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## Private Monitoring, Collusion and the Timing of Information

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# Private Monitoring, Collusion and the Timing of Information

## Abstract

When the information used by a principal to monitor an agent is private, and thus non-verifiable by a third party, the principal has a credibility issue with the agent. The agent should be concerned that the principal could misrepresent the information in order to collect a monetary penalty from him. Restoring credibility may lead to an extreme waste of resources—the so-called burning of money, where the monetary penalty is given away to a third party. We show that a more realistic and efficient outcome is feasible by exploiting the timing of private information. If the private information arrives before the agent has completed his effort, non-monetary tools like rescaling the project become optimal, and no money needs to be burned. We show that rescaling is more effective than pure monetary penalties because effort has different values to different parties, while money is equally valuable to all parties. An alternative way to solve the principal's credibility problem is to certify the private signal and make it public. When collusion between the certifier and the agent is an issue, we uncover interesting similarities between private signals and public (certified) signals vulnerable to collusive manipulation. We show that certification of private information by a third party may not always be in the interest of the principal if this certification raises the specter of collusion.

JEL-Code: D730, D820, D860.

Keywords: monitoring, unverifiable signal, certification, private communication, timing of information, collusion, non-monetary penalties, burning money.

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## **1. Introduction**

Contracting parties can obtain monitoring information at various times during an agency relationship. In this paper, we focus on monitoring information learned privately by the principal, i.e., private signals, and we show the importance of the timing of information. Private signals cannot be verified by a third party. For example, the signal could be the principal's subjective evaluation of the project's difficulty or of an agent's performance, a casual conversation with a customer or a co-worker about the agent's behavior, or the filtering of the agent's e-mail. Another example is the information that internal and external auditors may share with audit committees in executive sessions with details that go beyond what is documented in publicly released audit reports. In military procurement, national security concerns may prevent the Department of Defense from releasing information publicly. In these examples, the private information may arrive *ex post*, i.e., after the agent has performed his task, or *ex ante*, i.e., before or during the performance of tasks. The aim of this paper is to investigate to what extent the principal may use such private signals to improve the agent's incentives and to highlight the importance of the timing of information.

When a signal is privately observed by the principal, it provides an opportunity for the principal to abuse the private information. The principal's credibility becomes an issue. For instance, an employee should be concerned if the employer can impose and collect a penalty based on an unverifiable conversation with a customer. To resolve this concern, we consider two options. The principal can reassure the agent either by removing her own incentive to abuse private information, or by having the information publicly certified by a third party.

We begin by discussing the first option, where a new constraint is needed to ensure that the principal does not personally gain from a penalty based on such a signal. One way to satisfy this constraint is to force the principal to give away any collected penalty to a third party, the

so-called ‘burning money’ solution.<sup>1</sup> While this solution is quite intriguing, it is also a little disconcerting as it involves wasting resources with respect to the contracting parties.

In this paper, we argue that organizations will look for more efficient solutions than burning money to address the credibility issue with private signals. In particular, the principal may have additional alternatives depending on the timing of private information. If she obtains private information *ex ante*, then the agent’s required effort, i.e., the scale of the project itself, can be reduced as an alternative to burning money.<sup>2</sup> This rescaling of effort has two effects: it penalizes the agent and also affects the principal’s payoff to ensure that she does not gain from penalizing the agent. We show that relying on non-monetary penalties like rescaling is more effective than pure monetary penalties, which require money burning. The reason is that effort has different values to different parties, while money is equally valuable to all parties.

To present this intuition, we use an adverse-selection model where a principal observes the output and can use an imperfect private signal of the agent’s type to induce effort. The signal allows the principal to penalize the shirking agent but, because it is imperfect, complying agents are also penalized by mistake. If the penalty is lost to a third-party, as in the case when the penalty has to be burned, the signal is less effective. In the case of an *ex post* private signal, we show that the signal is not useful unless it is accurate enough. In the case of an *ex ante* private signal, we show that the principal can use the signal even if the level of accuracy is very low. The reason is that she can rely on rescaling rather than be limited to burning money. If the signal allows the principal to extract all rent for low accuracy, then there is no money burning in equilibrium. Otherwise, burning money can be used to extract the remaining rent.

Since rescaling cannot take place after the project has already been completed, the timing of information becomes critical: we find that the principal always prefers to receive

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<sup>1</sup> See the recent literature on subjective evaluation (Levin (2003), MacLeod (2003), Fuchs (2007)). Riordan and Sappington (1986) propose an analogous remedy: the principal may commit not to accept some portion of the output *ex post*, allowing it to go to waste.

<sup>2</sup> Holmstrom and Milgrom (1987) also challenge the assumption that the agent is required to make a once-for-all choice of his effort during the relationship without regard to the arrival of performance information. Their focus is on the agent, instead of the principal, receiving information and adjusting his effort continuously over time.

private information early. In other words, the principal may be willing to incur an extra cost to receive private signals *ex ante* rather than *ex post*.

A second option to resolve the principal's credibility problem is to certify the private signal, i.e., make it public (Farhi, Lerner and Tirole (2012)). If certification was free, this solution would always be preferred because, as is well-known from the literature, the first best can be reached irrespective of the timing of the signal.<sup>3</sup>

However, in a world where information can be manipulated, we consider an endogenous cost of certification due to the potential for collusion between the certifier (e.g., an auditor or a supervisor) and the agent. We contrast the two problems faced by a principal in the presence of manipulable information: her own credibility if the signal is privately learned by her, versus the threat of collusion if she certifies a signal to avoid her own credibility problem. First, we find that rescaling effort or, more generally, non-monetary penalties, can become ineffective as the certifier only values monetary bribes. The reason is that the certifier does not value the output in itself, and therefore, a monetary transfer is the only instrument that can impact both the agent and the certifier. Second, we find unexpected similarities between a public/certified signal and a private signal. The standard remedy to prevent bribery – rewarding the certifier for reporting unfavorable information about the agent<sup>4</sup> – has a similar effect to burning money. The principal does not collect any penalty from the agent because it is transferred to a third party, the certifier. Because of collusion, the signal is less effective and has to be accurate enough before it becomes useful. It is not clear *a priori* whether the principal should prefer a private signal suffering from a lack of credibility or a certified public signal vulnerable to collusion.

We show that each manipulation issue – lack of credibility and collusion – leads to the same outcome in the *ex post* signal case. In the *ex ante* signal case, the credibility issue is less severe, and a private signal is preferred to a public signal vulnerable to collusion. Once again, the timing of the signal is important. An implication of this result is that if the principal had the

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<sup>3</sup> See Cremer and McLean (1988) for *ex ante* signals, and Baron and Besanko (1984) and Riordan and Sappington (1988) for the case of *ex post* signals.

<sup>4</sup> See, e.g., Tirole (1986) and Kofman and Lawarrée (1993).

option of using a third-party to certify an ex ante private signal, i.e., make it public, she should choose not to do so if that opens the door for collusion. It is only when it is difficult for the agent and the certifier to collude (for instance due to large collusion costs between them) that a certified signal may be preferred to a private signal.

