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## Compensated discrete choice with particular reference to labor supply

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# MEMORANDUM

No 20/2013

## Compensated Discrete Choice with Particular Reference to Labor Supply

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**John K. Dagsvik, Steinar Strøm  
and Marilena Locatelli**

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# Compensated Discrete Choice with Particular Reference to Labor Supply

by

John K. Dagsvik<sup>1</sup>, Steinar Strøm<sup>2</sup> and Marilena Locatelli<sup>3</sup>

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(August 24, 2013)

## Abstract

Recently Dagsvik and Karlström (2005) have demonstrated how one can compute Compensating Variation and Compensated Choice Probabilities by means of analytic formulas in the context of discrete choice models. In this paper we offer a new and simplified derivation of the Compensated probabilities in the case with independent random utility models. Subsequently, we discuss the application of this methodology to compute compensated labor supply responses (elasticities) in discrete labor supply models. Whereas the Slutsky equation holds in the case of the standard microeconomic model with deterministic preferences, this is not so in the case of random utility models. Note that since the non-labor income elasticity is negative the Slutsky equation implies that the compensated wage elasticity is higher than the uncompensated one. With a random utility model we show empirically that in many cases the uncompensated wage elasticity is in fact the highest one.

**JEL classification:** J22, C51

**Keywords:** Female labor supply, compensated wage elasticities in RUM

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## 1. Introduction

In recent years labor supply analysis based on the theory of discrete choice and random utility formulations has become increasingly popular. A major reason is that discrete choice labor supply models are much more practical than the conventional continuous approach based on marginal calculus, see the surveys by Creedy and Kalb (2005) and Dagsvik et al. (2012). In the literature there are basically two versions of discrete models of labor supply that have been proposed. Van Soest (1995) and van Soest et al. (2002) proposed to analyze labor supply as a standard discrete choice problem. In contrast, Dagsvik et al. (1995), with further extension by Dagsvik and Strøm (2006), proposed to analyze labor supply as a job choice problem, where the set of feasible jobs is individual specific and latent. With the discrete choice approach, it is easy to deal with nonlinear and non-convex economic budget constraints, and to apply rather general functional forms of the utility representations. Whereas it is fairly straight forward to compute uncompensated responses and elasticities in discrete choice models, the computation of the corresponding compensated effects is, however, not trivial.

Dagsvik and Karlström (2005) have demonstrated how one can calculate Compensating Variation (CV) and compensated choice probabilities (Hicks choice probabilities) in random utility models, in the general case when the deterministic part of the utility function may be nonlinear in income. Dagsvik et al. (2009) applied the methodology of Dagsvik and Karlström (2005) to compute welfare measures such as CV in the context of selected tax reforms.

In this paper we show how one can apply the methodology of Dagsvik and Karlström (2005) to compute Hicks choice probabilities and corresponding compensated wage elasticities (Slutsky elasticities). To this end we base our analysis on the model proposed and estimated by Dagsvik and Strøm (2006). Specifically, we show how one can compute Hicks joint choice probabilities of being in particular states before and after wage rate changes or tax reforms. The states are non-working, working in specific sectors and with different working loads. Slutsky elasticities are derived from the Hicks joint probabilities.

We report uncompensated as well compensated wage elasticities, and also non-labor income elasticities. These calculations show that the compensated wage elasticities (Slutsky elasticities) tend to be higher at the extensive margin than at the intensive margin, while the opposite is true for uncompensated wage elasticities. The elasticities vary considerably with the wage level (the higher the wage is, the lower is the elasticity) and with household characteristics. Moreover, the calculation of the three elasticities demonstrates that to which extent the Slutsky equation is violated in this type of models.

Second, we calculate Slutsky elasticities based on the sample values that were used in estimating the model. It is shown that the mean value of the uncompensated elasticities of expected hours with respect to the wage exceed the compensated elasticities, which implies that the Slutsky equation does not hold.

The paper is organized as follows. In the next section we review some basic definitions and formulas given by Dagsvik and Karlström (2005) for computing compensated choice probabilities. Subsequently, we discuss special cases such as the binary and the ternary choice setting. This is done for the sake of bringing out the essentials of the methodology of Dagsvik and Karlström (2005), also when it is applied to labor supply. Section 3 gives a brief description of the empirical labor supply model that is the main focus of our application and reports numerical results. Section 4 reports elasticities and section 5 concludes.

## 2. Compensated choice in random utility models

The history of random utility models (RUM) dates back at least to Thurstone's (1927) analysis. It took more than 30 years before this type of models were applied in economics. See Quandt (1956), Luce (1959), and McFadden (1973, 1978, 1981, 1984) for extensive discussions on motivation, exposition and applications. An early discussion of the potential of the random utility approach within economics is provided by Quandt (1956). He confronts the established wisdom at the time by arguing that it might be better to define preferences in a probabilistic sense and thereby making the model more realistic by weakening the postulate of strict rationality.

In micro economics, the theory of compensated (Hicksian) demand and supply plays an important role. Until recently, there has been very little focus on a corresponding theory in the context of random utility models. To our knowledge, the first systematic analysis of compensated choice in random utility models was undertaken by Dagsvik and Karlström (2005). In this paper we offer a simplified proof of a fundamental result obtained by Dagsvik and Karlström (2005), which we shall now discuss.

Consider a setting with a finite number of alternatives. Assume that the utility of alternative  $j$  has the structure  $U_j(y) = \log v_j(y) + \epsilon_j$ , where  $v_j(y)$  is a positive, deterministic and monotonically increasing function of  $y$  and it may also depend on prices, non-pecuniary and alternative  $j$ -specific attributes, whereas  $\epsilon_j$  is a stochastic term that is supposed to account for the effect on preferences from variables that are not observed by the researcher.<sup>4</sup> For our purpose in this section we only need to introduce income in the notation. In the context of this paper we consider situations where a reform is introduced, and it is therefore convenient to introduce additional notation. To this end let  $U_j^0 = \log v_j^0 + \epsilon_j$  denote the ex ante utility of alternative  $j$ , where  $y^0$  denotes the initial income and  $v_j^0(\cdot)$  is the deterministic term associated with the utility of alternative  $j$  ex ante, and let  $U_j(y) = \log v_j(y) + \epsilon_j$  be the corresponding utility of alternative  $j$  ex post. The respective systematic terms  $v_j^0(\cdot)$  and  $v_j(\cdot)$  may differ due to exogenous changes in taxes or alternative specific attributes, suppressed in the notation here. Here it is assumed that the stochastic terms of the utility function are

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<sup>4</sup> With no loss of generality we apply the logarithm transformation for the sake of simplifying the expressions below.



not affected by the reform. Let  $J^0$  and  $J$  denote the ex ante and ex post choice given that the ex ante and ex post utility levels of the chosen alternatives are equal. Let  $P^H(j, k) = P(J^0 = j, J = k)$  denote the joint compensated (Hicksian) probability of choosing alternative  $j$  ex ante and alternative  $k$  ex post under the condition that the respective utility levels of the chosen alternatives before and after the reform are the same. In other words,

$$(2.1) \quad P^H(j, k) = P(U_j^0 = \max_r U_r^0, U_k(Y) = \max_r U_r(Y), \max_r U_r^0 = \max_r U_r(Y)),$$

where  $Y$  is the income required to maintain the utility level equal to the original utility level. Note that the income  $Y$  is stochastic due to the utility function containing a stochastic term. Assume in the following that the random error terms are iid with c.d.f.  $\exp(-\exp(-x))$ . Let  $y_j$  be determined by  $v_j^0(y^0) = v_j(y_j)$ . That is,  $y_j$  is the ex post income that ensures that the ex ante and ex post utility of alternative  $j$  are equal. Also let  $v_r(y) = \max(v_r^0(y^0), v_r(y))$ . For simplicity we shall in the following write  $v_j^0(y^0) = v_j^0$ . For  $j$  to be the most preferred alternative ex ante and  $k$  the most preferred alternative ex post this implies that  $U_k(Y) = U_j^0 = U_k^0$ , which implies that  $Y = y_k$ . Furthermore, since alternative  $k$  is the most preferred one ex post,  $U_k(Y) = U_j(Y)$ , which together with the above result yields  $U_j(Y) = U_j^0$ . The latter inequality implies that  $Y = y_j$ . Hence, for transitions from alternative  $j$  to alternative  $k$  to take place, under constant indirect utility level it must be the case that  $y_k = Y = y_j$ . The following result was obtained by Dagsvik and Karlström (2005, Corollary 3).

### Theorem 1

Assume an additive random utility model with independent and extreme value distributed error terms.

Then the joint density  $f(j, k, y)$  of  $(J^0, J, Y)$  is given by

$$(2.2) \quad f(j, k, y) = \frac{v_j^0 v_k(y)}{\left[ \sum_{r=1}^m v_r(y) \right]^2}$$

provided  $v_k(y)$  is differentiable,  $k = j$  and  $y_j = y = y_k$ .

The full proof of Theorem 1 is given in Appendix A. As mentioned above, the proof given in Appendix A is a very simplified version of the one given by Dagsvik and Karström (2005).

For the sake of bringing out the essential idea we shall provide a brief outline of the argument of the proof: Consider the case with  $m = 3$ . Assume that  $y_2 = y_1$  and let  $y = [y_2, y_1]$ . We have that

$$P(J^0 = 1, J = 2, Y = [y, y] | U_1^0 = x)$$

$$\| P(\max(U_2^0, U_3^0) \| U_1^0, \max(U_1(Y), U_3(Y)) \| U_2(Y), Y \| (y, y \| y) | U_1^0 \| x)$$

Since  $y_2 \| y \| y_1$  and  $U_1^0 \| x$  it follows that  $U_1(y) \| U_1^0$  and  $U_2(y) \| U_2^0$ . Hence

$$\begin{aligned} & P(J^0 \| 1, J^0 \| 2, Y \| [y, y \| y]) \| P(\max(U_3^0, U_3(y)) \| x, U_2(y) \| x \| U_2(y \| y) | U_1^0 \| x) \\ & \| P(\max(U_3^0, U_3(y)) \| x) P(U_2(y) \| x \| U_2(y \| dy) | U_1^0 \| x) \\ & \| \exp(-\beta x / \lambda_3(y)) \exp(-\beta x v_2(y)) e^{\beta x} dv_2(y). \end{aligned}$$

Thus, one can readily obtain the formula for  $P(J^0 \| 1, J^0 \| 2, Y \| [y, y \| y])$  from

$$\begin{aligned} & E P(\max(U_2^0, U_3^0) \| U_1^0, \max(U_1(Y), U_3(Y)) \| U_2(Y), Y \| [y, y \| dy) | U_1^0) \\ & \| \frac{v_1^0 dv_2(y)}{[v_1^0 \| v_2(y) \| \lambda_3(y)]^2} \end{aligned}$$

which is the desired expression.

