Boukouras, Aristotelis; Koufopoulos, Kostas

Working Paper
Political Competition, Ideology and Corruption

Courant Research Centre: Poverty, Equity and Growth - Discussion Papers, No. 58

Provided in Cooperation with:
Courant Research Centre 'Poverty, Equity and Growth in Developing and Transition Countries', University of Göttingen

Suggested Citation: Boukouras, Aristotelis; Koufopoulos, Kostas (2011) : Political Competition, Ideology and Corruption, Courant Research Centre: Poverty, Equity and Growth - Discussion Papers, No. 58, Courant Research Centre Poverty, Equity and Growth, Göttingen

This Version is available at:
http://hdl.handle.net/10419/90532

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.
If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Courant Research Centre
‘Poverty, Equity and Growth in Developing and Transition Countries: Statistical Methods and Empirical Analysis’

Georg-August-Universität Göttingen
(founded in 1737)

Discussion Papers

No. 58

Political Competition, Ideology and Corruption

Aristotelis Boukouras, Kostas Koufopoulos

January 2011
Political Competition, Ideology and Corruption*

Aristotelis Boukouras† Kostas Koufopoulos‡

April 24, 2008

Abstract

This paper presents a model of political competition, where voter decisions are affected by their ideological adherence to political parties. We derive a number of interesting results: First, we show that an equilibrium exists even though voting is fully deterministic. Second, although politicians, because of deterministic voting, can win an election with certainty by making concessions to voters, they choose to win the election only with some probability in order to maximize their expected rents. Third, if the distribution of ideology is asymmetric, then political parties follow different platforms in equilibrium. Finally, our model generates two novel empirical predictions, which, to the best of our knowledge, have not been tested yet: i) the higher the ideological adherence to a political party the more inefficient policies this party will follow, ii) the higher the number of extra votes required for election victory (the super-majority requirement) the higher the degree of corruption.

Keywords: corruption, political instability, voting behavior

JEL Classification: G21, G28, H32, P16, P43

*We would like to thank Theodoros Diasakos, Alex Stremme, Dezso Szalay, Spyros Vasilakis and especially Herakles Polemarchakis for their insightful discussions and comments. All remaining errors are ours.
†University of Warwick, Department of Economics, a.boukouras@warwick.ac.uk
‡University of Warwick, Warwick Business School, Kostas.Koufopoulos@wbs.ac.uk
Corresponding author: Kostas Koufopoulos
1 Introduction

The extent to which some market failures (monopoly of power, externalities and the free-rider problem) impair social welfare necessitates the use of political institutions of government selection and empowerment in order to limit their negative effects. But since political institutions themselves are man-made constructions they tend to suffer from a similar set of problems as their market counterparts so that a natural question arises: To what extent can we expect such social institutions to improve social welfare and how significant is the welfare loss associated with their functions?

Two different strands of literature have tried to answer this question. On one hand, the voting literature emphasizes the role of political competition between parties as an efficiency force, much like market competition (the Chicago school: Stigler 1971, Becker 1983, Wittman 1983), but also its shortcomings in the presence of probabilistic voting and information asymmetries (the Virginia school: Nelson 1976, Tullock 1983, Palfrey and Poole 1987). According to this point of view, if competition between parties is frictionless, then the desire of politicians to be elected in combination with the selfish motives of voters to support the most beneficial for them policies should be sufficient conditions for the selection of the most socially preferred outcomes. But if voters have heterogeneous preferences for politicians or for their representing ideologies and if other frictions, in the form of uncertainty or information asymmetry, exist, then political competition on its own can not guarantee socially desirable outcomes.

On the other hand, a growing body of literature assumes away the problem of policy selection and focuses on the issue of policy implementation instead. Even if government were comprised of benevolent leaders, who choose the most socially valuable objectives, they still need to realize these objectives through government agents, who may not share the same values. In addition, it is most likely that bureaucrats, through their occupation with governmental affairs, acquire some form of private information so that an agency problem arises between them and their hiring authority (Schleifer 1993, Banerjee 1997, Acemoglu, Verdier 2000). The immediate consequence of this is the emergence of informational rents for the bureaucrats, which usually take the form of corruption, and the reduction of social welfare.

Our paper is mostly related to the first strand of literature, though it touches some aspects of the second as well. We develop a simple yet very general model of political competition between two parties (or politicians), which make proposals over future policies to voters in order to earn their support and get elected to the government. Voters have heterogeneous preferences over the two parties, which may reflect their ideological adherence to each one of them or any other party characteristic that differentiates political contesters in the eyes of the public. In addition, each party requires a minimum amount of extra voters in comparison to its rival to secure election victory; otherwise it will win the elections only with some probability. We assume that this requirement

---

1A third strand of the economic literature is concerned with the transition process from an inefficient set of political institutions to more efficient institutions. For example, see Acemoglu (2003) or Acemoglu, Robinson (2000, 2001).
is an exogenous parameter and we carry out comparative statics exercises in order to make comparisons between different electoral rules.

The above conditions create an environment where securing political victory is costly for self-interested politicians and hence political competition is imperfect. Election candidates make inefficient proposals to voters because they understand that it is very costly for their opponent to propose more attractive policies. In other words, the combination of heterogeneous voters’ preferences and the super-majority requirement weaken political competition as contestants need not sacrifice all of their benefits from inefficient policies in order to receive enough votes for winning the election with certainty.

More specifically, we consider an economy where there are positive and negative net present value (NPV) projects, possessed by entrepreneurs, who have zero wealth and so they seek external funds to undertake their projects. For simplicity and without loss of generality, we assume that there are only state-owned banks, which have available funds and compete for attracting entrepreneurs with positive NPV projects \(^2\). If a politician (party) is elected in office, then he can exert pressure on the banks’ managers to channel a part of their funds to socially inefficient projects, which can earn him rents (or, alternatively, corruption bribes). Such an act will also imply a degree of taxation on the profits of the socially efficient projects so that the collected taxes are provided to state-owned banks to cover their losses.

Unfettered political competition acts as a discipline device which awards victory to the least corrupted politician, but the super-majority requirement and ideological adherence distort the outcome of elections. As they expect their opponents to do the same, candidates will make proposals with positive taxation level, implying a certain mis-allocation of funds and corruption level in the government. The greater the ideological adherence of individuals to party identities or the greater the super-majority requirement, the greater the frictions to political competition and the inefficiencies generated by the political system.

For expositional purposes, in section 2 of the paper we present the equilibrium outcome of the political game when ideological adherence of voters is symmetrically distributed between the two political contesters. The most interesting part of our paper is section 3, where we generalize our model for the case where the ideological distribution is potentially asymmetric. There we provide a novel insight on political competition.

In our model voting is deterministic, in the sense that given the platform a politician offers he knows exactly how many votes he will receive. As a result, in the asymmetric case, the politician (party) whose supporters exhibit the highest degree of ideological adherence could propose a platform implying lower (but still positive) rents for him, which at the same time secures him the victory in the election. However, the politician

---

\(^2\)All of our results would go through if we allow for both private and state-owned banks competing for attracting efficient projects. Clearly, private banks would never finance a negative NPV project. Of course, the project financing we use here is just an example. Our argument holds true in any situation where voters resort to the government to get a good or service, because they cannot acquire it in the marketplace.
strictly prefers to spend all his “ideological capital” in more inefficient policies (higher rents) and win the election with some probability (less than one). In this sense, the politician prefers a situation of political uncertainty with regard to the election outcome as opposed to a situation of political certainty, so that we can say that political uncertainty arises endogenously in the model.

Our model can also have a different interpretation. Political parties and electoral systems are well-defined concepts of smoothly functioning democracies, but in many regions of the world competing social groups vie for power and the control of government. In some cases it is not clear who rules the country (economy) as political instability means that the control of government changes hands frequently and unpredictably. Our framework can provide some insight on such complex phenomena.

In such environments, we can interpret parties as social groups, which represent different ideologies or social values, competing for the control of the government. The super-majority requirement can alternatively be seen as the minimum number of supporters that each side needs to consolidate its power and to remain unchallenged by the competing group. If a group does not receive enough support to pass this exogenous threshold, then it will prevail over its competitor only probabilistically. Political leaders offer policy plans to civilians to attract them on their side and achieve political stability, but their selfish incentives imply that they make inefficient proposals and prefer to remain in the politically unstable region of competition.

Though it is not in our intention to present a complete theory of institutional instability and regime shifts, our model generates a noteworthy prediction: the higher the polarization or the super-majority requirement, the higher the degree of corruption and social inefficiencies that will tend to prevail in a given society. This prediction is consistent with the empirical observations from highly unstable regions, and especially Africa (Levine, Easterly, 1997).

The paper makes the following contributions: First, from a technical point of view, we show that a Nash equilibrium exists even though voting is fully deterministic (in the sense that we have described above) and there are more than one policy dimensions (multi-dimensional setting). Many existing papers argue that probabilistic voting is necessary for the existence of a Nash equilibrium in multi-dimensional policy environments, even though they allow for infinitely many voters. Therefore, our result reveals that, if there are infinitely many agents (voters), the assumption of probabilistic voting is redundant.

---

3In contrast, in models of stochastic voting it is not the politicians’ choice to win the election with some probability. It is imposed to them by the structure of the model.

4See also Alesina, Baqir and Easterly, 1999.

5The assumption of a continuity of agents (voters) ensures the continuity of the response functions of politicians so that at least one Nash equilibrium of the voting game always exists, even if political proposals are multi-dimensional. On the contrary, Lindbeck and Weibull (1987) assume a finite number of agents and therefore require probabilistic voting in order to ensure the existence of a Nash equilibrium.

6It should be stressed that the stochastic electoral rule that we use in our model is not a disguise for stochastic voting. We have chosen this stochastic electoral rule because we want to treat the two parties
Second, the introduction of deterministic voting allows us to derive an interesting result. In our model, political uncertainty arises endogenously. A politician could have won the election with certainty by making more concessions to voters, but instead he prefers winning the election with some probability (less than one) because this strategy implies higher expected rents for him.

Third, regarding empirical implications, the model generates three novel predictions: i) political parties, which have greater ideological support, will tend to favor more inefficient policies and foster greater corruption than those parties with little ideological support, ii) the greater the asymmetry in the ideological adherence to parties, the greater the difference in the proposed policies. To the best of our knowledge, these predictions have not been empirically tested yet.

There exists, of course, a number of papers that is related to ours. Persson and Tabellini (1999) study the effects of different electoral systems on political competition. In their model voters exhibit ideological adherence to political parties and voting is probabilistic. They show that majoritarian systems imply tougher competition and hence lower political rents than proportional systems. This paper differs from theirs in two respects. First, in our case, voting is fully deterministic. Second, we focus on proportional systems, but we consider the effect of the number of extra votes a party needs to form a government on political competition (and political rents) 7. Persson and Tabellini (1999) do not consider this super-majority (extra votes) requirement, which is observed in many proportional electoral systems. Our approach allows us to establish a relation between the number of extra votes and political rents. A greater super-majority requirement weakens political competition and results in higher political rents.

Lindbeck and Weibull (1987) construct a model with heterogeneous voter preferences and competition between parties over redistribution policies and show that who wins the election depends on probabilistic behavior and parties may follow different redistribution policies. Building on this, Coate and Morris (1995) show that when politicians have private information over their types and over the value of the public good, then redistribution policies can target special interests and be socially inefficient. The model we propose can also replicate these results. If voters’ preferences are heterogeneous then parties will make different proposals, which will contain a degree of social inefficiency.

\[\text{symmetrically. In our model, a Nash equilibrium exists even if the electoral rule is deterministic and the politicians can perfectly predict the number of votes they receive given their proposed platforms. For more details see footnote 13 in p. 11}^{5}\text{In proportional systems the election outcome depends only on the aggregate number of votes as opposed to majoritarian systems, where the outcome depends on the number of districts gained. In many proportional systems the electoral rule is such that in order for the first party (in terms of votes received) to form a government, a certain number of extra votes is required over the second party. In our formal model this number of extra votes is captured by }\epsilon, \text{ which we call the super-majority requirement. This is especially relevant in countries where at least three parties compete for gaining power. Although in this version of the paper we do not consider the case of three or more parties, in an extended version of the paper we have shown that our results go through even in this case.}^{7}\]
Helpman and Grossman (1996) are also concerned with political competition and voting preferences in a two-party system, but their focus is mainly on special interests groups and electoral contributions as a means of affecting political platforms (See also Jackson and Kingdon, 1992, and Dixit and Londregan, 1996 on how special interests may affect policy formation and efficiency). Though the terminology is different, one could view contributions and corruption as two similar ways to express rents that politicians receive in order to use their power and serve the interests of specific social groups. An important assumption of their model, though, is that a subset of voters is relatively uninformed over party platforms and could be persuaded to vote for one candidate or the other if enough advertising takes place. In the absence of such a group, politicians would receive no rents, as contributions would not have an impact on the probability of election victory. On the other hand, if we were to allow for their structure to operate on our environment then no inefficiency would arise, as the special groups with the highest private value for the available funds (in our model the private and social values coincide) would be the ones with the highest contributions to parties.

