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Assisted Reproductive Technologies (ART) in a Model of Fertility Choice

Helmut Rainer, Geethanjali Selvaretnam, David Ulph

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ABSTRACT

This paper provides a simple theoretical framework to discuss the relationship between assisted reproductive technologies and the microeconomics of fertility choice. Individuals make choices of education and work along with decisions about whether and when to have children. Decisions regarding fertility are influenced by policy and labor market factors that affect the earnings opportunities of mothers and the costs of raising children. We show how observed differences in these economic factors across countries explain observed different fertility and childbearing age patterns. We then use the model to predict behavioral responses to biomedical improvements in assisted reproductive technologies, and hence the impact of these technologies on fertility.
NON-TECHNICAL SUMMARY

In virtually every industrialized country the total fertility rate has been declining for more than four decades, and ever since the 1980s it has been below the 2.1 children per woman needed for a population to replace itself. Countries with sub-replacement fertility fall into two distinct groups. In countries where birth rates are moderately below replacement level, the population size falls only slowly and, if considered necessary, can be supplemented with migration. In contrast, countries where the fertility rate has fallen below 1.5 births per woman and has stayed below this threshold are said to be locked into a “low fertility trap”. The anticipated economic consequences for countries in the low fertility trap are manifold, from a shortage of skilled young workers to a population ageing-related slowdown in the growth of GDP to an increase in age-related public spending. Demographers have recognized that a small increase or stabilization of total fertility could help countries in or at the brink of the low fertility trap prevent these economic problems from taking effect. Traditionally, most debates on stabilizing fertility rates focused on policies such as flexible working, maternity and paternity leave, and increasing benefits for second and third children. However, policymakers have recently started to take note of the potential role in stabilizing fertility rates of assisted reproductive technologies.

This paper pursues two related objectives. The first half of the paper develops a simple model that allows us to explain the different reproductive patterns observed across industrialized and emerging countries. In the second half of the paper, we use the model to assess the biomedical and behavioral effects of improvements in assisted reproductive technologies, and hence the impact of these technologies on fertility rates. Our analysis shows several interesting results. First, we show that cross-country differences in (i) institutional factors that affect the earnings opportunities of mothers and (ii) social policies that affect the cost of raising children are sufficient to rationalize the different reproductive patterns observed across industrialized and emerging countries. The model tells us, for example, that the demographic distinctiveness of Southern European countries can largely be attributed to a comparatively high wage penalty associated with motherhood. Cross-country variations in the quality and availability of formal childcare, in contrast, generate a positive correlation between fertility rates and proportions of births to older women, and hence can explain the most distinctive features of the reproductive patterns in Continental and Northern Europe on the one side and Eastern Europe on the other. Having established this, we look at the issue of assisted reproductive technologies (ART). We show that improvements in ART should have the direct biomedical effect of improving fertility rates. However, they could also cause indirect changes in behavior which could offset the direct effect. In particular, the availability of improved ART could cause some women who would otherwise have tried to have children earlier on in life to postpone childbirth to later in life when, despite ART, the conception success probability is lower. We show that this negative behavioral effect may offset the positive biomedical effect, and so lead to a reduction of the fertility rate. Finally, we use our model to assess the effects of ART from a macroeconomic perspective. We show that the negative behavioral effect that induces people to postpone childbirth is most pronounced in economic environments that are conducive to high levels of fertility. This, in turn, allows us to demonstrate that improvements in ART can have a negative effect in high-fertility countries, a positive effect in low-fertility countries, and so lead to a convergence of fertility rates across countries.
Assisted Reproductive Technologies (ART) in a Model of Fertility Choice

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Abstract

This paper provides a simple theoretical framework to discuss the relationship between assisted reproductive technologies and the microeconomics of fertility choice. Individuals make choices of education and work along with decisions about whether and when to have children. Decisions regarding fertility are influenced by policy and labor market factors that affect the earnings opportunities of mothers and the costs of raising children. We show how observed differences in these economic factors across countries explain observed different fertility and childbearing age patterns. We then use the model to predict behavioral responses to biomedical improvements in assisted reproductive technologies, and hence the impact of these technologies on fertility.

Keywords: Fertility Choice, Assisted Reproductive Technologies.

JEL Classifications: D10, J13.
1. Introduction

In virtually every industrialized country the total fertility rate has been declining for more than four decades, and ever since the 1980s it has been below the 2.1 children per woman needed for a population to replace itself (see Figure 1). Countries with sub-replacement fertility fall into two distinct groups. In countries where birth rates are moderately below replacement level, the population size falls only slowly and, if considered necessary, can be supplemented with migration. In contrast, countries where the fertility rate has fallen below 1.5 births per woman and has stayed below this threshold are said to be locked into a “low fertility trap” (McDonald [2006]).

The notion of a low fertility trap captures the idea that once fertility falls below a certain threshold and stays there for a while it can lead to self-reinforcing mechanisms that are difficult to reverse. One such mechanism is a negative population momentum, that is, the tendency of a population to decline due to the small cohorts born since the 1980s entering their reproductive ages (Lutz et al. [2003]). Another possible mechanism is behavioral: if very low fertility is sustained for a long period of time, preferences can begin to shift away from childbearing, and a reversal of low fertility becomes much more difficult (Goldstein et al. [2003], Rindfuss et al. [2004]).

The anticipated economic consequences for countries in the low fertility trap are manifold, from a shortage of skilled young workers to a population ageing-related slowdown in the growth of GDP to an increase in age-related public spending (Grant et al. [2006]). Demographers have recognized that a small increase or stabilization of total fertility could help countries in or at the brink of the low fertility trap prevent these economic problems from taking effect. Traditionally, most debates on stabilizing fertility rates focused on policies such as flexible working, maternity and paternity leave, and increasing benefits for second and third children. However, policymakers have recently started to take note of the potential role in stabilizing fertility rates of assisted reproductive technologies.

Assisted reproductive technologies (ART) cover a range of biomedical procedures [e.g., in vitro fertilization (IVF), intracytoplasmic sperm injection (ICSI), gamete intrafallopian transfer (GIFT), zygote intrafallopian transfer (ZIFT)], all of which have the ultimate aim of assisting subfertile couples who are having difficulties conceiving to become pregnant and achieve the birth of a healthy child. Since natural female

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1 The replacement fertility rate of 2.1 children per woman includes two children to replace the parents plus one-tenth of child to replace offspring who do not reach the age of 15.
fertility falls gradually after the age of 30, with a rapid decline after the age of 35, it is predominantly older people reaching the end of their fertile lives who are sensitive to fertility techniques. From a practical viewpoint, ART is an area of rapid technological change, and currently accounts for 1 to 3 percent of annual births in developed countries (Gosden et al. [2003]).

While fertility rates depend on biomedical factors that affect the ability to conceive, they are also heavily influenced by behavioral factors, particularly the interrelated decisions that are made about both the number of children to have and the age at which to try to have children. This is reflected in Figure 1 which shows that

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2Indeed, evidence suggests that in the age-range 35-39 (respectively, in the age range 40-44) it is almost 20 percent (respectively, 50 percent) less likely for a woman to conceive compared to women aged 25-29 years ceteris paribus (De la Rochebrochard [2001]). Male fecundity, in contrast, is at its highest between the ages of 30 and 34, after which it decreases, although not as quick as for women.
(i) in countries where fertility rates are below the replacement rate but above the low fertility trap, births to women aged 30 years or older account for more than 40 percent of the total fertility rate, implying that the mean age of all mothers who give birth (in a given year) is relatively high; (ii) while some countries in the low fertility trap are associated with a relatively high average age at which women give birth, in others women give birth at quite a young age. So while biomedical factors would have suggested a negative correlation between fertility and the average age at which women give birth, Figure 1 suggests a positive correlation.

Figure 1 also suggests that there are likely to be powerful socio-economic factors driving observed reproductive behavior since, broadly speaking, those countries that display both low fertility rates and low proportions of births to older women are mainly Eastern European (the “East”); those with very low fertility rates but high proportions of births to older women are mainly Southern European (the “South”), whereas those where both fertility rates and proportions of births to older women are high are largely Anglo-Saxon, Continental European, and Nordic countries (the “North”). Moreover, these groups of countries can be distinguished not only in terms of reproductive behavior, but also in terms of institutional factors that affect the labor cost of having children and the quality and availability of formal childcare.

Against this empirical background, this paper pursues two related objectives. The first half of the paper develops a simple model that allows us to explain the different reproductive patterns observed across industrialized and emerging countries. In the second half of the paper, we use the model to assess the biomedical and behavioral effects of improvements in assisted reproductive technologies, and hence the impact of these technologies on fertility rates.3

The proposed model is one in which women make decisions about education, work, and whether and when to have children. When assessing the “cost side” of having children, women take into account the impact of fertility choices on earnings and career opportunities. First, there is the direct loss of earnings through not working

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3The current paper therefore fits into the theoretical literature on the economic analysis of fertility. Earlier work in this literature has focused on the study of the total number of children born to a couple. This work includes the theory of the quantity-quality tradeoff developed by Gary Becker and his associates (Becker [1960], Becker and Lewis [1973], and Becker and Tomes [1976]). More recent theoretical studies (Cigno [1983], Ermisch and Cigno [1989]) have looked at the distribution of births over a woman’s lifetime. To our knowledge, the present paper contains the fullest analysis so far of (i) the economic determinants of differences in reproductive behavior across industrialized and emerging countries, and (ii) the biomedical environment within which women make fertility choices.
during the periods of child-rearing, the size of which may depend on the quality and availability of formal childcare. Second, there is the long-term impact that early career interruptions in order to have children can have on future earnings, which may manifest itself in a wage penalty associated with motherhood. On the “benefit side” of having children, the utility of parenthood is linked to basic aspects of preferences, which may reflect individual attitudes, social norms, and cultural climates.