Similarly to Corollary 3 of Dagsvik and Karlström (2005) it follows that

### Corollary 1

The Hicksian (Compensated) choice probabilities are given by

$$(2.3) \quad P^H(j, k) \| \frac{v_j^0 \int_{y_k}^{y_j} dv_k(y)}{[v_j^0 \| \int_{r=1}^m \lambda_r(y)]^2}$$

when  $k \| j$  and  $y_j \| y_k$ . In the case where  $j$  and  $k$  are distinct and  $y_j \| y_k$  then  $P^H(j, k) \| 0$ . Furthermore, when  $j = k$  then

$$(2.4) \quad P^H(j, j) \| \frac{v_j^0}{\int_{r=1}^m \lambda_r(y_j)}$$

It may be instructive to consider a couple of special cases, such as the binary and ternary cases, which we shall do below.

### 2.1. The binary case

For the sake of providing more intuition to the formalism displayed above we shall discuss the binary and subsequently the ternary case in more detail. We shall now consider the binary case in more detail. From (2.2) it follows by straight forward integration that when  $y_1 \| y_2$



$$(2.5) \quad P^H(1,2) = \frac{\int_{y_1^0}^{y_1} \frac{v_2(y) dy}{v_1^0 + v_2(y)}}{\frac{v_1^0}{v_1^0 + v_2^0}} = \frac{v_1^0}{v_1^0 + v_2(y_1)}$$

According to (2.3)  $P^H(2,2)$  is given by

$$(2.6) \quad P^H(2,2) = \frac{v_2^0}{v_1^0 + v_2^0}$$

Consequently, it follows that

$$(2.7) \quad P_2^H = P^H(1,2) + P^H(2,2) = \frac{v_2(y_1)}{v_1^0 + v_2(y_1)}$$

in this case. The corresponding ex ante probability of choosing alternative 2, is given by

$$(2.8) \quad P_2 = \frac{v_2^0}{v_1^0 + v_2^0}$$

Thus, it follows that the compensated effect, when measured by the relative change  $(P_2^H - P_2)/P_2$ , is given by

$$(2.9) \quad \frac{P_2^H - P_2}{P_2} = \frac{\frac{v_2(y_1)}{v_1^0 + v_2(y_1)} - \frac{v_2^0}{v_1^0 + v_2^0}}{\frac{v_2^0}{v_1^0 + v_2^0}} = \frac{v_2(y_1) + v_2^0}{v_1^0 + v_2(y_1)} - 1 > 0,$$

when  $y_1 > y_2$ .

In the case where  $y_1 < y_2$ ,  $P^H(1,2) < 0$ , and  $P_2^H < P^H(2,2)$ , so that we now obtain

$$(2.10) \quad P_2^H = P^H(2,2) = \frac{v_2^0}{v_2^0 + v_1(y_2)}$$

Hence, in the case with  $y_1 < y_2$

$$(2.11) \quad \frac{P_2^H - P_2}{P_2} = \frac{\frac{v_1^0 + v_1(y_2)}{v_2^0 + v_1(y_2)} - \frac{v_1(y_1) + v_1(y_2)}{v_2^0 + v_1(y_2)}}{\frac{v_2^0 + v_1(y_2)}{v_2^0 + v_1(y_2)}} < 0.$$

Consider next the special case with only a reduction in the price (or cost) of alternative 2 whereas other attributes (or tax rules remain fixed). This implies that  $v_1(y) = v_1^0(y)$  in this case so that  $y_1 = y^0$ . Since  $v_2(y) = v_2^0(y)$  for any  $y$ , and  $y_2$  is determined by  $v_2(y_2) = v_2^0(y^0)$ , it must be the case that  $y_2 = y^0 = y_1$ . According to the analysis above, this means that the compensated (Slutsky) price elasticity can never be negative in this case, which is intuitive.

## 2.2. The ternary case

Consider finally the case with 3 alternatives, i.e.,  $m = 3$ . Then (2.3) reduces to

$$(2.12) \quad P^H(j,k) = \frac{\int_{y_k}^{y_j} dv_k(y)}{\int_{y_1}^{y_3} \frac{dv_r(y)}{v_r^0}},$$

for  $j, k = 1, 2, 3$ , and distinct  $j$  and  $k$ . Suppose for example that  $y_1 < y_3 < y_2$ . Then it follows that

$P^H(1,2) > 0$ ,  $P^H(1,3) > 0$ ,  $P^H(3,2) > 0$ , whereas  $P^H(2,3) < 0$ ,  $P^H(3,1) < 0$ ,  $P^H(2,1) < 0$ . We obtain that

$$(2.13) \quad P^H(3,2) = \frac{\int_{y_3}^{y_2} dv_2(y)}{\int_{y_1}^{y_3} \frac{dv_r(y)}{v_r^0}} \\ = \frac{\int_{y_3}^{y_2} \frac{dv_2(y)}{v_2^0}}{\int_{y_1}^{y_3} \frac{dv_r(y)}{v_r^0}} \\ = \frac{\int_{y_3}^{y_2} \frac{dv_2(y)}{v_2^0}}{\int_{y_1}^{y_3} \frac{dv_1(y)}{v_1^0} + \int_{y_1}^{y_2} \frac{dv_2(y)}{v_2^0} + \int_{y_2}^{y_3} \frac{dv_3(y)}{v_3^0}}.$$

However, for  $P^H(1,2)$  one can in general not express the integral on closed form. The ex post Hicksian probability of choosing alternative 2 equals

$$(2.14) \quad P_2^H = P^H(1,2) + P^H(3,2) + P^H(2,2),$$

and similarly for the other cases. Since  $P^H(3,1) < 0$ ,  $P^H(2,1) < 0$ , we get for alternative 1 that

$$(2.15) \quad P_1^H = P^H(1,1) + \frac{v_1^0}{\int_{y_1}^{y_3} \frac{dv_r(y)}{v_r^0}}.$$

## 2.3. Examples of compensated effects in discrete labor supply models

Consider now the case of a model for labor force participation (binary case,  $j=1,2$ ) and subsequently a model for labor force participation and choice of working in one out of two sectors (ternary case,  $j=1,2,3$ ).

Example 1: Labor force participation (the binary case)

Consider first the choice of whether or not to work. Let  $w$  be the agent's wage,  $I$  the agent's non-labor income and let full time hours of work be normalized to one. Let  $f(hw, I)$  denote income after tax, where  $h = 1$  if working and  $h = 0$ , otherwise, and let  $u(C, h)$  denote the mean utility of disposable income and hours of work  $(C, h)$ .  $U_1(y) = u(f(0, y), 0)$ , be the agent's utility of not working and  $U_2(y) = u(f(w, y), 1)$ , the utility of working, were the random error term  $\epsilon_j$ ,  $j = 1, 2$ , are supposed to account for unobserved heterogeneity in preferences across alternatives and agents. The agent will work if  $U_2(y) > U_1(y)$ . In this example it is assumed

that the agent has no problem with finding a job, and consequently the event of being employed will therefore be determined by the agent's preferences.

Consider now the effect of a change in economic incentives (such as a wage increase or a tax reform) that makes the working alternative more (or less) attractive. We assume that the random part of the utility function is unaffected by the change in taxation. Let  $w^0, f^0$  be the ex ante wage and tax system, and  $w, f$ , the corresponding wage and tax system ex post. Moreover, let  $Y$  be the non-labor income that makes the ex post indirect utility equal to the ex ante indirect utility. Write  $\log v_j^0(y^0) \equiv \log v_j^0(u(f^0((j+1)w^0, y^0), j+1))$ ,  $U_j^0 \equiv \log v_j^0(y^0) \equiv \log v_j^0(u(f^0((j+1)w^0, y^0), j+1))$ , and  $U_j(Y) \equiv \log v_j(u(f((j+1)w, Y), j+1))$ , for  $j = 1, 2$ . Thus  $U_j^0$  is the ex ante utility of being in labor market state  $j$  and  $U_j(Y)$  the corresponding ex post utility. The random parts,  $\epsilon_j, j = 1, 2$ , are assumed iid extreme value distributed across alternatives. The deterministic utilities may also depend on other alternative specific attributes. Let  $y_j$  be defined by  $v_j^0(y^0) \equiv v_j(y_j), j = 1, 2$ . The income  $y_j$  is a money metric measure of the initial deterministic utilities. The compensated probability of "working" ("not working") can now be analyzed as outlined about in section 2.1. In particular, it follows that the compensated wage elasticity (Slutsky) will never be non-positive.

#### Example 2: A two sector labor supply model (the ternary case)

Consider next the following two sector labor supply model. In this case there are 3 alternatives, "not working" (1), "working in the public sector" (2) and "working in the private sector" (3). Here, and for expository reasons, in each sector the only option is to work full time. As above let  $U_1$  be the utility of not working,  $U_2$  the utility of working in sector 2 and  $U_3$  the utility of working in sector 2. Assume that  $U_j(y) \equiv u_j(f(w_j, y), 1) \equiv \log v_j(y)$  for  $j = 2, 3$ , where  $w_j$  is the wage of sector  $j$ . Note that the function  $u_j$  may depend on  $j$  because the systematic part of the utility function may depend on attributes of the sectors other than wage. Let  $\log v_1(y) \equiv u(f(0, y), 0)$  and  $\log v_j(y) \equiv u_j(f(w_j, y), 1)$ , for  $j = 2, 3$ , and  $U_j(y) \equiv \log v_j(y) \equiv \log v_j(y)$ . Similar to the analysis in the previous example, let  $U_j^0 \equiv \log v_j^0(y^0) \equiv \log v_j^0(y^0)$  be the ex ante utility representation and  $U_j(Y) \equiv \log v_j(Y) \equiv \log v_j(Y)$  the corresponding ex post utility representation. The analysis now proceeds similarly to the ternary case discussed above.