Our paper is related to the literature on subjective evaluations, which builds upon the theory of repeated games with asymmetric information (See, e.g., Kandori and Matsushima (1998). Kandori (2002) provides a survey). Because this literature emphasizes moral hazard (or hidden action), the principal's private information pertains directly to the agent's action and arrives *after* that action has been taken. MacLeod (2003) shows that it is impossible to induce effort under a single-period contract that is budget-balancing, hence the need to burn money. Levin (2003) considers an infinitely repeated game where, instead of burning money, the principal fires the agent when her privately observed measure of performance falls below a threshold. Also, in a dynamic environment, Fuchs (2007) characterizes equilibria in which the principal delays revealing to the agent what she has learned privately.<sup>5</sup>

Strausz (2006) and Dequiedt and Martimort (2012) consider adverse selection models and show the role of private information in restoring the continuity between correlated and uncorrelated environments. In these models, as in ours, the principal is able to use manipulable signals to extract rent. Strausz studies the impact of limited commitment due to private information in a multi-period game with private information arriving between periods. However, he assumes that the budget is in balance (burning money is not considered) and that collusion is not an issue. Dequiedt and Martimort (2012) study bilateral contracting in a model with multiple agents and correlated types instead of one agent and a supervisor. They do not consider the timing of information or burning money. In a sense, our result for the sub-optimality of burning money provides a rationale for their analysis. Also, in their model, if the agents can collude, multi-dimensional screening issues appear and collusion removes the value of correlated private information.

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<sup>5</sup> Demski and Sappington (1993) consider a double moral hazard problem with a private signal but rely on a second public signal in order to satisfy the principal's incentive constraint with a balanced budget.

## 2. The model

We consider a principal (she) - agent (he) model with adverse selection. The agent exerts an effort  $e$  which, together with a productivity parameter  $\theta$ , determines output  $x = \theta e$ . The parameter  $\theta$  takes one of two positive values, with  $\theta_1 < \theta_2$ , where the low type  $\theta_1$  obtains with probability  $\xi$ , and the high type  $\theta_2$  obtains with probability  $1 - \xi$ . We normalize  $\theta_2 = 1$  to simplify the exposition.<sup>6</sup> The agent's disutility of effort is given by the function  $\psi(e) = e^2/2$ . The output belongs to the principal, who compensates the agent with a transfer  $t$ . The principal maximizes the output net of transfers:  $\pi = x - t$ . The agent maximizes his utility  $U = t - \psi(e)$ , and we assume that the agent's reservation utility is zero.

The first-best contract equates the marginal cost and marginal product of the agent's effort ( $e_i^{FB} = \theta_i, i = 1, 2$ ) and pays the agent just enough to compensate for his disutility of effort ( $t_i^{FB} = \theta_i^2/2, i = 1, 2$ ). The first best cannot be implemented if  $\theta$  and  $e$  are private information of the agent. The principal must then offer a contract based on the observable output rather than the unobservable effort. As is well known (see, Baron-Myerson (1982) for example), the principal must solve the following problem:  $\text{Max } \xi(\theta_1 e_1 - t_1) + (1 - \xi)(e_2 - t_2)$  subject to the participation constraint of the low type,  $t_1 - \frac{(e_1)^2}{2} \geq 0$ , and the incentive compatibility constraint of the high type,  $t_2 - \frac{(e_2)^2}{2} \geq t_1 - \frac{(\theta_1 e_1)^2}{2}$ .<sup>7</sup> The optimal contract is the standard second-best contract. We refer to it as the *no-signal* contract and use the super-script *NS*. It distorts the effort of the low type ( $e_1^{NS} = \xi \theta_1 / \{\xi + (1 - \xi)(1 - \theta_1^2)\} < \theta_1 = e_1^{FB}$ ) to reduce the rent of the high type. The high type produces the first best level of effort ( $e_2^{NS} = 1 = e_2^{FB}$ ) and the low type receives no rent.

So far, we have a standard adverse selection model based on observable output, but we now introduce a new feature in the model. During the course of the principal-agent

<sup>6</sup> A high-type can mimic the low type by putting in a fraction  $\theta_1/\theta_2$  of the low-type's effort. So, a condition saying  $\theta_1 < 1$ , is really the condition  $\theta_1/\theta_2 < 1$ . If we used  $\theta_2$  instead of 1, we would need to replace  $\theta_1$  by  $\theta_1/\theta_2$  in the right hand side of the IC constraint below but our main results would be unaffected.

<sup>7</sup> The high type can mimic the observable output of the low type by exerting an effort  $\tilde{e}$  such that  $\theta_2 \tilde{e} = \theta_1 e_1$ . With  $\theta_2$  normalized at 1, the cost of mimicking is  $\frac{(\theta_1 e_1)^2}{2}$ .



relationship, the principal receives additional private information, which could enhance efficiency. For example, improved cost estimates about the project would allow better planning of the output. Here we focus on additional *private* information, and later in section 3, we will examine the certification of the private signal by a third party who can be bribed.

This additional information is learned privately by the principal and, therefore, not verifiable in a court of law.<sup>8</sup> The principal would certainly want to use the private information but the agent should worry about being ‘held up’ if the principal can alter the terms of the contract based on unverifiable information.<sup>9</sup> In such a case, one can ask whether a contract should be conditioned on this signal.

We use a standard way to model the principal’s signal, denoted by  $\sigma$ :  $\sigma \in \{\sigma_1, \sigma_2\}$  and  $\text{Prob}(\sigma_1|\theta_1) = \text{Prob}(\sigma_2|\theta_2) = \nu$ , and  $\text{Prob}(\sigma_1|\theta_2) = \text{Prob}(\sigma_2|\theta_1) = 1 - \nu$ , with  $\nu > \frac{1}{2}$ . That is, the signal is correct with probability  $\nu$  and wrong with probability  $(1 - \nu)$ . The signal is obtained at zero cost. Since the agent will have an incentive to misreport his type only when he is the high type, the signal is relevant only after the agent has claimed to be  $\theta_1$ .<sup>10</sup>

Because the signal is private information of the principal, the contract can only be based on the principal’s public assertion about the signal  $\sigma$ . We assume that the principal makes such a public announcement about the signal, and we label it  $\hat{\sigma}$ , where  $\hat{\sigma} \in \{\hat{\sigma}_1, \hat{\sigma}_2\}$ .

The timing of the signal and its announcement are important, and it matters whether the signal arrives before or after the agent makes his effort. When the signal arrives before the agent makes his effort, the principal can potentially use the contract to rescale the agent’s optimal effort based on the report. We call it the *ex ante information* case. When the signal arrives after the effort has already been taken, we call this the *ex post information* case. With *ex post* information, the only option for the principal is to use the report of the signal to affect the transfer to the agent, for instance by imposing a penalty on the agent.

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<sup>8</sup> Fuchs (2007) distinguishes between private signals and observable but unverifiable signals.

<sup>9</sup> For instance, suppose that the agent claims that the productivity parameter is low and the private signal confirms the claim. Because the information is private, nothing stops the principal from reporting that the signal contradicts the agent’s claim and penalizing him. In other words, the private information is soft.

<sup>10</sup> Signals received after the agent reports that he is a high type can be ignored.

Our main focus will be on the ex ante case but we will start by discussing briefly the ex post case to introduce the concept of burning money in the context of our model. We also derive a result on the accuracy of the signal that is used in the rest of the paper.

### *2.1. An ex post private signal: the optimality of burning money*

In the ex post case, the principal receives a private signal about the agent's type after the output has been produced. This case has been analyzed in the context of moral hazard by MacLeod (2003) and Fuchs (2007), and they show the need to burn money to make private signals useful. In this section, we extend their result to an adverse selection model, and we also highlight how the loss of the penalty by way of burning money implies that a private signal can only be used if it is accurate enough. We summarize the timing for the model of ex post signal as follows:

- (1) The agent observes  $\theta$ .
- (2) The principal offers the agent a contract; the agent rejects the contract, ending the game, or accepts the contract and reports  $\hat{\theta}$ .
- (3) The agent exerts effort, and output is publicly observed.
- (4) The principal observes the signal  $\sigma$  and reports  $\hat{\sigma}$ .
- (5) The transfers are realized.

