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Working Paper

Optimal monetary policy responses and welfare analysis within the highfrequency New-Keynesian framework

Economics Working Paper, No. 2014-03

Provided in Cooperation with:

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Suggested Citation: Sacht, Stephen (2014) : Optimal monetary policy responses and welfare analysis within the highfrequency New-Keynesian framework, Economics Working Paper, No. 2014-03, Kiel University, Department of Economics, Kiel

This Version is available at:

<https://hdl.handle.net/10419/90810>

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Christian-Albrechts-Universität zu Kiel

Department of Economics

Economics Working Paper
No 2014-03

optimal monetary policy responses and welfare analysis within the high- frequency new-keynesian framework

by Stephen Sacht

issn 2193-2476



Optimal Monetary Policy Responses and Welfare Analysis within the High-Frequency New-Keynesian Framework

Stephen Sacht*

January 23, 2014

Abstract

In this we investigate the welfare effects of optimal monetary policy measurements within a high-frequency New-Keynesian model i.e. under variation of the period length. Our results indicate that the policy maker faces a higher welfare loss on a higher relative to a lower frequency of the agents' decision making. While overall inertia in the model increases, we show that the more the pass-through of output gap movements into inflation rate dynamics is dampened on a higher frequency, this amplifies the trade-off of the central bank in case of a cost-push shock. This is caused by the impact of so-called frequency-dependent persistence effects, which mimic the impact of the increase in the amount of market days on the dynamics of the model. This result is less severe in the optimal monetary policy regime under Commitment because of a time-invariant history dependence effect with respect to the period length.

Keywords: Hybrid New-Keynesian model; high-frequency modelling; optimal monetary policy; frequency-dependent persistence.

JEL classification: C61, C63, E32, E52.

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I thank Hans-Werner Wohltmann for valuable discussions and comments. This paper is based on Chapter 3 of my doctoral thesis *High-Frequency Analysis and Moment-Matching Estimation of the Baseline New-Keynesian Model* (2014). This working paper version is preliminary.

1 Introduction

In this paper we investigate the *welfare effects* of monetary policy operations within an h-economy. Hereby we focus on the purely forward-looking baseline New-Keynesian model (NKM) and the impact of a *cost-push shock* on economic dynamics. The reasons for taking this experimental arrangement under consideration are manifold.

A cost-push shock has gained relative importance since it stands for the source of stagflation, i.e. a simultaneous increase and decrease in the inflation rate and the output gap, respectively. As we discussed in Sacht (2014), in order to dampen the increase in inflation, the monetary authority faces a policy trade-off: it must reduce the output gap as inflations' inherited driving force by a subsequent increase in the nominal (real) interest rate after the shock occurs. This upward movement in the policy instrument is the only reliable option the central bank will have within the baseline 3-equations NKM. This observation is confirmed by an overwhelming number of studies on *optimal* monetary policy (OMP) in the literature (see Woodford (2003), Winkler and Wohltmann (2008) and Walsh (2010) among others). The term 'optimal' refers to the aim of the central bank to minimize the value of an intertemporal criterion function. This function in its basic representation consists of the discounted variability of the inflation rate and the output gap. Since the central bank seeks to minimize these kinds of variability, the welfare function is simply called a *loss* function. In the following we refer to the welfare and loss function as being synonyms. Furthermore, it is assumed that there exists a prevalence of inflation rate over output gap stabilization by the central bank, expressed by a higher weight in the loss function on inflation rate movements. Obviously, the policy trade-off calls for a more elaborate discussion on the design of optimal monetary policy relative to demand (technology) or interest rate shocks. The latter both can be easily offset on a quarterly magnitude by the central bank, since here the dynamics in inflation and the output gap are rectified. Nevertheless, as we had seen (based on our analysis of IRFs in Sacht (2014)) there exist quantitative and qualitative differences across higher frequencies – at least due to the so-called frequency-dependent contraction persistence (FCP) effect – in all shocks. However, the *ineffectiveness* of monetary policy is much more apparent in the case of a cost-push shock.

From an empirical point of view, supply or cost-push shocks account to a large extent for the variability in economic dynamics, which holds at least for the two oil price shocks within the Great Inflation period in the 1970's (Hamilton and Herrera (2004)). Smets and Wouters (2007, pp. 598) identify supply shocks being significant driving forces in output growth and inflation dynamics over the time horizon from 1966:1 to 2004:4 in the US. In addition, Smets and Wouters (2003, p. 1162) show that cost-push shocks account mainly for the variation in inflation in the Euro Area between 1980:2 and 1999:4.¹

Throughout this paper we investigate how the OMP responses of the inflation rate and the output gap to a cost-push shock are going to change, as we increase the frequency in decision making. Therefore, again, IRFs are considered and an economic rationale based on different kinds of frequency-dependent persistency is given. In particular, we discuss the economic dynamics under two different OMP regimes. Essentially, in the regime *Discretion* the central bank must take the expectations of the households as given, while in the (pre-) *Commitment* regime expectations can be influenced by the monetary

¹The definition of a cost-push or supply shock is ambiguous and depends on the structural representation of the New-Keynesian Phillips Curve (NKPC). For example, Smets and Wouters (2007) investigate a NKM with sticky prices and wages, where supply shocks are represented as price and wage mark-up shocks. Galí et al. (2012) consider non-investment specific technology and price mark-up shocks, while wage mark-up shocks are sorted into labor shocks only. While this kind of differentiation is reasonable when a shock decomposition analysis is applied, however, for our theoretical discussion here a shock expressed by an AR(1) process is sufficient.

authority. It is well known that the outcome with respect to the value of the loss function in quarterly magnitudes, is higher under Discretion compared to Commitment. While this can be explained by the ability of the central bank to influence the expectation channel of monetary policy, it is an open question if this result prevails on a higher frequency. This is of serious concern, as we observe a more likely discretionary monetary policy conducted by the European Central Bank (ECB) in the aftermath of the financial and sovereign debt crisis in the Euro Area. Finally, we apply a sensitivity analysis, where we evaluate the values of the loss function across all policy regimes under variation of the Calvo parameter of price stickiness and the autocorrelation parameter in the shock process. Such a theoretical investigation is non-existent in the literature so far – except for the work by Sacht and Wohltmann (2013), where their contribution can be seen as closely related to ours.

There exist a large number of empirical studies, which deal with the impact of monetary policy on real variables under consideration of high-frequency data. For example, Faust et al. (2004) estimate the effect of monetary policy announcements on the expected future values of nominal interest rate and the price level. This is done via the matching of IRFs. In their work they use high-frequency data on the prices of Fed Funds futures contracts in a vector autoregression model. The authors report evidence for a small impact of monetary policy shocks on output. Rosa (2013) applies ordinary least squares under consideration of high-frequency data on energy prices. He shows that asset prices respond on a high-frequency due to sudden monetary policy announcements. Finally, Assenmacher-Wesche and Gerlach (2008) estimate a two-pillar NKPC of the European central bank (ECB), where they account for the ECBs simultaneous analysis of monetary and economic indicators. In their study they investigate the impact of high-frequency (e.g. exchange rates, import and fresh food prices and value-added taxes) as well as low-frequency (money and real output growth and the rate of velocity) components on the inflation process in the Euro Area. The authors check on co-integration of the inflation rate relative to these components under consideration of spectral regression techniques in order to account for the different time horizons. They find that low-frequency movements of money and real output growth as well as high-frequency fluctuations of the output gap account for the variation in the inflation rate over the period from 1970 to 2004.

In order to keep our analysis straightforward, the purely forward-looking specifications of both the baseline NKM and the h -economy model (as described in Sacht (2014) and Franke and Sacht (forthcoming)) are considered only. As we would turn to a hybrid specification of these model variants, backward-looking elements have to be considered explicitly. In this case the computation of the policy functions (as an outcome of dynamic programming) under Discretion becomes non-trivial. According to the seminal work of Woodford (2003), he shows that under Discretion a policy function, being optimal in a certain period, becomes non-optimal in subsequent periods as new information arrives. Hence, policy makers must be aware of the change in the optimal behaviour of private agents because the decisions of the latter are affected by previous states. Two technical approaches from the literature deal with this problem. The first one makes use of the Bellman equation within optimal control theory. Here the policy maker applies a single optimization in the initial period and sticks to the resulting policy function in all periods after. This procedure meets Bellman’s ‘principle of optimality’.² Problem-specific solution algorithms are provided by e.g. Söderlind (1999) among others. Note that here the decisions of policy makers and private agents are undertaken simultaneously.

In contradiction, as we allow the policy maker to make its decisions first, we end up

²Bellman (1957, p. 83) states that “[a]n optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.” The work on dynamic programming with respect to discretionary policy was pioneered by Oudiz and Sachs (1985), who study a finite instead of an infinite horizon problem.

in a Stackelberg-Nash equilibria, where the current policy maker is the Stackelberg leader and the private agents as well as the future policy maker are the Stackelberg follower. For this case, solution algorithms are provided by e.g. Dennis (2007) among others. Similar to Söderlind (1999), Dennis (2007) considers the structural representation of the model directly instead of its associated state-space representation. As far as we know, an intensive discussion on the outcome and the computational properties (in terms of convergence and computational speed) of both classes of algorithms is missing in the literature so far. Rather than undertake an exercise, which consists on the comparison of different solution techniques for discretionary monetary policy, we account for clearness of our approach. The purely forward-looking specification is sufficient in this respect since the solution is less time-consuming. As in Sacht (2014), across all policy regimes, the policy functions are computed via the method of undetermined coefficients in combination with the brute force iteration procedure. In general, this proceeding can be seen as a natural starting point for the analysis of OMP in DSGE models - independently, of course, of the underlying timing convention.³

The remainder of this paper is structured as follows. In the next section we discuss the design of OMP within a baseline purely forward-looking NKM in its h -economy variant. Furthermore, along with the presentation of the analytical results, we give an economic explanation for the upcoming results based on the change in the frequency-dependent persistence parameters in the model. Section 3 provides the numerical analysis of both OMP regimes in an h -economy environment based on IRFs. In section 4 we apply a numerical welfare analysis, where we investigate the value of the welfare loss (relations) together with a sensitivity analysis with respect to the degree of price stickiness and the autocorrelation parameter in the shock process. Section 5 concludes. The Appendix contains the reduced-form solutions to the corresponding minimization problems of both OMP regimes and all Figures.

2 Optimal Monetary Policy Responses in an h -Economy

2.1 The Design of Optimal Monetary Policy

For our analysis, we refer to purely forward-looking NKM in its (quarterized) high-frequency specification ($0 < h \leq 1$):

$$y_t^j = y_{t+h}^j - \delta_1(h)(i_t^j - \pi_{t+h}^j - \bar{i}) \quad (1)$$

$$\pi_t^j = \beta(h)\pi_{t+h}^j + \tilde{\kappa}(h)y_t^j + v_t^\pi \quad (2)$$

$$i_t^j = \phi_\pi \pi_t^j + \phi_y y_t^j \quad (3)$$

$$v_t^\pi = \rho(h)^\pi v_{t-h}^\pi + \varepsilon_t^\pi \quad (4)$$

where $j = \{D, C\}$ indicates the dynamics of the inflation rate and the output gap associated with the corresponding OMP regimes *D*iscretion and *C*ommitment. The definitions of the parameters can be found in Table 1 later on. In order to account for a(n auto-correlated) cost-push shock only, the autocorrelated nominal interest rate (v_t^i) and the technology (v_t^y) shocks are now discarded from the Taylor rule and the dynamic IS equation, respectively. We omit the expectation operator in front of the forward-looking terms since we only consider a *deterministic* shock. As already discussed, as the shock occurs,

³It must be emphasised that the above statement does not apply to the hybrid (baseline) NKM under Commitment due to an explicit consideration of the expectation channel in this OMP regime. This means that future states of the variables in the model can be controlled by the central bank based on previous realizations. For consistency and clearness of our study, where we compare the numerical outcomes given by the OMP regimes Discretion *and* Commitment, we leave this kind of investigation to further research.

this leads to a boost in (future) inflation on impact, which forces the central bank to increase the nominal interest rate according to the Taylor rule. As a result the output gap declines on impact.⁴ This leads to a decrease in the inflation rate and so forth. However, as the inflation rate and the output gap move in opposite directions, the central bank faces a policy trade-off if stagflation occurs, which can be interpreted as an output-inflation trade-off of monetary policy.

The welfare effects of OMP are in general measured by the following intertemporal function:

$$L_t^j = E_t \sum_{k=0}^{\infty} \beta(h)^{hk} \{h\bar{\alpha}_1(\pi_{t+hk}^j - \bar{\pi}^j)^2 + \bar{\alpha}_2(y_{t+hk}^j - \bar{y}^j)^2\} \quad (5)$$

where $\bar{\alpha}_1$ and $\bar{\alpha}_2$ denote the weights on inflation rate and output gap stabilization, respectively. It must be emphasized that we do not allow for nominal interest rate stabilisation as done in various papers on OMP. According to equation (5) the aim of the central bank is to minimize the quadratic deviations of inflation and the output gap from their fixed steady state values $\bar{\pi}$ and \bar{y} , respectively. Therefore, equation (5) can be denoted as a loss function. For simplicity we assume for both OMP regimes that $\bar{\pi}^j = \bar{y}^j = 0$ holds.⁵ Again, for a direct comparison of two h -economies, the stock variable π_{t+hk}^j is uniformly expressed in *quarterized* magnitudes. Note that

$$\pi_{t+hk}^j = (p_{t+hk}^j - p_{t+hk-h}^j)/h \quad (6)$$

is then considered with respect to the loss function (5), i.e. the latter must be augmented by h in order to ensure mathematical consistency. Therefore, based on the following definition

$$(p_{t+hk}^j - p_{t+hk-h}^j)^2 = (\Delta p_{t+hk}^j)^2 \quad (7)$$

it follows for $\bar{\pi}^j = 0$ that

$$h(\pi_{t+hk}^j)^2 = h \left(\frac{(\Delta p_{t+hk}^j)}{h} \right)^2 = h \frac{(\Delta p_{t+hk}^j)^2}{h^2} = \frac{(\Delta p_{t+hk}^j)^2}{h}$$

holds. Hence, according to the systematic/skip sampling aggregation scheme, the *squared deviations* of the inflation rate (from zero) are expressed in quarterized magnitudes.⁶ Since the output gap is a dimensionless adjustment rate, the weight $\bar{\alpha}_2$ is not augmented by h . According to optimal control theory, the loss function has to be minimized subjected to the equations for y_t^j and π_t^j . This approach differs across the monetary policy regimes to be considered (see also the next section as well as in the Appendix).

