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# **The dynamics of perception**

Modelling subjective well-being in a short panel

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## ABSTRACT

We consider the neglected issue of the dynamics of perceptions, as expressed in responses to survey questions on subjective well-being. We develop a simulated ML method for estimation for dynamic linear models, where the dependent variable is partially observed through ordinal scales. This latent autoregression (LAR) model is often more appropriate than the usual state-dependence (SD) model for attitudinal and interval variables. The paper contains an application to a model of households' perceptions of their financial well-being, demonstrating the superior fit of the LAR model to both the usual static model and the SD model.

**KEYWORDS:** Dynamic panel data models, ordinal variables, simulated maximum likelihood, GHK simulator, BHPS

## NON-TECHNICAL SUMMARY

There is much current interest in the broader concepts of “happiness”, “satisfaction” and “well-being” as alternatives to measures such as income and consumption, in evaluating social and economic outcomes. Analysis of these broader concepts is usually made by investigating the statistical relationship between the subjective assessments given by people interviewed in sample surveys and their personal and economic circumstances. This allows researchers to compare the additional well-being generated by higher income with that generated by happy events like the birth of a child or the losses from unhappy events like unemployment or bereavement.

The large applied literature on this topic is predominantly based on the strong assumption that the subjective assessments generated by survey interviews have a direct relationship with “true” well-being at the time of interview. This neglects the possibility that there may be slow adjustment of people’s perceptions of well-being to changes in their circumstances - in particular, that there may be some psychological inertia in moving away from last year’s assessment following a change in circumstances.

This methodological paper investigates the inertia effect and proposes a new approach to statistical analysis in the presence of inertia. We analyse the responses to a question in the British Household Panel Survey about perceived financial well-being and find strong evidence that perceptions adjust only slowly to changing circumstances. An implication of this finding is that researchers who fail to allow for the possibility of slow adjustment may reach misleading conclusions about the nature of well-being.

## 1 Introduction

This paper considers panel data methods in applications involving a combination of three statistical issues: (i) the panel (the British Household Panel Study (BHPS), in our case) is short in the sense that the number of waves is very much less than the number of individuals, so that asymptotic justifications rest on  $n \rightarrow \infty$  with  $T$  fixed; (ii) the possibility that perceptions display slow adjustment to changing circumstances; and (iii) the use of an ordinal Likert-type measure as dependent variable.

The panel data literature outside economics has two main strands: random-effects structures within the multi-level modelling approach (Goldstein, 2003), which generally deals with static models. The alternative structural equations (SEM) approach (Bollen, 1989) can be applied to short panels, with each wave represented by a different equation. Although the SEM approach can, in principle, capture rich dynamics by specifying cross-equation feedbacks, most applications are essentially static or accommodate change through latent growth curve models involving parametric time trends (Meredith and Tisak, 1990) or through temporal coefficient variation by allowing coefficients in the period-specific equations to differ.

From the viewpoint of the econometric literature these approaches have a rather cross-section ‘look’, contrasting with the econometric view of panel data as a collection of short realisations of individual time-series processes. In economics, these processes are sometimes derived from theoretical models of inter-temporal decision-making such as the life-cycle hypothesis (Hall, 1978) and they are frequently representable by an equation involving autoregressive elements, sometimes with a unit root, implying a stochastic trend quite different from the latent growth curve models widely used in other areas of social science. One potential area of convergence in panel data methods that appears largely absent outside economics, is the convergence of panel data models and time series models more generally.

In short panels, dynamic modelling changes the nature of inference procedures. Even in the simple regression model, fixed-effects estimation no longer gives consistent estimates (Nickell, 1981) and fixed-effects logit methods (Chamberlain, 1980) are not applicable in autoregressive models with covariates. When the dependent variable is discrete, the instrumental variable and generalised method-of-moments estimators developed by econometricians (see Hsiao 2003, chapter 4, for a survey) are not appropriate. There are two main reasons for taking explicit account of the ordinal nature of the dependent variable rather

than using regression methods. Firstly, the numerical scaling of responses is arbitrary and may impose an inappropriate cardinalisation on the estimates. Secondly, the logic of linear regression implies a residual distribution with a finite set of mass points, whose form varies with the values of the explanatory variables. Neglect of this complication makes standard inferential procedures unreliable. A further issue arising in dynamic models for discrete variables, is the ambiguity over the form that dynamic adjustment might take, since, in standard models, the dependent variable exists in two forms: a latent continuous form and a discrete observed form. Either of these might be specified to carry the process of dynamic adjustment.

The focus of this paper is the use of panel data to model subjective assessments of individual well-being. Subjective indicators have a potentially valuable role to play in studies of poverty and the distribution of welfare. There are well-known imperfections in the measurement of income, particularly in the extremes of the distribution, and a composite approach involving subjective well-being measures might moderate the distortions caused by income measurement error. This is particularly important in panel datasets, where consumption expenditure is rarely observed and income may be subject to erratic short-term movements. Moreover, the concepts of poverty and welfare are potentially much broader than that of low income or expenditure over a standard reference period (Sen, 1985). However, the value of subjective assessments and other non-income indicators of deprivation remains the subject of debate (Ravallion and Lokshin, 2001, 2002).

The availability of panel data makes it possible to allow for persistent individual effects which capture variations in the way that different individuals translate their perceptions into survey responses. It has become common practice to use random effects binary or ordered probit models (see Fréchet 2001, for a popular implementation). Ferrer-i-Carbonell and Frijters (2004) have extended the conditional logit estimator of Chamberlain (1980) to the ordered case, allowing the independence of individual effects and observed covariates to be tested. However, much of the applied literature on subjective well-being is static in nature and there has been little work so far on the dynamics of individual perceptions of well-being. In part this is due to the difficulty of dynamic modelling in short panels with discrete endogenous variables.

An important distinction can be made between inherent and observational discreteness. *Inherent discreteness* refers to a case where the variables of interest are naturally discrete. For example, an individual is either employed or not employed; she has a

university degree or not; she is married or not. *Observational discreteness* arises when the variables of interest are naturally continuous, but the survey instrument used to observe them imposes discreteness via a pre-specified ordinal scale of allowable responses. This applies to a wide range of attitudinal questions, which ask respondents to record their perceptions or beliefs on a Likert (1932) scale. Econometric analysis of attitudinal variables has grown enormously in recent years, with the development of the economic literature on happiness and satisfaction (see Van Praag and Ferrer-i-Carbonell, 2004, for a recent survey). There has so far been little discussion of the dynamics of perceptions or of the most appropriate type of dynamic model to use. Dynamic models typically involve lagged values of the dependent variable, implying long-range dependence between elements of each individual's realisation of the perception process. Observational discreteness does not only arise with attitudinal data. It may also occur in survey questions about more 'objective' entities like income, when respondents are required to place themselves within one of a number of given income ranges.

Most of the statistical literature dealing with discrete models for longitudinal data assumes inherent discreteness. The state dependence (SD) model of Heckman (1978, 1981a,b) in  $R$ -category ordinal form is:

$$y_{it}^* = \alpha_1 D_{it-1}^1 + \dots + \alpha_R D_{it-1}^R + \boldsymbol{\beta}' \mathbf{x}_{it} + u_i + \varepsilon_{it} \quad (1)$$

$$y_{it} = r \quad \text{iff} \quad y_{it}^* \in [\Gamma_{r-1}, \Gamma_r), \quad r = 1 \dots R \quad (2)$$

where:  $D_{it-1}^r$  is a dummy variable equal to 1 if  $y_{it-1} = r$ ;  $\mathbf{x}_{it}$  is a vector of strictly exogenous covariates;  $u_i$  is an unobserved individual effect uncorrelated with  $\mathbf{x}_{it}$ ;  $\varepsilon_{it}$  is a random residual uncorrelated across individuals and time; and  $\Gamma_1 \dots \Gamma_{R-1}$  are parameters, with  $\Gamma_0 = -\infty$  and  $\Gamma_R = \infty$ . This model was developed primarily for applications in labour economics, where discreteness is inherent in the problem and where past outcomes of  $y_{it}$ , in the form of dummy variables  $D_{it-1}^1 \dots D_{it-1}^R$ , represent state dependence. In these applications, the latent variable  $y_{it}^*$  is essentially an artificial construct and there is no reason why  $y_{it-1}^*$  should appear in (1).

