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COMPARISONS OF INCOME MOBILITY PROFILES

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ABSTRACT

Methods are developed for income mobility comparisons between countries or between population subgroups based on the construction of mobility profiles. Mobility profiles provide an evocative picture of both the magnitude of income changes in a population, and its distribution across the income range. Comparisons of mobility profiles permit assessments in which mobility among the poor is given greater weight than mobility among the rich. Non-intersection of mobility profiles is shown to correspond with unambiguous rankings according to a large class of functions for the social evaluation of mobility. Particular focus is put on generalized Gini social evaluation functions from which summary indices are derived to obtain complete orderings. An empirical application based on the European Community Household Panel survey illustrates the usefulness of the methods and show how they can be used to shed new light on “pro-poor growth” issues.

NON-TECHNICAL SUMMARY

Information about social mobility is important for assessing the prospects of individuals. It is not enough to observe the distribution of income at one point in time, or even at several points. These snapshot views do not tell us about how the income of each person evolves over time. To examine this, one needs longitudinal data, and these are increasingly available. One also needs appropriate tools with which to summarize mobility and, in contrast to the analysis of poverty or inequality at a point in time, there is relatively little consensus to date about how to describe and evaluate income mobility.

This paper argues that many of the most frequently used summary indices of mobility can be expressed as population averages of a statistic defined at the individual-level that captures the degree of mobility by that individual, for example the proportionate change in her income over time, the absolute value of the change, the change in her rank in the income distribution, etc. In the averaging process used to compile the overall summary measure, these indices treat each individual in the same way, and simply add up the individual mobilities. However, it is often implicit in the debate about mobility that it is a better thing from a societal point of view if the pattern of mobility is such that relatively poor people climb up the income ladder (and are replaced by others at the bottom of the distribution), rather than mobility being driven by improvements for the relatively rich. In other words, social evaluations of overall mobility depend on how income changes are distributed relative to people's positions in the base period income distribution -- whether income growth is relatively 'pro-poor' or not.

This paper develops the concept of a "mobility profile" to address these issues. It is shown, first, that mobility profiles provide an evocative graphical summary of the pattern of mobility according to base-period position, thereby also clarifying the relationship between different mobility concepts and indicating why different overall summary indices may lead to different conclusions about the extent of mobility. Second, the paper offers a framework for making comparisons of income mobility based on normative considerations that go beyond the comparison of population averages. It shows how non-intersection of mobility profiles for two societies is equivalent to rankings of their mobility according to a large class of summary mobility indices. A new family of mobility measures is also proposed.

The methods are applied to data for ten EU countries between 1996 and 2001 taken from the European Community Household Panel survey. It is shown that individual income growth in countries such as Ireland and Spain was particularly large among those at the bottom of the income distribution. The overall average level of income growth was much the same in Portugal but, by contrast with the other countries, it was much less 'pro-poor'. Portugal therefore performs worse than the other countries according to social evaluation functions in which concerns about the distribution of mobility across the income distribution are taken into account.

1 Introduction

The relevance of observing social mobility when one is willing to capture the long term prospects of people in a society has long been recognized; see, for example, Hart (1976) or Schiller (1977). It is not enough to observe aspects of the distribution of income such as inequality, poverty or the mean average income at one point in time. It is not even sufficient to make repeated observations of such an information. We also need to see the evolution of people's income *within* the distribution over time. For example, it is often argued that high 'snapshot' inequality is less of a concern if it is accompanied by high mobility: people's long term fortunes will tend to equalize themselves by the effect of mobility because those with unfavourable positions today will not be the same tomorrow. Similarly, as discussed by Gottschalk (1997), a rise in inequality may be compensated by a concomitant rise in mobility.

This recognition, coupled with the increasing availability of longitudinal data on income for households and individuals, has led to the emergence of a large body of theoretical and applied literature. Analysts typically proceed by (i) gathering panel data on income at two time periods for a sample of individuals, and (ii) computing summary statistics on the bivariate distribution of incomes that reflect some notion of mobility.¹ Theorists have developed a wide array of such summary statistics to choose from based on explicit axiomatic foundations, but, in contrast to the field of poverty or inequality measurement, a consensus about which summary measure should be used to best capture mobility is strikingly lacking (see Maasoumi (1998) or Fields & Ok (1999a) for surveys). As forcefully argued in Fields (2000) or Buchinsky *et al.* (2003), this is inherent to the "multi-faceted nature of mobility" which can be apprehended from different perspectives (e.g. directly as a magnitude of income changes or from changes in people's ranks, or indirectly from the inter-temporal independence of incomes or from comparisons inequality

¹Applications to the case of more than two time periods still represent a relatively small fraction of the analyses; see Burkhauser & Poupore (1997) or Maasoumi & Trede (2001). Approaches based on dominance criteria rather than summary indices (similar to Lorenz dominance used in static income distribution analyses) have been developed but remain rarely used (Dardanoni, 1993, Mitra & Ok, 1998, Fields *et al.*, 2002).

in short-term and long-term incomes).² It remains that the interpretation of, sometimes contradicting, results based on different approaches is frequently revealing uneasy.³ This paper proposes an additional instrument in the researcher’s toolbox to help apprehending income mobility, in particular when there is interest in making comparisons of mobility between two (or more) populations or over time. Pros and cons of capturing specific notions of mobility are not discussed directly. Instead, general methods that can be applied in a number of different settings are presented (as in Fields *et al.* (2002) or Van Kerm (2004), for example). The methods help depicting the underlying structure of mobility in finer details than what is typically done and may thereby help clarifying the relationship between different concepts and indicating why different aggregate indices lead to different conclusions.

Many of the most frequently used mobility indices can be expressed as population averages of some statistic defined at the individual-level and which capture the degree of “mobility” experienced by a person:

$$M(X, Y) = \int_{z_-}^{z_+} \int_{z_-}^{z_+} d(x, y; F) dF(x, y). \quad (1)$$

where X and Y are two correlated random variables (with joint cumulative distribution F and support $[z_-, z_+]$) that describe the incomes in a society at two time periods (a person’s base and final incomes is a realization (x, y) from F). $M(X, Y)$ is a statistic that captures the extent of mobility in the society, and $d(x, y; F)$ is a statistic that captures the degree of mobility experienced by an agent with incomes (x, y) . As shown in Cecchi & Dardanoni (2002), this class of measures covers many widely employed indices such as the measures advocated in Fields & Ok (1996, 1999b) or D’Agostino & Dardanoni (2006), measures based on the Pearson correlation coefficient like the Hart index (Hart, 1976, Shorrocks, 1993), measures based on the Spearman rank correlation coefficient, as well as ‘average jump’ statistics. These various measures can be shown to differ only in their definition of

²Fields (2000) identifies five different notions of income mobility. Differences in what summary indices capture can generally be tracked down to their specific properties and their respective axiomatic bases; see, *inter alia*, Chakravarty *et al.* (1985), Cowell (1985), Shorrocks (1993), Fields & Ok (1996), Mitra & Ok (1998), Fields & Ok (1999b), D’Agostino & Dardanoni (2006).

³See, for example, Cecchi & Dardanoni (2002), Buchinsky *et al.* (2003) or Van Kerm (2004) for illustrations of the sensitivity of results to the choice of mobility measures.