### 3. Compensated choice in a labor supply model with latent choice sets of available jobs

In this section we consider compensated choice probabilities for the discrete labor supply models with latent choice constraint developed by Dagsvik and Strøm (2006), see Dagsvik et al. (2012). Here we only give a brief review of the model. For expository reasons only the case of a one-sector labor supply for married females is considered here. The two sector labor supply model is outlined in Appendix B. The wage income of the husband is assumed exogenously given. The household derives utility from household consumption, here set equal to household disposable income, leisure and non pecuniary latent attributes of jobs.

Let  $z = 1, 2, \dots$ , be an indexation of the jobs and let  $z = 0$  represent not working. The utility function is assumed to have the form  $U(C, h, z) = \log v(C, h, z)$ , for  $z \in B$  where  $B$  is the set of available jobs,  $(C, h)$  denotes disposable income and annual hours of work,  $v(\cdot)$  is a suitable positive deterministic function. The set  $B$  is individual specific and latent. The terms  $\{\tau(z)\}$  are positive sector-and job-specific random taste shifters. The taste shifter accounts for unobserved individual characteristics and unobserved job-specific attributes. These taste shifters  $\{\tau(z)\}$ , are assumed to be i.i.d. across jobs and agents, with c.d.f.  $\exp(-e^{-x})$ , for real  $x$ . The reason why the index  $z$  enters the utility function is that job-specific attributes beyond wage and hours of work may affect the utility of the agents. For given hours of work  $h$  and wage rate  $w$ , disposable household income is given by  $C = f(hw, I)$ , where  $f(\cdot)$  is a function that transforms pre-tax incomes into after-tax incomes,  $w$  is the woman's wage and  $I$  denotes non-labor income. It is convenient to decompose the set  $B$  as the union of the sets  $\{B(h)\}$ , where  $B(h)$  is the set of available jobs with hours of work equal to  $h$ . Let  $\mu$  be a measure that represents the number of available jobs and  $g(h)$  the fraction of jobs with hours of work that are feasible. Thus  $\mu g(h)$  is the number of jobs in the latent set  $B(h)$ . We call  $\mu g(h)$  the opportunity measure of the agent. Let  $\pi(h)$  be the probability (uncompensated) of choosing hours of work  $h$  (for a utility maximizing agent), and let  $D$  be the set of feasible hours, which here is assumed to be finite. From the assumptions above it follows readily that

$$(3.1) \quad \pi(h) = \frac{v(f(hw, I), h)g(h)}{v(f(0, I), 0) + \sum_{x \in D, x \neq h} v(f(xw, I), x)g(x)}$$

for  $h \in D$ , see Dagsvik and Strøm (2006) or Dagsvik et al. (2012). For  $h = 0$ ,  $\pi(0)$  is obtained from (3.1) by replacing the numerator by  $v(f(0, I), 0)$ .

Similarly to Section 2, consider a setting where a tax reform, a wage change or some other change is introduced. For example, the framework above allows for changes in latent choice constraints through the

opportunity measure  $\frac{1}{g(h)}$ . The corresponding compensated effects can now be calculated in a similar way as outlined in Section 2. To this end, let

$$V^0(h, y) = \max_{z \in B(h)} (\log v^0(f^0(hw^0, y), h) + \beta(z))$$

where the zero superscript denotes the initial ex ante situation. Thus  $V^0(h, y)$  is the ex ante utility of working  $h$  hours when non-labor income is equal to  $y$ . Similarly, the corresponding ex post utility of working  $h$  hours can be expressed as

$$V(h, y) = \max_{z \in B(h)} (\log v(f(hw, y), h) + \beta(z)).$$

Let  $y(h)$  be the real number that solves:

$$v^0(f^0(hw^0, I), h) / \frac{1}{g^0(h)} = v(f(hw, y(h)), h) / g(h)$$

when  $h$  is positive and

$$v^0(f^0(0, I), 0) = v(f(0, y(0)), 0)$$

when  $h = 0$ . Furthermore, let  $\bar{D} = D \cup \{0\}$ . Define the joint compensated probability

$$(3.2) \quad P^H(h, \bar{h}) = P(\max_{x \in \bar{D}} V^0(x, I) = V^0(h, I), V(h, Y) = \max_{x \in \bar{D}} V(x, Y), \max_{x \in \bar{D}} V^0(x, I) = \max_{x \in \bar{D}} V(x, Y)).$$

The probability defined in (3.2) is entirely similar to the corresponding one defined in section 2. It is the probability that the ex ante labor supply is equal to  $h$  and the ex post supply is equal to  $\bar{h}$ , when the ex ante and ex post utility levels are equal. It follows from Corollary 1 that

$$(3.3) \quad P^H(h, \bar{h}) = \mathbb{1}\{y(h) = y(\bar{h})\} \int_{y(\bar{h})}^{y(h)} \frac{\frac{1}{g^0(h)} v^0(f^0(hw^0, I), h) v(f(\bar{h}w, dy), \bar{h})}{K(y)^2},$$

for  $h \in \bar{h}$  and  $h > 0$ ,  $\bar{h} \geq 0$ , where the indicator notation  $\mathbb{1}\{y = x\}$  means that  $\mathbb{1}\{y = x\}$  if  $y = x$  and zero otherwise and

$$(3.4) \quad K(y) = \max(v^0(f^0(0, I), 0), v(f(0, y), 0)) + \int_{x \in \bar{D}} \max[\frac{1}{g^0(x)} v^0(f^0(xw^0, I), h), \frac{1}{g(x)} v(f(xw, y), x)].$$

Note that (3.3) allows for changes in the opportunity measure, where  $\frac{1}{g^0(h)}$  and  $\frac{1}{g(h)}$  denote the ex ante and ex post opportunity measures, respectively. Furthermore, we get from Corollary 1 that

$$(3.5) \quad P^H(h, h) = \frac{\frac{1}{g^0(h)} v^0(f^0(hw^0, I), h)}{K(y(h))},$$

for  $h > 0$ . For  $h = 0$ ,  $\bar{h} \geq 0$  it follows that

$$(3.6) \quad P^H(0, \bar{h}) = \mathbb{1}\{y(0) = y(\bar{h})\} \int_{y(\bar{h})}^{y(0)} \frac{\frac{1}{g(\bar{h})} v^0(f^0(0, I), 0) v(f(\bar{h}w, dy), \bar{h})}{K(y)^2},$$



and similarly for  $h \leq 0, \bar{h} \leq 0$ . For  $h \leq \bar{h} \leq 0$ , we have that

$$(3.7) \quad P^H(0,0) = \frac{v^0(f^0(0,1),0)}{K(y(0))}.$$

Let  $\rho^0(h)$  denote the ex ante choice probability, which is obtained from (3.1) by inserting the initial tax system, wage and opportunity distribution. The marginal ex post compensated choice probability,  $\rho^H(h)$  is given by

$$(3.8) \quad \rho^H(h) = \sum_{x \in D} P(x,h).$$

Hence, the compensated relative change in the probability of choosing  $h$  hours of work induced by the reform equals

$$(3.9) \quad \frac{\rho^H(h) - \rho^0(h)}{\rho^0(h)}.$$

The corresponding compensated change in the mean labor supply is given by

$$(3.10) \quad \frac{\sum_{x \in D} x \rho^H(x) - \sum_{x \in D} x \rho^0(x)}{\sum_{x \in D} x \rho^0(x)}.$$

## 4. Elasticities

Here, we report numerical results based on the two sector discrete labor supply model developed by Dagsvik and Strøm (2006). In Appendix B we give a summary of the model with estimates. Summary statistics, tax functions and estimates are given in Appendix C.

### 4.1. Selected cases

We have calculated compensated wage elasticities and uncompensated wage and income elasticities for 48 different cases. We have chosen three wage levels (NOK 1994), which is low (NOK 70 per hour), high (NOK 200 per hour) and super-high (NOK 300 per hour). Non-labor incomes (which includes the income of the husband) are NOK 50 000 (low), NOK 100 000 (lower than average) and NOK 200 000 (above average). Household characteristics are no children or 2, and the age of the woman is either 30 or 40. The results below clearly show the extent of heterogeneity in the labor supply responses. Further detailed results can be found in Appendix D. The levels of participation and expected hours of work prior to the wage increase are given in Appendix E. The tables in Appendix E show that participation and hours worked, given the wage decline with non-labor income, number of children and age. Hours worked increase with the wage level.

The elasticities vary to a great extent. The highest uncompensated elasticity of conditional expected hours with respect to the wage is above 4 (public sector) and 2.5 (the private sector) times higher than the lowest elasticity (Table 1). For the uncompensated elasticity of unconditional hours the ratios are around 8.5 (public sector) and 6 (private sector). The differences between the highest and lowest compensated elasticities are

smaller (Table 2) than for the uncompensated. Table 3 shows that for the non-labor income elasticities the difference between the lowest and the highest elasticities is sizeable.

**Table 1. Uncompensated wage elasticities**

High: Woman aged 40, two children, non-labor income NOK 200 000, wage level NOK 70

Low: Woman aged 30, no children, non-labor income NOK 50 000, wage level NOK 70

Elasticity	Probability of working			Conditional mean hours			Unconditional mean hours		
	All	Public	Private	All	Public	Private	All	Public	Private
High	0.620	0.670	0.465	0.649	0.660	0.599	1.309	1.374	1.093
Low	0.000	0.007	-0.043	0.163	0.152	0.222	0.163	0.160	0.178

**Table 2. Compensated wage elasticities**

High: Woman aged 40, two children, non-labor income NOK 50 000, wage level NOK 300

Low: Woman aged 30, no children, non-labor income NOK 50 000, wage level NOK 70

Elasticity	Probability of working			Conditional mean hours			Unconditional mean hours		
	All	Public	Private	All	Public	Private	All	Public	Private
High	0.419	0.331	0.765	0.267	0.259	0.292	0.687	0.590	1.057
Low	0.131	0.135	0.119	0.186	0.174	0.254	0.318	0.310	0.368

**Table 3. Non-labor income elasticities**

High: Woman aged 30, no children, non-labor income NOK 200 000, wage level NOK 200

Low: Woman aged 30, two children, non-labor income NOK 100 000, wage level NOK 70

Elasticity	Probability of working			Conditional mean hours			Unconditional mean hours		
	All	Public	Private	All	Public	Private	All	Public	Private
High	0.000	0.000	0.000	-0.010	-0.010	-0.013	-0.010	-0.010	-0.013
Low	-0.126	-0.137	-0.075	-0.149	-0.148	-0.146	-0.273	-0.284	-0.221

The uncompensated wage elasticities are higher at the intensive margin than at the extensive margin, with only two exceptions (out of 48 cases). The compensated wage elasticities are more equal at the extensive and the intensive margin, but with a weak tendency to be higher at the extensive margin. The non-labor income elasticities tend to be lowest (more negative) at the intensive margin.