The key question is, then, how cross-country differences in these various cost and benefit factors determine not just the observed pattern of behavior in Figure 1, but also the response of women in different countries to the availability of new ART. Our answer to this question is derived in three steps.

First, we show that cross-country differences in (i) institutional factors that affect the earnings opportunities of mothers and (ii) social policies that affect the cost of raising children are sufficient to rationalize the different reproductive patterns observed in Figure 1. The model tells us, for example, that the demographic distinctiveness of the “South” can largely be attributed to a comparatively high wage penalty associated with motherhood. Cross-country variations in the quality and availability of formal childcare, in contrast, generate a positive correlation between fertility rates and proportions of births to older women, and hence can explain the most distinctive features of the reproductive patterns in the “North” and the “East”, respectively.

Having established this, we look at the issue of assisted reproductive technologies (ART). We show that improvements in ART should have the direct biomedical effect of improving fertility rates. However, they could also cause indirect changes in behavior which could offset the direct effect. In particular, the availability of improved ART could cause some women who would otherwise have tried to have children earlier on in life to postpone childbirth to later in life when, despite ART, the conception success probability is lower. We show that this negative behavioral effect may offset the positive biomedical effect, and so lead to a reduction of the fertility rate.

Finally, we use our model to assess the effects of ART from a macroeconomic

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4 A large body of evidence suggests that mother’s earnings suffer from a “family gap in pay” which refers to the differential in wages between women with and without children (see, e.g., Waldfogel [1998]). There are a number of channels through which such a family gap can arise: a reduction in labor market experience; the need to take part-time jobs or, more generally, reduced flexibility in hours and out-of-hours networking opportunities, which can affect promotion prospects.

5 As will become apparent later, by “sufficient” we mean that our model does not need to appeal to systematic cross-country differences in preferences to explain existing reproductive behavior.

6 As we shall demonstrate below, these predictions accord well with evidence from cross-country data.
perspective. We show that the negative behavioral effect that induces people to postpone childbirth is most pronounced in economic environments that are conducive to high levels of fertility. This, in turn, allows us to demonstrate that improvements in ART can have a negative effect in high-fertility countries, a positive effect in low-fertility countries, and so lead to a convergence of fertility rates across countries.

The organization of the remainder of the paper is as follows. In Section 2 we present the basic model and provide an analysis of individual choices regarding education, work and fertility. Section 3 derives a number of comparative statics results on the economic determinants of fertility and age patterns of childbearing. The results are used to explain differences in reproductive behavior across countries. In Section 4 we assess the impact of assisted reproductive technologies on fertility rates and spell out the implications from a macroeconomic perspective. Section 5 concludes.

2. The Model

The model we propose is one of individual decision-making, with women acting alone to make decisions about education, work, the number of children to have and when to have them. An obvious important direction for future research is to extend this to a family bargaining framework.

2.1. The Environment. The model has two time periods, 1 and 2, and a continuum of individuals. A key feature of the model is that individuals have knowledge of their decreasing biological fertility, in the sense that they know that their ability to conceive is higher in period 1 than in period 2. Thus, period 1 of our model sees younger people with a relatively high natural fecundity, while period has older people facing the end of their fertile lives. We assume that each individual wants at most one child. At the start of each period an individual can, if she wishes, make a single attempt to have a child. The probability that the attempt to have a child in period \( t = 1, 2 \) will be successful is \( p_t \), where \( 0 \leq p_t \leq 1 \). The success of the attempt to have a child is known at the start of each period before a woman needs to make any other decisions. Older individuals reaching the end of their fertile lives are sensitive to technological alternatives to conceiving naturally. Below, we shall therefore assume that the conception success probability of older individuals, \( p_2 \), can be increased by providing more or better ART treatments. We assume that the age-specific conception success probabilities are such that \( 0 \leq p_2 < p_1 \leq 1 \). This assumption captures the fact that a woman’s biological fertility starts to drop sharply over the age of 35,
and older women find it harder to conceive either naturally or with the help of ART. The parameters $p_1$ and $p_2$ capture the bio-medical features of the environment within which women make their decisions.

All education takes place in period 1. Let $e$, $0 \leq e \leq 1$, be the fraction of the first period spent in education, and let $w(e)$ be the wage rate that an individual gets as a result of education, where $w'(\cdot) > 0$ and $w''(\cdot) \leq 0$. The function $w(\cdot)$ captures the educational opportunities open to women.

Turning to the payoff consequences of parenthood, it is well understood that women’s earnings can suffer from a “price of motherhood” or “family gap in pay”, which refers to the differential in wages between women with and without children. After controlling for differences in characteristics such as education and work experience, researchers typically find a family penalty of 10-15 percent for women with children as compared to women without children (Korenman and Neumark [1992], Waldfogel [1997], Waldfogel [1998], Davies and Pierre [2005], Dupuy and Fernández-Kranz [2007]). However this family gap varies significantly across countries. Moreover there is evidence that this gap is larger for women who have children early than for women who have children later in life. Letting $\epsilon$, $0 < \epsilon < 1$, denote the family gap in pay, we capture the presence of such a family gap by assuming that if a woman has a child in period 1 her earnings in period 2 are a fraction $1 - \epsilon$ of those she would have obtained had she been childless.

There is no leisure so that the only activities to which time is ever allocated are education, child care, and work. We assume that if an individual has a child in any period, then she has to spend a fraction $c$, $0 < c < 1$, of that period in child care. The parameter $c$ captures the quality and availability of formal childcare. For example, the availability of pre-school education and childcare in nurseries would reduce the fraction $c$ of any period that a mother has to spend in childcare. Just as the family gap in pay, the supply of formal childcare also varies considerably across countries. For example, amongst industrialized and emerging countries the enrolment rate of children aged 0-3 ranges from 6 to 60 percent (OECD [2007b]). Below, we shall interpret the two parameters $c$ and $\epsilon$ as representing the direct and indirect costs of children, resulting from social policies that affect the cost of raising children and labor market factors that affect the earnings opportunities of mothers.

Finally, we assume that individuals directly derive utility from having a child. Let $\gamma_t$ be the perceived present value of utility that an individual will get if she succeeds in having a child in period $t$. Several factors may contribute to determine
the value of children. For example, the social-demographic literature (Friedman et al. [1994]) emphasizes the importance of individual attitudes, social norms, and cultural climates in affecting the value attached not just whether or not to have children but also when in life to have them.

To simplify the exposition, the following assumption on the parameters will be maintained throughout the paper.

**Assumption 1** \[ \gamma_1 > \gamma_2. \]

As will become apparent later, this assumption rules out trivial situations in which the equilibrium does not involve any individuals trying to have a child when young.

### 2.2. Analysis of Individual Decisions.

Recalling that an individual will know the outcome of a decision to have a child in any period right at the start of that period, the individual has to choose three things: (1) whether or not to try to have a child in period 1; (2) conditioning on the outcome of this decision the amount of education to have in period 1; (3) conditioning on the outcome of the first decision, whether or not to try to have a child in period 2. All these decisions are made at the start of period 1. This captures the complex interactions between education decisions and decisions about whether and when to have children. Future utility levels are discounted with discount factor \( \rho \) per period.

In order to make the first decision, the individual has to determine how the second and third decisions will be made conditional on the outcome of this first decision. Thus, let us suppose first of all that an individual does not give birth to a child in period 1. This can happen either because they chose not to try or because they tried and failed. They then have to decide whether or not they will try to have a child at the start of period 2 and the amount of education to have in period 1.