$d(x, y; F)$. For example, Fields & Ok (1996) advocate the use of

$$d(x, y; F) = |y - x|,$$

Fields & Ok (1999b) provide axiomatic foundations for the use of

$$d(x, y; F) = |\log(y) - \log(x)|,$$

and Cecchi & Dardanoni (2002) show how the Hart measure of mobility can be obtained by setting

$$d(x, y; F) = \frac{1}{2} \left(\left(\frac{\log(x) - m_X}{s_X} \right) - \left(\frac{\log(y) - m_Y}{s_Y} \right) \right)^2$$

where m_X , m_Y , s_X , s_Y are the means and standard deviations of log-incomes in each time period. Similarly, one can set

$$d(x, y; F) = (\log(y) - \log(x)),$$

as in Fields *et al.* (2002) and Buchinsky *et al.* (2003), and the aggregate mobility index becomes sensitive to the sign of the income changes and captures the average growth in log-income in the society. Fields *et al.* (2002) and Buchinsky *et al.* (2003) also consider functions such as the absolute number of centiles changed (reflecting *positional movement*), changes in an individual's share of total income (reflecting *share movement*), or the absolute value of changes in people's income (reflecting *income flux*).

Indices of the type $M(X, Y)$ have much to offer for empirical applications. Most important is probably their simplicity. Mobility is measured in two steps. In a first step –the ‘identification’ step–, the analyst selects a d function and measures the degree of mobility experienced by each person in the society. In a second step –the ‘aggregation’ step–, the individual mobilities are aggregated in a summary measure of mobility. This is a familiar procedure, reminiscent of approaches in the measurement of poverty (Sen, 1976) or discrimination (Jenkins, 1994). Clearly, the definition of the d function is crucial in this approach and consideration of the axioms underlying different choice is important.⁴ But once we think in terms of these two distinct steps, we are also lead to question the appropriateness of the aggregation rule. $M(X, Y)$ -like indices put all individuals on the

⁴D'Agostino & Dardanoni (2006), for example, provide a thorough discussion of the distinction between absolute and relative indices.

same footing and simply add up the individual mobilities. However, it is often implicit in the debate about mobility that mobility is good, from a societal point of view, if the pattern of mobility is such that relatively poor people climb up the income ladder (and are replaced by others at the bottom of the distribution), rather than mobility being driven by improvements for the relatively rich. In other words, social evaluations of overall mobility depend on how income changes are distributed relative to people's positions in the base period income distribution. It is easy to see that the most frequently used indices in the class of $M(X, Y)$ are not informative about this because, in the averaging process, they treat income changes of the (initially) rich in the same way as the income changes of the (initially) poor. The purpose of this paper is to circumvent this limitation and to present simple methods that (i) allow a depiction in finer details of the underlying structure of mobility, and (ii) offer a framework for making comparisons of mobility levels across societies based on normative considerations that go beyond the comparison of averages, while retaining the simplicity of the measures of the $M(X, Y)$ class.

Not all commonly used mobility measures belong to the class considered here. The most prominent measures that do not belong to this class are indices that capture mobility by comparing inequality in the short and in the long run (Shorrocks, 1978, Chakravarty *et al.*, 1985, Maasoumi & Zandvakili, 1986, Fields, 2005). Methods for analyzing these indices in ways similar to what is suggested here are developed in Schluter & Van de Gaer (2002) and Schluter & Trede (2003).

For clarity, I refer to "income" as the economic variable of interest throughout the paper, but the discussion and methods apply to any continuous measures of economic 'stature', such as wages and earnings, wealth, consumption or even to composite indicators of multi-dimensional economic resources. Discussion is also cast in terms of intra-generational mobility (i.e. following individuals over time), but the methods apply to inter-generational issues as well (when following dynasties and looking at the transmission of economic stature across generations).

The paper is structured as follows. Section 2 introduces the *mobility profile* which, it is argued, offers an appealing tool for depicting the structure of income mobility. Section 3 outlines normative underpinnings of the approach and presents three simple dominance checks that allow making comparisons of mobility that take into account the distribution

of income mobility along the income distribution. Section 4 illustrates the methods with an application to survey data for ten European countries. A discussion ends the paper.

2 Mobility profiles

As explained in the Introduction, the building block of the paper is to capture the degree of mobility experienced by an agent with incomes (x, y) with a (scalar) function $d(x, y; F)$ referred to as an agent's individual mobility. With F being an argument of the function, this approach is general because individual mobility may be partly determined by the evolution of other people's incomes in the society. It is assumed from now on that that a d function has been selected by the analyst. For illustrative purposes, I will use the change in log-income; more elaborated choices can be associated to mobility indices with more nuanced properties, but the methods are generally applicable to any d function.

Once the d function is defined, computing the indices defined by (1) is straightforward, yet, as argued above, it also discards much relevant information. Fortunately, the mathematical simplicity of this class permits to investigate the patterns and sources of mobility in detail. The stepping stone of this paper is to express the overall expected individual mobility (that is, the mobility index $M(X, Y)$) as a functional of a *mobility profile* which plots the expected individual mobility *conditionally* on a person's position in the base period distribution. In other words, separate mobility levels are estimated for each position in the initial income distribution, and the resulting mobility profile is plotted to obtain an evocative picture of the repartition of mobility levels across different parts of the distribution. Denote by F_X and F_Y the two marginal distribution functions, and $F_{X|y}$ and $F_{Y|x}$ the conditional distributions derived from F . The method simply consists in rewriting (1) as follows:

$$\begin{aligned}
 M(X, Y) &= \int_{z_-}^{z_+} \left(\int_{z_-}^{z_+} d(x, y; F) dF_{Y|x}(y) \right) dF_X(x) \\
 &\equiv \int_{z_-}^{z_+} m(X, Y|X = x) dF_X(x) \\
 &= \int_0^1 m(X, Y|X = x(p)) dp
 \end{aligned} \tag{2}$$

where $p = F_X(x)$ is the (normalized) rank corresponding to income x in the base period distribution, and $x(p) = F_X^{-1}(p)$ is the income corresponding to rank p in the base period

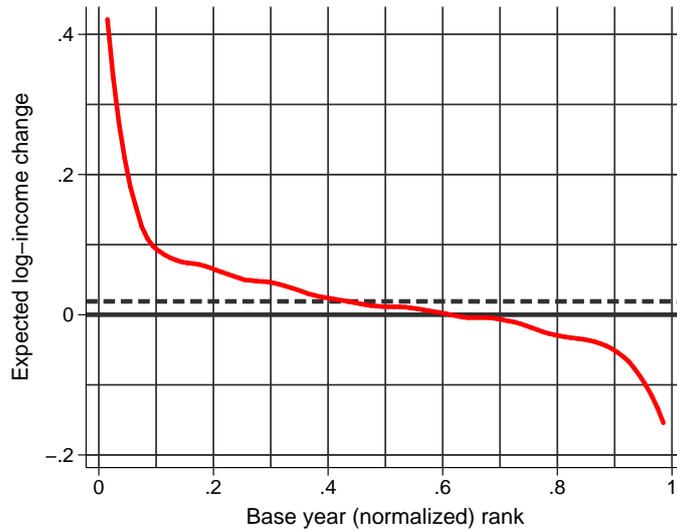
distribution. Focus is shifted from the overall expectation of individual income mobilities to their conditional expectations given agents' starting positions in the income ladder, $m(X, Y|X = x(p))$, to which, for notational clarity, I will henceforth refer as $m(p)$.

Plotting $m(p)$ against p gives the *mobility profile* –henceforth MP– and provides an evocative picture of the underlying mobility structure. Because the ranks are uniformly distributed, the aggregate mobility level corresponding to any MP is given by the area under the curve: integrating the MP produces $M(X, Y)$. The relationship between the profile and the associated aggregate measure is therefore visually direct. It makes it straightforward to identify the portions of the distribution that have the largest impact on the overall level of mobility, whether it is the rich, the poor or the middle class that experience the greatest mobility and to assess their respective impact on the overall mobility level.⁵

To fix ideas, an example of MP is presented in Figure 1. This is a profile of income mobility with $d(x, y; F) = \log(y) - \log(x)$ estimated from data for Italy drawn from the European Community Household Panel survey (see Section 4 for details). The aggregate index is estimated at 0.02 (marked by the dashed line on the plot). It is striking from the MP that this number hides large variation in the individual experiences depending on where one starts from. Because the aggregate index is obtained by simple integration of the profile, we directly see that, in fact, it is the result of substantial income gains (positive individual mobility) of people at the bottom of the distribution compensated by losses of people at the top end. Mobility, in this example, clearly involves a catching up of poor people. The aggregate index alone does not identify this and is indicative of neither the mobility among the poorest nor among the richest.