⁴Note that *ceteris paribus* the cost-push shock triggers an increase in the real interest rate denoted by $r_t^j = i_t^j - E_t[\pi_{t+h}^j]$. According to the Euler equation as theoretical basement of the dynamic IS curve, future consumption becomes more attractive, which would lead to a decrease in y_t^j . Hence $|di_t^j| > |dE_t[\pi_{t+h}^j]|$ holds if dynamic stability is ensured, i.e. as the Taylor principle $\phi_\pi > 1$ holds.

⁵We depart from the strand of the literature on OMP, where a second-order approximation of household's utility function is considered as a loss function (cf. Woodford (2003)). We also do not consider the so-called Ramsey approach (Kahn et al. (2003)). Instead, we follow an overwhelming number of studies, where an ad-hoc formulation of the loss function of type (5) is assumed. Essentially, equation (5) serves as a good compromise since its structure mimics a (microfounded) second-order approximation of household's utility function (cf. Walsh (2010) and Galí (2008)).

⁶It is easy to see that $(\Delta p_{t+hk}^j)^2$ should not be divided by h^2 , what probably would be a natural way to think of. Therefore, note that in this case we get $f^2 \cdot (\Delta p_{t+hk}^j)^2$, where $f = 1/h$ defines the frequency of decision making. In e.g. a monthly economy ($f = 3$) this leads to $f^2 \cdot (\Delta p_{t+hk}^j)^2 = 9 \cdot (\Delta p_{t+hk}^j)^2$ which is simply implausible since a quarter consists on 3 and not 9 months. In other words, in order to compare the values of the stock variables in an h -economy to its counterpart given in quarterly magnitudes, this requires the correct amount of subperiods to be considered on a higher frequency for aggregation. This holds regardless as we consider the level or squared deviation in the stock variable.

An important assumption must be made with respect to the solution of the shock process:

$$E_0 v_t^\pi = [\rho(h)^\pi]^{t/h} \varepsilon_0^\pi, \quad (8)$$

again, E_0 denotes the expectation operator conditional on the information given in the initial period. Note here that the parameter $\rho(h)^\pi$, which measures the FCP effect, is taken to the power of t/h instead of period t only. Hence, the period has to be multiplied by the frequency of decision making, i.e. $t \cdot f$ holds. This means that all points in time which correspond to a *quarter* on a higher frequency are *selected*. While this step is not necessary for the computation of the IRFs (where in Sacht (2014) we just consider period t only), it is required for the analysis of welfare effects on different frequencies. The reason is that in order to compare the values of the loss functions along the dimensions of the period length and the OMP regimes, the quarterly realizations of the variables have to be considered only. In particular, since the quarterly NKM with $h = 1$ serves as our baseline model, for a direct comparison of the corresponding loss functions for $0 < h \leq 1$, the shock process must be adjusted accordingly. Therefore, it must be emphasized that we take the entire stream of realizations of the variables over an infinite amount of periods, namely quarters, into account. Accordingly, the loss function is described by the weighted sum of these realizations. Without an adjustment of the shock process, this will lead to biased results as the period lengths is varied.

2.2 Analytical Results

In this section, the key equations for the OMP regimes Discretion and Commitment are revealed. Further explorations can be found in more detail in the Appendix. One of our *main results* indicates that the loss *increases* as the period length *decreases*. This general observation implies that welfare is *always* higher (based on the value of the loss function) on a quarterly magnitude ($h = 1$) relative to the case where subperiods ($h < 1$) are considered. This can be explained by the FCP effect, where the increase in the persistence of the shock process leads to a stronger destabilization effect on the economy – which holds independently in the OMP regimes Discretion and Commitment. In the following, we shed a light on the question how this increase in destabilisation can be explained by other (frequency-dependent) influences besides the FCP effect.

We start by discussing the case of *Discretion*. The minimization problem associated with this OMP regime is a static one. The reason is, that households' expectations on future movements of the output gap and the inflation rate are pre-existing for the central bank due to a lack of its credibility. Given the expectations, the monetary authority is forced to re-optimize every single period. Technically speaking, the loss function (5) is minimized with respect to the dynamic IS and NKPC, where all forward-looking terms are treated as being fixed expressions. As a result of the optimization, the relationship between the inflation rate and the output gap in case of a cost-push shock is described by the so-called *targeting rule*:

$$y_t^D = -\xi(h)\pi_t^D \quad (9)$$

with

$$\xi(h) = \frac{h\tilde{\kappa}(h)\bar{\alpha}_1}{\bar{\alpha}_2} = \frac{\kappa(h)\bar{\alpha}_1}{\bar{\alpha}_2}$$

and $\tilde{\kappa}(h) = \kappa(h)/h$. A targeting rule describes the optimal adjustment path of variables, which depend directly on each other. According to equation (9), an increase in the inflation rate requires a decrease in the output gap in order to dampen inflation after the shock occurs. Since inflation rate and output gap stabilisation being the target of the central bank, this type of rule is different to the so-called instrument rule. The standard

Taylor rule or an Optimal Simple Rule serve as examples of the latter.⁷ The (frequency-dependent) composite parameter $\xi(h)$ is known as the *stabilisation bias*. It measures the effectiveness of OMP, where a high value of $\xi(h)$ indicates a decrease in the stabilisation bias and a stronger reduction in the output gap, which leads to less pronounced increase in the inflation rate and vice versa. Hence, OMP becomes increasingly effective the higher $\xi(h)$ will be.

Under (pre-) *Commitment*, the central bank's reputation is confirmed, i.e. the households believe in the monetary policy announcement regarding the stabilisation of the inflation rate and the output gap. Hence, the monetary authority is able to control for the expectation channel of monetary policy, where private sector expectations on future movements in y_t^C and π_t^C can be influenced over time. It follows that the minimization problem becomes indeed intertemporal, allowing the central bank to manipulate households' expectations. The corresponding targeting rule reads

$$y_t^C = -\xi(h)\pi_t^C + \psi(h)y_{t-h}^C \quad (10)$$

with

$$\psi(h) = \beta(h)^{(1-h)}.$$

The previous equation accounts for *history dependence*, i.e. the optimal reaction of y_t^C to changes in π_t^C does not depend only on contemporaneous but also on past movements of the output gap. The reason is given by the *timeless perspective* policy (see Woodford (2003, p. 473)), which indicates that, by committing herself to a long-run optimal rule (linked to the reputation of the agents), the central bank is able to react smoothly to shocks. According to equation (10) the monetary authority is able to stabilize the inflation rate by allowing for a smaller decline in the output gap compared to the OMP regime Discretion. In total, the deviations in both variables are smaller under the OMP regime Commitment relative to Discretion since the central bank commits itself to lower future values of the output gap and, hence, the inflation rate (below their corresponding targets given by $\bar{\pi}^j = \bar{y}^j = 0$).

Setting $h = 1$ leads to the well-known expression for the reduced-form solutions and targeting rules in the Discretion and Commitment cases reported by an overwhelming amount of studies (see Woodford (2003), Galí (2008, his Chapters 3 and 4) and Walsh (2010, his Chapters 7 and 8) among others). From the same literature it is known, that due to the structure of the OMP regimes Discretion and Commitment, the former generates a higher welfare loss compared to the latter ($L_{(t)}^D > L_{(t)}^C$). In section 4 we will analytically (and numerically) show that this result prevails as we allow for a decrease in the period length.

It is worth mentioning that the weight $\psi(h)$ indicates a *frequency-dependent history (dependence) persistence* (FHP) effect. First of all, in his seminal paper, Leitemo (2008) shows that in a purely forward-looking model, the maximum degree of history dependence is required in order to achieve the lowest welfare loss in the OMP regime Commitment. While this means that $\psi(h) = 1$ holds, this condition is valid only in the baseline case of $h = 1$. In transition to an h-economy, we observe that the FHP effect is *dampened* as the period length declines.⁸ As a result, the stabilization effect on π_t^C is reduced as h decreases, i.e. on a higher frequency relative to a quarter, the inertia in the targeting rule (10) is

⁷See also Svensson (1999) as well as Svensson and Woodford (2005) for a discussion of targeting versus instrument rules. In the case of an Optimal Simple Rule, the optimal values for the monetary policy parameters ϕ_π and ϕ_y stem from the minimization of the loss function under consideration of the corresponding solution paths for the inflation rate and the output gap given the Taylor rule (cf. Sacht and Wohltmann (2013)).

⁸In his paper, Leitemo (2008) sets the discount factor given in quarterly magnitudes equal to unity i.e. $\beta(1) = 1$ holds. While he argues that this assumption is grounded on the matter of 'convenience' (p. 268), it is obviously the case, that this treatment is misleading as an h-economy is considered.

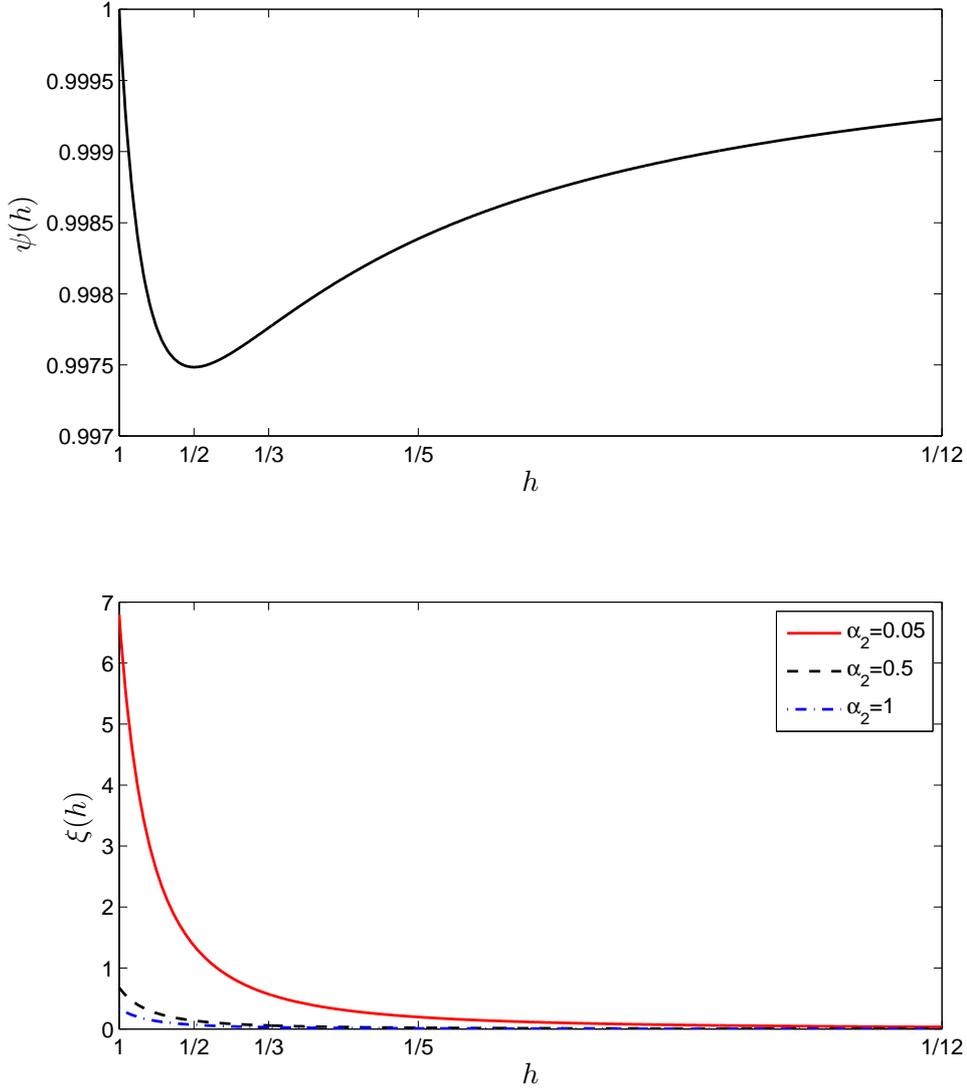


Figure 1: Development of the weights $\psi(h)$ and $\xi(h)$.

Note: The Figure depicts the development in the weights $\psi(h) = \beta(h)^{(1-h)}$ (upper panel) and $\xi(h)$ (lower panel) – as component of the targeting rule under the OMP regime Commitment (cf. equation (10)) – with respect to h . The scale of the horizontal axis is limited on the interval $h \in [1, \dots, 1/12]$, while it is confirmed that $\psi(h) = 1$ holds for $h = 0$. The *solid* line in the lower panel depicts the development of $\xi(h)$ for $\bar{\alpha}_2 = 0.05$. The *dashed* line in the lower panel depicts the development of $\xi(h)$ for $\bar{\alpha}_2 = 0.5$. The *dashed/dotted* line in the lower panel depicts the development of $\xi(h)$ for $\bar{\alpha}_2 = 1$. The associated parameter values are taken from Table 1 together with $\bar{\alpha}_1 = 1$.

less considered. However, the quantitative effects are quite small. This can be seen by the upper panel in Figure 1, which depicts the change in the degree of history dependence with respect to h . Here a non-linear development in the weight can be observed, where the minimum is reached at a value of $h = 0.5$. It follows from this that the dampening effect

on the history dependence and, therefore, the reduced stabilization effect is not dramatic as h decreases. Most importantly, a degree of history dependence below unity induces a long-run deviation in the price level from its initial steady state value. Hence, in an h-economy the central bank fails to stabilize the price level in OMP regime Commitment, which does not hold in the case $h = 1$ (therefore consider also the IRFs presented in the next section).

