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Estimating monetary policy rules when the zero lower bound on nominal interest rates is approached

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Estimating monetary policy rules when the zero lower bound on nominal interest rates is approached*

Konstantin Kiesel and Maik H. Wolters

Abstract:
Monetary policy rule parameters estimated with conventional estimation techniques can be severely biased if the estimation sample includes periods of low interest rates. Nominal interest rates cannot be negative, so that censored regression methods like Tobit estimation have to be used to achieve unbiased estimates. We use IV-Tobit regression to estimate monetary policy responses for Japan, the US and the Euro area. The estimation results show that the bias of conventional estimation methods is sizeable for the inflation response parameter, while it is very small for the output gap response and the interest rate smoothing parameter. We demonstrate how IV-Tobit estimation can be used to study how policy responses change when the zero lower bound is approached. Further, we show how one can use the IV-Tobit approach to distinguish between desired policy responses, that the central bank would implement if there was no zero lower bound, and the actual ones and provide estimates of both.

Keywords: monetary reaction function, zero lower bound, IV-Tobit estimator, censored regressions, non-linearity

JEL classification: E52, E58, E65

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1 Introduction

Studying monetary policy in terms of monetary policy rules has been in the interest of many researchers since Taylor (1993). He established the Taylor Rule which states that the interest rate set by the central bank can be explained as a linear function of two variables, inflation and the output gap. While the parameters of the original Taylor rule are calibrated, the response coefficients to inflation and the output gap can also be estimated. This is usually done by using ordinary least squares (OLS) or instrumental variables (IV) procedures like two-stage least squares (TSLS) in order to account for the simultaneity of the interest rate and inflation on the one hand and the interest rate and the output gap on the other hand. These methods, however, do not yield consistent estimates if the dependent variable is censored. Interest rates cannot fall below zero so that the usage of least squares estimators is problematic. The resulting bias is only neglectible as long as interest rates are high enough that reaching the zero lower bound is unlikely.

Figure 1 shows a plot of the short-term interest rates for Japan, the US and the Euro area from 1983 to 2013. It can be seen that the zero lower bound has become a constraint for all three central banks. Interest rates have been close to zero in Japan since the late 1990s, in the US since the end of 2008 and in the Euro area since 2013.

Figure 1: Policy rates in Japan, the US and the Euro area

Hence, monetary policy rules can no longer be estimated using standard methods. They would omit the obvious non-linearity that arises when the zero lower bound prevents the central bank to react to inflation and output gap dynamics as if there was no zero lower bound. Further, standard estimation methods do not deliver any information on how monetary policy responses change when the zero lower bound is approached. In this paper we show how censored estimation methods can be used to achieve consistent parameter estimates and we analyze how the estimated monetary policy responses change when the interest rate approaches zero.

A well-established procedure applied in many fields of economics that can deal with censoring is the Tobit estimator. Kato and Nishiyama (2005) and Kim and Mizen (2010) have been the first and, to the best of our knowledge, the only ones so far who applied the Tobit estimator to the

estimation of monetary policy rules. Kato and Nishiyama (2005) estimate monetary policy rules for Japan using a sample until the end of 2000. They use the Tobit estimator to achieve unbiased estimates in the low interest rate environment that prevails in Japan since the mid 1990s. Kim and Mizen (2010) also estimate a monetary policy rule for Japan and use a specification in which the interest rate responds to an inflation forecast and the output gap. Their sample ends in 2003. They use the IV-Tobit estimator to take into account the endogeneity of the inflation forecast and the output gap and they demonstrate that TSLS estimates are biased.

Our contribution is multifaceted. First, we estimate monetary policy responses not only for Japan, but also for the US and the Euro area as the zero lower bound has also become an issue for these economies after the Great Recession. Second, in contrast to Kato and Nishiyama (2005) and Kim and Mizen (2010) we account for the fact that most central banks change interest rates in a very gradual manner, which can be captured by including the lagged interest rate in the regression. The respective interest rate smoothing coefficient is usually close to one and highly significant (see e.g. Clarida et al., 1998; Orphanides, 2001; Orphanides and Wieland, 2008, among many others). Third, we are the first that use the IV-Tobit approach to analyse how the estimated monetary policy responses change when interest rates approach zero. To study this we exploit the non-linear dependence between monetary policy responses and the level of the interest rate, the inflation rate and the output gap that is captured by the IV-Tobit estimates. In this context we can also distinguish between the estimated desired monetary policy responses that the central bank would have set if there was no zero lower bound and the actual ones. While fitted values of the desired interest rate can become negative, the IV-Tobit estimator makes sure that fitted values of the actual interest rate remain above zero. Finally, we discuss whether the estimated change in policy responses when approaching zero interest rates is in line with predictions from theory. Overall, the explanations and results from this paper should help to understand how the IV-Tobit approach can be applied to monetary policy rule estimation, how the estimation results can be interpreted and also what the limitations of this approach are.

We find that conventional estimation techniques lead to a sizable bias in the estimated inflation response for all three economies, while the biases for the output gap response and the interest rate smoothing coefficients are small. The TSLS estimates overestimate the inflation response for Japan and the Euro area and underestimate it for the US. The IV-Tobit estimates of the desired monetary policy responses are larger than the estimates of the actual ones, because the latter mix policy responses in periods where the interest rate is far away from the zero lower bound—and policy can react as desired to inflation and the output gap—and estimates in periods of low interest rates where monetary policy responses are restricted by the zero lower bound. We show that the size of monetary policy responses depends directly on the estimated probability of observing an interest rate above zero conditional on inflation and the output gap. As long as this estimated probability is one, there is no change in monetary policy responses. This is the case for Japan until 1998, for the US until 2009 and for the Euro area until 2012 except for the year 2009. Once this estimated probability is below one, the actual monetary policy responses are lower than the desired ones. Our estimates show that the zero lower bound implies sharp restrictions for monetary policy responses in Japan and the US. While policy responses in the Euro area are currently smaller than desired, the

restrictions are smaller than for Japan and the US.