Our model differs from the aforementioned papers in two respects: First, in our case, differences in platforms are due to different degrees of voters’ ideological adherence to political parties. Second, we do not require any other friction (asymmetric information or probabilistic voting) to derive inefficiencies or rents to politicians.\textsuperscript{8}

In summary, in this paper we derive the following results: First, on technical grounds, we show that a Nash equilibrium exists even though voting is fully deterministic and there are more than one policy dimensions (multi-dimensional setting). Second, due to deterministic voting we are able to derive the following novel result: In our model, political uncertainty arises endogenously. A politician could have won the election with certainty by making more concessions to voters, but instead he prefers winning the election with some probability (less than one) because this strategy implies higher expected rents for him. Third, our model generates the following empirical predictions: i) The higher the degree of ideological adherence, the higher the political rents. This prediction is consistent with the empirical findings in Easterly and Levine(1997) and Svensson (2005). These authors report that political polarization is associated with less cost-efficient methods of production of public goods and is more likely to lead to higher corruption in the economy. We share this prediction with Persson and Tabellini (1999). ii) Another prediction in common with others (Coate and Morris (1995), Helpman and Grossman (1996)), although for different reasons, is that in equilibrium political platforms may be different. This prediction is consistent with the

\textsuperscript{8}In a different context, Alesina, Rosenthal (2000) show how divergent political platforms can be reconciled with political competition when institutional checks and balances imply that political parties must eventually bargain over the policy to be executed. In their paper politicians prefer polarized platforms because they give them greater flexibility and bargaining power in the negotiation stage. Anticipating that, voters understand that non-polarized proposals are not time consistent and hence vote for their most preferred party. Our model also predicts polarized platforms in equilibrium, though our focus is on the emergence of political rents and socially inefficient outcomes, rather on the asymmetries between contesters.
empirical findings in Poole and Rosenthal (1997) and Snyder (1996). iii) The greater the extra number of votes a party needs to form a government, the greater the political rents. iv) Political parties, which have greater ideological support, will tend to favor more inefficient policies and foster greater corruption than those parties with little ideological support. v) The greater the asymmetry in the ideological adherence to parties, the greater the difference in the proposed policies. To the best of our knowledge, the last three predictions have not been empirically tested yet.

The rest of the paper is organized as follows. Section 2 presents a simple model of political competition between two parties when the ideological preferences of voters are symmetric and derives a series of basic results. Section 3 generalizes the analysis of the previous section to the case where some or all groups of voters have asymmetric preferences over parties so that some party may be favored in a group over its rival. Section 4 discusses the results and concludes.

2 A symmetric model

The economy is assumed to last for a single period and consists of two basic categories of agents: On one hand, there is a continuum of entrepreneurs, whose projects differ with respect to their productivity. At the same time these entrepreneurs vote for the politician they prefer to be in power and his respective policy. On the other hand, there is a group of politicians, who propose several political measures and compete to get elected and receive the power-related benefits. Entrepreneurs have different preferred policies, according to their productivity, and different degrees of ideological adherence to a certain politician or political party, which allows for inefficient policies to be implemented and a certain degree of corruption to pertain in the economy. The timing of the model is shown in the next figure.

Figure 1: Timing of events

```
Figure 1: Timing of events
```
Entrepreneurs and Banks

There is a continuum of risk neutral entrepreneurs in the interval [0,1], who are divided into groups (types) according to the quality of their projects. There exist 2 types of entrepreneurs (projects), high quality (or efficient) projects ($\alpha_H$) and low quality (inefficient) projects ($\alpha_L$), with respective proportions $q$ and 1-$q$. Each project requires an amount $I$ of funds to start up, which will yield with certainty either $\alpha_H I$ or $\alpha_L I$ after 1 period, depending on the type of project. We assume that $\alpha_H > 1 > \alpha_L$. The quality of the project is publicly known and verifiable without any cost. Furthermore, by running the project the entrepreneur receives an unobserved, private benefit $b$, which has been deducted in the calculation of the net project-returns $\alpha_H I$ and $\alpha_L I$.

Let $\lambda$ denote the fraction of projects that can be undertaken by the available funds ($0 < \lambda \leq q$). These funds are provided to the entrepreneurs through state-owned banks, which do not operate exclusively under economic criteria, but are also subject to pressures by the political party in power \(^9\)\(^10\). Therefore the provision of funds for all efficient projects is not guaranteed, since politicians have an incentive to channel a part of the funds to low quality projects, from which they receive bribes (as long as $\lambda < q$) \(^11\).

If an inefficient project receives funds, after one period the entrepreneur will repay back to the bank an amount $r_L I$, which is determined by the politician, while the residual return from the project ($\beta = \alpha_L - r_L$) is repaid to the politician as a bribe. This means that if some inefficient projects receive funds, banks cannot recover the full amount of loans they provide to them and alternative means of financing are required through either taxation or higher repayment rates on good quality firms’ profits. Since the politicians ultimately have the power to extract profits from high quality entrepreneurs through taxation, they allow public banks to compete with each other for high quality projects, which makes high quality projects’ repayment equal to $I$, and use taxation as the sole means of rent extraction.

\(^9\)Khwaja and Mian (2005) and Dinc (2005) find that a significant portion of corruption transfers to politicians in emerging markets takes place through public lending to politically connected firms, which receive a premium rate compared to other borrowers. Our model is consistent with their empirical observations as well.

\(^10\)There is a growing body of empirical literature which relates corruption in developing countries with the activities of politically connected firms and seeks to determine the factors behind it. As example of this literature, see Mauro (1995), Svensson(1998, 2003), Ades and Di Tella (1999), Fisman (2001), Alt and Lassen (2006) and Faccio (2006).

\(^11\)Because we assume that all entrepreneurs receive the unobservable benefit $b$ from running a project, the social value of a high quality project will always be greater than the social value of a low quality project: $\alpha_H I + b > \alpha_L I + b$. In this case, the condition $\lambda \leq q$ implies that, because there are not sufficient funds for all the high quality projects to be undertaken, it is always suboptimal to fund any low quality project. We could relax this condition and let $\lambda$ be any value less or equal to 1, as long as another condition holds: $\alpha_L I + b < I \iff b < (1 - \alpha_L) I$, so that the social loss of lower productivity exceeds the private benefit of the entrepreneur. If this condition holds, then financing any low quality project is detrimental for social welfare. For reasons of expositional clarity, we stick to the former condition, though our results would go through if we assume the latter.
Politicians

There are two political players in this economy, $P_1$ and $P_2$, who can be interpreted as major political parties or more accurately as key politicians. These two vie for the control of the government. At the beginning of the period each one of them makes a public proposal of his intended policies to the set of voters, who, according to their preferences, choose who will govern. Once in government the elected politician has the power to set taxation for firms and the ability to exert influence over the decisions of the public banks on how to distribute the available funds for investment. If a certain level of funds is provided to inefficient projects in exchange for political support, then taxation is needed to cover the losses of banks.

The extent to which a politician can manipulate internal funds depends on the restrictions on the mismanagement of state-owned banks and his ability to remain in power despite these events. The assumption we maintain throughout the rest of the paper is that the politician has full discretion on how to allocate the available funds to the two types of projects. This means that the politician can only take decisions that do not lead state-owned banks to bankruptcy and this is a constraint he cannot manipulate. In addition, politicians have the power to bargain with low quality entrepreneurs on how the returns from their projects will be allocated between repayment to banks ($r_L$) and direct transfers to politicians (corruption bribes: $\beta$).

On the other hand, another set of restrictions is implicitly imposed by political competition and the fact that a pure predatory behavior from the politician’s part is unlikely to win him the election. More specifically, before the election takes place the two politicians publicly announce the set of politically controlled variables, namely the profit tax rate ($t$) and the number of efficient projects ($s^H$) that will be funded. We denote the political proposal by politician $P_i$ as: $P_i^R = \{t_i, s^H_i\}$

An implicit element of the proposal is the level of funds the politician appropriates from each low quality entrepreneur ($\beta_i I$), which can be considered as a political bribe or a reward for the politician for allowing inefficient projects to operate. This is never part of the public announcement but, given the proposal and the information structure of the environment, agents are able to infer the amount of proposed corruption as well as the repayment rate for inefficient projects ($r_L$) and the amount of low quality entrepreneurs who will receive funds ($s_L$). The total level of corruption in the economy is the bribe received from each low quality entrepreneur times the number of inefficient firms receiving funds: $B = \beta s_L I$.

In this framework we assume that political announcements are credible and that there are no commitment issues. Once in office, the politician implements his predetermined policy or otherwise he is thrown out of power and the other politician takes over $^{12}$.

$^{12}$We could alternatively obtain the same qualitative results by setting up a dynamic model with reputation effects and politicians competing for power given their credibility. This would give a more rigorous argument on why policies may be considered credible, but it would make the analysis much more complex and mathematically demanding.
Entrepreneurs as Voters

Entrepreneurs vote for their preferred candidate based on the expected utility they will derive from his policies and their ideological adherence. Each politician represents a specific ideology (ideology i for $P_i$ and ideology j for $P_j$, which are fixed for each politician), while voters have individual tastes which we assume that are uniformly distributed over the interval $[-M, M]$ within each group of types of entrepreneurs. $M$ shows the degree of adherence to the ideology i, or, in other words, it is the relative likeness or antipathy that an agent has to ideology i over j. We let the variable $d^i_k$ denote the relative preference of agent k ($k = \{h, l\}$) for ideology i. By definition the following equations hold:

$$d^i_h \sim \text{Uni}[-M, M]$$
$$d^i_l \sim \text{Uni}[-M, M]$$
$$d^i_k = -d^i_{\bar{k}}$$

where the subscripts h and l denote an entrepreneur of a high or low quality project respectively.

Figure 2: Ideological Distribution

Figure 2 shows the distribution of preferences for ideology i within each type of voters. We assume that the utility, which an agent derives from supporting his preferred ideology, is additively separable to the utility of expected wealth. The overall utility for a voter, conditional on his type, his ideological profile and the policy proposition of party i, is simply the sum of the expected profit and of his ideological adherence:

$$U_k(P^R_i) = E(\pi(P^R_i)) + \frac{d^i_k}{2}$$

For each type of agent utility can be written as:

$$U_h(P^R_i) = \frac{\frac{\beta^i_H}{q}[(\alpha_H - 1 - t^i)I + b] + d^i_h}{2}$$

(1)
The first multiplicative term is the probability of an entrepreneur to undertake the project under the proposed policy and the second term is the income he would receive in that case. The last term corresponds to the utility gain attached to ideological voting.

From the above equations and the fact that \( d_{ik} = -d_{jk} \) follows that an agent will vote for politician \( P_i \) (denoted as: \( v_k = 1 \)) iff:

\[
U_k(P_i^R) > U_k(P_j^R) \iff E(\pi_k(P_i^R)) - E(\pi_k(P_j^R)) + d_{ik} > 0
\]

**Elections and Politicians**

Let \( V^i \) be the total votes that \( P_i \) receives. In order to win the election we assume that the politician must receive a critical mass of \( \epsilon \) votes more than his competitor, where \( \epsilon \) can be an interval arbitrarily small. If \( P_i \) receives less votes than \( P_j \) by a difference at least as large as \( \epsilon \) then he loses the election, while if the difference is less than this threshold, he loses the election with only a probability equal to \( \frac{1}{2} \). The election rule is then specified as:

\[
p_{win}^i = 1, \text{ if } V^i - V^j \geq \epsilon
\]

\[
p_{win}^i = \frac{1}{2}, \text{ if } -\epsilon < V^i - V^j < \epsilon \text{ and}
\]

\[
p_{win}^i = 0, \text{ if } V^i - V^j \leq -\epsilon
\]

The critical mass of voters \( \epsilon \) is an important parameter of the problem at study. In many proportional systems the electoral rule is such that in order for the first party (in terms of votes received) to form a government, a certain number of extra votes

\[\text{It should be stressed that the stochastic electoral rule that we use in our model is not a disguise for stochastic voting. We have chosen this stochastic electoral rule because we want to treat the two parties symmetrically. To see this, suppose there are two parties, A and B. Consider the following election rule: Party B wins the election with certainty only if it gains a mass of } \epsilon \text{ votes more than party A. That is party A wins the election with certainty in all of the following cases: i) if party A receives more votes than party B, ii) if it receives the same mass of votes as B, iii) if party A receives less votes than B, but the difference is no more than } \epsilon \text{. In this case, in equilibrium, party B will propose a platform implying zero rents for itself and party A will propose a platform implying that it receives exactly a mass } \epsilon \text{ of votes less than party B and strictly positive political rents. Also, in equilibrium party A wins the election with certainty. Clearly, now, everything in our model is fully deterministic (including the election rule) and still equilibrium exists. In fact, the existence of the equilibrium is not related to stochastic voting or a stochastic election rule. Also, it does not depend on } \epsilon \text{. The only role of } \epsilon \text{ is to insure that political rents arise in equilibrium. If this } \epsilon \text{ is zero, then equilibrium still exists but political rents are also zero. Some papers argue that stochastic voting is required for the existence of an equilibrium, even if there is a continuum of voters. This argument is not quite right. In these papers stochastic voting is only needed for having an equilibrium with strictly positive political rents. However, if the number of voters is finite, then stochastic voting is indeed required for the existence of an equilibrium.} \]
is required over the second party. In our formal model this number of extra votes is captured by $\epsilon$, which we call the super-majority requirement. This is especially relevant in countries where at least three parties compete for gaining power. Higher values of this parameter mean that politicians need a higher majority to secure election and distort their proposals toward more inefficient policies.