On the contrary, the frequency-dependent inherited persistence (FIP) effect (Sacht (2014)) has a distinct negative impact on the effectiveness of OMP. The latter is weakened in transition from a lower to a higher frequency. In particular, as the period length h decreases – ceteris paribus – this leads to a decline in the slope of the NKPC measured by $\kappa(h)$. This can be explained by the increasing amount of transactions and reallocations the central bank has to react to more frequently in an h-economy (cf. Sacht (2014)). As a result the stabilisation bias is amplified, i.e. $\xi(h)$ declines for a given value of $\bar{\alpha}_2$. Compared to the FHP effect, the influence of the FIP effect on the stabilization of the inflation rate is remarkable. It can be seen from the solid line (where $\bar{\alpha}_2 = 0.05$ holds) in the lower panel of Figure 1 that the decline in $\xi(h)$ is quite strong as the period length decreases.

This result is obviously strengthened by high values for the weight on output gap stabilization measured by $\bar{\alpha}_2$. In reality, it can be stated that the ECB follows a *strict* inflation target ($\bar{\alpha}_1 > 0, \bar{\alpha}_2 \approx 0$), while the latter is more *flexible* with respect to the US Federal Reserve Bank (FED; $\bar{\alpha}_1 > 0, \bar{\alpha}_2 > 0$). As we can see from equations (9) and (10), the more flexible the inflation targeting is this leads to a more destabilizing effect. The reason is, that for high values of $\bar{\alpha}_2$, the central bank is cautious about a strong negative output gap needed for stabilizing the inflation rate. Hence, the drop in the output gap is less pronounced as being necessary, which leads to a smaller dampening effect on the inflation rate. Furthermore, high values of $\bar{\alpha}_2$ amplify the impact of the FIP effect on the stabilisation bias, i.e. $\xi(h)$ becomes close to zero even on a low frequency (that is, as h is close to 1). The dashed and dashed/dotted lines in the lower panel of Figure 1, which represent the developments in $\xi(h)$ for $\bar{\alpha}_2 = 0.5$ and $\bar{\alpha}_2 = 1$, respectively, show this explicitly. Technically, this observation can be easily explained: as we turn to a more flexible inflation targeting environment, $\xi(h)$ becomes more compressed as h decreases relative to low values of $\bar{\alpha}_2$ given.

As we jointly consider the impacts of the FIP and FHP effects together with the variation in the degree of inflation targeting, several observations are worth mentioning. First, while the increase in the stabilization bias (due to the FIP effect) has a stronger impact in the OMP regime Discretion compared to the Commitment one, in the latter case the FHP effect ensures the lowest welfare loss. Although, the reduction of the output gap is dampened on impact on a higher frequency, a decrease in h leads only to a small decrease in history dependence. Hence, the central bank is still able to reduce the inflation rate (and the output gap) below its target and, therefore, maintains a less destabilizing adjustment of the variable compared to the discretionary case. Second, in conjunction with the FCP effect, the almost unchanged degree in the history dependence leads to more pronounced humped-shaped adjustments in all variables under Commitment.

Finally, regarding the OMP regime Commitment only, the FHP effect *offsets* (or even dominates) the FIP effects in the case of strict inflation targeting, while the opposite holds under a flexible inflation targeting scenario. Therefore, note that a high value of $\xi(h)$ leads to a stronger reduction in the output gap relative to when this value is low. However, this strong reduction feeds back into the targeting rule (10) in the next period via the past value of the output gap, which is multiplied by $\psi(h)$. While this feedback effect is more pronounced for $h = 1$, we are able to see from Figure 1 that for $\bar{\alpha}_2 = 0.05$ the smooth decrease in $\xi(h)$ leads to a quantitatively strong consideration of the non-linear movement in $\psi(h)$ in the targeting rule (10). This observation helps to explain the corresponding non-

linear development we obtain within our welfare analysis, where we study the difference in L^D compared to L^C for the case $\bar{\alpha}_2 = 0.05$. As we turn to higher values of $\bar{\alpha}_2$, this non-linear movement in $\psi(h)$ is less considered since $\xi(h)$ decreases rapidly for even high values of h . Under flexible inflation targeting, the weak reduction in the output gap feeds less back into the targeting rule and, therefore, dampens the FHP effect.

3 Numerical Results: Impulse Response Functions

In the following we shed a light on the dynamics of the model variables as a result of the output-inflation trade-off linked to the cost-push shock. Furthermore, we discuss the implications for OMP based on our investigations undertaken in the previous section. First, we study the IRFs for all model variables based on the reduced-form solutions for y_t^j and π_t^j . Based on these solutions, the dynamics of the nominal interest rate is simply computed by solving the dynamic IS equation (1) for i_t^j . Therefore, the dynamics of the nominal interest rate mimics the monetary policy reaction given by the associated targeting rules (9) and (10). The real interest rate is then given by the difference ($r_t^j = i_t^j - \pi_{t-h}^j$) and the price level is simply computed by $p_t^j = \sum_{k=0}^t \pi_k^j$. We adopt the calibration of the parameters from Sacht (2014), where the corresponding values are given in the following Table.

ν	Households' time preference rate	0.010
σ	Inverse intertemporal elasticity of substitution in consumption	1.000
η	Intertemporal elasticity of substitution of labour	1.000
θ	Calvo degree of price stickiness	0.667
α	Degree of Price Indexation	0.500
χ	Habit-formation parameter in consumption	0.500
\bar{i}	Natural Interest Rate	0.000
ϕ_π	Weight on inflation in the Taylor rule	1.500
ϕ_y	Weight on the output gap in the Taylor rule	0.125
ρ^z	Persistence in the shock process	0.500
ε^z	Impulse associated with the shock process	1.000
$\bar{\alpha}_1$	Monetary policy reaction with respect to the inflation rate	1
$\bar{\alpha}_2$	Monetary policy reaction with respect to the output gap	0.2

Table 1: Numerical parameter scenario.

Note: In the purely forward-looking NKM the parameters χ and α are set to 0. In case of a non-autocorrelated shock ρ^z is set to 0. The parameters of the shock process ρ^z and ε^z are identical across all shocks $z = \{y, \pi, i\}$.

In addition we follow the ECB strategy when analyzing the IRFs by assuming strict inflation targeting, where $\bar{\alpha}_1 = 1$ and $\bar{\alpha}_2 = 0.05$ hold. However, as we turn to the investigation of the welfare effects of both OMP regimes in the next section, we apply a sensitivity analysis regarding different values of $\bar{\alpha}_2$. For a clear arrangement, the corresponding Figures for a non-autocorrelated shock process ($\rho^\pi = 0$) and the autocorrelated case ($\rho^\pi = 0.5$) can be found in the Appendix.

The IRFs for the OMP regime *Discretion* in case of a *non*-autocorrelated shock process

are depicted in Figure 2. In a quarterly economy ($h = 1$), we observe a large drop in y_t^D , which leads to a weak increase in π_t^D on impact according to the targeting rule (9). Since the latter exhibits no history dependence, all variables converge back to their long-run steady states in next quarter. As we turn to a monthly ($h = 1/3$) and weekly ($h = 1/12$) frequency, the impact effects on y_t^D become weaker the more h is reduced. This can be explained by the FIP effect, where the stabilization bias increases on a higher frequency. This leads to a more destabilizing impact effect regarding π_t^D . It can be seen that the quantitative differences are not negligible – even in the case of a transition from monthly to weekly magnitudes. In the same vein, the movements in i_t^D and r_t^D become ambiguous: both variables increase stronger on impact in the monthly economy compared to $h = 1$, while in a weekly economy even a strong drop on impact (in conjunction with hump-shaped movements afterwards) can be observed. The latter observation coincides with the quite weak impact effect on y_t^D . This can be interpreted as an indication for the central bank losing control over stabilizing the economy on a higher frequency. The increase in p_t^D emerges for all values of h , while the development is more pronounced quantitatively on a higher frequency. The dramatic upward-movement in p_t^D is grounded on the increase in the persistence of π_t^D due to the FCP effect.

In case of an *autocorrelated* shock (see Figure 3) the quantitative effects are more pronounced. Interestingly, y_t^D is further reduced on impact for $h = 1/3$ compared to $h = 1$. Accordingly, i_t^D and r_t^D exhibit a stronger increase on impact in a monthly compared to a quarterly economy. It can be guessed, that the increase in ρ^π amplifies the FCP effect, which leads to a dominance of this (now) *amplified* effect over the FIP one on relative high frequencies.

Figure 4 shows the dynamics in the OMP regime *Commitment* in case of a *non*-autocorrelated shock process. In the case $h = 1$, less strong reactions on impact for y_t^C and π_t^C are observed compared to Discretion. This can be explained by the history dependence in the targeting rule (10), which causes (more) persistence in the variables. The inertia in y_t^C mimics the commitment of the central bank to dampen future values of the inflation rate below its target. Furthermore, the p_t^C converges back to its steady state value due to history dependence. Again, on a higher frequency the reduction in y_t^C is less pronounced, while inertia is amplified (in all variables due to the FCP effect). Based on the FIP effect, the central bank fails to stabilize the inflation rate on impact also under Commitment. However, since the degree of history dependence is quantitatively less affected as h decreases, the corresponding FHP effect explains the increase in the persistence in y_t^C and π_t^C on higher frequencies. Especially, in the case $h = 1/12$ we see that π_t^C is pushed below its target for several periods. As we will see in the next section, the low impact effects together with the increase in inertia lead to a *lower* value of the loss function under Commitment relative to Discretion – even within an h-economy environment.

The hump-shaped behaviour of π_t^C leads also to the convergence of p_t^C back to its initial steady state value in the case $h = 1$. It is worth mentioning, that this result *does not* hold on a higher frequency. Again, due to the FHP effect, the decrease in the degree of history dependence for $0 < h < 1$ hinders the one-to-one response to y_{t-h}^C , i.e. as $\psi(h) < 1$ holds (cf. targeting rule (10)). Finally, like under Discretion, the movements in i_t^C and r_t^C change qualitatively in transition from a lower to a higher frequency.

In case of an *autocorrelated shock*, Figure 5 shows that the amplified FCP effect itself amplifies the FHP one: inertia increases rapidly, while the corresponding hump-shaped movements in all variables are more pronounced. It goes without saying that the quantitative effects are strengthened, while p_t^C (still) does not converge back to its initial steady state.⁹

⁹In Figure 4 (5) we limit the horizontal axis to 15 (20) quarters with respect to p_t^C , in order to account

We also make a comparison of the IRFs for a daily ($h = 1/90$) economy. Here we also compute the dynamics based on the baseline NKM with an ad-hoc Taylor rule assumed (labeled by *TR*). According to the Figures 6 ($\rho^\pi = 0$) and 7 ($\rho^\pi = 0.5$), not surprisingly, more pronounced effects are quantitatively observed for most of the variables (especially for the output gap) in the TR case compared to both OMP regimes. Since no minimization of the loss function is targeted by the central bank under consideration of an ad-hoc Taylor rule, it is well known that the corresponding loss is higher than in both OMP regimes – this result holds independently for any length of the period. Despite this observation, in general, across all specifications the quantitative effects on a daily frequency are quite strong due to the amplified FCP effect.

4 Numerical Results: Welfare Analysis

In this section we study the differences in the value of the loss functions given the reduced-form solutions (targeting rules) for the inflation rate and the output gap under Discretion and Commitment, respectively. All upcoming numerical results are obtained under consideration of the analytical solutions (for the loss functions) given in Sacht and Wohltmann (2013), which had been modified for the systematic/skip sampling instead of the temporal aggregation scheme. The reason for this is that our numerical solution approach presented here demand a large amount of main memory and time for execution. This can be explained by the computation of the loss function, where a discounted sum of all weighted squared deviations in the inflation rate and the output gap must be considered. However, the analytical solutions are grounded on the expression for the loss functions based on the relationship of all associated parameters of the model. In this case no streams of values over an infinite time horizon must be added up. However, it is confirmed that the numerical and analytical (based on Sacht and Wohltmann (2013)) solution procedures lead to the same results.¹⁰ All corresponding Figures can be found in the Appendix.

We consider the *absolute* and *relative* loss relations of Discretion compared to Commitment. The former is given by the difference $L^D - L^C$, while for the latter the percentaged change in the difference with respect to the loss under Commitment is computed, i.e. $(L^D - L^C)/L^C$ holds. We study the developments in these differences with respect to the variation in the period length for the cases of a (non-)autocorrelated shock. Figure 8 contains the related diagrams. According to both upper panels, the absolute loss relations are positive and increase monotonically as h decreases. The first observation confirms that the loss under Discretion is always higher compared to the one under Commitment – regardless of the length of the period. The second observation emphasizes one of our main results, that the more information and transactions (reallocations) is available/are observed on a higher frequency, the higher the welfare loss will be. This can be again explained by the FCP and FIP effects, where the former leads to an overall increase in the persistence of the shock, while the latter leads to an increase of the stabilisation bias. Due to the FHP effect this increase in the stabilisation bias is dampened under Commitment, where the weight on the past value of the output gap is only marginally affected by the decrease in h . Hence, the stabilisation of the economic indicators is much more successful under Commitment, which results in a positive value in the absolute loss relation. As we increase the weight on output stabilisation ($\bar{\alpha}_2$), the loss relation curve becomes flat-

for the less humped-shaped movements of the variable in the cases $h = 1$ and $h = 1/3$. However, it is confirmed that the price level does not converge back to zero as $0 < h < 1$ holds, i.e. in an h -economy.

¹⁰As already stated, this problem does not appear when computing all IRFs presented in this thesis. The related reduced-form solutions are given by the equations (21) and (22) for Discretion as well as (40) and (41) for Commitment – all given in the Appendix, respectively. The corresponding MATLAB codes can be downloaded from the authors' webpage at <http://www.makro-vwl.uni-kiel.de/de/team/dipl.-vw.-stephen-sacht>.

Non-Autocorrelated Cost-Push Shock ($\rho^\pi = 0$):					
$\bar{\alpha}_2 = 0.05$					
h	1	1/2	1/3	1/12	1/90
$L^D - L^C$	0.0555	0.6138	1.2747	4.8546	16.2144
$(L^D - L^C)/L^C$	22.53 %	85.47 %	96.68 %	57.81 %	18.94 %
$\bar{\alpha}_2 = 0.5$					
h	1	1/2	1/3	1/12	1/90
$L^D - L^C$	0.1888	0.5614	0.7973	1.8668	4.6486
$(L^D - L^C)/L^C$	30.28 %	39.04 %	33.65 %	16.15 %	4.78 %
$\bar{\alpha}_2 = 1$					
h	1	1/2	1/3	1/12	1/90
$L^D - L^C$	0.1805	0.4491	0.6090	1.3163	2.9710
$(L^D - L^C)/L^C$	25.21 %	27.98 %	23.46 %	10.86 %	3.00 %

Table 2: Selected values of the loss relations for $\rho^\pi = 0$.