⁵The direct link between the MP and $M(X, Y)$ distinguishes this approach from similar methods applied in Trede (1998) or Fields *et al.* (2003) who condition on initial income *levels* rather than income *ranks*, with the consequence that the aggregate measure depends on information not available in the picture, namely the density distribution of base-period incomes.

Figure 1: An example of mobility profile for the expected change in log-income



3 An extended class of mobility measures

A social evaluation of mobility

Measures expressed as $M(X, Y)$ do not, in general, give any special importance to who experience the greatest income mobility (by virtue of a form of anonymity or symmetry principle). Whether it is the rich or the poor that have the largest income changes (as measured by the d function) is irrelevant in measuring mobility. One may yet want to give greater weight to income changes for the poor since this indicates opportunity of escaping from an undesirable position. As emphasized by Dardanoni (1993, p.377) “(t)he symmetry (or anonymity) assumption is employed to guarantee that all individuals in the society are treated equally regardless to their ‘labeling.’ However, in this dynamic context there is a natural ‘label’ for each individual, namely their starting position in the income ranking.” The formulation of $M(X, Y)$ as in (2) combined with a concern about how mobility is distributed along the income ladder –in particular preference for observing greater mobility at the bottom of the distribution– leads to a straightforward generalization of this class of mobility measures:

$$M^w(X, Y) = \int_0^1 w(p) m(p) dp, \quad (3)$$

where $w(p)$ is an “ethical” weight function that determines the importance put on individuals of rank p when assessing overall mobility, and $\int w(p)dp = 1$. $M(X, Y)$ corresponds to a flat weighting scheme $w(p) = 1$ and other choices redistribute the weights differently across the population to reflect different views about how important are the experiences of people on different segments of the income ladder in aggregating mobility.⁶ Expression (3) is a form of Yaari social evaluation function (Yaari, 1987, 1988) where social evaluation is additive and linear in individual mobilities. Such a Yaari social evaluation function is particularly useful in this setting because the analyst is able to distinguish the setting of her preferences about the mobility of whom matters more (in $w(p)$), and the measurement of individual income mobilities themselves (in $m(p)$ and d); thereby providing a flexible framework to accommodate the variety of views about *what is* mobility.⁷

It seems natural to consider weights that decrease with p as this means that the ‘social marginal value’ of an individual’s mobility (which is given by $w(p)$) is higher the lower she starts in the base period income ladder. This is very similar in spirit to the principles used by Dardanoni (1993).⁸ However, without further qualification, the class $M^w(X, Y)$ probably remains too broad to be useful in practice. The next sub-sections identify two strategies to address this issue: selecting a well-known weight function likely to obtain broad support by analysts or looking for dominance relations that allow ordinal rankings of $M^w(X, Y)$ for large classes of $w(p)$ functions.

⁶This approach is closely related to the procedures presented in Schluter & Van de Gaer (2002) and Schluter & Trede (2003), but the approach is applied here in the different and largely simplified context of mobility measures defined by the class (1).

⁷Arguably, one could alternatively re-define $d^*(x, y; F) \equiv w(F_X^{-1}(x)) * d(x, y; F)$ in which case the class $M(X, Y)$ is no different from $M^w(X, Y)$. However, we will see that keeping separate the specification of the individual mobilities and the social weight associated to it allows us to make use of dominance relationships that do not require to pin down exactly the shape of w .

⁸Note that if there is a strong case for decreasing $w(p)$ when d is a directed measure –think for example of $d(x, y) = (y - x)$ –, this may be more open to question when d is ‘non-directed’ –for example $d(x, y) = |y - x|$ –.

The generalized Gini weight function

One specific weight function that appears particularly well suited for an implementation of $M^w(X, Y)$ is the weighting function which is implicit to (generalized) Gini coefficients –probably the most widely used inequality index–,

$$w(p) = v(1-p)^{v-1} \quad (4)$$

with $v > 1$. In the case of the Gini inequality index, weighted integration is over individual income shares while one integrates individual mobilities in the present context. Decreasing weight is attached to individuals when moving from poorest to richest, depending on their rank in the distribution (Donaldson & Weymark, 1980, Weymark, 1981, Donaldson & Weymark, 1983, Yitzhaki, 1983). The speed of decrease of the weight is controlled by v : $v = 2$ leads to weights that decrease linearly with p from 2 to 0 (this is the classical Gini index), $1 < v < 2$ gives a concave function, and $v > 2$ leads to a convex function.

Note that by selecting a specific weight function with $\int w(p)dp = 1$, $M^w(X, Y)$ has a neat interpretation as the “equally distributed equivalent” of $M(X, Y)$ (EDEM). The EDEM gives the individual mobility level that, if it were uniformly distributed along all base period ranks (a flat mobility profile), would have the same social value as the observed situation where expected individuals mobilities vary with the base period position. Typically, if $m(p)$ is decreasing with p and we give more weight to poorer individuals, then EDEM will be higher than $M(X, Y)$. The difference between the two statistics reflect by how much social welfare is improved by the unequal distribution of mobility experiences along the income ladder. It follows that the statistics $M^w(X, Y)/M(X, Y) - 1$ and $M^w(X, Y) - M(X, Y)$ can be used to provide assessments of the, respectively relative and absolute, welfare gains due to the ‘asymmetry’ of the distribution of mobility along the income line.

Dominance relations

Although (3) may provide a useful extension of simple mobility measures, it remains that the choice of the weight function is potentially arbitrary. The generalized Gini weight function is only one of many potential choices, and even within this class, one needs to

make a choice about the v parameter. However, the simple structure of $M^w(X, Y)$ leads to three dominance relations based on comparisons of MPs which can be invoked in order to compare mobility in two societies without the need to actually specify the shape of the weight function, $w(p)$.

First, if the MP of society A lies nowhere below the MP of society B then the social evaluation of mobility in society A is at least as high as in society B for any non-negative weight function. In other words, if $\forall p \in [0, 1], m^A(p) \geq m^B(p)$, then $M^w(X^A, Y^A) \geq M^w(X^B, Y^B)$ provided $w(p) \geq 0$. If, in addition, there exists $q \in [0, 1]$ where $w(q) > 0$ and $m^A(q) > m^B(q)$, then $M^w(X^A, Y^A) > M^w(X^B, Y^B)$. Let us call this Type A dominance. This is a strict criterion because, in order to be satisfied, the expected individual mobility in society A must be higher (or equal) to that in society B for any starting income rank. But in this case, it is clear that whatever one's concern about the mobility of whom matters more, the ordering of societies will remain the same. This bears much similarity to first-order stochastic dominance which is widely used in the context of income distribution comparisons (Hadar & Russell, 1969).

The second dominance relation is obtained by comparing integrals of the MPs. If the integral over $[0, p]$ of the MP of society A lies nowhere below the integral over $[0, p]$ of the MP of society B, then the social evaluation of mobility in society A is at least as high as in society B for any non-negative and non-increasing weight function. Define $G(p) = \int_0^p m(q) dq$. If $\forall p \in [0, 1], G^A(p) \geq G^B(p)$, then $M^w(X^A, Y^A) \geq M^w(X^B, Y^B)$ provided $w(p) \geq 0$ and $w'(p) \leq 0$. If, in addition, there exists $q \in [0, 1]$ where $w'(q) < 0$ and $G^A(q) > G^B(q)$, then $M^w(X^A, Y^A) > M^w(X^B, Y^B)$. Let us call this Type B dominance. (Derivation of this result is provided in the appendix.) This is a less stringent criterion than Type A dominance. It means that if the average expected mobility of people with a base period rank no greater than p is at least as high in society A than in society B, for any choice of p , then mobility in society A will be at least as high as in society B, according to M^w with non-increasing weights. This is similar to second-order stochastic dominance.