The elasticities tend to decline with the wage, in particular for women with two children. For example, for a woman aged 30 with two children and non-labor income NOK 200 000, the uncompensated elasticity of conditional expected hours (all sectors) declines from 0.634 (wage level NOK 70) to 0.244 (wage level NOK 300) and the uncompensated elasticity of unconditional expected hours from 1.177 to 0.344. For the same woman and for the same wage levels, the decline in compensated elasticity is smaller: The compensated elasticity of conditional expected hours declines from 0.410 to 0.266, and the compensated elasticity of unconditional expected hours drops from 1.093 to 0.635.



The participation elasticities are higher related to working in the private sector than in the public sector. The elasticities of conditional expected hours tend to be higher when the wage is low compared to when the wage is high. The reason is that when the wage is high the woman works long hours and hence the impact of a higher wage is lower compared to the case where the wage rate is low and initial hours are lower. This is particular the case given that the woman works in the private sector, where hours are less constrained.

If the woman has children the elasticities of conditional expected hours is higher compared to a childless woman. The reason is that the presence of small children reduces hours of work and hence the impact of wage increase becomes stronger compared to a case for a childless women who works initially longer hours.

Recall that in the traditional deterministic micro economic models the Slutsky equation will hold. That is, the difference between the uncompensated and the compensated wage elasticities equal the non-labor income elasticity multiplied by the ratio of non- labor income to hours of work times the marginal wage. In random utility models, however, the Slutsky equation does not necessarily hold. Table 4 below illustrates this feature. In particular, when non-labor income is equal to NOK 200,000 the Slutsky relation is very far from being met. See Appendix D for more detailed results.

**Table 4. Comparison between uncompensated, compensated and non-labor income elasticities.**

Woman aged 40, two children, wage level NOK 70.

Elasticity	Non-labor income NOK 50 000			Non-labor income NOK 200 000		
	Conditional mean hours			Conditional mean hours		
	All	Public	Private	All	Public	Private
Uncompensated	0.508	0.500	0.521	0.649	0.660	0.599
Compensated	0.566	0.557	0.571	0.412	0.376	0.487
Non-labor Income	-0.145	-0.143	-0.146	-0.116	-0.118	-0.108

## 4.2. Numerical results using sample values

This Section contains numerical results for compensated wage elasticities for the two-sector model using sample values. This allows us to show how the compensated elasticities vary across deciles of the (endogenous) income distribution.

The income deciles limits are calculated from the expected household disposable income, using the probabilities (in B.1, Appendix B). For each possible hour we have calculated the mean of the probabilities based on 50x50 draws from the normal distribution to account for the fact that the wage equations contain random terms. Summing over all possible hours and sectors, including not working, we are then able to identify

the expected income, inclusive of the exogenous incomes, and hence the income decile limits. Within each decile we then use (eqs. B.4- B.13) to calculate the relative change in compensated probabilities for each hours and sector and finally we calculate weighted average over hours, given the sector, using the compensated probabilities as weights. Again, we have to make draws from the distribution of the error terms in the wage equation since these enter in (eqs. B.4-B.13). In order to calculate the elasticity of working, we take the weighted average over all deciles and sectors. To calculate the elasticity of working in a specific sector we do an equivalent calculation.

In Tables 5-7 we give the uncompensated and the compensated elasticities related to an overall wage increase. Table 5 gives the elasticities of working, and working in the two sectors, Table 6 gives the elasticities of conditional expected hours, conditional on working and working in specific sectors, while Table 7 gives the elasticities of the unconditional expected hours (which is the sum of the two above).

**Table 5. Compensated wage elasticities of the probability of working, and of working in the public or private sector, across deciles in the household income distribution**

Sector	Uncompensated			Compensated		
	1 <sup>st</sup> decile	2-9 <sup>th</sup> decile	10 <sup>th</sup> decile	1 <sup>st</sup> decile	2-9 <sup>th</sup> decile	10 <sup>th</sup> decile
All sectors	0.2360	0.2805	0.2623	0.4170	0.4650	0.4752
Public	0.0790	0.0854	0.0811	0.2840	0.2670	0.2495
Private	0.3660	0.4880	0.4383	0.5232	0.6609	0.8180

**Table 6. Compensated wage elasticities of conditional expected hours in the public or private sector, across deciles in the household income distribution**

Sector	Uncompensated			Compensated		
	1 <sup>st</sup> decile	2-9 <sup>th</sup> decile	10 <sup>th</sup> decile	1 <sup>st</sup> decile	2-9 <sup>th</sup> decile	10 <sup>th</sup> decile
All sectors	0.3523	0.3529	0.3511	0.3069	0.2963	0.3256
Public	0.3700	0.3645	0.3655	0.3156	0.3052	0.324
Private	0.3373	0.3354	0.3337	0.3021	0.2903	0.3409

**Table 7. Compensated wage elasticities of unconditional expected hours in the public or private sector, across deciles in the household income distribution**

Sector	Uncompensated			Compensated		
	1 <sup>st</sup> decile	2-9 <sup>th</sup> decile	10 <sup>th</sup> decile	1 <sup>st</sup> decile	2-9 <sup>th</sup> decile	10 <sup>th</sup> decile
All sectors	0.5966	0.6433	0.6226	0.7238	0.7613	0.8009
Public	0.4519	0.4531	0.4495	0.6005	0.5772	0.5734
Private	0.7156	0.8399	0.7866	0.8253	0.9512	1.1590

A striking result is that both type of elasticities of conditional expected hours seem to be nearly the same across deciles and sectors. For the private sector the compensated elasticities at the extensive margin tend to be higher than at the intensive margin. This is in accordance with the results reported in the previous section. The elasticities indicate that a utility constant overall wage increase shift labor into the private sector and towards longer hours. Another string result is that the uncompensated elasticities of conditional hours exceed the compensated elasticities in both sectors and all deciles, except for the 10<sup>th</sup> decile in the private sector. Thus, for the mean elasticities in the sample the Slutsky equation does not hold.

## 5. Conclusion

We have demonstrated how Slutsky elasticities can be calculated in random utility models (RUM) and we also show numerical results based on a model estimated on Norwegian female labor supply data from 1994. We have compared the results when using the whole model with the results when only the deterministic part of the utility function is used. The elasticities of conditional expected hours, conditional on working, are around 1/3 in the RUM compared to in the case with deterministic preferences. Moreover, the individuals are assumed to choose between working or not, working in the public and private sector, and to work different hours of work. Thus, a constant utility wage increase may give rise to different substitutions.

The results in Section 4 and in Appendix D clearly indicate that the uncompensated and the compensated wage elasticities vary considerably between agents. Thus, heterogeneity seems to be an important issue in labor supply. The uncompensated wage elasticities is higher at the intensive margin than at the extensive margin, while the opposite tend to be the case for compensated elasticities. Both elasticities tend to decline with the wage level.

In standard microeconomic models with deterministic preferences the Slutsky equation implies that if the non-labor income elasticity is negative, then the compensated wage elasticity is higher than the uncompensated. With random utility models the Slutsky equation does not exist and we demonstrate empirically that in a many cases a negative non-labor elasticity does not imply that the compensated wage elasticity is the highest.

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## Appendix A

For convenience we first prove the following elementary Lemma.

### Lemma 1

$$\int_0^{\infty} \exp(-e^{-x}(a+b))e^{-bx}dx = \frac{b}{(a+b)^2}.$$

### Proof of Lemma 1:

Integration by parts yields

$$\begin{aligned} & \int_0^{\infty} \exp(-e^{-x}(a+b))e^{-bx}dx = \frac{b}{a+b} \int_0^{\infty} e^{-bx} \exp(-e^{-x}(a+b)) - \frac{b}{a+b} \int_0^{\infty} \exp(-e^{-x}(a+b))e^{-bx}dx \\ & = \frac{b}{a+b} \int_0^{\infty} \exp(-e^{-x}(a+b))e^{-bx}dx = \frac{b}{(a+b)^2}. \end{aligned}$$

### Proof of Theorem 1:

We shall first derive the proof of  $P^H(1,2)$  for the case with  $m=3$ . Assume that  $y_2 \geq y_1$  and let  $y \in [y_2, y_1]$ . We have that

$$\begin{aligned} & P(J^0 \in \{1, J \in \{2, Y \in [y, y_1 \cup y]\}) \\ & = P(\max(U_2^0, U_3^0) \leq U_1^0, \max(U_1(Y), U_3(Y)) \leq U_2(Y), Y \in [y, y_1 \cup y] | U_1^0 \leq x) \\ & = P(\max(U_2^0, U_3^0) \leq x, \max(U_1(Y), U_3(Y)) \leq U_2(Y), Y \in [y, y_1 \cup y] | U_1^0 \leq x) \\ & = P(\max(U_2^0, U_3^0) \leq x, \max(U_1(y), U_3(y)) \leq x, U_2(y) \leq x | U_2(y \in [y, y_1 \cup y]) | U_1^0 \leq x). \end{aligned}$$