Note: L^D and L^C denote the loss for the optimal policy regimes Discretion (D) and Commitment (C), respectively. The difference $L^D - L^C$ denotes the *absolute* loss relation. The difference $(L^D - L^C)/L^C$ denotes the *relative* loss relation.

ter, i.e. the absolute difference becomes less pronounced. The reason is that this change in the inflation targeting regime leads to a stronger consideration of the fluctuations in the output gap by the central bank. Since the FIP effect becomes amplified by this (the stabilisation bias increases much further), both OMP regimes are affected. For the case $\rho^\pi = 0.5$ we see stronger quantitative effects due to the (now) amplified FCP effect.¹¹

Both middle panels in Figure 8 depict the relative loss relations with respect to ρ^π . We observe a non-linear development of the relative loss relations, which is much more pronounced in the case $\bar{\alpha}_2 = 0.05$. As we already explained in section 2.2, the FHP effect offsets and even dominates the FIP effect under strict inflation targeting over a specific interval of h . As the period length decreases, the non-linear movement in the degree of history dependence becomes more considered the lower $\bar{\alpha}_2$ will be for high to moderate values of h . Due to the low value of the stabilisation bias, the strong reduction in the output gap on impact feeds into the targeting rule (10)) for Commitment via the past value of this variable. The additional degree of inertia due to the FHP effect leads to a stronger stabilisation effect under Commitment.

At a specific value of h , a maximum in the relative loss relation Discretion/Commitment is reached. We denote this value by \tilde{h} . Beyond this point, the FIP effect dominates the FHP one: the stabilization bias increases more rapidly the more h approaches to zero. Hence, the impact effect on the output gap becomes more and more negligible in

¹¹We limit the scale of the upper panels by $h \in [0.2, 1]$ to ensure a clear arrangement. We do so because for values of h close to 0, the loss relation curve, which represents the case $\bar{\alpha}_2 = 0.05$, increases rapidly compared to the other ones. However, in the Tables 2 and 3 we report also the values for the case $h = 1/90 = 0.01$.

Autocorrelated Cost-Push Shock ($\rho^\pi = 0.5$):					
$\bar{\alpha}_2 = 0.05$					
h	1	1/2	1/3	1/12	1/90
$L^D - L^C$	0.1300	2.6918	7.3873	40.0296	158.5244
$(L^D - L^C)/L^C$	30.58 %	152.00 %	203.60 %	133.24 %	40.90 %
$\bar{\alpha}_2 = 0.5$					
h	1	1/2	1/3	1/12	1/90
$L^D - L^C$	1.2859	4.7321	7.1605	18.6095	49.6553
$(L^D - L^C)/L^C$	74.18 %	89.58 %	76.33 %	34.89 %	10.00 %
$\bar{\alpha}_2 = 1$					
h	1	1/2	1/3	1/12	1/90
$L^D - L^C$	1.5672	4.1744	5.8594	13.5675	32.1513
$(L^D - L^C)/L^C$	68.66 %	64.56 %	52.72 %	23.20 %	6.25 %

Table 3: Selected values of the loss relations for $\rho^\pi = 0.5$.

Note: L^D and L^C denote the loss for the optimal policy regimes Discretion (D) and Commitment (C), respectively. The difference $L^D - L^C$ denotes the *absolute* loss relation. The difference $(L^D - L^C)/L^C$ denotes the *relative* loss relation.

this case (cf. Figure 4). Note that under Discretion this observation does not hold since history dependence is not considered in the corresponding targeting rule (9)). As we turn to a more flexible inflation targeting scenario, the FIP effect has a stronger impact on the relative loss relations. While the FHP effect is less considered (i.e. the non-linear movement in the relative loss relation almost vanishes), the differences between Discretion and Commitment decrease since the stabilization bias increases rapidly.

If an *autocorrelated* shock is assumed, the FCP effect amplifies the FHP effect, which can be seen especially in the case $\bar{\alpha}_2 = 0.05$. Here, the relative loss relation peaks at a value of 203.60 %. This value is taken from Table 3, where we together with Table 2 report the values of both loss relations for selected values of h . From these Tables, we can see that the absolute differences in the loss are low for the case $h = 1$, while it is increasing strongly as h decreases. However, this development is dampened as $\bar{\alpha}_2$ increases but it is still remarkable for values of h close to 0 (especially in the case of an autocorrelated shock, cf. Table 3). The values of the relative loss relations indicate a loss under Discretion being less than 100 % (higher) relative to Commitment – except for the strict inflation targeting scenario as an autocorrelated shock occurs. In this case the OMP regime Commitment is strongly superior to the Discretion one, which holds mainly on a monthly frequency.

As we can see from both middle panels, the value of \tilde{h} (especially in the case $\bar{\alpha}_2 = 0.05$) slightly moves to the left as ρ^π increases. The lower panel of Figure 8 shows this in more detail. In general, quite high values of ρ^π induce quite low values of h needed to maintain an (relative) advantage of the OMP regime Commitment over Discretion, i.e. \tilde{h} decreases as ρ^π increases. Furthermore, the higher the value of $\bar{\alpha}_2$, the higher \tilde{h} will be. Finally, at a maximum value of $\rho^\pi = 0.99$ we observe almost a convergence in \tilde{h} across different values

of $\bar{\alpha}_2$.¹² Based on these observations it can be stated that the (amplified) FCP effect has an impact on the interaction of the FIP and the FHP effect.

We close this section by investigating the impact of a variation in the price stickiness measured by the frequency-dependent Calvo parameter denoted by $\theta(h)$. The latter probability consists on the corresponding value of this parameter in quarterly magnitudes given by θ . So far, we discuss the FIP effect as being described by the change in the slope of the NKPC. However, besides the degree of price indexation (which is set to zero in this paper), the degree of price stickiness plays also an important role via the FIP effect. The higher the probability of the firm to *not* adjust their prices the lower the dampening effect of reducing the output gap by the central bank will be. From an economic point of view, a high degree of price stickiness makes it difficult for the policy maker to influence inflation rate dynamics. The reason is that the price setting scheme becomes increasingly degenerated from the movement in the real marginal cost (output gap) the higher the likelihood for sticky prices will be. As a result the welfare loss increases with a higher degree of price stickiness, i.e. as θ increases.

Applying a sensitivity analysis with respect to θ is meaningful since the empirical evidence regarding this parameter is ambiguous. While $\theta \approx 2/3$ is a generally accepted benchmark that is supported by several empirical studies (Galí et al. (2001, p. 1255), Christiano et al. (2005, p. 18) or Álvarez et al. (2006, p. 578)), one can also find lower and higher degrees of stickiness in the literature. Fabiani et al. (2007, p. 41) analyze data from surveys of 11000 firms which were conducted by the national banks of 9 European countries. The median number of price changes per year in these different countries is equal to one, which implies $\theta \approx 3/4$. By estimating a DSGE model with sticky prices and wages for the Euro Area with Bayesian techniques, Smets and Wouters (2003, p. 1144) obtain a much longer average duration of price contracts of two and a half years, or $\theta \approx 9/10$. In contrast, from the data of the US Bureau of Labor Statistics, Bils and Klenow (2004, p. 953) derive evidence for firms in the US changing their price every two quarters or even less, according to which θ can become as low as $1/2$ and less. Therefore, we consider for our simulation a range of θ between 0.5 and 0.9.¹³

In the following we repeat our computation exercise with respect to the relative loss relations under variation of θ .¹⁴ The results are depicted in Figure 9. The results from the two dimensional investigations are resembled: the FHP effect dominates the FIP effect with respect to h on a lower to a moderate frequency, i.e. the corresponding loss relation increases towards \tilde{h} , which represents a peak on a low value of h . This holds only for a low value of $\bar{\alpha}_2$. As the latter increases the differences are high for almost all values of h – but only for low values of θ . As the degree of price stickiness increases, the stabilization bias increases. In this case the advantage of the OMP regime Commitment over Discretion vanishes as monetary policy becomes more *ineffective*. This holds independently for the variation in $\bar{\alpha}_2$.

More pronounced effects can be observed if an *autocorrelated* shock occurs. Again, the values of the relative loss relation decreases in transition from more a flexible to a more sticky price scenario. If prices are close to being fixed, the OMP regime Commitment is nearly as being effective as the Discretion one. Therefore the FIP effect with respect to

¹²Note that we do not study the case $\rho^\pi = 1$ since we consider a *temporary* shock only. Therefore, we discard the possibility of a random walk in the shock process since in this case the dynamic system becomes obviously hard to evaluate empirically (which is not done in this thesis).

¹³Ahrens and Sacht (forthcoming) report a value of θ close to or even equal to zero in the case of Argentina, which stems from the estimation of a high-frequency (i.e. daily) NKPC. We omit this polar case in our sensitivity analysis here, where we consider observations for the industrialized countries, the United States and in the Euro Area only.

¹⁴Similar results can be found in Franke and Sacht (forthcoming). In particular, in their study an equivalent three dimensional numerical investigation (given in their Appendix) in the case of a nominal interest rate shock in the (baseline) NKM with an ad-hoc Taylor rule (regime *TR*) is applied.

h becomes amplified as θ increases. The central bank can be advised, first, to ensure a strict inflation targeting scenario and, second, to react more frequently to the fluctuations in the economic indicators than on a quarterly base. However, the effectiveness of such kinds of strategies can be still hindered if prices become more rigid. Furthermore, keep in mind that the absolute loss is *always higher* on a higher relative to a lower frequency due to the FCP effect.

Finally, we study the change in the value of \tilde{h} with respect to θ , $\bar{\alpha}_2$ and ρ^π . Figure 10 applies. We observe a convergence in \tilde{h} as ρ^π approaches to the value of 0.99. A decrease in the degree of price stickiness requires a low value of h in order to maintain a maximum in the relative loss relations. In the other way around, the more prices are rigid the more a lower frequency is required to observe a maximum. The reason is that in this case the FHP effect dominates the FIP one *only* for high values of h since the increase in θ leads to stronger increase in the stabilization bias due to the (now) reinforced FIP effect. This observations put an emphasis on the role of the degree of price stickiness – together with the FCP effect (see above) – for the interaction of the FIP and the FHP effect.

5 Conclusion

In this paper we study the OMP responses to a cost-push shock within an h-economy environment. Our analytical and numerical results reveal the impact of frequency-dependent persistence on the effectiveness of OMP in transition from a lower to a higher frequency. Therefore, the OMP regimes Discretion and Commitment are applied under consideration of a purely forward-looking NKM. We show that the central bank fails to stabilize the inflation rate and the output gap on a higher frequency – even if its response is stemming from welfare maximization. Our main observation indicates that the welfare loss is increasing rapidly as the period length declines. This can be explained by the increase in the information available, which causes rapid changes in the inflation rate and the output gap. This must lead to destabilisation in the economy, which is analogous to the impact of an anticipated shock in quarterly magnitudes as analyzed by Winkler and Wohltmann (2012).

Given that the welfare loss increases as the period length decreases, our results call for an optimal response of the central bank in terms of a timeless perspective. Here, the central bank makes use of the expectation channel of monetary policy in order to generate smooth dynamics of the economic indicators over time. In comparison, a discretionary optimal monetary policy response, where expectations of agents are given for the central bank, leads to a higher welfare loss. The overall increase in the welfare loss is triggered by the frequency-dependent contraction persistence (FCP) effect, where the persistence of the shock process is amplified by a decrease in the period length.

In particular, the magnitudes of the reduction of the output gap needed in order to dampen the increase of the inflation rate are small on a higher frequency. The reason is that the stabilization bias is increasing in this case due to the frequency-dependent inherited persistence (FIP) effect. While in this case the pass-through of changes in the output gap into inflation dynamics is hindered, this outcome is more severe in the OMP regime Discretion relative to Commitment. In the latter case, the frequency-dependent history (dependence) persistence (FHP), which describes the decrease in the history dependence with respect to the period length, is less affected by the frequency of decision making. Hence, the welfare loss under Commitment is lower relative to the one under Discretion, which holds independently of the period length. The difference is strengthened in transition from a strict to a flexible inflation targeting. This is confirmed by various robustness checks, where we also find evidence for an increase in the stabilization bias due to an increase in the price stickiness. In addition, as an interesting result we observe that the

price level does not converge back to its initial steady state level under Commitment due to the FCP effect, which does not hold on quarterly magnitudes. According to this, the recent discretionary policy conducted by the ECB in the aftermath of the financial and sovereign debt crisis in the Euro Area can be seen as being harmful with respect to welfare maximization – regardless of low- or high-frequent movements in the economic indicators being considered in general.

Sacht and Wohltmann (2013) show that similar results can be obtained when an Optimal Simple Rule is applied. Here, while the specification of an ad-hoc Taylor rule is assumed, the optimal values of the corresponding weights on the inflation rate and the output gap are given by the outcome of the minimization of the loss function. In their paper, the authors report qualitative equivalent results with respect to the absolute and relative loss relations, i.e. that the welfare loss under Commitment is lower compared to the case of an Optimal Simple Rule. Only small quantitative differences are observed in general for the remaining loss relations, which are stemming from the temporal aggregation scheme applied in their paper.

In particular, one seeks to identify an amount of Optimal Simple Rules, which represent a good approximation of the (in reality unobservable) targeting rule under Commitment. It means that as an outcome, this specific rule becomes as close as possible in describing the dynamics, which are obtained under the OMP regime Commitment (cf. Wohltmann and Winkler (2009) as well as Winkler and Wohltmann (2012)). It is meant by that a specific instrument rule must be imposed under consideration of the optimal weights in the loss function. An example is given by a Taylor rule with interest smoothing, where the central bank responses partly also to the previous value of the nominal interest rate. This has not been done so far when studying different frequencies of decision making. In addition, the volatility of the nominal interest rate can also be taken into account as part of the central bank’s welfare function. We leave this to further research.