Suppose an individual chooses not to try to have a child in period 2. Then

\[ V_{00}(e) = (1 - e)w(e) + \rho w(e) \]

gives the present value of utility of an individual who has no child in period 1, chooses not to have a child in period 2, and has an amount of education \( e \), where \( 0 \leq e \leq 1 \). Let

\[ \hat{V}_{00} = \max_{e \geq 0} V_{00}(e) \quad \text{and} \quad \hat{e}_{00} = \arg \max_{e \geq 0} V_{00}(e). \]

Then \( \hat{V}_{00} \) gives the utility an individual who is childless at the start of period 1 will get if she chooses not to have a child in period 2, and \( \hat{e}_{00} \) is the amount of education...
that such an individual will choose. In what follows it will be useful to let

\[
\hat{V}(z) = \max_{e \geq 0} (z - e)w(e) \quad \text{and} \quad \hat{e}(z) = \arg \max_{e \geq 0} (z - e)w(e).
\]

The first-order condition for \( e \) is

\[
(z - e)w'(e) = w(e).
\]

It is easy to see from (1) that \( \hat{e}'(z) > 0 \), and, from the envelope theorem, that \( \hat{V}'(z) = w[\hat{e}(z)] > 0 \). Furthermore, \( \hat{V}''(z) = w'[\hat{e}(z)] \hat{e}'(z) > 0 \). It now follows immediately that

\[
\hat{V}_{00} \equiv \hat{V}(1 + \rho) \quad \text{and} \quad \hat{e}_{00} \equiv \hat{e}(1 + \rho).
\]

Next suppose that an individual who does not have a child at the start of period 1 chooses to try to have a child at the start of period 2. Then

\[
V_{01}(e) = (1 - e)w(e) + \rho[p_2(1 - c)w(e) + (1 - p_2)w(e)] + p_2\gamma_2
\]

gives the expected present value of utility of such an individual if she has an amount of education \( e \), where \( 0 \leq e \leq 1 \). Let

\[
\hat{V}_{01} = \max_{e \geq 0} V_{01}(e) \quad \text{and} \quad \hat{e}_{01} = \arg \max_{e \geq 0} V_{01}(e).
\]

It then follows immediately that

\[
\hat{V}_{01} \equiv \hat{V}[1 + \rho(1 - cp_2)] + p_2\gamma_2 \quad \text{and} \quad \hat{e}_{01} \equiv \hat{e}[1 + \rho(1 - cp_2)].
\]

Notice that, if \( p_2 = 0 \), then \( \hat{V}_{01} = \hat{V}_{00} \) and \( \hat{e}_{01} = \hat{e}_{00} \), whereas if \( p_2 > 0 \) then \( \hat{e}_{01} < \hat{e}_{00} \). Ignoring things that happen on sets of measure zero, we can take it that an individual who is childless in period 1 will choose to try to have a child in period 2 if and only if \( \hat{V}_{01} > \hat{V}_{00} \), that is, if and only if

\[
\hat{V}[1 + \rho(1 - cp_2)] + p_2\gamma_2 > \hat{V}(1 + \rho).
\]

It now follows immediately that

\[
\hat{V}_0 = \max\{\hat{V}_{00}, \hat{V}_{01}\}
\]

is the (maximum) utility an individual will get if she does not have a child in period 1, taking account of the decisions that she will subsequently make.
Now consider an individual who succeeds in having a child at the start of period 1. She will not try to have a child in period 2 so, if she has an amount of education \( e \), \( 0 \leq e \leq 1 - c \), then her present value of utility will be

\[
V_1(e) = (1 - c - e)w(e) + \rho(1 - e)w(e) + \gamma_1.
\]

As before, let

\[
\hat{V}_1 = \max_{e \geq 0} V_1(e) \quad \text{and} \quad \hat{e}_1 = \arg \max_{e \geq 0} V_1(e).
\]

It then follows that

\[
\hat{V}_1 \equiv \hat{V}[1 + \rho(1 - \epsilon) - c] + \gamma_1 \quad \text{and} \quad \hat{e}_1 \equiv \hat{e}[1 + \rho(1 - \epsilon) - c].
\]

Finally, consider the decision as to whether or not to try to have a child in period 1. If an individual chooses not to try, then utility will be \( \hat{V}_0 \). If instead the individual chooses to try, the expected utility will be \( p_1\hat{V}_1 + (1 - p_1)\hat{V}_0 \). Therefore, an individual will try to have a child if and only if \( p_1\hat{V}_1 + (1 - p_1)\hat{V}_0 > \hat{V}_0 \), that is, if and only if

\[
\hat{V}_1 > \hat{V}_0
\]

which is independent of \( p_1 \).

Notice that \( 1 + \rho(1 - \epsilon) - c < 1 + \rho(1 - p_2c) < 1 + \rho \), and so

\[
\hat{e}_1 < \hat{e}_{01} < \hat{e}_{00}
\]

and

\[
\hat{V}[1 + \rho(1 - \epsilon) - c] < \hat{V}[(1 + \rho(1 - p_2c)] < \hat{V}(1 + \rho).
\]

### 2.3. Population Heterogeneity.

So far we have looked at a typical generic individual. Consider now a more explicit recognition of population heterogeneity by replacing the wage function \( w(e) \) by the function \( \omega(a, e) \equiv aw(e) \) where \( a > 0 \) is a parameter measuring ability. We assume hereafter that the \( a \)'s are distributed according to a probability density function \( f(a) \), with support \( (0, \infty) \) and associated cumulative distribution function \( F(a) \).

Notice that, from (1), education choices are independent of \( a \) and depend solely on childbirth outcomes: a person who is childless in period 1 and plans to remain childless will undertake more education than someone who is childless but plans to try for children in period 2, who in turn will undertake more education than someone who has given birth in period 1. So we have:

\[
\hat{V}_{00}(a) = a\hat{V}(1 + \rho)
\]
\[
\hat{V}_{01}(a) = a\hat{V}[1 + \rho(1 - p_2c)] + p_2\gamma_2
\]
\[
\hat{V}_0(a) = \max\{\hat{V}_{00}(a), \hat{V}_{01}(a)\}
\]
\[
\hat{V}_1(a) = a\hat{V}[1 + \rho(1 - \varepsilon) - c] + \gamma_1
\]

We now want to understand what decisions various types of women will make. To do this we define the following three variables. First, let \(a_0\) be the point of intersection between \(V_{00}(a)\) and \(V_{01}(a)\). From (4) and (5) we see that

\[
a_0 \equiv \frac{p_2\gamma_2}{\hat{V}(1 + \rho) - \hat{V}[1 + \rho(1 - p_2c)]}.
\]

Second, denoting the point of intersection between \(\hat{V}_1(a)\) and \(\hat{V}_{00}(a)\) by \(\tilde{a}\), then, from (4) and (7), we have

\[
\tilde{a} \equiv \frac{\gamma_1}{\hat{V}(1 + \rho) - \hat{V}[1 + \rho(1 - \varepsilon) - c]}.
\]

Notice that \(\tilde{a}\) does not depend on \(p_2\). Finally, if we denote the point of intersection between \(\hat{V}_1(a)\) and \(\hat{V}_{01}(a)\) by \(a_1\), then from (5) and (7) we have:

\[
a_1 \equiv \frac{\gamma_1 - p_2\gamma_2}{\hat{V}[1 + \rho(1 - p_2c)] - \hat{V}[1 + \rho(1 - \varepsilon) - c]}.
\]

We now note the following result:

Lemma 1 \(a_1 \geq \tilde{a}\) if and only if \(\tilde{a} \geq a_0\).

The proof of this result is straightforward and hence omitted. Building on this lemma, it is easy to establish:

Proposition 1 There are only two possible modes of individual decision-making:

Case “Early Childbearing”: If \(a_0 < \tilde{a} < a_1\), then the population can be divided into three subgroups, according to the ability parameter \(a\).

a. Individuals in the low ability range \([0, a_0]\) will try to have a child in period 1 and, if they fail, will try again in period 2.

b. Individuals in the medium ability range \([a_0, \tilde{a}]\) will also try to have a child in period 1, but, if they fail, will not try again in period 2.

c. Individuals in the high ability range \([\tilde{a}, \infty)\) will not try to have a child in either period 1 or period 2 and so will remain childless out of choice.
Figure 2: Two Modes of Individual Decision-Making.

Case “Postponed Childbearing”: If $a_1 < \tilde{a} < a_0$, then once again the population can be divided into three groups.

a. Individuals in the lowest ability range $[0, a_1]$ will try to have a child in period 1 and, if they fail will try again in period 2.

b. Individuals in the medium ability range $[a_1, a_0]$ will not try to have a child in period 1, but will postpone the attempt to have a child to period 2.

c. Individuals in the high ability range $[a_0, \infty)$ will not try to have a child in either period 1 or period 2 and so will be voluntarily childless.

The result is illustrated in Figure 2. Notice that the decisions of low ability women and high ability women are identical in the two cases: low ability women try to have children in both periods while high ability women try in neither; women with middle ability try to have a child in just one period. In the “Early Childbearing” case this is period 1, whereas in the “Postponed Childbearing” case this is in period 2.

From the definitions and results we have established above, it is also straightforward to determine the conditions under which a given society will fall into the “Early Childbearing” or the “Postponed Childbearing” scenario:
Lemma 2 Define

$$\Delta \equiv p_2 \left[ \frac{\gamma_2}{\gamma_1} \cdot \frac{\hat{V}(1 + \rho) - \hat{V}(1 + \rho(1 - \epsilon) - c)}{\hat{V}(1 + \rho) - \hat{V}(1 + \rho(1 - p_2 c))} \right]$$

(11)

a. If $\Delta \leq 1$, then $a_0 \leq \tilde{a} \leq a_1$ (Case “Early Childbearing”).

b. If $\Delta > 1$, then $a_1 < \tilde{a} < a_0$ (Case “Postponed Childbearing”).

Notice that if we take a linear approximation in both the numerator and the denominator of (11) then we can re-write the expression for $\Delta$ as

$$\Delta \simeq \left[ \frac{\gamma_2}{\rho \gamma_1} \right] \cdot \left[ 1 + \frac{\rho \epsilon}{c} \right].$$

(12)

On the one hand, if we fix all parameters other than $c$, then there exists a $c^* \in [0, 1]$ such that $\Delta > 1$ if and only if $c < c^*$. This means that case “Postponed Childbearing” is more likely to arise if the direct costs of childbearing, $c$, are relatively low ceteris paribus. The intuition runs as follows. While giving up time to raise children causes a loss of earned income whenever children are born, because women who have children in period 1 invest less in education, the income loss is smaller for early childbirth than for late childbirth. On the other hand, if we fix all parameters other than $\epsilon$, then there exists a $\epsilon^* \in [0, 1]$ such that $\Delta > 1$ if and only if $\epsilon > \epsilon^*$. So case “Postponed Childbearing” is more likely to emerge if the labor costs of childbearing, $\epsilon$, are relatively high ceteris paribus. Intuitively, if women anticipate that a significant family gap in pay kicks in if they have children early this tends to push them towards postponing childbearing. Finally, turning to the values attached to childbearing at different ages as captured by $\gamma_1$ and $\gamma_2$, a relatively high value attached to having children at older ages pushes a society towards the postponed childbearing regime.