However, attitudes, expectations and incomes are not inherently discrete and the use of models like (1)-(2) is questionable. If the discrete nature of  $y_{it}$  is only an artificial construct imposed by the questionnaire designer, then behaviour centres on the continuous variable  $y_{it}^*$ , rather than the observed indicator  $y_{it}$ . In these cases,  $y_{it-1}^*$  rather than  $y_{it-1}$ , should carry the dynamic feedback if the dynamic equation is to be a description of behaviour. This is an important point, largely neglected in the econometric literature, which focuses almost

exclusively on SD models when dynamic discrete models are considered. An exception to this is a paper by Bover and Arellano (1997). However, the context and model considered in that study is quite different from the case considered here, as is the approach to estimation.

This paper has several objectives. Firstly, (above and in section 2) we make the case for using dynamics in  $y_{it-1}^*$ , rather than  $y_{it-1}$ , in applications where the discreteness is observational rather than inherent and consider its dynamic implications. We propose a practical method of estimation in section 3 and, in section 4, this is applied to a panel data model of individuals' financial expectations, demonstrating the superior fit and different properties of the LAR model. Identification of the model is demonstrated in appendix 1.

## 2 The model

### 2.1 The statistical structure

We work with a behavioural model specified in terms of the 'natural' continuous variables as follows:

$$y_{it}^* = \alpha y_{it-1}^* + \boldsymbol{\beta}' \mathbf{x}_{it} + u_i + \varepsilon_{it} \quad (3)$$

We refer to this as the Latent Autoregression (LAR) model. The vector  $\mathbf{x}_{it}$  is assumed strictly exogenous and individuals are sampled independently from the underlying population. We make the standard assumption of Gaussian random effects so that the unobservables  $u_i$  and  $\varepsilon_{it}$  satisfy the following assumptions:

$$(u_i, \varepsilon_{it}) \perp \mathbf{X}_i \quad (4)$$

$$u_i \perp \varepsilon_{it} \quad (5)$$

$$\varepsilon_{it} \perp \varepsilon_{is} \quad \text{for every } s \neq t \quad (6)$$

$$\begin{pmatrix} u_i \\ \varepsilon_i \end{pmatrix} \sim N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & 0 \\ 0 & 1 \end{bmatrix} \right) \quad (7)$$

where  $\perp$  denotes statistical independence and  $\mathbf{X}_i = (\mathbf{x}_{i0}, \dots, \mathbf{x}_{iT})$ . We only observe  $y_{it}^*$  according to the grading scale defined by (2) above. Note that, since the scale and origin of  $y_{it}^*$  and  $\Gamma_r$  are arbitrary, the model is normalised by omitting the intercept from  $\mathbf{x}_{it}$  and setting  $\text{var}(\varepsilon_{it}) = 1$ , which is equivalent to dividing  $y_{it}^*$ ,  $y_{it-1}^*$ ,  $\boldsymbol{\beta}$ ,  $u_i$  and  $\varepsilon_i$  through by  $\sigma_\varepsilon$  in (2). Note that  $\alpha$  is not affected by this normalisation.

### 2.2 Interpretation of parameters

In models with unobserved grading thresholds, the scale of  $y_{it}^*$  is unobserved and we estimate  $\beta/\sigma_\varepsilon$  rather than  $\beta$ . Consequently, the estimated coefficients are interpretable as  $\partial E([y_{it}^*/\sigma_\varepsilon] | y_{it-1}^*, \mathbf{x}_{it}, u_i) / \partial \mathbf{x}_{it}$ . In applications to subjective well-being, this problem is more fundamental than a lack of identification induced by imperfect observation: there is a lack of natural units, which renders the scale of  $\beta$  inherently ambiguous. However, note that  $\alpha$  is identifiable independently of  $\sigma_\varepsilon$ . As a consequence, we can estimate unambiguously the speed of adjustment. For example, following a shock, the proportion of disequilibrium which is eliminated within  $s$  periods is  $1-\alpha^s$  and this is unaffected by normalisation.

### 2.3 Dynamics

The SD and LAR processes (1) and (3) imply different patterns of dynamic behaviour. Consider the following artificial example:

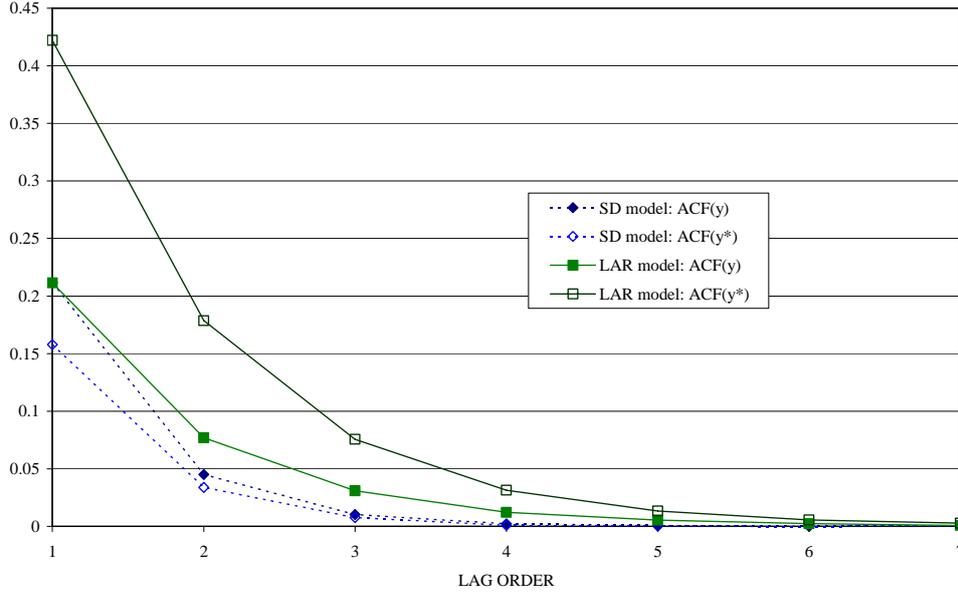
$$SD \text{ model: } y_t^* = 0.8y_{t-1} + x + \varepsilon_t \quad (8)$$

$$LAR \text{ model: } y_t^* = 0.422y_{t-1}^* + 0.355 + 0.770x + \varepsilon_t \quad (9)$$

where  $x = 0.5$ ,  $\varepsilon_t \sim N(0,1)$  and  $y_t = \mathbf{1}(y_t^* > 0)$ . The parameters of the LAR process (9) have been chosen to reproduce exactly three properties of the SD process (8):

- (i)  $\Pr(y = 1) = 0.877$ ;
- (ii)  $\partial \Pr(y = 1 | x) / \partial x = 0.246$ ;
- (iii)  $\Pr(y_t \neq y_{t-1}) = 0.170$ .

With the LAR parameters chosen in this way, the distributions of run lengths in states 0 and 1 are identical for the two processes. However, the relationship between successive run lengths is not. This is reflected in the autocorrelation functions (Figure 1). As we would expect, the LAR model has much higher autocorrelations than the SD model for  $y_t^*$ . For the observed  $y_t$ , the ACF decays faster for the SD than the LAR process, despite the fact that they have the same 1st-order autocorrelation by construction. Thus, an LAR model will display greater persistence than an observationally similar SD model, in this quite subtle sense.



**Figure 1** ACFs for the SD and LAR models

The two models also differ in terms of the implied dynamic multiplier effects of  $\mathbf{x}$  on  $y$ . To illustrate this, consider again the binary case and focus on two important features: the impact on  $\Pr(y_{it}=1 | y_{it-1}, \mathbf{X}_i, u_i)$  of switching the conditioning event from  $y_{it-1} = 0$  to  $y_{it-1} = 1$ ; and the impact of the history of  $\{\mathbf{x}_{it}\}$  on the probability of a positive response, without conditioning on  $y_{it-1}$ .