The remainder of the paper is structured as follows. Section 2 introduces the IV-Tobit estimation method in the context of monetary policy rules. In section 3 we describe the data used for the estimation. In section 4 we first explain how the estimates can be interpreted using a simple specification without interest rate smoothing. Afterwards we present the estimation results for the more realistic case with interest rate smoothing and discuss these. Section 5 relates the estimation results to predictions from economic theory about monetary policy responses close to the zero lower bound. Finally, section 6 concludes.

2 Censored regression and monetary policy rules

In the seminal paper by Taylor (1993) the interest rate responds to a weighted average of deviations of inflation from an inflation target and of output from potential output. In later work it has been found that rules which include an interest rate smoothing term and specifications where monetary policy responds to expectations about inflation (see e.g. Clarida et al., 2000) provide a good description of actual monetary policy. A general specification of this type of rules is given by:

$$i_t = \rho i_{t-1} + (1-\rho)\left(\bar{r} + \bar{\pi} + \gamma\left(\pi_{t+h|t} - \bar{\pi}\right) + \delta y_t\right) + \epsilon_t. \tag{1}$$

i_t denotes the nominal interest rate, $\bar{r}$ the long-run real interest rate, $\bar{\pi}$ the targeted inflation rate, $\pi_{t+h|t}$ an inflation forecast for horizon h based on information in period t, y_t an output gap and ϵ_t a monetary policy shock. The parameter ρ stands for the degree of interest rate smoothing, γ is the inflation response and δ is the response to the output gap.

For simplicity we will work with a linear version of equation (1) in what follows:

$$i_t = \alpha_0 + \alpha_i i_{t-1} + \alpha_\pi \pi_{t+h|t} + \alpha_y y_t + \epsilon_t = x_t\beta + \epsilon_t, \tag{2}$$

where $\alpha_0 = (1-\rho)(\bar{r}+(1-\gamma)\bar{\pi})$, $\alpha_i = \rho$, $\alpha_\pi = (1-\rho)\gamma$, $\alpha_y = (1-\rho)\delta$, $x_t = (1, i_{t-1}, \pi_{t+h|t}, y_t)$ and $\beta = (\alpha_0, \alpha_i, \alpha_\pi, \alpha_y)'$. The parameters of this equation can usually be estimated at the conditional expected value of the interest rate using standard methods like OLS or TSLS to handle endogeneity problems.

2.1 The Tobit model applied to monetary policy rules

When the nominal interest rate approaches the zero lower bound equation (2) is no longer a good description of actual policy setting as fitted values $\hat{i}_t$ can become negative. If the interest rate is restricted to positive values, i.e. $i_t \geq 0$, then assuming $E(i_t|x_t) = x_t\beta$ ignores the nonlinearity between i_t and x_t. The linear policy rule implies a constant partial effect, while the zero lower bound implies that the central bank cannot set the interest rate in response to inflation and the output gap as usual. Further, from an econometric point of view estimates of equation (2) will be biased as demonstrated in Kim and Mizen (2010) if the truncation of i_t is ignored. Conventional techniques for the estimation of monetary policy rules cannot be used and even for historical analyses cutting

the sample off before the zero lower bound is reached leads to inconsistent estimates (Wooldridge, 2010).

To deal with the truncation of i_t we rewrite equation (2):

$$i_t = min\{0, i_t^*\} \tag{3}$$

$$i_t^* = \alpha_0 + \alpha_i i_{t-1} + \alpha_\pi \pi_{t+h|t} + \alpha_y y_t + \epsilon_t = x_t\beta + \epsilon_t. \tag{4}$$

The notation has changed compared to equations (1) and (2) as we now need to distinguish the observed interest rate $i_t \geq 0$ and a new latent variable i_t^*. i_t^* is usually not directly interpreted in censored regression models of this type and only used to econometrically deal with the corner solution problem to get consistent estimates of $E_t(i_t|x_t)$. When estimating monetary policy rules, however, one might interpret i_t^* as the interest rate that the central bank would have liked to implement, if there was no zero lower bound and consistent estimates of $E_t(i_t^*|x_t)$ can be of interest to study the desired policy responses in addition to estimates of the actual ones, $E_t(i_t|x_t)$.

Assuming $\epsilon_t \sim N\left(0, \sigma^2\right)$ equations (3) and (4) resemble a standard censored Tobit model (Tobin, 1958) which can be consistently estimated as proven by Amemiya (1973). The Tobit model can be written as:

$$i_t = \begin{cases} i_t^* & \text{if} \quad i_t^* > 0 \\ 0 & \text{if} \quad i_t^* \leq 0 \end{cases} . \tag{5}$$

The conditional expected value for i_t is given by:

$$E(i_t|x_t) = P(i_t = 0|x_t)\, 0 + P(i_t > 0|x_t)\, E(i_t|x_t, i_t > 0). \tag{6}$$

$P(i_t > 0|x_t)$ can be written as a Probit model for the binary variable w which is defined as $w = 1$ if $i_t > 0, w = 0$ if $i_t = 0$ (the explanations here closely follow Wooldridge, 2010):

$$P(w = 1|x) = P(i_t^* > 0|x_t) = P(\epsilon_t > -x_t\beta|x_t) = P(\epsilon_t/\sigma > -x_t\beta/\sigma) = \Phi(x_t\beta/\sigma), \tag{7}$$

where $\Phi(.)$ denotes the *cdf* of the standard normal distribution. It can be shown that the last term of equation (6) is given by:

$$E(i_t|x_t, i_t > 0) = x_t\beta + E(\epsilon_t|\epsilon_t > -x_t\beta) = x_t\beta + \sigma\left[\frac{\phi(x_t\beta/\sigma)}{\Phi(x\beta/\sigma)}\right], \tag{8}$$

where $\phi(.)$ is the *pdf* of the standard normal distribution. Putting both terms together and simplifying we get a final expression for $E(i_t|x_t)$:

$$E(i_t|x_t) = \Phi(x_t\beta/\sigma)\left[x_t\beta + \sigma\frac{\phi(x_t\beta/\sigma)}{\Phi(x_t\beta/\sigma)}\right]. \tag{9}$$

In contrast to the latent model $E(i_t^*|x_t) = x_t\beta$, the conditional expectation $E(i_t|x_t)$ depends on the macroeconomic indicators x_t in a non-linear way.