The third dominance relation proceeds by integrating further the MPs. If the integral of the integral over $[0, p]$ of the MP of society A lies nowhere below the integral of the integral over $[0, p]$ of the MP of society B, then the social evaluation of

mobility in society A is at least as high as in society B for any non-negative, non-increasing, non-concave weight function and with $w(1) = 0$. Define $H(p) = \int_0^p G(q) dq$. If $\forall p \in [0, 1]$, $H^A(p) dp \geq H^B(p) dp$, then $M^w(X^A, Y^A) \geq M^w(X^B, Y^B)$ provided $w(p) \geq 0$, $w'(p) \leq 0$, $w''(p) \geq 0$ and $w(1) = 0$. Note that the last restriction on the shape of the weight function, $w(1) = 0$, can be relaxed if the additional condition $G^A(1) \geq G^B(1)$ is satisfied. As before, if, in addition, there exists $q \in [0, 1]$ where $w''(q) > 0$ and $H^A(q) > H^B(q)$, then $M^w(X^A, Y^A) > M^w(X^B, Y^B)$. Let us call this Type C dominance. (Derivation of this result is also provided in the appendix.) This is again a less stringent criterion than Type A and Type B dominance, but additionally imposes non-concavity of the weight function: the weights must be decreasing at a non-increasing rate. Note also the additional requirement that $w(1) = 0$ or the additional condition $G^A(1) \geq G^B(1)$ which is absent in Type A and Type B dominance.

The strength of these simple results is that partial orderings according to M^w can be obtained by comparing MPs without explicitly specifying a form for $w(p)$. If none of the three conditions hold, the mobility comparisons will depend on the specific functions used such as, for example, the weighting scheme underlying the generalized Gini inequality measure with selected v parameters. Note that the generalized Gini weights with $v > 1$ satisfy the four conditions required for using Type C dominance. This implies that if society A dominates society B according to Type C dominance, then M^w mobility indices with generalized Gini weights will be higher in society A for any choice of $v > 1$.

Dominance relations in mobility analysis are also proposed in Fields *et al.* (2002). It is important to realize that there is a fundamental difference with the present approach. In the framework developed here, the contribution to the social evaluation of mobility of an individual's $d(x, y; F)$ is determined by the rank of the person in the base period distribution whereas it is determined by the value of $d(x, y; F)$ itself in Fields *et al.* (2002). Fields *et al.* (2002) focus on classical stochastic dominance relations in the distribution of $d(x, y; F)$ with an 'anonymity principle', whereas we label individuals according to their base period income and let their social marginal utility depend on the label. These two approaches answer different questions and should be seen as complementary rather than substitutes.

A pro-poor growth interpretation

Social evaluation of mobility of the form (3) is very closely related to the measurement of ‘pro-poor growth’ (see e.g. Foster & Székely, 2000, Ravallion & Chen, 2003, Son, 2004). Consider cases such as $d(x, y; F) = y - x$ or $d(x, y; F) = \log(y) - \log(x)$ where the mobility simply reflects the growth of a person’s income (*directional income changes* in Fields *et al.* (2002)’s classification). Income mobility measures $M^w(X, Y)$ are weighted averages of individual income growth with larger weight given to poor individuals. This can clearly give rise to interpretation of the results in terms of pro-poor growth, since the more income growth is concentrated among (initially) poor people, the higher is $M^w(X, Y)$. Such a general point of view is the starting point in the literature of pro-poor growth too.⁹ There is however one key difference with the existing literature on pro-poor growth: it is income changes of individuals that are tracked, rather than income changes for income groups such as the poor or the income at given percentiles (as in Ravallion & Chen (2003) or Son (2004)). Whereas the pro-poor growth literature looks at change in the marginal distributions, the present approach considers the full bivariate distribution. To put it differently, if we focus on people with low income, we attempt to quantify what happens to the people starting with a low income, whereas much of the literature looking at pro-poor growth assesses whether the people in the low income group next year are better off or worse off than those who are in this group this year. The former approach recognizes that membership of income groups such as the poor and the rich changes over time, whereas this is not relevant for the latter.

This important distinction is discussed at length in Jenkins & Van Kerm (2006) who show that changes in the S-Gini coefficient over time can be meaningfully decomposed into two terms reflecting respectively (i) the progressivity of the income growth and (ii) the associated effect of re-ranking. This provides a framework to analyze jointly changes in income inequality, the progressivity of income growth (pro-poor growth), and mobility in the form of re-ranking. Jenkins & Van Kerm’s (2006) progressivity term is in fact a special case of $M^w(X, Y)$ where $d(x, y; F) = y/\mu_Y - x/\mu_X$ (capturing *income share movement* in Fields *et al.* (2002)’s classification) and the generalized Gini weighting function is adopted.

⁹For example, Foster & Székely (2000) use generalized means to this effect.

4 The pattern of family income growth in 10 EU countries

This section illustrates the application of the methods using data from the public-use file of the European Community Household Panel survey (ECHP). The application depicts and compares the profiles of income growth in ten EU countries over the period 1996-2001. This is probably the most direct application of the methods discussed above: individual mobilities are measured by $d(x, y; F) = \log(y) - \log(x)$; the mobility profile therefore exhibits the expected income growth rate as we move along the parade of people when ordered from poor to rich, and the social evaluation of mobility bears a “pro-poor growth” interpretation.

Data

The ECHP is a standardized multi-purpose annual longitudinal survey providing comparable micro-data about living conditions of the population living in EU-15 Member States. The topics covered in the survey include income, employment, housing, health, and education. An harmonized (E.U.-wide) questionnaire was designed at Eurostat, and the survey was implemented in each Member States by ‘National Data Collection Units’. The public-use database is derived from the data collected in each of the Member States and is created, maintained and centrally distributed by Eurostat.¹⁰ The survey therefore provides individual-level data on income and demographics which are comparable across countries and over time.

Sample data from ten countries are taken from the last five waves of the April 2004 release of the ECHP (covering the period 1996-2001).¹¹ Each household income datum is an estimate, provided by the person responsible for responding to the household question-

¹⁰See Eurostat (2003) or Lehmann & Wirtz (2003) for more information on the database, and Peracchi (2002) for an independent critical review. Also see the Eurostat website: <http://ec.europa.eu/eurostat>.

¹¹Results are not reported for Germany, Luxembourg, Sweden and the United Kingdom for which data are not derived from the original questionnaire but re-constructed *ex post* from other surveys, nor do we report results for France for which income data are reported before taxes where net income is reported in other countries.

naire, of the total current net monthly disposable income of the household.¹² All incomes are expressed at 1995 prices and are converted to a common currency using purchasing power parities. The modified OECD equivalence scale is applied to take into account differences in needs and economies of scale in larger households, and each individual in the household is attributed the single-adult equivalent income obtained after the application of the equivalence scale. To bound the potential leverage of extreme observations, equivalent incomes are top-coded at 5000 euros per month and bottom-coded at 75 euros per month at 1995 prices (each threshold affects the incomes of less than 0.25% of respondents in our sample).