We like also to emphasise, that the analysis of the OMP responses within the *hybrid* NKM under Commitment would be fruitful. In this case the investigation does not require the corresponding solution methods for the computation of the policy functions like in the case of Discretion – see our discussion with respect to the approaches provided by Söderlind (1999) and Dennis (2007) given in the Introduction. Note that, given a hybrid specification of the NKM under Commitment, the degree of price indexation now plays a significant role in the determination of the movements of the variables on a higher frequency. According to our analysis of the cost-push shock based on an ad-hoc Taylor rule (cf. Sacht (2014)), we guess that the existence of intrinsic persistence will amplify the FIP effect, which leads to a decrease in the effectiveness of OMP along with stronger hump-shaped adjustments of the variables being observed. However, those thoughts call for a more elaborate investigation in further research.

In addition, the question arises if the central bank has to be forced to intervene on a lower frequency when facing high welfare losses on a higher one. This implies that it could be welfare increasing to adjust the monetary policy instrument (via the targeting rule) on e.g. a monthly or quarterly basis as information for the remaining economic indicators are available on a, let’s say, daily basis. In this case the central bank monitors the adjustments on a higher frequency and react to them on a lower one in order to avoid strong destabilisation effects in case of a cost push shock. This treatment calls for an heterogeneous specification of the period length across agents in the economy. We leave such kind of investigation also to further research. We would like to emphasise that the availability of daily information on movements in the economic indicators is not an unrealistic assumption to make. This holds especially for the (Argentine) inflation rate as we discuss in Ahrens and Sacht (forthcoming).

6 Appendix

6.1 Solution of the OMP Regime Discretion in the Case $0 < h \leq 1$

For a clear arrangement, the expectation operator E_t on the future values of the variables is omitted throughout this Appendix. The corresponding Lagrangian under Discretion is given by

$$L_t^D = h\bar{\alpha}_1(\pi_t^D)^2 + \bar{\alpha}_2(y_t^D)^2 + \tilde{\theta}_t(y_t^D + \delta_1(h)(i_t^D - \bar{i}) - f_t) + \tilde{\rho}_t(\pi_t^D - \tilde{\kappa}(h)y_t^D - g_t) + F_t \quad (11)$$

where $\tilde{\theta}_t$ and $\tilde{\rho}_t$ denote the Lagrangian multipliers (which are predetermined under Discretion), where their initial values in $t = 0$ are equal to 0. In the OMP regime Discretion, the central bank must take all expectations regarding the future values of the economic indicators as given:

$$\begin{aligned} F_t &= E_t \sum_{k=1}^{\infty} \beta(h)^{hk} \{h\bar{\alpha}_1(\pi_{t+hk}^D)^2 + \bar{\alpha}_2(y_{t+hk}^D)^2\} \\ f_t &= y_{t+h}^D + \delta_1(h)\pi_{t+h}^D \\ g_t &= \beta(h)\pi_{t+h}^D + v_t^\pi. \end{aligned} \quad (12)$$

The first-order conditions read

$$\frac{\partial L_t^D}{\partial \pi_t^D} = 2h\bar{\alpha}_1\pi_t^D + \tilde{\rho}_t = 0 \quad (13)$$

$$\frac{\partial L_t^D}{\partial y_t^D} = 2\bar{\alpha}_2y_t^D + \tilde{\theta}_t - \tilde{\rho}_t\tilde{\kappa}(h) = 0 \quad (14)$$

$$\frac{\partial L_t^D}{\partial \tilde{\theta}_t} = y_t^D + \delta_1(h)(i_t^D - \bar{i}) - f_t = 0 \quad (15)$$

$$\frac{\partial L_t^D}{\partial \tilde{\rho}_t} = \pi_t^D - \tilde{\kappa}(h)y_t^D - g_t = 0. \quad (16)$$

In addition, $\frac{\partial L_t^D}{\partial i_t^D} = \tilde{\theta}_t = 0$ holds, since interest rate stabilization is not the aim of the central bank (cf. Walsh (2010, p. 358)). Solving equation (14) for $\tilde{\rho}_t$ and plug it into (13) results in the targeting rule (9) under Discretion:

$$\pi_t^D = -\frac{\bar{\alpha}_2}{h\tilde{\kappa}(h)\bar{\alpha}_1}y_t^D \Leftrightarrow y_t^D = -\frac{h\tilde{\kappa}(h)\bar{\alpha}_1}{\bar{\alpha}_2}\pi_t^D \left(= -\frac{\kappa(h)\bar{\alpha}_1}{\bar{\alpha}_2}\pi_t^D \right). \quad (17)$$

Note that $\tilde{\kappa}(h) = \kappa(h)/h$ holds. Inserting the first part of the targeting rule into the NKPC (cf. equation (16)) leads to

$$\begin{aligned} -\left(\frac{\bar{\alpha}_2}{h\tilde{\kappa}(h)\bar{\alpha}_1}\right)y_t^D + \left(\beta(h)\frac{\bar{\alpha}_2}{h\tilde{\kappa}(h)\bar{\alpha}_1}\right)y_{t+h}^D - \tilde{\kappa}(h)y_t^D - v_t^\pi &= 0 \\ \Rightarrow \left(\frac{\bar{\alpha}_2}{h\tilde{\kappa}(h)\bar{\alpha}_1} + \tilde{\kappa}(h)\right)y_t^D - \left(\beta(h)\frac{\bar{\alpha}_2}{h\tilde{\kappa}(h)\bar{\alpha}_1}\right)y_{t+h}^D + v_t^\pi &= 0 \\ \Rightarrow d_1y_t^D + d_2y_{t+h}^D + v_t^\pi &= 0 \end{aligned} \quad (18)$$

with

$$\begin{aligned} d_0 &= -\frac{\bar{\alpha}_2}{h\tilde{\kappa}(h)\bar{\alpha}_1} \\ d_1 &= \tilde{\kappa}(h) - d_0 \\ d_2 &= \beta(h)d_0. \end{aligned}$$

Based on the method of undetermined coefficients (McCallum (1983)), we guess that the dynamics in y_t^D are described by the following law of motion:

$$y_t^D = d_3 v_t^\pi. \quad (19)$$

By plugging equation (19) into equation (18) we get the solution for d_3 :

$$\begin{aligned} d_1 d_3 v_t^\pi + d_2 d_3 v_{t+h}^\pi + v_t^\pi &= 0 \\ \Rightarrow d_2 d_3 (\rho(h)^\pi v_t^\pi + \varepsilon_{t+h}^\pi) + (d_1 d_3 + 1) v_t^\pi &= 0 \\ \Rightarrow [(d_2 \rho(h)^\pi + d_1) d_3 + 1] v_t^\pi &= 0 \\ \Rightarrow -\frac{1}{d_1 + d_2 \rho(h)^\pi} &= d_3. \end{aligned} \quad (20)$$

Note that in case of an (deterministic) impulse shock $\varepsilon_{t+h}^\pi = 0$ holds. By plugging equation (19) together with equation (20) into the targeting rule (17) we obtain

$$x_t^D = \Gamma_D v_t^\pi = \begin{pmatrix} 1 \\ d_0 \end{pmatrix} d_3 v_t^\pi$$

with $x_t^D = (y_t^D, \pi_t^D)'$. The reduced-form solution for the output gap and the inflation rate under Discretion are then explicitly given by

$$y_t^D = -\frac{\kappa(h)\bar{\alpha}_1}{[1 - \beta(h)\rho(h)^\pi]\bar{\alpha}_2 + \{[\kappa(h)^2]/h\}\bar{\alpha}_1} [\rho(h)^\pi]^{t/h} \varepsilon_0^\pi \quad (21)$$

$$\pi_t^D = \frac{\bar{\alpha}_2}{[1 - \beta(h)\rho(h)^\pi]\bar{\alpha}_2 + \{[\kappa(h)^2]/h\}\bar{\alpha}_1} [\rho(h)^\pi]^{t/h} \varepsilon_0^\pi \quad (22)$$

where $\tilde{\kappa}(h) = \kappa(h)/h$ has been already applied. Therefore consider also the forward solution of the shock process for $0 < h \leq 1$ given by¹⁵

$$E_0 v_t^\pi = [\rho(h)^\pi]^{t/h} \varepsilon_0^\pi. \quad (23)$$

6.2 Solution of the OMP Regime Commitment in the Case $0 < h \leq 1$

The Lagrangian under Commitment is given by

$$\begin{aligned} L_t^C &= E_t \sum_{k=0}^{\infty} \beta(h)^{hk} [h\bar{\alpha}_1 (\pi_{t+hk}^C)^2 + \bar{\alpha}_2 (y_{t+hk}^C)^2 \\ &+ \bar{\theta}_{t+h+hk} (y_{t+hk}^C - y_{t+h+hk}^C + \delta_1(h) (i_{t+hk}^C - \pi_{t+h+hk}^C - \bar{i})) \\ &+ \bar{\rho}_{t+h+hk} (\pi_{t+hk}^C - \beta(h)\pi_{t+h+hk}^C - \tilde{\kappa}(h)y_{t+hk}^C + v_{t+hk}^\pi)] \end{aligned} \quad (24)$$

where $\bar{\theta}_{t+h+hk}$ and $\bar{\rho}_{t+h+hk}$, again, denote the corresponding Lagrangian multipliers (which are, again, predetermined since π_t and y_t are forward-looking), where their initial values

¹⁵Therefore, consider the derivation of the shock process under $0 < h \leq 1$ in Appendix A2.

are equal to 0. The first-order conditions read

$$\begin{aligned} \frac{\partial L_t^C}{\partial \pi_{t+hk}^C} &= \beta(h)^{hk} (2h\bar{\alpha}_1 \pi_{t+hk}^C) - \beta(h)^{hk-h} \bar{\theta}_{t+hk} \delta_1(h) \\ &+ \bar{\rho}_{t+h+hk} - \beta(h)^{hk-h+1} \bar{\rho}_{t+hk} = 0 \end{aligned} \quad (25)$$

$$\frac{\partial L_t^C}{\partial y_{t+hk}^C} = \beta(h)^{hk} (2\bar{\alpha}_2 y_{t+hk}^C + \bar{\theta}_{t+h+hk} - \tilde{\kappa}(h) \bar{\rho}_{t+h+hk}) - \beta(h)^{hk-h} \bar{\theta}_{t+hk} = 0 \quad (26)$$

$$\frac{\partial L_t^C}{\partial i_{t+hk}^C} = \bar{\theta}_{t+h+hk} = 0 \quad \forall h = 0, \dots, 1; k = 0, \dots, \infty \quad (27)$$

$$\frac{\partial L_t^C}{\partial \bar{\rho}_{t+h+hk}} = y_{t+hk}^C - y_{t+h+hk}^C + \delta_1(h) (i_{t+hk}^C - \pi_{t+h+hk}^C - \bar{i}) = 0 \quad (28)$$

$$\frac{\partial L_t^C}{\partial \bar{\rho}_{t+h+hk}} = \pi_{t+hk}^C - \beta(h) \pi_{t+h+hk}^C - \tilde{\kappa}(h) y_{t+hk}^C - v_{t+hk}^\pi = 0. \quad (29)$$

By dividing (25) and (26) by $\beta(h)^{hk}$ and taking into account that $k = 0$ holds, it follows

$$0 = 2h\bar{\alpha}_1 \pi_t^C + \bar{\rho}_{t+h} - \bar{\rho}_t \beta(h)^{1-h} \quad (30)$$

$$\bar{\rho}_{t+h} = \frac{2\bar{\alpha}_2}{\tilde{\kappa}(h)} y_t^C \Leftrightarrow \bar{\rho}_t = \frac{2\bar{\alpha}_2}{\tilde{\kappa}(h)} y_{t-h}^C. \quad (31)$$

Plugging (31) into (30) results in the targeting rule under Commitment:

$$\pi^C = -\frac{\bar{\alpha}_2}{h\bar{\alpha}_1 \tilde{\kappa}(h)} \left(y_t^C - \beta(h)^{(1-h)} y_{t-h}^C \right) \Leftrightarrow y_t^C = -\frac{\bar{\alpha}_1 \kappa(h)}{\bar{\alpha}_2} \pi_t^C + \beta(h)^{(1-h)} y_{t-h}^C. \quad (32)$$

Note, again, that $\tilde{\kappa}(h) = \kappa(h)/h$ holds. The expression in front of the brackets is equal to d_0 given under Discretion. Hence, the stabilization bias being the same in both OMP regimes. Inserting the first part of the previous equation into the NKPC (cf. equation (29) for $k = 0$) leads to

$$\begin{aligned} &- \frac{\bar{\alpha}_2}{h\bar{\alpha}_1 \tilde{\kappa}(h)} \left(y_t^C - \beta(h)^{(1-h)} y_{t-h}^C \right) - \tilde{\kappa}(h) y_t^C \\ &+ \frac{\beta(h) \bar{\alpha}_2}{h\bar{\alpha}_1 \tilde{\kappa}(h)} \left(y_{t+h}^C - \beta(h)^{(1-h)} y_t^C \right) = v_t^\pi \\ \Rightarrow &y_t^C \left(-\frac{\bar{\alpha}_2}{h\bar{\alpha}_1 \tilde{\kappa}(h)} - \tilde{\kappa}(h) - \beta(h)^{(1-h)} \frac{\beta(h) \bar{\alpha}_2}{h\bar{\alpha}_1 \tilde{\kappa}(h)} \right) \\ &+ y_{t-h}^C \left(\frac{\bar{\alpha}_2}{h\bar{\alpha}_1 \tilde{\kappa}(h)} \beta(h)^{(1-h)} \right) + y_{t+h}^C \left(\frac{\beta(h) \bar{\alpha}_2}{h\bar{\alpha}_1 \tilde{\kappa}(h)} \right) = v_t^\pi \\ \Rightarrow &y_t^C \left(1 + \frac{h\bar{\alpha}_1 \tilde{\kappa}(h)^2}{\bar{\alpha}_2} + \beta(h)^{(2-h)} \right) - \beta(h)^{(1-h)} y_{t-h}^C \\ &- \beta(h) y_{t+h}^C + \frac{h\bar{\alpha}_1 \tilde{\kappa}(h)}{\bar{\alpha}_2} v_t^\pi = 0 \\ \Rightarrow &c_1 y_t^C + c_2 y_{t-h}^C + c_3 y_{t+h}^C + c_0 v_t^\pi = 0 \end{aligned} \quad (33)$$

with

$$\begin{aligned} c_0 &= \frac{h\bar{\alpha}_1 \tilde{\kappa}(h)}{\bar{\alpha}_2} \\ c_1 &= 1 + \frac{h\bar{\alpha}_1 \tilde{\kappa}(h)^2}{\bar{\alpha}_2} + \beta(h)^{(2-h)} \\ c_2 &= -\beta(h)^{(1-h)} \\ c_3 &= -\beta(h). \end{aligned}$$