Given the above specifications of the economy, politicians try to maximize their expected wealth, which consists of the appropriated part of the low projects returns:

$$\max U_p(P^R_i, P^R_j) = p_{\text{win}}^i B_i^\gamma$$

where $\gamma$ is the coefficient of risk aversion for politicians ($0 \leq \gamma \leq 1$). Politicians essentially try to maximize the expected appropriation of funds under the limitations that they have been imposed to them:

–the allocation of funds condition:

$$\left(s_H^i + s_L^i\right) I \leq \lambda I$$

(the available funds for investment can either be invested in efficient projects or inefficient ones)

–the allocation of inefficient projects’ returns:

$$\beta_i + r_L^i \leq \alpha_L$$

–the profitability of public banks condition:

$$r_L s_L^i I + s_H^i I + t_i s_H^i I - B \geq \lambda I$$

(the sum of the repayments by inefficient and efficient projects and aggregate taxation must be at least equal to the initial funds available for investment)

–the Election Rule and the commitment of execution of proposed policies.

In terms of mathematical expressions, each candidate tries to solve the following problem (P.1):

$$\max_{s_H^i, s_L^i, \beta_i, r_L^i} U_p(P^R_i, P^R_j) = p_{\text{win}}^i B_i^\gamma \quad \text{s.t.}$$

$$B_i = \beta_i s_L^i I$$

$$(s_H^i + s_L^i) I \leq \lambda I$$

---

14Although in this version of the paper we do not consider the case of three or more parties, in an extended version of the paper we have shown that our results go through even in this case.
\[ \beta_i + r_i^L \leq \alpha_L \]
\[ r_i^L s_i L + s_i H I + t_i s_i H I \geq \lambda I \]
\[ 0 \leq t_i \leq \alpha_H - 1 \]
\[ p_{\text{win}}^i = 1, \text{if } V^i - V^j \geq \epsilon, \quad p_{\text{win}}^i = \frac{1}{2}, \text{if } -\epsilon \leq V^i - V^j \leq \epsilon, \quad p_{\text{win}}^i = 0, \text{if } V^i - V^j \leq -\epsilon \]

**Monopoly of Power Solution**

As a benchmark case, it is interesting to study the autocratic case, where there is only one politician and whose power remains undisputed for all possible policies that do not violate the public banks’ profitability condition. The solution to this problem will be used later on as a comparison base for the results under political competition.

If we were to assume that there is a dictator in the economy, whose power is unchallenged, then we implicitly impose that the politician does not face the fear of losing his position through elections and hence it would be equivalent to imposing the condition \( p_{\text{win}} = 1 \) to P.1. In other words, under the assumption of autocracy, P.1 is transformed into (P.1a):

\[
\max_{s_H, s_L, \beta, r_L} U_p = B^{\gamma} \quad \text{s.t.} \\
B = \beta s_L I \\
(s_H + s_L)I \leq \lambda I \\
\beta + r_L \leq \alpha_L \\
r_L s_L I + s_H I + t s_H I \geq \lambda I \\
0 \leq t \leq \alpha_H - 1
\]

The above problem is easy to solve. First, notice that all inequalities will hold with equality in the final solution. If that was not the case, the politician would always increase his utility by increasing either the share of inefficient projects which receive funds or the level of the bribe until the restrictions are satisfied with equality\(^{15}\).

Keeping this in mind and by recursively substituting these conditions into the utility function we can rewrite the problem as an unconstrained one:

\[
\max_{\beta, r_L, t} U_p = \left[ (\alpha_L - r_L) \left( \frac{\lambda}{1 + \lambda - r_L} \right) I \right]^{\gamma} \quad \text{s.t.}
\]

\[^{15}\text{A mathematical way to verify this is to set the Langrangian for the maximization problem and then show that all Langrange multipliers are strictly positive so that the constraints are binding.}\]
\[
\beta = \alpha_L - r_L
\]
\[s_H = \lambda - s_L\]
\[s_L = \frac{\lambda}{1+t-r_L}\]
\[0 \leq t \leq \alpha_H - 1\]

By taking first order conditions with respect to \(r_L\) and \(t\), we get the following expressions\(^{16}\):

\[
\frac{\partial U}{\partial r_L} = -\lambda t I \frac{\alpha_L(1+t-r_L)(-\lambda I)}{(1+t-r_L)^2} = \frac{-\lambda I(1+t-r_L)+(\alpha_L-r_L)\lambda I}{(1+t-r_L)^2} < 0
\]

and

\[
\frac{\partial U}{\partial t} = (\alpha_L - r_L)I \frac{\lambda(1+t-r_L)-\lambda I}{(1+t-r_L)^2} = (\alpha_L - r_L)I \frac{\lambda(1-r_L)}{(1+t-r_L)^2} \geq 0
\]

The above conditions imply that in order the politician to maximize his utility he must set the inefficient projects’ repayment as low as possible and the tax rate as high as possible. The full set of the monopoly of power solution is:

\[
r_L = 0, \beta = \alpha_L, t = \alpha_H - 1, s_L = \frac{\lambda(\alpha_H - 1)}{\alpha_H}, s_H = \frac{\lambda}{\alpha_H}
\]

and \(U_{mp}^m = \left[\frac{\lambda \alpha_L(\alpha_H - 1)I}{\alpha_H}\right]^\gamma\)

As expected, the dictator, since he does not face any real threat to his power, sets taxation for efficient projects to its maximum possible level, expropriating all their profits, imposes no repayment to inefficient entrepreneurs, in order to receive maximum possible bribes, and balances the proportion of high and low quality projects financed so as to maximize his wellbeing.

This is a straightforward result that shows the degree of inefficiency and corruption that is generated by autocratic regimes in the particular set up of our model and we will regularly refer back to it for comparison purposes.

**Political Equilibrium**

We now return to the case of political competition between two candidates. Their power is not unchecked, but subject to the constraints imposed by the political game as described earlier. If a politician tries to maximize his own wellbeing, as a dictator

\(^{16}\)Because \(U_p^{\frac{1}{2}}\) is a monotonic transformation of \(U_p\), both of them have their maximum for the same values of \(r_L\) and \(t\). For simplification reasons, we derive the first order conditions of the former expression.
would do, then he will be undercut by his competitor, who will offer a more attractive option to voters and will win the election.

But winning the election is costly itself. If the two politicians make exactly the same political proposal to voters then none of them would win with certainty. Since ideological dispersion is symmetric by assumption, half of the agents would vote for either politician. We call the set of voters who vote for one politician over another, when both of them offer the same proposal, as the natural support group, because of their exogenous ideological preference.

If a rival wants to win the election with certainty, a required mass of \( \epsilon \) voters is required and these voters have already a certain degree of ideological adherence to the other candidate. In order to win their votes a politician will have to offer greater concessions to them in terms of political proposal than what he would have offered to his own supporters and this cost is increasing in relative terms as the concessions required for election victory increase. Therefore there is a cut-off point, which makes politicians indifferent between winning elections or sticking to their support group and taking over power with probability \( \frac{1}{2} \). Given the above intuition, the rest of this section is focused on providing a diagrammatic and analytical exposition of the solution of the political game and the main arguments behind the results.

First, we try to describe the set of political proposals for politician \( P_i \), which win him the election for a specific proposal by \( P_j \) and then compare the utility from winning an election with the utility from playing the same strategy as the opponent and winning with probability \( \frac{1}{2} \). Essentially, the political contestants have 2 different strategies. They can either try to win the election, which implies a relative benefit from the certainty of rising into power and a relative cost in terms of higher concessions to voters, or they can mimic their opponent in terms of proposal and wait for luck to determine who gets the power.

If a politician mimics his opponent, then he will receive \( \frac{1}{2} - q \) votes from entrepreneurs with low quality projects and \( q \) votes from entrepreneurs with high quality projects, for a total of \( \frac{1}{2} \). If, on the other hand, \( P_i \) deviates, he requires at least \( \frac{1 + \epsilon}{2} \) votes to win the election \(^{17}\). The amount of extra low quality voters he will receive from such a deviation (which can also be negative) is: 

\[
\frac{(s_i^L - s_j^L)b}{1-q} M \cdot \frac{1-q}{2}.
\]

\( \frac{(s_i^L - s_j^L)b}{1-q} \) denotes the excess monetary utility low quality entrepreneurs will receive by \( P_i \)'s proposal and when divided by \( M \) it denotes the proportion of inefficient entrepreneurs for whom the differential monetary utility exceeds their ideological adherence. Of course, in order to be consistent, we assume that \(-M \leq \frac{(s_i^L - s_j^L)b}{1-q} \leq M \) since the fraction of the two values can not exceed 1. \( \frac{(1-q)}{2} \) is the amount of voters who can be attracted from or lost to the opponent. The total expression can be rewritten as \( \frac{(s_i^L - s_j^L)b}{2M} \) and, by including the natural supporters, the total number of inefficient type voters is:

\(^{17}\)Recall that if \( i \) receives an number of \( \frac{\epsilon}{2} \) extra votes by changing his proposal, his opponent loses these votes, so that the total vote difference is equal to \( \epsilon \).
\[ V^i(P^R_i, P^R_j) = \frac{(1 - q)}{2} + \frac{(s^j_L - s^j_i)b}{2M} \]

Similarly, the amount of extra high quality voters a politician can receive by \( P^R_i \neq P^R_j \) is 
\[ s_H^i[\frac{(\alpha_H - 1 - t_i)I + b)}{2M} - s_H^j[\frac{(\alpha_H - 1 - t_j)I + b)}{2M} \], while the total number of efficient type voters is:
\[ V^i(P^R_i, P^R_j) = \frac{q}{2} + \frac{s_H^i[(\alpha_H - 1 - t_i)I + b]}{2M} - \frac{s_H^j[(\alpha_H - 1 - t_j)I + b]}{2M} \]

All this implies that the total support politician \( P_i \) should expect is equal to:
\[ V^i(P^R_i, P^R_j) = \frac{1}{2} + \frac{s_H^i[(\alpha_H - 1 - t_i)I + b]}{2M} - \frac{s_H^j[(\alpha_H - 1 - t_j)I + b]}{2M} + \frac{(s^j_L - s^j_i)b}{2M} \]

If he were to win, then the following condition should be satisfied:
\[ V^i(P^R_i, P^R_j) \geq \frac{1}{2} + \epsilon \]

Essentially, if the politician were to win the election he should solve the modified problem P.1 with the addition of condition (7):
\[
\begin{align*}
\max_{s^i_H, s^i_L, t_i, \beta_i, r^i_L} & \quad U_p(P^R_i, P^R_j) = B^3_i \\
\text{s.t.} & \quad B_i = \beta_i s^i_L I \\
& \quad (s^i_H + s^i_L) I \leq \lambda I \\
& \quad \beta_i + r^i_L \leq \alpha_L \\
& \quad r^i_L s^i_L I + s^i_H I + t_i s^i_H I \geq \lambda I \\
& \quad \frac{s_H^i[(\alpha_H - 1 - t_i)I + b]}{2M} - \frac{s_H^j[(\alpha_H - 1 - t_j)I + b]}{2M} + \frac{b}{2M} \geq \epsilon \\
& \quad -M \leq \frac{(s^j_L - s^j_i)b}{1-q} \leq M
\end{align*}
\]
\[-M \leq \frac{s_H^i(\alpha_H - 1 - t_i)I + b}{q} - \frac{s_H^j(\alpha_H - 1 - t_j)I + b}{q} \leq M\]

\[0 \leq t_i \leq \alpha_H - 1\]

The solution to this problem is provided below\textsuperscript{18}:

\[\tilde{\beta} = \alpha_L, \quad \tilde{r}_L = 0, \quad \tilde{t}_i = \frac{\alpha_H^M}{\lambda + M e + s_H^i(\alpha_H - 1 - t_i)I} - 1,\]
\[\tilde{s}_L^i = \frac{\lambda(\alpha_H - 1)I - (M e + s_H^j(\alpha_H - 1 - t_j)I)}{\alpha_H^I}, \quad \tilde{s}_H^i = \frac{\lambda + M e + s_H^j(\alpha_H - 1 - t_j)I}{\alpha_H I}\]

The above variables comprise the elements of the politician’s proposal which, given his opponent’s proposal, maximize his utility under the condition that he wins the election with certainty. We denote this proposal as \(\tilde{P}_i^R\). The associated utility level from this proposal is expressed as:

\[\tilde{U}_p = B_i^\gamma = \left[\frac{\alpha_L(\alpha_H - 1)I}{\alpha_H} - \frac{\alpha_L}{\alpha_H} \left( M e + s_H^j(\alpha_H - 1 - t_j)I \right) \right]^{\gamma}\]

The above expression is very intuitive. The first term is the utility the politician would get if he was a dictator. The second term reflects the utility loss the politician must suffer in order to win the election and it is decreasing in \(s_H^j\). The more funds \(P_j\) provides to efficient projects, and hence the less corrupt he is, the greater are the concessions \(P_i\) has to make to secure victory.

