Boukouras, Aristotelis

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Separation of Ownership and Control: Delegation as a Commitment Device

Aristotelis Boukouras

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Abstract

This paper provides a theoretical model for explaining the separation of ownership and control in firms. An entrepreneur hires a worker, whose effort is necessary for running a project. The worker's effort determines the probability that the project will be completed on time, but the worker receives some unobservable benefit by continuing his employment in the project. Thus, motivating the worker requires an efficiency wage which is inflated by the private benefit. The entrepreneur would pay out a smaller wage if he could commit to terminate the project if a delay occurs, but this threat is not credible, because the project has positive continuation value. We show that hiring a manager can solve this time-inconsistency issue and reduce the efficiency wage. We extend the model to include managerial moral hazard and we examine the conditions under which separation of ownership and control is more likely to happen. The model is consistent with many of the findings of the empirical literature, while it generates some new predictions too.

Keywords: control structure, delegation, efficiency wage, entrepreneur, managerial contract, moral hazard, organizational hierarchy, private benefits, separation of ownership and control, time-inconsistency

JEL Classification: D86, G34, J31, L22, L26

*Courant Research Centre Poverty, Equity and Growth, Georg-August University Göttingen, aboukou@gwdg.de
1 Introduction

Separation of ownership and control has been an active research topic in corporate
governance for several decades now. Its traces go back to the seminal work of Berle
and Means (1932) and even Adam Smith (1776). The stylized fact of the topic is the
large firm, which is owned by many, relatively small stockholders and is run centrally
by professional managers, who have a negligible ownership of the corporation. The
associated agency costs and the corporate mechanisms to combat them have been the
central concerns of the literature since then. Examples of this literature are the pa-
pers by Monsen and Downs (1965), Alchian and Demsetz (1972), Jensen and Meckling

More recently this stylized fact has been challenged by empirical papers, which
argue that the ownership of stocks is not as widely dispersed as previously thought
in most countries (Porta, Lopez-De-Silanes, and Shleifer, 1999), that the dispersion
of ownership in US has changed over time (Holderness, Kroszner, and Sheehan, 1999)
and that modern US ownership is not different from the rest of the world (Holderness,
2009). The main argument is that modern corporations do not face as severe agency
costs as previously thought, because most firms have blockholders with sufficiently
high ownership stakes so as to ensure monitoring over the management (Demsetz and
Lehn, 1985).

Still, in the majority of cases, the top officers are different from the important block-
holders and they hold a small fraction of the total equity. Mikkelson and Partch (1989)
find that, for 60 percent of the companies in their sample, the top three officers own
less than 10 percent of the stock combined, while Holderness, Kroszner, and Sheehan
(1999) find that in their sample of US companies the average stock-holdings of a CEO
is 1.25 percent (the median is only 0.06). Similar findings are reported by Jensen and
Murphy (1990). Since top executives make the most important decisions, an interest-
ing question remains unanswered in the corporate governance literature. Why are the
agents who run firms (top executives, managers) usually different from the agents who
own them (blockholders)?

A common answer to this question is that the blockholders do not have the ability,
expertise or the knowledge to run firms, while managers do. Another answer is the
opportunity cost of time for large shareholders, namely that they may prefer leisure
or starting a new company than dealing with management issues. Though perfectly
valid, these answers offer little in terms of a theory, which relates the firms’ observable
characteristics with their control structure, so that one would expect that these two
(firm-characteristics and control structure) are unrelated. This is, however, disputed
by empirical studies (see for example Demsetz and Lehn (1985)).

We offer an alternative theoretical explanation on why investors may prefer to sep-
parate ownership from control, one that relates firm characteristics to the choice of the
optimal control structure. The main argument is that managers are better suited to

\footnote{See for example Jensen and Meckling (1976) and Tirole (2001).}
make certain decisions about the firm than its owners, exactly because they do not necessarily have a large stake on its long-run prospects. A manager’s payoff depends on his contract with the owner and this may cause him to value certain states differently than his principal. Though, this generates agency costs, it can actually be beneficial whenever the owner suffers from a time-inconsistency problem (this is actually similar to the reasoning of Acemoglu (1998)).

To make our argument as clear and stark as possible, in section 2 we consider a simple model with a single entrepreneur and worker. The entrepreneur is assumed to have full ownership and control of the firm and is not financially constrained. The worker, who suffers from moral hazard, exerts effort on a project, which increases the firm’s profits once completed. The probability of the project completing on time depends on the worker’s effort, who, however, receives a private benefit by working on it and prefers to have it delayed. As a result, the entrepreneur needs to compensate the worker for the loss of the private benefit, if he is to have the project completed on time.

We show that the entrepreneur would like to threaten the worker that he will terminate the project if it is delayed, in order to reduce the wage required for completing the project on time. But such a threat is not credible, because the continuation value of the project is strictly positive for the entrepreneur. As a result, he is forced to pay a high efficiency wage. This problem is solved by hiring a manager and giving him a payment conditional only on the first period profits. Thus, the manager is induced to terminate the project if it is delayed. This solves the time-inconsistency problem of the entrepreneur and allows him to reduce the wage paid to the worker. Delegation of partial control strictly increases the payoff of the owner.

In section 3 we extend the model by adding a moral hazard problem from the manager’s side and by making the manager the sole decision maker in the firm. That is, we assume that once the manager takes control of the firm, he makes all the relevant decisions, including the hiring of the worker and the wage contract. This allows us to study the more plausible case, where giving up control to the manager may generate undesirable consequences (agency costs). We provide the optimal managerial contract and we examine the conditions under which separation of ownership and control is optimal. We also provide and discuss the comparative statics between the parameters of the model and the optimal control structure.

The main comparative statics are as follows. Separation of ownership and control is more likely when: (i) the manager can not easily appropriate profits from the firm, (ii) the private benefits of the workers are high, (iii) the worker’s effort is important for the completion of tasks, (iv) the profit per worker is not very high, (v) the variance of

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In fact, it is irrelevant for our purposes if the firm has only one or many owners. We consider the case of an entrepreneur because it is the simplest possible and it allows us to distinguish the importance of the control structure (owners versus managers) from that of the ownership structure (large versus small shareholders). For the latter case, see the papers by Grossman and Hart (1980), Shleifer and Vishny (1986), Grossman and Hart (1988), Harris and Raviv (1988), Bebchuk (1994), Burkart, Gromb, and Panunzi (1997) and Burkart, Gromb, and Panunzi (1998).
profits is low. Some of these predictions ((i) and (v)) are intuitive and are consistent with empirical findings (Demsetz and Lehn, 1985), while the rest are new predictions and have not been empirically tested yet.