2.2 Monetary policy responses when the zero lower bound is approached

While the interpretation of the right-hand side terms of equation (9) is difficult, the implied partial effects have a very intuitive interpretation. Wooldridge (2010) shows that after some simplification the partial effects can be written as:

$$\frac{\partial E(i_t|x_t)}{\partial x_{j,t}} = \Phi(x_t\beta/\sigma)\beta_j. \tag{10}$$

For comparison the partial effects of the latent model are simply given by:

$$\frac{\partial E(i_t^*|x_t)}{\partial x_{j,t}} = \beta_j. \tag{11}$$

The response of the interest rate to inflation in equation (10) does therefore not only depend on $\beta_3 = \alpha_\pi$ as in the uncensored monetary policy rule, but it also depends non-linearly on the scale factor $\Phi(x_t\beta/\sigma)$. The estimated scale factor $\Phi(x_t\hat{\beta}/\hat{\sigma})$ denotes the estimated probability of observing a positive interest rate for a given x_t: $\Phi(x_t\hat{\beta}/\hat{\sigma}) = \hat{P}(i_t > 0|x_t)$. If $\Phi(x_t\hat{\beta}/\hat{\sigma})$ is close to one, then hitting the zero lower bound becomes unlikely and the partial effect $\Phi(x_t\beta/\sigma)\beta_j$ approaches β_j. $\Phi(x_t\hat{\beta}/\hat{\sigma})$ can be expected to increase with the values of the inflation forecast, the output gap and the lagged interest rate.

Kato and Nishiyama (2005) and Kim and Mizen (2010) use the Tobit estimator to achieve consistent estimates of β for monetary policy rules for Japan. Our analysis shows, however, that there are several other interesting parameters that can additionally be analyzed to study how monetary policy changes when the zero lower bound on nominal interest rates is approached. The objects of interest are:

1. *Partial effect in the latent model*: $\hat{\beta}$ denotes the estimated desired monetary policy response. In contrast to OLS the Tobit model yields consistent estimates of $\hat{\beta}$.

2. *Partial effect evaluated at the sample mean*: $\Phi(\bar{x}\hat{\beta}/\hat{\sigma})\hat{\beta}_j$ denotes the estimated actual monetary policy response evaluated at the sample mean $\bar{x}$ taking into account the zero lower bound. This object is, however, only partially informative as it mixes policy reactions when the zero lower bound is binding and during other times. Therefore, it is useful to study the policy responses at different values of x_t directly.

3. *Partial effect at different values of* x_t: $\Phi(x_t\hat{\beta}/\hat{\sigma})\hat{\beta}_j$ is an estimate of monetary policy responses for different realizations of the lagged interest rate, the inflation forecast and the output gap. It shows how monetary policy responses change when the zero lower bound is approached because inflation expectations are low and/or a recession occurs. When the probability of hitting the zero lower bound is low then $\Phi(x_t\hat{\beta}/\hat{\sigma})\hat{\beta}_j$ approaches $\hat{\beta}_j$.

2.3 IV-Tobit estimation

While the Tobit-model solves the non-linearity problem induced by the zero lower bound on nominal interest rates, the usual endogeneity problem caused by the two-way interaction of the interest

rate with expected inflation and the output gap persists. To solve this we use an IV-version of the Tobit estimator. Here, one can either run a two-step estimation (Newey, 1987) or a full maximum likelihood estimation that includes the instruments directly. The disadvantage of the two-step estimator is that it gives no estimate of σ which we need to compute estimates of $\Phi(x_t\beta/\sigma)$. Therefore, we use the full maximum likelihood estimator for which standard conditional maximum likelihood theory can be used to construct standard errors and test statistics. We use the Hubert-White estimator to get Heteroscedasticity-consistent standard errors.

3 Data

We use monthly data for Japan, the US and the Euro area. The policy rate for Japan is the uncollateralized overnight call rate which is directly available from the Bank of Japan. Data is available from July 1985 onwards, thus the sample includes 322 observations from 1985M7 to 2013M5. Regarding the inflation rate we compute year-on-year inflation rates based on the CPI index. As GDP data is not available on a monthly frequency we use industrial production instead. The output gap is computed using the HP-filter. Inflation and industrial production data are obtained from the OECD database.

For the US we also use CPI-inflation and industrial production data provided by the OECD. The effective federal funds rate is used as a proxy of the policy instrument. The sample for the US starts in 1983M1 and goes through 2013M6, which yields 354 observations. We do not start earlier to avoid a structural break in monetary policy responses to inflation and the output gap before and after Paul Volcker was chairman of the Fed.

As the Euro was introduced in 1999, we use monthly data for the Euro area from 1999M1 to 2013M6, which results in 162 observations. Data for CPI-inflation, industrial production and the EONIA rate are taken from the ECB data warehouse.

We follow Clarida et al. (1998) and Kim and Mizen (2010) and use 12-months-ahead ex-post inflation rates to approximate expected inflation. IV-estimators control for possible measurement error bias owing to the approximation of inflation forecasts with ex-post observations (see e.g. Clarida et al., 1998). We also experimented with forecasts of both, inflation and the output gap (see e.g. Orphanides, 2001) and we document in which cases the resulting estimates are similar and in which cases they differ from the baseline results.