This is not his best response function however. The politician might do better by mimicking his opponent’s proposal and not suffering the cost of higher concessions to voters. In order to verify if this is the case, the politician needs also to solve P.1 under the slightly modified condition:

\[V_i^i(P_i^R, P_j^R) = \frac{1}{2} \Leftrightarrow \frac{1}{2} + \frac{s_H^i(\alpha_H - 1 - t_i)I + b}{2M} - \frac{s_H^j(\alpha_H - 1 - t_j)I + b}{2M} + \frac{(s_L^i - s_L^j)b}{2M} = \frac{1}{2}\]

Of course, the above condition implies that: \(s_H^i = s_H^j, s_L^i = s_L^j, t_i = t_j\) which means that \(P_i\) mimics \(P_j\)’s proposal. In that case \(P_i\)’s utility would be:

\textsuperscript{18}The derivation of the solution is provided in Appendix A.
\[ U_p = \frac{1}{2} B_i = \frac{1}{2} (\beta_i s_i^j I) = \frac{1}{2} (\alpha_i s_i^L I) \quad 19 \]

Therefore, \( P_i \) prefers to win the election with probability \( \frac{1}{2} \) to a certain victory if and only if:

\[ U_p \geq \tilde{U}_p \iff \frac{1}{2} (\alpha_i s_i^L I) \geq \left[ \frac{\alpha_i (\alpha - 1) \lambda I}{\alpha} - \frac{\alpha_i}{\alpha} (M \epsilon + s_i^j (\alpha - 1 - t_j) I) \right] \gamma \]

\[ \iff s_i^j \leq \frac{2^{\gamma} M \epsilon}{\left( \frac{1}{2} - 1 \right) \alpha H I} \quad 20 \]

The above condition sets a critical level for the strategies \( P_i \) will play. If his opponent chooses \( s_i^j \) above this value, then \( P_i \) prefers to compete aggressively and win the election, while if it is below this value then he prefers to make exactly the same proposal as the other politician and at least get \( \frac{1}{2} \) chance of gaining power. But he would never choose to opt for high values of corruption and lose all chances of being elected, since, by the assumptions of the model, the only way he can gain some utility is through the potential bribes that come along with power.

Because of the symmetry of the political game, of course, \( P_j \) faces a similar condition and the same strategic issues:

\[ s_i^j \leq \frac{2^{\gamma} M \epsilon}{\left( \frac{1}{2} - 1 \right) \alpha H I} \]

Hence, the unique equilibrium of the game is:

\[ s_i^j = s_j^j = \frac{2^{\gamma} M \epsilon}{\left( \frac{1}{2} - 1 \right) \alpha H I} \]

The two politicians mimic each other’s proposals and chance determines who gets elected to power. If one of them deviates from this equilibrium and increases taxation by even an infinitesimal amount, the other one can do better by decreasing it and winning election for sure. On other hand, none of them has an incentive to decrease the level of funds that goes to low quality projects as winning the election with certainty, in

19 It is very easy to verify that if \( P_i \) mimics his opponent then his best response is to set \( \beta_i = \alpha_i \), by following exactly the same method as we did in the Appendix A for the case of pure election victory.

20 The derivation of this inequality is included in Appendix A.
this case, lowers the politician’s utility when compared with the utility level he derives by mimicking his opponent’s proposal. Therefore the symmetric equilibrium of this political game can be fully described as:

\[
\beta_i = \beta_j = \alpha_L, \quad r_i^L = r_j^L = 0, \quad s_i^L = s_j^L = \frac{2^{\frac{1}{\gamma}} M \epsilon}{(2^{\frac{1}{\gamma}} - 1) \alpha_H I},
\]

\[
s_i^H = s_j^H = \lambda - \frac{2^{\frac{1}{\gamma}} M \epsilon}{(2^{\frac{1}{\gamma}} - 1) \alpha_H I}, \quad t_i = t_j = \frac{2^{\frac{1}{\gamma}} M \epsilon}{(2^{\frac{1}{\gamma}} - 1) \alpha_H I - 2^{\frac{1}{\gamma}} M \epsilon}
\]

The equilibrium utility level politicians receive is:

\[
U^*_p = \frac{1}{2} \left[ \frac{2^{\frac{1}{\gamma}} - \frac{1}{\alpha_H} M \epsilon}{2^{\frac{1}{\gamma}} - 1} \right] \gamma
\]

Figure 3: Political Equilibria under Monopoly and Competition

The equilibrium is also represented diagrammatically in Figure 3. It is drawn in the \{t, s_L\} space and utility increases for the politician as \(t\) and \(s_L\) increase. The \(s_L = \frac{1}{1+t}\) curve corresponds to the level of low quality projects that can be undertaken as a function of the tax rate when the condition \(\beta = \alpha_L\) holds. It represents the profitability of banks condition (6) and shows the maximum number of low quality projects that can be financed for each level of taxation so that public banks do not go bankrupt.
Point A stands for the autocratic case, when the politician faces no challenges to his power and hence he can set the maximum possible tax in order to fund as many low quality projects as possible and reap the maximum level of bribes. On the other hand, point B in the diagram shows the case of political competition. Due to the fear of losing the election, politicians must provide some concessions to voters in terms of lower inefficiencies in the economy and lower corruption.

However, the level of corruption that will prevail in the economy depends on the degree of ideological adherence and the necessary majority to consolidate power. So long as $M$ and $\epsilon$ are sufficiently small, the cost of undermining a political opponent remains less than the benefit of winning the election and political competition has a bite on lowering the expropriating power of politicians. Specifically, a necessary condition for lower corruption under political competition than the autocratic case is:

$$s^*_L \leq s^m_L \iff \frac{2^{\frac{1}{2}} M \epsilon}{\left(2^{\frac{1}{2}} - 1\right) \alpha_H I} \leq \frac{\lambda (\alpha_H - 1)}{\alpha_H} \iff M \epsilon \leq \frac{\left(2^{\frac{1}{2}} - 1\right) \lambda (\alpha_H - 1) I}{2^{\frac{1}{2}}} \quad (8)$$

The left hand side of (8) represents the cost the politician must suffer in order to win the election, while the right side is the perceived benefit: the total profits that would be generated in the economy if all efficient projects were undertaken weighted by the increase in probability of winning the election and the degree of risk aversion of the politician. If $\epsilon = 0$ or $M = 0$, then attracting voters is unnecessary or costless for political victory so that there is perfect political competition and the economy can achieve the first best, where there is no corruption and only high quality entrepreneurs receive funds. On the other hand, if condition (8) is violated, then essentially the cost of political competition is so high that politicians can act unchallenged and impose political proposals that replicate the monopoly of power case. In other words, economies with higher degree of polarization or greater political instability, which require increased majorities for the implementation of power, give greater power to politicians to act according to their best interest and face the risk of higher inefficiencies and corruption.

Also, the degree of risk aversion of the politician works in favor of efficiency. The more risk averse the politicians are the more they seek to secure victory in elections and hence the greater the degree of political competition. As a result, the number of efficient projects funded and the implied level of corruption decrease on their political proposals.

In summary, in this section we derived the following results: First, we show that a Nash equilibrium exists even though voting is fully deterministic and there are more than one policy dimensions (multi-dimensional setting). Second, the greater the degree

---

21 Notice that if $\gamma = 1$, then $\frac{\left(2^{\frac{1}{2}} - 1\right)}{2^{\frac{1}{2}}} = \frac{1}{2}$. 
of ideological adherence of voters the greater the political rents. This prediction is consistent with the empirical findings in Easterly and Levine (1997) and Svensson (2005). These authors report that political polarization is associated with less cost-efficient methods of production of public goods and is more likely to lead to higher corruption in the economy. We share this prediction with Persson and Tabellini (1999). Third, the higher the number of extra votes required for election victory (the higher the $\epsilon$) the higher the degree of corruption. The latter prediction is novel and, to the best of our knowledge, has not been tested yet.

3 An Asymmetric Political Game

We now retain the framework as it was in the previous section, but generalize it to include different degrees of ideological adherence for each politician and within each type of entrepreneurs. More specifically, we assume that ideology is uniformly distributed over the interval $[M_h, \bar{M}_h]$ for the high quality group of entrepreneurs and over the interval $[M_l, \bar{M}_l]$ for the low quality entrepreneurs. In terms of notation:

$$d_i^h \sim Uni[M_h, \bar{M}_h]$$

$$d_i^l \sim Uni[M_l, \bar{M}_l]$$

Figure 4: Ideological Distribution under Asymmetry

An example of such a distribution can also be shown in Figure 4. The rest of the elements of the model remain the same as in section 1. Politicians, once again, compete with each other in terms of political proposals in an attempt to be elected into power. The main strategies they can use to achieve this also remain the same. They can either try to secure victory through greater concessions or they may opt to mimic their opponent. However, this generalization allows us to obtain new insights.
The main difference in the asymmetric environment is that mimicking does not mean offering the same political proposal anymore. Since the degree of ideological adherence varies among voters, the two politicians may not have equal chances of being elected if they follow the same policies. If the degree of ideological asymmetry is strong enough, then one politician may have sufficient mass of voters to win the election with certainty even if he makes a slightly different proposal than his competitor.

Assuming, for the rest of the analysis, that the natural supporters for politician $P_i$ have ideological adherence distributed over $[0, M_h]$ (the rest support $P_j$), the number of voters that will actually vote for $P_i$, depending on both political programs, can be expressed as:

$$V_i^i(P^R_i, P^R_j) = \frac{M_h}{M_h - M_L} q + \frac{M_l}{M_l - M_i} (1 - q) + \frac{s^i_l[(\alpha_H - 1 - t_i)I + b]}{M_h - M_h} - \frac{s^j_l[(\alpha_H - 1 - t_j)I + b]}{M_h - M_h} + \frac{(s^i_l - s^j_l)b}{M_l - M_l}$$

The first two terms represent the number of natural supporters for $P_i$, while the rest of the terms show the extra number of voters the politician can gain from both groups of entrepreneurs through his policy.