The optimal contract is based on the agent's report  $\hat{\theta}$  and the principal's report  $\hat{\sigma}$ . It specifies efforts  $e_1^p$  and  $e_2^p$  based on the agent's report and transfers based on both the agent's (first subscript) and the principal's (second subscript) reports:  $t_{11}^p, t_{12}^p$ . The superscript "p" reminds the reader that this signal comes ex post. Later we will use the superscript "a" for the ex ante case. Since the signal is not used after the agent reports high type, the corresponding transfer has only one subscript:  $t_2^p$ .

The principal would like to use the private signal to penalize the agent for under-reporting his type and reduce his information rent. At the same time, the agent needs to be assured that the principal will not abuse the private signal to penalize the agent. For example, suppose  $t_{11}^p > t_{12}^p$ . The principal would always report the private signal that leads him to pay

the lower transfer  $t_{12}^p$ . To induce the principal to tell the truth, the contract must satisfy two “Principal’s Incentive Compatibility” (PIC) constraints, one for each signal.<sup>11</sup> As known from the literature on moral hazard, these constraints cannot be satisfied by a ‘balanced budget’, and it is necessary to introduce a gap between what the principal pays and what the agent receives. We represent this gap by the variables  $B_{11}^p$  and  $B_{12}^p$ . When the principal pays  $t_{11}^p$  (resp.  $t_{12}^p$ ) to the agent, she also burns  $B_{11}^p$  (resp.  $B_{12}^p$ ).<sup>12</sup> The first constraint guarantees that the principal will not report state 2 ( $\hat{\sigma}_2$ ) when the signal is  $\sigma_1$  and the second guarantees that the principal will not report state 1 ( $\hat{\sigma}_1$ ) when the signal is  $\sigma_2$ . Therefore, the two (PIC) constraints for ex post information are

$$\begin{aligned} -(t_{11}^p + B_{11}^p) &\geq -(t_{12}^p + B_{12}^p) \\ -(t_{12}^p + B_{12}^p) &\geq -(t_{11}^p + B_{11}^p), \end{aligned}$$

and together they imply:

$$(PIC^p) \quad t_{11}^p - t_{12}^p = B_{12}^p - B_{11}^p,$$

which means that the principal cannot gain ex post from penalizing the agent. Note that even though the (PIC) requires the principal to be indifferent between reporting publicly  $\hat{\sigma}_1$  or  $\hat{\sigma}_2$ , that does not imply that the principal cannot benefit from the private information: it means that any benefit must come from its impact on the initial contract.

Some intuition about the impact of  $(PIC^p)$  on the principal’s problem can be presented by looking at the relevant participation and incentive compatibility constraints:<sup>13</sup>

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<sup>11</sup> The Revelation Principle can be extended to the case of private signals because the signal is free for the principal. An alternative is to use a mediated contract as in Strausz (2006). On the use of the revelation principle with private information, see also Dequiedt and Martimort (2012). If obtaining the private signal is costly, our results could be extended if the principal can commit to acquire the private signal. Otherwise, Rahman (2012) shows that mediated contracts relying on correlated equilibrium can improve efficiency.

<sup>12</sup> MacLeod (2003) suggests that an unbalanced budget is merely a tractable metaphor for equilibrium conflict, such as “work to rule” behavior by union members. What is important is that the principal does not collect  $B_{11}^p$  or  $B_{12}^p$ ; giving them to a third party (like a charity) would also satisfy (PIC). Fuchs (2007) cites an example from Jon Levin where fines collected from baseball players are given to charity. The same is true for football players in the NFL.

<sup>13</sup> The (IC) constraint for the low type is redundant and suppressed here and in the sections that follow.

$$(IR^a-1) \quad \nu t_{11}^p + (1 - \nu)t_{12}^p - \frac{1}{2}(e_1^p)^2 \geq 0$$

$$(IR^a-2) \quad t_2^p - \frac{1}{2}(e_2^p)^2 \geq 0,$$

$$(IC^p) \quad t_2^p - \frac{1}{2}(e_2^p)^2 \geq (1 - \nu)t_{11}^p + \nu t_{12}^p - \frac{1}{2}(\theta_1 e_1^p)^2.$$

As usual,  $(IR^a-1)$  and  $(IC^p)$  will be binding. Using the binding  $(IR^a-1)$  and re-arranging the terms, the high-type's rent, given by the right-hand-side of  $(IC^p)$ , can be re-written as:

$$u_2^p = \left[ \frac{1}{2}(e_1^p)^2 - \frac{1}{2}(\theta_1 e_1^p)^2 \right] - (2\nu - 1)(t_{11}^p - t_{12}^p)$$

The first terms in the brackets represent the cost differential between a truthful low type and a shirking high type. The second term represents a penalty effect created by the difference in the agent's transfers  $(t_{11}^p - t_{12}^p)$ ; the greater the difference in transfers, the lower the information rent.

We note first that  $(PIC^p)$  puts restrictions on the penalty by determining how much money must be burned ( $B_{11}^p$  and  $B_{12}^p$ ). It is proven in appendix 1 that  $B_{11}^p$  is zero. Intuitively, a positive  $B_{11}^p$  would make it harder to satisfy  $(PIC^p)$  by decreasing the penalty.<sup>14</sup> Thus, we can rewrite the rent expression as:

$$(Rent^p) \quad u_2^p = \left[ \frac{1}{2}(e_1^p)^2 - \frac{1}{2}(\theta_1 e_1^p)^2 \right] - (2\nu - 1)B_{12}^p.$$

While increasing  $B_{12}^p$  reduces rent by increasing the penalty, it is also costly: in equilibrium, the low-type is penalized by mistake (unless  $\nu = 1$ ), and this cost is borne by the principal because the agent's participation constraint  $(IR^p)$  binds. Thus the possibility of reducing the rent of a cheating high type has to be balanced against the cost of erroneous penalties on the truthful low type.

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<sup>14</sup> If  $B_{11}^p$  were strictly positive, the principal could reduce her cost without altering any constraint by lowering both  $B_{11}^p$  and  $B_{12}^p$  by equal amounts.

**Proposition 1:**

*An ex post private signal is useful only if the signal is accurate enough, i.e., when  $v > \frac{1}{2-\xi}$ ; money is burned and the agent earns zero rent. The no-signal contract is optimal when  $v \leq \frac{1}{2-\xi}$ .*

**Proof:** See appendix 1.

Since the penalty  $(t_{11}^p - t_{12}^p)$  is burned and therefore lost to the principal, there is a cost of increasing the penalty -- the truthful low-type is penalized by mistake. Without (PIC<sup>p</sup>) this cost would not be there since the principal could use the collected penalty to compensate the risk neutral agent for the erroneous penalties. It is useful to understand that the critical level of accuracy is determined by two first-order effects. A unit increase in the penalty reduces the information rent, which is paid with probability  $1 - \xi$ , by an amount  $2v - 1$  and requires a unit of money to be burnt with probability  $\xi(1 - v)$ . Therefore it is optimal to burn money to eliminate the rent if  $(1 - \xi)(2v - 1) > \xi(1 - v)$ , which can be rewritten as  $v > 1/(2 - \xi)$ .

The ex post information case has similarities with a pure moral hazard model since only the transfers can be modified based on the signal. However, in the moral hazard models used in the literature on subjective evaluations (Levin (2003, part IV), Fuchs (2007) and MacLeod (2003)), the output is not publicly observed. The principal receives a signal about the actual output, which the agent does not see. In our model, when the agent chooses his effort he knows what the publicly observable output will be but not the exogenous signal  $\sigma_i$  about the agent's type, which is privately observed by the principal. Since output can be contracted upon, there is a feasible contract even absent any signal, and the principal will only use the signal if it is accurate enough.

