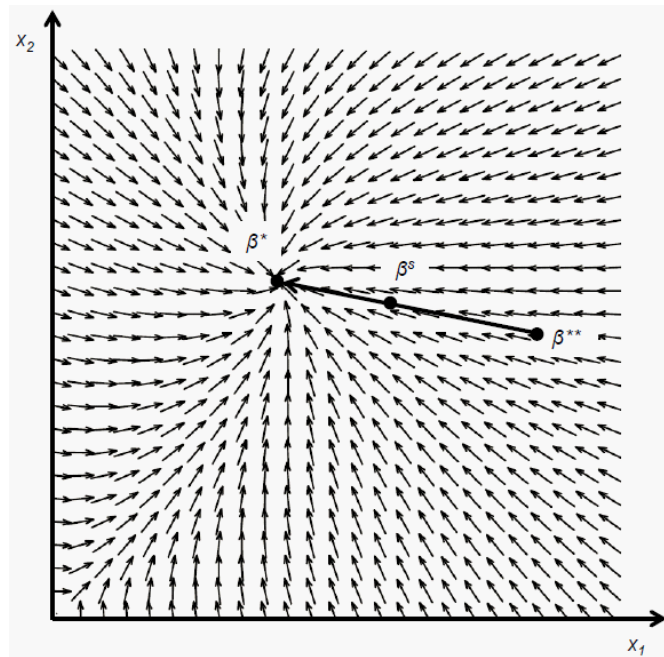


Figure 1: Illustration of Rosen's (1965) argument.

¹¹In the example drawn, the common valuation of winning is $v = 1$, and marginal costs are $c_1 = 0.6$ for player 1, and $c_2 = 0.4$ for player 2.

220 **3.2 Multiplicative separability**

221 The following assumption of multiplicative separability will be imposed on
222 players' valuation functions.

223 (C) There is a function $v : \Omega \rightarrow \mathbb{R}_{++}$ and, for each $i = 1, \dots, n$, a function
224 $\kappa_i : \Theta_i \rightarrow \mathbb{R}_{++}$ such that $v_i(\omega) = v(\omega) \cdot \kappa_i(t_i(\omega))$ for any ω .

225 This assumption encompasses many of the information structures that
226 have been used in the literature. In a setting with private valuations, for
227 instance, the common-value component can be normalized to unity, i.e., $v \equiv$
228 1. In this case, every player knows her valuation at the time of bidding.
229 Note that these private valuations can be either independently distributed
230 or correlated. In another example, a pure common value setting, each player's
231 private-value component is normalized to unity, i.e., $\kappa_i \equiv 1$ for $i = 1, \dots, n$. In
232 this case, the value of winning is the same for all players, yet each individual
233 player may possess only incomplete information about the value at the time
234 of bidding. Further settings are feasible.¹² A setting excluded by Assumption
235 (C) is one with truly interdependent valuations. In this case, $v_i(\omega)$ would
236 depend not only on $\theta_i = t_i(\omega)$ but also on $\theta_j = t_j(\omega)$, for at least one
237 $j \neq i$, and in such a way that a representation as in Assumption (C) becomes
238 impossible. However, we are aware of only one paper that has dealt with this
239 case so far, which is Wasser (2013b).¹³

¹²For example, in an international conflict about an oil field located under the Northern polar cap, $v(\omega)$ might correspond to the size of the oil field, and $\kappa_i(\theta_i)$ to a country-specific parameter. This setting is, in general, neither one of private values nor one of pure common value.

¹³While it must be conjectured that multiplicity of equilibria is a possibility with interdependent valuations, we have not managed to come up with a suitable example.

240 **3.3 The uniqueness theorem**

241 We shall now present the uniqueness argument more formally. We shall deal
 242 with the case of a pure common value first. Suppose there are two equilibria
 243 β^* and β^{**} with $\beta^* \neq \beta^{**}$. Consider the convex combination

$$244 \quad \beta^s = s\beta^* + (1-s)\beta^{**} \quad (5)$$

245 for $s \in [0, 1]$, and fix some type $\theta_i \in \Theta_i$ of some player i . If $x_i^{\max}(\theta_i) > 0$,
 246 then by Lemma A.1 in the Appendix, type θ_i 's expected payoff $\bar{\Pi}_i(x_i, \beta_{-i}^s, \theta_i)$
 247 is differentiable at $x_i = \beta_i^s(\theta_i)$ with

$$248 \quad \frac{\partial \bar{\Pi}_i(\beta_i^s(\theta_i), \beta_{-i}^s, \theta_i)}{\partial x_i} = \sum_{\omega \in \mathcal{P}_i(\theta_i)} q_i(\omega|\theta_i) \frac{\partial \Pi_i(\beta_i^s(\theta_i), \beta_{-i}^s(t_{-i}(\omega)), \omega)}{\partial x_i}. \quad (6)$$

249 Therefore, the “marginal payoff”

$$250 \quad \bar{\pi}_i(s, \theta_i) = \begin{cases} \frac{\partial \bar{\Pi}_i(\beta_i^s(\theta_i), \beta_{-i}^s, \theta_i)}{\partial x_i} & \text{if } x_i^{\max}(\theta_i) > 0 \\ 0 & \text{if } x_i^{\max}(\theta_i) = 0 \end{cases} \quad (7)$$

251 is well-defined for any i , any θ_i , and any $s \in [0, 1]$. Following Rosen (1965),
 252 consider now the “inner product”

$$253 \quad \gamma_s = \sum_{i=1}^n \sum_{\theta_i \in \Theta_i} q_i(\theta_i) \{\beta_i^*(\theta_i) - \beta_i^{**}(\theta_i)\} \bar{\pi}_i(s, \theta_i). \quad (8)$$

254 Recall that $\beta^0 = \beta^{**}$ and $\beta^1 = \beta^*$ are equilibria. Hence, for $s = 0$ or $s = 1$,
 255 from the necessary Kuhn-Tucker conditions, $\beta_i^s(\theta_i) = 0$ if $\bar{\pi}_i(s, \theta_i) < 0$ and
 256 $\beta_i^s(\theta_i) = x_i^{\max}(\theta_i)$ if $\bar{\pi}_i(s, \theta_i) > 0$, for any i and any θ_i . It follows that $\gamma_0 \leq 0$

257 and $\gamma_1 \geq 0$. To provoke a contradiction, we shall now show that $\gamma_1 - \gamma_0 < 0$.

258 Let $s \in [0, 1]$ be general again. Combining equations (6) and (7) delivers

$$259 \quad \bar{\pi}_i(s, \theta_i) = \sum_{\omega \in \mathcal{P}_i(\theta_i)} q_i(\omega | \theta_i) \pi_{i,\omega}(s), \quad (9)$$

260 with

$$261 \quad \pi_{i,\omega}(s) = \begin{cases} \frac{\partial \Pi_i(\beta^s(t(\omega)), \omega)}{\partial x_i} & \text{if } x_i^{\max}(t_i(\omega)) > 0 \\ 0 & \text{if } x_i^{\max}(t_i(\omega)) = 0. \end{cases} \quad (10)$$

262 Plugging equation (9) into (8), and subsequently exploiting $q_i(\theta_i)q_i(\omega | \theta_i) =$
 263 $q(\omega)$ for any $\omega \in \mathcal{P}_i(\theta_i)$ yields

$$264 \quad \gamma_s = \sum_{i=1}^n \sum_{\theta_i \in \Theta_i} \sum_{\omega \in \mathcal{P}_i(\theta_i)} q(\omega) \{\beta_i^*(\theta_i) - \beta_i^{**}(\theta_i)\} \pi_{i,\omega}(s). \quad (11)$$

265 Since player i 's possibility sets $\mathcal{P}_i(\theta_i)$, for signals $\theta_i \in \Theta_i$, form a partition of
 266 the state space Ω , we may write this more compactly as

$$267 \quad \gamma_s = \sum_{i=1}^n \sum_{\omega \in \Omega} q(\omega) z_i(\omega) \pi_{i,\omega}(s), \quad (12)$$

268 where $z_i(\omega) = \beta_i^*(t_i(\omega)) - \beta_i^{**}(t_i(\omega))$. For any ω satisfying $\beta^s(t(\omega)) = 0$ for
 269 some $s \in [0, 1]$, necessarily $\beta^*(t(\omega)) = 0$ or $\beta^{**}(t(\omega)) = 0$, so that by part (ii)
 270 of Lemma A.4 (and its obvious counterpart for β^{**}), $x_i^{\max}(t_i(\omega)) = 0$ for any
 271 i , and hence, $\pi_{i,\omega}(s) = 0$ for any i . Therefore,

$$272 \quad \gamma_s = \sum_{i=1}^n \sum_{\omega \in \Omega_0} q(\omega) z_i(\omega) \pi_{i,\omega}(s), \quad (13)$$