Through the construction of expected inflation measures we lose twelve observations for each sample. In addition six further observations are lost because we use six lags of inflation and the output gap as instruments. These lagged variables are correlated with expected inflation and the output gap. They can be assumed to not be influenced by the period t interest rate as they refer to macroeconomic developments in periods $t-1$ to $t-6$.

4 Estimation results

We start with the estimation of the simple case without interest rate smoothing, i.e. $\alpha_i = 0$, to demonstrate how the different estimated objects can be used to describe monetary policy above the

zero lower bound and also when approaching the zero lower bound. This case has also been studied by Kato and Nishiyama (2005) and Kim and Mizen (2010) for Japan. Afterwards, we study the more realistic case without restriction on the interest rate smoothing parameter.

4.1 A simple benchmark case without interest rate smoothing

Table 1 shows the estimated partial effects for the case without interest rate smoothing. The first column for each of the three economies refers to the TSLS-estimates of equation (2). The second column shows the unbiased counterpart estimated at $E(i_t^*|x_t)$ which is informative if the interest rate is well above zero. At low interest rates, this estimate can be interpreted as the desired interest rate response that the central bank would have implemented if there was no zero lower bound. Finally, the third column shows the estimates for $E(i_t|x_t)$ evaluated at the sample mean $\bar{x}$. We will study $E(i_t|x_t)$ for alternative values of x_t below. The table further shows estimates of σ and the number of observations.

The estimates show that the Taylor principle of increasing the nominal interest rate more than one-to-one in response to changes in inflation is fulfilled for all three central banks. The inflation response coefficients are well above one and they are highly significant. The output gap coefficient estimates are insignificant and close to zero for Japan. Similarly, Clarida et al. (1998), Kuttner and Posen (2004) and Kim and Mizen (2010) find a response to the output gap for Japan that is insignificant on the 5% level. The output gap responses are positive and significant for the US. For the Euro area we use a slightly different specification than for Japan and the US. We include an ex-post output gap forecast—constructed in the same way as the inflation forecast—instead of the actual output gap. Using outcomes instead of forecasts for the output gap would yield a significant negative inflation coefficient. We regard this as implausible. With the output gap forecast specification the inflation coefficient has the expected sign, but the output gap coefficient turns out to be negative and significant. So, overall the results without interest rate smoothing for the Euro area have to be interpreted with caution as these are signs for possible misspecification. The more realistic results with interest rate smoothing which are discussed in the next section yield plausible parameter estimates for the inflation and the output gap response.

For all three central banks the inflation response parameter is higher for the IV-Tobit estimates ($\hat{\alpha}_\pi$) than for the TSLS estimates ($\hat{\alpha}_\pi^{\text{TSLS}}$). Intuitively, the larger IV-Tobit estimates make sense as the TSLS estimates include periods where the interest rate needs to stay constant even if inflation decreases further, which lowers estimates of the inflation response. The differences between the TSLS and IV-Tobit estimates are largest for the US and smallest for the Euro area. For the Euro area the zero lower bound is not binding yet and due to the construction of the inflation forecasts we lose the last 12 observations with the interest rate observations close to zero. The difference between the TSLS and IV-Tobit estimates of α_y are small for all three economies. The bias thus mainly shows up in the inflation response parameter estimates.

Comparing the TSLS estimates with interest rate responses estimated at $E_t(i_t|\bar{x})$ using IV-Tobit which correctly includes the non-linearity, confirms the upward bias of conventional estimates

	Japan			US			Euro area		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	TSLS	*IV-Tobit*	*IV-Tobit*	*TSLS*	*IV-Tobit*	*IV-Tobit*	*TSLS*	*IV-Tobit*	*IV-Tobit*
	$E(i_t\|x_t)$	$E(i_t^*\|x_t)$	$E(i_t\|\bar{x})$	$E(i_t\|x_t)$	$E(i_t^*\|x_t)$	$E(i_t\|\bar{x})$	$E(i_t\|x_t)$	$E(i_t^*\|x_t)$	$E(i_t\|\bar{x})$
	$\hat{\beta}_j^{\text{TSLS}}$	$\hat{\beta}_j$	$\Phi(\bar{x}\hat{\beta}/\hat{\sigma})\hat{\beta}_j$	$\hat{\beta}_j^{\text{TSLS}}$	$\hat{\beta}_j$	$\Phi(\bar{x}\hat{\beta}/\hat{\sigma})\hat{\beta}_j$	$\hat{\beta}_j^{\text{TSLS}}$	$\hat{\beta}_j$	$\Phi(\bar{x}\hat{\beta}/\hat{\sigma})\hat{\beta}_j$
inflation response	2.689***	2.843***	2.118***	1.838***	3.557***	3.096***	2.903***	2.929***	2.641***
	(0.182)	(0.500)	(0.400)	(0.243)	(0.293)	(0.240)	(0.728)	(0.166)	(0.169)
output gap response	-0.038	-0.041	-0.031	0.453***	0.433***	0.376**	-0.770***	-0.799***	-0.720***
	(0.032)	(0.048)	(0.035)	(0.073)	(0.167)	(0.147)	(0.159)	(0.077)	(0.084)
constant	0.605***	0.531**		-0.674	-5.628***		-3.598**	-3.649***	
	(0.148)	(0.235)		(0.723)	(0.961)		(1.504)	(0.387)	
$\hat{\sigma}$			2.663			4.117			1.940
Observations	317	317	317	348	348	348	162	162	162

*,**,*** indicate significance at the 10%-, 5%- and 1%-level, respectively.