Rent can be fully extracted as soon as the signal is useful, and while the distortion in effort can be reduced, effort is less than the first best unless  $v = 1$ . Therefore, the classic separation result of Baron and Besanko (1984), which says that rent is extracted before effort is restored towards the first best, can be extended to the case of ex post private information.

Next, we analyze ex ante private signals and show that burning money is no longer optimal.

## 2.2. The ex ante private signal case

### 2.2.1. Preliminaries

In this section, we consider the case where the principal receives a private signal before the agent takes his action, i.e., we flip points 3 and 4 in the timing of the ex post case above. The main difference with respect to the ex post signal case is that the agent's effort may now depend on both the agent's (first subscript) and the principal's (second subscript) reports:  $e_{11}^a$ ,  $e_{12}^a$ . The superscript "a" denotes the ex ante case. The individual rationality and incentive compatibility constraints now reflect this dependence of effort on the principal's report (the participation constraint of the high type is unchanged):

$$(IR^a-1) \quad v \left( t_{11}^a - \frac{1}{2} (e_{11}^a)^2 \right) + (1 - v) \left( t_{12}^a - \frac{1}{2} (e_{12}^a)^2 \right) \geq 0$$

$$(IR^a-2) \quad t_2^a - \frac{1}{2} (e_2^a)^2 \geq 0,$$

$$(IC^a) \quad t_2^a - \frac{1}{2} (e_2^a)^2 \geq (1 - v) \left( t_{11}^a - \frac{1}{2} (\theta_1 e_{11}^a)^2 \right) + v \left( t_{12}^a - \frac{1}{2} (\theta_1 e_{12}^a)^2 \right).$$

The principal's incentive compatibility constraints also now reflect the dependence of effort on the principal's public report:<sup>15</sup>

$$\theta_1 e_{11}^a - t_{11}^a - B_{11}^a \geq \theta_1 e_{12}^a - t_{12}^a - B_{12}^a$$

$$\theta_1 e_{12}^a - B_{12}^a - t_{12}^a \geq \theta_1 e_{11}^a - t_{11}^a - B_{11}^a.$$

These two conditions can be satisfied simultaneously only if  $\theta_1 e_{11}^a - t_{11}^a - B_{11}^a = \theta_1 e_{12}^a - t_{12}^a - B_{12}^a$ , which can be re-written in the more intuitive form,

$$(PIC^a) \quad t_{11}^a - t_{12}^a = B_{12}^a - B_{11}^a + \theta_1 (e_{11}^a - e_{12}^a)$$

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<sup>15</sup> Note that the principal infers that the agent is of low type when computing her payoffs in (PIC). This is due to the (IC) constraints and the fact that the signal is only relevant after the agent claims to be a low-type.

The principal wants to set  $t_{11}^a > t_{12}^a$  to penalize the high-type agent who under-reports. To be credible, the principal must then either burn more money ( $B_{12}^a > B_{11}^a$ ) or scale down effort ( $e_{12}^a < e_{11}^a$ ) after paying the lower transfer. The choice between these two instruments to satisfy (PIC<sup>a</sup>) captures the main departure from the ex post case. A key issue for this section is whether the principal will rely on rescaling efforts or burning money.

### 2.2.2. The optimal contract under an ex ante private signal: the sub-optimality of burning money

Having characterized the feasible set of contracts, we are ready to state the principal's problem in the ex ante private signal case. First, like in the ex post case, we can anticipate that  $B_{11}^a = 0$ , and we prove it formally in appendix 2. The principal chooses the contract  $\{e_{11}^a, e_{12}^a, e_2^a, t_{11}^a, t_{12}^a, t_2^a, B_{12}^a\}$  to solve the following problem:

$$(P) \quad \text{maximize } \xi [\nu(\theta_1 e_{11}^a - t_{11}^a) + (1 - \nu)(\theta_1 e_{12}^a - t_{12}^a - B_{12}^a)] + (1 - \xi)(e_2^a - t_2^a)$$

subject to (IC<sup>a</sup>), (IR<sup>a</sup>-1), (IR<sup>a</sup>-2), and (PIC<sup>a</sup>).

As usual, (IR<sup>a</sup>-1) and (IC<sup>a</sup>) will be binding. We can rewrite the high-type's rent, given by the right-hand-side of (IC<sup>a</sup>), by using the binding (IR<sup>a</sup>-1) and by replacing the 'penalty' ( $t_{11}^a - t_{12}^a$ ) using (PIC<sup>a</sup>) as in the ex post signal case:

$$\begin{aligned} (\text{Rent}^a) \quad u_2^a = & \left[ \nu \frac{1}{2} (e_{11}^a)^2 + (1 - \nu) \frac{1}{2} (e_{12}^a)^2 \right] - \left[ (1 - \nu) \frac{1}{2} (\theta_1 e_{11}^a)^2 + \nu \frac{1}{2} (\theta_1 e_{12}^a)^2 \right] \\ & - (2\nu - 1)[(e_{11}^a - e_{12}^a) \theta_1 + B_{12}^a] \end{aligned}$$

The first two terms in brackets represent the cost differential between a truthful low type and a shirking high type. The last term differs from the ex post case because the (PIC<sup>a</sup>) in the ex ante case also includes the difference in outputs,  $(e_{11}^a - e_{12}^a) \theta_1$ , besides  $B_{12}^a$ . While the principal could rely on  $B_{12}$  as in the ex post case, it is unclear how she might use the effort terms. Requiring a lower effort when paying a lower transfer ( $e_{11}^a > e_{12}^a$ ) affects the cost differentials, and the net impact on the rent has to be examined.<sup>16</sup>

<sup>16</sup> The agent is paid less when the signal contradicts his report, but he is also given a less arduous assignment.

We present the main results in Proposition 2, and we explain them below.

**Proposition 2:**

- i. *An ex ante private signal is useful for all  $\nu$ . The agent's rent decreases with  $\nu$ , and it becomes zero if the signal is accurate enough ( $\nu > \tilde{\nu}$ , where  $\tilde{\nu} < 1/(2 - \xi)$ ).*
- ii. *After a private signal contradicts the agent's claim, the project is rescaled ( $e_{11}^a > e_{12}^a$ ) and the low type agent is penalized:  $t_{11}^a - \frac{1}{2}e_{11}^{a^2} > 0 > t_{12}^a - \frac{1}{2}e_{12}^{a^2}$ . No money is ever burned, i.e.,  $B_{11}^a = B_{12}^a = 0$ .*
- iii. *The principal prefers an ex ante private signal to an ex post private signal.*

**Proof:** See appendix 2

The first remarkable result is that the private signal is useful for the full range of the signal, i.e., for all  $\nu > 1/2$ , whereas in the ex post case (Proposition 1), a private signal is useful only if it is accurate enough, i.e.,  $\nu > 1/(2 - \xi)$ . The second striking departure from the ex post case is that the principal does not have to burn money in equilibrium. In our ex ante information model, the principal has a choice between rescaling the effort and burning money, and she chooses the first option exclusively.

To provide some intuition for both results, note that, after replacing the transfers using the binding constraints, the principal's problem boils down to choosing  $e_{11}^a, e_{12}^a, e_2^a, B_{12}^a$  to maximize

$$\xi \left[ \nu \left( \theta_1 e_{11}^a - \frac{(e_{11}^a)^2}{2} \right) + (1 - \nu) \left( \theta_1 e_{12}^a - \frac{(e_{12}^a)^2}{2} - B_{12}^a \right) \right] + (1 - \xi) \left[ e_2^a - \frac{(e_2^a)^2}{2} - u_2^a \right]$$

where the rent  $u_2^a$  is given above. Observe first that  $B_{12}^a$  appears as a cost in the objective function (term multiplied by  $\xi(1 - \nu)$ ) but as a benefit as a reduction in the rent (term multiplied by  $(1 - \xi)(2\nu - 1)$ ). Only when  $\nu > \frac{1}{2 - \xi}$ , is the benefit higher than the cost. Money burning has a similar trade-off as in the ex post signal case; it can be optimal only if the signal is accurate enough.