273 where Ω_0 is the set of ω such that $\beta^s(t(\omega)) \neq 0$ for all $s \in [0, 1]$. Consider
 274 next the difference

$$275 \quad \gamma_1 - \gamma_0 = \sum_{i=1}^n \sum_{\omega \in \Omega_0} q(\omega) z_i(\omega) (\pi_{i,\omega}(1) - \pi_{i,\omega}(0)) \quad (14)$$

$$276 \quad = \sum_{i=1}^n \sum_{\omega \in \Omega_0} q(\omega) \int_0^1 \frac{\partial \pi_{i,\omega}(s)}{\partial s} z_i(\omega) ds. \quad (15)$$

277 Fix for the moment some i and $\omega \in \Omega_0$. If $x_i^{\max}(t_i(\omega)) > 0$, then

$$278 \quad \pi_{i,\omega}(s) = v_i(\omega) \frac{\partial p_{i,\omega}(\beta_1^s(t_1(\omega)), \dots, \beta_n^s(t_n(\omega)))}{\partial x_i} - \frac{\partial c_{i,\omega}(\beta_i^s(t_i(\omega)))}{\partial x_i}, \quad (16)$$

279 and a straightforward application of the chain rule for differentiation, using
 280 $\beta_j^s(t_j(\omega)) = \beta_j^{**}(t_j(\omega)) + s \cdot z_j(\omega)$ for $j = 1, \dots, n$, leads to

$$281 \quad \frac{\partial \pi_{i,\omega}(s)}{\partial s} = v_i(\omega) \sum_{j=1}^n \frac{\partial^2 p_{i,\omega}(\beta_1^s(t_1(\omega)), \dots, \beta_n^s(t_n(\omega)))}{\partial x_j \partial x_i} z_j(\omega) \\ 282 \quad - \frac{\partial^2 c_{i,\omega}(\beta_i^s(t_i(\omega)))}{\partial x_i^2} z_i(\omega). \quad (17)$$

283 Multiplying this through with $z_i(\omega)$, and subsequently exploiting Assumption
 284 (B), one obtains the inequality

$$285 \quad \frac{\partial \pi_{i,\omega}(s)}{\partial s} z_i(\omega) \leq v_i(\omega) \sum_{j=1}^n \frac{\partial^2 p_{i,\omega}(\beta^s(t(\omega)))}{\partial x_j \partial x_i} z_j(\omega) z_i(\omega). \quad (18)$$

286 If, on the other hand, $x_i^{\max}(t_i) = 0$, then $z_i(\omega) = \beta_i^{**}(t_i(\omega)) - \beta_i^*(t_i(\omega)) = 0$,
 287 and (18) holds likewise. Thus, (18) holds for any i and any $\omega \in \Omega_0$. Plugging

288 this into (15), and exploiting that $v_i(\omega) = v(\omega)$ for $i = 1, \dots, n$, one arrives at

$$289 \quad \leq \sum_{i=1}^n \sum_{\omega \in \Omega_0} q(\omega) v_i(\omega) \int_0^1 \sum_{j=1}^n \frac{\partial^2 p_{i,\omega}(\beta^s(t(\omega)))}{\partial x_j \partial x_i} z_j(\omega) z_i(\omega) ds \quad (19)$$

$$291 \quad = \sum_{\omega \in \Omega_0} q(\omega) v(\omega) \int_0^1 z(\omega)^T J_{p,\omega}(\beta^s(t(\omega))) z(\omega) ds, \quad (20)$$

292 where $z(\omega) = (z_1(\omega), \dots, z_n(\omega))^T$, and $J_{p,\omega}(x)$ is the $n \times n$ -matrix whose ele-
 293 ments are $\partial^2 p_{i,\omega}(x) / \partial x_i \partial x_j$. It suffices to show now that the right-hand side
 294 of (20) is negative. By part (iii) of Lemma A.2, $J_{p,\omega}(\beta^s(t(\omega))) + J_{p,\omega}(\beta^s(t(\omega)))^T$
 295 is negative semi-definite for any $\omega \in \Omega_0$. Hence, by Lemma A.5,

$$296 \quad z(\omega)^T J_{p,\omega}(\beta^s(t(\omega))) z(\omega) \leq 0, \quad (21)$$

297 for any $s \in [0, 1]$ and for any $\omega \in \Omega_0$. Moreover, by Lemma A.3, there is
 298 a state ω_0 such that $z(\omega_0) \neq 0$ and such that $\beta^s(t(\omega_0))$ has two or more
 299 nonzero components. Hence, using the respective strict versions of part (iii)
 300 of Lemma A.2 and of Lemma A.5, we obtain even

$$301 \quad z(\omega_0)^T J_{p,\omega_0}(\beta^s(t(\omega_0))) z(\omega_0) < 0, \quad (22)$$

302 for any $s \in [0, 1]$. Since $\omega_0 \in \Omega_0$, $q(\omega_0) > 0$, and $v(\omega_0) > 0$, this implies that
 303 the right-hand side of (20) is indeed negative. Thus, $\gamma_1 - \gamma_0 < 0$, as claimed,
 304 which is inconsistent with $\gamma_0 \leq 0$ and $\gamma_1 \geq 0$. The contradiction shows that
 305 there cannot be two distinct equilibria.

306 We arrive at the following result:

307 **Theorem 1** Under Assumptions (A), (B) and (C), there exists at most
 308 one pure-strategy Nash equilibrium in the contest with incomplete informa-
 309 tion.

310 **Proof.** The case of pure common value has been proved in the text
 311 above. The general case can be reduced to the case of pure common value
 312 as follows. Maximizing type θ_i 's expected payoff

$$\begin{aligned}
 & \bar{\Pi}_i(x_i, \beta_{-i}, \theta_i) & (23) \\
 = & \sum_{\omega \in \mathcal{P}_i(\theta_i)} q_i(\omega|\theta_i) \left(\frac{f_{i,\omega}(x_i)}{f_{i,\omega}(x_i) + \sum_{j \neq i} f_{j,\omega}(\beta_j(t_j(\omega)))} v_i(\omega) - c_{i,\omega}(x_i) \right)
 \end{aligned}$$

315 is, by the virtue of Assumption (C), equivalent to maximizing type θ_i 's ex-
 316 pected payoff in units of her private-valuation component $\kappa_i(\theta_i) > 0$,

$$\begin{aligned}
 & \frac{\bar{\Pi}_i(x_i, \beta_{-i}, \theta_i)}{\kappa_i(\theta_i)} & (24) \\
 = & \sum_{\omega \in \mathcal{P}_i(\theta_i)} q_i(\omega|\theta_i) \left(\frac{f_{i,\omega}(x_i)}{f_{i,\omega}(x_i) + \sum_{j \neq i} f_{j,\omega}(\beta_j(t_j(\omega)))} v(\omega) - \frac{c_{i,\omega}(x_i)}{\kappa_i(\theta_i)} \right).
 \end{aligned}$$

319 The normalized contest \hat{C} with valuations $\hat{v}_i(\omega) = v(\omega)$ and cost functions
 320 $\hat{c}_{i,\omega}(x_i) = c_{i,\omega}(x_i)/\kappa_i(t_i(\omega))$, all else equal, is one of pure common value.
 321 Therefore, as shown above, there is at most one pure-strategy equilibrium in
 322 the contest \hat{C} . Hence, there is at most one pure-strategy equilibrium in the
 323 original contest. \square

324 Theorem 1 subsumes and extends a variety of conditions that have been
 325 proposed for the existence of at most one pure-strategy Nash equilibrium.
 326 In particular, this is true for conditions used by Szidarovszky and Okuguchi

327 (1997), Clark (1997), Schoonbeek and Winkel (2006), Münster (2006), and
328 Einy et al. (2013).

329 **4 Existence**

330 **4.1 Approach to existence**

331 To prove existence, we will deal more explicitly with the discontinuity of the
332 contest success function (1). Indeed, since there is no optimal choice for a
333 player if all of her opponents remain inactive, the best-response correspon-
334 dence does not possess a closed graph, which conflicts with the usual way of
335 constructing an equilibrium.¹⁴ To circumvent the problem, a number of pa-
336 pers, including Fey (2008), Ryvkin (2010), and Einy et al. (2013), introduce
337 a minimum bid $\varepsilon > 0$, and let ε go to zero. Our approach is similar.