We would like to comment on (iv), because it seems counter-intuitive. In our model there is only one worker and as a result the total profitability of the firm is equivalent to the profitability per worker. But, as we discuss in subsection 3.2, in a model with multiple workers under the manager, the value of delegation may increase with the size of the firm (number of workers or total profits), as one would expect, but it will decrease with the profitability per worker. In our model, this is because the manager can appropriate less resources as the profitability per worker goes down and, given everything else, agency costs decrease. We believe that it is interesting to examine if this prediction is also confirmed by the data.

Finally, in section 4 we discuss certain theoretical issues regarding the model. We discuss how the results change by the inclusion of participation constraints, we discuss the issue of renegotiation-proofness of the managerial contract and we examine other potential solutions to the entrepreneur’s problem. There, we show that, under certain conditions, both financial securities and government intervention are sub-optimal to separation of ownership and control.

Other papers have also examined the issue of separation of ownership and control from a theoretical perspective. The early literature (Jensen and Meckling (1976), Fama (1980), Fama and Jensen (1983), and Demsetz (1983)) recognized the existence of agency costs in the firm and examined how the organizational ownership and control structure were used in order to combat them. But they did not explain why the decision power had to be delegated to managers in the first place.

More recently, Acemoglu (1998) explains the separation of ownership and control as a signal of the entrepreneur to financial markets about the quality of his project. Our paper does not relate the presence of managers to financial markets but to the internal workings of the firm. Their role is also different: they are not used as signaling devices but as commitment devices.

Also Ferreira, Ornelas, and Turner (2010), based on Ornelas and Turner (2007), examine the separation of ownership and control in a model of optimal dissolution of partnership. Two partners allocate ex-ante and ex-post ownership rights in order to optimize ex-post incentives in revealing their type and allocating optimal control rights. Thus, their model is one of shareholders reaching an agreement on who should run the firm and it is not a principal-agent model, like ours. Moreover, the friction in their model is one of hidden types, while ours is one of hidden actions.

To summarize, we believe that the main contributions of our paper are the following: (i) We show how the separation of ownership and control can act as a commitment device, which reduces the efficiency wage and increases the value of the firm. (ii) Delegation of decision power is an endogenous decision in our model, with both costs and benefits. (iii) This trade-off relates firm-characteristics to the optimal choice of control structure and generates several predictions. (iv) Some of these predictions are consistent with the findings of the empirical literature, while others remain to be tested.
2 A simple model

A risk neutral entrepreneur (E) employs a risk neutral worker (W) in order to complete a project. The worker exerts effort in the beginning of period one. Effort is unobservable and there are two effort levels: high \( e_H \) and low \( e_L \). The effort level chosen by the worker determines the probability that the project will be completed on time. If the worker exerts high effort then he incurs cost \( c_H \) and the project is completed in the end of period one with certainty, yielding revenues equal to \( V \) for the entrepreneur. If, on the other hand, the worker exerts low effort, then he incurs a cost \( c_L \) and the project is completed on time with probability \( p \) and is delayed with probability \( 1 - p \). If the project is delayed, then it requires an additional period to complete, yielding revenues \( V \) in the end of period two.

However, delay is costly for the entrepreneur. At \( t = 1/2 \) E finds out whether the project will be delayed or not. Then, he can choose either to terminate the project \( T = 1 \) and forgo the future revenues or to let the project continue \( T = 0 \) but suffer a cost \( k \) in period one. This cost can have multiple interpretations. For example, it can reflect the cost of procuring additional inputs (excluding labor) or the additional effort required by E for renewing agreements with suppliers and customers. If the delay shock is publicly available (e.g. launch of a new product), it can also reflect the loss in terms of reputation, market value or customers. For this section we assume that the delay shock is private information to the relevant decision maker at time \( t = 1/2 \) (E in this case), but the completion date and status of the project are verifiable. Let \( k < V \).

Assuming that the entrepreneur can not impose financial penalties to the worker if delay occurs and that there is no participation constraint, he offers a wage schedule, conditional on the completion status of the project at the end of period one, in order to induce W to exert high effort. In addition, the worker enjoys a private benefit \( b \) for every time period that he is employed in the firm. Again, the private benefit can have multiple interpretations. The one usually provided by the literature is that it is an unobservable part of the output, which is appropriated by the worker\(^4\). However, this interpretation is not necessary for our purposes. It would also do if the worker has some bargaining power when negotiating the wage with the entrepreneur or if it reflects the psychological benefit of being employed. What is important is that it accrues over time and that it is not in the control of the entrepreneur whether to provide it or not.

Let \( \{w_1, w_0\} \) be the wage contract offered to W, conditional on the project being completed or not in period one respectively. The workers decision on whether to exert

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\(^3\)The fact that \( k \) is positive is not crucial for the results of this section. This assumption facilitates the derivation of the optimal managerial contract in section 3.1. It is also more intuitive to assume that the continuation value of the project decreases if it is delayed rather than it increases, which would be the case if \( k \) is negative.

\(^4\)see for example the papers by Holmstrom and Tirole (1997) and Pagano and Volpin (2006)
high or low effort depends on the wage schedule and the termination decision. This is because, when the project suffers from the delay shock and E decides to terminate it, then W loses the private benefit from the operation of the firm in period 2. More specifically, if the worker exerts high effort, then his payoff is the private benefit in period one, plus the wage he receives for the timely completion of the project minus the cost of effort: \( b + w_1 - c_H \). If W exerts low effort then with probability \( p \) he receives \( b \) and \( w_1 \). With probability \( 1 - p \) the project is delayed and then he receives \( w_0 \), \( b \) from the operation of the firm in the first period and \( b \) from the operation of the firm in the second period if E decides to continue the project (\( T = 0 \)) and zero otherwise (\( T = 1 \)). Since W incurs \( c_L \) if he exerts low effort, his payoff in this case is: 

\[
b + pw_1 + (1 - p)[w_0 + (1 - T)b] - c_L.
\]

Clearly, the worker will exert high effort iff:

\[
w_1 \geq w_0 + (1 - T)b + \frac{c_H - c_L}{1 - p}
\]

E decides the wage contract and the termination of the project if the delay shock occurs. His expected payoff is: 

\[
p(e_w)(V - w_1) + [1 - p(e_w)][(1 - T)(V - k) - w_0].
\]

\( p(e_w) \) denotes the fact that the probability of delay depends on the effort level of W. However, because \( V > k \), the optimal choice is to always let the project continue at \( t = 1/2 \). Therefore, inducing high effort requires that:

\[
w_1 \geq w_0 + b + \frac{c_H - c_L}{1 - p}
\]

Because reducing \( w_0 \) relaxes the incentive compatibility constraint and at the same time reduces entrepreneur's expected payment, the optimal choice for E is either to set \( w_0 = 0 \) and \( w_1 = b + \frac{c_H - c_L}{1 - p} \), if he wants W to exert high effort, or to set \( w_1 = w_0 = 0 \), if he wants W to exert low effort. His payoff in the first case is \( V - w_1 \) and in the second case \( V - (1 - p)k \). Therefore, E will choose to provide high incentives to W iff \( k \geq w_1/(1 - p) \). However, E would do better than that, if he were to decide to terminate the project if the delay occurs. In this case, W would not receive the second period private benefit and his incentive compatibility would be relaxed. As a result, E would be able to lower the wage required for inducing high effort (it would fall to \( w_1 = \frac{c_H - c_L}{1 - p} \), so the difference is equal to the private benefit), which would benefit him whenever \( k \geq (c_H - c_L)/(1 - p)^2 \) (if \( k \) is below this threshold, E prefers to induce low effort anyway).

