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Working Paper

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ISER Working Paper Series, No. 2013-26

Provided in Cooperation with:

Institute for Social and Economic Research (ISER), University of Essex

Suggested Citation: Hancock, Ruth; Morciano, Marcello; Pudney, Stephen (2013) : Nonparametric estimation of a compensating variation: The cost of disability, ISER Working Paper Series, No. 2013-26, University of Essex, Institute for Social and Economic Research (ISER), Colchester

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Nonparametric estimation of a compensating variation: the cost of disability

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No. 2013-26
December 2013



INSTITUTE FOR SOCIAL
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Non-technical summary

Good design of public policy requires that the social benefit of a policy intervention should be at least as great as the cost of the policy. But the objective of policy is to improve national wellbeing in some sense, and it is difficult to compare an intangible improvement in wellbeing with the measurable money cost of the policy. A good example is public support (in the form of care services and disability benefit) for disabled older people. The cost of providing this support is clearly measured in the public accounts, but the improvement in the wellbeing of disabled people is not easily measured because there exists no “market for disability” which can be used to measure personal costs of disability in the same money units that we use to measure the cost of public disability support.

Economists often use the idea of a “compensating variation” (CV) as a way of measuring intangibles in money terms, to allow the impact of policy to be compared with its financial cost. For a disabled individual, the CV is defined as the amount of additional income he or she would require to be as well off in some sense as he or she would be without disability. The CV can be thought of as a measure of the additional costs a disabled person faces by virtue of his or her disability. It represents the additional money required to pay for care needs, more expensive transport requirements, etc.

The usual method of estimating the average CV is to analyse individual-level survey data and build a statistical model of the relationship between standard-of-living, income and disability. Once that relationship has been estimated statistically, the CV for each disabled individual can be inferred indirectly by hypothetically setting the degree of disability to zero and solving for the lower level of income that would be needed to give the same predicted level of living standards. In this paper, we show that this indirect method is highly unstable in the sense that small errors in the way the statistical modelling is done can cause very large errors in the estimated CV – in particular, there is a great risk of over-estimating the cost of disability.

We propose instead a direct method of estimating the CV, in which we match each disabled person in the survey with a comparator non-disabled person chosen to be as close as possible in terms of all personal characteristics and their achieved standard of living. The difference in their incomes is then a direct approximation to that person’s cost of disability. We show that this new direct method produces much more robust results than the usual indirect approach. When we apply the new method to data on over 34,000 older people, drawn from the nationally representative *Family Resources Survey* for 2004-8, we do indeed find that it gives smaller, more plausible estimates of disability costs than the standard method.

Despite this, we find that disability costs are large, averaging around £48-61 a week (in 2007 prices) across all disabled people over state pension age. This level exceeds the mean level of state disability benefit received (£28 a week) and the total of disability benefit and care services (£47 a week). We also find that disability costs rise steeply with the severity of disability – much more so than the progression built into the rate structure of the *Attendance Allowance* and *Disability Living Allowance* benefits, so there is a case for redesigning the disability benefit system for older people to deliver relatively more support to those with particularly severe disability.

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Abstract

We propose a nonparametric matching approach to estimation of implicit costs based on the compensating variation (CV) principle. We apply the method to estimate the additional personal costs experienced by disabled older people in Great Britain, finding that those costs are substantial, averaging in the range £48-61 a week, compared with the mean level of state disability benefit (£28) or total public support (£47) received. Estimated costs rise strongly with the severity of disability. We compare the nonparametric approach with the standard parametric method, finding that the latter tends to generate large overestimates unless conditions are ideal. The nonparametric approach has much to recommend it.

Keywords: compensating variation, disability, matching, nonparametric.

JEL codes: C81, D1, I1

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We are grateful to the Nuffield Foundation for financial support through project “The role and effectiveness of disability benefits for older people” (grant no. R16499), the Economic and Social Research Council (grant no. ES/K003852/1: “Disability and care needs in the older population: disability benefits, social care and well-being”) and the European Research Council (project no. 269874 [DEVHEALTH]) for support of Pudney’s contribution to the research. We are grateful to the Melbourne Institute for research support while Pudney was an academic visitor.

1 Introduction

The principal policy instruments available to public authorities involve the use of tangible resources measurable in money terms. But the objectives of policy are often intangibles like better health, greater personal security or improved wellbeing, and coherent decision-making is impossible without some way of comparing tangible costs with anticipated intangible benefits. The compensating variation (CV) is a standard microeconomic tool for measuring the impact of changed conditions on individual welfare and thus for evaluating in monetary terms the projected outcome of policy. For someone in an adverse situation, it is defined as the additional income that would be required to leave him or her as well off in some defined sense as in a standard situation. The definition involves explicit comparison of an individual in one set of circumstances with an otherwise identical individual in some counterfactual setting, but it is usually estimated indirectly by fitting a structural welfare model and inferring the compensating income variation from estimates of its parameters. Estimated CVs are consequently vulnerable to bias from misspecification of the model, and it is surprising that so little attention has been paid by econometricians to alternative methods of estimating CVs. By virtue of its comparative nature, the CV lends itself naturally to a nonparametric approach using statistical matching but, as far as we know, matching methods have not previously been considered for this purpose in the research literature. Our primary aim in this paper is to introduce the matching approach, compare its properties with those of the conventional indirect parametric approach, and demonstrate its application in an important policy area.

Although often defined in relation to price changes, the CV, or the related concept of the equivalent variation, can be applied in many other contexts. In the ‘happiness’ literature, it has been used to estimate the income variation equivalent to events or resources like marriage, divorce, childbirth, unemployment and social capital (see, for example, Blanchflower and

Oswald 2004, Di Tella and MacCulloch 2008 and Groot et al 2007). In health, it has been used by Ferrer-i-Carbonell and van Praag (2002), Groot and Maassen van den Brink (2004, 2006, 2007) and Mentzakis (2011) to estimate the costs of a range of disease types, and by Mentzakis et al (2012) to estimate the impact of caring responsibilities on informal carers. In this paper, we follow Zaidi and Burchardt (2005) and Morciano et al (2013) in applying the CV principle to the problem of evaluating personal disability costs, which are important in themselves, but also because coherent design of social security and health policy requires an ability to compare the economic welfare of groups facing different circumstances. The CV is one possible measure of what Sen (1992,1999) calls the “conversion handicap” that a disabled person has in “converting money into good living”. Failing to allow for these additional costs can lead to seriously misleading conclusions about the targeting of public programmes, as documented by Hancock and Pudney (2013).

Figure 1 illustrates the use of the CV principle for estimating disability costs. We plot the curve relating standard of living (S) to income (Y) for a person with given disability level ($D = 1$, say) and an otherwise identical non-disabled person ($D = 0$); then choose a reference level for S ($S = 0$ and $S = 1$ are illustrated) and observe the additional income (Δ_0 or Δ_1) that would be required to bring the disabled person to the same standard of living as the comparable non-disabled person. The location of each (S, Y) curve depends on all relevant observable and unobservable characteristics, X and U , of the individual, so the CV is a function $\Delta(D, S; X, U)$.

Note that, apart from its sign, the CV for a disabled person on income Y_1 is also the equivalent variation, or income loss that is welfare-equivalent to the onset of disability, for a non-disabled person initially on income $Y_1 + \Delta_1$. However, it is more natural to work in terms of the CV, since disability policy is concerned with the appropriate level of support for people who are disabled.

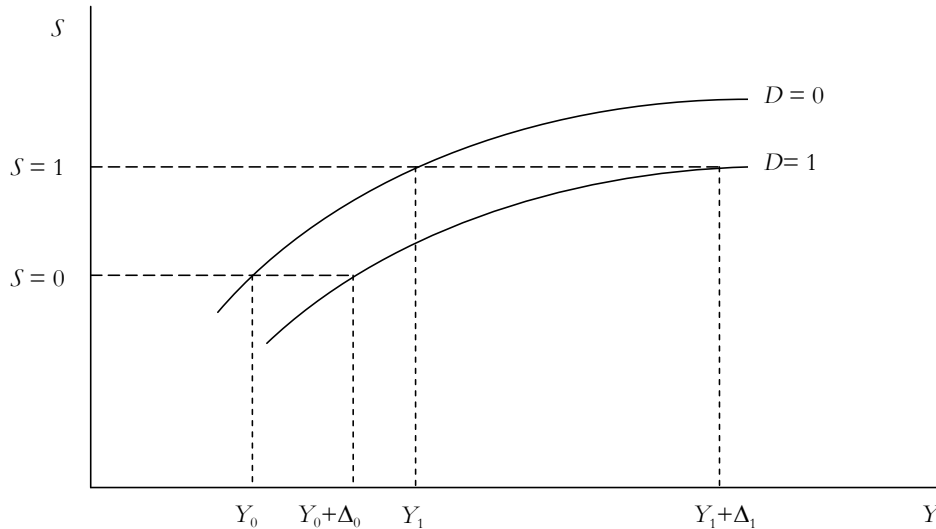


Figure 1 The Standard-of-Living/Income profile and compensating variations

2 Estimation approaches

Conventional estimation of the CV involves parametric econometric modelling of living standards to construct the empirical (S, Y) curves. This carries an attendant risk of specification error, which may have a large impact on the results. In particular, it is clear from Figure 1 that any misspecification bias leading to a flattening of the curves may lead to great over-estimation of the CV. Mentzakis (2011) and Morciano et al (2013) have demonstrated the importance of model specification in relation to parameter heterogeneity and functional form respectively. Other specification errors may also be important; for example, the additive structures typically built into parametric models have the implausible implication that the CV is invariant to X and U . In practice, it is hard to judge the robustness of parametric CV estimates, and nonparametric or semiparametric estimation of the living standards-income-disability profile is unpromising because of its dimensionality.

Our matching approach is flexible in the sense that it does not require *a priori* specification of the shape of the (S, Y) curves, and it also captures more directly the basic definition of

the CV. We will show that neither the standard parametric approach nor the nonparametric approach gives unbiased estimates of the CV under the conditions likely to be encountered in practice, but that the nonparametric estimator provides much more stable estimates and is consequently more reliable for policy applications.

When we estimate the CV at a particular point $D = d, S = s$, the statistical matching procedure differs from the standard form set out by Rosenbaum and Rubin (1983). We not only match disabled (“treated”) to non-disabled (“control”) individuals in terms of their relevant characteristics, but also simultaneously match the former to the specified reference level ($D = d$) of disability and both individuals to the specified reference level ($S = s$) of the standard of living. This means, for example, that if propensity score matching (Rosenbaum and Rubin 1983) is used, the propensity score must be constructed conditional on the joint event $D \in \{0, d\}, S = s$ for each (d, s) reference point.