Table 1: Monetary policy rule parameter estimates without interest rate smoothing for Japan, the US and the Euro area.

found by Kim and Mizen (2010) for Japan.[1] TSLS Euro area estimates also show an upward bias, while the bias for the US estimates is negative.[2] These differences in the inflation response coefficients and to a much smaller extent in the output gap response coefficients show that the estimation of monetary policy rules for these samples leads to unreliable estimates if the zero lower bound is not taken into account.

Finally, when comparing the second and third column, the results show that the desired interest rate responses, $\hat{\beta}$, to inflation and in the case of the US also to the output gap are always larger than the actual ones, $\Phi(\bar{x}\hat{\beta}/\hat{\sigma})\hat{\beta}$. This intuitively makes sense, because the actual interest rate response estimates take into account the constraints on monetary policy that prevent central banks from reacting as strongly to inflation and the output gap as they desire.

The analysis so far has shown how the Tobit framework can be used to achieve consistent estimates of monetary policy rule parameters. These techniques can hence be used in the future to conduct historical monetary policy analysis. Now, we go one step further and study how the policy response parameters change, when the interest rate approaches the zero lower bound.

The solid line in figure 2 shows the estimated inflation response for different values of inflation: $\Phi((1, \pi_t, \bar{y})\hat{\beta}/\hat{\sigma})\hat{\alpha}_\pi$. For the output gap we again take the sample mean. The circles mark the estimated inflation response at the sample mean as shown in table 1. For comparison the dotted lines show the desired inflation responses, i.e. $\hat{\alpha}_\pi$ estimated at $E(i_t^*|x_t)$, and the dashed-dotted lines show the (biased) TSLS estimates $\hat{\alpha}_\pi^{\text{TSLS}}$. Both do not depend on the level of inflation so that they are depicted as horizontal lines.

The solid line reveals the full non-linearity of the inflation response when the zero lower bound is approached as a result of decreasing inflation. Very low inflation rates are usually accompanied by very low interest rates, so that central banks cannot react to these by decreasing the policy rate further. The estimated inflation response parameter therefore converges to zero. Comparing the solid line with the TSLS estimates shows that for most inflation rates the TSLS estimates are upward biased for Japan and the Euro area. Only for inflation rates above about 1.3% the bias becomes negative for Japan. For the Euro area the bias diminishes for inflation rates above 2.5%. For the US the bias is negative for inflation above 1.5% and positive for inflation rates below 1.5%. Comparing the solid lines with the desired inflation responses (dotted lines) shows that already for inflation rates below 2% for Japan, below 4% for the US and below 2.5% for the Euro area the actual inflation responses start to deviate from the desired ones. So, at least for monetary policy rule estimates without interest rate smoothing accounting for the non-linearity induced by the zero lower bound is of importance, not only directly at the zero lower bound but also above. Policy responses deviate from the desired ones even for inflation rates as high as the sample means (sample means of inflation are indicated by the circles).

[1] The sample mean for inflation is 0.4%, 3% and 2.1% for Japan, the US and the Euro area, respectively. The sample mean of the output gap is 0 by construction for all three economies.

[2] We refer the reader to Kim and Mizen (2010) for the exact econometric conditions for the bias to be positive or negative.

Figure 2: Inflation responses for different levels of inflation

Finally, we can check what the different parameter estimates imply for the fitted interest rate. Figure 3 shows a scatter plot of the observed interest rates and the inflation forecasts together with the fitted interest rate for different levels of inflation (the output gap is hold constant at the sample mean). The solid lines show the implied interest rates when taking into account the non-linearity induced by the zero lower bound. The two straight lines show the fitted interest rates implied by the TSLS estimates (dashed-dotted) and the implied latent or desired interest rate $\hat{i}_t^*$ (dotted). The difference between the two is particularly large for the US reflecting the large bias of the TSLS estimates in this case. The fit even for the fully non-linear IV-Tobit estimator is not particularly good, because we hold the output gap fixed at zero, while low inflation and low interest rates are often observed for negative output gaps. For inflation rates above about 1% for Japan and above about 2% for the US and the Euro area the IV-Tobit estimates for $\hat{i}_t$ and $\hat{i}_t^*$ coincide as $\hat{P}(i_t > 0|x_t) \longrightarrow 1$. Yet even below these values the linear estimates do not provide a good description of actual interest rate responses. For inflation rates below 0% for Japan and below 1%

for the US and the Euro area the TSLS estimates imply negative interest rates and the unbiased IV-Tobit estimates for $\hat{i}_t^*$ confirm that the central banks would have set negative interest rates if they could. In contrast, the IV-Tobit estimates for $\hat{i}_t$ take into account the zero lower bound and converge to zero for low positive and negative inflation rates.

Figure 3: Expected central bank rate for different inflation expectations

4.2 Monetary policy rule estimates with interest rate smoothing

Having demonstrated the non-linearities of monetary policy responses when the interest rate approaches zero for the simple case without interest rate smoothing, we now turn to the more realistic estimates with interest rate smoothing. Table 2 shows the estimated partial effects. The table is structured exactly as for the case without interest rate smoothing but additionally reports the esti-

mated response to the lagged interest rate.[3]

It is apparent that the response to the lagged interest rate is large and highly significant for all three economies. The ECB sets interest rates most gradually with a coefficient very close to one. The interest rate smoothing coefficient is only slightly lower for the US, but quite a bit lower for Japan. The inflation response is positive and highly significant for all three central banks. From the table it is not clear whether the Taylor principle is satisfied because we report estimates of $\alpha_\pi = (1-\rho)\gamma$. If the structural inflation response coefficient $\gamma = \alpha_\pi/(1-\rho)$ are computed it can be seen that the Taylor principle is fulfilled for all three central banks. The structural inflation response coefficient is largest for the Euro area owing to the very large estimate of ρ in the denominator: $\hat{\gamma}^{EA} = 0.225/(1-0.997) = 75$. For the US and Japan the coefficients are smaller and in a more reasonable range: $\hat{\gamma}^{US} = 4.03$, $\hat{\gamma}^{JAP} = 2.67$. The output gap response is close to zero and insignificant for Japan, but positive and highly significant for the US and the Euro area. The estimation results are roughly in line with what previous literature has found for rules with an interest rate smoothing term.