338 **4.2 An assumption**

339 We will say that player i is the only player with a positive budget in state
340 ω if $x_i^{\max}(t_i(\omega)) > 0$ and $x_j^{\max}(t_j(\omega)) = 0$ for any $j \neq i$. Clearly, to obtain
341 an equilibrium, we need to require that a player cannot be certain to be the
342 only one with a positive budget. We therefore assume that

343 (D) there is no state ω in which a player is certain to be the only player
344 with a positive budget.

¹⁴Note that changing the value of the contest success function at the origin would not solve the problem. Also, restricting attention to symmetric contests with independent types does not help. While any equilibrium is then necessarily interior, a proof of existence is still needed.

345 Certainty is defined here in the usual way. I.e., a type θ_i is certain of
346 an arbitrary event $E \subseteq \Omega$ when she assigns probability one to E , i.e., if
347 $\sum_{\omega \in E} q_i(\omega|\theta_i) = 1$.

348 4.3 The existence theorem

349 Consider a modified contest C^m in which each type θ_i of any player i with a
350 positive budget is obligated to bid at least $\varepsilon_m > 0$, for some positive integer
351 m , where $\lim_{m \rightarrow \infty} \varepsilon_m = 0$. We assume that m is sufficiently large so that
352 the minimum bid is not in conflict with the budget constraint, i.e., such that
353 $\varepsilon_m < x_i^{\max}(\theta_i)$ for all i and all θ_i with $x_i^{\max}(\theta_i) > 0$. Then, clearly, there
354 exists a pure-strategy Nash equilibrium β^m in C^m .¹⁵

355 To construct an equilibrium in the original rent-seeking game, we let m
356 go to infinity. Since strategy spaces are closed and bounded, we may assume
357 without loss of generality that $\{\beta^m\}_{m=1}^{\infty}$ is convergent to some profile β^* (if
358 $\{\beta^m\}_{m=1}^{\infty}$ is not convergent, then one may select a converging subsequence).

359 We first claim that, in each state ω_0 in which at least one player has
360 a positive budget, there is at least one player active in the profile β^* . To
361 see this, fix ω_0 and let $\delta > 0$ be sufficiently small, in all what follows. To
362 provoke a contradiction, suppose that for any sufficiently large m , all bids
363 in the profile $\beta^m(t(\omega_0))$ are smaller than or equal to δ . Consider now some
364 player i_0 that has a positive budget in state ω_0 . By Assumption (D), player

¹⁵Here is a sketch of the proof. Consider the agent-normal form of the game C^m , i.e., the game in strategic form in which each type realization $\theta_i \in \Theta_i$ of each player $i = 1, \dots, n$ is considered a separate player. In that game, type θ_i 's expected payoff $\bar{\Pi}_i$ is continuous in the profile β . Moreover, by part (i) of Lemma A.1 in the Appendix, $\bar{\Pi}_i$ is concave in x_i . Therefore, invoking standard existence theorems (see, e.g., Fudenberg and Tirole, 1991, Theorem 2.1), there is indeed a pure-strategy Nash equilibrium β^m in C^m .

365 i_0 must not be certain in state ω_0 to be the only one with a positive budget.
366 Therefore, there is another player $i_1 \neq i_0$ and a state ω_1 consistent with
367 player i_0 's information in state ω_0 such that player i_1 's budget is positive in
368 state ω_1 . Note that player i_0 has a positive budget at state ω_1 as well, because
369 she cannot distinguish ω_0 and ω_1 . Thus, there are at least two players with
370 a positive budget in state ω_1 .

371 Write now $\beta_j^m = \beta_j^m(t_j(\omega_1))$ for $j = 1, \dots, n$, and consider, among the
372 players that have a positive budget in state ω_1 , the player i with the smallest
373 number of "lottery tickets" $f_{i,\omega_1}(b_i^m)$ (if there are several such players, any
374 one of those can be chosen). The expected marginal payoff of type $\theta_i = t_i(\omega)$
375 can now be bounded from below as follows:

$$\begin{aligned}
376 \quad & \sum_{\omega \in \mathcal{P}_i(\theta_i)} q_i(\omega|\theta_i) \left(\frac{\partial p_{i,\omega}(\beta^m(t(\omega)))}{\partial x_i} v_i(\omega) - \frac{\partial c_{i,\omega}(\beta_i^m(\theta_i))}{\partial x_i} \right) \quad (25) \\
377 \quad & \geq q_i(\omega_1|\theta_i) \frac{\partial p_{i,\omega_1}(\beta^m(t(\omega_1)))}{\partial x_i} v_i(\omega_1) - \sum_{\omega \in \mathcal{P}_i(\theta_i)} q_i(\omega|\theta_i) \frac{\partial c_{i,\omega}(\delta)}{\partial x_i}
\end{aligned}$$

378 Thus, we dropped all marginal revenue terms corresponding to states $\omega \in$
379 $\mathcal{P}_i(\theta_i)$ different from ω_1 (these terms are weakly positive by Assumption (A)),
380 and made use of the fact that $\beta_i^m(\theta_i) \leq \delta$, in combination with Assumption
381 (B). By Assumption (A),

$$\begin{aligned}
382 \quad & \frac{\partial p_{i,\omega_1}(\beta^m(t(\omega_1)))}{\partial x_i} = \frac{\partial f_{i,\omega_1}(b_i^m)/\partial x_i}{\sum_{j=1}^n f_{j,\omega_1}(b_j^m)} \cdot \frac{\sum_{j \neq i} f_{j,\omega_1}(b_j^m)}{\sum_{j=1}^n f_{j,\omega_1}(b_j^m)} \quad (26) \\
383 \quad & \geq \frac{\partial f_{i,\omega_1}(x_i^{\max}(\theta_i))/\partial x_i}{\sum_{j=1}^n f_{j,\omega_1}(\delta)} \cdot \frac{1}{2}, \quad (27)
\end{aligned}$$

384 because $f_{i,\omega_1}(b_i^m) > 0$, and because there are at least two players with a

385 positive budget in state ω_1 . Now, if δ is sufficiently small then, by continuity,
 386 the denominator in (27) can be made arbitrarily small, while the numerator
 387 is bounded away from zero by Assumption (A). Since $q_i(\omega_1|\theta_i) > 0$ and
 388 $v_i(\omega_1) > 0$, the right-hand side of inequality (25) will become positive for m
 389 sufficiently large, in contradiction to the Kuhn-Tucker condition associated
 390 with type θ_i 's optimal bid, which requires the left-hand side of inequality
 391 (25) to be weakly negative for δ sufficiently small, because then $\beta_i^m(\theta_i) \leq$
 392 $\delta < x_i^{\max}(\theta_i)$.

393 The contradiction shows that there are infinitely many m such that some
 394 bid in $\beta^m(t(\omega_0))$ exceeds δ . Since the number of players is finite, there must
 395 be some player that expends more than δ in state ω_0 for infinitely many m .
 396 But $\{\beta^m\}_{m=1}^\infty$ is known to converge. Hence, indeed, $\beta^*(t(\omega_0)) \neq 0$ in any
 397 state ω_0 in which not all players are forced to be remain inactive.

398 We finally show that β^* is an equilibrium in the original contest. Since
 399 β^m is an equilibrium in C^m ,

$$\begin{aligned}
 400 \quad & \sum_{\omega \in \mathcal{P}_i(\theta_i)} q_i(\omega|\theta_i) \Pi_i(\beta_i^m(\theta_i), \beta_{-i}^m(t_{-i}(\omega)), \omega) \\
 401 \quad & \geq \sum_{\omega \in \mathcal{P}_i(\theta_i)} q_i(\omega|\theta_i) \Pi_i(x_i, \beta_{-i}^m(t_{-i}(\omega)), \omega) \tag{28}
 \end{aligned}$$

402 holds for any i , θ_i and $x_i \in [\varepsilon_m, x_i^{\max}(\theta_i)]$. Consider first some positive bid
 403 $x_i \in (0, x_i^{\max}(\theta_i)]$. Then, $x_i \in [\varepsilon_m, x_i^{\max}(\theta_i)]$ for m sufficiently large. By the
 404 continuity of the mapping $x \mapsto \Pi_i(x, \omega)$ over $\mathbb{R}_+^n \setminus \{0\}$, taking the limit in

405 inequality (28) yields

$$\begin{aligned}
406 \quad & \sum_{\omega \in \mathcal{P}_i(\theta_i)} q_i(\omega|\theta_i) \Pi_i(\beta_i^*(\theta_i), \beta_{-i}^*(t_{-i}(\omega)), \omega) \\
407 \quad & \geq \sum_{\omega \in \mathcal{P}_i(\theta_i)} q_i(\omega|\theta_i) \Pi_i(x_i, \beta_{-i}^*(t_{-i}(\omega)), \omega) \tag{29}
\end{aligned}$$

408 for any i , θ_i and any $x_i \in (0, x_i^{\max}(\theta_i)]$. Consider now a deviation to $x_i = 0$.
409 Suppose that $\beta_{-i}^*(t_{-i}(\omega)) = 0$ for some $\omega \in \mathcal{P}_i(\theta_i)$. Then the deviation to
410 zero yields a lower expected payoff than a sufficiently small positive bid,
411 and we can argue as before. On the other hand, if $\beta_{-i}^*(t_{-i}(\omega)) \neq 0$ for any
412 $\omega \in \mathcal{P}_i(\theta_i)$, then the right-hand side of inequality (29) vanishes. Moreover,
413 because a bidder could always choose the minimum bid in C_m , which would
414 ensure an expected payoff of at least $-\varepsilon_m$, the left-hand side of inequality
415 (29) weakly exceeds $\lim_{m \rightarrow \infty} (-\varepsilon_m) = 0$. Hence, β^* is indeed a pure-strategy
416 Nash equilibrium in the original rent-seeking game.