The conditions for the politician to draw or win an election, respectively, remain:

$$V_i^i(P^R_i, P^R_j) = \frac{1}{2}$$

$$V_i^i(P^R_i, P^R_j) \geq \frac{1 + \epsilon}{2}$$

The politician will decide which strategy to follow, depending on the utility he would receive by doing so. If $P_i$ were to guarantee election victory then his best response to his opponent’s proposal would be (let $M_H = M_h - M_h$ and $M_L = M_l - M_l$):

if $\left(\frac{M_H}{M_L} - 1\right) b < (\alpha_H - \alpha_L)I$, then:

$$s^i_L = s^j_L - \frac{1 + \epsilon}{2} M_H M_L - \frac{M_h M_L q - M_l M_H (1 - q)}{M_l \alpha_H I - (M_H - M_L) b}$$

if $\left(\frac{M_H}{M_L} - 1\right) b \geq (\alpha_H - \alpha_L)I$, then:

$$s^i_L = s^j_L + \frac{1 + \epsilon}{2} M_H M_L - \frac{M_h M_L q - M_l M_H (1 - q)}{(M_H - M_L) b}$$

If he were to draw instead, his best response to his opponent’s proposal would be:

if $\left(\frac{M_H}{M_L} - 1\right) b < (\alpha_H - \alpha_L)I$, then:

$$s^i_L = s^j_L - \frac{1}{2} M_H M_L - \frac{M_h M_L q - M_l M_H (1 - q)}{M_l \alpha_H I - (M_H - M_L) b}$$

\[22\text{See also Appendix B}\]
\[
\text{if } \left( \frac{M_H}{M_L} - 1 \right) b \geq (\alpha_H - \alpha_L)I, \text{ then:}
\]
\[
s^i_L = s^j_L + \frac{\frac{1}{2}M_H M_L - \overline{M}_h M_L q - \overline{M}_l M_H (1 - q)}{(M_H - M_L)b}
\]  

(14)

In a similar fashion, \( P_j \)'s strategies as a response to his opponent's proposal would be:

\[
\text{if } \left( \frac{M_H}{M_L} - 1 \right) b < (\alpha_H - \alpha_L)I, \text{ then:}
\]
\[
s^j_L = s^i_L - \frac{\frac{1}{2}M_H M_L - M_h M_L q - M_l M_H (1 - q)}{M_L\alpha_H I - (M_H - M_L)b}
\]  

(15)

\[
\text{if } \left( \frac{M_H}{M_L} - 1 \right) b \geq (\alpha_H - \alpha_L)I, \text{ then:}
\]
\[
s^j_L = s^i_L + \frac{\frac{1}{2}M_H M_L - M_h M_L q - M_l M_H (1 - q)}{(M_H - M_L)b}
\]  

for the case of victory, while for the case of draw:

\[
\text{if } \left( \frac{M_H}{M_L} - 1 \right) b < (\alpha_H - \alpha_L)I, \text{ then:}
\]
\[
s^j_L = s^i_L - \frac{\frac{1}{2}M_H M_L - M_h M_L q - M_l M_H (1 - q)}{M_L\alpha_H I - (M_H - M_L)b}
\]  

(17)

\[
\text{if } \left( \frac{M_H}{M_L} - 1 \right) b \geq (\alpha_H - \alpha_L)I, \text{ then:}
\]
\[
s^j_L = s^i_L + \frac{\frac{1}{2}M_H M_L - M_h M_L q - M_l M_H (1 - q)}{(M_H - M_L)b}
\]  

(18)

Therefore, the asymmetric political game has two equilibria, depending on a single condition.

**Case 1:** \( \left( \frac{M_H}{M_L} - 1 \right) b < (\alpha_H - \alpha_L)I \)

In this case the cost of high quality supporters relative to low quality, reflected by the left hand side of the inequality, is lower than the extra benefit these supporters can provide the politician in terms of higher efficiency and greater profits in the economy. Both politicians prefer to compete for the support of the high quality group in the economy, irrespective of their natural support group, and they adopt policies that reduce inefficiency and corruption. Notice that, if \( M_H = M_L = 2M \), which corresponds to the symmetric game, this condition would always hold, so that the problem of section 1 is a special case of this case.
Both of them face a threshold value of inefficient projects their opponent is proposing, below which they would prefer not to directly compete for victory, but to draw the election. This value, however, varies between the two politicians, depending on their natural support groups and determines a relative bargaining power in terms of aggressiveness. The threshold value for politician $P_i$ and $P_j$ respectively is:

$$s_j^L \leq \frac{\frac{1}{2}M_H M_L - M_h M_L q - M_l M_H (1 - q)}{M_L \alpha_H I - (M_H - M_L) b} + \frac{2^{\frac{1-\gamma}{\gamma}}}{\left(\frac{2^{\frac{1}{\gamma}}}{\gamma} - 1\right)} \left(\frac{M_H M_L \epsilon}{M_L \alpha_H I - (M_H - M_L) b}\right)$$

(19)

$$s_i^L \leq \frac{\frac{1}{2}M_H M_L - M_h M_L q - M_l M_H (1 - q)}{M_L \alpha_H I - (M_H - M_L) b} + \frac{2^{\frac{1-\gamma}{\gamma}}}{\left(\frac{2^{\frac{1}{\gamma}}}{\gamma} - 1\right)} \left(\frac{M_H M_L \epsilon}{M_L \alpha_H I - (M_H - M_L) b}\right)$$

(20)

The politician who has the strongest support groups has more to gain from a victory than his opponent and so he is more willing to win, even for proposals of his competitor with so low inefficiency, that if the other politician were in his position he would not accept.

For expositional reasons, let’s assume that $P_i$ has greater natural support groups, so that $s_j^L < s_i^L$. The following diagram depicts the strategic situation:

![Diagram](image.png)

**Figure 5: Critical Values Comparison**

If both contestants make proposals above $s_i^L$ then they would prefer to deviate, propose lower inefficiency in the economy and win elections with certainty. Once $P_j$ reaches his critical value, however, he prefers the more defensive strategy of winning the election with probability $\frac{1}{2}$, while his opponent continues to deviate. The equilibrium of the game is reached when $P_j$ offers $s_j^L$ and $P_i$ offers:

$$s_i^L = s_j^L - \frac{\frac{1}{2}M_H M_L - M_h M_L q - M_l M_H (1 - q)}{M_L \alpha_H I - (M_H - M_L) b} \Rightarrow$$

$$s_i^L = \frac{2^{\frac{1-\gamma}{\gamma}}}{\left(\frac{2^{\frac{1}{\gamma}}}{\gamma} - 1\right)} \left(\frac{M_H M_L \epsilon}{M_L \alpha_H I - (M_H - M_L) b}\right)$$

(21)
At this point, none of the contestants has an incentive to deviate, since this would decrease his payoff without increasing his chances of victory. Notice also that we have assumed that $P_i$ has a larger group of natural supporters. This means that:

$$\frac{M_H}{M_H} q + \frac{M_L}{M_L} (1 - q) > \frac{1}{2} \Leftrightarrow M_L M_H q + M_H M_L (1 - q) > \frac{1}{2} M_H M_L \Leftrightarrow$$

$$\frac{1}{2} M_H M_L - M_L M_H q - M_H M_L (1 - q) < 0$$

In other words, the equilibrium political proposal of $P_i$ contains greater inefficiency than the equilibrium proposal of his opponent, which reflects the relative superiority of his bargaining position. Since the politician with the greater support worries less about the possibility to lose the election, he can reap greater benefits for himself than his opponent. Hence, at equilibrium there are two different proposals, one more efficient than the other, and politicians face the same probability of gaining power. In either case $\tau_L = 0, \beta = \alpha_L$ while the taxation rate under the politician with greater support is higher than the taxation of the other \(^{23}\).

This result also highlights another interesting implication of our model. The politician with the greater support group could have won the election with certainty while retaining some rents if he were willing to propose a more efficient policy. However, he prefers more rents and winning the election with probability one half. What makes this result interesting is that, due to deterministic voting, in our model political instability arises endogenously.

**Case 2:** \(\left(\frac{M_H}{M_L} - 1\right) b \geq (\alpha_H - \alpha_L) I\)

This is the opposite case. The relative cost of high quality supporters outweighs the extra benefit, so that both politicians prefer to compete for the support of the low quality group. They achieve that by making offers with greater extent of taxation and funding for inefficient projects. The threshold value for the two politicians under this case is:

$$s_L^* = \frac{\lambda (\alpha_H - 1)}{\alpha_H - \alpha_L} - \frac{1}{2} M_H M_L - \frac{M_L M_H q - M_H M_L (1 - q)}{(M_H - M_L)b} - \frac{2\gamma}{\left(\frac{2}{\gamma} - 1\right)} \left[ \frac{M_H M_L \epsilon}{(M_H - M_L)b} \right]$$

\(22\)

$$s_L^* = \frac{\lambda (\alpha_H - 1)}{\alpha_H - \alpha_L} - \frac{1}{2} M_H M_L - \frac{M_L M_H q - M_H M_L (1 - q)}{(M_H - M_L)b} - \frac{2\gamma}{\left(\frac{2}{\gamma} - 1\right)} \left[ \frac{M_H M_L \epsilon}{(M_H - M_L)b} \right]$$

\(23\)

\(^{23}\)Recall that at equilibrium $t = \frac{s_L^*}{\lambda - s_L}$.
Now the politician with the larger support group is the one with the highest threshold value, as the competition takes place in terms of more inefficiency rather than less, which was the previous case. Assuming that $P_i$ has a larger support group, the game reaches equilibrium when $P_j$ offers $s_j^*$ and $P_i$ offers:

$$s_i^L = s_j^L + \frac{1}{2} M_H M_L - \frac{M_h M_L q - M_l M_H (1 - q)}{(M_H - M_L)b}$$

$$s_i^L = \frac{\lambda (\alpha_H - 1)}{\alpha_H - \alpha_L} - \frac{2^{1+\gamma} q}{\gamma (2^{1+\gamma} - 1)} \left[ \frac{M_H M_L \epsilon}{(M_H - M_L)b} \right]$$

Notice that, in this case, the entrepreneur with the greater group of natural supporters makes more efficient proposals than his opponent, in the sense that he proposes to fund a smaller number of low quality projects. This result, however, is not surprising. Given that political competition is distorted in such a way, so that all profits from high quality projects are taxed, the real trade-off is the way this surplus will be divided between the politician in power and low quality entrepreneurs. The politician who has more supporters is also the one with higher degree of bargaining power, so that he has equal chances of winning the election even if he promises a smaller share of the surplus to entrepreneurs with inefficient projects than his opponent. Under this perspective, his proposal is closer to the policy that a selfish dictator would impose, as described in section 2. Nevertheless, from a societal point of view he would seem as a more efficient candidate.

This is an interesting case on its own right, because it exemplifies a situation where ideological adherence not only creates inefficiencies in the economy, but also distorts the field of competition from the high quality entrepreneurs to the low quality ones, increasing even more the degree of corruption in equilibrium. It also affects politicians strategies so that those, who receive relatively greater support by voters, make more efficient proposals than the rest. This case is more relevant in countries where the legal protection of outside investors is weak and hence the private benefits for entrepreneurs are high (high $b$).

In summary, the introduction of asymmetry in ideological adherence gives rise to the following predictions: First, in equilibrium political platforms are different. We share this prediction with others (Coate and Morris (1995), Helpman and Grossman (1996)), although for different reasons. This is consistent with the empirical findings in Poole and Rosenthal (1997) and Snyder (1996). Second, the greater the asymmetry in the ideological adherence to parties, the greater the difference in the proposed policies. Third, political parties, which have greater ideological support, will tend to favor more
inefficient policies and foster greater corruption than those parties with little ideological support. To the best of our knowledge, the last two predictions have not been empirically tested yet.

Conclusion

There is a number of points this paper is trying to make, which we draw as our final conclusions. First, on theoretical grounds we show that a Nash equilibrium exists even though voting is fully deterministic and there are more than one policy dimensions. Many existing papers argue that probabilistic voting is necessary for the existence of a Nash equilibrium in multi-dimensional policy environments, even though they allow for infinitely many voters. Our result reveals that, if there are infinitely many agents, the assumption of probabilistic voting is redundant for the existence of a Nash equilibrium or political rents. Institutional frictions, associated with electoral rules along with ideological adherence to parties, can equally well create suitable environments for political opportunism and inefficient social policies.

Second, deterministic voting allows us to derive another interesting result. In equilibrium, parties generally win the elections probabilistically, even if political proposals exist that could ensure victory for a party with certainty. This means that uncertainty over the results of an election is endogenously determined by the nature of political competition and the cost of luring supporters from the opposition.

Third, the higher the number of extra votes required for election victory (the higher the \(\epsilon\)) the higher the degree of corruption. Fourth, parties who have the highest degree of societal support will tend to favor more inefficient policies than less well supported groups. In other words, politicians will capitalize on their favorable situation to extract as many political rents as possible. These two predictions are novel and, to the best of our knowledge, have not been tested yet.

Fifth, the greater the degree of ideological adherence of voters the greater the political rents. This prediction is consistent with the empirical findings in Easterly and Levine (1997) and Svensson (2005). These authors report that political polarization is associated with less cost-efficient methods of production of public goods and is more likely to lead to higher corruption in the economy. We share this prediction with Persson and Tabellini (1999).

Sixth, political competition does not necessarily lead to convergence of political platforms. If political preferences over social groups are unevenly distributed, then political platforms will be radically different. This is consistent with the empirical findings in Poole and Rosenthal (1997) and Snyder (1996).