The problem is that, because the continuation value of the project is strictly positive, he can not credibly promise to the worker that he will terminate it in case of delay. In other words, the entrepreneur suffers from a time-inconsistency problem, which is similar in nature to a soft-budget constraint \(^5\). How can he deal with it? The solution is to hire a manager under a contract which provides a payment conditional on short-run (period-one) profits\(^6\).


\(^6\)For other possible solutions to this problem and a discussion on why our solution is optimal see
To see this, suppose that E can not impose financial penalties to M (the manager), that the latter does not face a participation constraint, and consider the following contract. M takes over the control of the firm at the beginning of the period one (the authority to determine T at time $t = 1/2$) just after the worker is hired, and receives reward $y(\Pi_1) = \epsilon_1$ if the profit at the end of period one is equal to $\Pi_1 = V - \frac{c_H - c_L}{1-p}$, $y(\Pi_1) = \epsilon_2$ if $\Pi_1 = 0$ and $y(\Pi_1) = 0$ if $\Pi_1 = -k$, with $\epsilon_1 > \epsilon_2$ strictly positive but arbitrarily close to zero. Under this contract, M prefers to let the project continue if there is no delay, since $\epsilon_1 > \epsilon_2 > 0$, while he prefers to terminate the project if the delay occurs ($\epsilon_2 > 0$). This solves the time-inconsistency problem of the entrepreneur and allows him to offer a lower wage to W in order to induce him to exert high effort. As a result, E strictly prefers to hire the manager and provide high incentives to the worker if $k \geq \frac{c_H - c_L}{(1-p)^2} + \epsilon_1$ and to run the firm by himself and provide no wage to the worker otherwise.

The main intuition is that the manager does not suffer from the time-inconsistency problem that the entrepreneur faces, because his payoff is constructed through the contract and does not depend on the primitives of the economy. As a result, the delegation of control to the manager can relax the incentive compatibility of the worker and this increases the entrepreneur’s payoff. In other words, the separation of ownership and control is optimal from the entrepreneur’s point of view in this economy.

Note that this does not necessarily mean that separation of ownership and control is optimal from a societal point of view as well. One can easily find values of the parameters such that the induced effort level diverges from the societal optimal. Of course, given the presence of incomplete information, this should come as no surprise. But one can show that the delegation of control shrinks the range of parameters values for which suboptimal incentives are induced.

3 A model with managerial moral hazard

3.1 Optimal managerial contract

In this section we present a modified version of the model of section 2, by extending it in two directions. First, we allow the manager to suffer from a moral hazard problem as well. Second, when control is delegated to the manager, we assume that he makes all the decisions relevant for the operation of the firm thereafter. This means that, apart from making the termination decision, the manager is the one who provides the terms of the contract to the worker and not the entrepreneur. Moreover, the entrepreneur does not directly observe the decisions of the manager but only the realized profits and he must, therefore, design the managerial contract accordingly.

We are interested in the first direction because the model of section two generates a rather implausible result: separation of ownership and control always generates positive value for the entrepreneur and is essentially costless. In this section we allow for more
general results by including a moral hazard problem from the manager’s side. Thus the rents earned by the manager reduce the value of delegation for the entrepreneur and generate a trade-off between cheaper worker incentives and better control incentives. We consider the second extension in order to show that our results do not depend crucially on the manager having limited control of the firm, in other words, that our results survive even under full delegation of decision power.

In order to make these points, we consider an extreme form of moral hazard, where the manager can appropriate a part of profits from the firm and transform them into private benefits or non-pecuniary rewards at an exogenously given and constant return factor $q$, with $0 < q < 1$. That is, for every single unit of profit that the manager appropriates, proportion $q$ is transformed into utility for the manager and proportion $1 - q$ is lost as appropriation cost.

More specifically, we now consider a firm which exists for two periods. The firm is owned in its entirety in the beginning of period one by E. The firm generates a random stream of profits in each period, $\rho_t$, which is normally distributed with mean $\tau$ and variance $\sigma^2$. On top of that, E is given the option to undertake a project which will increase the profits of the firm when it is completed. The details of this project are left as in section 2 (the project is defined by the same variables: $\{p, e_w, c_w, V, k, T\}$, and their interpretation and interactions remain the same as before).

In addition, we now assume that the status of the project is non-verifiable for the entrepreneur if he has delegated control, but remains verifiable for the manager. Hence, the wage contract can be made conditional on the status of the project, but the managerial contract can not. Furthermore, E does not observe the wage contract provided by M. These assumptions, apart from plausible (in many cases the shareholders do not have access to the monitoring and control devices of the management or it is too costly to obtain it), makes the manager’s incentive problem more interesting. The entrepreneur must now make sure that the manager has the incentives to provide correct effort incentives to the worker, as E can not control it directly or explore it in order to control M. In other words, since in our model we have two layers of contracts, one from the E to M and one from M to W, the second layer is treated as an additional incentive compatibility constraint in the design of the first contract. Therefore, by allowing this more complicated (and realistic) structure of incentives we can show that our results are robust to other types of asymmetries in the information structure.

The only observable and verifiable variable from E’s point of view, when he is not the decision maker, is the profit level of the firm (after the potential extraction of rents). Hence, the managerial contract can be made conditional on this variable: $y_t(\pi_e)$. On the other hand, the decision maker can make the wage contract conditional on both the profit level and the status of the project. For simplicity, and without loss of generality, due to risk-neutrality, we assume that the wage contract is conditional only on the status: $\{w_s\}$. The variable $s$ denotes the status of the project at the end of period one, and it takes the value 1 if the project is completed and 0 otherwise. This makes the formulation of the incentives of the worker identical to the problem in section 2 and, at the same time, it implies that W receives his reward at the end of period one.
At the end of each period the decision maker (M or E) privately observes the realized return $\pi^m_t$ (firm profits plus project profits minus worker’s wage) and, if it is positive, decides how much to transform into private (unobservable) benefits $r(\pi^m_t)$ and how much to keep as verifiable profits: $\pi^e_t = \pi^m_t - r(\pi^m_t)$. Because the profits of the firm are random, $\pi^m_t$ is a normally distributed variable as well, with variance $\sigma^2$ and mean, which depends on the status of the project and the termination decision: $\mu_t(s,T)$. More specifically, $\mu_1 = \tau + s[(1-T)V - w_1] + (1-s)[-(1-T)k - w_0]$, $\mu_2 = \tau + (1-s)(1-T)V$.