For the former, the SD model is relatively simple:

$$\Pr(y_{it} = 1 | y_{it-1} = 1, \mathbf{X}_i, u_i) - \Pr(y_{it} = 1 | y_{it-1} = 0, \mathbf{X}_i, u_i) = \Phi(\alpha + \boldsymbol{\beta}' \mathbf{x}_{it} + u_i) - \Phi(\boldsymbol{\beta}' \mathbf{x}_{it} + u_i) \quad (10)$$

where  $\Phi(\cdot)$  is the cdf of the  $N(0,1)$  distribution. For the LAR model, we have instead:

$$\begin{aligned} & \Pr(y_{it} = 1 | y_{it-1} = 1, \mathbf{X}_i, u_i) - \Pr(y_{it} = 1 | y_{it-1} = 0, \mathbf{X}_i, u_i) \\ &= \frac{\Pr(y_{it} = 1, y_{it-1} = 1 | \mathbf{X}_i, u_i)}{\Pr(y_{it-1} = 1 | \mathbf{X}_i, u_i)} - \frac{\Pr(y_{it} = 1 | \mathbf{X}_i, u_i) - \Pr(y_{it} = 1, y_{it-1} = 1 | \mathbf{X}_i, u_i)}{1 - \Pr(y_{it-1} = 1 | \mathbf{X}_i, u_i)} \\ &= \frac{\Pr(y_{it} = 1, y_{it-1} = 1 | \mathbf{X}_i, u_i) - \Pr(y_{it} = 1 | \mathbf{X}_i, u_i) \Pr(y_{it-1} = 1 | \mathbf{X}_i, u_i)}{\Pr(y_{it-1} = 1 | \mathbf{X}_i, u_i) [1 - \Pr(y_{it-1} = 1 | \mathbf{X}_i, u_i)]} \end{aligned} \quad (11)$$

Assume the process (2) is stable and long-established. Then:

$$y_{it}^* = \sum_{s=0}^{\infty} \alpha^s \boldsymbol{\beta}' \mathbf{x}_{it-s} + \frac{u_i}{1 - \alpha} + \sum_{s=0}^{\infty} \alpha^s \varepsilon_{it-s} \quad (12)$$

and therefore  $\Pr(y_{it}=1, y_{it-1}=1 \mid \mathbf{X}_i, u_i) = \Phi^*(\mu_{it}, \mu_{it-1}; \alpha)$  and  $\Pr(y_{it}=1 \mid \mathbf{X}_i, u_i) = \Phi(\mu_{it})$ , where  $\Phi^*(\cdot, \cdot; \alpha)$  is the bivariate standard normal cdf with correlation  $\alpha$  and  $\mu_{it}$  is the scaled conditional mean  $(1-\alpha^2)^{1/2}[\sum_s \alpha^s \boldsymbol{\beta}' \mathbf{x}_{it-s} + u_i / (1-\alpha)]$ . Thus:

$$\Pr(y_{it} = 1 \mid y_{it-1} = 1, \mathbf{X}_i, u_i) - \Pr(y_{it} = 1 \mid y_{it-1} = 0, \mathbf{X}_i, u_i) = \frac{\Phi(\mu_{it}, \mu_{it-1}; \alpha) - \Phi(\mu_{it})\Phi(\mu_{it-1})}{\Phi(\mu_{it-1})[1 - \Phi(\mu_{it-1})]} \quad (1)$$

3)

The important difference between (10) and (13) is that the former depends only on the current vector  $\mathbf{x}_{it}$ , whereas the latter depends on the entire history of  $\mathbf{x}_{it}$ .

Consider now the alternative summary measure,  $\Pr(y_{it}=1 \mid \mathbf{X}_i, u_i)$ . The LAR process gives a relatively simple form:

$$\Pr(y_{it} = 1 \mid \mathbf{X}_i, u_i) = \Phi(\mu_{it}) \quad (14)$$

implying that the lagged marginal response decays geometrically:

$$\frac{\partial \Pr(y_{it} = 1 \mid \mathbf{X}_i, u_i)}{\partial \mathbf{x}_{it-s}} = \phi(\mu_{it}) \alpha^s \sqrt{1-\alpha^2} \boldsymbol{\beta} \quad (15)$$

where  $\phi(\cdot)$  is the standard normal pdf.

For the state-dependence model, we can write:

$$\begin{aligned} \Pr(y_{it} = 1 \mid \mathbf{X}_i, u_i) &= \Pr(y_{it} = 1 \mid y_{it-1} = 0, \mathbf{X}_i, u_i) \Pr(y_{it-1} = 0 \mid \mathbf{X}_i, u_i) \\ &+ \Pr(y_{it} = 1 \mid y_{it-1} = 1, \mathbf{X}_i, u_i) \Pr(y_{it-1} = 1 \mid \mathbf{X}_i, u_i) \end{aligned} \quad (16)$$

Rearrange and write this as a recursion:

$$P_{it} = P_{it-1} \delta_{it} + \rho_{it} \quad (17)$$

where:  $P_{it} = \Pr(y_{it}=1 \mid \mathbf{X}_i, u_i)$ ;  $\delta_{it} = \Phi(\alpha + \boldsymbol{\beta}' \mathbf{x}_{it} + u_i) - \Phi(\boldsymbol{\beta}' \mathbf{x}_{it} + u_i)$ ; and  $\rho_{it} = \Phi(\boldsymbol{\beta}' \mathbf{x}_{it} + u_i)$ .

Solving back to an arbitrary period 0:

$$P_{it} = P_{it0} \prod_{j=0}^{t-1} \delta_{it-j} + \sum_{s=0}^{t-1} \rho_{it-s} \prod_{j=0}^{s-1} \delta_{it-j} \quad (18)$$

where we use the convention  $\prod_{j=0}^{j=-1} \delta_{it-j} \equiv 1$ . On reasonable assumptions about the  $\mathbf{x}$ -process, solving back indefinitely leads to the following representation:

$$P_{it} = \sum_{s=0}^{\infty} \rho_{it-s} \prod_{j=0}^{s-1} \delta_{it-j} \quad (19)$$

Thus:

$$\begin{aligned}
\frac{\partial \Pr(y_{it} = 1 | \mathbf{X}_i, u_i)}{\partial \mathbf{x}_{it-s}} &= \frac{\partial \rho_{it-s}}{\partial \mathbf{x}_{it-s}} \prod_{j=0}^{s-1} \delta_{it-j} + \sum_{k=s+1}^{\infty} \frac{\rho_{it-k}}{\delta_{it-s}} \prod_{j=0}^{k-1} \delta_{it-j} \frac{\partial \delta_{it-s}}{\partial \mathbf{x}_{it-s}} \\
&= \left( \prod_{j=0}^{s-1} \delta_{it-j} \right) \phi(\boldsymbol{\beta}' \mathbf{x}_{it-s} + u_i) \boldsymbol{\beta} \\
&\quad + \sum_{k=s+1}^{\infty} \left( \frac{\rho_{it-k}}{\delta_{it-s}} \prod_{j=0}^{k-1} \delta_{it-j} \right) [\phi(\alpha + \boldsymbol{\beta}' \mathbf{x}_{it-s} + u_i) - \phi(\boldsymbol{\beta}' \mathbf{x}_{it-s} + u_i)] \boldsymbol{\beta}
\end{aligned} \tag{20}$$

The profile of  $\partial \Pr(y_{it}=1 | \mathbf{X}_i, u_i) / \partial \mathbf{x}_{it-s}$  is thus considerably more complicated than the geometric decay implied by the SD model (1).