Our model can be extended in a number of interesting ways. First, the entrepreneurs’ private benefit can be reduced by strengthening the legal protection for outside investors. Of course, this is a political decision and can be derived as the outcome of the political game. Second, the degree of ideological adherence can be also endogenized. Because a higher degree of ideological adherence leads to higher political rents, through
campaigns, politicians may increase polarization and hence strengthen their bargaining position. We are currently working on the first issue, while we leave the second one for future research.
Appendix A

Solution to problem P.1b

Given that $P_i$ wants to win the election with certainty, he solves the problem:

$$\max_{s_{H_i}, s_{L_i}, t_i, \beta_i, r_{L_i}^i} U_p(P_i^R, P_j^R) = B_\gamma^i$$

s.t.

$$B_i = \beta_i s_{L_i}^I$$

$$\left( s_{H_i}^i + s_{L_i}^i \right) I \leq \lambda I$$

$$\beta_i + r_{L_i}^i \leq \alpha_L$$

$$r_{L_i}^i s_{L_i}^I + s_{H_i}^i I + t_i s_{H_i}^i I \geq \lambda I$$

$$\frac{s_{H_i}^i (\alpha_H - 1 - t_i) I + b}{2M} - \frac{s_{H_i}^j (\alpha_H - 1 - t_j) I + b}{2M} + \frac{(s_{L_i}^i - s_{L_i}^j)b}{2M} \geq \frac{\epsilon}{2}$$

$$-M \leq \frac{(s_{L_i}^i - s_{L_i}^j)b}{1 - \rho} \leq M$$

$$-M \leq \frac{s_{H_i}^i (\alpha_H - 1 - t_i) I + b}{q} - \frac{s_{H_i}^j (\alpha_H - 1 - t_j) I + b}{q} \leq M$$

$$0 \leq t_i \leq \alpha_H - 1$$

Using the same type of argument as in the monopoly of power case, it is obvious that constraints (26), (27), (28) and (29) of this maximization problem will hold with equality at the solution. Otherwise the politician can do better by changing one of the variables of the problem and receiving strictly higher utility. First, condition (29) can be written:

$$\frac{s_{H_i}^i (\alpha_H - 1 - t_i) I + b}{2M} - \frac{s_{H_i}^j (\alpha_H - 1 - t_j) I + b}{2M} + \frac{(s_{L_i}^i - s_{L_i}^j)b}{2M} = \frac{\epsilon}{2} \iff$$

$$s_{H_i}^i (\alpha_H - 1 - t_i) I + b - s_{H_i}^j (\alpha_H - 1 - t_j) I + b + (s_{L_i}^i - s_{L_i}^j)b = M \epsilon \iff$$

29
\[ s^i_H(\alpha_H - 1 - t_i)I + s^i_Hb + s^i_Lb = M\epsilon + s^i_H(\alpha_H - 1 - t_j)I + s^i_Hb + s^i_Lb \]

But now notice that, since \( s^i_Hb + s^i_Lb = s^i_Hb + s^i_Lb = \lambda b \), the last equation is equivalent to:

\[ s^i_H(\alpha_H - 1 - t_i)I = M\epsilon + s^i_H(\alpha_H - 1 - t_j)I \]  

(33)

Now let \( M(s^i_H, t_j) = M^* = M\epsilon + s^i_H(\alpha_H - 1 - t_j)I \). Since the politician can not directly affect his opponents’ choice variables, they are considered as exogenous from his point of view, so that \( M^* \) is treated as a constant in his maximization problem. So:

\[ s^i_H(\alpha_H - 1 - t_i)I = M^* \]  

(34)

By (26) and (34):

\( (\lambda - s^i_L)(\alpha_H - 1 - t_i)I = M^* \iff (\lambda - s^i_L)(\alpha_H - 1)I - (\lambda - s^i_L)t_iI = M^* \iff \)

\[ t_i = \alpha_H - 1 - \frac{M^*}{(\lambda - s^i_L)I} \]  

(35)

Also, by substituting (26) and (27) into (28) and solving for \( \beta_i \) we get the following expression:

\( (\alpha_L - \beta_i)s^i_L + (1 + t_i)(\lambda - s^i_L) = \lambda \iff \alpha_Ls^i_L - \beta_is^i_L + \lambda + \lambda t_i - (1 + t_i)s^i_L = \lambda \iff \)

\[ \lambda t_i + (\alpha_L - 1 - t_i)s^i_L = \beta_is^i_L \iff \beta_i = (\alpha_L - 1 - t_i) + \frac{\lambda t_i}{s^i_L} \]  

(36)

and by (35):

\[ \beta_i = \left( \alpha_L - 1 - \alpha_H + 1 + \frac{M^*}{(\lambda - s^i_L)I} \right) + \frac{\lambda (\alpha_H - 1 - \frac{M^*}{(\lambda - s^i_L)I})}{s^i_L} \iff \]

\[ \beta_i = \frac{(\alpha_L - \alpha_H)(\lambda - s^i_L)I + M^*}{(\lambda - s^i_L)I} + \frac{\lambda(\alpha_H - 1)(\lambda - s^i_L)I}{s^i_L} \iff \]

\[ \beta_i = \frac{(\alpha_L - \alpha_H)(\lambda - s^i_L)Is^i_L + M^*s^i_L}{(\lambda - s^i_L)Is^i_L} + \frac{\lambda(\alpha_H - 1)(\lambda - s^i_L)I}{(\lambda - s^i_L)Is^i_L} \iff \]

\[ \beta_i = -(\alpha_H - \alpha_L) + \frac{\lambda(\alpha_H - 1)}{s^i_L} - \frac{M^*}{s^i_LI} \]  

(37)
Using equations (25) and (37) we substitute back to the objective function to rewrite it as a function of only one choice variable:

\[
\max_{s^i_L} B_i^\gamma = (\beta_i s^i_L I)^\gamma = \left[ -\alpha_H - \alpha_L + \frac{\lambda(\alpha_H - 1)}{s^i_L} - \frac{M^*}{s^i_L I} \right] s^i_L I \]

\[
= -\alpha_H - \alpha_L s^i_L I + (\alpha_H - 1)\lambda I - M^* \]

Since the objective function is monotonic in \( \gamma \), the maximum of the above expression is attained at the same level of \( s^i_L \) as the maximum of \( B_i \). The F.O.C. for this simpler problem is:

\[
\frac{\partial B_i}{\partial s^i_L} = -\alpha_H - \alpha_L I < 0, \text{ since } \alpha_H > \alpha_L, \ I > 0
\]

This implies that the politician must reduce \( s^i_L \) as much as possible in order to maximize his own utility, and this holds irrespectively of the political proposal of the opponent. Notice that, by equation (37), as \( s^i_L \) decreases \( \beta_i \) increases (and also notice that because \( t_i \geq 0 \), it must hold that \( \lambda(\alpha_H - 1)I - M^* > 0 \), by equation (35)). Therefore, the minimum possible level for \( s^i_L \) is the one that makes \( \beta_i \) the maximum possible. This implies that at the optimum \( \beta_i = \alpha_L \). By using equation (37) once more we get the solution for \( s^i_L \):

\[
\alpha_L = -\alpha_H - \alpha_L + \frac{\lambda(\alpha_H - 1)}{s^i_L} - \frac{M^*}{s^i_L I} \iff 0 = -\alpha_H + \frac{\lambda(\alpha_H - 1)I - M^*}{s^i_L I} \iff
\]

\[
s^i_L = \frac{\lambda(\alpha_H - 1)I - M^*}{\alpha_H I} \tag{38}
\]

And, by (35):

\[
t_i = \alpha_H - 1 - \frac{M^*}{(\lambda - s^i_L)I} \iff t_i = \alpha_H - 1 - \frac{M^*}{(\lambda - \frac{\lambda(\alpha_H - 1)I - M^*}{\alpha_H I}) I} \iff
\]

\[
t_i = \alpha_H - 1 - \frac{M^*}{\frac{\lambda(\alpha_H - 1)I - \lambda M + M^*}{\alpha_H I} I} \iff t_i = \alpha_H - 1 - \frac{\alpha_H M^*}{\lambda I + M^*} \iff
\]

\[
t_i = \frac{\alpha_H \lambda I}{\lambda I + M^*} - 1 \tag{39}
\]

In order to complete the solution, note that \( \beta_i = \alpha_L \Rightarrow \gamma_L = 0 \) and that:

\[
s^i_H = \lambda - s^i_L \Rightarrow s^i_L = \lambda - \frac{\lambda(\alpha_H - 1)I - M^*}{\alpha_H I} \iff s^i_H = \frac{\lambda I + M^*}{\alpha_H I}
\]
The utility the politician will derive by winning the elections as a response to his opponent’s proposal is:

\[
\widetilde{B}_i^\gamma = (\beta_i s_L^i I)^\gamma = \left[ \frac{\alpha_L (\alpha_H - 1) \lambda I}{\alpha_H} - \frac{\alpha_L}{\alpha_H} M^i \right]^\gamma \Rightarrow \\
\widetilde{B}_i^\gamma = \left[ \frac{\alpha_L (\alpha_H - 1) \lambda I}{\alpha_H} - \frac{\alpha_L}{\alpha_H} (M\epsilon + s_H^i (\alpha_H - 1 - t_j)I) \right]^\gamma
\]

If the politician decides to mimic he gets utility (see also the political equilibrium section of the paper):

\[
U_p = \frac{1}{2} B_i^\gamma = \frac{1}{2} (\beta_i s_L^i I)^\gamma = \frac{1}{2} (\alpha L s_L^i I)^\gamma
\]

Politician decides not to pursue victory iff:

\[
U_p \geq \widetilde{U}_p \iff \frac{1}{2} (\alpha L s_L^i I)^\gamma \geq \left[ \frac{\alpha_L (\alpha_H - 1) \lambda I}{\alpha_H} - \frac{\alpha_L}{\alpha_H} (M\epsilon + s_H^i (\alpha_H - 1 - t_j)I) \right]^\gamma \Rightarrow \\
\alpha L s_L^i I \geq 2^{\frac{1}{\gamma}} \left[ \frac{\alpha_L (\alpha_H - 1) \lambda I}{\alpha_H} - \frac{\alpha_L}{\alpha_H} (M\epsilon + (\lambda - s_L^i)(\alpha_H - 1 - t_j)I) \right]
\]

At this point notice that, since the problem is symmetric, \( P_j \) will also set \( \beta_j = \alpha_L \) in order to maximize his utility irrespectively of what \( P_i \) will do and by equation (36) we have:

\[
\alpha_L = (\alpha_L - 1 - t_j) + \frac{\lambda t_j}{s_L^i} \iff 1 + t_j = \frac{\lambda t_j}{s_L^i} \iff s_L^i = \lambda t_j - s_L^i t_j \iff t_j = \frac{s_L^i}{\lambda - s_L^i}
\]

We use this into the preceding expression:

\[
s_L^i I \geq 2^{\frac{1}{\gamma}} \left[ \frac{\alpha_L (\alpha_H - 1) \lambda I}{\alpha_H} - \frac{1}{\alpha_H} \left( M\epsilon + \left( \frac{\lambda - s_L^i}{\alpha_H} - \frac{s_L^i}{\alpha_H} \right) \right) I \right] \Rightarrow \\
\alpha_L s_L^i I \geq 2^{\frac{1}{\gamma}} \frac{\lambda (\alpha_H - 1)}{\alpha_H} - 2^{\frac{1}{\gamma}} M\epsilon I - 2^{\frac{1}{\gamma}} \frac{\lambda (\alpha_H - 1)}{\alpha_H} s_L^i + 2^{\frac{1}{\gamma}} \frac{1}{\alpha_H} s_L^i \Rightarrow \\
\alpha_L s_L^i I \geq -2^{\frac{1}{\gamma}} M\epsilon I + 2^{\frac{1}{\gamma}} s_L^i \iff \left( 1 - 2^{\frac{1}{\gamma}} \right) s_L^i \geq -2^{\frac{1}{\gamma}} M\epsilon I \iff \\
\alpha_L s_L^i I \leq \frac{2^{\frac{1}{\gamma}} M\epsilon}{\left( 2^{\frac{1}{\gamma}} - 1 \right) \alpha_H I}
\]
Appendix B

Solution to problem P.1b under asymmetry

We follow the same procedure as in the symmetric case. If $P_i$ wants to win the election then he needs to solve the modified problem P.2:

$$\max_{s^i_H, s^i_L, t_i, b^i_L} U_p(P^R_i, P^R_j) = B^\gamma_i$$ \hspace{1cm} (42)

s.t.

$$B_i = \beta_i s^i_L I$$ \hspace{1cm} (43)

$$\left(s^i_H + s^i_L\right) I \leq \lambda I$$ \hspace{1cm} (44)

$$\beta_i + r^i_L \leq \alpha_L$$ \hspace{1cm} (45)

$$r^i_L s^i_L I + s^i_H I + t_i s^i_H I \geq \lambda I$$ \hspace{1cm} (46)

$$\frac{M_h}{M_h - M_h}q + \frac{M_l}{M_l - M_l}(1-q) + \frac{s^i_H[(\alpha_H - 1 - t_i)I + b]}{M_h - M_h} - \frac{s^j_H[(\alpha_H - 1 - t_j)I + b]}{M_h - M_h} + \frac{(s^i_L - s^j_L)b}{M_l - M_l} \geq \frac{1 + \epsilon}{2}$$ \hspace{1cm} (47)

$$-M_h \leq \frac{(s^i_L - s^j_L)b}{1 - q} \leq M_l$$ \hspace{1cm} (48)

$$-M_h \leq \frac{s^i_H[(\alpha_H - 1 - t_i)I + b]}{q} - \frac{s^j_H[\alpha_H - 1 - t_j)I + b]}{q} \leq M_h$$ \hspace{1cm} (49)

$$0 \leq t_i \leq \alpha_H - 1$$ \hspace{1cm} (50)