3. Explaining Fertility and Age Patterns of Childbearing

The first main benefit of our model is that it provides a clear-cut way for thinking about the economic determinants of observed fertility and childbearing age patterns. However, before we can proceed, we need to first determine two society level variables – the fertility rate and the incidence of childbearing at older ages – that arise from the individual decisions characterized above.
3.1. Fertility Rate and Incidence of Childbearing at Older Ages. The fertility rate, $Φ$, is the population-weighted number of births. A natural measure for the age pattern of childbearing is the incidence of childbearing at older ages, i.e., the proportion of births that occur in period 2 - which we denote by $Ψ$. Applying these measures to the two cases identified above, it is readily checked that when a society is characterized by “Early Childbearing”, then the fertility rate is

$$Φ = F(\bar{a})p_1 + F(a_0)(1 - p_1)p_2,$$

while the incidence of childbearing at old ages is

$$Ψ = \frac{F(a_0)[1 - p_1]p_2}{F(\bar{a})p_1 + F(a_0)(1 - p_1)p_2}.$$  

In a society characterized by “Postponed Childbearing” the fertility rate is

$$Φ = F(a_1)p_1 + [F(a_0) - p_1F(a_1)]p_2,$$

while the incidence of childbearing at old ages is

$$Ψ = \frac{[F(a_0) - p_1F(a_1)]p_2}{F(a_1)p_1 + [F(a_0) - p_1F(a_1)]p_2}.$$  

3.2. Comparative Statics. Building on these measures, in this subsection we perform comparative statics and unveil factors that help explain existing reproductive patterns. The two key economic parameters in our analysis are $c$ and $\epsilon$. As we noted earlier, the parameter $c$ could be thought of as a measure of the quality and availability of formal child care: a well functioning child care system reduces the fraction $c$ of any period that a parent has to spend in child care. The parameter $\epsilon$ is a measure of the wage penalty associated with entering motherhood at a young age. As a key result, we establish that the behavior observed in Figure 1 arises endogenously from our model when comparative static results on $c$ and $\epsilon$ are combined.

It is useful to conduct our analysis in elasticity form. One particular elasticity that turns out to be important is the elasticity of the cumulative distribution function $F(a)$ with respect to $a$. We will need to know how this elasticity varies at different points on the ability distribution. For ease of notation, we will denote this particular elasticity by $g(a) \equiv \frac{af(a)}{F'(a)}$. Most commonly-used distributions that have closed-form cumulative distribution functions have the property that $g(a) \equiv \frac{af(a)}{F'(a)}$ is decreasing.
We therefore assume:

**Assumption 2** $g'(a) < 0$.

Lastly, we denote by $\eta_x \equiv \frac{\partial x}{\partial y} \cdot \frac{y}{x}$ the elasticity of some variable $y$ with respect to another variable $x$. Now, for any $x \in \{c, \epsilon\}$, it is straightforward to see from (13) and (14) that in the case of “Early Childbearing”:

$$
\eta_x^\Phi = (1 - \Psi)g(\tilde{a})\eta_x^\tilde{a} + \Psi g(a_0)\eta_x^{a_0} \tag{17}
$$

and

$$
\eta_x^\Psi = (1 - \Psi)\left[g(\tilde{a}) (\eta_x^\tilde{a}) - g(a_0) (\eta_x^{a_0})\right]. \tag{18}
$$

It is also readily checked from (15) and (16) that in the “Postponed Childbearing” case:

$$
\eta_x^\Phi = [1 - (1 - p_2)(1 - \Psi)] g(a_0)\eta_x^{a_0} + [(1 - p_2)(1 - \Psi)] g(a_1)\eta_x^{a_1} \tag{19}
$$

and

$$
\eta_x^\Psi = (1 - \Psi)\frac{F(a_0)}{F(a_0) - p_1 F(a_1)} \left[g(a_1)(\eta_x^{a_1}) - g(a_0)(\eta_x^{a_0})\right]. \tag{20}
$$

Notice that the sign of each of these elasticities depends solely on the effect of $x \in \{c, \epsilon\}$ on either $a_0$ and $\tilde{a}$ (“Early Childbearing”) or on $a_0$ and $a_1$ (“Postponed Childbearing”). For simplicity and ease of exposition, let us use linear approximations to rewrite the ability thresholds $(a_0, a_1, \tilde{a})$ as:

$$
a_0 \approx \frac{\gamma_2}{\hat{V}'(1 + \rho)c} \tag{21}
$$

$$
a_1 \approx \frac{\gamma_1 - p_2 \gamma_2}{\hat{V}'(1 + \rho)[\rho c + c(1 - \rho p_2)]} \tag{22}
$$

---

7For example, distributions such as the exponential distribution, the Pareto distribution, the Weibull distribution, and the arc-sine distribution all have the property that $g(a)$ is decreasing in $a$. For distributions that lack a closed-form representation for the c.d.f. and for the function $g(a)$, a necessary condition for Assumption 2 to be satisfied is that the function $\ln F(a)$ is concave. As shown by Bergstrom and Bagnoli [2005], this condition is satisfied by the family of probability distributions that have log-concave density functions, which includes the normal distribution, the gamma distribution, the chi-squared distribution, and the beta distribution.

8We use these linear approximations purely to simplify the exposition of some of our results, but, it should be emphasized, without affecting our main insights. Indeed, it can be verified that all comparative static results in this subsection are robust to using the non-linearized expressions of $(a_0, a_1, \tilde{a})$. 
Using these linear approximations, it follows immediately that:

\[
\eta^a_{c^0} = -1, \quad \eta^a_{\epsilon^0} = 0, \\
\eta^a_{c^1} = -\frac{c(1 - \rho p_2)}{\rho \epsilon + c(1 - \rho p_2)}, \quad \eta^a_{\epsilon^1} = -\frac{\rho \epsilon}{\rho \epsilon + c(1 - \rho p_2)}, \\
\eta^{\tilde{a}}_{c} = -\frac{c}{\rho \epsilon + c}, \quad \eta^{\tilde{a}}_{\epsilon} = -\frac{\rho \epsilon}{\rho \epsilon + c}.
\]  

(24)

It is now straightforward to establish:

**Proposition 2** Assume that Assumption 2 is satisfied. Then the following comparative static results are obtained:

In the “Early Childbearing” regime \((\Delta \leq 1)\), the fertility rate, \(\Phi\), is strictly decreasing in both \(\epsilon\) and \(c\). The proportion of births to older women, \(\Psi\), is strictly increasing in \(\epsilon\) and strictly decreasing in \(c\).

In the “Postponed Childbearing” regime \((\Delta > 1)\), the fertility rate, \(\Phi\), is strictly decreasing in both \(\epsilon\) and \(c\). The proportion of births to older women, \(\Psi_p\), is strictly increasing in \(\epsilon\). Moreover, \(\Psi\) is strictly decreasing in \(c\) provided \(-\eta^a_{c^1} g(a_1) < -\eta^a_{c^0} g(a_0)\).

Notice that the comparative static results are similar across the two regimes. However, there are no clear-cut qualitative predictions about the impact of \(c\) on \(\Psi\) in the “Postponed Childbearing” case. Indeed, since in this case \(a_0 > a_1\), it follows from Assumption 1 that \(g(a_0) < g(a_1)\), while from (24) it can be readily seen that \(-\eta^a_{c^0} > -\eta^a_{c^1}\). Thus all we can say is that close to the boundary of the “Postponed Childbearing” regime – where \(a_0 \approx a_1\) – it will certainly be true that the proportion of births to older women is decreasing in \(c\). However it is not clear what can be said further away from the boundary. In what follows we will simply impose:

**Assumption 3** Assume that in the “Postponed Childbearing” scenario the proportion of births to older women is a decreasing function of \(c\).

There are a number of justifications for such an assumption. First, the assumption is consistent with the fact that the larger is \(c\) the less likely it is that a society will be in the “Postponed Childbearing” regime. Second, the assumption is also consistent with the within-regime comparative static prediction for the “Early Childbearing” case. Finally, having a negative relation between \(\Psi\) and \(c\) generates the observed positive correlation between fertility and the proportion of older women giving birth (more about this below).
3.3. Economic Determinants of Differences in Reproductive Behavior. An interesting practical implication of our model arises when the comparative static results on \( c \) and \( \epsilon \) are combined. The following proposition does not appeal to any particular regime and hence applies to both the “Early Childbearing” and the “Postponed Childbearing” case:

**Proposition 3** Suppose we compare three countries (“North”, “South”, “East”) that are otherwise identical but differ in the quality and availability of formal childcare \( (c) \) and the family gap in pay \( (\epsilon) \). Suppose we observe that in the “North” both the fertility rate and the average age of giving birth are high \( (\Phi^H, \Psi^H) \); in the “South” the fertility rate is low but the average age of giving birth is high \( (\Phi^L, \Psi^H) \); and in the “East” both the fertility rate and the average age of giving birth are low \( (\Phi^L, \Psi^L) \). Then:

a. \( c^{north} < c^{south} < c^{east} \);

b. \( \epsilon^{east} < c^{north} < \epsilon^{south} \).