In the beginning of period one the entrepreneur decides whether to delegate control to a manager or not. If he does not delegate control, then he remains the decision maker for the firm. He then decides the terms of the wage contract to the worker, $\{w_1, w_0\}$, and whether to terminate the project or not at $t = 1/2$ (after having observed whether there is a delay of the project or not). If, however, E delegates control to M, it is the latter that makes these decisions. In this case, the entrepreneur offers a managerial contract which provides a payment $y$ to the manager for every profit level $\pi^e_t$ that E observes: $y_t(\pi^e_t)$. Through the design of this contract, E tries to indirectly induce M to take the decisions that are optimal for the former. Once again we assume away participation constraints for M or W and limited liability from their part (non-negative rewards). The full time-line of events is presented in figure 1.

![Figure 1: Timing of events](image)

Therefore, the entrepreneur has to decide whether to delegate decision authority to the manager or not and, if so, what contract he should provide. However, if the entrepreneur decides not to separate ownership from control, his decision problem remains identical to the one of section 2. To see this more clearly, note that the payment required for inducing the worker to exert high incentives remains the same as before: $w_1 \geq b + \frac{cH - cL}{1-p}$. Also, E would never extract any rents from his firm, as this destroys value for him. Hence, his utility under the combination of ownership with control is: $\max\{\tau + V - (b + \frac{cH - cL}{1-p}), \tau + V - (1-p)k\}$. Suppose that the first term in the brackets is higher than the second, which implies that the entrepreneur strictly prefers to induce the worker to exert high effort when he is in control of the firm. This will facilitate the comparison to the case of delegation.
Now, our task is to find what is the optimal contract for E when he decides to delegate control to M and under what conditions is delegation preferred to entrepreneurial control. The optimal contract is the solution to the following problem:

$$\max_{y_1(\pi^e_1),y_2(\pi^e_2)} \left\{ \sum_{s=0}^{1} p_s(e_w) \left[ \sum_{t=1}^{\infty} \int_{-\infty}^{\infty} (\pi^e_t - y_t(\pi^e_t)) f(\pi_t^e|\mu_t) d\pi^e_t \right] \right\}$$

subject to:

$$\{w_s, r_t(\pi^m_t), T\} = \arg\max \left\{ \sum_{s=0}^{1} p_s(e_w) \left[ \sum_{t=1}^{\infty} \int_{-\infty}^{\infty} (y_t(\pi^e_t) + qr_t(\pi^m_t)) f(\pi_t^m|\mu_t) d\pi^m_t \right] \right\}$$

$$e_w = \arg\max \left\{ p(e_w)(b + w_1) + (1 - p(e_w))(b + w_0 + (1 - T)b) - c(e_w) \right\}$$

$$\pi^e_t = \pi^m_t - r_t(\pi^m_t)$$

In the expressions above, $p_s(e_w)$ denotes the probability of state $s$ as a function of the worker’s effort and $f(\pi^m_t|\mu_t)$ is the conditional probability of the firm having profits equal to $\pi^m_t$ (similarly for $f(\pi^e_t|\mu_t)$ and $\pi^e_t$).

Given the assumptions and the analysis so far, it is clear that E designs the managerial contract so as to induce M to terminate the project if a delay occurs and so as to induce the worker to exert high effort. As we have stated before, the last effect comes indirectly, by giving the manager appropriate incentives, so that M prefers to offer a similar wage contract to W as E would. Essentially, problem (1)-(4) can be reduced to maximizing (1) under the following incentive compatibility constraints:

$$r_t(\pi^m_t) = \arg\max \{y_t(\pi^m_t - r_t) + qr_t\}, \quad \forall \pi_t \in (0, +\infty), t \in \{1, 2\}$$

$$T = 1 | s = 0 \Leftrightarrow \int_{-\infty}^{+\infty} \left[ y_1(\pi^m_1 - r(\pi^m_1)) + qr(\pi^m_1) \right] f(\pi^m_1|\tau - \hat{w}_0) d\pi^m_1 \geq$$
\[
\int_{-\infty}^{+\infty} [y_1(\pi_1^m - r(\pi_1^m)) + qr(\pi_1^m)] f(\pi_1^m) d\pi_1^m + \int_{-\infty}^{+\infty} [y_2(\pi_2^m - r(\pi_2^m)) + qr(\pi_2^m)] f(\pi_2^m) d\pi_2^m \\
\text{(IC}_2) \]

\[
T = 0 \mid s = 1 \iff \int_{-\infty}^{+\infty} [y_1(\pi_1^m - r(\pi_1^m)) + qr(\pi_1^m)] f(\pi_1^m) d\pi_1^m \geq 0 \\
\int_{-\infty}^{+\infty} [y_1(\pi_1^m - r(\pi_1^m)) + qr(\pi_1^m)] f(\pi_1^m) d\pi_1^m \\
\text{(IC}_3) \\
\]

\[
p \int_{-\infty}^{+\infty} [y_1(\pi_1^m - r(\pi_1^m)) + qr(\pi_1^m)] f(\pi_1^m) d\pi_1^m + (1-p) \int_{-\infty}^{+\infty} [y_1(\pi_1^m - r(\pi_1^m)) + qr(\pi_1^m)] f(\pi_1^m) d\pi_1^m \\
\text{(IC}_4) \]

In the incentive compatibility constraints above, \( \hat{w}_1 = \frac{\mu - \epsilon_L}{1-p} \) and \( \hat{w}_0 = 0 \). This is the optimal contract that M provides to W in order to induce him to exert high effort, given that the threat of the termination of the project is credible, and it is the same as in section 2.

**IC**1 gives the level of profits that M appropriates for the purpose of providing private benefits as a function of the managerial contract and the technological parameter \( q \). **IC**2 is the condition that must hold in order to make the threat of termination of the project credible if there is delay, while **IC**3 requires that the project is not terminated if it is not delayed. Finally, **IC**4 is the incentive compatibility condition, which is required if M is to provide high effort incentives to W rather than low.

In order to simplify the analysis and provide sharp comparative statics, we now assume that the probability of the project not suffering from delay when W exerts low effort is sufficiently small. Technically, we require that \( p \leq p^* \), where \( p^* \) is the solution to the following equation:

\[
(V - \hat{w}_1)(1 - F(-\tau - V)) + \tau(F(-\tau) - F(\hat{w}_1 - \tau - V)) + \int_{\hat{w}_1 - \tau - V}^{-\tau} \rho f(\rho) d\rho = \\
p \left[ (1 - F(-\tau - V)) + \tau(F(-\tau) - F(-\tau - V)) + \int_{-\tau - V}^{-\tau} \rho f(\rho) d\rho \right] 
\]

11
As we show in the Appendix, if \( p \) is below this threshold, then a linear managerial contract satisfies IC\(_4\) and this simplifies the exposition considerably\(^7\). Given this assumption, we provide the following proposition, the proof of which is included in the Appendix.