Let $M_h - M_h = M_H$ and $M_l - M_l = M_L$. Taking into account that at equilibrium inequalities (44),(45),(46),(47) will hold with equality, (47) can be rewritten as:

$$\frac{M_h}{M_h - M_h}q + \frac{M_l}{M_l - M_l}(1-q) + \frac{s^i_H[(\alpha_H - 1 - t_i)I + b]}{M_H} - \frac{s^j_H[(\alpha_H - 1 - t_j)I + b]}{M_H} + \frac{(s^i_L - s^j_L)b}{M_L} = \frac{1 + \epsilon}{2} \Leftrightarrow$$
\[
\overline{M}_h M L q + \overline{M}_H M (1 - q) + M_L s_H^i [(\alpha_H - 1 - t_i) I + b] - M_L s_H^j [(\alpha_H - 1 - t_j) I + b] + M_H (s_L^i - s_L^j) b = \frac{1 + \epsilon}{2} M_H M_L \Leftrightarrow
\]

\[
M_L s_H^i [(\alpha_H - 1 - t_i) I + b] - M_L s_H^j [(\alpha_H - 1 - t_j) I + b] + M_H (s_L^i - s_L^j) b = E
\]

where: \( E = \frac{1 + \epsilon}{2} M_H M_L - \overline{M}_h M L q - \overline{M}_H M (1 - q) \) (51)

By using (44):

\[
M_L (\lambda - s_L^i) [(\alpha_H - 1 - t_i) I + b] - M_L (\lambda - s_L^j) [(\alpha_H - 1 - t_j) I + b] + M_H (s_L^i - s_L^j) b = E \Leftrightarrow
\]

\[-M_L \lambda t_i I + (M_H - M_L) b s_L^i - M_L s_L^i (\alpha_H - 1 - t_i) I + M_L \lambda t_j I - (M_H - M_L) b s_L^j + M_L s_L^j (\alpha_H - 1 - t_j) I = E \Leftrightarrow
\]

\[-M_L \lambda t_i - M_L s_L^i (\alpha_H - 1 - t_i) I + (M_H - M_L) b s_L^i = -M_L \lambda t_j I - M_L s_L^j (\alpha_H - 1 - t_j) I + (M_H - M_L) b s_L^j
\]

(52)

Let: \( Z(t_j, s_L^j) = Z = E - M_L \lambda t_j I - M_L s_L^j (\alpha_H - 1 - t_j) I + (M_H - M_L) b s_L^j \) (53)

Then:

\[-M_L \lambda t_i I - M_L s_L^i (\alpha_H - 1 - t_i) I + (M_H - M_L) b s_L^i = Z \] (54)

Solving (54) for \( t_i \) we get:

\[
t_i = \frac{Z + [M_L I (\alpha_H - 1) - (M_H - M_L) b] s_L^i}{-M_L I (\lambda - s_L^i)} \Rightarrow
\]

\[
t_i = \frac{Z + \Theta s_L^i}{-M_L I (\lambda - s_L^i)}
\]

where, \( \Theta = M_L I (\alpha_H - 1) - (M_H - M_L) b \) (56)

Also, by using (44),(45) and (46):

\[
r_L^i s_L^i I + s_H^i I + t_i s_H^i I = \lambda I \Leftrightarrow \alpha_L - \beta_i) s_L^i I + (\lambda - s_L^i)(1 + t_i) I = \lambda I \Leftrightarrow
\]

\[
\alpha_L s_L^i - \beta_i s_L^i + (\lambda - s_L^i) t_i - s_L^i = 0
\]
Substituting the value of $t_i$, by (55), in the above expression:

\[(\alpha_L - 1)s_L^i - \beta_is_L^i + (\lambda - s_L^i)\frac{Z + \Theta s_L^i}{M_LI(\lambda - s_L^i)} = 0 \iff \beta_is_L^i = (\alpha_L - 1)s_L^i - \frac{\Theta s_L^i}{M_L} \frac{Z}{M_LI} \iff (57)\]

\[\beta_is_L^i = (\alpha_L - 1)s_L^i - \frac{\Theta s_L^i}{M_L} \frac{Z}{M_LI} \iff \beta_is_L^iI = (\alpha_L - 1)Is_L^i - \frac{\Theta s_L^i}{M_L} \frac{Z}{M_L} \iff (58)\]

\[B_i = \left[(\alpha_L - 1)I - \frac{\Theta}{M_L}\right] s_L^i - \frac{Z}{M_L} \]

Since $B_i^\gamma$ is monotonic in $\gamma$, the value of $s_L^i$ that maximizes $B_i^\gamma$ is the same with the one that maximizes (58). Hence, by FOC:

\[\frac{\partial B_i}{\partial s_L^i} = (\alpha_L - 1)I - \frac{\Theta}{M_L} \]

(59)

We now examine two different cases depending on the sign of the first derivative.

**Case 1: $\frac{\partial B_i}{\partial s_L^i} < 0$**

\[\frac{\partial B_i}{\partial s_L^i} < 0 \iff (\alpha_L - 1)I - \frac{\Theta}{M_L} < 0 \iff (\alpha_L - 1)I - \frac{M_LI(\alpha_H - 1) - (M_H - M_L)b}{M_L} < 0 \iff (\alpha_L - 1)I - (\alpha_H - 1)I + \left(\frac{M_LI}{M_LI}\right) b < 0 \iff \left(\frac{M_H}{M_LI}\right) b < (\alpha_H - \alpha_L)I \]

(60)

First, notice that condition (60) means that $P_i$ must minimize $s_L^i$ to maximize his utility. Second, notice also that this condition does not depend on $\overline{M}_h$ or $\overline{M}_l$, the degree of ideological adherence to $P_i$, but on $M_H$ and $M_L$, which reflect the total ideological dispersion between groups and which are identical for $P_j$ as well. In other words, condition (60) is symmetric and either holds for both politicians or for none. Therefore, if it is optimal for one of the two to minimize $s_L^i$, it will also be optimal for the other.

We proceed in our analysis assuming that condition (60) is satisfied and that $Z < 0$. We will later return to show that the condition $Z < 0$ is indeed satisfied if (60) holds. Using equation (57) we get:

\[\beta_is_L^i = (\alpha_L - 1)s_L^i - \frac{\Theta s_L^i}{M_L} \frac{Z}{M_LI} \iff \beta_i = (\alpha_L - 1) - \frac{\Theta s_L^i}{M_L} - \frac{Z}{M_LI s_L^i} \iff \frac{\partial \beta_i}{\partial s_L^i} < 0 \]
As \( P_i \) tries to minimize \( s^i_L \), the amount of the low quality project’s return that will be provided as a bribe will increase until it reaches its maximum value (\( \beta_i = \alpha_L \)), similarly to the symmetric case. At that point:

\[
\alpha_L = (\alpha_L - 1) - \frac{\Theta}{M_L I} - \frac{Z}{M_L I s^i_L} \iff \frac{Z}{M_L I} = -\frac{M_L I + \Theta}{M_L I} \iff s^i_L = -\frac{Z}{M_L I + \Theta}
\]

\[
s^i_L = -\frac{E - M_L \lambda t_j I - M_L s^i_L (\alpha_H - 1 - t_j) I + (M_H - M_L) b s^j_L}{M_L I + M_L I (\alpha_H - 1) - (M_H - M_L) b} \iff
\]

\[
s^i_L = -\frac{E - M_L \lambda t_j I - M_L s^i_L (\alpha_H - 1 - t_j) I + (M_H - M_L) b s^j_L}{M_L \alpha_H I - (M_H - M_L) b} \iff
\]

Because (57) holds for both politicians, it also holds that \( \beta_j = \alpha_L \). Using this fact and the respective condition (44),(45),(46) for politician \( P_j \), we get:

\[
\alpha_L s^j_L + (\lambda - s^j_L) t_j - s^i_L = 0 \iff t_j = \frac{s^j_L}{(\lambda - s^j_L)} \tag{61}
\]

Substituting back to the previous expression:

\[
s^i_L = -\frac{E - M_L \lambda \left( \frac{s^i_L}{\lambda - s^j_L} \right) I - M_L s^j_L (\alpha_H - 1 - \frac{s^i_L}{\lambda - s^j_L}) I + (M_H - M_L) b s^j_L}{M_L \alpha_H I - (M_H - M_L) b} \iff
\]

\[
s^i_L = -\frac{E - M_L (\lambda - s^j_L) \frac{s^i_L}{(\lambda - s^j_L)} I - M_L s^j_L (\alpha_H - 1) I + (M_H - M_L) b s^j_L}{M_L \alpha_H I - (M_H - M_L) b} \iff
\]

\[
s^i_L = -\frac{E - M_L s^i_L I - M_L s^j_L \alpha_H I + M_L s^j_L I + (M_H - M_L) b s^j_L}{M_L \alpha_H I - (M_H - M_L) b} \iff
\]

\[
s^i_L = -\frac{E - M_L \alpha_H I + (M_H - M_L) b s^j_L}{M_L \alpha_H I - (M_H - M_L) b} \iff
\]

\[
s^i_L = s^j_L - \frac{E}{M_L \alpha_H I - (M_H - M_L) b} \iff
\]

And by substituting the value of \( E \):

\[
s^i_L = s^j_L - \frac{1 + q}{2} M_H M_L - \overline{M}_H M_L q - \overline{M}_I M_H (1 - q) \]

\[
\Rightarrow s^i_L = \frac{1 + q}{2} M_H M_L - \overline{M}_H M_L q - \overline{M}_I M_H (1 - q) \tag{62}
\]
Equation (62) gives the amount of low quality projects $P_i$ needs to finance as a response to his opponents strategy in order to win the election and maximize his payoff. In a similar manner as in section 1, we denote this level of inefficiency as $s^i_L$. If the politician were to draw, instead, condition (47) would be rewritten as:

$$\frac{M_H}{M_H} q + \frac{M_i}{M_L}(1 - q) + \frac{s^i_H[(\alpha_H - 1 - t_i)I + b]}{M_H} - \frac{s^i_H[(\alpha_H - 1 - t_j)I + b]}{M_H} + \frac{(s^i_L - s^j_L)b}{M_L} = \frac{1}{2}$$  \hspace{1cm} (63)

The only difference now is that the politician does not require the $\epsilon$ mass of extra voters. Hence, by following the same steps as above we get a similar condition to (62), which expresses the amount of low quality projects $P_i$ needs to finance in order to maximize his utility given that he draws the election:

$$s^i_L = s^j_L - \frac{1}{2}M_H M_L - \frac{M_H M_L}{M_L \alpha H I - (M_H - M_L)b}$$  \hspace{1cm} (64)

The politician will prefer to draw the election rather than win with certainty if and only if:

$$U_{\bar{P}} \geq U_P \Leftrightarrow \frac{1}{2} B_i^j \geq B_i^L \Leftrightarrow B_i \geq 2^\frac{1}{2} B_i^L \Leftrightarrow \alpha LS_L^i I \geq 2^\frac{1}{2} \alpha LS_L^i I \Leftrightarrow$$

$$s^j_L - \frac{1}{2}M_H M_L - \frac{M_H M_L}{M_L \alpha H I - (M_H - M_L)b} \geq 2^\frac{1}{2} \left[ s^j_L - \frac{1}{2} M_H M_L - \frac{M_H M_L}{M_L \alpha H I - (M_H - M_L)b} \right] \Leftrightarrow$$

$$2^\frac{1}{2} \left[ \frac{1}{2} M_H M_L - \frac{M_H M_L}{M_L \alpha H I - (M_H - M_L)b} \right] - \frac{1}{2}M_H M_L - \frac{M_H M_L}{M_L \alpha H I - (M_H - M_L)b} \geq \left( 2^\frac{1}{2} - 1 \right) s^j_L \Leftrightarrow$$

$$\left( 2^\frac{1}{2} - 1 \right) \left[ \frac{1}{2} M_H M_L - \frac{M_H M_L}{M_L \alpha H I - (M_H - M_L)b} \right] + 2^\frac{1}{2} \left( \frac{M_H M_L}{M_L \alpha H I - (M_H - M_L)b} \right) \geq \left( 2^\frac{1}{2} - 1 \right) s^j_L \Leftrightarrow$$

$$s^j_L \leq \frac{1}{2} M_H M_L - \frac{M_H M_L}{M_L \alpha H I - (M_H - M_L)b} + \frac{2^\frac{1}{2} \left( \frac{M_H M_L}{M_L \alpha H I - (M_H - M_L)b} \right)}{2^\frac{1}{2} - 1} \left( \frac{M_H M_L}{M_L \alpha H I - (M_H - M_L)b} \right)$$