Figure 3 illustrates the result. Define an iso-fertility curve (respectively, iso-age curve) to be a line in \([0,1]^2\) on which all \((c, \epsilon)\)-combinations generate the same fertility \( \Phi \) (respectively, the same average age of giving birth \( \Psi \)). Notice that as we let \((\Phi, \Psi)\) vary we get a whole family of iso-fertility and iso-age curves. From the results we have
established in the previous subsection, and from a straightforward application of the implicit function theorem, it is easy to see that iso-fertility curves (respectively, iso-age curves) must have a negative slope (respectively, positive slope), and that lower iso-fertility curves (respectively, higher iso-age curves) are associated with higher fertility (respectively, higher average age of giving birth). Now let \( i, j \in \{ \text{Low, High} \} \). It then follows immediately that any observed \((\Phi^i, \Psi^j)\) can be explained by the \((c, \epsilon)\)-pair at which the iso-fertility curve associated with fertility \(\Phi^i\) intersects the iso-age curve associated with average age of giving birth \(\Psi^j\).

The above result is very important for understanding the evidence reported in Figure 1, which shows that amongst countries with similar very low fertility rates (the “East” and “South”) there is a considerable variation in the proportion of births to older women, while amongst countries with similar high proportion of births to older women (the “South” and the “North”), there is a considerable variation in fertility rates. Two points should be emphasized in this regard. First, it is interesting to note that variations in \(c\) alone are potentially sufficient to explain the most distinctive features of the reproductive patterns in the “East” on the one side and in the “North” on the other. Indeed if we compare two countries that are otherwise identical but one has well established formal daycare for younger children (a low \(c\)) while the other has low public spending on childcare (a high \(c\)), then the latter country will have a lower total fertility rate and childbearing will predominantly occur at younger ages, while the former country will have a higher total fertility rate and higher proportions of childbearing at older ages. Stated differently, cross-country variations in the quality and availability of formal childcare generate a positive correlation between fertility and the incidence of childbearing at older ages, as observed in Figure 1. Moreover, while only suggestive, these results are consistent with some more detailed facts gleaned from cross-sectional data (Table 1). For, compared to the countries of Eastern Europe, public expenditure on childcare is much larger in the countries of Northern and Western Europe.\(^9\) Just as public expenditure on childcare differs greatly between countries, enrolment in daycare for children aged 0-3 also varies significantly, with most of the countries of Eastern Europe displaying the lowest enrolment rates. To the extent that the Eastern European countries display very low total fertility rates (TFRs) and proportions of births to older women (PBOs), and to the extent that Northern and Western European countries have relatively high TFRs and PBOs, our

\(^9\)Public expenditure on childcare is all public financial support for families participating in formal day-care services (e.g. day care centers and family care for children under the age of 3).
Table 1: Quality and Availability of Childcare in Industrialized and Emerging Countries

<table>
<thead>
<tr>
<th>Region</th>
<th>Public Expenditure on Childcare (% of GDP)</th>
<th>Enrolment Rate of Children Aged 0-3 in Daycare (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northern Europe:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Denmark</td>
<td>1.0</td>
<td>61.7</td>
</tr>
<tr>
<td>Finland</td>
<td>1.0</td>
<td>35.0</td>
</tr>
<tr>
<td>Norway</td>
<td>0.7</td>
<td>43.7</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.8</td>
<td>39.5</td>
</tr>
<tr>
<td>Western Europe:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>0.5</td>
<td>26.0</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.2</td>
<td>38.5</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.2</td>
<td>25.8</td>
</tr>
<tr>
<td>Germany</td>
<td>0.0</td>
<td>9.0</td>
</tr>
<tr>
<td>Eastern Europe:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poland</td>
<td>0.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>0.1</td>
<td>3.0</td>
</tr>
<tr>
<td>Hungary</td>
<td>0.1</td>
<td>6.9</td>
</tr>
<tr>
<td>Slovak Republic</td>
<td>0.1</td>
<td>17.7</td>
</tr>
<tr>
<td>Southern Europe:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>0.1</td>
<td>6.3</td>
</tr>
<tr>
<td>Spain</td>
<td>0.1</td>
<td>20.7</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.4</td>
<td>23.5</td>
</tr>
<tr>
<td>Greece</td>
<td>0.2</td>
<td>7.0</td>
</tr>
</tbody>
</table>

1Source: OECD [2007a]. 2Source: OECD [2007b].

model is consistent with this evidence.

Second, while variations in \( c \) can explain the most distinctive features of the reproductive patterns in the “North” and the “East” respectively, they are not sufficient to account for the demographic distinctiveness of Southern European countries, where very low TFRs (as in the “East”) go hand in hand with high PBOs (as in the “North”). To account for this distinctiveness, we need to recognize that countries vary simultaneously in more than one economic factor. Our theory tells us that the distinctive reproductive patterns in the “South” can be explained by a comparatively high “price of motherhood” or “family gap in pay”, as captured by a high value of \( \epsilon \). Several studies have recently looked at variations in the family gap across Europe (Davies and Pierre [2005], Dupuy and Fernández-Kranz [2007]). These studies find that the penalty in pay associated with motherhood is at its largest in the countries of Southern Europe. For example, Dupuy and Fernández-Kranz [2007] estimate that mothers in Southern Europe suffer a wage penalty up to two times as large as mothers in Eastern European and Nordic countries (see Table 2). To the extent that the countries of Southern Europe display a systematic pattern of lowest-low TFRs combined with high PBOs, our model is consistent with this estimate.

10See Waldfogel [1998] for a comprehensive survey.
3.4. An Example. We have already seen that our model provides a simple theory of differences in fertility and age patterns of childbearing, and of some of their main economic correlates. To illustrate this in more detail, we now take as a reference point the “Postponded Childbearing” regime and consider a simple numerical example. Let the wage that an individual receives be given by $w(e) = e$. Assume that ability $a$ is distributed according to the Pareto distribution $F(a) = 1 - a^{-\beta}$. Empirical evidence suggests that a value of $\beta$ between 1.5 and 2 fits the mid-upper range of most income distributions. For example, a value of 1.688 has been estimated on IRS data for the United States. Thus, assume that $\beta = 2$. To complete the parametric setup, let $p_1 = 0.95$, $p_2 = 0.8$, and $\rho = 0.9$. Now, for this example, the fertility rate and the incidence of childbearing at older ages are respectively given by

\begin{align*}
\Phi &= 0.99 - 1.52 \left[ \frac{(\gamma_1 - 0.8\gamma_2)(0.9\epsilon + 0.28c)^{-1}}{(3.8 - 1.72c - 0.9\epsilon)} \right]^{-2} - 0.8 \left[ \frac{4\gamma_2}{0.9c(3.8 - 0.72c)} \right]^{-2} \\
\Psi &= \frac{0.04}{\Phi} + 6.08 \left[ \frac{(\gamma_1 - 0.8\gamma_2)(0.9\epsilon + 0.28c)^{-1}}{(3.8 - 1.72c - 0.9\epsilon)} \right]^{-2} - 0.8 \left[ \frac{4\gamma_2}{0.9c(3.8 - 0.72c)} \right]^{-2}
\end{align*}

We now proceed as follows. First, we fix empirically observed values of $(\Phi, \Psi)$ and $(c, \epsilon)$ for a particular group of countries (say the “East”), and use (25) and (26) to derive preference parameters $(\gamma_1, \gamma_2)$ that rationalize $(\Phi, \Psi)$ and $(c, \epsilon)$. Having done that, we fix the preference parameters $(\gamma_1, \gamma_2)$ at their derived values, allow $(\Phi, \Psi)$ to vary to fit empirically observed values for a different group of countries (the “South” or the “North”), and predict the corresponding values of $(c, \epsilon)$. Throughout, we will normalize observed rates of fertility relative to the replacement rate of 2.1.

Suppose we take it from Figure 1 that the fertility rate of a representative country in the “East” and in the “South” is $\Phi = \frac{13}{27}$, while the fertility rate of a representative country in the “North” is $\Phi = \frac{18}{27}$. Next, we take it from Figure 1 that the proportion of births to older women is $\Psi = 0.3$ in the “East”, and $\Psi = 0.5$ in the “South” and the “North”. Finally, Table 1 indicates that the quality and availability of childcare in the “East” is very low, and as a point of comparison we set $c = 0.95$; Table 2 suggests a penalty in pay of 18 percent for mothers in the “East” and hence we choose $\epsilon = 0.18$. Using (25) and (26), we can now compute preferences parameters, $(\gamma_1, \gamma_2)$, that rationalize the values of $(\Phi, \Psi)$ and $(c, \epsilon)$ chosen for the “East”, and then use the derived values of $(\gamma_1, \gamma_2)$ to predict values of $(c, \epsilon)$ that explain reproductive patterns in the “South” and the “North”, respectively.