Note that since  $y_2 \geq y \geq y_1$  and  $U_1^0 \leq x$  it follows that  $U_1(y) \leq U_1^0$  and  $U_2(y) \leq U_2^0$ . Hence

$$\begin{aligned} & P(J^0 \in \{1, J \in \{2, Y \in [y, y_1 \cup y]\}) = P(\max(U_3^0, U_3(y)) \leq x, U_2(y) \leq x | U_2(y \in [y, y_1 \cup y]) | U_1^0 \leq x) \\ & = P(\max(U_3^0, U_3(y)) \leq x) P(U_2(y) \leq x | U_2(y \in [y, y_1 \cup y]) | U_1^0 \leq x) \\ & = P(v_3(y) \leq v_3 | x) P(v_2(y) \leq v_2 | x | v_2(y \in [y, y_1 \cup y]) \leq v_2) \\ & = \exp(-e^{-x}/3(y)) (\exp(-e^{-x}v_2(y)) \exp(-e^{-x}v_2(y \in [y, y_1 \cup y]))) \\ & = \exp(-e^{-x}/3(y)) \exp(-e^{-x}v_2(y)) e^{-x}(v_2(y \in [y, y_1 \cup y]) - v_2(y)) = o(1/y). \end{aligned}$$



The last expression above implies that

$$(A.1) \quad P(\max(U_2^0, U_3^0) \mid \mathbf{x}, \max(U_1(Y), U_3(Y)) \mid U_2(Y), Y \mid dy) \mid U_1^0 \mid \mathbf{x}) \\ \mid \exp(\mid e^{\square x} / \int_3(y)) \exp(\mid e^{\square x} v_2(y)) e^{\square x} dv_2(y).$$

Thus, using (A.1) and Lemma 1 we obtain that

$$(A.2) \quad P(J^0 \mid \mid 1, J \mid \mid 2, Y \mid \mid [y, y \mid \mid y]) \mid \mid EP(\max(U_2^0, U_3^0) \mid U_1^0, \max(U_1(Y), U_3(Y)) \mid U_2(Y), Y \mid dy) \mid U_1^0) \\ dv_2(y) \mid \mid \exp(\mid e^{\square x} / \int_3(y)) \exp(\mid e^{\square x} v_2(y)) e^{\square x} \exp(\mid e^{\square x} v_1^0) e^{\square x} v_1^0 dx \\ \mid \mid \frac{v_1^0 dv_2(y)}{[v_1^0 \mid \mid v_2(y) \mid \mid / \int_3(y)]^2}$$

from which Theorem 1 follows. In the general case with more than 3 alternatives we have that

$$P(J^0 \mid \mid 1, J \mid \mid 2, Y \mid \mid [y, y \mid \mid y]) \\ \mid \mid P(\max_{r \in \square} U_r^0 \mid \mid \mathbf{x}, \max_{r \in \square} U_r(Y) \mid U_2(Y), Y \mid \mid (y, y \mid \mid y) \mid U_1^0 \mid \mid \mathbf{x}) \\ \mid \mid P(\max_{r \in \square} U_r^0 \mid \mid \mathbf{x}, \max_{r \in \square} U_r(y) \mid \mid \mathbf{x}, U_2(y) \mid \mid \mathbf{x} \mid U_2(y \mid \mid y) \mid U_1^0 \mid \mid \mathbf{x}). \\ \mid \mid P(\max_{r \in \{1,2\}} (U_r^0, U_r(y)) \mid \mid \mathbf{x}, U_2(y) \mid \mid \mathbf{x} \mid U_2(y \mid \mid y) \mid U_1^0 \mid \mid \mathbf{x}) \\ \mid \mid P(\max_{r \in \{1,2\}} (\log / /_r(y) \mid \mid / /_r) \mid \mid \mathbf{x}, U_2(y) \mid \mid \mathbf{x} \mid U_2(y \mid \mid y) \mid U_1^0 \mid \mid \mathbf{x}) \\ \mid \mid P(\max_{r \in \{1,2\}} (\log / /_r(y) \mid \mid / /_r) \mid \mid \mathbf{x}) P(U_2(y) \mid \mid \mathbf{x} \mid U_2(y \mid \mid y) \mid U_1^0 \mid \mid \mathbf{x}).$$

The rest of the proof in the general case is entirely similar to the ternary case.

Consider finally Corollary 1. The result for  $P^H(j, k)$  for distinct  $j$  and  $k$  follows immediately from Theorem 1. Consider  $P^H(j, j)$ . For this to happen it must be the case that  $U_j^0 \mid U_j(Y)$  which implies that  $Y \mid y_j$ . Furthermore, one must have that  $U_j^0 \mid \max_{r \in \square} (U_r^0, U_r(y_j)) \mid \max_{r \in \square} (\log / /_r(y_j) \mid / /_r)$ . The corresponding probability is equal to

$$(A.3) \quad P^H(j, j) \mid \mid \frac{v_j^0}{v_j^0 \mid \mid \square_{r \in \square, r \neq j}^m / /_r(y_j)} \mid \mid \frac{v_j^0}{\square_{r \in \square}^m / /_r(y_j)}.$$

The last equality follows from the fact that  $v_j^0 \mid v_j(y_j)$ .

Q.E.D.

## Appendix B. The two sector discrete labor supply model

This section outlines the two sector job choice model for married females. Here it is assumed that the female takes her husband's income as given. Let  $w_k$  denote the wage the female receives when working in sector  $k$ ,  $k = 1, 2$ , and let  $w = (w_1, w_2)$ . The budget constraint when working in sector  $k$ ,  $k = 1, 2$ , is given by

$$C_k = f(hw_k, I) + hw_k + T(hw_k) + I,$$

where  $T$  is the tax function,  $h$  is hours of work and  $I$  is the sum of three income components. These three incomes are the after tax wage income of the husband, the capital income (taxed at 28 per cent) of the household and child allowances, which vary with the number of children up to the age of 18. Child allowances are not taxed. All details of the tax structure  $T(\cdot)$  are taken into account in the estimation and simulation of the model.

Let  $\pi_k(h)$  be the uncompensated probability of choosing a job in sector  $k$  with hours of work  $h$  (for an utility maximizing agent), and let  $D$  be the set of feasible hours (assumed to be the same across sectors). Similarly to Section 3, it is demonstrated by Dagsvik and Strøm (2006) that

$$(B.1) \quad \pi_k(h) = \frac{v(f(hw_k, I) + hw_k) g_k(h) / \pi_k}{\sum_{r=1}^2 \sum_{x=0, x \in D} v(f(xw_r, I) + x) g_r(x) / \pi_r}$$

for  $h \geq 0$ ,  $k = 1, 2$ , where  $g_k(h)$  denotes the fraction of jobs with hours of work that are available in sector  $k$  (available to the agent). The term  $\pi_k$  is a measure of the total amount of jobs available to the female in sector  $k$ ,  $k = 1, 2$ . For  $h = 0$ ,  $\pi_k(0)$  is obtained from (B.1) by replacing the numerator by  $v(f(0, y) + 0)$ .

The deterministic part of the utility function is specified as a Box-Cox transformation of consumption and leisure:

$$(B.2) \quad \log v(C, h) = \frac{1}{2} \left( \frac{10^{\alpha_1} (C - C_0)^{\alpha_1} + 1}{\alpha_1} \right)^{\beta_1} + \frac{1}{9} \left( \frac{10^{\alpha_2} (C - C_0)^{\alpha_2} + 1}{\alpha_2} \right)^{\beta_2} + \frac{1}{3} \left( \frac{(1 - h/3640)^{\alpha_3} + 1}{\alpha_3} \right)^{\beta_3} + \left( \frac{1}{4} + \frac{1}{5} \log A + \frac{1}{6} (\log A)^2 + \frac{1}{7} X_{0,6} + \frac{1}{8} X_{7,17} \right) \left( \frac{(1 - h/3640)^{\alpha_3} + 1}{\alpha_3} \right)^{\beta_3}$$

Here  $C_0$  is minimum consumption;  $A$  is age and  $X_{0,6}$ ,  $X_{7,17}$  are the number of children below 6 and between 7 and 17, respectively. The  $\alpha$ -s are unknown coefficients. If  $\alpha_1$  and  $\alpha_3$  are below 1, the deterministic part of the utility function is strictly concave. In order to make the paper self-contained, information about data, tax functions and estimates is appended (see Tables C.1 ÷ C.4).

Consider now the calculation of the compensated choice probabilities in the context of a reform of the tax system or a change in the wages. Here we shall assume that the opportunity measures  $\pi_k g_k(h)$  remain



unaffected by the reform. Let  $f^0$  and  $f$  represent the initial and ex post tax system and  $w_k^0$  and  $w_k$  the initial and ex post wage in sector  $k$ . Let  $y_k(h)$  be defined by  $v(f^0(hw_k^0, I), h) = v(f(hw, y_k(h)), h)$ , for positive  $h$  and  $k$ , and let  $y_0(0) = y(0)$ , for  $h = k = 0$ . The function  $y_k(h)$  is the ex post non-labor income that makes the ex ante deterministic part of utility equal to the corresponding ex post part. Since the function  $v$  and the opportunity measures are unaffected by the reform it follows that the equation above is equivalent to

$$(B.3) \quad f^0(hw_k^0, I) = f(hw_k, y_k(h)).$$

Similarly to the definition in (3.2) let  $P^H(j, h, k, \bar{h})$  denote the joint probability of being in sector  $j$  working  $h$  hours ex ante and in sector  $k$  working  $\bar{h}$  hours, given that the ex ante and ex post maximal utilities are the same. Here  $j, k = 0, 1, 2$ , where by sector 0 we understand the alternative not working. As in (3.3) it follows from Dagsvik and Karlström (2005) that

$$(B.4) \quad P^H(j, h, k, \bar{h}) = \frac{1\{y_j(h) = y_k(\bar{h})\} \int_{y_k(\bar{h})}^{y_j(h)} g_j(h) g_k(\bar{h}) / j / k v(f^0(hw_j^0, I), h) v(f(hw_k, dy), \bar{h})}{K(y)^2},$$

which is valid for positive hours and  $k > 0, j > 0, (j, h) \neq (k, \bar{h})$ . Due to (A.2) it follows that

$$(B.5) \quad K(y) = \max(v(f^0(0, I), 0), v(f(0, y), 0)) \\ + \sum_{r=1}^2 \sum_{h \in D} \max(g_r(h) / r v(f^0(hw_r^0, I), h), g_r(h) / r v(f(hw_r, y), h))$$

and

$$(B.6) \quad v(f(hw_k, dy), h) = \int_{\frac{1}{2}}^{\frac{1}{9}} \int_{\frac{1}{9}}^{\frac{1}{3}} \frac{(1 - h/3640)^{-3}}{\sqrt{3}} \int_{\frac{1}{9}}^{\frac{1}{3}} 10^{-3h} (f(hw_k, y) - C_0)^{-3} v(f(hw_k, y), h) dy.$$

The cases where one or two alternatives are “not working”, are given by:

$$(B.7) \quad P^H(j, h, 0, 0) = \frac{1\{y_j(h) = y(0)\} \int_{y(0)}^{y_j(h)} g_j(h^0) / j v(f^0(hw_k^0, I), h) v(f(0, dy), 0)}{K(y)^2},$$

$$(B.8) \quad P^H(0, 0, k, \bar{h}) = \frac{1\{y(0) = y_k(\bar{h})\} \int_{y_k(\bar{h})}^{y(0)} g_k(\bar{h}^0) / k v(f^0(0, I), 0) v(f(hw_k, dy))}{K(y)^2},$$

$$(B.9) \quad P^H(j, h, j, h) = \frac{g_j(h) / j v(f^0(hw_k^0, I), h)}{K(y_j(h))},$$

for positive  $h$  and  $j$ , and

$$(B.10) \quad P^H(0, 0, 0, 0) = \frac{v(f^0(0, I), 0)}{K(y(0))}$$

for  $j = h = 0$ .