### 3 Estimation

#### 3.1 Initial conditions

In the SD model, there are two alternative approaches for dealing with the random effects  $u$ . Heckman (1981b) specifies an approximation to the distribution of  $y_{i0} | \mathbf{X}_i, u_i$ , and then derives the distribution of  $y_{i1} \dots y_{iT} | y_{i0}, \mathbf{X}_i, u_i$  using sequential conditioning. The random effects are then integrated out by numerical quadrature. The alternative approach, used by Wooldridge (2000) is to specify instead the distribution of  $u_i | y_{i0}, \mathbf{X}_i$ . A semi-parametric variant due to Arellano and Carrasco (2003) involves the sequence of conditional means  $\lambda_{it} = E(u_i | y_{i0} \dots y_{it}, \mathbf{x}_{i0} \dots \mathbf{x}_{it})$ , which are estimated as nuisance parameters. The latter approach has many advantages in models like (1) but is less attractive in LAR models, where the variable of interest,  $y_t^*$ , is not observable and cannot be conditioned on. Conditioning on its observable counterpart does not lead to useful simplification. For this reason, we use the Heckman treatment of initial conditions.

Assume that we observe  $y$  and  $\mathbf{x}$  over a period  $t = 0 \dots T$ . The LAR process (2) implies the following distributed lag representation:

$$y_{it}^* = \alpha^t y_{i0}^* + \sum_{s=0}^{t-1} \alpha^s \boldsymbol{\beta}' \mathbf{x}_{it-s} + \frac{1 - \alpha^t}{1 - \alpha} u_i + \sum_{s=0}^{t-1} \alpha^s \varepsilon_{it-s} \tag{21}$$

This is a useful basis for estimation if either  $t$  is sufficiently large and  $\alpha^t$  decays sufficiently rapidly with  $t$  or if we can find a good empirical approximation for  $y_{i0}^*$ .

Write this approximation to  $y_{i0}^* | \mathbf{X}_i, u_i$  as:

$$y_{i0}^* = \boldsymbol{\delta}' \mathbf{w}_i + \gamma u_i + \eta_i \tag{22}$$

$$y_{i0} = r \quad \text{iff} \quad y_{i0}^* \in [\Gamma_{r-1}^0, \Gamma_r^0), \quad r = 1 \dots R \tag{23}$$

where  $\mathbf{w}_i$  is a vector constructed from  $\mathbf{X}_i$ ;  $\delta$  and  $\gamma$  are parameters and, in the ordered probit case,  $\Gamma_r^0$  may differ from  $\Gamma_r$ . The random term  $\eta_i$  satisfies the following assumptions:

$$\eta_i \perp u_i \mid \mathbf{X}_i \quad (24)$$

$$\eta_i \perp \varepsilon_{it} \mid \mathbf{X}_i \quad \text{for every } t > 0 \quad (25)$$

$$\eta_i \mid \mathbf{X}_i \sim \text{N}(0, \sigma_\eta^2) \quad (26)$$

Note that, unlike  $\varepsilon_{it}$ ,  $\eta_i$  is not normalised to have unit variance.

In principle, the vector  $\mathbf{w}_i$  may contain all distinct elements of  $\{\mathbf{x}_{i0}, \mathbf{X}_i\}$ . However, in practice it may be found that  $\mathbf{w}_i = \mathbf{x}_{i0}$  is adequate, or that limited summaries, such as  $\mathbf{w}_i = \{\mathbf{x}_{i0}, T^{-1} \sum_1^T \mathbf{x}_{it}\}$ , work well. This is essentially an empirical issue.

With approximation (22)-(23), equation (21) becomes:

$$y_{it}^* = \alpha^t \delta' \mathbf{w}_i + \sum_{s=0}^{t-1} \alpha^s \beta' \mathbf{x}_{it-s} + c_t u_i + \sum_{s=0}^{t-1} \alpha^s \varepsilon_{it-s} + \alpha^t \eta_i \quad (27)$$

where  $c_t = (1 - \alpha^t) / (1 - \alpha) + \alpha^t \gamma$ .

The model now consists of equation (22) and a set of equations (27) for any collection of periods  $t > 0$ . In practice, the initial conditions model (22) is only an approximation and is a potential source of specification error. However, if  $|\alpha| < 1$  so that  $\alpha^t \rightarrow 0$  as  $t \rightarrow \infty$ , then the influence of the initial conditions declines as we consider later periods. There is, therefore, a case for leaving a gap (of  $S$  periods) between the initial period 0 and the subsequent periods used to estimate the LAR model. Consequently, we work with a system of  $(T-S+1)$  equations consisting of (22) and (27) for  $t = S+1 \dots T$ . Data on  $\{y_{i1} \dots y_{iS}\}$  are not used. The choice of  $S$  involves a trade-off between possible misspecification bias and efficiency, since increasing  $S$  reduces both the influence of initial conditions and the amount of data used for estimation. Increasing  $S$  also reduces the scale of the computational problem. This system is nonlinear in its parameters  $\theta = \{\alpha, \beta, \delta, \gamma, \sigma_u, \Gamma_1 \dots \Gamma_R, \Gamma_1^0 \dots \Gamma_R^0\}$ . Appendix 1 establishes that the model is identified provided the sample contains at least three waves.

### 3.3 SML estimation

This identification argument does not lead to an efficient estimator, since it does not impose all the restrictions on the coefficients ( $a_t, \mathbf{b}_t, \mathbf{d}_{0t}, \dots, \mathbf{d}_{t-1,t}$ ) in (29), nor does it exploit the relationship between the residual correlation  $\rho_{12}$  and the model parameters. Instead we use a simulated ML procedure. Let the observed outcome for  $y_{it}$  be  $r_{it}$ , implying  $y_{it}^* \in [\Gamma_{r_{it}-1}, \Gamma_{r_{it}})$ . The likelihood for this set of events is:

$$\Pr(y_{i0} = r_{i0}, y_{iS+1} = r_{iS+1}, \dots, y_{iT} = r_{iT} \mid \mathbf{X}_i) = \Pr(v_{i0} \in A_{i0}, v_{iS+1} \in A_{iS+1}, \dots, v_{iT} \in A_{iT}) \quad (28)$$

where  $v_{it} = c_t u_i + \sum_0^{t-1} \alpha^s \varepsilon_{it-s} + \alpha^t \eta_i$ ,  $\mu_{it} = \alpha^t \boldsymbol{\delta}' \mathbf{w}_i + \sum_0^{t-1} \alpha^s \boldsymbol{\beta}' \mathbf{x}_{it-s}$  and  $A_{it}$  is the interval  $[\Gamma_{r_{it}} - \mu_{it}, \Gamma_{r_{it+1}} - \mu_{it})$ . The residual vector  $\mathbf{v}_i = (v_{i0}, v_{iS+1} \dots v_{iT})$  has a covariance matrix with elements:

$$\omega_{00} = \gamma^2 \sigma_u^2 + \sigma_\eta^2 \quad (29)$$

$$\omega_{0t} = \gamma c_t \sigma_u^2 + \sigma_\eta^2 \alpha^t, \quad S < t \leq T \quad (30)$$

$$\omega_{st} = c_s c_t \sigma_u^2 + \sum_{p=0}^{\min(s,t)-1} \alpha^{s+t-2p} \sigma_\varepsilon^2 + \alpha^{s+t} \sigma_\eta^2, \quad S < (s, t) \leq T \quad (31)$$

The probability (28) is a  $(T-S+1)$ -dimensional rectangle probability. Under normality, probabilities of this kind can be calculated using the GHK simulator (Hajivassiliou and Ruud, 1994), with antithetic acceleration used to improve simulation precision. We construct the following simulated log-likelihood function:

$$\ln \hat{L}(\boldsymbol{\theta}) = \sum_{i=1}^n \ln \hat{P}_i(\boldsymbol{\theta}) \quad (38)$$

where  $\hat{P}_i(\boldsymbol{\theta})$  is the predicted probability (28) for individual  $i$ , estimated using the GHK algorithm. The simulated likelihood is maximised numerically with respect to  $\boldsymbol{\theta}$ .

#### 4 An application to individual expectations data

The British Household Panel Survey (BHPS) is the principal source of nationally-representative household- and individual-level panel data in the UK. This application is based on 11 waves, relating to the years 1993-2003. Each year, BHPS participants are asked a series of questions about their attitudes. Here we work with the set of observations on 2,219 males who were household heads in the year 1992. The resulting panel dataset is unbalanced but has a common initial period  $t = 0$  in 1993 (the year 1992 is lost through the need to construct certain differenced variables).