The trade-off is different when the principal uses effort rescaling instead of burning money. To see this, observe that the benefit of effort rescaling appears in the last term in the rent with  $\theta_1(e_{11}^a - e_{12}^a)$ , which acts as a penalty and reduces rent. Consider first the case where the signal is uninformative, i.e.,  $\nu = \frac{1}{2}$ ; the optimal efforts would be  $e_{11}^a = e_{12}^a = e_1^{NS}$ , and the principal would obtain the no-signal payoff. Now consider a small increase in accuracy. As  $\nu < \frac{1}{2-\xi}$ , burning money is not optimal, but the principal can obtain a first order gain in her payoff by creating a penalty when making  $e_{11}^a$  higher than  $e_{12}^a$ . There is also a cost of distorting the effort away from the no-signal level but it is of second order as the expected effort  $[\nu e_{11}^a + (1 - \nu)e_{12}^a]$  stays close to the effort of the no-signal contract  $e_1^{NS}$ .

What makes effort rescaling optimal is its asymmetric effect on the output, that impacts the principal, and on the cost of effort, that impacts the agent. The impact on the principal is determined by (PIC<sup>a</sup>),  $t_{11}^a - t_{12}^a = \theta_1(e_{11}^a - e_{12}^a)$ , and (PIC<sup>a</sup>) limits the maximum penalty to the difference in the output levels. However, due to under-production in equilibrium, a rescaling of efforts leads to a larger impact on the agent since the difference in outputs is larger than the difference in costs:

$$\theta_1(e_{11}^a - e_{12}^a) > \frac{1}{2}[(e_{11}^a)^2 - (e_{12}^a)^2].$$

Hence the principal can use the penalty to reduce rent, which can be seen by the fact that the lying high-type faces a penalty if the signal contradicts his claim of being a low type,

$$\left(t_{11}^a - \frac{1}{2}(\theta_1 e_{11}^a)^2\right) > \left(t_{12}^a - \frac{1}{2}(\theta_1 e_{12}^a)^2\right)$$

even when the (PIC<sup>a</sup>) is satisfied.<sup>17</sup> Thus, rescaling is more effective than pure monetary penalties because effort has different values to different parties, while money is equally valuable to all parties.

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<sup>17</sup> Substitute  $t_{11}^a - t_{12}^a$  from (PIC<sup>a</sup>) in the above inequality to obtain  $\left(1 - \frac{\theta_1}{2}(e_{11}^a + e_{12}^a)\right) > 0$ , which is true as there is under-production in equilibrium ( $e_{12}^a < e_{11}^a < \theta_1$ ).

Rescaling effort allows the principal to extract the entire rent for a value of  $\nu$  lower than the one that would make burning money profitable, namely  $\nu = 1/(2 - \xi)$ . Even when the signal is more accurate than  $1/(2 - \xi)$ , the principal continues to rely on the gap in efforts to satisfy (PIC) rather than burn money.

Comparing with Proposition 1, we can conclude that, if it were a choice, the principal would prefer an ex ante private signal to an ex post private signal. Technically, the principal's problem for the ex post signal case is a special case of the ex ante signal problem. The optimal contract in the previous ex post information case satisfies all constraints in the ex ante information case. Moreover, since  $e_{11}^a > e_{12}^a$ , which was impossible in the ex post case, we can infer that ex ante information is strictly preferred. This result suggests that the principal will want to look for an alternative to burning money when relying on private signals. She will try to induce reporting of private signals as early as possible and rescale the project size rather than be forced to burn money.

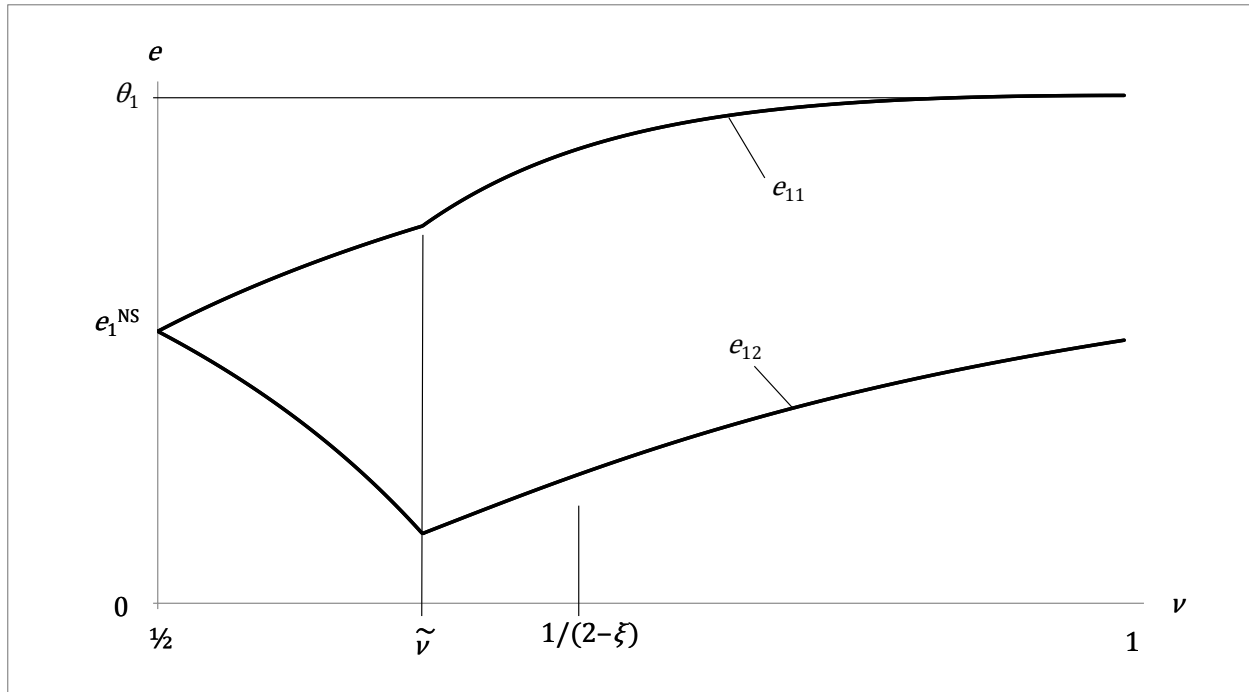


Figure 1—Optimal effort levels ( $e_{11}^a > e_{12}^a$ ) with respect to  $\nu$ .<sup>18</sup>

<sup>18</sup> Figure 1 is based on the example, where  $\xi = 0.5$  and  $\theta_1 = 0.36$ . Note that both  $e_1^{SB}$  and  $\tilde{\nu}$  are increasing with respect to  $\xi$ .