417 We have proved the following result.

418 **Theorem 2** *Under Assumptions (A), (B) and (D), there exists a pure-*
419 *strategy Nash equilibrium in the contest with incomplete information. If As-*
420 *sumption (C) is added, the equilibrium is also unique.*

421 4.4 Mixed equilibria

422 One might wonder whether there could be additional equilibria in mixed
423 strategies. A mixed extension of the contest could be defined, for instance,
424 by assuming that each type θ_i chooses, rather than a bid $x_i \in [0, x_i(\theta_i)]$, a

425 probability distribution on the Borel subsets of $[0, x_i(\theta_i)]$.¹⁶ We claim that
 426 all equilibrium strategies in the mixed extension would be degenerate, i.e.,
 427 give probability one to a specific bid $x_i \in [0, x_i(\theta_i)]$. To see this, consider
 428 some state ω and fix a type of some player i . Indeed, under Assumptions
 429 (A) and (B), provided the opponents profile of bids is not zero, i.e., $x_{-i} \neq 0$,
 430 we have $\partial^2 \Pi_i(x_i, x_{-i}, \omega) / \partial x_i^2 < 0$ for any $x_i \in [0, x_i^{\max}(\theta_i)]$. On the other
 431 hand, when $x_{-i} = 0$, then $\partial^2 \Pi_i(x_i, x_{-i}, \omega) / \partial x_i^2 \leq 0$ for any $x_i \in (0, x_i^{\max}(\theta_i)]$.
 432 Moreover, $\Pi_i(x_i, 0, \omega)$ jumps up at $x_i = 0$. Since, in a mixed equilibrium,
 433 $x_{-i} = 0$ cannot occur with probability one (there would be no best response),
 434 the interim expected payoff of each type θ_i against any mixed equilibrium
 435 strategy profile of her opponents has a negative second derivative over the
 436 interval $(0, x_i^{\max}(\theta_i)]$, and jumps up at $x_i = 0$. Thus, any equilibrium strategy
 437 is necessarily a degenerate distribution, and therefore, any equilibrium must
 438 be in pure strategies.

439 **5 Rent dissipation**

440 In prior work, Szidarovszky and Okuguchi (1997) had documented the fact
 441 that the expected net rent in rent-seeking games of complete information is
 442 always weakly positive for all players. This result and its proof obviously
 443 extend to the class of contests with incomplete information considered here,
 444 since it is always possible for a player to remain passive, which incurs zero
 445 costs to her as a consequence of Assumption (B).

446 We examine this point further. From the proof of the existence theorem,

¹⁶See Dasgupta and Maskin (1986) for further details.

447 we know that in any state ω_0 admitting a positive budget for some player
448 i , there is some player j (not necessarily identical to i) that bids a positive
449 amount at ω_0 .¹⁷ To make this positive bid a best response for type $\theta_j =$
450 $t_j(\omega_0)$, there must be a state ω_1 consistent with θ_j 's information, and a
451 player k , different from j , that bids a positive amount in state ω_1 . Thus,
452 unless budgets are zero for all players in all states, there are at least two
453 players that are active with positive probability.

454 There is more that can be deduced here. As explained above, any active
455 type expects in equilibrium that at least one other player is active with
456 positive probability. Therefore, the expected payoff function for this type,
457 when playing against the opponents' equilibrium profile, is a weighted average
458 of concave payoff functions, with positive weight on functions that have a
459 negative second derivative. As a consequence, the expected payoff for any
460 active type is strictly concave. Since inactivity generates a certain payoff of
461 zero, activity must (as a consequence of the first-order condition and strict
462 concavity of interim expected payoffs) yield a positive payoff. In particular,
463 any player that is active with positive probability (and there are at least two)
464 will realize a positive expected net rent from an ex-ante perspective.

465 We summarize the discussion as follows.

466 **Theorem 3** *Impose Assumptions (A), (B) and (D). Assume also that*
467 *there exists a state in which not all budgets are zero. Then the following*
468 *statements hold true: (i) In each state of the world in which not all bud-*
469 *gets are zero, at least one player is active. (ii) There are two players that*

¹⁷Alternatively, this can be seen from part (ii) of Lemma A.4.

470 are active with positive probability. **(iii)** Any active type earns a positive
471 expected net rent. In particular, there are at least two players that earn a
472 positive expected net rent. **(iv)** If the contest is symmetric and, in addition,
473 Assumption (C) holds, then all players earn a positive expected net rent.

474 **Proof.** See the text before the statement of the theorem. \square

475 **6 Some implications**

476 This section derives some implications of our main results in a number of
477 specific settings that have been considered in the literature.

478 **6.1 Two-sided asymmetric information**

479 Hurley and Shogren (1998b) study the comparative statics of the unique in-
480 terior equilibrium. Since their setting is one of private valuations, it follows
481 from our results that there is at most one equilibrium. In particular, when-
482 ever there is an interior equilibrium for certain parameter values, additional
483 equilibria at the boundary can be definitely excluded.

484 **6.2 Symmetric contests**

485 Malueg and Yates (2004) study a class of symmetric Tullock contests, where
486 $f_i(x_i) = x_i^r$ for some parameter $r > 0$. In their set-up, a player's private value
487 is either v_L or v_H , where $v_H \geq v_L > 0$. Moreover, $\text{pr}\{v_1 = v_2 = v_L\} = \text{pr}\{v_1 =$
488 $v_2 = v_H\} = \frac{\sigma}{2}$ and $\text{pr}\{v_1 = v_L \text{ and } v_2 = v_H\} = \text{pr}\{v_1 = v_H \text{ and } v_2 = v_L\} =$
489 $\frac{1-\sigma}{2}$, where $\sigma \in [0, 1]$ is a parameter that measures the correlation between

490 the contestants' value distributions. As Malueg and Yates (2004) note, the
 491 corresponding Bayesian game need not in general possess a symmetric Nash
 492 equilibrium in pure strategies. Rather,

$$493 \quad r \leq 2 \left(\frac{\sigma + (2 - \sigma)\rho^r}{1 + \rho^r} \right) \left(\frac{2 + \rho^{-r} + \rho^r}{2(2 - \sigma) + \sigma(\rho^{-r} + \rho^r)} \right) \quad (30)$$

494 is a necessary and sufficient condition for existence of a symmetric equilibrium
 495 in terms of the parameters r , $\rho = v_L/v_H$ and σ . However, condition (30)
 496 holds for any $r \leq 1$.¹⁸ Thus, there exists a unique symmetric equilibrium in
 497 Malueg and Yates' (2004) framework when $r \leq 1$. We may now use Theorem
 498 1 to conclude that, in fact, the game considered by Malueg and Yates (2004)
 499 does not allow any asymmetric equilibria for $r \leq 1$.¹⁹ Thus, the symmetric
 500 equilibrium studied in this context is indeed the only Nash equilibrium of
 501 this game.