We analyse responses to the following question: “How well would you say you yourself are managing financially these days?” Responses have been recoded as:  $y = 1$  “Finding it very difficult”;  $y = 2$  “Finding it quite difficult”;  $y = 3$  “Just about getting by”,  $y = 4$  “Doing alright”;  $y = 5$  “Living comfortably”. Under the LAR model, the individual’s

underlying assessment of his financial position at time  $t$  is a naturally continuous variable,  $y_{it}^*$ , which we assume to be generated according to the panel autoregression (3). The respondent is then assumed to translate  $y_{it}^*$  into a response to the categorical survey question according to the rule (2).

The final parameter estimates for this LAR model are given in Table 1. Computation was done using the GHK simulator, using successive passes, initially with 50 replications, rising to 500 once the neighbourhood of the optimum was reached. Following convergence, a single iteration was performed with 2000 replications as a check on convergence and the optimised likelihood value. Antithetic variance reduction is used throughout.

The variables used in the model are summarised in Appendix Table A1. Following initial experimentation with alternative specifications, our model for the initial condition  $y_{it}^*$  used a vector of covariates  $\mathbf{w}_i$  comprising the current values  $\mathbf{x}_{i0}$  and overall sample means  $\bar{\mathbf{x}}_i$  for a subset of the variables. Of the latter averaged variables, only the unemployment variable is significant in the model for  $y_{it}^*$ . Estimation results appear not to be very sensitive to the specification of the model for  $y_{it}^*$  in this application. The results reported here use a skip rate of  $S = 0$ , so that our final specification uses all available waves of data. Consequently, the rectangle probabilities involved in SML estimation are 11-dimensional. We also computed estimates, not reported here, based on observations of  $y$  for waves 0, 6...10 (a skip rate of  $S = 5$ ), which shortened computing times considerably but made no important change to the estimates (see also Pudney, 2005).

Table 1 summarises the sample fit of the LAR, SD and static random-effects ordered probit models. Full parameter estimates for all three models are given in the appendix. The static and SD model estimates were computed using 48-point Gauss-Hermite quadrature. Both dynamic models give a much higher likelihood value than the static model, which is overwhelmingly rejected by a likelihood ratio test. Dynamic adjustment of perceptions is clearly important here.

Despite the fact that the SD model has two more parameters than the LAR model, the latter achieves a substantially higher log-likelihood. This is also true of other BHPS samples and model specifications not presented here (see Pudney, 2005). Thus the conventional type of dynamic response embedded in the SD model is clearly not the best way of capturing dynamic adjustment. The comparison between the LAR estimates on one hand and the static and SD estimates on the other, shows that the latter models generate too little persistence

through dynamic adjustment and the model fitting process compensates for this misspecification by overestimating the variance of the individual effect. In our application, the share of residual variance attributable to the persistent effect,  $\sigma_u^2/(1+\sigma_u^2)$ , is estimated to be only 27% for the LAR model, compared to 46% for the SD model and 54% for the static model. If we interpret the individual effects  $u_i$  as the result of inter-individual differences in the interpretation of the response scales or in psychological characteristics, then correcting the misspecification inherent in the SD and static models dramatically reduces the importance of such differences. Instead of inherent between-individual differences in perception, our results emphasise a general tendency towards inertia in all individuals' updating of perceptions.

**Table 1** Sample fit and intra-individual squared correlation of alternative models

	<b>LAR model</b>	<b>SD model</b>	<b>Static model</b>
Log-likelihood	-19,427.53	-19,456.38	-19,822.90
Number of parameters	98	100	96
$\rho^2 = \sigma_u^2 / (1+\sigma_u^2)$	0.274	0.456	0.541

It is sometimes assumed that the neglect of dynamics is unimportant if we are mainly interested only in long-term equilibrium effects. Our results demonstrate that this is not a reliable assumption. In the LAR model, the steady-state equilibrium relationship between  $y^*$  and  $\mathbf{x}$  is given by the coefficients  $\beta/(1-\alpha)$ . Comparing these with the coefficients from the static model, we find that the coefficients of the level variables in the principal equation are subject to proportionate biases averaging  $\pm 25\%$  for those coefficients which are significant at the 5% level in the LAR model. There are particularly large positive biases of 38-45% in the coefficients of the education variables. For the variables reflecting changes (such as marriage dissolution, job loss, *etc.* within the last year) the biases are also large, with the significant coefficients biased downwards by 34% on average, using a direct comparison of the short-run coefficients  $\beta$  in the two models. In particular, the static model gives severe underestimation of the short-term impact of job loss. The year effects are also heavily biased: by 57% absolute on average. It is clear from this comparison that, if we are seeking good estimates of either short- or long-run influences well-being, it is important to allow appropriately for the dynamic adjustment of perceptions.

The estimated LAR model is set out in Table A2 of appendix 2; it largely conforms to *a priori* expectations. Economic circumstances are represented by the level of household

income per capita, the proportion of household income earned by the respondent himself, and a dummy for owner-occupation, together with the estimated value of the equity in the house. As expected, the level of household per capita income has a significant positive effect on perceptions of financial well-being. The respondent's share in household income has a negative influence on his reported perceptions, which is consistent with the idea that diversification of income sources reduces risk and thus increases perceived financial security. However, the diversification effect is not very strongly significant. There is stronger evidence to support the widely-held view that homeowners' perceptions respond to rising house values. Human capital also appears to be an important element in perceived financial well-being, with a strong positive influence of educational attainment.

However, these 'objective' financial factors are not sufficient to explain the determination and evolution of perceived financial well-being. Other explanatory variables are mostly time-invariant. Ethnicity is represented by dummies for the Black and Asian groups and there is weak evidence of a negative difference. The effect of marital status is captured by dummies for being married/cohabiting, divorced/separated or widowed. A further dummy identifies those who have made a transition into the divorced/separated group within the last year. Other status transitions were insignificant or too few in number to permit reliable estimation. There is a significant positive influence of a marital or cohabitation relationship. Divorce or separation reduces perceived well-being, with a very large temporary reduction in the year of separation.

Labour market status is important, with significant positive effects for employment and self-employment. The status of unemployment has a significant negative effect, with a further temporary effect in the year of transition into unemployment.

Differences in household size and structure are important. There are significant negative coefficients for the number of school-age children and working-age adult household members, but no detectable impact of a new birth on financial perceptions. These effects are in addition to the per capita equalisation used for the household income variables. The relationship between perceived well-being and birth cohort of the household head has a convex form, decreasing over the cohorts covered by the sample. The year dummies show a strongly rising trend up to 1998, then a two-year dip with a recovery in 2001.

## **5 Conclusions**

We have considered an alternative to the discrete state dependence (SD) model for dynamic modelling of ordinal variables from panel data. The alternative LAR model involves ordinal observation of a latent autoregression, rather than lagged feedback of the previous period's discrete outcome. It is argued that this specification is more appropriate for a range of applications involving observational, rather than inherent, discreteness. Examples include interval regressions and models of expectations, and subjective well-being.

The method has been applied to a model of individual perceptions of financial well-being, applied to UK household panel data. The LAR model provides a robust description of the evolution of financial perceptions over time, with a significant role for lagged adjustment. The LAR model fits the data considerably better than a static ordered model or the conventional dynamic extension of SD form. The LAR model has quite different equilibrium and dynamic properties than both static and SD models. In particular, the static and SD models display less persistence than the LAR model, and when misused to model data with LAR-type dynamics, overcompensate by grossly overestimating the variance of the individual effect. This has the important practical consequence of exaggerating the importance of inter-individual differences in perceptions of well-being. Neglect of the slow adjustment of perceptions in modelling well-being leads to substantial biases in the estimates of both short-run impact effects and steady-state equilibrium effects.