The CV is defined in relation to some theoretically-justified measure of welfare or living standards and different welfare concepts will imply different CVs. Several different survey indicators of personal welfare have been used in the applied literature; for example Groot and Maassen van den Brink (2004, 2006) use subjective life satisfaction and Ferrer-i-Carbonell and van Praag (2002) use a set of domain-specific satisfaction variables. Measures of this kind capture a broad concept of wellbeing, and raise important conceptual and practical issues. In our application, we use a welfare concept limited to the idea of material living standards, which has two advantages. First, it matches the rationale for realistic government policy better than broader notions of happiness or life satisfaction. For example, the official explanation of the purpose of the UK Attendance Allowance benefit programme states that it is intended “*to help with personal care because you’re physically or mentally disabled and you’re aged 65 or over*” (DWP 2013). This is essentially a commitment to meet part of the material costs associated with disability. It is hard to imagine any government making the

larger – and possibly unbounded – commitment to help restore the level of happiness that disabled people would have enjoyed in the absence of disability.

A second potential problem with broad concepts of wellbeing is that the corresponding CV may not exist at all. Assume the individual makes consumption decisions consistent with maximisation of a utility function $v(C; D)$, where C is the vector of market commodities (including care services), and that experienced wellbeing has the following structure:

$$\omega(\sigma(p, Y; D); D) \tag{1}$$

where $\sigma(\cdot)$ is the indirect utility function resulting from maximisation of $v(\cdot)$, and interpreted as the achieved material standard of living given commodity prices p and income Y . Given this structure, the ‘standard of living’ CV is defined as the income variation Δ satisfying $\sigma(p, Y + \Delta; D) = \sigma(p, Y; 0)$, which must exist if $\sigma(\cdot)$ is unbounded, continuous and strictly increasing in Y for each D .

A broader ‘experienced wellbeing’ CV can be defined as the income variation, Δ^* , satisfying $\omega(\sigma(p, Y + \Delta^*; D); D) = \omega(\sigma(p, Y; 0); 0)$. Even if $\sigma(\cdot)$ has the properties required for existence of Δ , this condition may have no solution for Δ^* if $\omega(\cdot)$ is not unbounded and strictly monotonic in σ for every D . In other words, while it may be possible to cover the additional costs of disability to reach a reference material living standard s , there may be no income addition, however large, that would restore a severely disabled person to the level of happiness or satisfaction with life that would be enjoyed in the absence of disability. The linear or loglinear parametric models typically used in applied work impose the existence of a finite CV as a consequence of the functional form assumption, whereas a matching approach would automatically reveal non-existence as a failure of the common support requirement.

2.1 Bias in the nonparametric estimator

Change notation slightly by rewriting the achieved living standard S as a function:

$$S = \sigma(D, Y; X, U) \quad (2)$$

where we have suppressed the argument p and assumed prices are either uniform or vary in a way captured by X . Assuming that the CV exists globally, $\sigma(\cdot)$ is continuous and strictly increasing in income, with inverse function:

$$Y = h(D, S; X, U) \quad (3)$$

The CV comparing non-disability ($D = 0$) with some positive degree of disability ($D = d$) is defined implicitly by the condition $\sigma(0, Y; X, U) = \sigma(d, Y + \Delta; X, U) = s$, which has solution:

$$\Delta(d, s; X, U) = h(d, s; X, U) - h(0, s; X, U) \quad (4)$$

Definition (4) implies that the mean CV at disability d and living standards s is:

$$\Delta(d, s) = E_{x|ds} \left\{ \int [h(d, s; X, U) - h(0, s; X, U)] dF(U|D = d, S = s, X) \right\} \quad (5)$$

where $F(U|D = d, S = s, X = x)$ is the conditional distribution function of U and the expectation is with respect to the distribution of $X|D = d, S = s$.

Instead, matching only on the observables $S = s, X = x$ identifies the function:

$$\begin{aligned} \tilde{\Delta}(d, s) = E_{x|ds} \left\{ \int h(d, s; X, U) dF(U|D = d, S = s, X) \right. \\ \left. - \int h(0, s; X, U) dF(U|D = 0, S = s, X) \right\} \quad (6) \end{aligned}$$

If the distributions $F(U|D = d, S = s, X = x)$ and $F(U|D = 0, S = s, X = x)$ coincide, the resulting $\tilde{\Delta}(d, s)$ is identical to the true mean CV given by (5). Otherwise, $\tilde{\Delta}(d, s)$

and $\Delta(d, s)$ may differ. Thus a sufficient condition for matching to identify $\Delta(d, s)$ is the conditional independence of U and D :

$$U \perp\!\!\!\perp D \mid S = s, X \tag{7}$$

This differs from the ‘classical’ exogeneity assumption $U \perp\!\!\!\perp D, Y, X$, which is standard in the econometric literature on disability costs. Both exogeneity assumptions are questionable, and there are four sources of correlation that have a bearing on their validity:

(i) U and D must covary positively along any curve $\sigma(D, Y; X, U) = s$ for given Y and X since $\sigma(\cdot)$ is increasing in U and decreasing in D and so, for given income Y , a disabled person requires a higher value of U to reach the living standard s than a similar non-disabled person. This only affects the nonparametric estimator, which conditions on $S = s$.

(ii) The system of public disability support induces positive covariation between disability D and the relevant income concept Y , since the latter may include disability benefit and the value of public subsidy on disability services. Our empirical application involves people over state pension age, for whom earnings are a very small component of income, so the limiting effects of disability have no significant negative income consequences. Correlation between D and Y only affects the efficiency of the parametric estimator, but it affects the bias of the nonparametric estimator, since positive correlation between D and Y tends to offset the negative covariation induced by the condition $S = s$.

(iii) There may be dependence between the unobservable U and disability D . For example: U may contain components of disability which are positively correlated with the observable component D but are not fully revealed by survey questions. Alternatively, some types of disability may impair the individual’s capacity to reach the maximal level of living standards from given financial resources. Either case results in negative covariation between D and U .

(iv) Correlation between Y and U may be induced by endogenous take-up of public support.

Disability benefit or subsidised care services have to be applied for and, if such support is included in income Y and U has a component representing the cost of claiming support, then Y and U covary negatively. This endogeneity problem has been neglected in much of the applied literature on CV estimation.

A standard remedy is instrumental variables. For given d, s, x , the match inside the expectation in (5) can be represented as a regression of Y on the dummy variable $\mathbb{1}(D = d)$ for the subpopulation satisfying $D = 0$ or $d, S = s, X = x$. That regression could in principle be estimated using an instrument Z , which would correctly identify the mean CV $\Delta(d, s, x)$, provided the residual $V = h(0, s, x, U) - \bar{h}(0, s, x) + \mathbb{1}(D = d)[\Delta(d, s, x, U) - \Delta(d, s, x)]$ is mean-independent of Z in the relevant subpopulation, where $\bar{h}(0, s, x)$ is defined as $E[h(0, s, x, U)|D = d, S = s, X = x]$. It is hard to see how such an instrument could be found without introducing additional very strong assumptions (see Heckman 1997 for a discussion of this in the standard evaluation case). The same difficulty arises in finding plausible instruments to deal with endogeneity problems in the parametric approach.

Rewrite the bias in the matching estimator (6) as:

$$\begin{aligned} \tilde{\Delta}(d, s) &= \Delta(d, s) \\ &+ E_{x|ds} \int h(0, s; X, U)[dF(U|D = d, S = s, X) - dF(U|D = 0, S = s, X)] \quad (8) \end{aligned}$$

Since $h(0, s; X, U)$ is decreasing in U , the bias must be negative if the distribution of U conditional on reaching living standards s without disability stochastically dominates the distribution conditioned on disability level d . First-order stochastic dominance is guaranteed trivially if U has no variation, but it is not necessarily satisfied otherwise. Matching does not generally deliver an unbiased estimate of the CV, but bias is only one of the important properties of an estimator: robustness to small deviations from the ideal assumptions used to justify conventional estimation is considerably more important, and we show that the

matching estimator performs much better than the parametric estimator in that respect. This is very clear in the special case of a Gaussian-linear setup, which we now consider.

2.2 The Gaussian-linear case

Assume:

$$S = \alpha Y - \beta D + X\gamma + U \quad (9)$$

$$U, Y|D, X \sim N([\mu_u(D, X), \mu_y(D, X)], [\sigma_u^2, \sigma_y^2, \sigma_{uy}]) \quad (10)$$

The following sign assumptions are plausible: $\sigma_{uy} \leq 0$ (endogenous benefit take-up); $\mu_u(D, X)$ is decreasing in D (disability-impaired resource use); $\mu_y(D, X)$ is increasing in D (disability-targeted benefit income). The true CV is $\Delta(d, s) = \beta d/\alpha$ and the bias in the matching estimator is:

$$\tilde{\Delta}(d, s) - \Delta(d, s) = -\frac{1}{\alpha} E_{x|ds} \{E(U|D = d, S = s, X) - E(U|D = 0, S = s, X)\} \quad (11)$$

Structure (9)-(10) implies that $U, S|D, X$ are jointly normal and that S has mean function $\mu_s(D, X) = \alpha\mu_y(D, X) - \beta D + X\gamma + \mu_u(D, X)$, variance $\sigma_s^2 = \alpha^2\sigma_y^2 + \sigma_u^2 + 2\alpha\sigma_{uy}$ and covariance $\sigma_{us} = \alpha\sigma_{uy} + \sigma_u^2$. Therefore $U|D = d, S = s, X$ is normal with mean function $\mu_u(D, X) + B_{us}[s - \mu_s(D, X)]$, where $B_{us} = \sigma_{us}/\sigma_s^2$ is the regression slope of U on S within any D, X cell. Note that B_{us} always lies in the unit interval if $\sigma_{uy} \leq 0$ and, if σ_{uy} is zero, B_{us} is 1 minus the R^2 from a regression of S on Y within any D, X cell. The bias (11) has three components:

$$\tilde{\Delta}(d, s) - \Delta(d, s) = -B_{us} \left\{ \frac{\beta d}{\alpha} \right\} - (1 - B_{us}) \left\{ \frac{\mu_u(d, X) - \mu_u(0, X)}{\alpha} \right\} + B_{us} \{ \mu_y(d, X) - \mu_y(0, X) \} \quad (12)$$

The first arises from the condition $S = s$; it is unambiguously negative and proportional to the true CV. Under our sign assumptions, the second and third terms are both positive, so the downward bias is offset to some degree. All three bias components (12) are invariant to s , so the bias does not distort the shape of the empirical relationship between the CV and

the chosen reference point s . Note that, if $\sigma_u^2/[\alpha^2\sigma_y^2 + \sigma_u^2] = 0.5$, then B_{us} is also 0.5 and is invariant to σ_{uy} . Thus, for B_{us} close to this quite plausible value, the nonparametric CV estimate is approximately unaffected by endogenous take-up.