To calculate the compensated elasticity, we have to sum the transitions probabilities from one initial state to all possible states. Let  $\pi_k^H(h)$  be the probability that the agent chooses (k,h) ex post, given that utility is the same as before the reform. Then for  $h > 0$ ,

$$(B.11) \quad \pi_k^H(h) = \sum_{j \in k} \sum_{x \in D \setminus \{h\}} P^H(j, x, k, h) + \sum_{x \in D \setminus \{h\}} P^H(k, x, k, h) + \sum_{j \in k} P^H(j, h, k, h) + P^H(k, h, k, h),$$

for  $h, k > 0$ , and

$$(B.12) \quad \pi^H(0) = \sum_{x \in D} \sum_{j \in \emptyset} P^H(j, x, 0, 0) + P^H(0, 0, 0, 0).$$

The compensated and the relative change, for all (h, k) is given by

$$(B.13) \quad \pi_k^H(h) \pi_k^0(h) \quad \text{and} \quad \frac{\pi_k^H(h) / \pi_k^0(h)}{\pi_k^0(h)}.$$

## Appendix C. Data, tax functions and estimates

Data on the labor supply of married women in Norway used in this study consist of a merged sample of the “Survey of Income and Wealth, 1994” and the “Level of living conditions, 1995” (Statistics Norway, 1994 and 1995, respectively). Data cover married couples as well as cohabiting couples with common children. The ages of the spouses range from 25 to 64. None of the spouses is self-employed and none of them is on disability or other type of benefits. A person is classified as a wageworker if their income from wage work is higher than their income from self-employment. All taxes paid are observed and in the assessment of disposable income, at hours not observed, all details of the tax system are accounted for. Hours of work are calculated as the sum of hours of the main job as well as those of any side jobs. A large majority of the women have only one job.

Wage rates above NOK 350 or below NOK 40<sup>5</sup> are not utilized when estimating the wage equations. The wage rates are computed as the ratio of annual wage income to hours worked. When computing annual wage income, we take into account the fact that some women have multiple jobs. The size of the sample used in estimating the labor supply model is 810. Descriptions of variables and summary statistics are given in Table C.1.

**Table C.1. Descriptive statistics, number of observations = 810 (values in NOK, 1994)**

Variable	Mean	Std. Dev.	Min	Max
Number of children (age 0-2)	0.23	0.45	0.00	2.00
Number of children (age 0-6)	0.54	0.77	0.00	3.00
Number of children (age 3-6)	0.30	0.56	0.00	3.00

<sup>5</sup> As of June 2013, 1 USD = NOK 5,80

Number of children (age 7-17)	0.66	0.85	0.00	4.00
Age in year (men)	42.80	9.17	25.00	66.00
Education in year (men)	12.05	2.49	9.00	19.00
Age in year (women)	40.07	9.04	25.00	64.00
Education in year (women)	11.61	2.15	9.00	17.00
Sector (1=Public, 2=Private)	1.34	0.61	0.00	2.00
Work experience (woman age – woman education in years)	22.45	9.63	2.00	49.00
Capital income (child allowances included)	32306.71	42378.48	0.00	568403.00
Child allowances	13094.37	12154.01	0.00	60084.00
Women wage income per year	149751.97	83060.53	0.00	581693.00
Men wage income per year	274372.89	106239.67	17312.00	1184861.00
Woman hourly wage in public sector	89.36	12.09	64.88	132.34
Woman hourly wage in private sector	109.77	13.68	80.14	156.44

**Table C.2. Tax function in 1994 for a married nonworking woman whose husband is working, 1994.**

Male income, $Y_{\text{male}}$	Tax T
0–41907	0
41907–140500	$0.302Y_{\text{male}} - 12656$
140500–252000	$0.358Y_{\text{male}} - 20524$
252000–263000	$0.453Y_{\text{male}} - 44464$
263000–	$0.495Y_{\text{male}} - 55510$

**Table C. 3. Tax function in 1994 for a married working woman or man, NOK 1994**

Wage income, Y	Tax T
0–20954	0
20954–140500	$0.302Y - 6328$
140500–208000	$0.358Y - 14196$
208000–236500	$0.453Y - 33956$
236500–	$0.495Y - 43889$

**Table C.4. Estimates**

Variables	Parameters	Estimates	t-values
<b>Preferences:</b>			
Consumption:			
Exponent	$\beta_1$	0.64	7.6
Scale $10^4$	$\beta_2$	1.77	4.2
Subsistence level $C_0$ in NOK per year	$\beta_3$	60 000	
Leisure:			
Exponent	$\beta_3$	-0.53	-2.1
Constant	$\beta_4$	111.66	3.2
Log age	$\beta_5$	-63.61	-3.2
$(\log \text{age})^2$	$\beta_6$	9.2	3.3
# children 0-6	$\beta_7$	1.27	4.0
# children 7-17	$\beta_8$	0.97	4.1
Consumption and Leisure, interaction	$\beta_9$	-0.12	-2.7
Subsistence level of leisure in hours per year	$\beta_{10}$	5120	
<b>The parameters <math>\beta_1</math> and <math>\beta_2</math>; <math>\log \beta_j = \mathbf{f}_{j1} + \mathbf{f}_{j2}\mathbf{S}</math></b>			
Constant, public sector (sector 1)	$\mathbf{f}_{11}$	-4.2	-4.7
Constant, private sector (sector 2)	$\mathbf{f}_{21}$	1.14	1.0
Education, public sector (sector 1)	$\mathbf{f}_{12}$	0.22	2.9
Education, private sector (sector 2)	$\mathbf{f}_{22}$	-0.34	-3.3
<b>Opportunity density of offered hours, <math>g_{k2}(\mathbf{h})</math>, <math>k=1,2</math></b>			
Full-time peak, public sector (sector 1)*	$\log(g_{12}(h_{\text{Full}}) / g_{12}(h_0))$	1.58	11.8
Full-time peak, private sector (sector 2)	$\log(g_{22}(h_{\text{Full}}) / g_{22}(h_0))$	1.06	7.4
Part-time peak, public Sector	$\log(g_{12}(h_{\text{Par}}) / g_{12}(h_0))$	0.68	4.4
Part-time peak, private Sector	$\log(g_{22}(h_{\text{Par}}) / g_{22}(h_0))$	0.8	5.2
# observations		824	
<b>Log likelihood</b>		-1760.9	

\* The notation  $h_0$  refers to an arbitrary level of hours of work different from full-time and part-time hours.

To check the concavity of the deterministic part of the utility function we let  $u(c,h)=\log v(c,h)$ , where  $c=10^4(C-C_0)$ .

$$\text{Let } B = \beta_4 + \beta_5 \log A + \beta_6 (\log A)^2 + \beta_7 X_{0,6} + \beta_8 X_{7,17}.$$

The age of a female in the sample ranges from 25 to 64. With no children  $B > 0$ , and with children  $B$  becomes more positive.

Let

$$\frac{\partial u(c,h)}{\partial c} = u_c$$

and let

$$f(h) = \frac{(1 + h/3640)^{\beta_3} - 1}{\beta_3}.$$

Because  $\beta_3$  is estimated to be negative,  $f(h)$  is increasing in  $h$ . The highest feasible hours is 2600, and given the estimates  $f(2600)=1.98$ , thus  $f(h)$  is positive for all feasible  $h$ .

The marginal utility of consumption is then  $u_c = c^{\beta_1 - 1} f(h) > 0$  for all  $h$ .

The second derivative denoted  $u_{cc}$  is given by

$$u_{cc} = (\beta_1 - 1) c^{\beta_1 - 2} f(h) < 0, \text{ because } \beta_1 < 0.$$

Let the cross second derivatives be denoted  $u_{ch} = u_{hc}$ .

$$\text{We then have } u_{ch} = \frac{\beta_3}{3640} \left(1 + \frac{h}{3640}\right)^{\beta_3 - 1} c^{\beta_1 - 1} < 0$$

Let  $u_h$  be the partial derivative of  $u$  with respect to  $h$ .

Then,

$$u_h = \frac{(1 + h/3640)^{\beta_3 - 1}}{3640} \left( \beta_3 \frac{c^{\beta_1 - 1}}{\beta_1} + B \right).$$

Given the estimates and given the smallest value of  $B$  (for a woman aged 31 and with no children):  $u_h$  is negative for  $C < \text{NOK } 432\,000$ , which in 1994 was a rather high disposable income. Thus for higher consumption levels the negative interaction term implies that utility is increasing in hours. For the highest possible value of  $B$  (woman aged 49 with 3 children below 6, and 4 children above 6),  $u_h$  is negative for  $C < \text{NOK } 689\,524$ . This is a very high disposable income. In the sample, the marginal utility of hours is negative. This is also the case for the cases shown in Appendix D.