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## Appendix 1: Identification

Partition the covariates into a common set of time-invariant variables  $\zeta_i$  and a sequence of time-varying covariates  $\xi_{it}$ , so that  $\mathbf{x}_{it} = (\zeta_i, \xi_{it})$ . Assume a full specification of the initial condition (9), so that  $\mathbf{w}_i = (\zeta_i, \xi_{i1} \dots \xi_{iT})$ . Make the further assumption that the matrix  $\text{plim}(n^{-1} \sum \mathbf{w}_i \mathbf{w}_i')$  is positive definite. An ordered probit model for  $y_{i0}$  on  $\mathbf{w}_i$  will consistently estimate the normed coefficient vector  $\delta/v_0$ , where  $v_0^2 = \sigma_\eta^2 + \gamma^2 \sigma_u^2$ .

Consider equation (27), for any period,  $t > 0$ . Rewrite it in standardised form:

$$\begin{aligned} \left( \frac{y_{it}^*}{v_t} \right) = & \left( \frac{\alpha^t v_0}{v_t} \right) \omega_i + \left( \frac{(1-\alpha^t) \boldsymbol{\beta}_\zeta}{(1-\alpha) v_t} \right)' \zeta_i + \left( \frac{\boldsymbol{\beta}_\xi}{v_t} \right)' \xi_{it} + \left( \frac{\alpha \boldsymbol{\beta}_\xi}{v_t} \right)' \xi_{it-1} + \dots + \left( \frac{\alpha^{t-1} \boldsymbol{\beta}_\xi}{v_t} \right)' \xi_{i1} \\ & + \left[ c_t u_i + \sum_{s=0}^{t-1} \alpha^s \varepsilon_{it-s} + \alpha^t \eta_i \right] / v_t \end{aligned} \quad (\text{A1})$$

where  $\boldsymbol{\beta}' = (\boldsymbol{\beta}_\zeta', \boldsymbol{\beta}_\xi')$ ,  $v_t^2 = c_t^2 \sigma_u^2 + (1-\alpha^{2t})/(1-\alpha^2) + \alpha^{2t} \sigma_\eta^2$  and  $\omega_i$  is the variable  $\delta' \mathbf{w}_i / v_0$  which can be constructed from the coefficients of the initial conditions model (22). Rewrite (28) in simplified notation as:

$$y_{it}^* / v_t = a_t \omega_i + \mathbf{b}_t' \zeta_i + \mathbf{d}_{0t}' \xi_{it} + \mathbf{d}_{1t}' \xi_{it-1} + \dots + \mathbf{d}_{t-1,t}' \xi_{i1} + v_{it} \quad (\text{A2})$$

Note that the covariates  $(\omega_i, \zeta_i, \xi_{i1} \dots \xi_{it})$  are (asymptotically) non-collinear. Thus, ordered probit estimation of (29) will generate consistent estimates of the scaled coefficients  $(a_t, \mathbf{b}_t, \mathbf{d}_{0t}, \dots, \mathbf{d}_{t-1,t})$ . Identification then proceeds as follows. First, the value of  $\alpha$  can be constructed as any element of any of the vectors of ratios  $\mathbf{d}_{st} / \mathbf{d}_{s-1,t}$ . If  $\alpha$  is zero, the model becomes a static random effects ordered probit, so there is no new identification issue; we consider the case  $\alpha \neq 0$  henceforth. With  $\alpha$  known,  $\boldsymbol{\beta}$  can be inferred up to scale as  $\mathbf{g} / \|\mathbf{g}\|$  where  $\mathbf{g} = [\mathbf{b}_t(1-\alpha)/(1-\alpha^t), \mathbf{d}_{0t}]$ . Thus, the key behavioural parameters  $\alpha$  and the direction of the vector  $\boldsymbol{\beta}$  are essentially identifiable from only two waves of the panel.

The ratio,  $R_t$ , of  $a_t$  to  $\alpha^t$  gives the value  $v_0/v_t$ , thus:

$$R_t v_t = v_0 \quad (\text{A3})$$

The correlation between the random errors in equations (22) and (27), which can be estimated consistently by joint estimation or from the generalised residuals, is  $\rho_{0t}$  satisfying the following:

$$\rho_{0t} v_0 v_t = c_t \gamma \sigma_u^2 + \alpha^t \sigma_\eta^2 \quad (\text{A4})$$

Equations (30) and (31) are clearly insufficient to determine the three remaining unknowns,  $\gamma$ ,  $\sigma_u^2$  and  $\sigma_\eta^2$ , so full identification requires at least two waves of data, in addition to wave 0.

Consider the 3-wave case, where we have data for  $t = 0, 1, 2$ . Calculate each of the ratios  $(v_t/v_0)^2$  as  $\alpha^{2t}/a_t^2$ . Using the definition (31), after some manipulation the quantity  $\gamma \sigma_u^2 / v_0^2$  can be expressed as:

$$\frac{\gamma \sigma_u^2}{v_0^2} = \alpha^2 \left[ \frac{\rho_{02}}{a_2} - \frac{\rho_{01}}{a_1} \right] \quad (\text{A5})$$

Note that  $a_t \neq 0$  for  $\alpha \neq 0$ , so (32) is well-defined. Now express  $v_t^2$  as  $(A_t + \alpha^t \gamma)^2 \sigma_u^2 + B_t + \alpha^{2t} \sigma_\eta^2$ , where  $A_t = (1 - \alpha^t)/(1 - \alpha)$  and  $B_t = (1 - \alpha^{2t})/(1 - \alpha^2)$ . Thus:

$$\left( \frac{v_t}{v_0} \right)^2 = A_t^2 \left( \frac{\sigma_u^2}{v_0^2} \right) + 2A_t \alpha^t \left( \frac{\gamma \sigma_u^2}{v_0^2} \right) + \alpha^{2t} \left( \frac{\gamma^2 \sigma_u^2 + \sigma_\eta^2}{v_0^2} \right) + B_t \left( \frac{1}{v_0^2} \right) \quad (\text{A6})$$

We know the value of  $\gamma \sigma_u^2 / v_0^2$  from (32) and we know *a priori* that  $(\gamma^2 \sigma_u^2 + \sigma_\eta^2)/v_0^2$  is equal to 1. This gives the following pair of equations with known right-hand sides:

$$A_t^2 \left( \frac{\sigma_u^2}{v_0^2} \right) + B_t \left( \frac{1}{v_0^2} \right) = \left( \frac{v_t}{v_0} \right)^2 - 2A_t \alpha^t \left( \frac{\gamma \sigma_u^2}{v_0^2} \right) - \alpha^{2t}, \quad t = 1, 2 \quad (\text{A7})$$

Note that the matrix  $\begin{pmatrix} A_1^2 & B_1 \\ A_2^2 & B_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ (1 + \alpha)^2 & (1 + \alpha^2) \end{pmatrix}$  is non-singular for all  $\alpha \neq 0$ , so there is a unique solution for  $(\sigma_u^2/v_0^2)$  and  $(1/v_0^2)$ . From these,  $\sigma_u^2$  and  $v_0^2$  are determined. The value of  $\gamma$  is then given by (32) and  $\sigma_\eta^2$  by  $v_0^2 - \gamma^2 \sigma_u^2$ , so all parameters are identified.