The parametric approach regresses S on Y, D, X , identifying the following relationship:

$$E(S|Y, D, X) = \alpha Y - \beta D + X\gamma + \mu_u(D, X) + B_{uy}[Y - \mu_y(D, X)] \quad (13)$$

where $B_{uy} = \sigma_{uy}/\sigma_y^2 \leq 0$. If the mean functions are approximately linear:

$$\begin{aligned} \mu_u(D, X) &= a_1 D + X a_2; & a_1 \leq 0 \\ \mu_y(D, X) &= b_1 D + X b_2; & b_1 \geq 0 \end{aligned}$$

the CV constructed from the coefficients of Y and D in $E(S|Y, D, X)$ is:

$$\hat{C}\hat{V} = \left\{ \frac{\beta - a_1 + B_{uy}b_1}{\alpha + B_{uy}} \right\} d \quad (14)$$

If there is no income endogeneity ($\sigma_{uy} = 0$), the bias is $-a_1 d/\alpha$, which is unambiguously positive. Moreover, the bias increases with the magnitude of B_{uy} if the relationship between Y and D as measured by $b_1 d$ is no larger than the estimated CV, since $\partial \hat{C}\hat{V} / \partial B_{uy} = [b_1 d - \hat{C}\hat{V}] / [\alpha + B_{uy}]$. In this sense, the parametric approach is highly sensitive to exogeneity problems relating to income.

We illustrate the performance of the parametric and nonparametric approaches through a set of bias calculations using (12) and (14). Assume a true CV averaging £90 per week in the disabled population, in line with the estimates of Morciano et al 2013, and specify the following values which are broadly consistent with the data we use in the empirical application: $\sigma_s = 1.2$; $\sigma_y = 290$; $b_1 = 35$; $E(D|D > 0) = 1.8$; $\sigma_d = 1$. We then make alternative assumptions about the correlations between U and Y (ρ_{uy}) and U and $D|D > 0$ (ρ_{ud}) and about the signal-noise ratio in the living standards relation, by taking a grid of values for $(\rho_{uy}, \rho_{ud}, B_{us})$ over $[-0.5, 0.2] \times [-0.5, 0.2] \times [0.25, 0.95]$. For each of the 448 points

in this grid, we solve for the implied values of $\alpha, \beta, \sigma_{uy}, \sigma_u, a_1$ and evaluate the CV estimates defined by (12) and (14) at the point $d = E(D|D > 0)$.

Figure 2 summarises the poor robustness properties of the parametric approach, which may account for the surprisingly large CV estimates sometimes reported in the ‘happiness’ literature. Even quite modest deviations from the classical assumptions $\rho_{uy} = \rho_{ud} = 0$ produce large positive biases in the estimated CV: for example, with $\rho_{uy} = \rho_{ud} = -0.1$ and $B_{us} = 0.5$, the CV identified by the parametric approach is a 69.6% overestimate (£153), while the nonparametric approach overestimates by 14% (£102.60). In the circumstances most favourable to the parametric approach ($\rho_{uy} = \rho_{ud} = 0$), the nonparametric approach still works quite well: over the grid $0.05 \leq B_{us} \leq 0.75$, its average bias is +£6.61 and its RMSE is £17.50. Over the whole of the grid $-0.5 \leq \rho_{uy}, \rho_{ud} \leq 0.2; 0.05 \leq B_{us} \leq 0.75$, the average biases of the parametric and nonparametric estimators are £222 and £25 respectively. The parametric approach is highly sensitive, the bias varying enormously over the grid with a root mean squared error (RMSE) of almost £400, compared to less than £45 for the nonparametric approach.¹

¹Many applied studies use log income in the living standards equation, in which case the CV is $\Delta(d, s) = [e^{\beta d/\alpha} - 1]E(Y|D = d, S = s)$. This expression involves the same ratio $\beta d/\alpha$ as the linear CV and is even more prone to large positive biases due to the curvature of the exponential function.

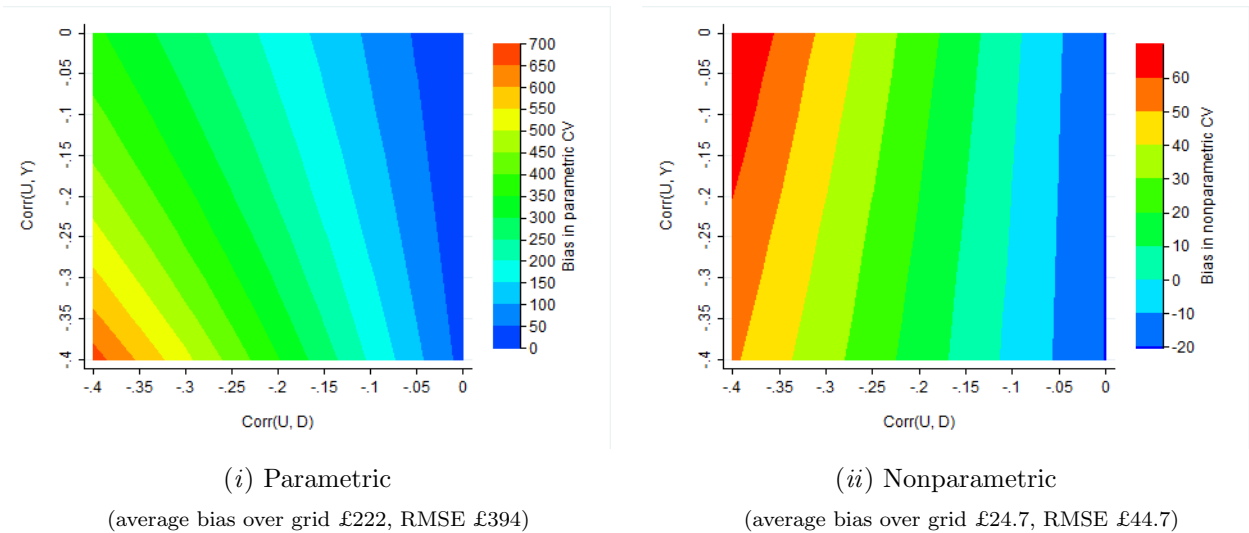


Figure 2 Bias in the parametric and nonparametric CV estimates

In the remainder of this paper we apply the parametric and nonparametric approaches to CV estimation using British survey data on people over state retirement age. The estimates are then compared with the value of external support that older disabled people receive in the form of nationally-administered cash disability benefits, publicly-funded formal care services organised by local government and informal care received from friends and family outside the household. We first consider the choice of dataset for the analysis in section 3, then present the main results in section 4 and consider their robustness in section 5.

3 Data: the Family Resources Survey

There are few surveys that meet the requirement to measure disability and economic welfare alongside details of cash and in-kind resources linked to disability received from sources outside the household. We use data from the Family Resources Survey (FRS), a repeated cross-section household survey, and currently the only nationally representative British survey meeting that requirement. We select FRS data for the five financial years 2004/5 to 2007/8 in preference to more recent years because of the stable design of questions on standard of living in that period. We restrict analysis to members of households in Great Britain

where at least one member is aged 65 or over, no-one is under state pension age (65 for men; 60 for women at the time of the surveys) and the household contains only a single person or a couple.

The characteristics X used for matching should cover all observable characteristics and circumstances that affect the capacity to achieve a high level of material living standards. We use region, age, education, household structure, and measures of capital assets, including housing tenure, the estimated house value (for homeowners) and financial wealth. The variables are defined and summarised in Appendix Table A3. We retain in the sample individuals for whom there is a proxy interview, since the proxy questionnaire covers all relevant variables, and excluding proxy cases would bias the sample away from the most severely disabled people, especially those with cognitive impairments. After dropping the small proportion of cases where critical information is missing, the resulting sample contains a maximum 34,184 individuals.

It is critically important to construct good measures of living standards (S), disability (D) and income Y , which we consider now. The important issue of measurement error is discussed in sections 5.1 and 5.3.

3.1 The living standards measure

Unlike the analysis of Jones and O'Donnell (1995), which is based on Engel curves and thus focuses on the intensive margin, our approach works at the extensive margin, using basic goods which are selected to be sensitive to living standards in the lower part of the income distribution, where policy concern is greatest. These standard of living indicators can be reconciled with formal demand theory. Applying Deaton et al's (1989) notion of demographic separability to the disability context, we assume that the cost function corresponding to

utility index $v(C; D)$ has the separable form:

$$c(p, s; D) = c_1(s, p_1) + c_2(s, p_2; D) \quad (15)$$

where $p = (p_1, p_2)$, and p_1 is the subvector containing prices of disability-separable goods. Consider the extensive margin for these goods; since c_1 is increasing in living standards s , demand for any good will be positive if s exceeds some critical threshold. The thresholds for separable goods are independent of disability, so those goods can be used as indicators of material living standards for both disabled and non-disabled people.

FRS indicators of living standards are based on a list of ten items or activities viewed as potential ‘necessities’. Households which did not have the items or do the activities were asked whether it was because they did not want them or because they could not afford them. Our measure S is derived from the pattern of items/activities that the household can afford, among the subset of six listed in the lower section of Table A1. This list excludes four of the FRS questionnaire items/activities, for which the disability-separability assumption is particularly suspect.² We use a value-weighted index, based on estimated average weekly costs for each of the basic goods/activities specified in the FRS deprivation module, computed from the 2002 report of the Family Budget Unit (FBU 2002). The FBU estimated detailed categories of living costs at the “modest but adequate” standard for six typical household types with members aged 65-74 years, which we converted to 2007 prices using the relevant Consumer Price Index category. Weekly costs by household type are reported in Table A2. The index S was then constructed as the proportion of the cost of six indicator goods/activities which the household reported being able to afford. The distribution of the resulting measure S is shown in Figure 3; the vertical lines indicate caliper ranges centred on

²For example, the ability to afford two pairs of all-weather shoes or holidays away from home, which may be strongly related to difficulties with mobility. In fact, the results are remarkably robust to the choice of goods to use as indicators of living standards.

six reference points s which are used in the next section to test the hypothesis of invariance of the CV to s .