**Proposition 1:** Suppose that \( p \leq p^* \). Then, under IC\(_1\)-IC\(_4\), the optimal managerial contract is a linear contract with slope equal to \( q \) and a large bonus for the manager if profits reach an arbitrarily large threshold.

The exact form of the managerial contract is provided by equations 7 and 8 in the Appendix. We leave the comparative statics for the next subsection and we make a few notes on the form and interpretation of the managerial contract. The managerial contract is similar to a call option with exercise price zero. This is because it makes no payment to the manager, if profits are negative, and starts to pay-out when profits are positive. Moreover, the manager receives a constant proportion of the profits up to an arbitrarily large maximum level. The first part of the managerial contract is a direct implication of limited liability (non-negative rewards), while the second part is due to the ability of the manager to divert profits into private benefits. As a result, the contract treats the manager as if he is a debt-holder when profits are negative, but as if he is an equity-holder when profits are positive. This does not, of course, make the manager a residual claimant in the firm. Since the entrepreneur retains all profits after paying out manager’s reward, it is E who is the residual claimant in the firm and not M.

Finally, we note that the shape of the managerial contract depends on the assumptions of risk-neutrality and limited liability. If limited liability is not assumed, but we retain the assumption of risk-neutrality, then one can find contracts, which make the expected payment to the manager arbitrarily close to zero. However, if the manager is risk-averse, the expected managerial reward will be strictly positive, even in the absence of limited liability.

### 3.2 Optimal control structure and comparative statics

We can now evaluate the conditions under which the entrepreneur prefers to separate ownership from control. As we have already noted, if E maintains the control of the firm, he receives a payoff equal to:

\[
V_E^{EW} = \tau + V - \left( b + \frac{c_H - c_L}{1 - p} \right)
\]

On the other hand, if he delegates the control to the manager he receives: \( V_E^{EMW} = \)

\(^7\)The results regarding the optimal managerial contract when this assumption is not satisfied are available from the author upon request.
\[
\int_{-\infty}^{+\infty} (\pi_1^e - y_1(\pi_1^e)) f(\pi_1^e | \tau + V - \hat{w}_1) d\pi_1^e.
\]
Given the optimal managerial contract, \(\pi_1^e = \pi_1^m\) and \(\hat{w}_1 = \frac{c_H - c_L}{1 - p}\) and the above expression rewrites as:

\[
V_{E}^{EMW} = \tau + V - \frac{c_H - c_L}{1 - p} - q \int_{0}^{+\infty} \pi_1^m f \left( \pi_1^m | \tau + V - \frac{c_H - c_L}{1 - p} \right) d\pi_1^m
\]

By directly comparing \(V_{E}^{EW}\) to \(V_{E}^{EMW}\), we see that \(E\) prefers to separate ownership from control iff:

\[
b \geq E[y_1(\pi_1^m)]
\]

where \(E[y_1(\pi_1^m)] = q \int_{0}^{+\infty} \pi_1^m f \left( \pi_1^m | \tau + V - \frac{c_H - c_L}{1 - p} \right) d\pi_1^m\)

The comparative statics of equation 5 are clear and we summarize them in the following corollary of Proposition 1:

**Corollary 1**: The entrepreneur is more likely to separate ownership from control if:

- The ability of the manager to appropriate profits (\(q\)) decreases.
- The private benefit of the worker (\(b\)) increases.
- The probability of delay (\(1 - p\)) decreases or the differential cost of effort (\(c_H - c_L\)) increases. More generally, whenever the efficiency wage of the worker (\(\hat{w}_1\)) increases.
- The average value of the firm (\(\tau\)) or of the project (\(V\)) decreases.
- The variance of the firm profits (\(\sigma^2\)) increases.

While the interpretation of most of these comparative statics is straightforward, for some of them it is actually counter-intuitive. We consider each one in turn.

---

8Here, the term “likely” refers to whether the set of parameters that satisfy equation 5 increases or not as one of them changes. One can justify this term by imagining that there is a probability distribution over the set of parameters value, which provides the percentage of firms with the same characteristics and which gives the total probability of a firm belonging to one control structure or the other, as evaluated by the cumulative distribution conditional on 5. The mental exercise is, therefore, to examine what happens to this probability, conditioning on a small change around a specific value of one parameter. As similar interpretation is given by Tirole (2001).
3.2.1 The ability of the manager to appropriate profits

This is the most straightforward implication of the model. The higher the ability of the manager to hide profits and transform them into managerial benefits, the greater the cost of delegation and the smaller the willingness of the entrepreneur to separate ownership from control. It is also one of the predictions of the model which fits with both the model of Demsetz (1983) and the empirical findings of Demsetz and Lehn (1985). Firms with greater opaqueness of operation, where it is impossible or very costly to verify whether the incurred expenditure is necessary for the operation of the firm (businesses in service sector, research oriented institutions), are better kept under the tight control of the entrepreneur, even if this implies higher rents to employees. On the other hand, businesses with established management practices and good monitoring devices (manufacturing sector) lend themselves more easily to the separation of ownership and control.

This is also related to the finding that young firms are much more likely to be controlled by entrepreneurs than managers, while the opposite is true for old firms. A potential explanation is that, in the former case entrepreneurs have not yet acquired the necessary experience and skills for setting-up appropriate monitoring devices in their absence, while the reverse is true for the latter case. In other words, if one lets \( q \) to be a decreasing function of the experience of the entrepreneur, then our model can be made consistent with this empirical finding.

3.2.2 The private benefit of the worker

The fact that the value of delegation increases with \( b \) is a direct implication of the way we set-up the model. The real purpose of the manager and his true distinction from the entrepreneur is that he is initially an outsider, who has no stake in the firm. Through the design of the managerial contract, he is employed in order to make the threat of termination of the delayed project credible and, therefore, reduce the efficiency wage by the level of the private benefit that \( W \) enjoys in each period. Therefore, it comes as no surprise that the higher is the value of the private benefit, the higher is value of delegation for the entrepreneur.

Note that our model presents a distinctive feature here in comparison with the early literature. While the opaqueness of managerial decisions (\( q \)) reduces the value of delegation, the opaqueness of the worker’s decisions (\( b \)) actually increases it. In the early theories of ownership and control (Jensen and Meckling, 1976; Demsetz, 1983) this distinction is not present. According to these theories, increasing the opaqueness of the organization at any layer reduces the willingness of the owner to relinquish control. In our model, this prediction holds for the higher layers of the organization but not for the lower ones. Therefore, our model generates a new empirical prediction: firms whose workers gain more by being employed are more likely to be run by managers than entrepreneurs.