## Appendix 2: Additional tables

**Table A1** Summary statistics of variables  
(Unbalanced panel of male household heads, 1992-2001;  $n = 2,219$ )

Variable	Definition	Sample mean
<i>Level variables</i>		
Birth cohort	(Year of birth-1940)/10	3.138
Black	Self-assessed membership of “black” ethnic group	0.0074
Asian	Self-assessed membership of “Indian”, “Pakistani” or “Bangladeshi” ethnic group	0.0096
Married/cohabiting	Living as married or cohabiting	0.649
Divorced/separated	Divorced or separated	0.107
Widower	Widower	0.141
Employed	In full-time employment (30 hours or more per week)	0.458
Self-employed	Self-employed	0.085
Retired	Self-assessed as retired	0.308
Unemployed	Self-assessed as unemployed	0.029
Long-term sick	Self-assessed as not working due to long-term sickness/incapacity	0.047
Degree	University degree qualification	0.113
Certificate/Diploma	Has HND, HNC or comparable qualification	0.070
A-level	Has at least one A-level	0.141
O-level/GCSE	Has at least one O-level, CSE or GCSE	0.258
Vocational qualifications	Has a vocational qualification	0.340
Pre-school child	Household contains at least one pre-school child	0.107
# pre-school children	Number of pre-school children in household	0.134
# school-age children	Number of school-age children in household	0.458
# retired members	Number of retired household members	0.434
# working age adults	Number of non-retired adults in household	1.469
Owner-occupier	House owned outright or with a mortgage	0.730
Private rental	House rented from a private landlord	0.066
Housing equity	Housing equity (constant 2001 prices ÷ £100,000)	0.579
Household income pc	Annual household income per household member (constant 2001 prices ÷ £1,000)	1.107
Income share	Income of respondent as proportion of total household income	0.657
<i>Change variables</i>		
Newly divorced	Change of marital status to divorced/separated	0.0077
Newly widowed	Change of marital status to widower	0.0038
Newly retired	Change of activity status to retired	0.0128
Job loss	Change of activity status to unemployed	0.0100
Newly sick	Change of activity status to long-term sick	0.0037
New child	New pre-school child in last 12 months	0.0231

**Table A2a** LAR model parameter estimates: principal equation

<b>Parameter</b>	<b>Estimate</b>	<b>Std.err.</b>	<b>Parameter</b>	<b>Estimate</b>	<b>Std.err.</b>
<i>Level variables</i>			<i>Autoregressive coefficient</i>		
Black	-0.5622	0.4318	$\alpha_1$	0.3288	0.0115
Asian	-0.1604	0.1612	<i>Difference variables</i>		
Married/cohabiting	0.1812	0.0519	Newly divorced	-0.3466	0.0852
Divorced/separated	-0.1091	0.0630	Newly widowed	0.1679	0.1338
Widower	0.1181	0.0726	Newly retired	-0.0513	0.0680
Employed	0.3398	0.0898	Job loss	-0.2311	0.0869
Self-employed	0.3665	0.0945	Newly sick	-0.7199	0.1166
Retired	0.1968	0.1036	New child	-0.0805	0.0600
Unemployed	-0.2888	0.1025	<i>Year effects (reference year = 2003)</i>		
Long-term sick	-0.0435	0.0997	1994	-0.4395	0.1544
Degree	0.2812	0.0529	1995	-0.1575	0.0394
Certificate/Diploma	0.2149	0.0658	1996	-0.0471	0.0391
A-level	0.1613	0.0525	1997	0.0112	0.0387
O-level/GCSE	0.1494	0.0451	1998	0.0263	0.0390
Vocational qualifications	-0.0493	0.0359	1999	-0.0846	0.0397
# pre-school children	-0.0336	0.0550	2000	-0.0943	0.0415
# school-age children	-0.0294	0.0156	2001	0.0338	0.0425
# retired members	0.0061	0.0385	2002	0.0213	0.0419
# working age adults	-0.0413	0.0195	<i>Random effect variance</i>		
Pre-school child	-0.0466	0.0717	$\sigma_u^2$	0.3775	0.0499
Owner-occupier	0.2431	0.0436	<i>Threshold parameters</i>		
Private rental	0.1018	0.0524	$\Gamma_1$	-1.8172	0.2110
Housing equity	0.0184	0.0086	$\Gamma_2$	-1.0357	0.1905
Household income pc	0.1998	0.0134	$\Gamma_3$	0.4400	0.1856
Income share	-0.0601	0.0467	$\Gamma_4$	1.6734	0.2163
Birth cohort	-0.1058	0.0173			
Birth cohort <sup>2</sup>	0.0184	0.0070			

**Table A2b** LAR model parameter estimates: initial conditions

<b>Parameter</b>	<b>Estimate</b>	<b>Std.err.</b>	<b>Parameter</b>	<b>Estimate</b>	<b>Std.err.</b>
<i>Variables dated wave 0</i>			<i>Variables averaged over all time periods</i>		
Black	-0.5697	0.5803	Married/cohabiting	-0.2032	0.2645
Asian	-0.5131	0.2474	Divorced/separated	-0.3175	0.3342
Married/cohabiting	0.5491	0.2248	Widower	-0.3186	0.3721
Divorced/separated	-0.1216	0.2968	Employed	-0.4039	0.4899
Widower	0.4690	0.3568	Self-employed	-0.3266	0.5072
Employed	0.7814	0.2233	Retired	-0.1717	0.5490
Self-employed	0.4753	0.2501	Unemployed	-1.1242	0.5637
Retired	0.5480	0.3111	Long-term sick	-0.4084	0.5139
Unemployed	0.1408	0.2449	# pre-school children	0.1966	0.1473
Long-term sick	0.2833	0.2789	# school-age children	-0.0554	0.0953
Degree	0.1649	0.1206	# retired members	-0.0525	0.2104
Certificate/Diploma	0.2107	0.1407	# working age adults	-0.1752	0.1099
A-level	0.3121	0.1078	Owner-occupier	0.2091	0.2774
O-level/GCSE	0.1106	0.0898	Private rental	0.2617	0.3191
Vocational qualifications	-0.0300	0.0724	Housing equity	0.0138	0.0855
# pre-school children	0.0051	0.1694	Household income pc	0.1251	0.0849
# school-age children	0.0518	0.0739	Income share	-0.1655	0.2788
# retired members	-0.0200	0.1607	<i>Coefficient of random effect</i>		
# working age adults	0.0607	0.0698	$\gamma$	1.3624	0.0899
Pre-school child	-0.1864	0.2091	<i>Threshold parameters</i>		
Owner-occupier	0.1886	0.2431	$\Gamma_1^0$	-1.1674	0.4863
Private rental	0.0913	0.2613	$\Gamma_2^0$	-0.2905	0.4849
Housing equity	0.1259	0.0994	$\Gamma_3^0$	1.2811	0.4833
Household income pc	0.4188	0.0666	$\Gamma_4^0$	2.4494	0.4845
Income share	0.3924	0.1727	<i>Initial condition variance</i>		
Birth cohort	-0.1140	0.0462	$\sigma_\eta^2$	0.7049	0.0910
Birth cohort <sup>2</sup>	0.0332	0.0154			

**Table A3a** SD model parameter estimates: principal equation

Parameter	Estimate	Std.err.	Parameter	Estimate	Std.err.
<i>Level variables</i>			<i>Autoregressive coefficients</i>		
Black	-0.7717	0.5392	$\alpha_1$	0.3108	0.0661
Asian	-0.2343	0.2115	$\alpha_2$	0.6081	0.0629
Married/cohabiting	0.2505	0.0646	$\alpha_3$	1.1026	0.0659
Divorced/separated	-0.1325	0.0794	$\alpha_4$	1.4757	0.0690
Widower	0.1730	0.0917	<i>Difference variables</i>		
Employed	0.4437	0.1067	Newly divorced	-0.3808	0.0965
Self-employed	0.4726	0.1122	Newly widowed	0.1842	0.1572
Retired	0.2537	0.1262	Newly retired	-0.0372	0.0807
Unemployed	-0.3526	0.1240	Job loss	-0.2102	0.1016
Long-term sick	-0.0579	0.1227	Newly sick	-0.7937	0.1266
Degree	0.4058	0.0664	New child	-0.0722	0.0693
Certificate/Diploma	0.3089	0.0847	<i>Year effects (reference year = 2003)</i>		
A-level	0.2327	0.0680	1994	-0.2489	0.0443
O-level/GCSE	0.2136	0.0583	1995	-0.2198	0.0455
Vocational qualifications	-0.0729	0.0477	1996	-0.0838	0.0458
# pre-school children	-0.0499	0.0672	1997	-0.0025	0.0452
# school-age children	-0.0420	0.0196	1998	0.0241	0.0456
# retired members	0.0013	0.0474	1999	-0.1021	0.0465
# working age adults	-0.0475	0.0240	2000	-0.1177	0.0486
Pre-school child	-0.0637	0.0873	2001	0.0303	0.0500
Owner-occupier	0.3260	0.0525	2002	0.0260	0.0476
Private rental	0.1504	0.0653	<i>Random effect variance</i>		
Housing equity	0.0237	0.0105	$\sigma_u^2$	0.8386	0.0230
Household income pc	0.2423	0.0047	<i>Threshold parameters</i>		
Income share	-0.0454	0.0559	$\Gamma_1$	-1.4030	0.1640
Birth cohort	-0.1497	0.0205	$\Gamma_2$	-0.4740	0.1621
Birth cohort <sup>2</sup>	0.0249	0.0092	$\Gamma_3$	1.2797	0.1608
			$\Gamma_4$	2.7424	0.1612