Comparing the TSLS and IV-Tobit estimates ($\hat{\beta}^{\text{TSLS}}$ and $\hat{\beta}$) shows that the TSLS estimates are biased. In contrast to the results without interest rate smoothing, the bias of the inflation response is now negative for all three central banks. As in the previous section the bias of the output gap response estimates is very small. Regarding the interest rate smoothing coefficient, the TSLS estimates overestimate the degree of interest rate smoothing somewhat for Japan and the US and underestimate it for the Euro area.

Comparing the desired interest rate responses, $E(i_t^*|x_t)$, with the actual ones evaluated at the sample mean, $E(i_t|\bar{x})$, shows that there is no difference at all. These results are very different from the estimation results without interest rate smoothing in the previous section. The explanation is that the IV-Tobit estimates are evaluated at the sample mean of x_t. The sample mean for the interest rate which is included in x_t via the lagged interest rate is quite a bit above zero (1.79%, 4.56% and 2.33% for Japan, the US and the Euro area). Thus at the sample mean the IV-Tobit estimates cannot reveal any non-linearities as the central banks can implement the desired monetary policy responses. Therefore, we now turn to the evaluation of the interest rate responses at different values for x_t including those close to zero to study the non-linearity of policy responses.

Figure 4 shows how the inflation response changes with the level of expected inflation. We hold the output gap constant at zero and the interest rate at 0.25%. Holding the interest rate constant at the sample mean would prevent any non-linearities in the graph as this is too far away from the zero lower bound to change the inflation response even for deflationary forecasts.

[3] For the case with interest rate smoothing there are no miss-specification problems for the Euro area estimates leading to negative inflation response estimates as in the previous section so that we can report estimates for all three economies for the baseline specification where the interest rate responds to forecasts of inflation, but to outcomes of the output gap.

	Japan			US			Euro area		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	TSLS	*IV-Tobit*	*IV-Tobit*	*TSLS*	*IV-Tobit*	*IV-Tobit*	*TSLS*	*IV-Tobit*	*IV-Tobit*
	$E(i_t\|x_t)$	$E(i_t^*\|x_t)$	$E(i_t\|\bar{x})$	$E(i_t\|x_t)$	$E(i_t^*\|x_t)$	$E(i_t\|\bar{x})$	$E(i_t\|x_t)$	$E(i_t^*\|x_t)$	$E(i_t\|\bar{x})$
	$\hat{\beta}_j^{\text{TSLS}}$	$\hat{\beta}_j$	$\Phi(\bar{x}\hat{\beta}/\hat{\sigma})\hat{\beta}_j$	$\hat{\beta}_j^{\text{TSLS}}$	$\hat{\beta}_j$	$\Phi(\bar{x}\hat{\beta}/\hat{\sigma})\hat{\beta}_j$	$\hat{\beta}_j^{\text{TSLS}}$	$\hat{\beta}_j$	$\Phi(\bar{x}\hat{\beta}/\hat{\sigma})\hat{\beta}_j$
inflation response	0.255***	0.360***	0.360***	0.095***	0.137***	0.137***	0.151***	0.225***	0.225***
	(0.068)	(0.127)	(0.127)	(0.032)	(0.004)	(0.004)	(0.042)	(0.002)	(0.002)
output gap response	-0.002	-0.003	-0.003	0.016*	0.020***	0.019***	0.018***	0.017***	0.017***
	(0.003)	(0.096)	(0.096)	(0.008)	(0.006)	(0.006)	(0.005)	(0.000)	(0.000)
interest rate response	0.900***	0.865***	0.865***	0.975***	0.966***	0.966***	0.991***	0.997***	0.997***
	(0.024)	(0.026)	(0.026)	(0.009)	(0.003)	(0.003)	(0.011)	(0.001)	(0.001)
constant	0.043**	0.059		-0.185***	-0.264***		-0.316***	-0.486***	
	(0.018)	(0.395)		(0.065)	(0.029)		(0.100)	(0.004)	
$\hat{\sigma}$			0.320			0.247			0.193
Observations	317	317	317	348	348	348	162	162	162

*,**,*** indicate significance at the 10%-, 5%- and 1%-level, respectively.

Table 2: Monetary policy rule parameter estimates with interest rate smoothing for Japan, the US and the Euro area.

Figure 4: Inflation responses for different levels of inflation

In Japan, since the mid-1990s when the interest rate approached zero actual observed inflation has been in a range from about -2% to 2%. The graph shows that for this range the inflation response varies from 0 to 0.4 and coincides with the desired response only for inflation rates above 1%. For the US observed inflation ranges from about -2% to 4% since the zero lower bound became an issue in 2010. For this whole range the actual inflation response is lower than the desired one and is close to zero for $\pi_{t+12|t} = -2\%$. Finally, inflation rates for the Euro area for the two periods of low interest rates from the middle of 2009 to the end of 2010 and again from 2012 onwards range from about 1.5% to 3%. For this range the actual inflation responses are lower than the desired ones, though they do not reach zero.

So far, we have studied non-linearities close to the zero lower bound caused by different inflation forecasts in isolation. To study how monetary policy responses change when the zero lower bound is approached not only through changes in inflation, but the combination of previously low interest rates, changes in inflation forecasts and changes in the output gap, we compute the partial effects for each point in time t evaluated at the specific values i_{t-1}, $\pi_{t+12|t}$ and y_t. In addition we can compute

the estimated probability of observing an interest rate above zero given the lagged interest rate, the inflation forecast and the output gap: $\Phi(x_t\hat{\beta}/\hat{\sigma}) = \hat{P}(i_t > 0|x_t)$. The monetary policy responses at each point in time equal this probability times the estimated policy response parameters $\hat{\alpha}_i$, $\hat{\alpha}_\pi$ and $\hat{\alpha}_y$ as shown in equation (10).