However, one needs to exercise caution when interpreting the term “gains”. It may refer to non-monetary rewards that employees receive or extract from the organization.
These could be psychological benefits, like esteem from working in a very well-known and established firm, or non-pecuniary rents, like free access to phone services, which are provided by the nature of the job. But the term may also refer to monetary rewards, as long as they are not directly controlled by the firm. This could be due to the workers having bargaining power in negotiating their wages (strong presence of unionized labor) or due to minimum wage policies that increase the wage level above what the firm would otherwise pay. In any of these cases, our model predicts that running the firm by managers may actually decrease effective wage cost, by reducing the incidents of work being delayed, and, therefore, that delegation of control should be more frequently present under these conditions.

### 3.2.3 The other factors of efficiency wages

More interesting is the fact that the other factors, which determine the efficiency wage (the probability of delay, \(1-p\), and the differential cost of effort, \(c_H - c_L\)), also contribute in the same way as \(b\). One might think that these factors play no role, as this part of the efficiency wage has to be paid by both the entrepreneur and the manager, if they are to induce \(W\) to exert high effort. However, a higher efficiency wage leaves less profits for appropriation by the manager and makes him cheaper to hire. As a result, holding everything else constant, a higher cost of employment makes the option of delegation of control more attractive to the entrepreneur.

While this result seems rather counter-intuitive, its empirical implication is clear: Firms with lower profits per worker (or perhaps, more clearly, per effective unit of control) provide less potential for rent extraction by the managers and delegation of control should be observed more frequently in these organizations. That does not, however, mean that smaller firms are more likely to be run by managers than larger firms. To see this consider the following slight extension of the model.

Suppose that the firm has \(n\) available projects, each one of which operates as the project of section 2 and 3.1. The total value of all projects is equal to: \(V(n) = nV - \psi(n)\), where \(\psi(n)\) is an increasing and convex function of the number of workers, representing the dis-economies of scale generated by having multiple workers in the firm. Hence, \(V(n)\) is a concave function. As a result, equation 5 now writes as:

\[
nb \geq q \int_{0}^{+\infty} \pi_1^m f(\pi_1^m|\mu(n)) d\pi_1^m
\]

where \(\mu(n) = \tau + nV - \psi(n) - \frac{c_H - c_L}{1 - p}\)

An increase in the number of available projects increases the cost of the firm by \(b\) in the case it is run by the entrepreneur, while it increases the cost of the firm by

\[
q \frac{\partial \mu(n)}{\partial n} \int_{0}^{+\infty} \pi_1^m f(\pi_1^m|\mu(n)) \frac{\pi_1^m - \mu(n)}{\sigma^2} d\pi_1^m,
\]

if it is run by a manager. Clearly, there exists a
cut-off value \( n^m \) above which the marginal cost of an E-run firm is higher than the marginal cost of a M-run firm, which means that, above \( n^m \) the set of parameters that satisfy equation 6 increases. The interpretation, then, is that firms which have more projects or employees than some threshold are more likely to be manager-controlled and entrepreneur-controlled. In other words, our model is consistent with the empirical finding that separation of ownership and control is more likely for larger firms (Demsetz and Lehn, 1985).

In order to be as clear as possible, we recapitulate the empirical prediction of our model regarding efficiency wages: Then lower is the profit per worker for the firm, the more likely separation of ownership and control is. Therefore, given everything else, an increase in the efficiency wage increases the possibility of delegation of control to the manager. On the other hand, an increase in the number of control units (workers and control units are equivalent in our model) increases the possibility of delegation, if this number is sufficiently high, and has ambiguous effects, if this number is below a certain threshold (\( n^m \)).

### 3.2.4 The average profitability of the firm and the project

A similar kind of reasoning as above applies with regards to the parameters \( \tau \) and \( V \). If the average profits of the firm or the profits of the project increase, then there is greater scope for rent extraction by the manager and the value of delegation goes down. Again, in terms of empirical predictions, the distinction between the effects of an increase of total profits due to an increase in the total number of projects or due to an increase on the profitability of each project applies in this case as well (see also the discussion in the previous subsection).

### 3.2.5 The variance of profits

Finally, an increase in the variance of profits (\( \sigma^2 \)) increases the expected payoff of the manager, \( E[y_1(\pi_1^m)] \), and decreases the value of separation of ownership and control. This is because an increase in \( \sigma^2 \) makes high-profit states more likely. Since the managerial reward is an increasing function of profits, so as to prevent rent extraction, higher variance increases the expected reward of the manager and, hence, the expected cost for the entrepreneur. This prediction of the model is also consistent with many of the empirical studies on this topic (Demsetz and Lehn, 1985).

### 4 Discussion

There are some theoretical issues of the model of section 3.1, which we have left for discussion in this section for the interested reader. We discuss each one in turn in order to demonstrate that our model is robust to certain theoretical concerns.
4.1 Participation constraints

In section 3 we simplified the analysis by omitting the participation constraints of the manager and the worker. Since both of them have a non-negative utility in equilibrium, the results of the previous section remain the same if we were to assume that the outside option for both M and W is equal to zero. Here, we discuss how these results change with the inclusion of more general participation constraints.

First, let $o^w$ and $o^m$ denote the outside options for the worker and the manager respectively. Clearly, if the outside option of the worker is less than the efficiency wage under delegation ($\hat{w}_1$), then the cost of hiring him remains unchanged under both control structures. Therefore, which one is preferred also depends on the outside option of the manager. If $o^m$ is below the expected value of the optimal contract (as provided by equations 7 and 8 in the Appendix), then our results remain unchanged. If $o^m$ lies above the expected value of the managerial contract, then the cost of hiring the manager is $o^m$ and delegation is still the optimal control structure if $o^m \leq b$.

In the case where $o^w$ is equal or above $\hat{w}_1$, but below $\hat{w}_1 + b$, then the benefit from delegation falls to $\hat{w}_1 + b - o^w$ and the optimal control structure is determined by the comparison between $\hat{w}_1 + b - o^w$ and $\max\{E[y_1(\pi_1^m)] , o^m\}$. Finally, in the case where $o^w$ is equal or above $\hat{w}_1 + b$, then there is no benefit from delegation and the only optimal control structure is the combination of ownership with control.

How comparative statics change or remain the same as in section 3.2 is fairly straightforward and, for the sake of brevity, is left to the interested reader\textsuperscript{9}.

4.2 Renegotiation proofness of the managerial contract

In the models of section 2 and 3.1, the issue of the renegotiation-proofness of the managerial contract arises after the delay shock has occurred. This is because, after the manager finds out that the project will be delayed, he realizes that he needs to undertake a sub-optimal action: to terminate a project which has a strictly positive continuation value. He can improve on his situation by communicating with E in order to change the terms of the contract. The renegotiated contract would provide an increase to the payoff of both parties by giving incentives to the manager not to terminate the project and sharing the surplus generated in period two.