The results of this exercise are summarized in Figure 4. Several interesting observations can be derived from this numerical example. First, the figure confirms...
that appropriate parameters, $c$ and $\epsilon$, can be found to rationalize the different reproductive patterns observed in Figure 1. Interestingly enough, the differences in the availability of childcare, $c$, and the family gap in pay, $\epsilon$, predicted by our model are qualitatively in line with the evidence reported in Tables 1 and 2, respectively.

Second, our simple framework does a good job of explaining existing reproductive patterns without appealing to systematic cross-country differences in the preference parameters $\gamma_1$ and $\gamma_2$. Naturally, this explanation is excessively simple, and neither the model’s message, nor its practical implication, is that socio-cultural attitudes and behavioral norms prevalent in different countries are not important for existing reproductive patterns.\footnote{Indeed, while our model provides a theory of differences in reproductive behavior and some of their main economic determinants, to precisely predict country-specific outcomes one may need to allow for variations in $\gamma_1$ and $\gamma_2$. There are several reasons to believe that preferences and attitudes affecting the utility from entering parenthood at different ages might vary across countries. Here we mention two. First, there are considerable differences in the emancipation of the youth between...}

However, the simplicity of our framework makes it clear...
that labor market factors that affect the earnings opportunities of mothers and social policies that affect the costs of raising children are key forces that shape reproductive behavior. Finally, it is worth emphasizing that the conclusions drawn in the previous subsection applied to both the “Early Childbearing” and the “Postponed Childbearing” regime. Here we have seen that the reproductive patterns in industrialized and emerging countries can be broadly rationalized within the “Postponed Childbearing” regime, where a fraction of the population deliberately postpones the attempt to have a child to older ages. We also considered a numerical example in which we took the “Early Childbearing” regime as a reference point. The reproductive patterns that emerged in this case were much harder to justify on statistical grounds, at least in the context of industrialized and emerging countries. The explanation for this discrepancy is that the “Early Childbearing” regime generates proportions of births to older women that are considerably lower than those observed in industrialized and emerging countries.

4. The Effects of Improvements in Assisted Reproduction

Having established that our simple framework does a good job of explaining existing reproductive patterns across countries, we now turn our attention to the next issue of concern, namely, to analyze the effects of improvements in assisted reproductive technologies (ART) on fertility rates. We also ask whether the availability of new ART can have different effects for countries that already have low levels of fertility than for those with a higher level of fertility. Interestingly, the answer is yes: under the “weak family countries” of Northern and Western Europe and the “strong family countries” of Southern Europe: in Northern Europe the preferred mode of reaching independence and autonomy is for the youth to leave the parental home early, which in fact constitutes a precondition for making individual choices on parenthood (Billari et al. [2001]); in Southern Europe, the young stay with their parents well into adulthood, and leave only at the time of marriage (Giuliano [2007]); parents discourage an early departure from the family by placing minimal restrictions on their children’s comings and going, giving them limited domestic responsibility, and paying their household expenses, making it almost unreasonable to leave (Dalla Zuanna [2001], Martinez-Granado and Ruiz-Castillo [2002]). The cover offered by the parental family leads to increased opportunity costs of leaving the parental home, which, in turn, is an obstacle to entering parenthood at a young age. Second, an indirect effect of the late departure from the parental home is that young men, never having had domestic responsibility, have no experience of housework, and thus do not help out in the home, even if their wives are in full-time employment (Dalla Zuanna [2001], Sobotka [2004]). The attitude of male partners, and the resulting excessive burden for women, can be considered important in reducing the utility females derive from entering parenthood at a young age.
conditions that we shall identify, improvements in assisted reproduction can have a positive effect in low-fertility countries, a negative effect in high-fertility countries, and so lead to a convergence of fertility rates across countries.

4.1. Biomedical versus Behavioral Effects. We characterize improvements in assisted reproduction in a simple reduced form, by supposing that they lead to an exogenous increase in the probability of conception success at older ages, captured by the parameter $p_2$.

- **Case EC: “Early Childbearing”**. We begin by examining the effects that would arise in the “early childbearing” scenario; that is, we study improvements in assisted reproduction with $\Delta \leq 1$. Here, as we saw in the previous subsection, a fraction $F(\tilde{a})$ of individuals will try to have a child in period 1, with the conception success probability being $p_1$; of those who fail to conceive, a fraction $F(a_0)$ will try again in period 2, the conception success probability being $p_2$, while the remaining fraction $[F(\tilde{a}) - F(a_0)]$ will choose to stay childless. We now show for $\Delta \leq 1$ that improvements in ART will lead to higher fertility rates:

**Proposition 4** In the “Early Childbearing” regime ($\Delta \leq 1$), an improvement in assisted reproduction, represented by an increase in $p_2$, raises the fertility rate.

To see this, observe from (13) that

$$\eta^\Phi_{p_2} = \Psi \left[ 1 + g(a_0)\eta^a_{p_2} \right].$$

(27)

From (8) it follows that

$$\eta^a_{p_2} = \left[ 1 - \frac{(z_{00} - z_{01})\hat{V}'(z_{00})}{\hat{V}(z_{00}) - \hat{V}(z_{01})} \right].$$

(28)

where $z_{00} \equiv 1 + \rho$ and $z_{01} \equiv 1 + \rho(1 - p_2c)$. Given the convexity of $\hat{V}(z)$, it is straightforward to see that $\eta^a_{p_2} > 1$.\textsuperscript{12} It is clear from (??) that the total demographic effect of an increase in $p_2$ is the sum of two positive effects:

(i) The first term, $\Psi$, represents the percentage change in fertility due to the direct biomedical effect of new ART. After the rise in $p_2$, individuals who failed to have a child in period 1 and choose to try again in period 2 have a larger chance of conception success, and so the total fertility rate increases.

\textsuperscript{12}Formally, since $\hat{V}'(z) > 0$ and $\hat{V}''(z) > 0$, and since $z_{00} - z_{01} > 0$, it follows that $\hat{V}'(z_{01}) < [\hat{V}(z_{00}) - \hat{V}(z_{01})]/(z_{00} - z_{01})$, and hence that $\eta^a_{p_2} > 0$.
(ii) The second term, $[\Psi g(a_0)\eta p_2^{a_0}]$, represents the change in fertility that arises from an indirect behavioral effect which increases the proportion of individuals who try to have a child in both period 1 and period 2. Before the rise in $p_2$, a fraction $(1 - p_1)\{F(\bar{a}) - F(a_0)\}$ of individuals who tried but failed to have a child in period 1 did not try again for a child in period 2 and so chose to remain childless. Now, as $p_2$ rises, some individuals switch their behavior in period 2 from “not trying” to “trying”. As a consequence, there now exists a group of individuals who end up having a child that they would otherwise not have had.

Notice that the indirect behavioral effect works through $a_0$, the critical ability level at which individuals who tried to have a child in period 1 and failed decide to try again for a child in period 2. In predicting how an increase in $p_2$ will affect $a_0$, there are two opposing effects to consider. On the one hand, an increase in $p_2$ raises the expected utility from having a child in period 2, and so pushes individuals who would otherwise not have tried to have child in period 2 into trying to have one – and so increases $a_0$. On the other hand, an increase in $p_2$ reduces the expected returns to education and so lowers the amount of education that people trying to have child in period 2 will get. This raises the earnings gap between trying and not trying in period 2, and so reduces $a_0$. However, since the former effect always dominates the latter, the net effect of improvements in assisted reproduction is to increase the critical ability level $a_0$, causing the population-weighted number of births to go up.

- **Case PC: “Postponed Childbearing”**. In the case of “Early Childbearing” we found that fertility rates are boosted when new ART becomes available. The more interesting question is whether ART is desirable in the case of “Postponed Childbearing”, which is broadly descriptive of the reproductive behavior observed in industrialized and emerging countries. Therefore, let us now turn to the analysis of our model with $\Delta > 1$. As we noted earlier, the requirement $\Delta > 1$ embodies the conditions under which a fraction $[F(a_0) - F(a_1)]$ of individuals will postpone the attempt to have a child to period 2. Aside from this group of “late childbearers”, a fraction $F(a_1)$ of individuals will try to have a child in period 1 and, if unsuccessful, will try again in period 2.

**Proposition 5** In the “Postponed Childbearing” regime ($\Delta > 1$), an improvement in assisted reproduction, represented by an increase in $p_2$, has the potential to either increase or to reduce the fertility rate.

To gain intuition for the effects that arise from improvements in ART, it is useful
to examine the formula for the elasticity of the fertility rate with respect to \( p_2 \). For \( \Delta > 1 \), this elasticity can be written in simplified form as

\[
\eta_{p_2}^\Phi = \Psi + [1 - (1 - p_2)(1 - \Psi)]g(a_0)\eta_{p_2}^{a_0} + [(1 - p_2)(1 - \Psi)]g(a_1)\eta_{p_2}^{a_1}.
\] (29)

We have already observed that \( \eta_{p_2}^{a_0} > 0 \). From (9) and (28) it is straightforward to establish that

\[
\eta_{p_2}^{a_1} = \left[ \frac{p_2 \gamma_2}{\gamma_1 - p_2 \gamma_2} \right] \left[ (a_1 - a_0)V'(z_{01})(z_{00} - z_{01}) - p_2 \gamma_2 \eta_{p_2}^{a_0} \right],
\] (30)

where \( z_{00} \equiv 1 + \rho \) and \( z_{01} \equiv 1 + \rho(1 - p_2 c) \). Since, to be in the “Postponed Childbearing” regime, it must be that \( a_1 < a_0 \), it follows immediately that \( \eta_{p_2}^{a_1} < 0 \). In assessing the case for or against ART, there are now three effects to consider:

(i) The first term, \( \Psi \), represents the percentage change in fertility due to the direct biomedical effect of new ART; its sign is positive.