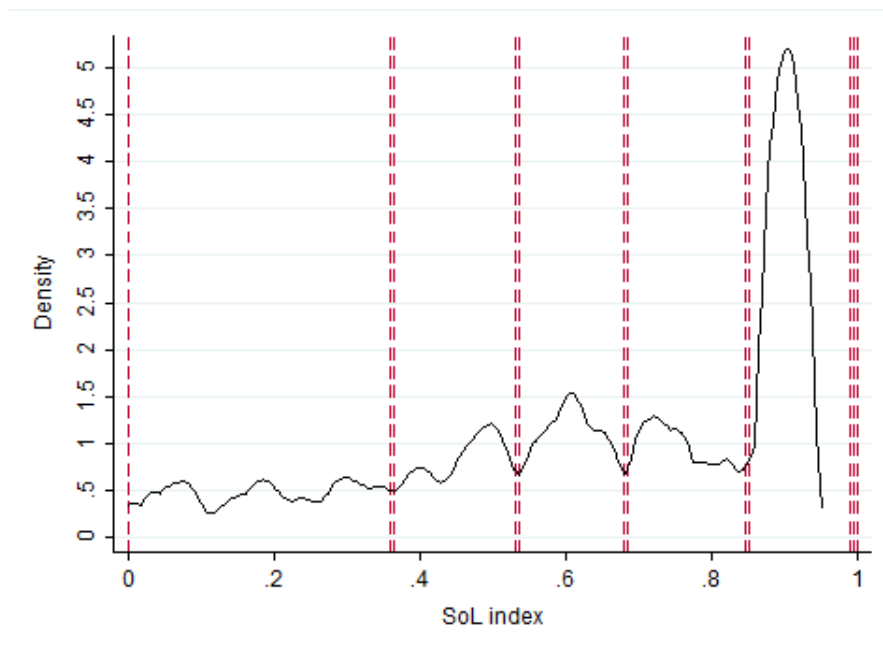


Figure 3 The distribution of the living standards index
(All FRS respondents; Epanechnikov kernel, bandwidth = 0.02)

3.2 The disability measure

A good index of disability needs to weight the observed disability indicators in some way to reflect their different sensitivities to the severity of disability. There are various ways of doing this, and we use the pattern of decision-making by public administrators and potential applicants for public support as a guide to construction of an index. FRS respondents are asked whether they have a health problem or disability and, if so, whether they have significant difficulties in each of the nine areas of life listed in the upper part of Appendix Table A1. Our disability measures D are derived from the responses to these questions. We

fit a logit model for receipt of any disability-tested public support using the nine disability indicators and also include the covariates X to capture other influences on benefit receipt (results in Appendix Table A4).³ We construct the index D as a weighted sum of the nine disability indicators, with weights specified as the relevant logit coefficients. This gives an index in which mobility and appreciation of danger are heavily weighted, reflecting the importance assigned to these dimensions of disability by the public system of support and potential claimants' own perceptions of need. Memory problems appear much less important. We plot the estimated CV against disability level at six points chosen to capture the main concentrations of data, to give adequate sample numbers. The caliper ranges centred on these six points are indicated by vertical lines superimposed on the distribution shown in Figure 4.

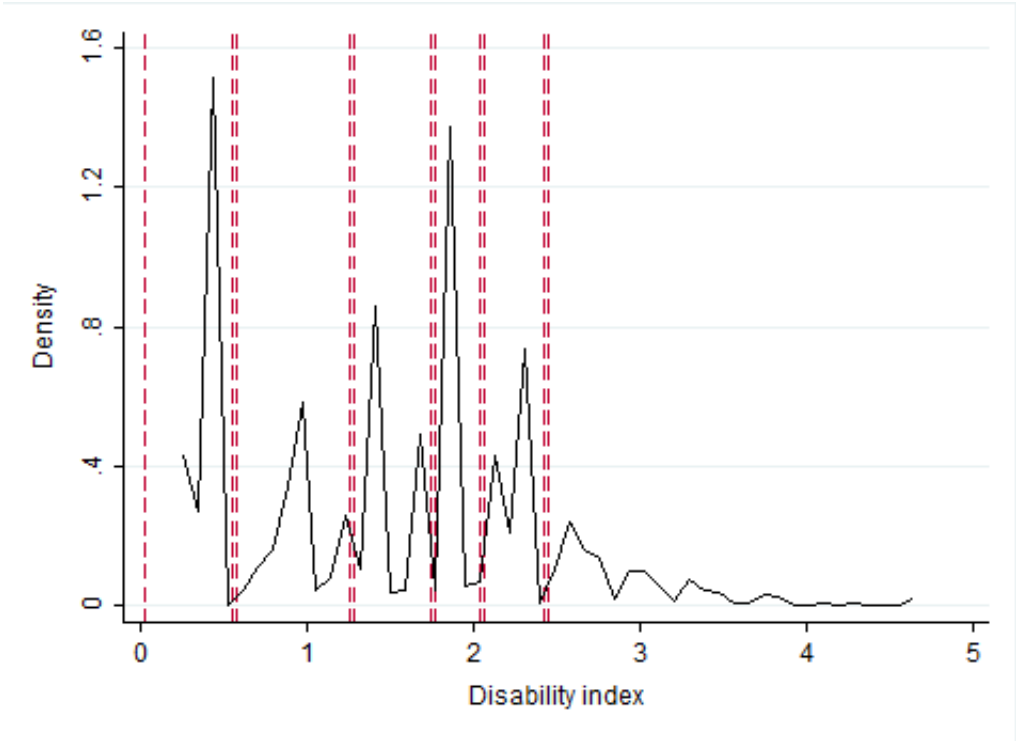


Figure 4 The distribution of an eligibility-weighted disability index (All FRS respondents reporting any disability; Epanechnikov kernel, bandwidth = 0.02)

³Note that it is only the relative magnitudes of the disability coefficients that matter, not their absolute size. The index is remarkably robust to the specification of this logit model.

3.3 The income measure

It is important to include both receipt of disability benefit and the value of external care services in the net income variable Y , since it represents access to all economic resources that can contribute to the realised standard of living. Without doing so, it would not be possible to specify adequately the standard-of-living function $\sigma(\cdot)$ which is the basis of the estimated CV. Our main income definition includes net cash income from all sources, comprising mainly net investment income (interest, rent, dividends, private pensions, annuities) and state pension income, plus disability benefits and other social security benefits.⁴ To this we add an estimate of the value of state-subsidised care services. Previous work based on the standard-of-living approach by Zaidi and Burchardt (2005) and Morciano et al (2013) included cash benefits but not services, corresponding to the implicit assumption of zero substitutability between care services and other basic consumption.

Two variations on this income definitions are used as a robustness check in section 5.3; one follows the published literature by excluding the imputed value of formal care services. The other adds an estimate of the equivalent market value of informal care received from family and friends outside the household, thus relaxing the assumption of zero substitutability between informal care and basic consumption. We express all income variables net of housing costs (comprising rent, local property tax, mortgage interest and property insurance), reflecting an assumption that housing is treated as an essentially fixed cost by most members of this age group; it gives a disposable income concept similar to the “After Housing Cost” measure used in the official Households Below Average Income analysis (DWP, 2009) but with wider scope.

⁴Disability benefits include non means-tested disability benefits (mainly Attendance Allowance or Disability Living Allowance) and an estimate of any disability -related addition to means-tested benefits.

The FRS identifies weekly hours of two types of formal care: social care from a Local Authority (LA) and (in very few cases) nursing care provided at home. Recipients of LA care are liable for a charge which depends on their income and assets. We value hours of care received from the LA at £18 per hour (in 2007 prices) estimated as an average for a home care worker (see Curtis 2008, Table 9.5.) We then assess how much of that would be met by the LA given the recipient’s income and assets. This calculation takes account of the radical difference between social care policy in Scotland (where there is no social care means test) and the rest of Great Britain. Although the precise form of the means test varies across LAs within England and Wales, national guidance enables us to estimate a maximum and minimum value, which we then average. Care from a nurse is assumed to be provided free by the National Health Service and is valued at £64 per hour (Curtis, 2008; Table 8.1). It is less easy to value unpaid informal care. Although informal carers are generally untrained and unqualified, it is plausible to argue that the close personal relationship that typically exists between cared-for and carer means that such care is of high average quality. Moreover, care workers are generally low-paid and the shadow market value of the time of informal carers is unlikely to be significantly lower than that of publicly-provided carers. In constructing our alternative broader income definition, we have therefore also valued externally-sourced informal care at £18 per hour. Appendix Table A5 summarises the resulting income variables by disability status.

4 Estimates of the compensating variation

4.1 Matching estimators

If all variables are discrete and exact matching is feasible, the definition (6) has a natural empirical analogue which substitutes sample means for expected values. If exact matching is infeasible because of continuous variation or inadequate cell sample sizes, we can use a

hybrid approach, with stratification giving exact matching in important discrete dimensions and approximate matching accepted in others. Define the ‘treatment’ variable $T = \mathbb{1}(D > 0)$ where $\mathbb{1}(\cdot)$ is the indicator function, and partition the vector X as $\{V, W\}$, where $V \in \mathbb{V}$ contains covariates for which an exact match is feasible, and W contains the covariates which will be approximately matched. The main results presented here are based on nearest neighbour Mahalanobis matching (with replacement); other methods give substantially the same results (see section 4.4 below).

We use three different matching estimators for different purposes. In section 4.2, we test for invariance of $\Delta(d, s)$ to s , by estimating the CV at a 6×6 grid of points, with approximate matching of observed (D, S) to each (d, s) , within separate calipers, κ_s, κ_d and κ_ρ , for D, S and W . For each stratum $v \in \mathbb{V}$, define sets $\mathbb{K}(0, s, v) = \{k : D_k = 0, |S_k - s| < \kappa_s, V_k = v\}$ and $\mathbb{I}(d, s, v) = \{i : |D_i - d| < \kappa_d, |S_i - s| < \kappa_s, \rho(W_i, W_{k(i,s,v)}) < \kappa_\rho, V_i = v\}$, where $\rho(W_i, W_{k(i,s,v)})$ is a distance function. For each $i \in \mathbb{I}(d, s, v)$, select a non-disabled match as the individual $k(i, s, v) \in \mathbb{K}(0, s, v)$ who minimises $\rho(W_i, W_{k(i,s,v)})$. The mean CV is estimated as:

$$\hat{\Delta}(d, s) = N(d, s)^{-1} \sum_{v \in \mathbb{V}} \sum_{i \in \mathbb{I}(d, s, v)} (Y_i - Y_{k(i,s,v)}) \quad (16)$$

where $N(d, s)$ is the total number of disabled individuals for whom a match is found.