¹⁸This fact is suggested by the intuitive discussion of Malueg and Yates (2004, pp. 168-169). For a proof, note that it suffices to show that the right-hand side of inequality (30) weakly exceeds unity, i.e., it suffices to show that

$$1 \leq 2 \left(\frac{\sigma + (2 - \sigma)\rho^r}{1 + \rho^r} \right) \left(\frac{2 + \rho^{-r} + \rho^r}{2(2 - \sigma) + \sigma(\rho^{-r} + \rho^r)} \right).$$

Re-arranging the ratios individually yields

$$1 \leq 2 \left(\frac{2(1 - \sigma)}{1 + \rho^{-r}} + \sigma \right) \left(\frac{2 + \rho^{-r} + \rho^r}{4(1 - \sigma) + \sigma(2 + \rho^{-r} + \rho^r)} \right).$$

Multiplying through with the reciprocal of the second ratio, one arrives at

$$\frac{4(1 - \sigma)}{2 + \rho^{-r} + \rho^r} \leq \frac{4(1 - \sigma)}{1 + \rho^{-r}} + \sigma,$$

which is obviously satisfied for any $\sigma \in [0, 1]$ and any $\rho \in (0, 1)$.

¹⁹While for $r < 1$, the function $f_i(x_i) = x_i^r$ is not differentiable at $x_i = 0$, a simple change of variables turns the Tullock contest into a lottery contest with $r = 1$ and cost functions $c_i(x_i) = x_i^{1/r}$, which then satisfies Assumptions (A) and (B).

502 **6.3 Contests with resale**

503 Sui (2009) extends the framework of Malueg and Yates (2004) by allowing for
504 the possibility of resale. In substance this means that, should a player with
505 a low valuation v_L win the contest, she may offer the prize to the loser at
506 price (marginally below) v_H , who will accept the offer if and only if she has
507 valuation v_H . Thus, each player's value of winning is v_L if both players have
508 a low valuation of the prize, and v_H otherwise. In other words, the possibility
509 of resale turns the game into one of a pure common value. As a consequence,
510 Theorem 1 applies and we find that there is a unique equilibrium. Thus,
511 again, the restriction to symmetric equilibria is not necessary.

512 **6.4 Asymmetric information about ability**

513 Epstein and Mealem (2013) derive conditions under which, in an interior equi-
514 librium, an extreme type that is privately informed about her ability would
515 prefer the uninformed player to become informed. Without uniqueness, how-
516 ever, the ranking result for more than two types might be overturned by the
517 presence of additional equilibria at the boundary. Precisely this possibility
518 is, again, excluded by Theorem 1.

519 **7 Conclusion**

520 It has been shown that uniqueness of the Nash equilibrium is a pervasive
521 property of contests with weakly decreasing returns and incomplete informa-
522 tion. Our conditions subsume and extend various prior conditions for the

523 existence of a unique equilibrium. As we illustrated in specific settings, our
524 results allow to exclude the possibility of additional equilibria (e.g., at the
525 boundary), or the possibility of asymmetric equilibria in symmetric games.
526 This type of result is useful for both comparative statics and policy analysis.
527 Further, our results should encourage further numerical calculations of equi-
528 libria and experiments. In fact, experimental contests with incomplete infor-
529 mation such as those reported in Brookins and Ryvkin (2013) and Grosskopf
530 et al. (2010) seem to be a particularly interesting object of research, and
531 should be studied much more thoroughly in our view.

532 8 Appendix

533 This Appendix contains several lemmas that are used in the proof of Theorem
534 1. We will assume throughout that β^* and β^{**} denote two distinct equilibria,
535 and write $\beta^s = s\beta^* + (1-s)\beta^{**}$ and $z(\omega) = \beta^{**}(t(\omega)) - \beta^*(t(\omega))$, as before.

536 The first lemma deals with the differentiability of interim expected pay-
537 offs.

538 **Lemma A.1.** *If $x_i^{\max}(\theta_i) > 0$, then the function $x_i \mapsto \bar{\Pi}_i(x_i, \beta_{-i}^s, \theta_i)$ is*
539 *differentiable at $x_i = \beta_i^s(\theta_i)$, with*

$$540 \quad \frac{\partial \bar{\Pi}_i(\beta_i^s(\theta_i), \beta_{-i}^s, \theta_i)}{\partial x_i} = \sum_{\omega \in \mathcal{P}_i(\theta_i)} q_i(\omega | \theta_i) \frac{\partial \Pi_i(\beta_i^s(\theta_i), \beta_{-i}^s(t_{-i}(\omega)), \omega)}{\partial x_i}. \quad (31)$$

541 **Proof.** The claim follows directly from equation (3) if $\beta_i^s(\theta_i) > 0$. Sup-
542 pose, therefore, that $\beta_i^s(\theta_i) = 0$. Assume first that $s = 1$, so that $\beta^s = \beta^*$.
543 Since $x_i^{\max}(\theta_i) > 0$, part (ii) of Lemma A.4 below implies that $\beta_{-i}^*(t_{-i}(\omega)) \neq 0$

544 for any $\omega \in \mathcal{P}_i(\theta_i)$, so that the claim follows in this case. Assume now that
545 $s \in [0, 1)$. Then $\beta_i^s(\theta_i) = 0$ implies $\beta_i^{**}(\theta_i) = 0$. Applying part (ii) of Lemma
546 A.4 again (with β^* replaced by β^{**}), it follows that $\beta_{-i}^{**}(t_{-i}(\omega)) \neq 0$ for any
547 $\omega \in \mathcal{P}_i(\theta_i)$. Hence, $\beta_{-i}^s(t_{-i}(\omega)) \neq 0$ for any $\omega \in \mathcal{P}_i(\theta_i)$, which proves the
548 lemma. \square

549 The next lemma applies an argument due to Goodman (1980) to contests.
550 Recall that $J_{p,\omega}(x)$ is the $n \times n$ -matrix whose elements are $\partial^2 p_{i,\omega}(x)/\partial x_i \partial x_j$.

551 **Lemma A.2.** *Impose Assumption (A), and let $\omega \in \Omega$. Then the following*
552 *statements hold true: (i) For any i , the function $p_{i,\omega}$ is twice continuously dif-*
553 *ferentiable on $\mathbb{R}_+^n \setminus \{0\}$ with $\partial^2 p_{i,\omega}/\partial x_i^2 \leq 0$. Moreover, $\partial^2 p_{i,\omega}(x_i, x_{-i})/\partial x_i^2 <$
554 0 for any $x_i \geq 0$ and any $x_{-i} \in \mathbb{R}_+^n \setminus \{0\}$. (ii) For any i and any $x_i > 0$,
555 the function $x_{-i} \mapsto p_{i,\omega}(x_i, x_{-i})$ is convex over \mathbb{R}_+^{n-1} . (iii) $J_{p,\omega}(x) + J_{p,\omega}(x)^T$
556 is negative semi-definite for any $x \in \mathbb{R}_+^n \setminus \{0\}$, and negative definite for any
557 $x \in \mathbb{R}_+^n$ possessing two or more nonzero components.*

558 **Proof.** (i) For any $x \in \mathbb{R}_+^n \setminus \{0\}$, by Assumption (A),

$$559 \quad \frac{\partial^2 p_{i,\omega}(x)}{\partial x_i^2} = \frac{\partial^2}{\partial x_i^2} \frac{f_{i,\omega}(x_i)}{f_{i,\omega}(x_i) + \sum_{j \neq i} f_{j,\omega}(x_j)}$$

$$560 \quad = \frac{\partial}{\partial x_i} \frac{(\partial f_{i,\omega}(x_i)/\partial x_i) \sum_{j \neq i} f_{j,\omega}(x_j)}{\left(f_{i,\omega}(x_i) + \sum_{j \neq i} f_{j,\omega}(x_j)\right)^2} \quad (32)$$

$$561 \quad = -\frac{2(\partial f_{i,\omega}(x_i)/\partial x_i)^2 \sum_{j \neq i} f_{j,\omega}(x_j)}{\left(f_{i,\omega}(x_i) + \sum_{j \neq i} f_{j,\omega}(x_j)\right)^3} \quad (33)$$

$$562 \quad + \frac{(\partial^2 f_{i,\omega}(x_i)/\partial x_i^2) \sum_{j \neq i} f_{j,\omega}(x_j)}{\left(f_{i,\omega}(x_i) + \sum_{j \neq i} f_{j,\omega}(x_j)\right)^2}$$

$$563 \quad \leq 0. \quad (34)$$

564 Moreover, if $x_{-i} \in \mathbb{R}_+^n \setminus \{0\}$, then $\sum_{j \neq i} f_{j,\omega}(x_j) > 0$, and hence, $\partial^2 p_{i,\omega}(x) / \partial x_i^2 <$
 565 0, as claimed.

566 **(ii)** The mapping $x_{-i} \mapsto \sum_{j \neq i} f_{j,\omega}(x_j)$ is concave over \mathbb{R}_+^{n-1} , as can be seen
 567 by noting that the corresponding Hessian is a diagonal matrix with weakly
 568 negative entries. Further, for any fixed $x_i > 0$, the mapping $y \mapsto \frac{x_i}{x_i + y}$ is
 569 convex and decreasing on \mathbb{R}_{++} . The claim follows now from standard results
 570 on the concatenation of convex mappings (e.g., Th. 5.1 in Rockafellar, 1970).