**Table A3b** SD model parameter estimates: initial conditions

<b>Parameter</b>	<b>Estimate</b>	<b>Std.err.</b>	<b>Parameter</b>	<b>Estimate</b>	<b>Std.err.</b>
<i>Variables dated wave 0</i>			<i>Variables averaged over all time periods</i>		
Black	-0.6027	0.5805	Married/cohabiting	-0.2121	0.2623
Asian	-0.5187	0.2501	Divorced/separated	-0.2696	0.3303
Married/cohabiting	0.5662	0.2224	Widower	-0.2742	0.3706
Divorced/separated	-0.1176	0.2931	Employed	-0.3188	0.4735
Widower	0.4480	0.3538	Self-employed	-0.2403	0.4910
Employed	0.7733	0.2184	Retired	-0.1274	0.5345
Self-employed	0.4562	0.2457	Unemployed	-1.0164	0.5492
Retired	0.5518	0.3051	Long-term sick	-0.3218	0.4963
Unemployed	0.1112	0.2395	# pre-school children	0.1921	0.1461
Long-term sick	0.2839	0.2715	# school-age children	-0.0545	0.0940
Degree	0.2279	0.1218	# retired members	-0.0474	0.2099
Certificate/Diploma	0.2357	0.1417	# working age adults	-0.1620	0.1093
A-level	0.3318	0.1089	Owner-occupier	0.1959	0.2722
O-level/GCSE	0.1291	0.0908	Private rental	0.2873	0.3115
Vocational qualifications	-0.0379	0.0731	Housing equity	-0.0001	0.0858
# pre-school children	0.0038	0.1672	Household income pc	0.0984	0.0847
# school-age children	0.0444	0.0733	Income share	-0.1499	0.2768
# retired members	-0.0183	0.1590	<i>Coefficient of random effect</i>		
# working age adults	0.0580	0.0696	$\gamma$	1.0250	0.0540
Pre-school child	-0.1901	0.2061	<i>Threshold parameters</i>		
Owner-occupier	0.2187	0.2382	$\Gamma_1^0$	-1.1313	0.4727
Private rental	0.0777	0.2544	$\Gamma_2^0$	-0.2319	0.4715
Housing equity	0.1275	0.0994	$\Gamma_3^0$	1.3551	0.4700
Household income pc	0.4105	0.0669	$\Gamma_4^0$	2.5267	0.4709
Income share	0.3765	0.1703			
Birth cohort	-0.1210	0.0464			
Birth cohort <sup>2</sup>	0.0343	0.0155			

**Table A4a** Static model parameter estimates: principal equation

<b>Parameter</b>	<b>Estimate</b>	<b>Std.err.</b>	<b>Parameter</b>	<b>Estimate</b>	<b>Std.err.</b>
<i>Level variables</i>			<i>Difference variables</i>		
Black	-1.0763	0.6809	Newly divorced	-0.3303	0.0981
Asian	-0.3223	0.2749	Newly widowed	0.1849	0.1691
Married/cohabiting	0.3163	0.0659	Newly retired	0.0125	0.0804
Divorced/separated	-0.1891	0.0825	Job loss	-0.0266	0.1052
Widower	0.2159	0.0945	Newly sick	-0.6609	0.1330
Employed	0.5676	0.1054	New child	-0.0203	0.0711
Self-employed	0.6002	0.1104	<i>Year effects (reference year = 2003)</i>		
Retired	0.3076	0.1248	1994	-0.3920	0.0438
Unemployed	-0.4192	0.1232	1995	-0.3467	0.0452
Long-term sick	-0.1080	0.1215	1996	-0.1986	0.0455
Degree	0.5833	0.0823	1997	-0.0746	0.0449
Certificate/Diploma	0.4413	0.1081	1998	-0.0172	0.0452
A-level	0.3415	0.0822	1999	-0.1246	0.0460
O-level/GCSE	0.3234	0.0729	2000	-0.1631	0.0479
Vocational qualifications	-0.1036	0.0609	2001	-0.0244	0.0501
# pre-school children	-0.0869	0.0633	2002	0.0142	0.0524
# school-age children	-0.0541	0.0190	<i>Random effect variance</i>		
# retired members	0.0044	0.0475	$\sigma_u^2$	1.1793	0.0240
# working age adults	-0.0355	0.0236	<i>Threshold parameters</i>		
Pre-school child	-0.0829	0.0825	$\Gamma_1$	-2.1604	0.1630
Owner-occupier	0.4113	0.0567	$\Gamma_2$	-1.2251	0.1607
Private rental	0.2166	0.0668	$\Gamma_3$	0.5391	0.1592
Housing equity	0.0093	0.0114	$\Gamma_4$	2.0100	0.1601
Household income pc	0.2696	0.0043			
Income share	0.0550	0.0552			
Birth cohort	-0.2077	0.0240			
Birth cohort <sup>2</sup>	0.0334	0.0116			

**Table A4b** Static model parameter estimates: initial conditions

<b>Parameter</b>	<b>Estimate</b>	<b>Std.err.</b>	<b>Parameter</b>	<b>Estimate</b>	<b>Std.err.</b>
<i>Variables dated wave 0</i>			<i>Variables averaged over all time periods</i>		
Black	-0.7198	0.5883	Married/cohabiting	-0.2051	0.2591
Asian	-0.5254	0.2556	Divorced/separated	-0.2091	0.3249
Married/cohabiting	0.5457	0.2174	Widower	-0.2604	0.3723
Divorced/separated	-0.1688	0.2853	Employed	-0.2899	0.4459
Widower	0.4296	0.3514	Self-employed	-0.2247	0.4648
Employed	0.7329	0.2105	Retired	-0.1292	0.5079
Self-employed	0.4272	0.2380	Unemployed	-0.9367	0.5227
Retired	0.5125	0.2971	Long-term sick	-0.2843	0.4697
Unemployed	0.0618	0.2322	# pre-school children	0.1771	0.1451
Long-term sick	0.2328	0.2642	# school-age children	-0.0625	0.0931
Degree	0.3014	0.1225	# retired members	-0.0632	0.2078
Certificate/Diploma	0.2897	0.1429	# working age adults	-0.1471	0.1077
A-level	0.3648	0.1079	Owner-occupier	0.1578	0.2717
O-level/GCSE	0.1748	0.0912	Private rental	0.2894	0.3069
Vocational qualifications	-0.0536	0.0736	Housing equity	-0.0199	0.0852
# pre-school children	0.0128	0.1643	Household income pc	0.0419	0.0840
# school-age children	0.0482	0.0726	Income share	-0.1199	0.2715
# retired members	-0.0172	0.1576	<i>Coefficient of random effect</i>		
# working age adults	0.0545	0.0686	$\gamma$	0.7818	0.0335
Pre-school child	-0.1959	0.2029	<i>Threshold parameters</i>		
Owner-occupier	0.2515	0.2361	$\Gamma_1^0$	-1.2788	0.4489
Private rental	0.0997	0.2493	$\Gamma_2^0$	-0.3627	0.4483
Housing equity	0.1218	0.0978	$\Gamma_3^0$	1.2672	0.4468
Household income pc	0.4057	0.0663	$\Gamma_4^0$	2.4719	0.4477
Income share	0.3604	0.1669			
Birth cohort	-0.1401	0.0461			
Birth cohort <sup>2</sup>	0.0344	0.0157			