A variant of this is used in section 4.3 to estimate mean disability profiles of the CV, using local linear regression smoothing. Here, we find a match for each disabled person and then regress the income difference of the matched pair on the observed disability level. Redefine $\mathbb{K}(0, v)$ as $\{k : D_k = 0, V_k = v\}$ and, for each individual with $D_i > 0, V_i = v$ select a match as the individual $k(i, v) \in \mathbb{K}(0, v)$ who minimises an extended distance measure $\rho([S_i, W_i], [S_{k(i,v)}, W_{k(i,v)}])$, within an overall caliper κ_ρ . The estimated disability-specific CV $\tilde{\Delta}(d)$ is the conditional mean estimated at the point d by a nonparametric regression of $Y_i - Y_{k(i,v)}$ on D_i . Confidence bands are constructed by bootstrapping the whole procedure.

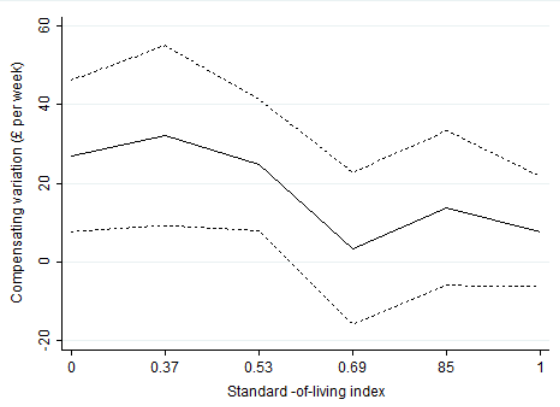
The third variant is a simple matching estimator, used in section 4.4, for the average CV in the disabled population, using $\mathbb{1}(D_i > 0)$ as the “treatment” indicator and caliper-matching on (S_i, W_i) , with stratification by V .

4.2 Testing income-invariance of the CV

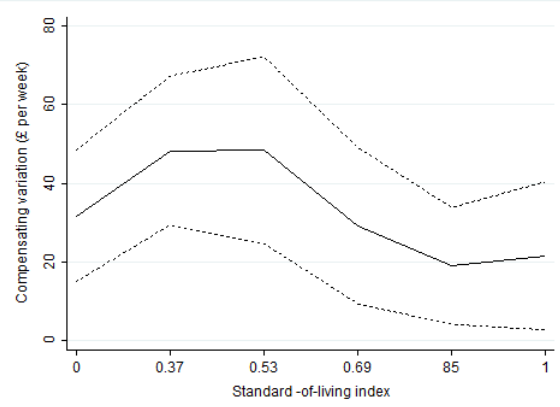
The CV results from the matching analysis are shown separately in Figure 5 for six levels of disability. In each plot, the non-disabled group is defined as the set of individuals reporting no difficulties caused by health problems, and we perform the matching at the six points marked in the living standards distribution of Figure 2 above. Matching on the Mahalanobis criterion is restricted to the common support using the variable calipers illustrated in Figure 2, and is stratified by a vector V containing two binary variables: ownership of financial assets over £1,000; and membership of a couple rather than a single-person household.⁵ The remaining covariates in W include age, education, housing tenure, housing wealth, financial assets, household size and location (Appendix Table A3).

We use a $\chi^2(5)$ test of the hypothesis that the CV is invariant to the reference level of living standards for matches within each of the disability ranges shown in Figure 4. The P -values range from 0.14 to 0.62, so we do not reject invariance. This also has the important implication that the CV, when expressed in terms of income rather than living standards, as $\Delta(d, \sigma(d, Y; X, U); X, U)$, is income-invariant, so that a disability equivalence adjustment should have an additive form rather than the multiplicative scale conventionally used for demographic adjustment of family income. Invariance corresponds to the universal nature of the main disability benefits (Attendance Allowance and Disability Living Allowance), which are disability-tested but not means-tested.

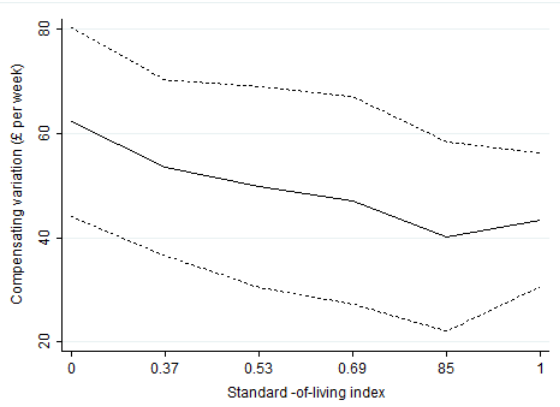
⁵Implementation is through Leuven and Sianesi’s (2003) *PSMATCH2* software.



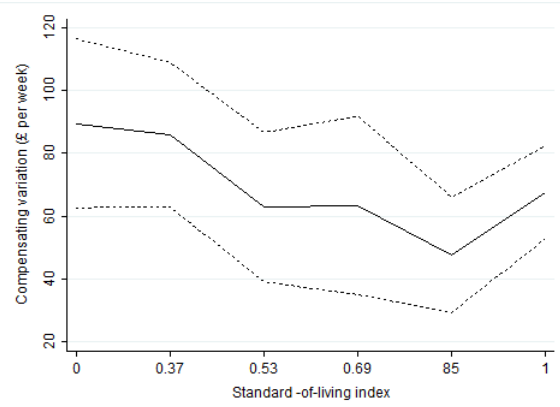
(a) $d \approx 0.38$; $\chi^2(5) = 5.66$ ($P = 0.340$)



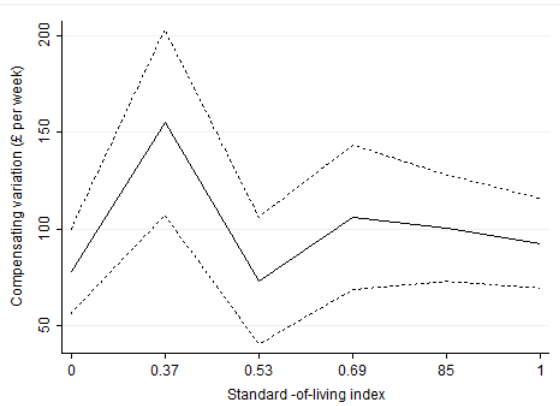
(b) $d \approx 0.93$; $\chi^2(5) = 6.60$ ($P = 0.252$)



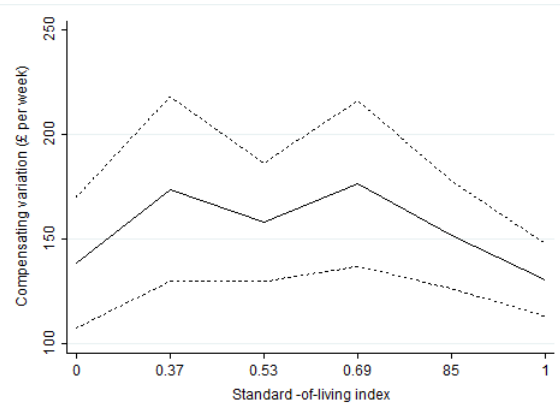
(c) $d \approx 1.51$; $\chi^2(5) = 3.50$ ($P = 0.624$)



(d) $d \approx 1.90$; $\chi^2(5) = 6.56$ ($P = 0.256$)



(e) $d \approx 2.25$; $\chi^2(5) = 8.28$ ($P = 0.142$)



(f) $d \approx 2.92$; $\chi^2(5) = 5.67$ ($P = 0.340$)

Figure 5 Estimated CVs profiles at six disability levels
(with asymptotic 90% confidence bands and χ^2 invariance tests)

4.3 Disability cost profiles under income-invariance

Under the invariance hypothesis, the estimates for different s can be pooled to improve efficiency and plotted against the reported level of disability. Figure 6 shows the outcome of this and confirms the anticipated statistically significant monotonic increase of the CV with respect to the severity of disability. The estimate of mean implicit disability costs is substantial, rising from around £20 a week at the lowest level of disability to almost £120 at the highest.

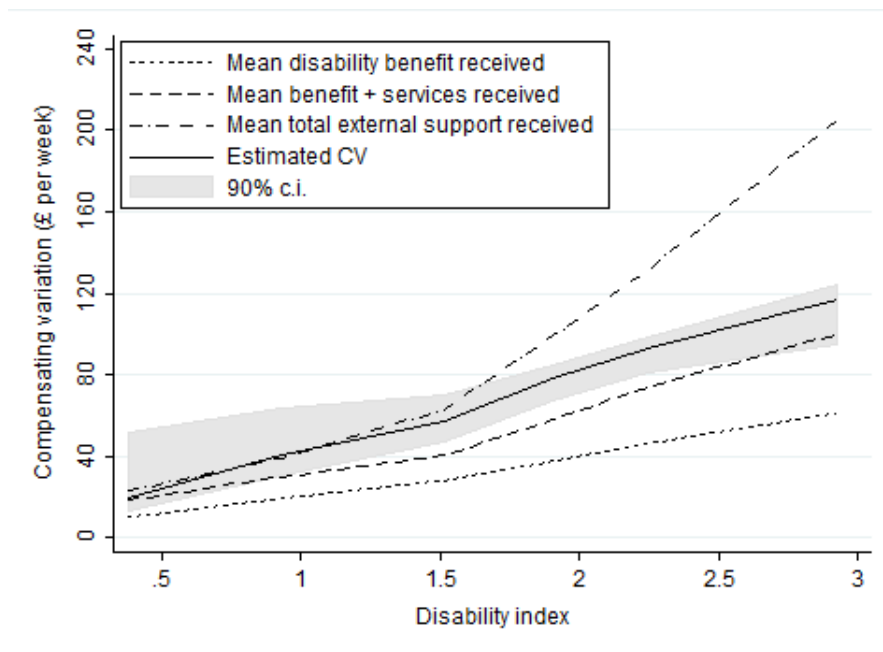


Figure 6 Estimated CV by disability level: weighted disability index (90% bootstrap confidence intervals: 500 replications)

These estimates compare with average receipts of disability-tested cash benefit rising from approximately £10 to £60, total publicly-funded support (benefit and services) from £20 to almost £100. Thus the average scale of public support is less than, but rises with severity of disability roughly in proportion to the estimated CV. The main cash disability benefit available to older people is *Attendance Allowance* which, in 2007/8, was payable at

rates £43.15 or £64.50, which thus rise much less strongly with severity. The implication is that targeting of support depends much more on potential claimants' take-up behaviour and the authorities' claim adjudication procedures than on tailoring of the benefit profile to need.