Data from waves 3 to 8 are pooled and year-on-year mobility is considered. To prevent cross-country differences to emerge because of different sample compositions over time, the data are re-weighted so that each wave is equally represented. Typically, because of panel attrition, observations from earlier waves receive a lower weight than observations from later waves.¹³

Estimation methods

Implementation of the methods require reliable estimation of $m(p)$ which is, in fact, a conditional expectation (see (2)). The estimation problem is therefore one of regression of $d(x, y; F)$ on p . Obviously, as we do not want to impose *a priori* parametric restrictions on the estimates, non-parametric regression function estimation methods are called for. A wide array of techniques have been proposed recently and various methods could be fruitfully applied in this context; e.g., Nadaraya-Watson kernel regressions, local polynomial fitting, or smoothing splines (see Härdle (1990), Fan & Gijbels (1996), Pagan & Ullah (1999) for reviews).

Locally weighted regression (LOESS) introduced by Cleveland (1979) is applied in this illustration. The method is detailed in Cleveland (1979), Cleveland & Grosse (1991), Cleveland *et al.* (1991) and Hastie & Loader (1993). As many non-parametric methods,

¹²The respondent is asked to take into consideration all income sources from all household members in his global assessment of household income.

¹³These weights are used in conjunction with the standard sample weights provided in the ECHP which correct other forms of differential non-response.

the technique involves determining a local neighbourhood around p and using sample observations falling in this neighbourhood to estimate $m(p)$ using (locally) weighted least squares regression. Ease of implementation is the first advantage of this method. The second advantage is that the methods, using local polynomial fitting, correctly handle estimation at the boundary of the support of p (as opposed to Nadaraya-Watson estimators, for example). The third advantage is the availability of a ‘robust’ version of the technique. The robust LOESS estimation guards against deviant points that may affect estimation of $m(p)$ by attaching smaller weights in the estimation process to outlying observations (i.e. observations with an extremely large absolute ‘distance’ between initial and final income). Applying the robust procedure permits to keep under control the potential effect of data contamination.¹⁴

Both local linear and local quadratic LOESS fitting have been tested. The quadratic fitting did not yield distinctively better results, hence only the results obtained by the less computationally demanding linear fitting algorithm are presented. A ‘nearest neighbours’ bandwidth with sample fractions of between 15% or 22% were used as nearest-neighbours, depending on applications. Sensitivity analysis suggested that the results obtained are robust to alternative choices of non-parametric local regression smoothers. Because initial income ranks are uniformly distributed, the estimates obtained do not differ from alternative approaches based on fixed bandwidth.

The profile of family income growth in the EU

Figure 2 depicts the estimated mobility profile for the ten countries analyzed. The overall pattern is the same in all countries and show regression to the mean: people among the poorest 10 percent achieve substantial income growth rates and account for the largest share of the aggregate mobility. The expected growth rate then falls regularly up to the highest 10 percent of the population among whom the average growth rate is low (and often negative). The overall mean growth rate is positive but there is clearly substantial variation in the individual experiences depending on the starting income rank. Looking

¹⁴See Cowell & Schluter (1998) on the estimation of income mobility measures with dirty data. Indirect estimation of $M(X, Y)$ by integration of the robust estimate of $m(p)$ makes it also robust to contamination, in contrast to standard direct estimation based on unit record data.

just at the average income growth in a country conceals substantial information. Such MPs stress that analyses of poverty need to be complemented with mobility consideration given the substantial income growth among the poor. The overall pattern is common across all countries, but closer scrutiny also reveals cross-country differences in the shape of the profiles (to which we return when looking at dominance relationships.)

The MPs show that income grows faster for poor individuals, there is therefore evidence of a catching up. But the speed of convergence towards higher incomes, i.e. whether the regression to the mean implies substantial redistribution of incomes over time, can not be assessed directly from Figure 2 since the initial income levels corresponding to the different starting positions are not shown. Figure 3 provides this information. The plotted lines are (i) the base period income parade which shows the period 1 income corresponding to each base year percentile (solid line), and (ii) the expected future income for each base year percentile as given by $F^{-1}(p) \times \exp(m(p))$ where $F^{-1}(p)$ is the base period income at rank p and $m(p)$ is the expected growth of log income at rank p shown in Figure 2 (dashed line). Figure 3 reveals that, despite the magnitude of the income growth at the bottom of the distribution, the redistributive effect of mobility is relatively limited since the expected second period incomes are not high enough to lead to marked catching up.

Consider now cross-country differences in more details. It is apparent from the figures that, for example, the income growth among the poorest is higher in Ireland, Spain or Denmark than in most other countries, and that the expected income losses of the richest are much smaller in Portugal than in countries such as Greece or Denmark. The main strength of using MPs is that they lend themselves to meaningful dominance relations, and these will help us in assessing the differences in the performance of the various countries. Results for the three dominance relations are presented in Table 1. Blank entries indicate no dominance: the profiles (for Type A) or cumulated profiles (for Type B and Type C) of the two countries cross at least once. Entries with A, B or C indicate respectively Type A, Type B, or Type C dominance of the row country over the column country. (Remember that Type A dominance implies Type B and Type C dominance, and that Type B implies Type C.) Dominance of the column country over the row country is indicated by a minus sign.

Expectedly, evidence of Type A dominance is scarce: only Ireland is performing un-