The example in Figure 1 illustrates how the optimal efforts change with the accuracy of the signal. When  $\nu = \frac{1}{2}$ , the no-signal contract is optimal as the signal is of no value. We have  $e_{11}^a = e_{12}^a = e_1^{NS}$ , and the high-type receives the no-signal information rent. For  $\nu > \frac{1}{2}$ , the principal uses the signal to impose a penalty ( $t_{11}^a - t_{12}^a$ ), but due to (PIC<sup>a</sup>), there must also be a rescaling of efforts, but no money is burned. As long as the agent's rent is not fully extracted, we show that the gap between efforts is formed by increasing  $e_{11}^a$  and decreasing  $e_{12}^a$ , as this is mainly done to reinforce the penalty effect, defined above, and reduce the rent.<sup>19</sup> Note that, despite the upward rescaling,  $e_{11}^a$  is always less than the first best level of effort.<sup>20</sup>

When the private information is sufficiently accurate ( $\nu > \tilde{\nu}$ ), the entire rent is extracted. Any further increase in the accuracy of the signal (higher  $\nu$ ) allows the principal to restore the effort levels toward the first best while keeping the rent at zero.<sup>21</sup> Both efforts can now be increased toward the first best, but to maintain the penalty, a gap between the two efforts is necessary. Thus, as  $\nu$  approaches one,  $e_{11}^a$  approaches the first-best level while  $e_{12}^a$  does not. Even when relying on very precise private information, the principal must rescale the project when his private information contradicts the agent's claim about his type, although the inefficient project vanishes in expectation as  $\nu$  approaches 1.<sup>22</sup>

### 2.2.3. Can burning money be optimal?

As rescaling effort is sufficient to extract all rent before burning money is profitable, i.e., before  $\nu = 1/(2 - \xi)$ , no money is burned at all. This suggests that if we modify the model such that the agent retains rent even when  $\nu = 1/(2 - \xi)$ , we may find money burning to be optimal.

<sup>19</sup> Increasing  $e_{11}^a$  also has a negative effect on the cost differential (it increases the rent) while decreasing  $e_{12}^a$  has an ambiguous effect on the cost-differential. Overall, we show that the penalty effect is stronger than the cost differential effect and the principal succeeds in decreasing the rent by increasing  $e_{11}^a$  and decreasing  $e_{12}^a$ .

<sup>20</sup> In contrast to the ex post signal case (Proposition 1), the traditional separation result (Baron and Besanko (1984)) breaks down since the low type's efforts,  $e_{11}^a$  and  $e_{12}^a$ , are adjusted as soon as  $\nu > \frac{1}{2}$ . However, since  $e_{12}^a$  is only restored once rent is fully extracted, the intuition of the traditional separation result remains largely valid: restoring efficiency in effort is not a priority for the principal as long as the rent is positive.

<sup>21</sup> In other words, the private nature of information restores continuity between correlated and uncorrelated environments as noted by Dequiedt and Martimort (2012).

<sup>22</sup> Rescaling  $e_{12}^a$  is costly for the principal since, in equilibrium, the agent is always truthful and the signal is simply incorrect.

We show next that money burning can be optimal under an ex ante signal only if rescaling effort is not efficient enough to extract rent. That is, the instruments are ranked.

We modify the model to reduce the efficacy of rescaling effort as a rent-extraction device. Consider a situation where the principal is limited in her ability to rescale the project due to a minimum scale constraint. We assume that  $e_{12}^a$  cannot fall below some level  $e_0$ . For example, this would be the case for a construction project that requires a minimum safety or quality standard. In appendix 3.1, we derive the minimum value of  $e_0$  such that burning money occurs in equilibrium. Basically,  $e_0$  must be such that the rent is not fully extracted before  $\nu = 1/(2 - \xi)$ .<sup>23</sup> Note that burning money only occurs for  $\nu \geq 1/(2 - \xi)$ , and it is used in conjunction with rescaling effort. For smaller values of  $\nu$ , rescaling effort is the only instrument used.<sup>24</sup>

#### 2.3.4. Asymmetric information about the timing of the signal

When arguing that an ex ante signal is better than an ex post signal, we assumed that the agent knew that the principal has obtained her signal ex ante. In this section we explore the case where the agent does not necessarily know when the principal receives the signal. The timing of the signal matters because the contract with an ex ante signal yields higher profit for the principal. The principal may have an incentive to claim that she received the signal ex ante whether it is true or not.

To model the agent's uncertainty about the signal, we assume that the principal privately receives just one signal about the agent's type, and that the principal and the agent share a common belief about when and whether the signal will arrive. The principal's credibility problem is even more severe than before because constraints have to be added to prevent the principal from lying about *whether* and *when* she received the signal, which further

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<sup>23</sup> The key idea is to restrict downward adjustment in  $e_{12}$ . For instance, a model with shut-down of effort would be another example. This could be achieved through a non-standard marginal cost function that violates Inada conditions, e.g.,  $\psi(e) = \frac{ae+e^2}{2}$ , with a high enough  $a > 0$ .

<sup>24</sup> Other modifications of the model that limits the principal's ability to extract rent can also make money burning optimal. For example, suppose there is a lower bound on the transfer such that  $t \geq \underline{t}$ . Then, for  $\underline{t}$  high enough, we can show that rent will remain positive at  $\nu = 1/(2 - \xi)$  and money burning may occur.

limit her ability to rescale effort based on an ex ante signal. However, we show in appendix 3.2 that the principal still benefits from the possibility of obtaining the signal ex ante. In other words, despite the additional credibility issues due to the agent's not knowing the timing of the signal, the principal is better off than in the case with no signal.

In the next section we study the certification of the private signal as an alternative to imposing (PIC).

### ***3. Certifying the private signal to make it public information***

#### ***3.1. Public information***

In the previous section, we learned how the principal's credibility problem (PIC) makes it costly for her to use private information. An alternative to (PIC) for the principal would be to certify the private signal and make it public, i.e., verifiable in court. For instance, the principal could hire a third party and have the private information checked by an auditor or certified by a certification agency (Farhi, Lerner and Tirole (2012)). The principal then no longer faces a credibility issue and can explicitly contract on  $\sigma$  without imposing (PIC).

With a certified public signal, our model becomes a straightforward application of the public monitoring literature (e.g., Crémer and McLean (1988), Riordan and Sappington (1988)). If certification is free, we obtain the first best since the agent is risk neutral. Since the difference in the transfers is not restricted by (PIC), it is easy to find transfers satisfying  $t_{11} > t_{12}$  such that the relevant (IR) and (IC) constraints are satisfied for the first-best contract, and the agent earns no rent. So, unless there are costs in certifying the signal, the principal should always certify a private signal and make it public, and the timing of signals would be irrelevant.

We note two points. First, the contract under ex ante private information has more realistic features than the contract under public information. The latter contract does not rescale the level of efforts – maintained at the first-best levels – with better information, but adjusts only the transfers. Under our private information contract, the principal adjusts the

transfers *and* effort levels, scaling upward the project when the private information corroborates the agent's claim about his type and scaling it down otherwise. In actual contracts, various features of the project itself (not just transfers) are often adjusted as new information arrives. Our results suggest that one motivation for doing so might be to satisfy the principal's incentive constraints.

Second, we should note that certification is, in general, costly. In particular, when information can be manipulated, we should consider the cost of certification arising from collusion between the certifier (e.g., an auditor or a supervisor) and the agent. Since the certifier makes the private signal public, his identity can be known by the agent who could attempt to bribe him to induce him to report a more favorable signal. Collusion will make the certified public signal less attractive and comparing it with a private signal is no longer straightforward.<sup>25</sup>

### *3.2. Collusion versus credibility*

The main question in this section is to study whether a principal would prefer having a private signal certified even if that would introduce a risk of collusion. In order to focus on the cost due to collusion, we assume that certification is free. With a private signal, the principal either has to burn money or rescale efforts to address her credibility problem. With a certified signal vulnerable to collusion, she has to give a reward to the certifier to deter collusion.<sup>26</sup> There are similarities as both problems are due to issues of manipulating signals. In particular, rewards given to deter bribes go to a third party, the certifier, and have a similar effect to money burning. On the other hand, while the credibility problem is due to potential manipulation by the principal, the collusion problem is due to potential manipulation by the agent.