Moreover, the terms of the contract could be such that E would infer from them that only a manager, who knows that the delay will occur, has an incentive to propose such terms, and this would make M’s proposal credible. But in this case, the termination of the project will not happen in equilibrium, and knowing this, W will shirk unless provided with an increase in his wage equal to $b$. In other words, the prospect of the renegotiation of the managerial contract undoes the credibility of the threat of project termination in case of delay and destroys any benefits generated by the delegation of control.

\textsuperscript{9}They are also available by the author upon request.
The issue of renegotiation-proofness has generated its own theoretical literature and we do not intend to address it here in its full generality\textsuperscript{10}. We note, however, that one can find theoretical solutions around this problem. A solution, which is relevant to our context is the use of a golden parachute. The interpretation here is that part of the terms of the original managerial contract is a clause which defines a large compensation for the manager if the other terms of the contract are renegotiated or modified in any way. Such a clause could make the managerial contract too costly to renegotiate for the entrepreneur and forestall any change in its terms.

The catch is, of course, that the manager should not be able to rescind this clause along with the other terms of the contract. This would require a legal system where the golden parachute is recognized as a “senior” right to any other contractual rights of the manager, or, in other words, that, even if the manager rescinds his right to the golden parachute with some later contract, any court will recognize his claim to it, if he asks to. Such an institutional arrangement would ensure that managers always demand their compensation after any alteration of the original agreement and would stop the entrepreneurs’ efforts to change them (as long as changing the institution itself is much more costly for E and M than continuing with their current arrangement).

In reality, we do not observe laws of this kind. But since such a theoretical solution exists, the fact that we do not observe it can mean that either there are other, more efficient, solutions to this problem or that it is not nearly as important as economic theory suggests. In either case, we believe that it is not a crucial concern for our model.

4.3 Optimality of the mechanism

We now consider two alternative solutions to the problem we have presented in sections 2 and 3.1. We also discuss under what conditions they are optimal to the solution of delegation and vice versa. The first one is a governmental policy, which taxes away all profits of the firm in period two. The second one is issuing claims on the profits of the firm to financial markets.

As far the the first one is concerned, taxing away the profits of period two destroys the continuation value of the project and makes the threat of termination credible. This is because, as soon as E finds out that the project will be delayed, he prefers to shut it down in order to avoid the continuation costs. The result is the same as delegating decision power to the manager: E can reduce the efficiency wage of the worker and increase his payoff by \( b \).

However, the policy is not costless for the entrepreneur. Since the continuation of the project increases average profits for the firm in period two, the governmental policy can solve the time inconsistency problem of E only if it taxes away \textit{all} of the profits of the firm in this period. Therefore, the expected cost of the policy is equal to \( \tau \). Hence, taxing away profits is a better mechanism than delegation only if the expected profits of the firm in period two (excluding the project value) are less than the expected payment

\textsuperscript{10}See, for example, Dewatripont and Maskin (1990) and Maskin and Moore (1999).
of the manager. Otherwise, separation of ownership and control remains an optimal solution.

The second solution is more interesting. According to it, the entrepreneur issues financial claims on the future stream of profits of the firm in the beginning of period one and sells them to financial markets. One such financial claim is for instance a claim on all profits of period two, which can be sold for a price equal to $\tau$. Another potential claim is the one which replicates the state-contingent payoff of the entrepreneur in period one when delegation is used, which is sold for a price equal to $V_{E}^{EMW}$.\(^{11}\)

Both of these claims work equally well. The first one replicates the effects of the governmental policy, discussed above, while the second one replicates the incentives provided by the managerial contract. Moreover, both of them have the additional benefit that the entrepreneur retains the value of the claims he is selling by receiving the price. Indeed, for the models of section 2 and 3, selling financial claims is a costless way for the entrepreneur to commit not to continue the project, if a delay occurs. In other words, delegation is useless in terms of our model, if well functioning financial markets are available.

The main issue, however, with this solution is that financial markets usually suffer from adverse selection. While we have avoided to complicate the model of 3.1 for the sake of expository clarity, it is easy to make the point here. If the initial profitability of the firm is observable to the entrepreneur but not to outsiders, then any claim issued by a high quality entrepreneur will suffer a market discount and this is the true cost of financial markets. If this is sufficiently high then the entrepreneur may still prefer to delegate control to a manager, who suffers from moral hazard, than issue underpriced securities. In other words, we effectively handicapped delegation as a potential solution to E’s problem when we added managerial moral hazard to the problem, while we assumed perfect information about the quality of the firm. But a fair comparison between delegation and the use of financial markets requires to consider the information problems on both sides\(^{12}\).

From the above discussion, we conclude that delegation is an optimal way to solve the time-consistency problem of the entrepreneur (are, at least, a subset of entrepreneurs) if the managerial moral hazard is not severe enough ($q$ is low), if the expected profits

\(^{11}\)It is essentially $\pi_{t}^{e} - y_{t}(\pi_{t}^{e})$, where $y_{t}(\pi_{t}^{e})$ is provided by equation 7 in the Appendix.

\(^{12}\)The model of section 3.1 can easily accommodate both sides of the problem. Just let two different types of firms, one with high profits, $\tau_{H}$, and one with low profits, $\tau_{L}$, where the type of the firm is private information to the entrepreneur at the start of the period. Then, for a sufficiently high enough difference of profits between the two types, type H entrepreneurs find it optimal to hire a manager and avoid the mispricing of their securities, while type L entrepreneurs prefer the financial markets. Note that even though one prediction of this extension is the same as in Acemoglu (1998) (high quality firms hire a manager, low quality firms do not), there is an important difference. In our model firms do not require external capital and only low quality firms sell securities to the markets, while in Acemoglu (1998) firms need financial capital for investment and, in equilibrium, all of them borrow from financial markets. In other words, in Acemoglu (1998), delegation is used as a signaling device towards financial markets, while for us markets is a competing mechanism to delegation and operates as a commitment device.
of the firms are high ($\tau$ is high), and if financial markets suffer from severe adverse selection. Therefore, even though the solution we propose is not always an optimal solution, it remains the only optimal solution under specific parameter values.

5 Conclusion

The paper presents a simple model of delegation of corporate control from an entrepreneur to a manager. We thus provide a theoretical reasoning for the separation of ownership and control in modern firms. The main reasoning behind our model is that managers can impose penalties to procrastinating workers more credibly than entrepreneurs, because, by construction of their contract, they do not care about the long-run value of the firm as much as its owners. On one hand, this reduces the efficiency wage paid to workers and generates firm value. On the other hand, the introduction of managers in the firm generates agency costs in the form of appropriation of profits for the provision of private benefits to the top management. This trade-off between low-tier and higher-tier benefits characterizes the optimal choice of control structure and provides interesting comparative statics. Some of our predictions are consistent with the findings of the empirical literature, while others remain to be tested yet.