Figure 2: Mobility profiles: Expected change in log-income

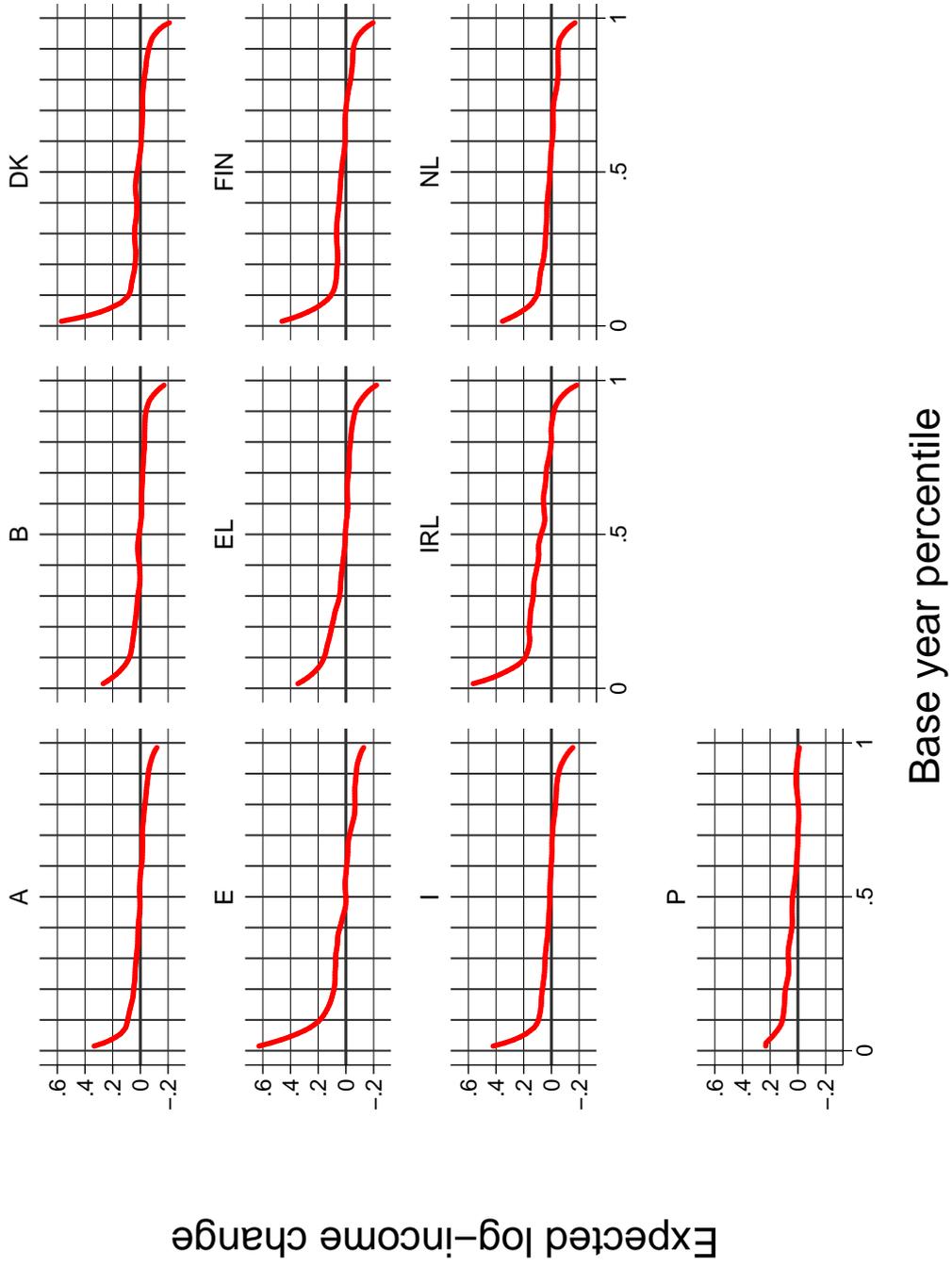
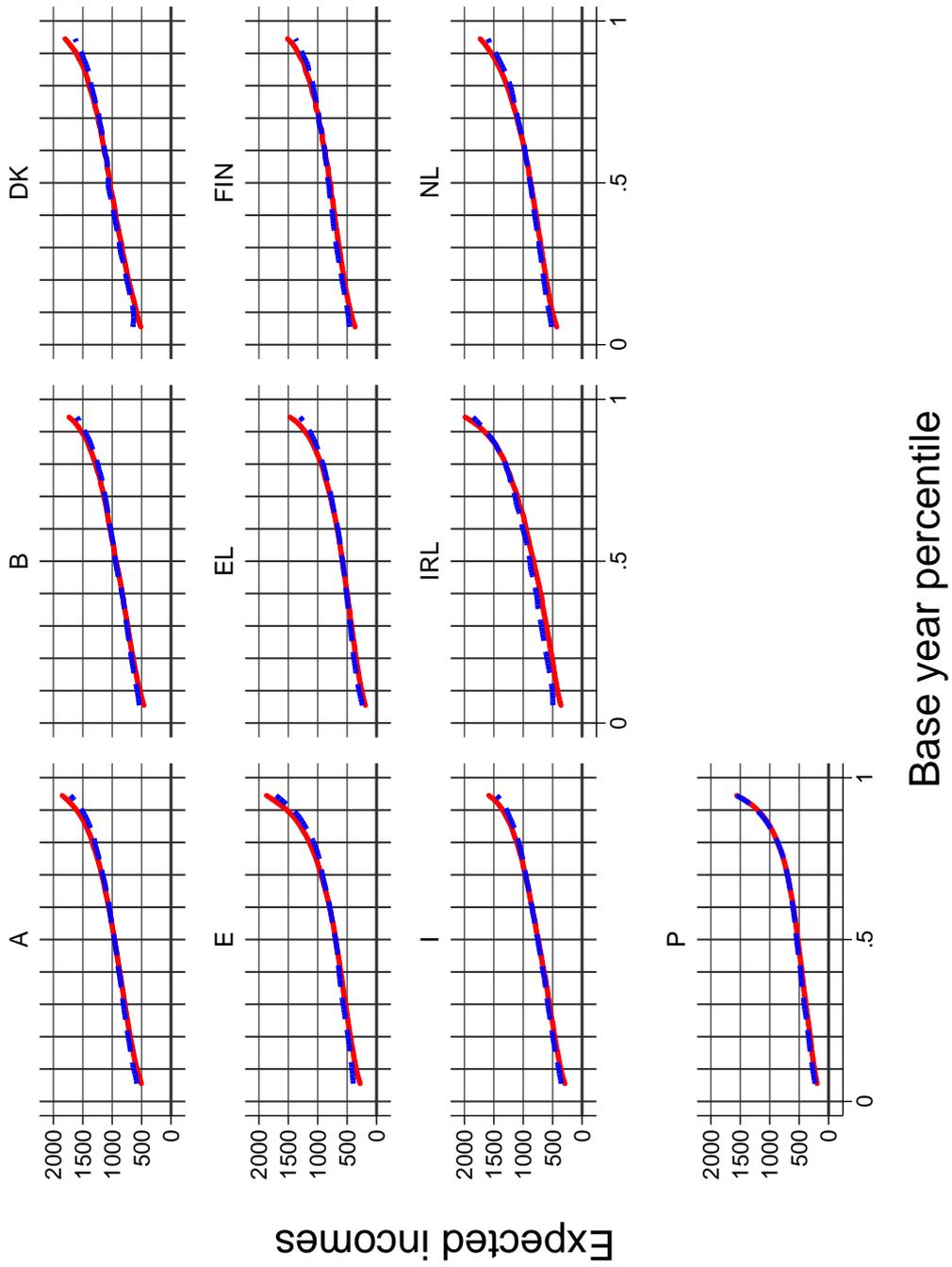


Figure 3: Base period income (solid line) and expected income at Base+1 as simulated from the expected change in log-income profile (dashed line)



ambiguously better than Greece and Finland in the period covered by the data – irrespective of one’s starting income rank, the expected income growth rate is higher in Ireland than in the other two countries. Type B dominance relations are much more frequent. The best performing countries appear to be Ireland and Spain which do better than 7 out of 9 countries and worse than none. On the contrary, Belgium is doing better than no country and is doing unambiguously worse than 8 out of 9 countries. Interestingly, note that Portugal, with its flat profile, is doing better than no country but is only dominated by Ireland. This means that it is not sufficient to put decreasing weights to income growth as we move up the income ladder to unambiguously assess the performance of Portugal. We need considering Type C dominance relations to break further ties. Portugal is now being dominated by several countries. However, it is clearly when moving from Type A to Type B dominance that most of the results appear.

Table 1: Dominance relations in expected change in log-income

	DK	NL	B	IRL	IT	GR	SP	PT	A	FIN
Denmark	–	B	B		B		-B		B	
Netherlands	-B	–	B	-B	-B		-B			-B
Belgium	-B	-B	–	-B	-B	-B	-B		-B	-B
Ireland		B	B	–	B	A		B	B	A
Italy	-B	B	B	-B	–		-B		B	-B
Greece			B	-A		–	-B	C		
Spain	B	B	B		B	B	–	C	B	B
Portugal				-B		-C	-C	–		-C
Austria	-B		B	-B	-B		-B		–	-B
Finland		B	B	-A	B		-B	C	B	–

Note: A, B and C respectively indicate Type A, Type B or Type C dominance of row country over columns country. Minus signs indicate dominance of the column country over row country.

To complete the analysis, equally distributed equivalent growth rates (EDEGR) (that is, mobility measures of the M^w -type) based on the generalized Gini weight function are reported in Table 2 for five values of the v parameter. $v = 1$ simply gives the average growth rate. The other parameters give more weight to income growth at low income ranks, and the higher the value of the parameter, the more convex is the weight function (i.e. the faster the decline of the “ethical” weight as we move to higher incomes). Adopting a particular parameter for these “ethically weighted” income growth rates allows us to obtain a complete ordering of countries. Because the growth rate is higher for low income

people, the EDEGR increases with the weight given to low income ranks.