Informal care from non-co-resident friends and relatives rises much more strongly with disability than formal public support, and the total of all external support from outside the household rises to over £200 at the highest disability levels. This exceeds the estimated CV at high disability levels. There are three plausible explanations for this. First, our estimated CV is likely to be an underestimate because of measurement error in disability and living standards (section 5.1). Second, there is no market for informal care, so its valuation is highly uncertain and our valuation of £18 an hour may be too high. Third, the suppliers of informal care generally have strong ties of affection, and their objective may be to restore some of the wider wellbeing ("happiness") lost through disability, rather than the more limited objective of public authorities to guarantee a reasonable level of material wellbeing. For that reason, informal care may be supplied at a level well above that necessary to cover the material-standards CV.

4.4 Alternative matching methods

As an overall summary and robustness check, Table 2 gives estimates of mean disability costs among the whole of the older disabled household population, using matching based on two different distance metrics (Mahalanobis and propensity score), and three different degrees of stringency of the calipers for each. The alternative estimates of mean costs are remarkably close. Allowing for sampling error and systematic differences in method, the average lies somewhere in the range £48-61 a week, which is large: approximately 21-26% of the after housing costs net disposable income of matched non-disabled people (£231) and greater

than the mean levels of state disability benefit (£28) or total public support (£47). Given that the standard rates of disability benefit are substantially higher than £28 and may be supplemented by additions to means-tested benefits, these low levels of mean public support imply that many people with substantial disabilities are receiving no disability benefit at all, despite the universal nature of the disability benefit system.

Table 2 also shows estimates of the mean CV from four parametric models involving D, X and either Y or $\ln Y$ as covariates in either a linear regression model of the continuous standard of living index or an ordered probit model of an unweighted count index for S . The parametric CV estimates are much larger and more erratic than the nonparametric estimates – ranging from £167 to £515 – they are 2.7-8.4 times larger than the largest conceivable nonparametric estimate.

Table 2 Estimated overall mean disability costs

	Mahalanobis ¹			Propensity score ¹		
	$\kappa_w = 2$	$\kappa_w = 4$	$\kappa_w = \infty$	$\kappa_w = 0.005$	$\kappa_w = 0.01$	$\kappa_w = 0.02$
CV	£55.99	£55.36	£54.88	£57.25	£57.31	£57.52
Std err	(3.07)	(3.47)	(3.50)	(3.10)	(3.10)	(3.10)
Support % ²	92.2	95.5	100.0	97.8	99.0	99.4
<i>Parametric estimates (standard error)</i>						
Linear regression model (continuous living standards index) ³				£515.29	(44.65)	
Log regression model (continuous living standards index) ⁴				£311.97	(26.32)	
Linear ordered probit (count index of living standards) ³				£167.41	(8.45)	
Logarithmic ordered probit (count index of living standards) ⁴				£228.87	(15.31)	

¹ Nearest-neighbour matching, with replacement. ² Proportion of “treated” sample retained after applying caliper and common support. ³ Constructed as $-\hat{b}_d \bar{D} / \hat{b}_y$, where b_y, b_d are coefficients in a linear ordered probit model of S on Y, D, X and age^2 , and \bar{D} is the mean among those reporting disability. ⁴ Constructed as the mean of $Y[\exp(-b_d D / b_y) - 1]$, where b_y, b_d are coefficients in the logarithmic model.

Note three further aspects of these results. First, there is no common support problem linked to the ability of disabled people to reach any reference level of living standards, as measured by the FRS deprivation questions. The common support restriction causes minimal loss of the “treated” (disabled) cases even with exact matching on two variables in X and a stringent caliper for the remaining covariates. The estimates in Table 2 use matching with

replacement from the control sample; when matching without replacement is used instead, the off-support loss rate rises to around 20% of the disabled sample, but there is otherwise little change in the results. There is little reason for concern about non-existence of the CV based on material deprivation, since even severely disabled people appearing in the FRS sample are clearly able to avoid material deprivation with sufficiently high income and external support.

Second, our estimates of material disability costs substantially exceed total publicly-funded support for people with high levels of disability. The mean gap may be as much as half of the cost for this group, so the policy ambition to “help with personal care” (DWP 2013) could be said to be achieved on average, but not with overwhelming generosity. Our analysis of measurement error (section 5) indicates that even this modest contribution may be an overestimate as a proportion of true disability cost.

Finally, it is striking that the mean level of support from all sources external to the household, roughly half of which comes from informal carers, is close to estimated disability cost and significantly exceeds it for the most severely disabled. We would expect that close friends and relatives acting as carers have concern for a deeper concept of wellbeing of the cared-for person than the public authorities’ concern for material wellbeing. If so, our estimated CV will (greatly) underestimate the degree of need that is perceived by informal carers, and this is likely to be part of the explanation for the excess of total support over estimated disability costs for the most severely disabled.

5 Robustness: sources of bias

5.1 Measurement error in D and S

In practice, both disability and standard of living are prone to significant measurement error. Relatively little is known about the properties of matching estimators when the treatment indicator and matching variables (in our application, D and S) are subject to error, but we can establish the nature of the bias under simple conditions. Consider the use of count indexes for disability and living standards to estimate Δ at a single point (d, s, x) so that both true and measured disability and living standards are binary. Let D^* and S^* be the observed measures and assume their misclassification errors are independent of each other and of (D, S, X) , and:

$$\begin{aligned} Pr(D^* = 1|D = 0) &= p^+; & Pr(D^* = 0|D = 1) &= p^- \\ Pr(S^* = 1|S = 0) &= q^+; & Pr(S^* = 0|S = 1) &= q^- \end{aligned} \quad (17)$$

where p^+, p^-, q^+ and q^- are constants. Define $\mu(d, s, x) = E(Y|D = d, S = s, X = x)$ and consider matching at level $S^* = 0$. The expected value of income conditional on each of the observable matches is:

$$\begin{aligned} E(Y|D^* = 1, S^* = 0, X = x) &= \mu(0, 0, x)p^+(1 - q^+) + \mu(1, 0, x)(1 - p^-)(1 - q^+) \\ &+ \mu(0, 1, x)p^+q^- + \mu(1, 1, x)(1 - p^-)q^- \end{aligned} \quad (18)$$

$$\begin{aligned} E(Y|D^* = 0, S^* = 0, X = x) &= \mu(0, 0, x)(1 - p^+)(1 - q^+) + \mu(1, 0, x)p^-(1 - q^+) \\ &+ \mu(0, 1, x)(1 - p^+)q^- + \mu(1, 1, x)p^-q^- \end{aligned} \quad (19)$$

These imply that the empirical CV based on D^* and $S^* = 0$ at $X = x$ is:

$$\begin{aligned} \hat{\Delta}(0, x) &= E(Y|D^* = 1, S^* = 0, X = x) - E(Y|D^* = 0, S^* = 0, X = x) \\ &= (1 - 2p^-) [(1 - q^+)\Delta(0, x) + q^-\Delta(1, x)] + 2(p^+ - p^-) [(1 - q^+)\mu(0, 0, x) + q^-\mu(0, 1, x)] \end{aligned} \quad (20)$$

A similar derivation gives the empirical CV at level $S^* = 1$:

$$\hat{\Delta}(1, x) = (1 - 2p^-) [q^+ \Delta(0, x) + (1 - q^-) \Delta(1, x)] + 2(p^+ - p^-) [q^+ \mu(0, 0, x) + (1 - q^-) \mu(0, 1, x)] \quad (21)$$

Define the gradient of Δ with respect to s as $G(x) = \Delta(1, x) - \Delta(0, x)$ with empirical analogue $\hat{G}(x) = \hat{\Delta}(1, x) - \hat{\Delta}(0, x)$, which can be expressed as:

$$\begin{aligned} \hat{G}(x) &= (1 - 2p^-)(1 - 2q^-)G(x) + 2(1 - 2p^-)(q^+ - q^-)\Delta(0, x) \\ &+ 2(p^+ - p^-)(1 - 2q^-)(\mu(0, 1, x) - \mu(0, 0, x)) + 4(p^+ - p^-)(q^+ - q^-)\mu(0, 0, x) \quad (22) \end{aligned}$$

Suppose reporting error is symmetric in the sense that $p^+ = p^-$ and $q^+ = q^-$. Then the bias in the CV (20) or (21) is predominantly downward because of the factor $(1 - 2p^-)$, but may be modified by variation in $\Delta(s, x)$ with respect to the reference level s . The gradient (22) becomes $\hat{G}(x) = (1 - 2p^-)(1 - 2q^-)G$, so any non-zero gradient of the CV with respect to the reference level of living standards is unambiguously attenuated quite strongly – a misclassification rate of, say, 10% in D and S would reduce the empirical gradient to 64% of its true value. However, note that the proportionality of \hat{G} and G implies that, if we find a zero gradient in the empirical CV and are prepared to assume symmetric reporting errors, the true CV gradient must also be zero.

The estimated mean CV is the value of $\hat{\Delta}(S^*, X)$ averaged with respect to the distribution of $S^*, X|D^* = 1$. That distribution is:

$$F(S^*, X|D^* = 1) = \begin{cases} F_{00}(X)p(1 - q)\pi_{00} + F_{01}(X)pq\pi_{01} + F_{10}(X)(1 - p)(1 - q)\pi_{10} + F_{11}(X)(1 - p)q\pi_{11} & \text{if } S^* = 0 \\ F_{00}(X)pq\pi_{00} + F_{01}(X)p(1 - q)\pi_{01} + F_{10}(X)(1 - p)q\pi_{10} + F_{11}(X)(1 - p)(1 - q)\pi_{11} & \text{if } S^* = 1 \end{cases} \quad (23)$$

where $F_{ds}(x)$ is $Pr(X \leq x|D = d, S = s)$ and $\pi_{ds} = Pr(D = d, S = s)$. The resulting bias is complex. However, suppose that reporting error is symmetric and the true gradient $G(x)$

is approximately zero. Then the bias is unaffected by the distribution used to average $\hat{\Delta}(S^*, X)$, and the CV is attenuated by the simple factor $(1 - 2p)$. For example, a 10% misclassification rate in D will lead to disability costs being underestimated by around 20%.

5.2 Alternative income concepts

The main analysis is based on an income measure comprising net cash income and the imputed value of public subsidies on care services. This definition is appropriate if the disabled person would buy exactly the same amount of formal care privately at the same prices, given the same support in cash rather than kind. Here we consider two alternatives: a narrower income definition restricted to net cash income; and a wider definition treating informal care as equivalent to its imputed value in cash terms. In our view, these are both less easily justified than our principal income definition. The former ignores the fact that public services are generally only available in cases of great need, so it is implausible that there would be no consumption loss were they to be withdrawn. The latter overlooks the fact that informal care is linked to strong family and friendship ties and thus often takes forms (companionship, reassurance, etc.) which are not marketed or substitutable for consumption. Despite their drawbacks, these alternative income definitions are useful as an indication of theoretical robustness.