(ii) The second term, \( [\cdot]g(a_0)\eta_{p_2}^{a_0} \), captures an indirect behavioral effect that reinforces the direct biomedical effect of improved ART. It arises from individuals who switch behavior from “voluntary childlessness” to “postponed childbearing” and so represents a reduction in the incidence of voluntary childlessness. Before the rise in \( p_2 \), a fraction \( [1 - F(a_0)] \) of individuals made the deliberate choice to remain childless and so contributed nothing to the total fertility rate. Now, as \( p_2 \) rises, the availability of improved ART causes some women who would otherwise have not tried to have children later in life to try to do so, and this increases the fertility rate.

(iii) The third term, \( [\cdot]g(a_1)\eta_{p_2}^{a_1} \), is negative, and hence is the one that could potentially undo the positive effects of ART and result in a reduction of the total fertility rate. Indeed, for ART to have negative demographic consequences, the third term must be larger in absolute magnitude than the first and the second term. The third term represents an indirect behavioral effect that arises from individuals that switch from the “early childbearing” to the “postponed childbearing” group. Before the rise in \( p_2 \), these individuals tried to have a child in period 1 and, if the failed, tried again in period 2. Their lifetime conception success probability was \( p_1 + (1 - p_1)p_2 \). Now, as these individuals postpone childbearing to period 2, their lifetime conception success probability falls to \( p_2 \), which means that with probability \( p_1(1 - p_2) \) they will end up not having the
child that they would otherwise have had. Thus, the availability of improved ART causes some women who would otherwise have tried to have children earlier on in life to postpone childbirth to later in life when, despite ART, the probability of conception is lower. This results in a reduction of the fertility rate.

Thus, while improvements in assisted reproduction should have a direct biomedical effect of raising fertility, they could cause indirect changes in behavior which could either reinforce or offset the direct effect.

4.2. Macroeconomic Perspectives. We have just observed that the effects of improvements in ART are ambiguous in the case of “Postponed Childbearing”. Suppose we take it that this case applies to industrialized and emerging countries. Then we know that different groups of countries exhibit significant variations with respect to fertility rates and age patterns of childbearing. An important question in this regard is whether the economic factors that give rise to these variations will also create systematic differences in the responsiveness of different groups of countries to ART, and, if so, whether the responsiveness is higher or lower in countries where the fertility rate is already low. In particular, is it possible that improvements in ART cause the fertility rate to rise in one group of countries and to fall in another, and therefore lead to a convergence or divergence of fertility rates across countries?

The simplest way to get at this question is to investigate how the elasticity of the fertility rate with respect to ART varies between countries with different reproductive patterns. We do so by making two simplifying assumptions. First, we assume that ability \( a \) is distributed according to the Pareto distribution \( F(a) = 1 - a^{-\beta} \). Notice that this assumption allows us to rewrite \( g(a) \) as

\[
g(a) = \beta \left[ \frac{1 - F(a)}{F(a)} \right]. \tag{31}
\]

Second, suppose we use the linear approximations to \( a_0 \) and \( a_1 \) as characterized in equations (21) and (22), respectively. It is readily established that these approximations imply that

\[
\eta_{p_2}^{a_0} \approx 0 \quad \text{and} \quad \eta_{p_2}^{a_1} \approx \left[ \frac{p_2 \gamma_2}{\gamma_1 - \gamma_2 p_2} \right] \left[ \frac{a_1}{a_0} - 1 \right]. \tag{32}
\]

\(^{13}\text{This is consistent with the fact, from (28), that when we take linear approximations to } \hat{V}(z_{00}) \text{ and } \hat{V}(z_{01}), \text{ then } \eta_{p_2}^{a_0} \approx 0.\)
Now define

\[ k \equiv \beta (1 - p_2) p_1 \left( \frac{p_2 \gamma_2}{\gamma_1 - p_2 \gamma_2} \right), \]

noticing that \( k \) is positive. Then, using (31) and (32), we can rewrite (29) as

\[ \eta_{p_2}^\Phi = \Psi - \frac{k (1 - \Psi)}{p_1} \frac{1 - F(a_1)}{F(a_1)} \left[ 1 - \frac{a_1}{a_0} \right]. \]

(33)

Let us think of this elasticity as being a measure for the responsiveness of the fertility rate in a given country to ART. What remains to be established is the correlation between this elasticity and a country’s underlying fertility rate and age pattern of childbearing. With the specific functional form of \( F(\cdot) \), and with the definitions of \( \Phi \) and \( \Psi \) in (15) and (16), it is straightforward to check that

\[
F(a_0) = \frac{\Phi[\Psi + p_2(1 - \Psi)]}{p_2} \quad a_0 = \left[ 1 - \frac{\Phi[\Psi + p_2(1 - \Psi)]}{p_2} \right]^{-\frac{1}{\gamma}}, \\
F(a_1) = \frac{\Phi(1 - \Psi)}{p_1} \quad a_1 = \left[ 1 - \frac{\Phi(1 - \Psi)}{p_1} \right]^{-\frac{1}{\gamma}},
\]

(34)

making explicit the dependence of the elasticity of the fertility rate on \((\Phi, \Psi)\).\textsuperscript{14}

Formally, substituting (34) into (33), we get

\[
\eta_{p_2}^\Phi(\Phi, \Psi) = \Psi - \frac{k}{\Phi} \left[ 1 - \frac{\Phi(1 - \Psi)}{p_1} \right] \left[ 1 - \left( \frac{1 - \frac{\Phi[\Psi + p_2(1 - \Psi)]}{p_2}}{1 - \frac{\Phi(1 - \Psi)}{p_1}} \right)^{\frac{1}{\gamma}} \right].
\]

(35)

A first question is whether improvements in ART can have different effects for countries with a low level of fertility than for those with a higher level of fertility. To begin with, suppose that the coefficient \( k \) is constant across countries. This amounts to assuming that there are no systematic differences in preferences \((\gamma_1, \gamma_2)\) and conception success probabilities \((p_1, p_2)\) across countries. Next define the threshold \( \hat{k}(\Phi, \Psi) \) to be that value of \( k \) such that the elasticity of the fertility rate with respect to ART is zero. Formally, let \( \hat{k}(\Phi, \Psi) \) be the solution to \( \eta_{p_2}^\Phi(\Phi, \Psi) = 0 \). Comparing \( \hat{k}(\Phi, \Psi) \) across countries with different reproductive patterns, we have the following proposition.

\textbf{Proposition 6} Consider two countries, one with a high level of fertility \((\Phi^H)\), the other with a low level of fertility \((\Phi^L)\). Assuming that the two countries are otherwise

\textsuperscript{14}Notice that since \( F(a_0) \leq 1 \), then \( F(a_0) = \frac{\Phi[\Psi + p_2(1 - \Psi)]}{p_2} \) implies the restriction \( \Phi \leq \frac{p_2}{p_2 + \Psi(1 - p_2)} \).

In what follows, we will therefore have to be careful in how we choose to normalize the fertility rate and set the value for \( p_2 \).
identical, then for any $\beta > 1$ the two country-specific thresholds on $k$ are such that $\hat{k}(\Phi^H, \cdot) < \hat{k}(\Phi^L, \cdot)$.

a. If $k < \hat{k}(\Phi^H, \cdot)$, then an improvement in assisted reproduction raises the fertility rate in both the high-fertility and the low-fertility country;

b. If $k > \hat{k}(\Phi^L, \cdot)$, then an improvement in assisted reproduction reduces the fertility rate in both the high-fertility and the low-fertility country;

c. However, if $\hat{k}(\Phi^H, \cdot) < k < \hat{k}(\Phi^L, \cdot)$, then an improvement in assisted reproduction reduces fertility in the high-fertility country, and raises fertility in the low-fertility country.

Proving this proposition amounts to showing that the elasticity of the fertility rate with respect to ART is strictly decreasing in $\Phi$.\textsuperscript{15} The details of the proof are relegated to the appendix. The main message here is as follows. In general, countries with high levels of fertility are less responsive to improvements in ART than countries with low levels of fertility. Moreover, depending on the value of $k$, the availability of new ART can have differential effects for countries with low levels of fertility than for those with high levels of fertility. On one side, if $k$ is small enough, then the effect of ART will be positive for both high- and low-fertility countries; if instead $k$ is large enough, then the effect of ART will be negative for both high- and low-fertility countries. Interestingly enough, there are good reasons for thinking that these two cases will arise only for extreme preference configurations: the former case ($k$ small) will emerge only if individuals have strong preferences for having children at a young age ($\frac{\gamma_1}{\gamma_2}$ large), while the latter case ($k$ large) will emerge only if individuals have strong preferences for having children at an old age ($\frac{\gamma_1}{\gamma_2}$ small). Finally, there exists an intermediate range of values of $k$ in which improvements in assisted reproduction will have a positive effect in low-fertility countries and a negative effect in high-fertility countries. At face value, this suggests that improvements in assisted reproduction could produce a convergence of fertility rates across high- and low-fertility countries.