571 **(iii)** Take some $x \in \mathbb{R}_+^n \setminus \{0\}$. We wish to show that

$$572 \quad J_{p,\omega}(x) + J_{p,\omega}(x)^T \tag{35}$$

$$573 \quad = \begin{pmatrix} 2 \frac{\partial^2 p_{1,\omega}(x)}{\partial x_1^2} & \frac{\partial^2 p_{2,\omega}(x)}{\partial x_2 \partial x_1} + \frac{\partial^2 p_{1,\omega}(x)}{\partial x_2 \partial x_1} & \cdots & \frac{\partial^2 p_{n,\omega}(x)}{\partial x_n \partial x_1} + \frac{\partial^2 p_{1,\omega}(x)}{\partial x_n \partial x_1} \\ \frac{\partial^2 p_{1,\omega}(x)}{\partial x_1 \partial x_2} + \frac{\partial^2 p_{2,\omega}(x)}{\partial x_1 \partial x_2} & 2 \frac{\partial^2 p_{2,\omega}(x)}{\partial x_2^2} & \cdots & \frac{\partial^2 p_{n,\omega}(x)}{\partial x_n \partial x_2} + \frac{\partial^2 p_{2,\omega}(x)}{\partial x_n \partial x_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 p_{1,\omega}(x)}{\partial x_1 \partial x_n} + \frac{\partial^2 p_{n,\omega}(x)}{\partial x_1 \partial x_n} & \frac{\partial^2 p_{2,\omega}(x)}{\partial x_2 \partial x_n} + \frac{\partial^2 p_{n,\omega}(x)}{\partial x_2 \partial x_n} & \cdots & 2 \frac{\partial^2 p_{n,\omega}(x)}{\partial x_n^2} \end{pmatrix}$$

574

575 is negative semi-definite. By part (i) of this lemma, $\partial^2 p_{i,\omega} / \partial x_i^2 \leq 0$ over
 576 $\mathbb{R}_+^n \setminus \{0\}$, for all i . Therefore, the diagonal matrix

$$577 \quad M_\omega(x) = \begin{pmatrix} \frac{\partial^2 p_{1,\omega}(x)}{\partial x_1^2} & 0 & \cdots & 0 \\ 0 & \frac{\partial^2 p_{2,\omega}(x)}{\partial x_2^2} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \frac{\partial^2 p_{n,\omega}(x)}{\partial x_n^2} \end{pmatrix} \tag{36}$$

578 is negative semi-definite. Fix some $k = 1, \dots, n$. From part (ii) of this lemma,
 579 for $x_k > 0$, the mapping $x_{-k} \mapsto p_{k,\omega}(x_k, x_{-k})$ is convex over \mathbb{R}_+^{n-1} . Likewise,

580 for $x_k = 0$, the mapping $x_{-k} \mapsto p_{k,\omega}(0, x_{-k}) \equiv 0$ is convex over $\mathbb{R}_+^{n-1} \setminus \{0\}$.

581 Therefore, the corresponding Hessian matrix, i.e.,

$$582 \quad H_{k,\omega}(x) \tag{37}$$

$$583 \quad = \begin{pmatrix} \frac{\partial^2 p_{k,\omega}(x)}{\partial x_1^2} & \cdots & \frac{\partial^2 p_{k,\omega}(x)}{\partial x_{k-1} \partial x_1} & \frac{\partial^2 p_{k,\omega}(x)}{\partial x_{k+1} \partial x_1} & \cdots & \frac{\partial^2 p_{k,\omega}(x)}{\partial x_n \partial x_1} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial^2 p_{k,\omega}(x)}{\partial x_1 \partial x_{k-1}} & \cdots & \frac{\partial^2 p_{k,\omega}(x)}{\partial x_{k-1}^2} & \frac{\partial^2 p_{k,\omega}(x)}{\partial x_{k+1} \partial x_{k-1}} & \cdots & \frac{\partial^2 p_{k,\omega}(x)}{\partial x_n \partial x_{k-1}} \\ 584 \quad \frac{\partial^2 p_{k,\omega}(x)}{\partial x_1 \partial x_{k+1}} & \cdots & \frac{\partial^2 p_{k,\omega}(x)}{\partial x_{k-1} \partial x_{k+1}} & \frac{\partial^2 p_{k,\omega}(x)}{\partial x_{k+1}^2} & \cdots & \frac{\partial^2 p_{k,\omega}(x)}{\partial x_n \partial x_{k+1}} \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 p_{k,\omega}(x)}{\partial x_1 \partial x_n} & \cdots & \frac{\partial^2 p_{k,\omega}(x)}{\partial x_{k-1} \partial x_n} & \frac{\partial^2 p_{k,\omega}(x)}{\partial x_{k+1} \partial x_n} & \cdots & \frac{\partial^2 p_{k,\omega}(x)}{\partial x_n^2} \end{pmatrix},$$

585 is positive semi-definite. Thus, $z_{-k}^T H_{k,\omega}(x) z_{-k} \geq 0$ for any

$$586 \quad z_{-k} = (z_1, \dots, z_{k-1}, z_{k+1}, \dots, z_n)^T \in \mathbb{R}^{n-1}. \tag{38}$$

587 Consider now the matrix

$$588 \quad H_{k,\omega}^0(x) \tag{39}$$

$$589 \quad = \begin{pmatrix} \frac{\partial^2 p_k(x)}{\partial x_1^2} & \cdots & \frac{\partial^2 p_k(x)}{\partial x_{k-1} \partial x_1} & 0 & \frac{\partial^2 p_k(x)}{\partial x_{k+1} \partial x_1} & \cdots & \frac{\partial^2 p_k(x)}{\partial x_n \partial x_1} \\ \vdots & \ddots & \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial^2 p_k(x)}{\partial x_1 \partial x_{k-1}} & \cdots & \frac{\partial^2 p_k(x)}{\partial x_{k-1}^2} & 0 & \frac{\partial^2 p_k(x)}{\partial x_{k+1} \partial x_{k-1}} & \cdots & \frac{\partial^2 p_k(x)}{\partial x_n \partial x_{k-1}} \\ 590 \quad 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \frac{\partial^2 p_k(x)}{\partial x_1 \partial x_{k+1}} & \cdots & \frac{\partial^2 p_k(x)}{\partial x_{k-1} \partial x_{k+1}} & 0 & \frac{\partial^2 p_k(x)}{\partial x_{k+1}^2} & \cdots & \frac{\partial^2 p_k(x)}{\partial x_n \partial x_{k+1}} \\ \vdots & & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 p_k(x)}{\partial x_1 \partial x_n} & \cdots & \frac{\partial^2 p_k(x)}{\partial x_{k-1} \partial x_n} & 0 & \frac{\partial^2 p_k(x)}{\partial x_{k+1} \partial x_n} & \cdots & \frac{\partial^2 p_k(x)}{\partial x_n^2} \end{pmatrix}.$$

591 It is straightforward to check that $z^T H_{k,\omega}^0(x) z = z_{-k}^T H_{k,\omega}(x) z_{-k} \geq 0$ for any
 592 $z = (z_1, \dots, z_n)^T \in \mathbb{R}^n$. Thus, the matrix $H_{k,\omega}^0(x)$ is likewise positive semi-
 593 definite. Summing now over $k = 1, \dots, n$, we obtain that

$$594 \quad \sum_{k=1, \dots, n} H_{k,\omega}^0(x) \tag{40}$$

$$595 \quad = \begin{pmatrix} \sum_{\substack{k=1, \dots, n \\ k \neq 1}} \frac{\partial^2 p_{k,\omega}(x)}{\partial x_1^2} & \sum_{\substack{k=1, \dots, n \\ k \neq 2, 1}} \frac{\partial^2 p_{k,\omega}(x)}{\partial x_2 \partial x_1} & \cdots & \sum_{\substack{k=1, \dots, n \\ k \neq n, 1}} \frac{\partial^2 p_{k,\omega}(x)}{\partial x_n \partial x_1} \\ \sum_{\substack{k=1, \dots, n \\ k \neq 1, 2}} \frac{\partial^2 p_{k,\omega}(x)}{\partial x_1 \partial x_2} & \sum_{\substack{k=1, \dots, n \\ k \neq 2}} \frac{\partial^2 p_{k,\omega}(x)}{\partial x_2^2} & & \vdots \\ \vdots & & \ddots & \vdots \\ \sum_{\substack{k=1, \dots, n \\ k \neq 1, n}} \frac{\partial^2 p_{k,\omega}(x)}{\partial x_1 \partial x_n} & \cdots & \cdots & \sum_{\substack{k=1, \dots, n \\ k \neq n}} \frac{\partial^2 p_{k,\omega}(x)}{\partial x_n^2} \end{pmatrix}.$$