(65)

The above condition determines a critical value for $P_j$ in the same way as in the symmetric political game. For values of $s^j_L$ above this critical value, the politician prefers to play aggressively and win the election with certainty, while for values below the critical threshold he prefers to draw the election. Let this critical value be $s^j_L^*$. Similarly, there is an equivalent critical value for $P_j$:

$$s^i_L^* = \frac{1}{2} M_H M_L - \frac{M_H M_L}{M_L \alpha H I - (M_H - M_L)b} + \frac{2^\frac{1}{2} \left( \frac{M_H M_L}{M_L \alpha H I - (M_H - M_L)b} \right)}{2^\frac{1}{2} - 1} \left( \frac{M_H M_L}{M_L \alpha H I - (M_H - M_L)b} \right)$$

(66)
Conditions (65) and (66) determine the equilibrium of the asymmetric game, as it is described in section 2. Also, if condition (60) is satisfied, then:

\[
\left( \frac{M_H}{M_L} - 1 \right) < \frac{(\alpha_H - \alpha_L)I}{b} \iff (M_H - M_L)b < M_LI(\alpha_H - \alpha_L) \iff \\
M_LI(\alpha_H - \alpha_L) - (M_H - M_L)b > 0
\]

But if this condition holds then also \( M_LI(\alpha_H - \alpha_L) - (M_H - M_L)b > 0 \) must also hold since:

\[
M_LI(\alpha_H - \alpha_L) - (M_H - M_L)b > 0 \iff M_LI(\alpha_H - \alpha_L) - (M_H - M_L)b > M_LI\alpha_L > 0
\]

This means that the solution we have provided above is well defined. Finally, we show that if (60) is satisfied then also the assumption \( Z < 0 \) holds. To prove this, first observe that the value of \( Z \) is equivalent to the left hand side of equation (54):

\[
-Z = -M_L\lambda t_i I - M_Ls^i_L(\alpha_H - 1 - t_i)I + (M_H - M_L)bs^i_L = Z
\]

Also observe that, from conditions (44), (45), (46) we had previously derived that:

\[
\alpha_Ls^i_L - \beta_is^i_L + (\lambda - s^i_L)t_i - s^i_L = 0
\]

Solving for \( t_i \) and substituting in the previous relation gives:

\[
t_i = \frac{(1 + \beta_i - \alpha_L)s^i_L}{(\lambda - s^i_L)}
\]

\[
Z = -M_L\lambda t_i I - M_Ls^i_L(\alpha_H - 1 - t_i)I + (M_H - M_L)bs^i_L \iff \\
Z = -M_L(\lambda - s^i_L)t_i - M_Ls^i_L(\alpha_H - 1)I + (M_H - M_L)bs^i_L \iff \\
Z = -M_LI(\lambda - s^i_L)\frac{(1 + \beta_i - \alpha_L)s^i_L}{(\lambda - s^i_L)} - M_LI(\alpha_H - 1) + (M_H - M_L)bs^i_L \iff \\
Z = -M_LI(\alpha_H - \alpha_L) + (M_H - M_L)bs^i_L - M_LI\beta_is^i_L
\]

But condition (60) implies that the sum of the first two terms is negative and the third term is non-positive, so \( Z < 0 \). This concludes the exposition of case 1.
Case 2: $\frac{\partial B_i}{\partial s^i_L} > 0$

$$\frac{\partial B_i}{\partial s^i_L} > 0 \iff \left( \frac{M_H}{M_L} - 1 \right) b > (\alpha_H - \alpha_L)I$$

The politician maximizes his utility when $s^i_L$ takes the highest possible value, in this case. But, as $s^i_L$ increases, the tax rate must also increase in order to cover the losses of state-owned banks. More formally, note that, the tax rate as a function of $s^i_L$ is by equation (55):

$$t_i = \frac{Z + \Theta s^i_L}{-M_LI(\lambda - s^i_L)}$$

By taking partial derivative of $t_i$ with respect to $s^i_L$, we get the following expression:

$$\frac{\partial t_i}{\partial s^i_L} = \frac{-M_LI\Theta(\lambda - s^i_L) - (Z + \Theta s^i_L)M_LI}{[-M_LI(\lambda - s^i_L)]^2} \iff$$

$$\frac{\partial t_i}{\partial s^i_L} = \frac{-M_LI(\lambda\Theta + Z)}{[-M_LI(\lambda - s^i_L)]^2}$$

Notice that the sign of this derivative depends on the sign of the expression $\lambda\Theta + Z$. We use equations (54) and (56) to substitute for the values of $Z$ and $\Theta$, respectively:

$$\lambda\Theta + Z = \lambda M_LI(\alpha_H - 1) - \lambda(M_H - M_L)b - \lambda M_LIt_i + (M_H - M_L)b s^i_L - s^i_L M_LI(\alpha_H - 1 - t_i) \iff$$

$$\lambda\Theta + Z = M_LI(\alpha_H - 1 - t_i)(\lambda - s^i_L) - (M_H - M_L)b(\lambda - s^i_L) \iff$$

$$\lambda\Theta + Z = (\lambda - s^i_L)(M_LI(\alpha_H - 1 - t_i) - (M_H - M_L)b)$$

Because we have assume that $\frac{\partial B_i}{\partial s^i_L} > 0$ holds, then it also true that $(M_H - M_L)b > M_LI(\alpha_H - \alpha_L)$ or $M_LI(\alpha_H - \alpha_L) - (M_H - M_L)b < 0$. However, it is also true that: $(\alpha_H - \alpha_L) > (\alpha_H - 1 - t_i)$. Therefore:

$$M_LI(\alpha_H - \alpha_L) > M_LI(\alpha_H - 1 - t_i) \iff$$

$$M_LI(\alpha_H - 1 - t_i) - (M_H - M_L)b < M_LI(\alpha_H - \alpha_L) - (M_H - M_L)b < 0$$

Hence $\lambda\Theta + Z < 0$ and this implies that: $\frac{\partial t_i}{\partial s^i_L} > 0$. As $s^i_L$ increases, the required taxation increases as well. Hence, the maximum value that $s^i_L$ can attain is when
\[ t_i = \alpha_H - 1. \] The above conditions hold for both politicians and this means that 
\[ t_j = \alpha_H - 1 \] as well. Under these conditions (53) and (54) transform into:

\[ Z(t_j, s^j_L) = Z = E - M_L I \lambda (\alpha_H - 1) + (M_H - M_L) b s^j_L \] (67)

\[-M_L I \lambda (\alpha_H - 1) + (M_H - M_L) b s^j_L = Z \] (68)

Hence:

\[ s^j_L = \frac{Z + M_L I \lambda (\alpha_H - 1)}{(M_H - M_L) b} \iff s^j_L = \frac{E - M_L I \lambda (\alpha_H - 1) + (M_H - M_L) b s^j_L + M_L I \lambda (\alpha_H - 1)}{(M_H - M_L) b} \]

\[ s^j_L = s^j_L + \frac{E}{(M_H - M_L) b} \iff s^j_L = s^j_L + \frac{\frac{1}{2} M_H M_L - M_h M_L q - M_l M_H (1 - q)}{(M_H - M_L) b} \] (69)

Because \( \left( \frac{M_H}{M_L} - 1 \right) b > (\alpha_H - \alpha_L) I \iff (M_H - M_L) b - M_L I (\alpha_H - \alpha_L) > 0 \) it also holds that \( (M_H - M_L) b > 0 \), so this solution is well defined. This is the value of \( s^j_L \) which \( P_i \) needs to offer to win the election and we denote it as \( s^i_L \).

Also, By (44), (45) and (46):

\[ \alpha_L s^i_L - \beta_i s^i_L + (\lambda - s^i_L) t_i - s^i_L = 0 \iff \alpha_L s^i_L - \beta_i s^i_L + (\lambda - s^i_L)(\alpha_H - 1) - s^i_L = 0 \iff
\]

\[ \beta_i s^i_L = \lambda (\alpha_H - 1) - (\alpha_H - \alpha_L) s^i_L \iff \tilde{\beta}_i = \frac{\lambda (\alpha_H - 1)}{s^i_L} - (\alpha_H - \alpha_L) \] (70)

The overall utility to the politician by this strategy is given by:

\[ \tilde{B}^\gamma_{i} = (\tilde{\beta}_i s^i_L I)^\gamma = \left[ \lambda (\alpha_H - 1) I - (\alpha_H - \alpha_L) s^i_L I \right]^\gamma \] (71)

If the politician wants to draw the election, the respective conditions of (69), (70) and (71) are:

\[ s^i_L = s^j_L + \frac{1}{2} M_H M_L - M_h M_L q - M_l M_H (1 - q)}{(M_H - M_L) b} \] (72)

\[ \beta_i = \frac{\lambda (\alpha_H - 1)}{s^i_L} - (\alpha_H - \alpha_L) \] (73)

\[ B^\gamma_{i} = (\beta_i s^i_L I)^\gamma \Rightarrow B^\gamma_{i} = \left[ \lambda (\alpha_H - 1) I - (\alpha_H - \alpha_L) s^i_L I \right]^\gamma \] (74)

The politician will prefer draw over victory iff:
\[ U_p \geq \tilde{U}_p \iff \frac{1}{2} B_i^p \geq \tilde{B}_i^p \iff B_i \geq 2^{\frac{1}{2}} \tilde{B}_i \iff \]

\[
\lambda (\alpha_H - 1) I - (\alpha_H - \alpha_L) s_i^L I \geq 2 \frac{1}{2} \left[ \lambda (\alpha_H - 1) I - (\alpha_H - \alpha_L) \tilde{s}_i^L I \right] \iff
\]

\[
2^{\frac{1}{2}} (\alpha_H - \alpha_L) \tilde{s}_i^L - (\alpha_H - \alpha_L) s_i^L \geq \left( 2^{\frac{1}{2}} - 1 \right) \lambda (\alpha_H - 1) \iff
\]

\[
2^{\frac{1}{2}} \left( s_i^j + \frac{\frac{1}{2} M_H M_L - \frac{1}{2} M_h M_L q - \frac{1}{2} M_l M_l (1 - q) \epsilon}{(M_H - M_L) \epsilon} \right) - s_i^j \geq \frac{\frac{1}{2} M_H M_L - \frac{1}{2} M_h M_L q - \frac{1}{2} M_l M_l (1 - q) \epsilon}{(M_H - M_L) \epsilon} \geq \frac{\left( 2^{\frac{1}{2}} - 1 \right) \lambda (\alpha_H - 1)}{(\alpha_H - \alpha_L)} \iff
\]

\[
(2^{\frac{1}{2}} - 1) s_i^j + (2^{\frac{1}{2}} - 1) \frac{\frac{1}{2} M_H M_L - \frac{1}{2} M_h M_L q - \frac{1}{2} M_l M_l (1 - q) \epsilon}{(M_H - M_L) \epsilon} + 2^{\frac{1}{2}} \frac{\frac{1}{2} M_H M_L}{(M_H - M_L) \epsilon} \geq \frac{\left( 2^{\frac{1}{2}} - 1 \right) \lambda (\alpha_H - 1)}{(\alpha_H - \alpha_L)} \iff
\]

\[
s_i^j \geq \frac{\lambda (\alpha_H - 1)}{(\alpha_H - \alpha_L)} - \frac{\frac{1}{2} M_H M_L - \frac{1}{2} M_h M_L q - \frac{1}{2} M_l M_l (1 - q) \epsilon}{(M_H - M_L) \epsilon} - 2^{\frac{1}{2}} - 1 \left[ \frac{M_H M_L \epsilon}{(M_H - M_L) \epsilon} \right] \quad (75)
\]

As in case 1, the above condition determines a critical value for \( P_i \), below which the politician prefers to play aggressively and win the election with certainty, while for values above he prefers to draw. Once again, we denote the critical value as \( s_i^L \). The equivalent critical value for \( P_j \) is:

\[
s_i^L = \frac{\lambda (\alpha_H - 1)}{(\alpha_H - \alpha_L)} - \frac{\frac{1}{2} M_H M_L - \frac{1}{2} M_h M_L q - \frac{1}{2} M_l M_l (1 - q) \epsilon}{(M_H - M_L) \epsilon} - 2^{\frac{1}{2}} - 1 \left[ \frac{M_H M_L \epsilon}{(M_H - M_L) \epsilon} \right] \quad (76)
\]

Conditions (75) and (76) determine the equilibrium of the asymmetric game under case 2, as it is described in section 2.
References