Based on the method of undetermined coefficients, we guess that the dynamics in y_t^C are described by the following law of motion:

$$y_t^C = \tilde{c}_1 y_{t-h}^C + \tilde{c}_2 v_t^\pi. \quad (34)$$

By plugging equation (34) into equation (33) we are able to compute the solution for \tilde{c}_1 and \tilde{c}_2 :

$$\begin{aligned} c_1(\tilde{c}_1 y_{t-h}^C + \tilde{c}_2 v_t^\pi) + c_2 y_{t-h}^C + c_3[\tilde{c}_1 y_t^C + \tilde{c}_2(\rho(h)^\pi v_t^\pi + \varepsilon_{t+h}^\pi)] + c_0 v_t^\pi &= 0 \\ \Rightarrow c_1(\tilde{c}_1 y_{t-h}^C + \tilde{c}_2 v_t^\pi) + c_2 y_{t-h}^C + c_3[\tilde{c}_1(\tilde{c}_1 y_{t-h}^C + \tilde{c}_2 v_t^\pi) + \tilde{c}_2 \rho(h)^\pi v_t^\pi] + c_0 v_t^\pi &= 0 \\ \Rightarrow (c_1 \tilde{c}_1 + c_2 + c_3 \tilde{c}_1^2) y_{t-h}^C + [c_1 \tilde{c}_2 + c_3(\tilde{c}_1 \tilde{c}_2 + \tilde{c}_2 \rho(h)^\pi) + c_0] v_t^\pi &= 0. \end{aligned} \quad (35)$$

Note, again, that in case of a (deterministic) impulse shock $\varepsilon_{t+h}^\pi = 0$ holds. The first term in equation (35) displays a characteristic polynomial:

$$c_3 \tilde{c}_1^2 + c_1 \tilde{c}_1 + c_2 = 0 \Leftrightarrow \tilde{c}_1^2 + \frac{c_1}{c_3} \tilde{c}_1 + \frac{c_2}{c_3} = 0 \quad (36)$$

with

$$\begin{aligned} \frac{c1}{c3} &= -\frac{1 + [h\bar{\alpha}_1 \tilde{\kappa}(h)^2] \bar{\alpha}_2^{-1} + \beta(h)^{(2-h)}}{\beta(h)} \\ \frac{c2}{c3} &= \frac{1}{\beta(h)^h}. \end{aligned}$$

Based on the numerical parameter scenario given, we choose the specific solution of (36) which ensures dynamic stability, i.e. $0 < \tilde{c}_1 < 1$ holds.¹⁶ Given the solution of \tilde{c}_1 , we are able to compute the one of \tilde{c}_2 , which follows from the second term in equation (35):

$$c_1 \tilde{c}_2 + c_3[\tilde{c}_1 \tilde{c}_2 + \tilde{c}_2 \rho(h)^\pi] + c_0 \Leftrightarrow \tilde{c}_2 = -\frac{c_0}{c_1 + [\tilde{c}_1 + \rho(h)^\pi] c_3}. \quad (37)$$

The dynamics in the inflation rate under Commitment are now described by (cf. targeting rule (10))

$$\pi_t^C = -\frac{1}{c_0} \left[(\tilde{c}_1 - \beta(h)^{(1-h)}) y_{t-h}^C + \tilde{c}_2 v_t^\pi \right] \quad (38)$$

The laws of motion in x_t^C can be summarized by

$$x_t^C = \Gamma_{C,1} x_{t-h}^C + \Gamma_{C,2} v_t^\pi \quad (39)$$

where

$$\begin{aligned} \Gamma_{C,1} &= \begin{pmatrix} \tilde{c}_1 & 0 \\ -\frac{\tilde{c}_1 - \beta(h)^{(1-h)}}{c_0} & 0 \end{pmatrix} \\ \Gamma_{C,2} &= \begin{pmatrix} \tilde{c}_2 \\ -\frac{\tilde{c}_2}{c_0} \end{pmatrix} \end{aligned}$$

¹⁶Obviously, this observation serves as a necessary but not sufficient condition since it depends on the underlying parameter scenario. In our case we obtain numerically that $\tilde{c}_{1,1}$ is the *unstable* root of (36), while $\tilde{c}_{1,2}$ it the *stable* one, e.g. $0 < \tilde{c}_{1,2} < 1$ holds. This is true for all values of h over its admissible range. It is also confirmed that when plugging both solutions into the characteristic polynomial above, this results in 0 in both cases. However, in general, analytical solutions for $\tilde{c}_{1,1}$ and $\tilde{c}_{1,2}$ are hard to find since the non-trivial expressions $\frac{c1}{c3}$ and $\frac{c2}{c3}$ are considered to be part of the associated quadratic formula. Therefore, we rely on the numerical computation of the roots but give the strong advise to check on the (stable) solutions for any given parameter scenario. For a clear arrangement in the following we define $\tilde{c}_1 := \tilde{c}_{1,2}$.

with $x_t^C = (y_t^C, \pi_t^C)'$ and $x_{t-h}^C = (y_{t-h}^C, \pi_{t-h}^C)'$ hold. More explicitly, the reduced-form solution for the output gap can be written as (cf. equation (34))

$$y_t^C = \frac{\tilde{c}_1^{(t+1)/h} - [\rho(h)^\pi]^{(t+1)/h}}{\tilde{c}_1 - \rho(h)^\pi} \tilde{c}_2 \varepsilon_0^\pi. \quad (40)$$

Plugging this law of motion into equation (38), after some re-arrangement we get the reduced-form solution for the inflation rate:

$$\pi_t^C = -\frac{\tilde{c} \tilde{c}_1^{t/h} - (\tilde{c} - \tilde{c}_2) [\rho(h)^\pi]^{t/h}}{c_0} \varepsilon_0^\pi \quad (41)$$

with

$$\tilde{c} = \frac{(\tilde{c}_1 - \beta(h)^{(1-h)}) \tilde{c}_2}{\tilde{c}_1 - \rho(h)^\pi}.$$

The previous expression make, again, use of the forward solution of the shock process for $0 < h \leq 1$ given by

$$E_0 v_t^\pi = [\rho(h)^\pi]^{t/h} \varepsilon_0^\pi. \quad (42)$$

6.3 Impulse Response Functions (D,C) in the Case $0 < h \leq 1$

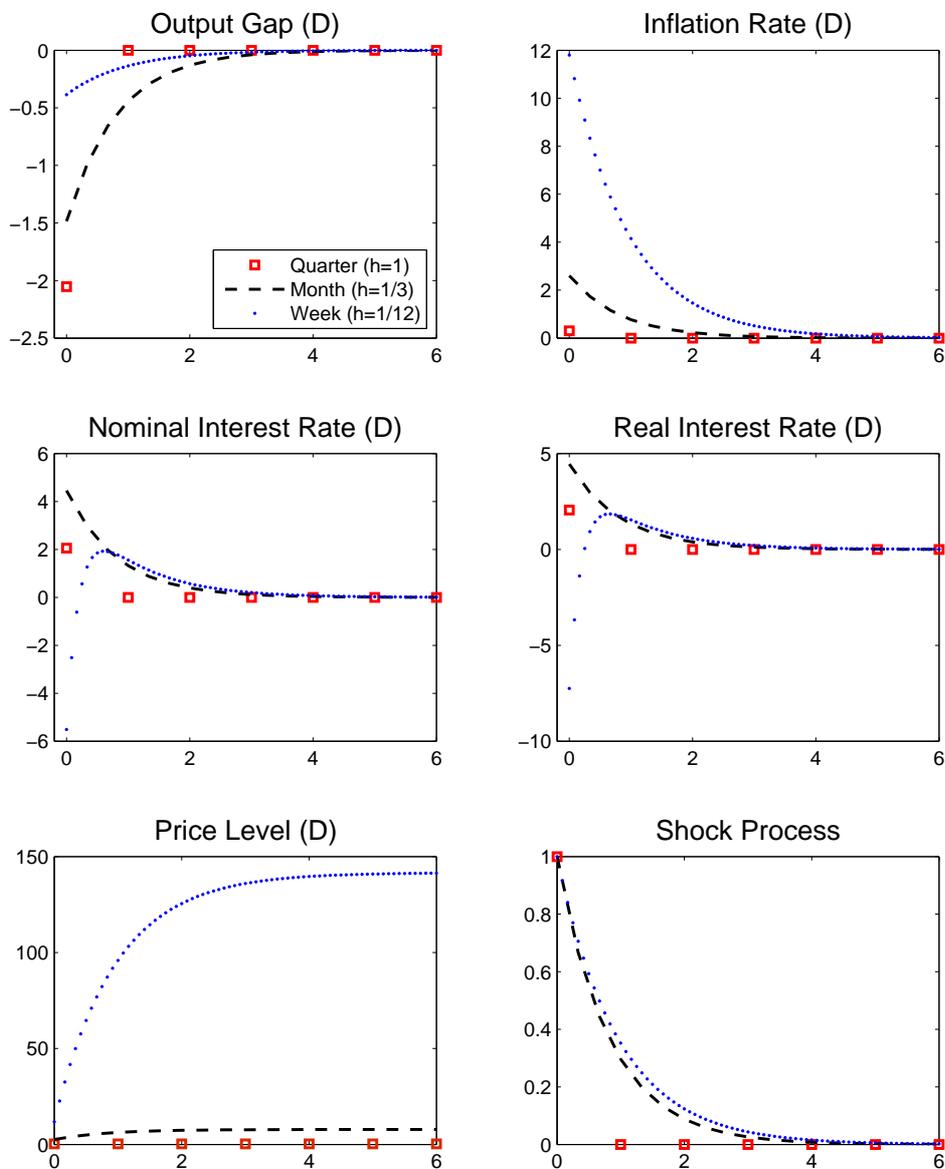


Figure 2: IRFs in case of a *non*-autocorrelated cost-push shock in the NKM under the OMP regime Discretion (D).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent (age points) are displayed on the horizontal and vertical axis, respectively.

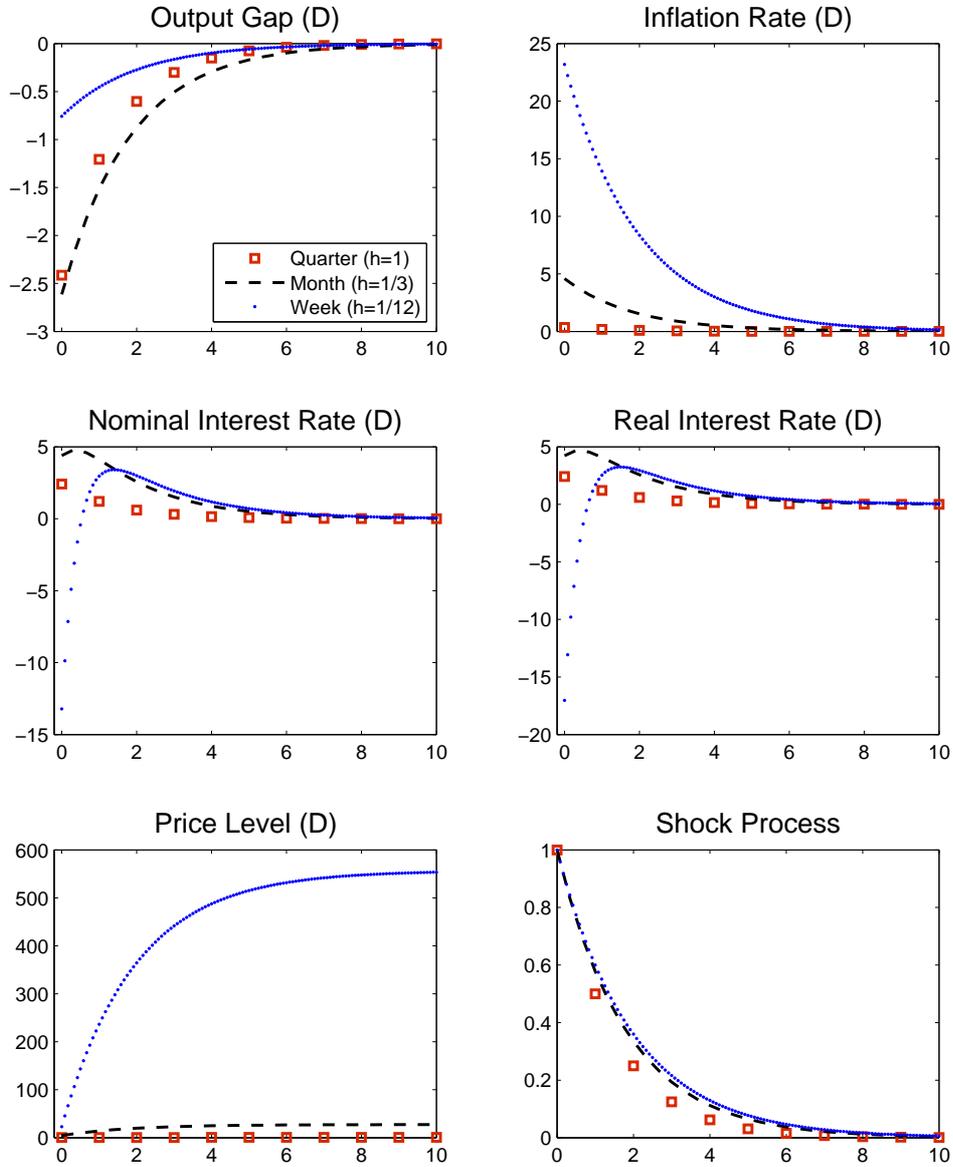


Figure 3: IRFs in case of an *autocorrelated* cost-push shock in the NKM under the OMP regime Discretion (D).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