597 Since $x \neq 0$, we have $\sum_{k=1}^n p_{k,\omega}(x) = 1$ and, consequently, $\sum_{k=1}^n \frac{\partial^2 p_{k,\omega}(x)}{\partial x_i \partial x_j} = 0$

598 for arbitrary i and j . Hence,

$$599 \quad - \sum_{k=1, \dots, n} H_{k, \omega}^0(x) \quad (41)$$

$$600 \quad = \begin{pmatrix} \frac{\partial^2 p_{1, \omega}(x)}{\partial x_1^2} & \frac{\partial^2 p_{1, \omega}(x)}{\partial x_2 \partial x_1} + \frac{\partial^2 p_{2, \omega}(x)}{\partial x_2 \partial x_1} & \dots & \frac{\partial^2 p_{1, \omega}(x)}{\partial x_n \partial x_1} + \frac{\partial^2 p_{n, \omega}(x)}{\partial x_n \partial x_1} \\ \frac{\partial^2 p_{1, \omega}(x)}{\partial x_1 \partial x_2} + \frac{\partial^2 p_{2, \omega}(x)}{\partial x_1 \partial x_2} & \frac{\partial^2 p_{2, \omega}(x)}{\partial x_2^2} & & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{\partial^2 p_{1, \omega}(x)}{\partial x_1 \partial x_n} + \frac{\partial^2 p_{n, \omega}(x)}{\partial x_1 \partial x_n} & \dots & \dots & \frac{\partial^2 p_{n, \omega}(x)}{\partial x_n^2} \end{pmatrix}.$$

602 Thus, $J_{p, \omega}(x) + J_{p, \omega}(x)^T = M_\omega(x) - \sum_{k=1}^n H_{k, \omega}^0(x)$ is indeed negative semi-
603 definite for any $x \in \mathbb{R}_+^n \setminus \{0\}$. Let now $x \in \mathbb{R}_+^n$ possess two or more nonzero
604 components. Then, $x_{-i} \in \mathbb{R}_+^{n-1} \setminus \{0\}$ for all $i = 1, \dots, n$. In this case, therefore,
605 $M_\omega(x)$ is negative definite, and so is $J_{p, \omega}(x) + J_{p, \omega}(x)^T$. \square

606 The following lemma says that any two distinct equilibria must assign, in
607 some state of the world, different bids for at least two players.

608 **Lemma A.3** *There is a state $\omega \in \Omega$ such that $z(\omega) \neq 0$ and such that*
609 *$\beta^s(t(\omega))$ has two or more nonzero components, for any $s \in (0, 1)$.*

610 **Proof.** Since $\beta^* \neq \beta^{**}$, there is some i and θ_i such that $\beta_i^*(\theta_i) \neq \beta_i^{**}(\theta_i)$.
611 Take an arbitrary $\omega \in \mathcal{P}_i(\theta_i)$. Then $z_i(\omega) = \beta_i^*(t_i(\omega)) - \beta_i^{**}(t_i(\omega)) \neq 0$
612 and, hence, $z(\omega) \neq 0$. Moreover, $\beta_i^s(t_i(\omega)) > 0$ for any $s \in (0, 1)$, so that the
613 profile $\beta^s(t(\omega))$ has at least one non-zero component. We are done if $\beta^s(t(\omega))$
614 has two or more nonzero components, for any $s \in (0, 1)$. We may assume
615 therefore that there is some $s \in (0, 1)$ such that $\beta_j^s(t_j(\omega)) = 0$ for all $j \neq i$.
616 Then, clearly, also $\beta_j^*(t_j(\omega)) = 0$ for all $j \neq i$. In other words, $\beta_{-i}^*(t_{-i}(\omega)) =$

617 0. To provoke a contradiction, suppose that, in fact, $\beta_{-i}^*(t_{-i}(\tilde{\omega})) = 0$ for any
618 state $\tilde{\omega} \in \mathcal{P}_i(\theta_i)$. Then necessarily $x_i^{\max}(\theta_i) = 0$ by part (i) of Lemma A.4,
619 and consequently $\beta_i^*(\theta_i) = 0$. Thus, $\beta^*(t(\omega)) = 0$. Analogously, one proves
620 that $\beta^{**}(t(\omega)) = 0$, so that $z(\omega) = \beta^*(t(\omega)) - \beta^{**}(t(\omega)) = 0$. It follows from
621 the contradiction that there is a state $\tilde{\omega} \in \mathcal{P}_i(\theta_i)$ such that $\beta_{-i}^*(t_{-i}(\tilde{\omega})) \neq 0$.
622 Hence, there is a $j \neq i$ such that $\beta_j^*(t_j(\tilde{\omega})) > 0$ and, therefore, $\beta_j^s(t_j(\tilde{\omega})) > 0$
623 for any $s \in (0, 1)$. Since also $\beta_i^s(t_i(\tilde{\omega})) = \beta_i^s(t_i(\omega)) > 0$ for any $s \in (0, 1)$,
624 the profile $\beta^s(t(\tilde{\omega}))$ has two or more nonzero components for any $s \in (0, 1)$.
625 Moreover, $z_i(\tilde{\omega}) = \beta_i^*(t_i(\tilde{\omega})) - \beta_i^{**}(t_i(\tilde{\omega})) = \beta_i^*(t_i(\omega)) - \beta_i^{**}(t_i(\omega)) = z_i(\omega) \neq 0$.
626 Therefore, the assertion holds with ω replaced by $\tilde{\omega}$. \square

627 The next lemma collects a number of useful facts related to the logit form
628 of the contest success function.

629 **Lemma A.4** *For any $i \in \{1, \dots, n\}$, the following statements hold true:*

630 **(i)** *For any $\theta_i \in \Theta_i$ with $x_i^{\max}(\theta_i) > 0$, there is a state $\omega \in \mathcal{P}_i(\theta_i)$ such that*
631 *$\beta_{-i}^*(t_{-i}(\omega)) \neq 0$. **(ii)** For any $\theta_i \in \Theta_i$ with $x_i^{\max}(\theta_i) > 0$ and $\beta_i^*(\theta_i) = 0$, we*
632 *have $\beta_{-i}^*(t_{-i}(\omega)) \neq 0$ for any $\omega \in \mathcal{P}_i(\theta_i)$. **(iii)** For any $\omega \in \Omega$ with $z(\omega) \neq 0$,*
633 *we have $\beta^s(t(\omega)) \neq 0$ for any $s \in [0, 1]$.*

634 **Proof.** **(i)** Suppose that $\beta_{-i}^*(t_{-i}(\omega)) = 0$ for any $\omega \in \mathcal{P}_i(\theta_i)$. Then,
635 $x_i = 0$ is not an optimal response to β_{-i}^* for type θ_i because increasing
636 x_i marginally above zero (which is feasible because of $x_i^{\max}(\theta_i) > 0$) would
637 raise, in each state $\omega \in \mathcal{P}_i(\theta_i)$, the probability of winning the rent $v_i(\omega) > 0$
638 discontinuously from $p_i(0, \dots, 0) < 1$ to unity. But there is also no $x_i \in$
639 $(0, x_i^{\max}(\theta_i)]$ that could be a best response because, when type θ_i is certain to
640 face zero bids from her opponents, type θ_i 's expected payoff $\bar{\Pi}_i(x_i, \beta_{-i}^*, \theta_i) =$

641 $\sum_{\omega \in \mathcal{P}_i(\theta_i)} q_i(\omega|\theta_i)(v_i(\omega) - c_{i,\omega}(x_i))$ is strictly declining over $(0, x_i^{\max}(\theta_i)]$ by
642 Assumption (B). Thus, $\beta_i^*(\theta_i)$ is not a best response to β_{-i}^* for type θ_i , which
643 is a contradiction.