In what follows, we find that non-monetary penalties are no longer effective due to the threat of collusion. We show that if the certifier is potentially collusive, the principal may as well rely on private information when it comes *ex post*. But if the information comes *ex ante*,

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<sup>25</sup> We assume that private information is not vulnerable to collusion.

<sup>26</sup> See, for example, Tirole (1986), Kofman-Lawarree (1993), or Laffont-Martimort (1997).

certifying a private signal could make the principal worse off if the certifier and agent can easily collude.

Before providing details of the argument, we clarify the setting under which collusion takes place. We first note that the information provided by the certifier's signal cannot be completely "soft," or the certifier would be useless. The information is soft if the certifier can manipulate the information by himself. Therefore, we make an assumption similar to Kofman-Lawarrée (1993) and assume that the certifier can only manipulate the signal with the help of the agent.<sup>27, 28</sup> The principal can then exploit the difference in objectives between the certifier and agent to deter collusion. We assume collusion between the agent and certifier occurs after the signal has been observed by the certifier but before he makes his report to the principal.<sup>29</sup> We also assume that both the agent and the certifier observe the signal, and therefore the two parties negotiate the bribe under symmetric information.<sup>30</sup> As is standard in this type of model, the side-contract is assumed to be enforceable (Tirole (1986, 1992), Kofman-Lawarrée (1993)).

We begin by first considering the effect of collusion on ex post public information. Referring to the timing of section 2.1, step 4 is now modified: the certifier observes the signal, negotiates a side-contract with the agent, and then makes a report to the principal. If the agent reports  $\theta_1$  and the certifier reports  $\sigma_2$ , the agent stands to lose  $(t_{11}^p - t_{12}^p)$ , which is the stake of collusion. To deter collusion, the optimal contract must satisfy a coalition incentive compatibility (CIC) constraint, which says that the principal has to pay a reward to the certifier,

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<sup>27</sup> When the signal is  $\sigma_2$ , the agent will try to bribe the certifier to report  $\sigma_1$ . However, when the signal is  $\sigma_1$ , the agent will not bribe the certifier who is then unable to report  $\sigma_2$  on his own. Therefore, this certification technology rules out extortion. For a technology where extortion is possible, see Khalil, Lawarree and Yun (2010).

<sup>28</sup> Without this assumption, the soft information provided by the certifier is useless in this model. Soft information can be useful in specific settings (for instance see Baliga (1999), Faure-Grimaud, Laffont and Martimort (2003), Khalil, Lawarree and Yun (2010), or Angelucci and Russo (2012)). Rahman (2012) shows how mediated contracts can also address monitoring with soft information when there is no collusion between the agent and the monitor. His focus is on the supervisor's moral hazard, while ours is on inducing truth-telling by the principal. In Rahman, the principal instructs the agent to shirk occasionally and thereby creates shared observations between herself and the monitor. The shared observations allow the principal to verify the monitor's report.

<sup>29</sup> For a model where the timing of collusion has strategic value to the principal, see Faure-Grimaud et al (2002), and Faure-Grimaud and Martimort (2003).

<sup>30</sup> See Laffont and Martimort (1997, 2000) or Che and Kim (2006) or Celik (2009) for collusion under asymmetric information.

denoted by  $W^p$ , that is at least as high as the stake of collusion. Thus, the (CIC<sup>p</sup>) constraint can be written as:

$$(CIC^p) \quad W^p \geq t_{11}^p - t_{12}^p.$$

The reward is paid to the certifier whenever  $t_{12}^p$  is paid to the agent. The principal's problem is given in appendix 4.1. Since the (CIC<sup>p</sup>) is binding in equilibrium, imposing this constraint effectively transfers the penalty  $(t_{11}^p - t_{12}^p)$  to the certifier who acts as a bounty-hunter.

While this result is standard in the collusion literature, it is remarkable that it is identical to the result in the case of ex post *private* information where the principal had to transfer the penalty to a third party to address her credibility problem (satisfy the (PIC<sup>p</sup>) constraint). With collusion, the third party is the certifier. Thus, an ex post *private* signal is equivalent to an ex post *public* signal that is vulnerable to collusion. In either case, the signal is only useful if it is accurate enough, i.e.,  $\nu > 1/(2 - \xi)$ . Therefore, the principal does not benefit from certifying a private signal if it comes ex post.

Now consider the effect of collusion on ex ante *public* information. Since the efforts can now be based on the signal, we can study the effectiveness of rescaling effort as a tool to fight collusion. The coalition incentive contract for this case (CIC<sup>a</sup>) has to account for the possibility of rescaling the effort levels. If the high-type agent under-reports and the certifier detects it, the stake of collusion is:

$$\left[ t_{11}^a - \frac{1}{2}(\theta_1 e_{11}^a)^2 \right] - \left[ t_{12}^a - \frac{1}{2}(\theta_1 e_{12}^a)^2 \right].$$

Therefore, the reward to the certifier must be at least as high as the stake of collusion and the (CIC<sup>a</sup>) constraint can be written as:<sup>31</sup>

$$(CIC^a) \quad W^a \geq t_{11}^a - t_{12}^a - \frac{1}{2}\theta_1^2((e_{11}^a)^2 - (e_{12}^a)^2).$$

We characterize in appendix 4.2 the main features of the optimal contract for ex ante public information with collusion. We show that the principal does not benefit from a gap in efforts,

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<sup>31</sup> In appendix 4.2, we show that the analogous constraint for a truthful low type is also satisfied.



and the optimal contract involves  $e_{11}^a = e_{12}^a = e_1^a$ . Therefore, the  $(CIC^a)$  is identical to  $(CIC^p)$ , and the optimal contracts are identical whether the public signal comes ex ante or ex post.

This confirms our earlier intuition that non-monetary tools are effective only when the parties value them differently. Despite some similarity in form,  $(CIC^a)$  captures very different incentives than  $(PIC^a)$  under an ex ante private signal in section 2.2. Namely,  $(CIC^a)$  removes the agent's incentive to manipulate (under-report) the signal while  $(PIC^a)$  eliminates the principal's incentive to manipulate (over-report) the signal. In the private signal case, the principal evaluated effort in terms of output, while the agent evaluated it in terms of cost, and a gap in efforts could be used to address both  $(PIC)$  and  $(IC)$ . In the public signal case with collusion, both agent and certifier evaluate effort similarly because the agent's rent, which depends on the difference in cost of effort, is also the source of the stake of collusion. A gap in efforts is ineffective in relaxing the  $(IC)$  and  $(CIC)$ . The only tool that can affect both the  $(IC)$  and the  $(CIC)$  is monetary transfers. In other words, because the certifier does not care about the output per se, a monetary transfer is the only instrument that can impact both the agent and the certifier.

Hence, we have shown that ex ante private information is strictly better than public information that is vulnerable to collusion. We have already shown in proposition 2 that ex ante private information is better than ex post private information, and in this section we have argued that ex post private signals are equivalent to public signals (ex ante or ex post) vulnerable to collusion. In short, ex ante private  $>$  ex post private  $\sim$  ex post public vulnerable to collusion  $\sim$  ex ante public vulnerable to collusion.