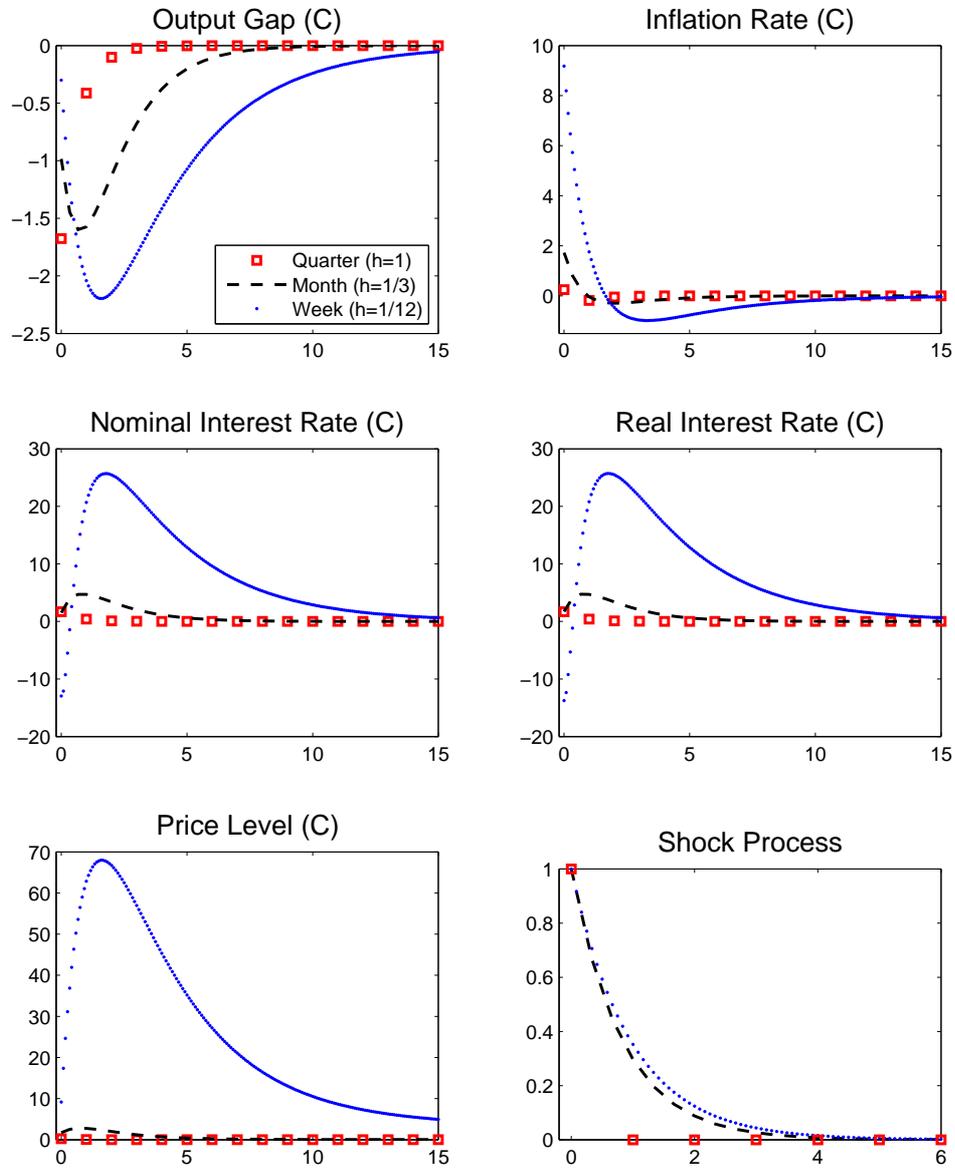


Figure 4: IRFs in case of a *non*-autocorrelated cost-push shock in the NKM under the OMP regime Commitment (C).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

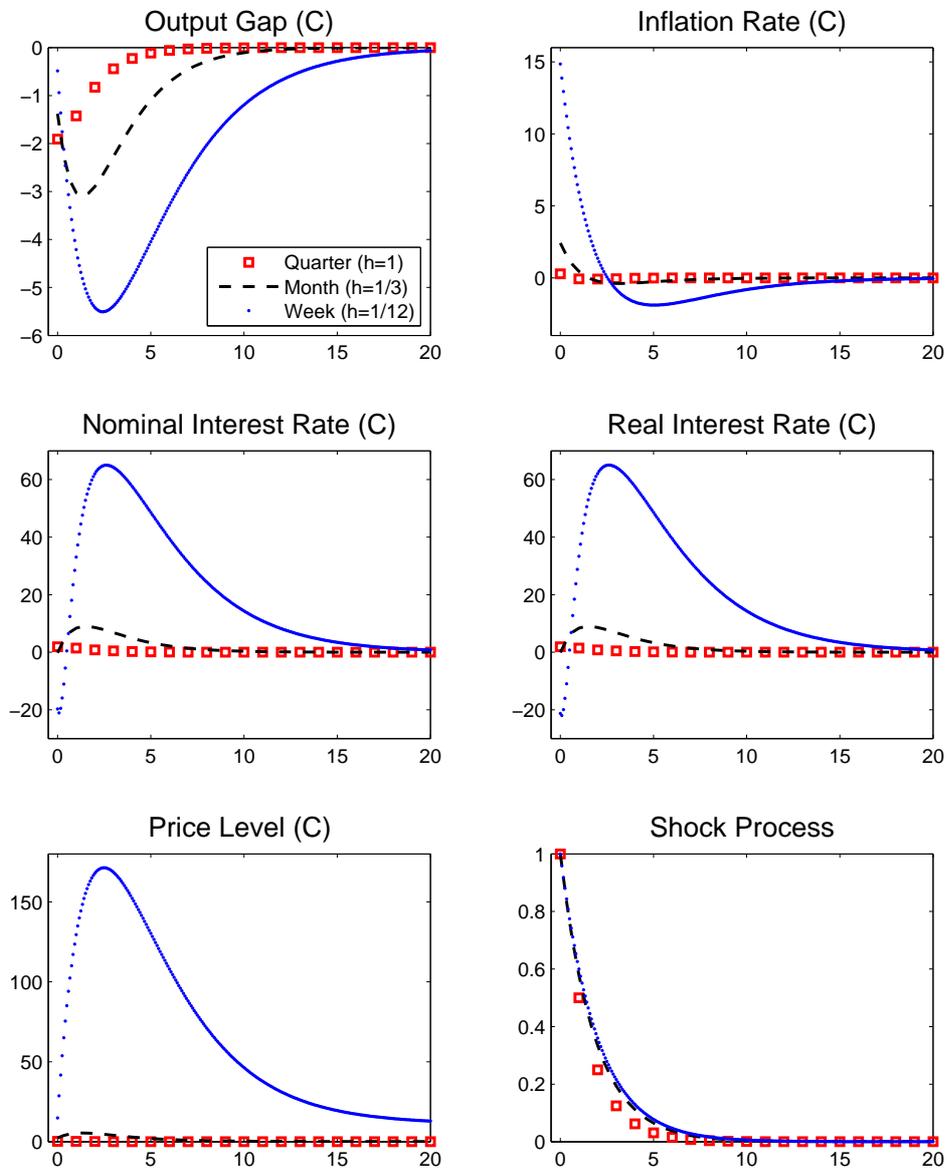


Figure 5: IRFs in case of an *autocorrelated* cost-push shock in the NKM under the OMP regime Commitment (C).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

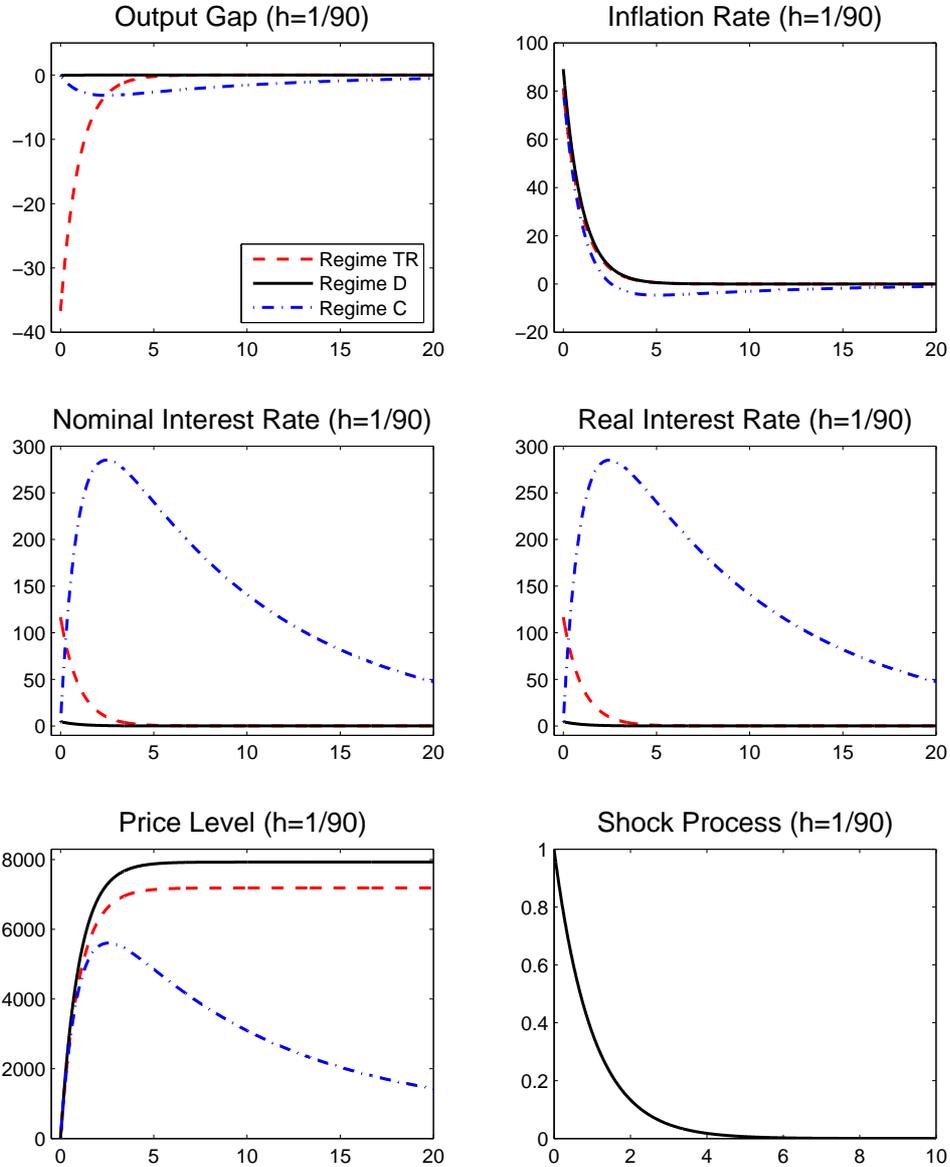


Figure 6: *Daily* ($h = 1/90$) IRFs in case of a *non*-autocorrelated cost-push shock in the baseline NKM (TR) and under the OMP regimes Discretion (D) and Commitment (C).

Note: The *dashed* line represents the daily IRFs under the regime *TR*. The *solid* line represents the daily IRFs under the regime Discretion (D). The *dashed/dotted* line represents the daily IRFs under the regime Commitment (C). The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

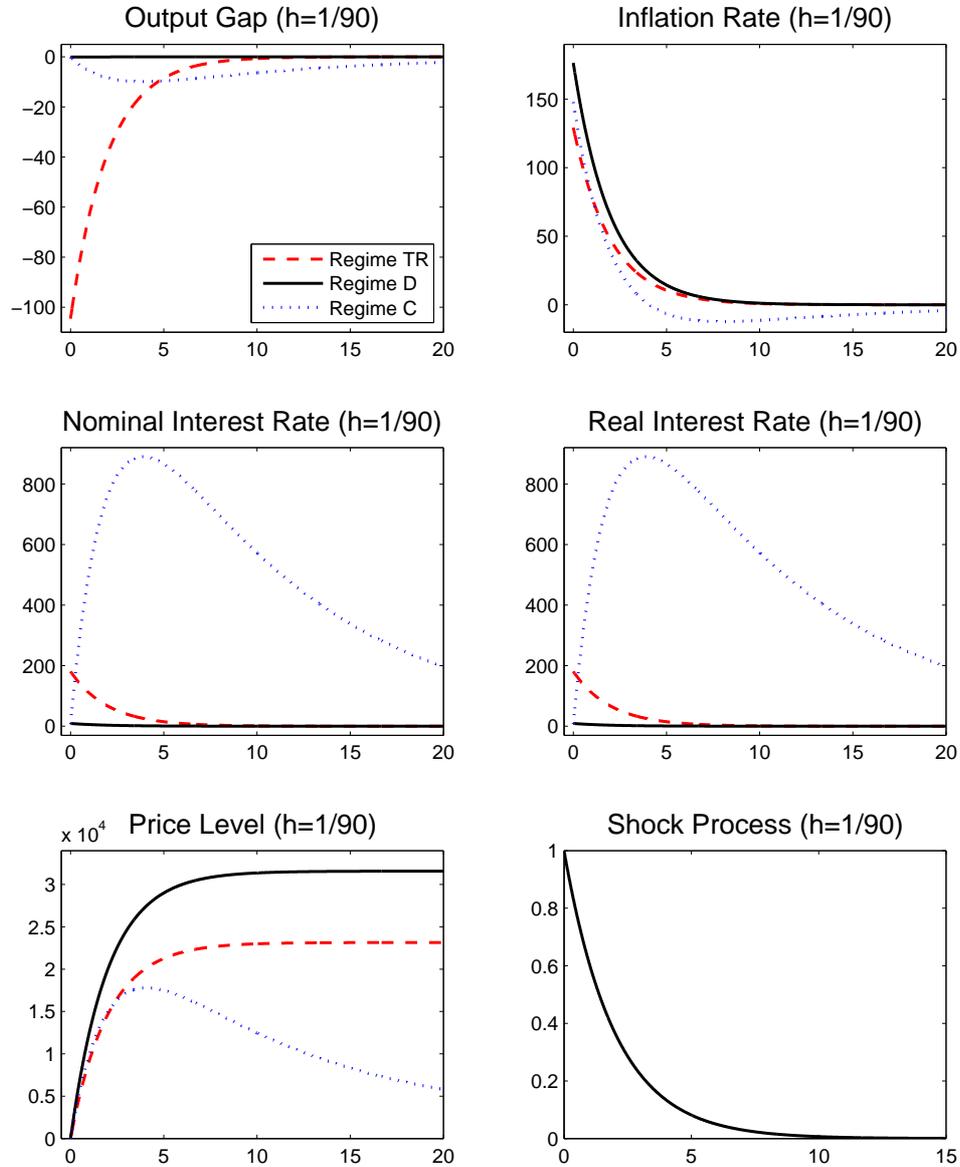


Figure 7: Daily ($h = 1/90$) IRFs in case of an *autocorrelated* cost-push shock in the baseline NKM (TR) and under the OMP regimes Discretion (D) and Commitment (C).