Figure 5 shows the results for Japan. In addition to the policy response coefficients and the estimated probability of observing a strictly positive interest rate given x_t the figure also shows data for the three macroeconomic variables contained in x_t. The first graph of figure 5 shows the estimated probability of observing an interest rate above zero, $\Phi(x_t\hat{\beta}/\hat{\sigma})$. This term was equal to one until 1998. The second graph shows the nominal interest rate. It dropped to 0.5% in 1995. This was, however, not sufficient to change the monetary policy response as can be seen in the third, fifth and seventh graph of the figure. In 1998 the decrease in the inflation forecast led to a drop in the probability of the interest rate being above zero. From this point onwards the smoothing coefficient, the inflation response and the output gap response are lower than the desired responses that were in place until 1998. In 1999 following further interest rate decreases the probability of hitting the zero lower bound increased and the monetary policy responses to inflation and the output gap approached values close to zero. Additionally the interest rate smoothing coefficient decreased substantially. From then on there is only one minor change in the interest rate. The interest rate increased from values close to zero to up to 0.5% between the middle of 2006 and the end of 2008. During this period $\hat{P}(i_t > 0|x_t)$ went back to one and the monetary policy responses were equal to the desired ones. For the remaining periods $\hat{P}(i_t > 0|x_t)$ and hence the strength of monetary policy responses closely reflect the inflation developments. While there are large movements in the output gap as well—in particular the output gap dropped below -20% during the financial crisis—this has almost no impact on the policy response as the estimates show no reaction of the Japanese policy rate to the output gap.

Figure 6 shows that the US central bank was able to implement the desired interest rate responses for the largest part of the sample. Only since 2009 the estimated probability of the interest rate being above zero deviates from one and dropped sharply in 2009 because of the highly negative output gap caused by the financial crisis and the following interest rate reductions. Interestingly, the drop in the inflation forecast for 2009 that occurred in 2008 did not reduce the probability of the interest rate staying above zero. Here, the limitations of the approach of approximating forecasts with actual ex-post inflation observations becomes visible. Actual inflation forecasts in 2008 for 2009 were probably not as pessimistic and therefore the interest rate was only lowered once the financial crisis caused the large negative output gap in 2009. The drop in the probability of the interest rate being above zero led to a change in the monetary policy responses. The inflation response decreased from 0.14 to 0.05 and the output gap response dropped from 0.02 to 0.01.[4] In 2010 inflation forecasts increased again (because of the actual inflation increase in 2011) and the probability of the interest rate being above zero returned to values close to one. Accordingly, the

[4] One should keep in mind that these are combined coefficients that include $1 - \rho$ and not the structural coefficients. Though these coefficients seem to be very small, their effect is amplified over time through interest rate smoothing.

Figure 5: Monetary policy responses for Japan over time

Figure 6: Monetary policy responses for the US over time

Figure 7: Monetary policy responses for the Euro area over time

policy responses to the lagged interest rate, inflation and the output gap increased. However, the interest rate smoothing coefficient was so large, that despite this increase in inflation the interest rate remained at zero. After 2010 the probability of the interest rate being above zero was closely related to inflation and output gap dynamics and equaled about 0.6. Inflation remained somewhat below 2% and the estimated output gap was around 1%. Any future decrease of inflation or the output gap would decrease the probability of the interest rate being above zero even further and lower monetary policy responses to inflation and the output gap.

Finally, figure 7 shows monetary policy responses over time for the Euro area. It is apparent that the zero lower bound has changed monetary policy responses only to some extent in 2009 and from 2012 onwards. In 2009 the output gap was low owing to the financial crisis and the ECB lowered the interest rate accordingly. The probability of the interest rate being above zero dropped from 1 to 0.7. Accordingly, the inflation response decreased from 0.23 to 0.15. In 2010 the increase in inflation and the output gap led to normal interest rate responses again and the interest rate increased slightly in 2011. In 2012 the ECB lowered the interest rate again as the inflation forecast and the output gap decreased because of the weak economic dynamics caused by the sovereign debt crisis. The probability of the interest rate being above zero dropped to about 0.8 so that monetary policy responses were weakened somewhat. They are, however, in contrast to some periods in Japan and the US still largely above zero.

5 The IV-Tobit estimates and predictions from economic theory

The estimation results of the previous section showed that actual policy responses to inflation, the output gap and the lagged interest rate will start to deviate from the desired ones, once the estimated probability of observing strictly positive interest rates conditional on the lagged interest rate, the inflation forecast and the output gap decreases below one. The estimated monetary policy responses decrease proportionally to this probability when the zero lower bound is approached. By definition the IV-Tobit estimates of monetary policy responses must become smaller when the zero lower bound is approached and cannot become larger.

Now, we want to compare this finding with predictions from economic theory on optimal monetary policy responses when the zero lower bound is approached. Orphanides and Wieland (2000), Kato and Nishiyama (2005), Adam and Billi (2006) and Oda and Nagahata (2008) find that the reaction to inflation and the output gap should increase when the danger of reaching the zero lower bound becomes larger to decrease the interest rate pre-emptively. For example Orphanides and Wieland (2000) find that in a model where the optimal inflation response equals 2.0 in the absence of the zero lower bound, when accounting for the zero lower bound the inflation response increases gradually to a coefficient of almost 3 when inflation decreases from 3% to 0.5%. If inflation drops even further then the inflation response decreases very quickly and converges to zero as the zero lower bound on nominal interest rates is approached. Similar results are obtained by the other cited papers. The intuition is that central banks should lower the interest rate more quickly than in other times, i.e. respond more aggressively to decreasing inflation and negative output gaps, to stimulate the economy as much as possible early on to fight the danger of running into deflation, reaching the

zero lower bound and loosing conventional monetary policy as an instrument.