We would also like to note that our paper is related with the theoretical literature regarding delegation. The literature so far focuses on how to optimally design the action-set of the agent, but takes delegation of decision power as given. Examples of this literature are Holmstrom (1982), Faure-Grimaud, Laffont, and Martimort (2003) and Alonso and Matouschek (2008). However, as we noted above, delegation of decision power is endogenously determined in our model. The same claim, of course, can be made for Acemoglu (1998). We hope that we can extend these theoretical examples to more general cases and provide a more complete theory of endogenous delegation.
References


Appendix

Proof of Proposition 1

We find the optimal contract by minimizing the expected payment to the entrepreneur conditional on IC$_1$-IC$_4$. First, IC$_1$ implies that the managerial contract is a strictly increasing function of profits. Otherwise, the manager has the incentive to destroy profits in the neighborhood of any realized profit where his contract is non-increasing. By doing so, he does not reduce his compensation while extracting private benefits for himself. In fact, by differentiating IC$_1$ with respect to $r_t$, we find that the manager is indifferent between extracting more rents or reporting the true profits if \( \frac{\partial m}{\partial \pi_t} = q \). This means that the managerial contract is an increasing function of reported profits, with slope at least equal to $q$ in order to prevent the manager from extracting private benefits. Clearly, the expected payment is minimized when $y_t = q\pi_t^e$.

Due to risk neutrality, IC$_2$ can be satisfied with approximately zero cost to the entrepreneur by using an argument similar to Mirrlees (1975). Since we do not allow for negative rewards, E needs to provide very high rewards to M whenever profits reach some arbitrarily large threshold. To see this more clearly, consider the linear contract with slope equal to $q$, if profits are between zero and some threshold $\bar{\pi}$, at which point the manager receives a bonus $M$, and zero otherwise:

\[
y_1 = \begin{cases} 
0, & \text{if } \pi_t^e < 0 \\
q\pi_t^e, & \text{if } 0 \leq \pi_t^e < \bar{\pi} \\
q\pi_t^e + M, & \text{if } \pi_t^e = \bar{\pi} \\
0, & \text{if } \pi_t^e > \bar{\pi}
\end{cases}
\]

\[
y_2 = \begin{cases} 
0, & \text{if } \pi_t^e < 0 \\
q\pi_t^e, & \text{if } \pi_t^e \geq 0
\end{cases}
\]

Using the above managerial contract and the fact that $\hat{w}_0 = 0$, IC$_2$ is rewritten as:

\[
\int_0^{+\infty} q\pi_1^m f(\pi_1^m | \tau) d\pi_1^m + M f(\bar{\pi} | \tau) \geq \int_0^{+\infty} q\pi_1^m f(\pi_1^m | \tau-k) d\pi_1^m + M f(\bar{\pi} | \tau-k) + \int_0^{+\infty} q\pi_2^m f(\pi_2^m | \tau+V) d\pi_2^m \Rightarrow
\]

\[
M(\bar{\pi}) = \frac{qP}{f(\bar{\pi} | \tau) - f(\bar{\pi} | \tau-k)}
\]

24
\[ P = \int_{0}^{+\infty} q\pi_1^m f(\pi_1^m | \tau - k)d\pi_1^m + \int_{0}^{+\infty} q\pi_2^m f(\pi_2^m | \tau + V)d\pi_2^m - \int_{0}^{+\infty} q\pi_1^m f(\pi_1^m | \tau)d\pi_1^m \]

The additional expected loss for the entrepreneur by offering the above managerial contract (as opposed to the simple linear contract \( y = q\pi \)) is equal to:

\[ \text{Additional Expected Loss} = f(\pi | \tau)M(\pi) + (1 - q) \int_{\pi}^{+\infty} \pi_1^m f(\pi_1^m | \tau)d\pi_1^m \]

\[ = \frac{f(\pi | \tau)P}{f(\pi | \tau) - f(\pi | \tau - k)} + (1 - q) \int_{\pi}^{+\infty} \pi_1^m f(\pi_1^m | \tau)d\pi_1^m \]

Now, because \( \frac{\partial}{\partial \pi} \left( \frac{f(\pi | \tau)}{f(\pi | \tau) - f(\pi | \tau - k)} \right) < 0 \) (recall that \( f \) is normally distributed), we have that \( \lim_{\pi \to +\infty} \left( f(\pi | \tau) - f(\pi | \tau - k) \right) = 0 \), and, therefore, the additional expected loss for the entrepreneur approaches zero as the threshold value becomes arbitrarily large. For the remainder of the proof, we assume that \( \pi \) is large enough, so that the expected loss of the entrepreneur can be neglected.

Furthermore, it is clear that the same contract also satisfies IC3, as the left hand side assigns higher probability to higher managerial payoffs than the right-hand side. In other words, if there is no delay in the completion of the project, the manager strictly prefers not to terminate it, as this generates a probability distribution over payoffs which first-order stochastically dominates the distribution generated by termination.

The last incentive compatibility constraint can be written as:

\[ \int_{0}^{+\infty} q\pi_1^m f(\pi_1^m | \tau + V - \hat{\omega}_1)d\pi_1^m \geq p \int_{0}^{+\infty} q\pi_1^m f(\pi_1^m | \tau + V)d\pi_1^m + (1 - p) \int_{0}^{+\infty} q\pi_1^m f(\pi_1^m | \tau)d\pi_1^m \]

Substituting \( \rho \) for \( \pi_1^m (\pi_1^m = \mu_1 - \tau + \rho) \) and doing the calculations, we get:

\[ (V - \hat{\omega}_1)(1 - F(\hat{\omega}_1 - \tau - V)) + \tau F(-\tau) - F(\hat{\omega}_1 - \tau - V) + \int_{\hat{\omega}_1 - \tau - V}^{-\tau} \rho f(\rho)d\rho \geq p \left( V(1 - F(-\tau - V)) + \tau F(-\tau) - F(-\tau - V) + \int_{-\tau - V}^{-\tau} \rho f(\rho)d\rho \right) \]

\[ \Leftrightarrow \]

\[ p \leq \frac{\int_{\hat{\omega}_1 - \tau - V}^{-\tau} \rho f(\rho)d\rho}{V(1 - F(-\tau - V)) + \tau F(-\tau) - F(-\tau - V) + \int_{-\tau - V}^{-\tau} \rho f(\rho)d\rho} \]

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The left-hand side of the above inequality is an increasing function of $p$, while the right-hand side is positive, but decreasing function of $p$. Therefore, by continuity, there exists a threshold $p^*$ such that for every $p < p^*$, the above inequality holds. Therefore, we have demonstrated that below this threshold value of $p$, the managerial contract described by 7 and 8 satisfies $\mathbf{IC}_1$-\(\mathbf{IC}_4\) and at the same time minimizes E’s expected payment. ■