The second derivate, denoted  $u_{hh}$ , is given by

$$u_{hh} = (1 - \beta) \frac{(1 + h/3640)^{-\beta}}{3640^2} \left( \frac{c^{-1}}{1} \right) \quad (B)$$

Because  $\beta < 1$ ,  $\text{sign } u_{hh} = \text{sign } u_h$ .

The deterministic part of the utility function is strictly concave if  $u_{cc} < 0$ ,  $u_{hh} < 0$  and  $u_{cc}u_{hh} - (u_{ch})^2 > 0$ .

The latter is given by

$$u_{cc}u_{hh} - (u_{ch})^2 = \frac{c^{-1}(1 + h/3640)^{-\beta}}{3640^2} \left[ \frac{f(h)}{c(1 + h/3640)} \right]^2$$

The lowest value of the bracket term occurs when  $h=0$ . In that case, and with very few exceptions, the term is positive for sample values. Thus, in the sample and for the case studied in Appendix D, the deterministic part of the utility function is strictly concave.

From the probability given in (A.1) we get

$$\frac{\partial u_c(h)}{\partial I} = u_c(h) \left[ \int_{r=1}^2 \int_{x=0}^1 u_c(x) g_r(x) f_r(x) \right]$$

where

$$u_c(h) = c(h)^{-1} f(h) > 0.$$

$$\frac{\partial E(h)}{\partial I} = \frac{\int_{r=1}^2 \int_{x=0}^1 x f_r(x)}{\int_{r=1}^2 \int_{x=0}^1 x} - \frac{\int_{r=1}^2 \int_{x=0}^1 x \frac{\partial u_c(x)}{\partial I}}{\int_{r=1}^2 \int_{x=0}^1 x}.$$

The sign of the derivative  $\frac{\partial u_c(h)}{\partial I}$  could be positive for high values of  $u_c(h)$ , which could occur for high value of  $I$ , which was also the reason for  $u_h$  increasing with  $h$  and for  $u_{hh} > 0$ . For sample values and for the elasticities shown in Appendix D this does not occur.

## Appendix D. Elasticities

M=uncompensated wage elasticity (Marshall), S=compensated wage elasticity (Slutsky), I= income elasticity

**Table D.1. Married woman aged 30, no children**

<b>Wage NOK 70</b>										
Income I	Elasticity	Probability of working			Conditional mean hours			Unconditional mean hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	0.007	-0.043	0.163	0.152	0.222	0.163	0.160	0.178
	S	0.131	0.135	0.111	0.186	0.174	0.254	0.318	0.310	0.365
	I	0.000	-0.007	0.040	-0.060	-0.057	-0.081	-0.060	-0.064	-0.041
100000	M	0.029	0.057	-0.089	0.269	0.254	0.352	0.299	0.306	0.259
	S	0.183	0.205	0.061	0.254	0.239	0.335	0.438	0.445	0.397
	I	-0.022	-0.031	0.028	-0.080	-0.076	-0.101	-0.102	-0.107	-0.073
200000	M	0.119	0.149	-0.038	0.355	0.338	0.441	0.479	0.493	0.401
	S	0.276	0.324	0.015	0.319	0.305	0.389	0.596	0.629	0.404
	I	-0.046	-0.054	-0.009	-0.062	-0.060	-0.074	-0.109	-0.114	-0.083

  

<b>Wage NOK 200</b>										
Income I	Elasticity	Probability of working			Conditional mean hours			Unconditional mean hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	-0.046	0.245	0.192	0.190	0.194	0.192	0.142	0.445
	S	0.295	0.237	0.588	0.224	0.223	0.222	0.519	0.461	0.810
	I	0.000	0.001	-0.007	-0.008	-0.008	-0.009	-0.008	-0.007	-0.017
100000	M	0.000	-0.043	0.231	0.207	0.203	0.220	0.207	0.158	0.457
	S	0.272	0.219	0.546	0.221	0.219	0.225	0.493	0.438	0.771
	I	0.000	0.001	-0.007	-0.012	-0.011	-0.014	-0.012	-0.010	-0.021
200000	M	0.000	-0.039	0.202	0.216	0.211	0.239	0.216	0.172	0.453
	S	0.240	0.194	0.481	0.215	0.212	0.227	0.456	0.406	0.709
	I	0.000	0.000	0.000	-0.010	-0.010	-0.013	-0.010	-0.010	-0.013



**Wage NOK 300**

Income I	Elasticity	Probability of working			Conditional mean hours			Unconditional mean hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	-0.049	0.229	0.178	0.184	0.147	0.178	0.134	0.380
	S	0.347	0.297	0.574	0.169	0.175	0.138	0.516	0.472	0.713
	I	0.000	0.001	-0.006	-0.004	-0.005	-0.004	-0.004	-0.003	-0.010
100000	M	0.000	-0.051	0.240	0.186	0.192	0.155	0.186	0.140	0.399
	S	0.338	0.286	0.570	0.173	0.179	0.142	0.511	0.465	0.713
	I	0.000	0.002	-0.009	-0.007	-0.007	-0.006	-0.007	-0.005	-0.015
200000	M	0.000	-0.052	0.249	0.195	0.200	0.165	0.195	0.147	0.418
	S	0.315	0.263	0.549	0.175	0.181	0.146	0.490	0.444	0.695
	I	0.000	0.001	-0.008	-0.007	-0.007	-0.006	-0.007	-0.005	-0.015

**Table D.2. Woman aged 30, two children**

**Wage NOK 70**

Income I	Elasticity	Probability of working			Conditional mean hours			Unconditional mean hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.047	0.096	-0.190	0.462	0.451	0.498	0.511	0.551	0.299
	S	0.327	0.394	-0.012	0.494	0.481	0.536	0.821	0.876	0.523
	I	-0.046	-0.059	0.014	-0.131	-0.128	-0.137	-0.177	-0.187	-0.122
100000	M	0.254	0.304	0.021	0.593	0.589	0.595	0.862	0.911	0.618
	S	0.581	0.704	-0.017	0.667	0.675	0.569	1.249	1.379	0.551
	I	-0.126	-0.137	-0.075	-0.149	-0.148	-0.146	-0.273	-0.284	-0.221
200000	M	0.511	0.554	0.319	0.634	0.637	0.603	1.177	1.227	0.942
	S	0.682	0.830	-0.027	0.410	0.382	0.508	1.093	1.212	0.480
	I	-0.167	-0.173	-0.141	-0.111	-0.112	-0.105	-0.277	-0.283	-0.246

**Wage NOK 200**

Income I	Elasticity	Probability of working			Conditional mean hours			Unconditional mean hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	0.001	-0.006	0.201	0.188	0.277	0.201	0.189	0.271
	S	0.246	0.232	0.325	0.212	0.200	0.287	0.458	0.432	0.613
	I	0.000	-0.001	0.009	-0.014	-0.013	-0.020	-0.014	-0.015	-0.011
100000	M	0.000	0.008	-0.048	0.231	0.216	0.319	0.231	0.222	0.270
	S	0.247	0.241	0.281	0.227	0.213	0.310	0.475	0.455	0.591
	I	0.000	-0.003	0.018	-0.025	-0.020	-0.034	-0.025	-0.027	-0.016
200000	M	0.001	0.019	-0.101	0.278	0.261	0.381	0.280	0.281	0.276
	S	0.249	0.253	0.221	0.254	0.238	0.347	0.503	0.492	0.568
	I	0.000	-0.006	0.027	-0.035	-0.032	-0.046	-0.035	-0.038	-0.019

**Wage NOK 300**

Income I	Elasticity	Probability of working			Conditional mean hours			Unconditional mean hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	-0.053	0.296	0.229	0.223	0.252	0.229	0.169	0.556
	S	0.440	0.367	0.826	0.273	0.270	0.284	0.714	0.637	1.110
	I	0.000	0.001	-0.005	-0.006	-0.006	-0.007	-0.006	-0.005	-0.013
100000	M	0.000	-0.050	0.282	0.234	0.228	0.264	0.234	0.176	0.553
	S	0.413	0.345	0.778	0.268	0.263	0.284	0.681	0.608	1.063
	I	0.000	0.001	-0.006	-0.010	-0.009	-0.012	-0.010	-0.008	-0.019
200000	M	0.000	-0.045	0.253	0.244	0.235	0.284	0.244	0.189	0.544
	S	0.375	0.315	0.702	0.260	0.253	0.285	0.635	0.568	0.988
	I	0.000	0.000	-0.002	-0.013	-0.012	-0.016	-0.013	-0.011	-0.019

**Table D.3 Married woman, aged 40, no children**

<b>Wage NOK 70</b>										
Income I	Elasticity	Probability of working			Conditional mean hours			Unconditional mean hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	0.019	-0.078	0.187	0.172	0.249	0.187	0.192	0.168
	S	0.148	0.164	0.083	0.211	0.195	0.283	0.360	0.359	0.366
	I	0.000	-0.012	0.050	-0.076	-0.070	-0.098	-0.076	-0.082	-0.048
100000	M	0.045	0.085	-0.107	0.319	0.298	0.399	0.366	0.384	0.287
	S	0.225	0.271	0.042	0.303	0.282	0.380	0.582	0.553	0.422
	I	-0.034	-0.049	0.022	-0.100	-0.094	-0.120	-0.134	-0.143	-0.098
200000	M	0.177	0.226	-0.004	0.417	0.395	0.490	0.601	0.631	0.486
	S	0.364	0.458	0.000	0.389	0.372	0.433	0.753	0.831	0.433
	I	-0.069	-0.080	-0.030	-0.078	-0.075	-0.088	-0.147	-0.155	-0.119

  

<b>Wage NOK 200</b>										
Income I	Elasticity	Probability of working			Conditional mean hours			Unconditional mean hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	-0.053	0.209	0.199	0.192	0.218	0.199	0.138	0.432
	S	0.291	0.220	0.563	0.229	0.224	0.240	0.521	0.445	0.804
	I	0.000	0.001	-0.005	-0.009	-0.009	-0.011	-0.009	-0.007	-0.016
100000	M	0.000	-0.049	0.193	0.209	0.200	0.236	0.209	0.150	0.434
	S	0.270	0.205	0.517	0.226	0.220	0.244	0.498	0.426	0.762
	I	0.000	0.001	-0.004	-0.013	-0.013	-0.016	-0.013	-0.011	-0.021
200000	M	0.000	-0.041	0.165	0.223	0.212	0.260	0.223	0.170	0.430
	S	0.241	0.187	0.450	0.223	0.214	0.249	0.464	0.402	0.699
	I	0.000	0.000	0.003	-0.014	-0.013	-0.018	-0.014	-0.013	-0.014