Countries are ranked in Table 2 in decreasing order of average annual income growth rate ($v = 1$).¹⁵ Ireland stands on top with an average annual income growth rate estimated at 9% in real terms. Belgium and Austria, by contrast, reached less than 2%. But the ranking of countries is affected once we take into account the distribution of the income growth along the income distribution. The most striking situation is in Portugal. Portugal is second only to Ireland in terms of average growth rate, but its relative performance deteriorates substantially once ethical weights are introduced. It ends up just above Belgium and Austria for v at 4 or 5. It is indeed clear from the mobility profile of Portugal that growth was not much concentrated on the low income people. Conversely, the situation of Spain is improving substantially to catch up with the performance of Ireland with v at 4 or 5. The relative performance of Greece is also improved once the average growth figures are made sensitive to how growth is distributed. Belgium and Austria, on the other hand, remain the worst performers. Table 2 clearly indicates that looking at the average growth rate does not give a complete picture of the benefits of income growth, and that the relative performance of countries varies widely if one incorporates ‘pro-poor’ concerns about the distribution of the income growth.

Table 2: Equally distributed equivalent income growth rates

	v parameter:				
	1	2	3	4	5
Ireland	0.091	0.152	0.187	0.212	0.232
Portugal	0.048	0.075	0.094	0.107	0.116
Spain	0.044	0.110	0.153	0.186	0.214
Finland	0.039	0.087	0.115	0.135	0.152
Greece	0.027	0.079	0.110	0.132	0.149
Denmark	0.026	0.075	0.105	0.129	0.149
Italy	0.026	0.068	0.093	0.112	0.128
Netherlands	0.023	0.065	0.091	0.109	0.124
Austria	0.015	0.052	0.074	0.090	0.104
Belgium	0.011	0.044	0.064	0.078	0.091

¹⁵The EDEGR reported in Table 2 have been estimated by numerical integration of each country’s estimated mobility profile (plotted in Figure 2). See StataCorp (2005) for a description of the numerical integration algorithm.

Sensitivity to measurement error

Before concluding the illustration, it is worthwhile considering the potential effect of measurement error on the estimates obtained. Profiles such as shown in Figure 2 could indeed be driven by measurement error in individual incomes. If incomes are mis-measured, and if the measurement errors at the two time periods are not correlated, a spurious correlation between the base period income and the estimated individual distance $d(x_i, y_i; F)$ will be introduced because, e.g., a low observed x_i due to negative measurement error is more likely to be associated with a high income change (a correction effect), which will in turn lead to a high $d(x_i, y_i; F)$ if y_i is not affected by the underestimation of the base-period income. The $m(p)$ for low p 's will then be over-estimated and the estimated MP will be biased. The $m(p)$ for high incomes will be biased similarly, although the sign of the bias will depend on the particular distance function (whether $d(x, y; F)$ is a directed distance or not).¹⁶ This is a typical problem of regression to the mean due to income mis-measurement. (Note that it must be borne in mind that regression to the mean may also be a genuine feature of the income change process which we want to capture in the MP.)

One potential treatment to this problem when estimating the MP is to 'instrument' the base period income observations by estimating the MP using non-parametric methods that use data points in a local neighbourhood determined by *predictions* of base period income rather than observed base period income itself (see Fields *et al.*, 2003). Predictions can be based on a regression model, in which case, if the explanatory variables used to predict the base period income are uncorrelated with the measurement error, then the suggested treatment removes the spurious component of the association between the $m(p)$ and p . An alternative strategy is to use a proxy variable for base period income to estimate the base period rank. The proxy variable should be highly correlated with the income variable but should not be correlated with the measurement error in the latter.

The second route is followed here to check the sensitivity of the results. Estimation has been repeated with individuals ranked according to a 'proxy' of their base period income. This proxy is a measure of total annual incomes of households constructed by adding up the income components of each adults in the surveyed households, divided by twelve. Because the period over which the individual income components are measured

¹⁶Downward bias is expected in the present application looking at average income growth rate.

(the calendar year preceding the date of the survey) differs from the date of the assessment of the current monthly income, we use as a proxy the annual income variable constructed using year t survey responses or year $t + 1$ survey responses, whichever is closer to the current monthly income assessment.

Ranking individuals according to the proxy and re-computing the mobility profiles (that is, the expected change in log-income using the current monthly log-income change conditional on having a proxied rank p) yields the profiles of Figure 4. The overall picture is reassuring us that measurement error is not the main factor driving the shape of the mobility profile. Unexpectedly, the profiles are now flatter and the peaks in the expected income growth rates at the two very ends of the distribution are eroded. This suggests that mobility at the very highest and lowest incomes are affected by measurement error. However, the overall interpretation remains valid: the expected growth rates fall rapidly at first, then slowly thereafter, and expected growth rates turn negative for a significant fraction of the population in most countries as we move up to higher income ranks. However, the speed of convergence of poor individuals toward higher incomes appears to be somewhat smaller.

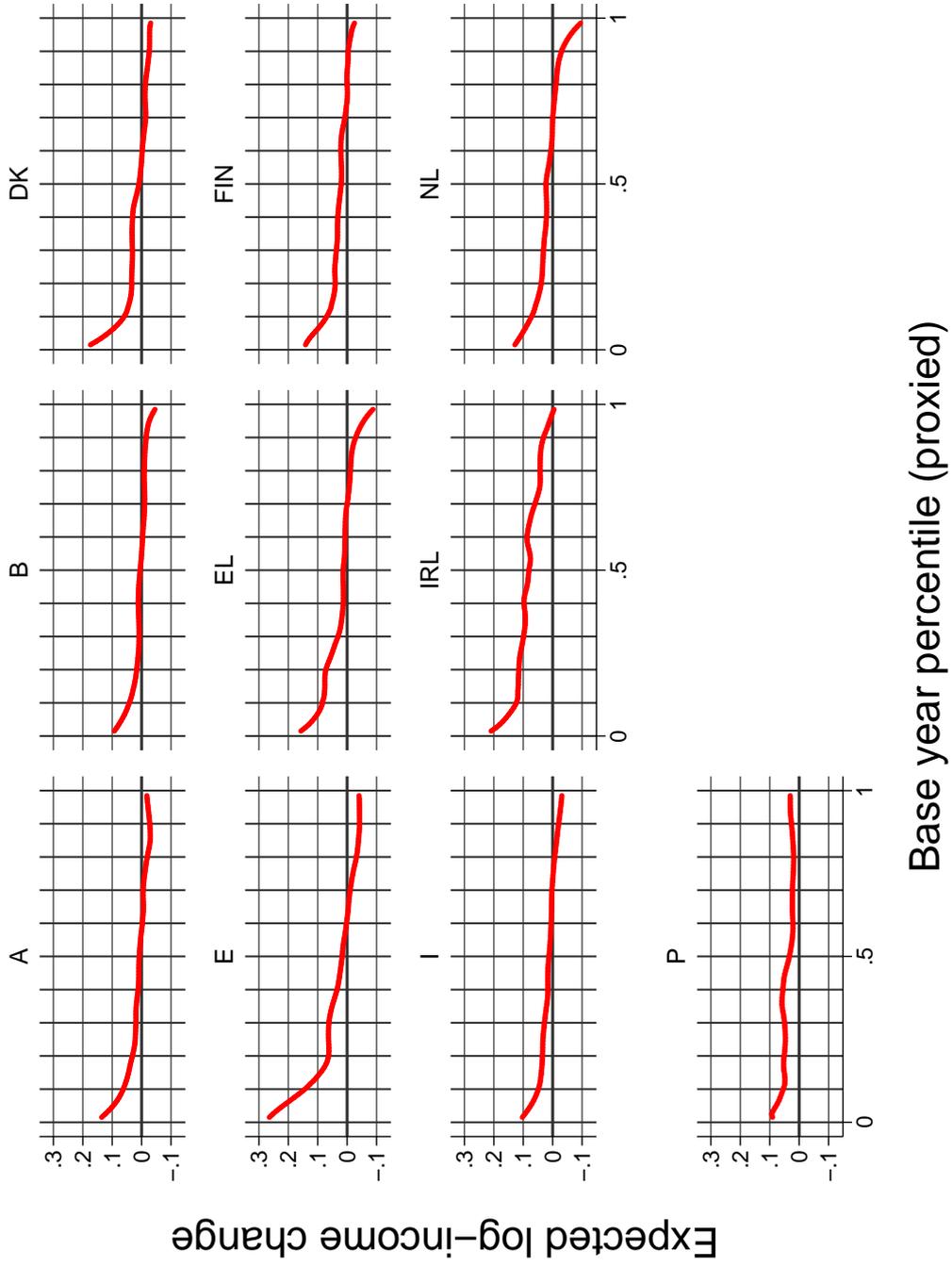
Table 3: Dominance relations in expected change in log-income with profiles corrected for measurement error