Note: The *dashed* line represents the daily IRFs under the regime *TR*. The *solid* line represents the daily IRFs under the regime Discretion (D). The *dashed/dotted* line represents the daily IRFs under the regime Commitment (C). The time in quarters and the change in percent (age points) are displayed on the horizontal and vertical axis, respectively.

6.4 Development in the Loss Relations D/C

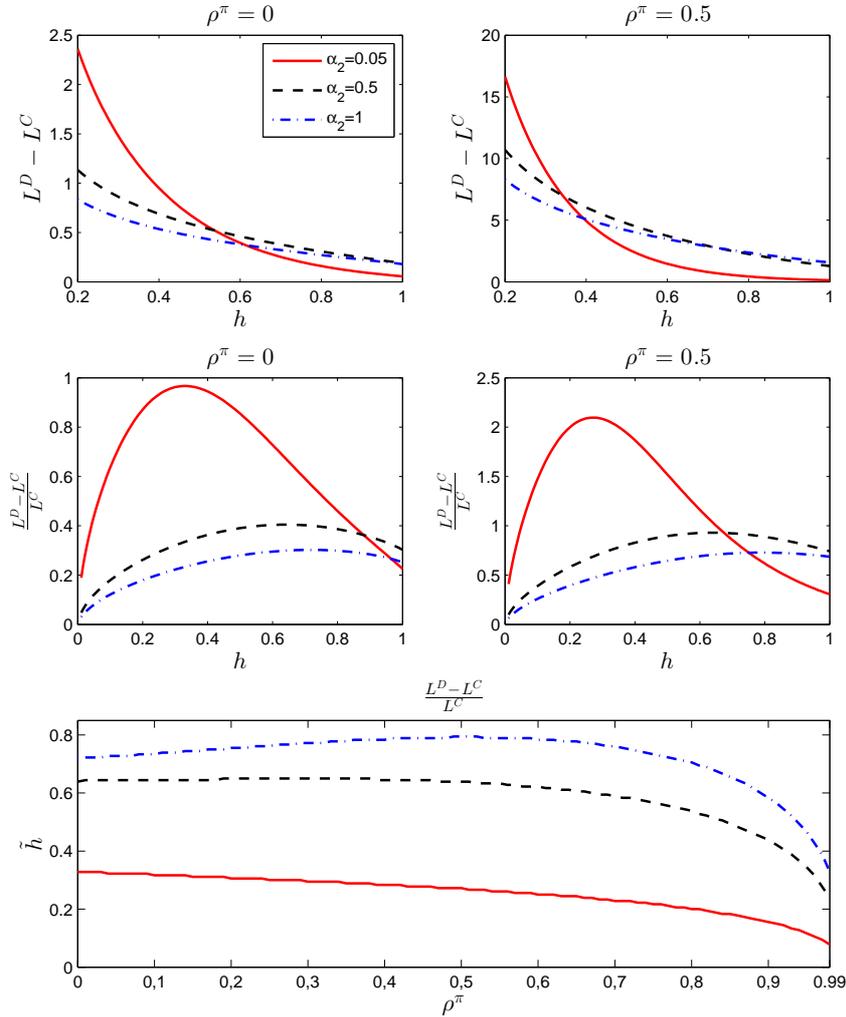


Figure 8: Development and maximum of the loss relation(s) Discretion(D)/Commitment(C).

Note: The *upper* and *middle* panels depict the development of the absolute and relative *loss relations* Discretion (D)/Commitment (C) with respect to h and under variation of $\bar{\alpha}_2$ in the case of a non-autocorrelated ($\rho^\pi = 0$; left panels) and an autocorrelated ($\rho^\pi = 0.5$; right panels) cost-push shock. L^D and L^C denote the loss for the OMP regimes Discretion and Commitment, respectively. The *lower* panel depicts the value of h associated with the maximum in the relative loss relation Discretion/Commitment (denoted as \tilde{h}) under variation of $\bar{\alpha}_2$ and ρ^π . The *solid* line represents the loss relations/changes in \tilde{h} for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 0.05$. The *dashed* line represents the loss relations/changes in \tilde{h} for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 0.5$. The *dashed/dotted* line represents the loss relations/changes in \tilde{h} for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 1$.

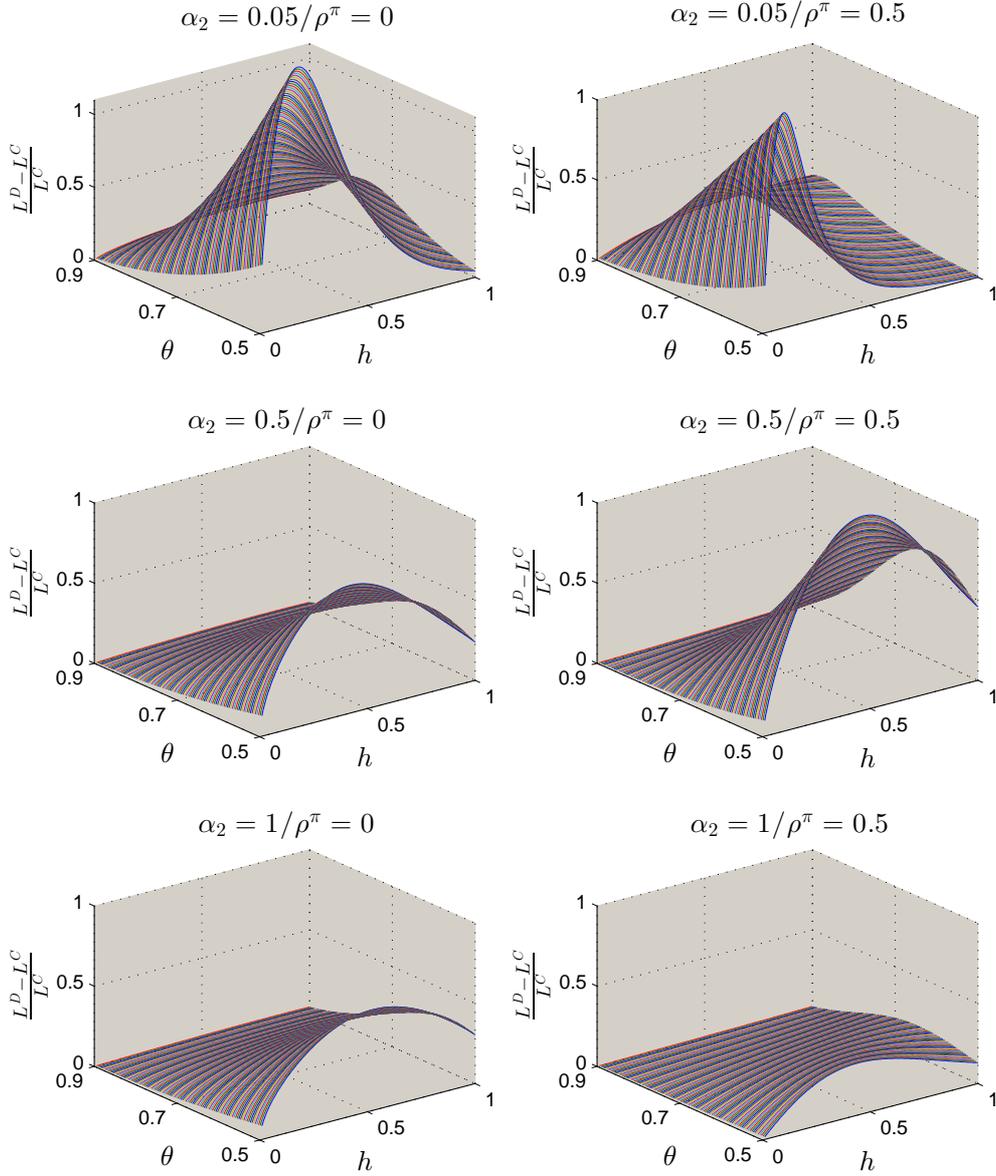


Figure 9: Development of the (relative) loss relation Discretion (D)/Commitment (C) in the case of an (*non*)-autocorrelated cost-push shock under variation of θ .

Note: The Figure depicts the development of the relative loss relation Discretion (D)/Commitment (C) with respect to h and under variation of $\bar{\alpha}_2$ and θ in the case of a non-autocorrelated ($\rho^\pi = 0$; left panels) and an autocorrelated ($\rho^\pi = 0.5$; right panels) cost-push shock. L^D and L^C denote the loss for the OMP regimes Discretion and Commitment, respectively. The *upper panels* represent the loss relations for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 0.05$. The *middle panels* represent the loss relations for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 0.5$. The *lower panels* represent the loss relations for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 1$.

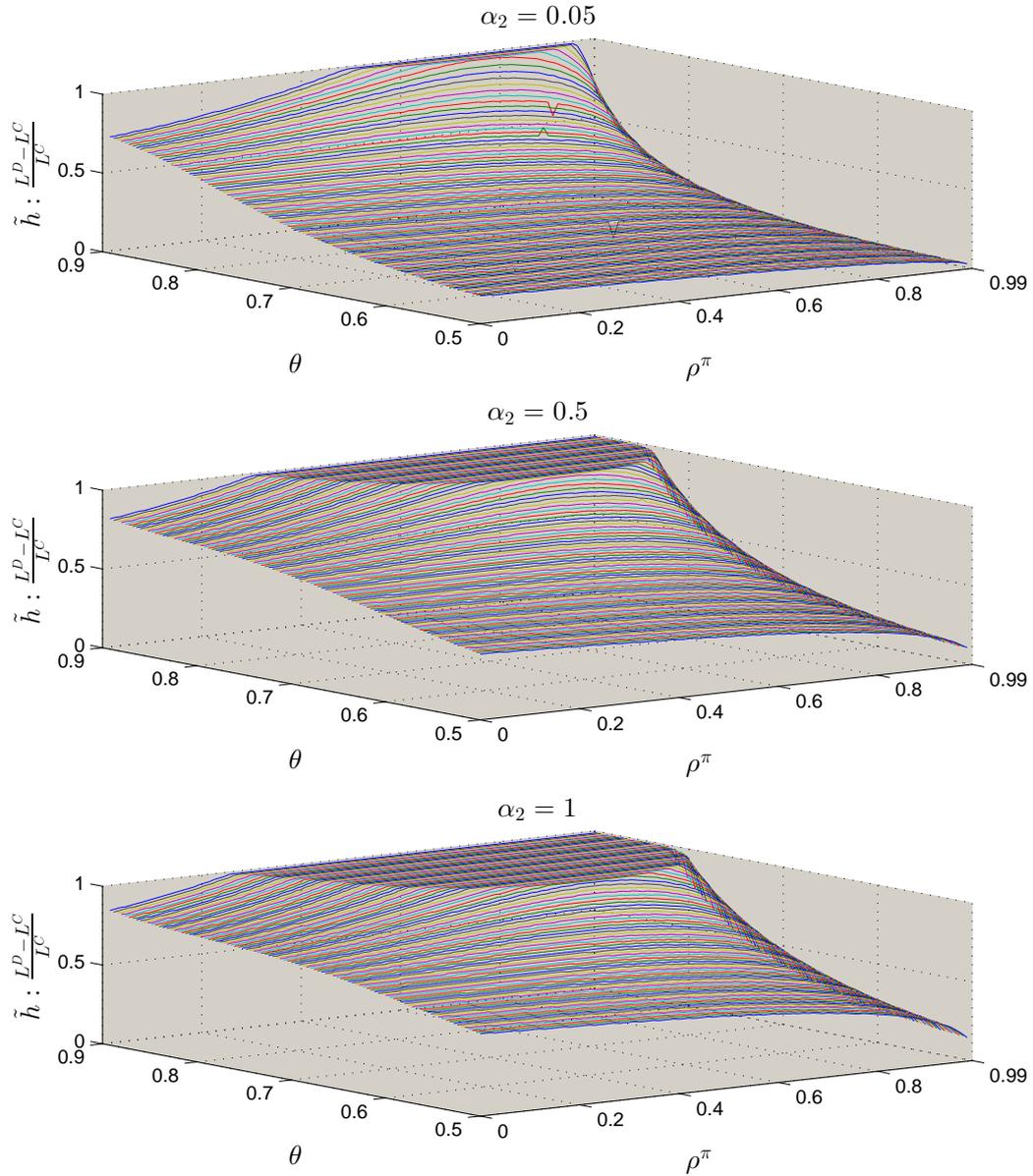


Figure 10: Value of h associated with the maximum in the loss relation Discretion (D)/Commitment (C) under variation of $\bar{\alpha}_2$, θ and ρ^π .

Note: The Figure depicts the value of h associated with the maximum in the loss relation (\tilde{h}) Discretion (D)/Commitment (C) under variation of $\bar{\alpha}_2$, θ and ρ^π . L^D and L^C denotes the loss for the OMP regimes Discretion and Commitment, respectively. The *upper panel* represents the changes in \tilde{h} for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 0.05$. The *middle panel* represents the changes in \tilde{h} for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 0.5$. The *lower panel* represents the changes in \tilde{h} for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 1$.

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