**Wage NOK 300**

Income I	Elasticity	Probability of working			Conditional mean hours			Unconditional mean hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	-0.070	0.258	0.200	0.207	0.172	0.200	0.138	0.427
	S	0.393	0.315	0.650	0.198	0.206	0.167	0.591	0.521	0.817
	I	0.000	0.002	-0.007	-0.005	-0.005	-0.004	-0.005	-0.003	-0.011
100000	M	0.000	-0.074	0.259	0.208	0.214	0.181	0.208	0.138	0.446
	S	0.379	0.301	0.640	0.201	0.208	0.171	0.581	0.510	0.812
	I	0.000	0.002	-0.010	-0.008	-0.008	-0.007	-0.008	-0.005	-0.017
200000	M	0.000	-0.075	0.265	0.217	0.222	0.193	0.217	0.145	0.463
	S	0.351	0.276	0.610	0.202	0.209	0.175	0.554	0.483	0.786
	I	0.000	0.002	-0.009	-0.008	-0.008	-0.007	-0.008	-0.005	-0.016

**Table D.4. Married woman aged 40, two children**

**Wage NOK 70**

Income I	Elasticity	Probability of working			Conditional mean hours			Unconditional mean hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.065	0.131	-0.153	0.508	0.500	0.521	0.578	0.638	0.360
	S	0.371	0.474	0.011	0.566	0.557	0.571	0.938	1.031	0.582
	I	-0.065	-0.082	-0.010	-0.145	-0.143	-0.146	-0.210	-0.224	-0.156
100000	M	0.331	0.393	0.130	0.629	0.631	0.605	0.981	1.049	0.744
	S	0.714	0.920	0.006	0.784	0.820	0.576	1.498	1.740	0.582
	I	-0.163	-0.176	-0.122	-0.158	-0.160	-0.151	-0.320	-0.334	-0.271
200000	M	0.620	0.670	0.465	0.649	0.660	0.599	1.309	1.374	1.093
	S	0.821	1.063	-0.008	0.412	0.376	0.487	1.233	1.440	0.478
	I	-0.203	-0.209	-0.183	-0.116	-0.118	-0.108	-0.317	-0.325	-0.290

**Wage NOK 200**

Income I	Elasticity	Probability of working			Conditional mean hours			Unconditional mean hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	0.014	-0.062	0.218	0.199	0.297	0.218	0.214	0.233
	S	0.257	0.253	0.273	0.223	0.204	0.301	0.480	0.458	0.575
	I	0.000	-0.003	0.013	-0.018	-0.016	-0.024	-0.018	-0.019	-0.011
100000	M	0.000	0.026	-0.105	0.257	0.235	0.348	0.258	0.265	0.239
	S	0.265	0.272	0.234	0.245	0.225	0.332	0.511	0.497	0.567
	I	0.000	-0.005	0.023	-0.031	-0.028	-0.041	-0.031	-0.034	-0.018
200000	M	0.002	0.042	-0.156	0.316	0.291	0.419	0.319	0.335	0.255
	S	0.273	0.295	0.181	0.284	0.261	0.380	0.557	0.557	0.561
	I	-0.001	-0.009	0.032	-0.044	-0.040	-0.056	-0.045	-0.050	-0.024

**Wage NOK 300**

Income I	Elasticity	Probability of working			Conditional mean hours			Unconditional mean hours		
		All	Public	Private	All	Public	Private	All	Public	Private
50000	M	0.000	-0.059	0.244	0.221	0.211	0.255	0.221	0.149	0.506
	S	0.419	0.331	0.765	0.267	0.259	0.292	0.687	0.590	1.057
	I	0.000	0.000	-0.003	-0.006	-0.006	-0.008	-0.006	-0.005	-0.012
100000	M	0.000	-0.054	0.225	0.228	0.216	0.269	0.228	0.160	0.501
	S	0.395	0.315	0.715	0.261	0.252	0.292	0.657	0.567	1.008
	I	0.000	0.000	-0.003	-0.011	-0.010	-0.014	-0.011	-0.009	-0.018
200000	M	0.000	-0.046	0.190	0.243	0.228	0.295	0.243	0.180	0.491
	S	0.363	0.294	0.640	0.255	0.244	0.295	0.619	0.538	0.935
	I	0.000	0.000	0.001	-0.015	-0.014	-0.020	-0.015	-0.014	-0.018

**Appendix E. Uncompensated probabilities, conditional and unconditional mean hours**

**Table E.1 Married woman aged 30, no children**

Income I	Probability of working			Conditional mean hours			Unconditional mean hours		
	All	Public	Private	All	Public	Private	All	Public	Private
	<b>Wage NOK 70</b>								
50000	1.0000	0.8504	0.1496	1907	1912	1878	1907	1626	281
100000	0.9936	0.8399	0.1537	1810	1821	1755	1799	1529	270
200000	0.9691	0.8140	0.1551	1711	1725	1637	1658	1404	254

Income I	Probability of working			Conditional mean hours			Unconditional mean hours		
	All	Public	Private	All	Public	Private	All	Public	Private
	<b>Wage NOK 200</b>								
50000	1.0000	0.8410	0.1590	2185	2176	2230	2185	1830	355
100000	1.0000	0.8419	0.1581	2168	2160	2211	2168	1819	350
200000	1.0000	0.8424	0.1576	2149	2142	2187	2149	1804	345

Income I	Probability of working			Conditional mean hours			Unconditional mean hours		
	All	Public	Private	All	Public	Private	All	Public	Private
	<b>Wage NOK 300</b>								
50000	1.0000	0.8228	0.1772	2377	2369	2415	2377	1949	428
100000	1.0000	0.8238	0.1762	2367	2359	2406	2367	1943	424
200000	1.0000	0.8251	0.1749	2354	2346	2395	2354	1935	419

**Table E.2 Married woman aged 30, two children**

Income I	Probability of working			Conditional mean hours			Unconditional mean hours		
	All	Public	Private	All	Public	Private	All	Public	Private
	<b>Wage NOK 70</b>								
50000	0.9894	0.8200	0.1693	1496	1515	1403	1480	1243	238
100000	0.9331	0.7664	0.1667	1348	1366	1261	1257	1047	210
200000	0.8352	0.6814	0.1537	1218	1235	1144	1017	842	176

Income I	Probability of working			Conditional mean hours			Unconditional mean hours		
	All	Public	Private	All	Public	Private	All	Public	Private
	<b>Wage NOK 200</b>								
50000	1.0000	0.8540	0.1460	1956	1958	1945	1956	1672	284
100000	1.0000	0.8525	0.1474	1929	1933	1907	1929	1648	281
200000	0.9997	0.8498	0.1499	1887	1894	1851	1887	1609	278

**Wage NOK 300**

Income I	Probability of working			Conditional mean hours			Unconditional mean hours		
	All	Public	Private	All	Public	Private	All	Public	Private
50000	1.0000	0.8470	0.1530	2132	2124	2173	2132	1799	332
100000	1.0000	0.8477	0.1523	2119	2112	2157	2119	1791	329
200000	1.0000	0.8483	0.1517	2101	2095	2135	2101	1777	324

**Table E.3 Married woman aged 40, no children**

<b>Wage NOK 70</b>									
Income I	Probability of working			Conditional mean hours			Unconditional mean hours		
	All	Public	Private	All	Public	Private	All	Public	Private
50000	1.0000	0.8020	0.1980	1855	1865	1812	1855	1496	359
100000	0.9898	0.7858	0.2040	1739	1756	1671	1721	1380	341
200000	0.9529	0.7490	0.2039	1621	1643	1540	1545	1231	314

<b>Wage NOK 200</b>									
Income I	Probability of working			Conditional mean hours			Unconditional mean hours		
	All	Public	Private	All	Public	Private	All	Public	Private
50000	1.0000	0.7967	0.2033	2141	2131	2179	2141	1698	443
100000	1.0000	0.7975	0.2025	2123	2115	2157	2123	1686	437
200000	1.0000	0.7977	0.2023	2101	2094	2128	2101	1670	431

<b>Wage NOK 300</b>									
Income I	Probability of working			Conditional mean hours			Unconditional mean hours		
	All	Public	Private	All	Public	Private	All	Public	Private
50000	1.0000	0.7745	0.2255	2337	2326	2378	2337	1801	536
100000	1.0000	0.7759	0.2241	2326	2314	2367	2326	1795	531
200000	1.0000	0.7776	0.2224	2311	2299	2354	2311	1788	524

**Table E.4 Married woman aged 40, two children**

<b>Wage NOK 70</b>									
Income I	Probability of working			Conditional mean hours			Unconditional mean hours		
	All	Public	Private	All	Public	Private	All	Public	Private
50000	0.9848	0.7594	0.2254	1416	1440	1332	1394	1094	300
100000	0.9099	0.6937	0.2161	1263	1286	1191	1149	892	257
200000	0.7916	0.5990	0.1926	1136	1154	1078	899	691	208

<b>Wage NOK 200</b>									
Income I	Probability of working			Conditional mean hours			Unconditional mean hours		
	All	Public	Private	All	Public	Private	All	Public	Private



	All	Public	Private	All	Public	Private	All	Public	Private
50000	1.0000	0.8075	0.1925	1919	1925	1896	1919	1554	365
100000	1.0000	0.8051	0.1949	1887	1895	1853	1887	1526	361
200000	0.9995	0.8006	0.1989	1836	1848	1788	1835	1480	356

### Wage NOK 300

Income I	Probability of working			Conditional mean hours			Unconditional mean hours		
	All	Public	Private	All	Public	Private	All	Public	Private
50000	1.0000	0.8034	0.1966	2096	2088	2129	2096	1678	418
100000	1.0000	0.8040	0.1960	2083	2076	2112	2083	1669	414
200000	1.0000	0.8043	0.1957	2063	2058	2086	2063	1655	408