644 (ii) Suppose that $\beta_{-i}^*(t_{-i}(\omega)) = 0$ at some state $\omega \in \mathcal{P}_i(\theta_i)$. Then,
645 for any such state ω , player i 's payoff $\Pi_i(x_i, 0, \omega)$ jumps up discontinuously
646 when the bid x_i is increased marginally from zero, because $v_i(\omega) > 0$. More-
647 over, for any state $\omega \in \mathcal{P}_i(\theta_i)$ with $\beta_{-i}^*(t_{-i}(\omega)) \neq 0$, if any, player i 's pay-
648 off $\Pi_i(x_i, \beta_{-i}^*(t_{-i}(\omega)), \omega)$ is continuous at $x_i = 0$. Since $q_i(\omega|\theta_i) > 0$ for all
649 states $\omega \in \mathcal{P}_i(\theta_i)$, type θ_i 's expected payoff $\bar{\Pi}_i(x_i, \beta_{-i}^*, \theta_i)$ jumps up discontin-
650 uously when the bid x_i is increased marginally from zero. Hence, $\beta_i^*(\theta_i) > 0$.

651 (iii) Take some ω , and suppose that $\beta^s(t(\omega)) = 0$ for some $s \in [0, 1]$.
652 Then necessarily $\beta^*(t(\omega)) = 0$ or $\beta^{**}(t(\omega)) = 0$. Assume first that $\beta^*(t(\omega)) =$
653 0 . Writing $\theta_i = t_i(\omega)$ for $i = 1, \dots, n$, we obtain $\omega \in \mathcal{P}_i(\theta_i)$, $\beta_i^*(\theta_i) = 0$, and
654 $\beta_{-i}^*(t_{-i}(\omega)) = 0$. By part (ii) of this lemma, $x_i^{\max}(\theta_i) = 0$ for any i . But then
655 also $\beta_i^{**}(\theta_i) = 0$, so that $z_i(\omega) = \beta_i^*(t_i(\omega)) - \beta_i^{**}(t_i(\omega)) = \beta_i^*(\theta_i) - \beta_i^{**}(\theta_i) = 0$
656 for any i , which contradicts the assumption that $z(\omega) \neq 0$. The proof is
657 analogous if $\beta^{**}(t(\omega)) = 0$. \square

658 The final lemma states two simple matrix-theoretic facts.

659 **Lemma A.5.** *Let J be an $n \times n$ -matrix such that $J + J^T$ is negative*
660 *semi-definite. Then $z^T J z \leq 0$ for any $z \in \mathbb{R}^n$. If $J + J^T$ is even negative*
661 *definite, then $z^T J z < 0$ for any $z \in \mathbb{R}^n \setminus \{0\}$.*

662 **Proof.** Since $z^T J z$ is a real number, $z^T J z = ((z^T J)z)^T = z^T (z^T J)^T =$
663 $z^T J^T (z^T)^T = z^T J^T z$. Therefore, $2z^T J z = z^T J z + z^T J^T z = z^T (J z + J^T z) =$
664 $z^T (J + J^T) z$. The two assertions are now immediate. \square

665 **References**

- 666 [1] Baik, K.H., Shogren, J.F., 1995. Contests with spying. *Eur. J. Polit.*
667 *Econ.*11, 441–451.
- 668 [2] Bolle, F., 1996. Contests with spying: A Comment. *Eur. J. Polit.*
669 *Econ.*12, 729–734.
- 670 [3] Brookins, P., Ryvkin, D., 2013. An experimental study of bidding
671 in contests of incomplete information. *Exp. Econ.* forthcoming: DOI
672 10.1007/s10683-013-9365-9.
- 673 [4] Clark, D.J., 1997. Learning the structure of a simple rent-seeking game.
674 *Public Choice* 93(1-2), 119–130.
- 675 [5] Corchón, L., 2007. The theory of contests: A survey. *Rev. Econ. Design*
676 11, 69–100.
- 677 [6] Cornes, R., Hartley, R., 2005. Asymmetric contests with general tech-
678 nologies. *Econ. Theory* 26(4), 923–946.
- 679 [7] Dasgupta, P., Maskin, E., 1986. The existence of equilibrium in discon-
680 tinuous economic games, I: Theory. *Rev. Econ. Stud.* 53, 1–26.
- 681 [8] Einy, E., Haimanko, O., Moreno, D., Sela, A., Shitovitz, B., 2013. Tul-
682 lock Contests with Asymmetric Information. Mimeo.
- 683 [9] Epstein, G.S., Mealem, Y., 2013. Who gains from information asymme-
684 try? *Theor. Decis.* 75, 305–337.

- 685 [10] Fey, M., 2008. Rent-seeking contests with incomplete information. Pub-
686 lic Choice 135(3), 225–236.
- 687 [11] Fu, Q., Jiao, Q., Lu, J., 2011. On disclosure policy in contests with
688 stochastic entry. Public Choice 148, 419–434.
- 689 [12] Fudenberg, D., Tirole, J., 1991, Game Theory, MIT Press.
- 690 [13] Goodman, J.C., 1980. A note on existence and uniqueness of equilibrium
691 points for concave n -person games. Econometrica 48(1), 251.
- 692 [14] Grosskopf, B., Rentschler, L., Sarin, R., 2010. Asymmetric information
693 in contests: theory and experiments. Working paper. Texas A&M Uni-
694 versity.
- 695 [15] Hurley, T.M., Shogren, J.F., 1998a. Effort levels in a Cournot Nash
696 contest with asymmetric information. J. Public Econ. 69(2), 195–210.
- 697 [16] Hurley, T.M., Shogren, J.F., 1998b. Asymmetric information contests.
698 Eur. J. Polit. Econ. 14(4), 645–665.
- 699 [17] Lim, W., Matros, A., 2009. Contests with a stochastic number of players.
700 Games Econ. Behav. 67(2), 584–597.
- 701 [18] Malueg, D.A., Yates, A.J., 2004. Rent seeking with private values. Public
702 Choice 119(1), 161–178.
- 703 [19] Münster, J., 2006. Contests with an unknown number of contestants.
704 Public Choice 129(3-4), 353–368.

- 705 [20] Münster, J., 2009. Repeated contests with asymmetric information. J.
706 Public Econ. Theory 11(1), 89–118.
- 707 [21] Myerson, R.B., Wärneryd, K., 2006. Population uncertainty in contests.
708 Econ. Theory 27(2), 469–474.
- 709 [22] Nti, K.O., 1999. Rent-seeking with asymmetric valuations. Public
710 Choice 98(3), 415–430.
- 711 [23] Pérez-Castrillo, J., Verdier, R.D., 1992. A general analysis of rent-
712 seeking games. Public Choice 73(3), 335–350.
- 713 [24] Rentschler, L., 2009. Incumbency in imperfectly discriminating contests.
714 Mimeo, Texas A&M University.
- 715 [25] Rockafellar, R.T., 1970, Convex Analysis, Princeton University Press.
- 716 [26] Rosen, J.B., 1965. Existence and uniqueness of equilibrium points for
717 concave n -person games. Econometrica 33(3), 520–534.
- 718 [27] Ryvkin, D., 2010. Contests with private costs: Beyond two players. Eur.
719 J. Polit. Econ. 26(4), 558–567.
- 720 [28] Schoonbeek, L., Winkel, B., 2006. Activity and inactivity in a rent-
721 seeking contest with private information. Public Choice 127(1-2), 123–
722 132.
- 723 [29] Sui, Y., 2009. Rent-seeking with private values and resale. Public Choice
724 138(3-4), 409–422.

- 725 [30] Szidarovszky, F., Okuguchi, K., 1997. On the existence and uniqueness
726 of pure Nash equilibrium in rent-seeking games. *Games Econ. Behav.*
727 18(1), 135–140.
- 728 [31] Tullock, G., 1980. Efficient rent-seeking, in: Buchanan, J.M., Tollison,
729 R.D., Tullock, G. (Eds.), *Toward a theory of the rent-seeking society.*
730 Texas A&M University Press, College Station, pp. 97–112.
- 731 [32] Wärneryd, K., 2003. Information in conflicts. *J. Econ. Theory* 110(1),
732 121–136.
- 733 [33] Wärneryd, K., 2012. Multi-player contests with asymmetric information.
734 *Econ. Theory* 51(2), 277–287.
- 735 [34] Wasser, C., 2013a. Incomplete information in rent-seeking contests.
736 *Econ. Theory* 53(1), 239–268.
- 737 [35] Wasser, C., 2013b. A note on Bayesian Nash equilibria in imper-
738 fectly discriminating contests. *Math. Soc. Sci.*, forthcoming: DOI
739 10.1016/j.mathsocsci.2013.03.001.
- 740 [36] Yamazaki, T., 2008. On the existence and uniqueness of pure-strategy
741 Nash equilibrium in asymmetric rent-seeking contests. *J. Public Econ.*
742 *Theory* 10(2), 317–327.
- 743 [37] Yamazaki, T., 2009. The uniqueness of pure-strategy Nash equilibrium
744 in rent-seeking games with risk-averse players. *Public Choice* 139, 335–
745 342.