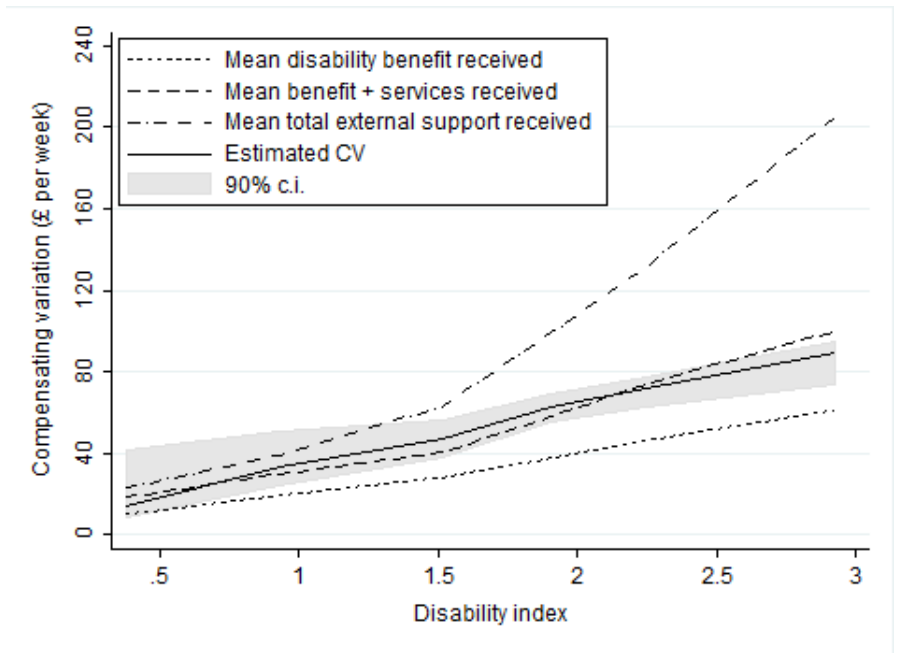
Table 3 summarises the overall mean CVs estimated under the three alternative income definitions. The estimated CV is reduced by about 20% when we use the narrower definition, and increased by about 55% with the wider definition. The parametrically-estimated CVs are affected in the same direction by changes of income definition, by much larger absolute amounts (which are slightly smaller in proportionate terms: -9% and +35% for the linear model and -17% and +44% for the logarithmic model).

Table 3 Estimated mean CVs for alternative income definitions

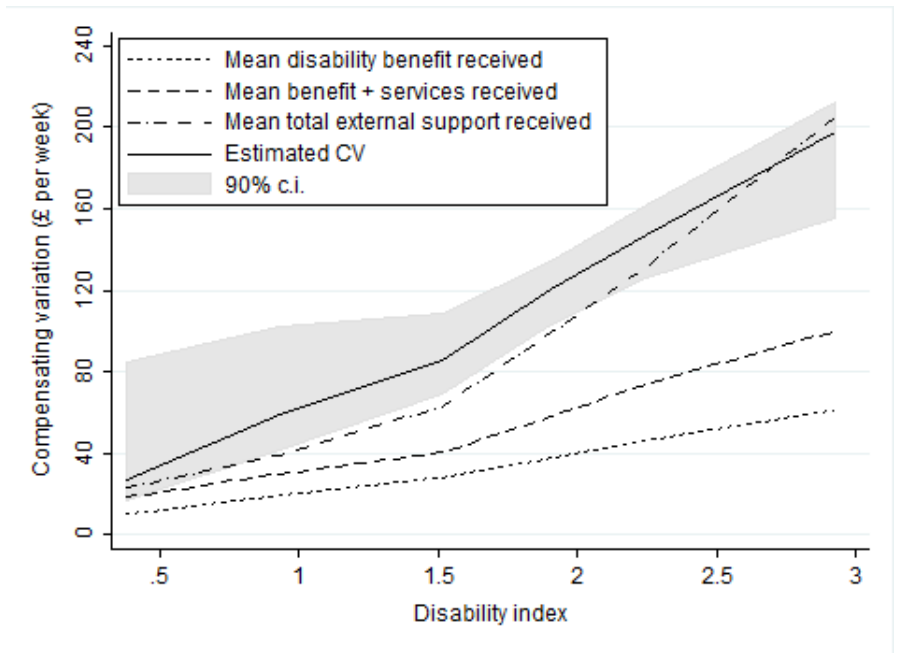
Estimator	Income definition		
	cash income only	cash income + formal care	cash income + formal + informal care
Nonparametric ¹	£44.85 (2.93)	£55.99 (3.07)	£86.97 (3.48)
Parametric: linear ²	£470.80 (41.12)	£515.29 (44.65)	£696.65 (66.83)
Parametric: logarithmic ²	£257.39 (21.23)	£311.97 (26.32)	£450.67 (41.75)

¹ Mahalanobis matching, $\kappa_w = 2$. ² Regression with continuous disability and living standards indicators.

Figure 7 shows the CV profiles under the two alternative income definitions. Both show the same steep rise of disability costs with the measured degree of disability.



(i) $Y = \text{net cash income}$



(ii) $Y = \text{net income including imputed value of all external support}$

Figure 7 Estimated CV profiles for alternative income definitions
(90% bootstrap confidence intervals: 500 replications)

5.3 Income reporting error

The parametric and nonparametric approaches to estimation of the CV are affected in quite different ways by measurement error in income. First consider the case of ‘classical’ measurement noise in income. This would cause no bias in the nonparametric CV estimates, whereas it would cause systematic bias in parametric estimates. Consider a linear model of S : classical measurement error in Y would attenuate the income coefficient and thus tend to generate upward bias in the implied CV. However, there is evidence of non-classical income measurement error, particularly among those reporting very low incomes, who may have failed to report an important income source. Our standard-of-living indicators are useful in identifying the range where income under-reporting is likely to be most serious. Figure 7 illustrates this by plotting the nonparametric regressions of the sum of the six binary standard of living indicators on each of two income variables: *original income* which excludes all means-tested and disability-tested public support and *full income* which includes all benefit income and the imputed market value of subsidised care services, and subtracts housing costs.

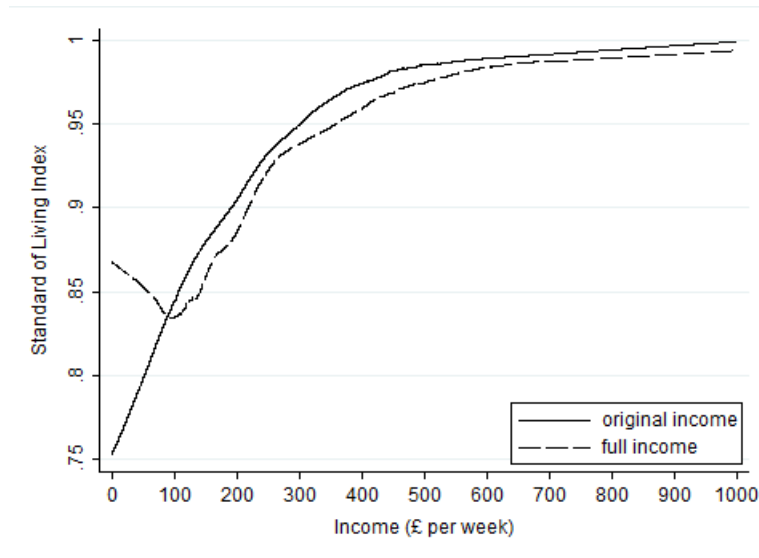


Figure 8 The empirical income-standard of living profile
 (Individuals reporting no disabilities; FRS 2006/7-2007/8;
 income at 2007 prices after housing costs;
 local linear regression; tricube weights; bandwidth=0.3)

We observe the expected monotonic concave shape over much of the income range, but there are anomalies among households with very low reported incomes. This problem is especially serious for the benefit-inclusive income definition, suggesting under-reporting of benefit income for some proportion of households (see Lynn et al (2006) for evidence of benefit under-reporting in UK survey data). We also find that, at high income levels, the profile is nearly flat, so the deprivation questions offer little identifying information about the impact of disability for high-income households.⁶

To assess robustness, we exclude a subset of low- and high-income individuals using cut-off points of £100 and £700 for non-disability net disposable income defined as base income plus non-disability-related means-tested benefit less housing costs. This criterion excludes 3,318 (20.3%) of the sampled individuals. While sample truncation is not a complete solution to the measurement error problem, comparison of results from the full and truncated samples gives

⁶The income-deprivation profile appearing in Figure 7 was used formally by Pudney and Francavilla (2006) to identify a measurement error model for income in Albania. Similar patterns have also been found for Australia by Saunders (2005) and the UK by Brewer and O’Dea (2012).

a good indication of the robustness of our CV estimates with respect to these types of income measurement error. Table 4 shows the effect of this sample truncation on the nonparametric CV estimates, comparing estimates from the full sample with similar estimates computed from the income-truncated sample. There are only minor variations in the estimated costs of disability, with no evidence of a systematic upward or downward bias, except in the case of the weighted disability index, where sample truncation reduces the estimated CV by around 20% at higher levels of disability.

We also investigate income misreporting in another way. Comparisons of the prevalence of receipt of disability benefit in survey data and administrative records suggest a degree of under-reporting in surveys (Hancock et al 2013). In the FRS sample there is a small but significant proportion of disabled people who report receiving care services but no disability benefit; this is surprising, because Local Authorities not only provide services but typically also give advice and assistance to promote benefit claims. To explore this, we construct an enhanced income variable by adding to full income for such people an imputed amount of disability benefit constructed as the sample mean level of disability benefit among those who reported receipt of both benefit and care services. We then repeat the analysis, showing the results in Table 4: there is very little change in the nonparametric estimates.

The stability with respect to income measurement that we see in the nonparametric estimates contrasts with the instability of parametric estimates. The lower panel of Table 4 shows the CV estimates derived from alternative ordered probit regressions of S on Y, D, X or on $\ln Y, D, X$. Income truncation produces a large fall in the estimated CV, especially for the logarithmic model.