Figure 5 illustrates this possibility, extending the numerical example developed in the previous section. The elasticity of the fertility rate with respect to ART can be computed in two different ways. The first method, which is based on linear approximations to $(a_0, a_1)$, follows the macro-perspective developed in the present

\textsuperscript{15}Notice that the proposition holds only for parameters satisfying $\beta > 1$. As we noted earlier, however, empirical evidence suggests that a value of $\beta$ between 1.5 and 2 fits most earnings distributions. Thus, the restriction $\beta > 1$ is justified on statistical grounds.
The "East" $\eta_{\Phi_p^2} = 0.072$

The "South" $\eta_{\Phi_p^2} = 0.068$

The "North" $\eta_{\Phi_p^2} = -0.074$

Cost of Raising Children $c$

Family Gap in Pay

Figure 5: Numerical Example Revisited. The example is based on the following parameter values: $p_1 = 0.95$, $p_2 = 0.8$, $\rho = 0.9$, $\beta = 2$, $\gamma_1 \simeq 1.21$, $\gamma_2 \simeq 1.15$. The implied value of $k \equiv \beta(1 - p_2)p_1\left[\frac{p_2\gamma_2}{\gamma_1 - p_2\gamma_2}\right]$ is $k = 1.2$.

section. The strategy is to use biomedical figures on $(p_1, p_2)$, observed values for $(\Phi, \Psi)$, and a value of $k$ that is consistent with the parametric setup from the earlier example to calculate (35) for a typical country in the “North”, the “South”, and the “East”. The results are illustrated in Figure 5.A. In general, the $(\Phi, \Psi)$-space can be divided into two regions, indicating whether improvements in ART have a negative or positive effect on fertility. Notice that the boundary between the two regions, $\Phi(\Psi)$, is strictly decreasing in $\Psi$. This suggests that, for the parameters under consideration, the elasticity of the fertility rate with respect to ART is not only strictly decreasing in $\Phi$, but also strictly decreasing in $\Psi$. Thus, our model predicts that we are most likely to see negative effects of ART in countries where both fertility and the incidence of childbearing at older ages are high. For example, for a typical country in the “North”, we calculate that a 10 percent increase in the conception success probability $p_2$ would reduce the fertility rate by roughly 1 percent. Conversely, for representative countries in the “South” and in the “East”, a 10 percent increase in the effectiveness of assisted production would raise the fertility rate by roughly 1 percent.

An alternative way of calculating the elasticity of the fertility rate with respect
to ART is to look at the differences in \((c, \epsilon)\) that give rise to observed variations in \((\Phi, \Psi)\). Unlike the above approach, this method does not rely on linear approximations to \((a_0, a_1)\), and uses (25) as the basis for all computations. The strategy is to compute the elasticity of the fertility rate based on \((c, \epsilon)\)-combinations that rationalize the reproductive patterns observed in the “North”, the “South”, and the “East”. The results are illustrated in Figure 5.B. The figure divides the \((c, \epsilon)\)-space into two subregions, indicating whether improvements in assisted reproduction have a negative or positive effect on fertility. Notice first that the elasticities computed for the three hypothetical countries are qualitatively similar to ones reported in Figure 5.A. This suggests the macro-perspective developed in this section provides a reliable short-cut to predicting the effects of improvements in assisted reproduction. In line with our previous observations, the results suggest that improvements in ART are most likely to have negative effects on the fertility rate for \((c, \epsilon)\)-configurations that give rise to high fertility rates and high proportions of births to older women. Intuitively, the negative behavioral effect that induces people to postpone childbirth is more pronounced in economic environments that are conducive to high levels of fertility in the first place. Indeed, the better the availability of formal childcare (low \(c\)), and the lower the family gap in pay (low \(\epsilon\)), the more likely it is that improvements in assisted reproduction lead to worse fertility outcomes.

5. Conclusion

This paper has pursued two related themes. Our primary concern has been to examine the relationship between assisted reproductive technologies (ART) and the microeconomics of fertility choice. Along the way we have put forward a simple model that does a good job in explaining existing reproductive patterns across industrialized and emerging countries.

We have demonstrated that the model’s predictions about the main economic correlates of reproductive behavior accord well with a considerable body of cross-country facts. Our model, in particular, suggests that cross-country differences in (i) labor market factors that affect the earnings opportunities of mothers and (ii) social policies that affect the cost of raising children are key forces shaping differences in reproductive behavior.

Our analysis of ART has centered around the distinction between biomedical and behavioral effects. While improvements in assisted reproduction should have the direct biomedical effect of raising fertility, they could cause indirect changes
in behavior which could offset the direct effect. In particular, the availability of improved ART could cause some women who would otherwise have tried to have children earlier on in life to postpone childbirth to later in life when, despite ART, the conception success probability is lower. We have shown that this behavioral effect of postponement may result in a reduction of the fertility rate.

From a practical standpoint, the findings in this paper suggest that improving ART is no panacea for the problem of stabilizing fertility rates. The negative behavioral effect that induces people to postpone childbirth is most pronounced in economic environments that are conducive to high levels of fertility. Indeed, we have demonstrated that an improvement in assisted reproduction can have a negative effect in high-fertility countries, a positive effect in low-fertility countries, and so lead to a convergence of fertility rates across countries. Overall, our results suggest that understanding the consequences of ART does require exploring its behavioral implications in different economic environment.

This paper is only a first step towards an analysis of assisted reproductive technologies from an economic perspective and leaves open many questions. From a macroeconomic perspective, it would be interesting to embed the present framework in a model of economic growth (Barro and Becker [1989], Becker et al. [1990]), and use the model to assess the effects of ART and other population policy measures on population and income growth. From a microeconomic perspective, it would be interesting to assess the effects of ART in a dynamic bargaining model of the household, which recognizes that a household’s choices about whether and when to have children, and the impact of these choices on women’s earnings, can affect the household’s balance of power (Basu [2006]). Many other interesting questions, such as the effects of ART on the formation and duration of relationships, make up a rather rich agenda for further research.

References


Appendix

**Proof of Proposition 6.** Let \( \hat{k}(\Phi, \Psi) \) be the solution to \( \eta_{k}^{p^2} = 0 \), where \( \eta_{k}^{p^2} \) is defined in (35). Since \( \eta_{k}^{p^2} \) is strictly decreasing in \( k \), to establish that \( \hat{k}(\Phi, \Psi) \) is strictly decreasing in \( \Phi \), it is sufficient to show that \( \eta_{k}^{p^2} \) is strictly decreasing in \( \Phi \).

Differentiating \( \eta_{k}^{p^2} \) with respect to \( \Phi \), we obtain

\[
\frac{\partial \eta_{k}^{p^2}}{\partial \Phi} = \frac{k}{\beta \Phi^2} \left[ \beta \left( 1 - \frac{a_1}{a_0} \right) + F(a_1) \frac{a_1}{a_0} - F(a_0) \frac{1 - F(a_1)}{1 - F(a_0)} \frac{a_1}{a_0} \right]. \tag{A.1}
\]

After substituting \( F(a) = 1 - a^{-\beta} \) into the right-hand side of (A.1), simplifying, and rearranging, it follows that:

\[
\frac{\partial \eta_{k}^{p^2}}{\partial \Phi} \leq 0 \iff \Gamma(\beta) \equiv \beta \left( \frac{a_0}{a_1} \right)^{1-\beta} + (1 - \beta) \left( \frac{a_1}{a_0} \right) \beta \lesssim 1. \tag{A.2}
\]

We now note that

\[
\Gamma'(\beta) = \left( \frac{a_0}{a_1} \right)^{1-\beta} \left[ 1 - \beta \ln \left( \frac{a_0}{a_1} \right) \right] - \left( \frac{a_1}{a_0} \right)^\beta \left[ 1 - (1 - \beta) \ln \left( \frac{a_1}{a_0} \right) \right]. \tag{A.3}
\]

Hence, it follows that the function \( \Gamma(\beta) \) has unique stationary point, namely at

\[
\hat{\beta} = \frac{\left( \frac{a_0}{a_1} - 1 \right) + \ln \left( \frac{a_1}{a_0} \right)}{\frac{a_0}{a_1} \ln \left( \frac{a_0}{a_1} \right) + \ln \left( \frac{a_1}{a_0} \right)}. \tag{A.4}
\]

It is now readily checked that, for any \( \frac{a_0}{a_1} > 1 \), we have:

\[
\Gamma'(0) > 0 \quad \text{and} \quad \Gamma'(1) < 0. \tag{A.5}
\]

Hence, it follows that the unique stationary point \( \hat{\beta} \) occurs in the interval \((0, 1)\) and is the point at which \( \Gamma(\cdot) \) achieves a maximum. Finally, since \( \Gamma(0) = 1 \) and \( \Gamma(1) = 1 \), and since \( \Gamma'(0) > 1 \) and \( \Gamma'(1) < 0 \), it follows immediately that

\[
\Gamma(\beta) \leq 1 \iff \beta \lesssim 1. \tag{A.6}
\]

This, in turn, implies that \( \eta_{k}^{p^2} \) is strictly decreasing [increasing] in \( \Phi \) if \( \beta > [<] 1 \), which establishes the proposition. ■