	DK	NL	B	IRL	IT	GR	SP	PT	A	FIN
Denmark	–		B	-A	B		-B		B	
Netherlands		–	B	-A	B	-B	-B			-B
Belgium	-B	-B	–	-A	-B	-B	-B		-B	-A
Ireland	A	A	A	–	A	A		B	A	A
Italy	-B	-B	B	-A	–	-B	-B		-C	-B
Greece			B	B	-B	B	–	-B	B	C
Spain	B	B	B		B	B	–	C	B	B
Portugal				-B			-C	–		
Austria	-B		B	-B	C	-B	-B		–	-B
Finland		B	B	-B	B	-C	-B		B	–

Note: A, B and C respectively indicate Type A, Type B or Type C dominance of row country over columns country. Minus signs indicate dominance of the column country over row country.

Dominance relations based on the profiles corrected for measurement error are reported in Table 3. It is again reassuring to see that the dominance relations with or without correction for measurement error largely coincide. More Type A dominance relations are observed (in particular for Ireland), and some pairs of countries are now being ranked

Figure 4: Mobility profiles corrected for measurement error: Expected change in log-income



differently, but one can remain confident that the main results of this illustration are not driven by measurement error.

5 Discussion and conclusion

The paper has proceeded in two steps. First, a graphical approach –the use of mobility profiles– is suggested to make more detailed representations of mobility in the context of distance-based measurement of mobility. Second, an extension is derived from the profiles to incorporate concerns about the distribution of individual mobilities. Dominance relations are presented and an extended class of mobility measures is suggested, in which the class of measures we started from, $M(X, Y)$, becomes a special case.

Mobility profiles are appealing for describing the patterns of income mobility along the income distribution. The visual impact of the curves conveys an intuitive understanding of the underlying structure mobility measures across the income range. This feature is inherent to other approaches using quantile-based transition matrices but is otherwise lost in standard applications of distance-based summary measures of type $M(X, Y)$ as advocated, for example, in Fields & Ok (1996) or Fields & Ok (1999b). The MP takes the best of both approaches in this regard. Admittedly, the measures belonging to $M(X, Y)$ are special cases of more general classes which may not be expressible as simple population averages (see for example Cowell, 1985, Mitra & Ok, 1998). However, empirical applications tend to restrict focus to these special cases. Interest is therefore perceived for developing methods for finer analysis of this ‘restricted’ class.

Importantly, the MP allows us to incorporate easily more elaborated normative concerns. This is particularly helpful for income mobility comparisons. Robust (but partial) orderings can be obtained from simple dominance criteria, while complete orderings can be obtained by introducing explicit social welfare weights of a form familiar to inequality measurement literature. This approach requires no new normative concepts. What is different from existing approaches of ‘mobility dominance’ is that, as advocated by Dardanoni (1993), individual mobility is not anonymous but rather weighted according to people’s base period rank in the society.

The methods also have promising potential in the context of pro-poor growth assess-

ment, as illustrated in the empirical application, provided one is willing to track individual income growth rather than the evolution of anonymous groups (such as ‘the poor’) as advocated in Jenkins & Van Kerm (2006).

Several directions require further research. First, this paper is limited to the usual two time periods framework. Although the majority of mobility measurement approaches (most notably transition/mobility matrices) still focus on a two periods framework, further research will be devoted to an extension of the methods to multi-period income flows. Long running panel survey data are now available and offer opportunity for multi-period flows analysis of mobility. A promising extension to a multi-period framework proceeds by conditioning the mobility profile at time T on the rank in the distribution of incomes cumulated over periods 1 to $T - 1$. Again, such an approach follows Dardanoni (1993).

Second, statistics on average income change are sensitive to the presence of a few large outlying observations. These variations may be due to measurement error or to genuine income variability (in particular for the self-employed individuals), but they can influence substantially (conditional) means estimates. The robust LOESS estimator is a candidate solution to this problem. A more general approach could be to use (conditional) quantile regressions as suggested, for example, in Yu & Jones (1998). Such an approach would also allow for a more detailed description of the mobility profiles as in Trede (1998). However research should be done to clarify the normative underpinnings of mobility measures obtained by integrating conditional quantiles rather than conditional means.

Third, for more substantive analysis than the illustration presented here, adequate statistical inference methods should be employed in order to be able to obtain standard errors and confidence intervals for the computed statistics, to compute variability bands around the MPs, and to test the statistical significance of the dominance results (Davidson & Duclos, 2000). Resampling-based approaches seem best suited given the potential complexity of analytical methods in this context.

Finally, the applications in this paper have been descriptive. Research is required to identify the causes of the observed cross-national differences in income mobility, e.g. why did Ireland and Spain perform well, what is behind Portugal’s unusual mobility profile? One potential pathway to gain insights on these questions is to explore decompositions

by population subgroups of the mobility profiles. These would allow one to identify the groups experiencing higher mobility and/or the events (demographic, economic) that are associated to increased mobility. Decomposition by income sources could also be considered in order to provide other lines of explanations, like what is the role of labour income changes, spouse's wages, replacement income variations, *etc.* However, no generally applicable decomposability properties can be outlined as these will be dependent on the specific choice of d function.

Appendix: Derivation of dominance relations

$m^A(p)$ and $m^B(p)$ are the mobility profiles of society A and society B respectively. Define $A(p) = m^A(p) - m^B(p)$. The difference in aggregate mobility according to M^w can be written

$$\Delta = M^w(X^A, Y^A) - M^w(X^B, Y^B) = \int_0^1 w(p)A(p)dp. \quad (\text{A-1})$$

Clearly, if $w(p) \geq 0$, $A(p) \geq 0$ for all p implies $\Delta \geq 0$ (Type A dominance).

To derive Type B dominance, define $B(p) = \int_0^p m^A(q)dq - \int_0^p m^B(q)dq = \int_0^p A(q)dq$, and integrate (A-1) by parts as follows:

$$\Delta = w(1)B(1) - w(0)B(0) - \int_0^1 w'(p)B(p)dp \quad (\text{A-2})$$

where $B(0) = 0$ by definition. If one chooses $w(p) \geq 0$ and $w'(p) \leq 0$, then $B(p) \geq 0$ for all p implies $\Delta \geq 0$ (Type B dominance)

Type C dominance is derived similarly. Defining $C(p) = \int_0^p B(q)dq$ and integrating the last term of equation (A-2) by parts yields

$$\Delta = w(1)B(1) - w(0)B(0) - w'(1)C(1) + w'(0)C(0) + \int_0^1 w''(p)C(p)dp \quad (\text{A-3})$$

where $B(0)$ and $C(0)$ are zero by definition. If one chooses $w(p) \geq 0$, $w'(p) \leq 0$, and $w''(p) \geq 0$, the condition $C(p) \geq 0$ for all p ensures that all but the first term in (A-3) are non-negative. However, the first term, $w(1)B(1)$, can be negative. It is therefore necessary to impose also $w(1) = 0$ or to observe that $B(1) \geq 0$ in addition to $C(p) \geq 0$ to conclude that $\Delta \geq 0$ (Type C dominance).

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