Such predictions from theory cannot be captured or tested using the Tobit approach applied to an otherwise linear policy rule. The Tobit approach can only capture the final convergence of policy responses to zero when the zero lower bound is approached. There are two important assumptions for the Tobit approach that prevent an increase in policy responses. First, it is assumed that the desired interest rate that would be implemented if there was no zero lower bound is a linear function of the lagged interest rate, inflation and the output gap. The linearity prevents any systematic changes in desired interest rate responses when the zero lower bound is approached. Second, it is assumed that the monetary policy shock to the desired interest rate is normally distributed. This prevents any discretionary asymmetric policy responses when the zero lower bound is approached.

One possibility to check for pre-emptive interest rate decreases when approaching the zero lower bound is to include non-linear terms in the equation for the desired interest rate. Kato and Nishiyama (2005) include squared terms of inflation and the output gap and estimate indeed negative coefficients for these using Tobit regression without instruments. So, the response of the interest rate to inflation increases if inflation decreases. As they do not provide estimates of the inflation response for different levels of inflation it remains unclear, whether these negative coefficients or the decrease of $\Phi(x_t\hat{\beta}/\hat{\sigma})$ dominate when approaching the zero lower bound. So, the results could imply a decrease or an increase in the inflation and output gap responses when interest rates are low. We also included squares of inflation and the output gap in our IV-Tobit estimates of a rule without interest rate smoothing and in contrast to Kato and Nishiyama (2005) also in a rule with interest rate smoothing. For the rule without interest rate smoothing we find a negative, but insignificant coefficient on squared inflation for Japan, a positive significant coefficient for the US and the Euro area. The coefficients on the squared output gap are positive and significant for Japan, negative and insignificant for the US and negative and significant for the Euro area. Some of the estimates of the remaining parameters were, however, hardly plausible so that we are very careful in interpreting these results. For the more realistic specification with interest rate smoothing, the estimator had convergence problems for all three economies. Already without the squared inflation and output gap terms, the maximization of the likelihood for the IV-Tobit model is not easy and can lead to numerical problems. As it is not clear whether the IV-Tobit approach with additional squared terms of inflation and the output gap can deliver reliable results we discuss in the following two other approaches that might be useful to test for pre-emptive interest rate decreases near the zero lower bound.

Gerlach (2011) estimates a monetary policy rule for the ECB for the period 1999 to 2009 using an ordered Logit model. To study whether interest rate decreases from 4.25% in September 2008 to 1% in May 2009 were standard responses to worsening macroeconomic conditions or whether in addition interest rates were decreased pre-emptively, he allows for a smooth transition from one policy response parameter set to the next (see Teräsvirta, 2004, for an explanation of the smooth transition approach). He indeed finds a change in the monetary policy rule. The parameter on the lagged interest rate increased substantially, making it more likely that a decrease in the interest rate is followed by another one. While this result indicates pre-emptive interest rates decrases, Gerlach

finds no change in the output response.[5] Gerlach and Lewis (2013) use the smooth transition regression method to estimate a monetary policy rule for the ECB from 1999 to 2010. They find a change in monetary policy in 2008 and a lower interest rate than implied by the pre-crisis rule after 2008. Before 2008 monetary policy responses to inflation and the output gap are significant with the expected sign, but not afterwards, so that pre-emptive interest rate decreases were not caused by larger policy responses to inflation and the output gap.

Another possibility to test for larger inflation and output gap responses when the zero lower bound is approached is to use censored quantile regression. Chevapatrakul et al. (2009) and Wolters (2012) show that uncensored quantile regression can be used to analyse asymmetric deviations of monetary policy responses from a linear rule. Using this framework one can estimate policy response parameters for each quantile of the conditional interest rate distribution. This includes cases where the interest rate is set higher or lower than on average given inflation and output gap developments. Directly at the zero lower bound the interest rate is set higher than at the conditional mean and inflation and output gap responses are captured by the estimates in the upper conditional quantiles. This is clear also from our IV-Tobit estimates: the actual interest rate is higher then the desired one. But if there are pre-emptive interest rate decreases slightly above the zero lower bound, then these would be captured by inflation and output gap responses estimated in the lower conditional quantiles. The interest rate would in this case be set lower than estimates at the conditional mean imply and so the reactions to decreasing inflation and the output gap would be higher than those at the conditional mean. While the work of Chevapatrakul et al. (2009) and Wolters (2012) using quantile regression is useful to capture such asymmetric reactions to inflation and the output gap in normal times, their method needs to be extended to a censored quantile regression approach to guarantee unbiased estimates in samples with low interest rates.

6 Conclusion

We have shown how the IV-Tobit estimator can be used to achieve consistent estimates of monetary policy rule parameters accounting for the zero lower bound on nominal interest rates. The approach has been applied to three large economies: Japan, the US and the Euro area. In all three economies policy rates have reached values close to zero in recent years. The comparison of the IV-Tobit estimates with conventional two-stage least squares estimates shows that the latter are biased. In addition, we have demonstrated how estimated monetary policy responses change when the zero lower bound is approached and how they deviate from the desired responses that the central bank would implement if there was no zero lower bound. Overall, the analysis in this paper is useful to understand how the IV-Tobit estimator can be used in the future for the estimation of monetary policy rules in samples that include low interest rates. Researchers do not need to wait until there are enough new observations of interest rates above the zero lower bound, but they can use the entire sample including periods of almost zero interest rates. We have shown how the various parameters

[5] He dropped inflation altogether from the equation as the estimated inflation responses were insignificant.

can be interpreted as policy responses in normal times, desired policy responses that the central bank would implement if there was no zero lower bound and actual estimated policy responses when the zero lower bound is approached.

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