Table 4 Nonparametric estimation and income measurement error: the effect of sample truncation and income enhancement

<i>Matching estimates</i>			
Index point	Full sample	Income-truncated ¹	Income-enhanced ²
0.38	£12.28 (7.11)	£16.44 (3.82)	£7.15 (13.69)
0.93	£28.8 (5.73)	£24.45 (3.77)	£29.52 (5.76)
1.51	£41.88 (5.96)	£42.98 (3.73)	£42.20 (5.99)
1.90	£62.59 (6.84)	£59.52 (5.08)	£63.93 (6.89)
2.25	£92.05 (7.28)	£90.65 (6.06)	£93.34 (7.33)
2.92	£125.69 (12.15)	£131.87 (6.68)	£126.63 (12.18)
Mean CV	57.31 (3.10)	55.38 (2.01)	58.22 (3.12)
<i>Parametric estimates</i>			
Linear model	515.97 (44.65)	205.08 (12.26)	521.87 (45.40)
Log-linear model	311.97 (26.32)	206.16 (14.16)	311.97 (26.32)

Nearest-neighbour propensity score matching, caliper = 0.01; standard errors in parentheses. ¹ Only cases with original + means-tested income less housing costs in range (£100, £700). ² Income enhanced by mean disability benefit amount if receipt of services is reported but not disability benefit.

6 Conclusions

This paper makes five main contributions. First, we have developed a new and simple approach to estimation of compensating variations (CV), using statistical matching estimators. This provides a flexible, largely nonparametric method which is less vulnerable than standard modelling techniques to functional form misspecification and other modelling errors which we have shown to cause serious biases. The new approach and the standard parametric approach rest on different identifying assumptions, and neither set of identifying assumptions

is obviously weaker or stronger than the other. The vulnerability to bias of the standard parametric approach may account for some of the surprisingly large CV estimates reported in the ‘happiness’ literature, and we expect the new approach to be a valuable complement to conventional econometric modelling for analysts seeking to assess the robustness of their estimates of the CV.

Second, we have clarified the theoretical basis for the estimation of CV measures, emphasising the important distinction between a CV concept based on material living standards and the CV derived from broader concepts of wellbeing. We have argued that the former CV concept is closer to the concerns of current disability policy and less likely to suffer from existence problems, and that the parametric methods used so far to estimate CVs based on subjective wellbeing scales are questionable because they impose existence of the CV a priori, as a by-product of the choice of functional form. Existence of the CV can be more directly investigated in the new approach by checking the common support restriction. The absence of any substantial difficulty with common support in our application confirms that there is no existence problem for the CV based on material living standards.

Third, we have emphasised the importance of using a full income measure to reflect access to economic resources more accurately than has been done in the existing literature on disability costs. We have argued that it is impossible to capture the correct relationship between achieved living standards and disability without incorporating both disability-tested welfare payments and the value of external care services (formal and informal) in measured income. Few surveys have sufficiently comprehensive coverage to meet these needs, but the Family Resources Survey used here does cover income and receipt of external support, together with indicators of disability and living standards.

Fourth, exploiting the appropriate full measure of access to economic resources and using the extensive margin for a set of basic goods and activities to indicate material living stan-

dards, we have been able to estimate the costs of disability. These estimates are conceptually conservative in the sense that they relate only to the material costs of disability, and thus exclude potentially large psycho-social costs. Despite this conservatism, we find disability costs to be high, with an overall mean of around £48-61 per week in the older disabled population, equivalent to about 21-26% of the disposable income that is available before any external disability support is received. The costs rise strongly with the extent of disability, but we find no evidence that the compensating variation is related to the reference level for living standards. This implies that the additional material costs of disability are uniform across the income distribution: a finding which contradicts the proportionality relationship embodied in conventional demographic equivalence scales.

Finally, we have shown that the method is robust to the most likely forms of income measurement error, but that it may yield underestimates if there are significant problems of misclassification of disability/living standards, or of unobserved confounding. The latter source of bias may particularly affect estimated disability costs at the moderate levels of severity that are common in the older population. Despite this additional source of conservatism in our analysis, comparing mean costs of disability with amounts of public support from disability benefit and subsidised care services suggests that public provision falls considerably short of total disability costs in Great Britain. Personal material costs of disability greatly exceed the value of state support for moderate to severe degrees of disability, and are only slightly below total external support when we include the value of informal care from friends and relatives outside the household, valued at market prices of carers' time.

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Appendix: Additional tables

Table A1 Disability and standard-of-living indicators by gender and household structure (%)

	Lone female ¹	Lone male ²	Couple ³	
			Female	Male
Has difficulty with...				
...mobility (moving about)	38.3	35.3	25.3	28.6
...lifting, carrying or moving objects	36.5	30.7	23.4	26.6
...manual dexterity using hands for daily tasks	15.1	11.2	10.2	9.7
...continence (bladder/bowel control)	7.2	8.7	5.0	7.4
...communication (speech, hearing or eyesight)	9.4	9.8	5.6	9.4
...memory/concentration/learning/understanding	6.0	6.8	3.8	6.2
...recognising when in physical danger	1.5	1.2	1.1	1.4
...physical co-ordination	12.9	11.7	7.4	9.0
...other area of life	12.3	14.0	9.0	10.7
% individuals giving 1 or more indications of disability	52.4	52.2	37.6	43.8
% individuals giving 2 or more indications of disability	37.9	34.5	24.2	28.9
Has enough money or does not want...				
...to keep home in a decent state of decoration?	88.5	89.9	94.2	
...to have household contents insurance?	93.3	90.9	96.8	
...to make savings of £10 a month or more?	75.3	78.9	80.6	
...to replace any worn out furniture?	80.5	84.8	88.3	
...to replace or repair broken electrical goods?	86.1	87.6	92.9	
...to spend each week on yourself, not on your family?	91.0	94.6	92.6	
% households with 1 or more indications of deprivation	36.6	30.8	26.6	
% households with 2 or more indications of deprivation	22.5	18.6	14.0	

Sample proportions for FRS 2004/5-2007/8 respondents. ¹ n = 8,910; ² n = 4,176; ³ n = 10,549.

Table A2 Weekly costs of indicative consumption activities

Deprivation indicator	Single woman	Single man	Married couple
Keep home in a decent state of decoration ¹	£8.13	£7.91	£8.13
Household contents insurance ²	£1.64	£1.64	£1.64
Make savings of £10 a month or more ³	£2.50	£2.50	£2.50
Replace any worn out furniture ⁴	£6.97	£6.97	£7.03
Replace/repair broken electrical goods ⁵	£3.74	£3.74	£3.74
Money to spend each week on self, not family ⁶	£2.59	£1.11	£3.01
Total weekly costs	£25.58	£23.88	£26.06

¹ *Enough money to keep your home in a decent state of decoration*: includes homeowners' costs for floor coverings, curtains, lampshades and textiles, gardening and DIY tools and home security (FBU 2002; Table 5 p. 18), converted to 2007 prices by CPI index "regular maintenance and repair of the dwelling".

² *Household contents insurance*: home contents policy for homeowners (FBU 2002, table 14 pg. 34), converted to 2007 prices by CPI index "insurance connected with the dwelling". ³ *Make savings of £10 a month or more*: £2.50. ⁴ *Replace any worn out furniture*: costs for a minimum set of furniture (FBU 2002; table 5 pg. 18), revalued by CPI index "furniture, furnishings". ⁵ *Replace or repair broken electrical goods such as fridge, washing machine*: costs for owner occupiers of replacing microwave, fridge freezer, etc. (FBU 2002; table 5 pg. 18), revalued by CPI index "major HH appliances & small electrical household appliances". ⁶ *Money to spend each week on yourself, not on your family*: costs of cosmetics and personal accessories such as suitcases, handbags, a modest selection of gold jewellery, umbrellas, etc. (FBU 2002; table 4 pg. 15), revalued by CPI index "appliances, articles & products for personal care".

Table A3 Covariates used for matching

Covariate	Definition	Mean ¹
<i>Age</i>	Age in years at last birthday	75.8
<i>Education</i>	Number of years' schooling beyond legal minimum	0.89
<i>Homeowner</i>	Homeowner with or without mortgage	0.67
<i>Social tenant</i>	Rents home from local authority or housing association	0.28
<i>House value</i> ²	Estimated market value of house (£'00,000)	1.89 ³
<i>Household type</i> ⁴	Single people	0.44
<i>Gender</i>	Female	0.56
<i>Asset owner</i> ⁴	Has savings over £1,000	0.60
<i>Financial assets</i>	Value of financial asset holdings per capita (£'000)	21.1 ⁵
<i>Scotland</i>	Lives in Scotland	0.18
<i>London</i>	Lives in London	0.06
<i>SE</i>	Lives in south-east England	0.11

¹ Mean in sample of individuals declaring at least one disability. ² See Morciano et al (2013) for details of construction. ³ Mean house value for homeowners only (median = 1.64). ⁴ Variable used for stratification (exact matching) in analysis. ⁵ Mean asset holding for those with positive amounts (median = 5.2). Monetary values in 2007 prices.

Table A4 Logit model for receipt of public disability support

Disability indicators	Coefficient (standard error)	Other covariates	Coefficient (standard error)
Mobility	0.935 (0.044)	Assets < £1,000	0.258 (0.039)
Lifting	0.444 (0.042)	Financial assets (£'000)	-0.000 (0.0001)
Dexterity	0.455 (0.044)	Single person	-0.482 (0.040)
Incontinence	0.319 (0.052)	Home owner	0.287 (0.094)
Communication	0.278 (0.049)	House value (£'00,000)	-0.262 (0.026)
Memory	0.377 (0.057)	Social housing	0.426 (0.086)
Danger	0.866 (0.123)	Years post-compulsory education	-0.054 (0.012)
Co-ordination	0.463 (0.046)	Age/10	-3.925 (0.512)
Other	0.476 (0.044)	(Age/10) squared	0.271 (0.033)
		Female	0.146 (0.038)
		Scotland	0.137 (0.048)
		London	-0.119 (0.080)
		SE	-0.280 (0.062)

Sample size = 13,796 individuals with at least one reported disability.

Table A5 Means of alternative income concepts by disability status

Income definition	Single Female		Single Male		Couple	
	$D = 0$	$D > 0$	$D = 0$	$D > 0$	$D = 0$	$D > 0$
Base income	177.81	177.07	216.61	197.63	394.07	335.02
(standard deviation)	(245.84)	(106.91)	(211.79)	(137.88)	(346.25)	(216.31)
Cash income	179.57	207.02	218.09	220.85	400.41	366.24
(standard deviation)	(246.43)	(126.13)	(212.12)	(147.95)	(345.96)	(224.8)
Cash income + value of formal care	182.59	234.5	220.5	242.78	402.87	376.26
(standard deviation)	(247.6)	(177.46)	(212.83)	(173.75)	(347.19)	(237.41)
Cash income + value of formal & external informal care (standard deviation)	185.24	303.01	223.7	291.47	402.43	383.82
	(252.64)	(296.82)	(219.26)	(257.73)	(346.96)	(250.41)

£per week at April 2007 prices.