We summarize our findings in Proposition 3:

**Proposition 3:**

- i. *Under a public signal, the timing of information does not matter regardless of whether or not there is a threat of collusion.*
- ii. *An ex post private signal is equivalent to a public signal vulnerable to collusion; the signal is only useful if it is accurate enough.*
- iii. *Certification of a private signal is not useful if it raises the specter of collusion – an ex ante private signal is preferable to an ex post public signal vulnerable to collusion.*

**Proof:** See appendix 4.2.

Next, we briefly consider the case where the principal can also bribe the certifier to report  $\sigma_2$  when the signal is  $\sigma_1$ . It is not clear a priori whether the optimal contract above is also collusion-proof to a coalition between the principal and certifier. First, it is worth pointing out that this re-introduces elements of (PIC) as in the private signal case of section 2 since the principal would need to assure the agent that she will not engage in bribery. This is reminiscent of Strausz (1997), who notes that if the principal can bribe the certifier, the signal is akin to a private signal. In particular, in his paper, the principal can costlessly remove the agent's incentive to bribe using off-the-equilibrium path payments, which is not possible in our case. Therefore, in our model, the contract must deal simultaneously with bribery by the agent and by the principal and the extensive form of the bribery game matters. Thus, whether the optimal contract above is also collusion-proof to a coalition between the principal and certifier will depend on the specific extensive form. In any case, our key point in Proposition 3 on the comparative advantage of a private signal is made stronger as the certified public signal can only lose value with the possibility of bribery by the principal.

### 3.3. Collusion costs

If collusion costs are present (due, for instance, to the psychological burden of participating in an illegal activity (see Tirole (1992)), transaction costs (Faure-Grimaud et al. (2003)) or the fear of being detected in the future (Khalil and Lawarree (2006)), the public signal becomes more profitable for the principal. Technically, the (CIC) constraint is easier to satisfy.

Denote by  $k$ , the collusion costs, i.e., one unit of bribe is only worth  $k$  dollars to the certifier, where  $k \in [0,1]$ . So far we have assumed that  $k=1$ . With  $k < 1$ , the agent can offer at most a fraction  $k$  of the benefit of misreporting the agent's type (the stake of collusion). If  $k = 0$ , the public signal is collusion-free, and the first best can be reached. For the ex post signal, the (CIC) is now  $W^p \geq k(t_{11}^p - t_{12}^p)$ , and for an ex ante signal, the (CIC) becomes  $W^a \geq k \left( \left( t_{11}^a - \frac{1}{2}(\theta_1 e_{11}^a)^2 \right) - \left( t_{12}^a - \frac{1}{2}(\theta_1 e_{12}^a)^2 \right) \right)$ .

Let us now re-examine Proposition 3 when collusion costs are present ( $k < 1$ ).<sup>32</sup> Part (i) of Proposition 3 remains intact. With  $k < 1$ , the reward to deter collusion is less but the lower reward applies both ex ante and ex post. Therefore, the timing of information for a public signal still does not matter. In Part (ii) of Proposition 3, we showed that the ex post private signal was equivalent to a public signal vulnerable to collusion. This is no longer true. With collusion costs, the public signal is more profitable and, therefore, the public signal dominates the private signal.

Finally, the principal still prefers to keep an ex ante signal private (Part (iii) of Proposition 3 is still true) unless collusion costs are large. With no collusion costs ( $k = 1$ ), we showed in Proposition 3 that an ex ante private signal was better than a public signal. With large enough collusion cost (i.e., small enough  $k$ ), a public signal could become more profitable than the private signal. Indeed, if  $k = 0$ , the first best can be reached with a public signal since the signal is collusion-free. Therefore, there is a critical  $k$  – say  $k^p$  – such that an ex ante private signal dominates a public signal for all  $k \geq k^p$  and is dominated by a public signal otherwise.

#### **4. Conclusion**

In this paper, we show that the timing of information can be critical to solve the credibility issue related to private information received by a principal. If the principal can collect her private

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<sup>32</sup> For the rest of the section, we just outline the arguments as the techniques are similar to earlier steps.

information before the project is completed, she prefers to use non-monetary tools like rescaling the project instead of monetary penalties which induce burning money. The intuition relies on the fact that money is equally valuable to all parties, while a non-monetary tool, like rescaling effort, exploits the fact that parties evaluate effort differently. An immediate consequence is that the principal always prefers to obtain private information before the project is over since rescaling the project is more efficient than burning money.

By comparing private signals and certified public signals vulnerable to collusion, we have learned that relying on non-monetary penalties, like rescaling, is not effective to deter collusion; instead, the principal must use monetary penalties. Intuitively, it is because the certifier does not care about the output per se, and only a monetary transfer can affect both the agent and the certifier. For ex post signals, only monetary penalties are available, and private signals are equivalent to certified public signals vulnerable to collusion. However, if the signal arrives ex ante, a private signal is preferred because non-monetary penalties are available. Therefore, certification of an ex ante private signal by a third part may not be in the interest of the principal if this certification raises the specter of collusion.

## **Appendix**

### **Appendix 1: Proof of Proposition 1**

For ex post monitoring, the principal chooses the contract  $\{e_1^p, t_{11}^p, t_{12}^p, e_2^p, t_2^p, B_{11}^p, B_{12}^p\}$  to maximize the profit function

$$\xi \left( \theta_1 e_1^p - v(t_{11}^p + B_{11}^p) - (1 - v)(t_{12}^p + B_{12}^p) \right) + (1 - \xi)(e_2^p - t_2^p)$$

subject to

$$(IR-1) \quad vt_{11}^p + (1 - v)t_{12}^p - \frac{(e_1^p)^2}{2} \geq 0$$

$$(IR-2) \quad t_2^p - \frac{(e_2^p)^2}{2} = u_2^p \geq 0$$

$$(IC) \quad u_2^p \geq (1 - v)t_{11}^p + vt_{12}^p - \frac{(\theta_1 e_1^p)^2}{2}$$

$$(PIC) \quad t_{11}^p - t_{12}^p = B_{12}^p - B_{11}^p$$

and non-negativity conditions for  $e_1^p, e_2^p, t_2^p, B_{11}^p$ , and  $B_{12}^p$ .<sup>33</sup>

We show that, in the ex post signal case, the certifier is used only if  $v > \frac{1}{2-\xi}$ . Note that the (IR-1) and (IC) are binding in the above problem.

Using these and (PIC) to replace the transfers, the principal's problem is to choose  $e_1^p, e_2^p, B_{11}^p, B_{12}^p$ , and  $u_2^p$  to maximize

$$\xi \left( \theta_1 e_1^p - \frac{(e_1^p)^2}{2} - vB_{11}^p - (1 - v)B_{12}^p \right) + (1 - \xi) \left( e_2^p - \frac{(e_2^p)^2}{2} - u_2^p \right)$$

subject to

$$(IC) \quad u_2^p = \frac{(e_1^p)^2}{2} - \frac{(\theta_1 e_1^p)^2}{2} - (2v - 1)(B_{12}^p - B_{11}^p)$$

and non-negativity conditions on  $e_1^p, e_2^p, B_{11}^p, B_{12}^p$ , and  $u_2^p$ .

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<sup>33</sup> There is no need for an upper limit on  $B_{1j}$  as that is not relevant, but a negative  $B_{1j}$  is not meaningful. It would mean that a third party (and not the agent) pays the principal.

Denoting by  $\lambda$ , the non-negative multiplier on the (IC), the first order conditions for a maximum are:

$$(ep1) \quad \frac{\partial L}{\partial e_1^p} = \xi(\theta_1 - e_1^p) - \lambda(1 - \theta_1^2)e_1^p \leq 0; \quad e_1^p \frac{\partial L}{\partial e_1^p} = 0$$

$$(ep2) \quad \frac{\partial L}{\partial e_2^p} = (1 - \xi)(1 - e_{12}^p) \leq 0; \quad e_2^p \frac{\partial L}{\partial e_2^p} = 0$$

$$(ep3) \quad \frac{\partial L}{\partial u_2^p} = -(1 - \xi) + \lambda \leq 0; \quad u_2^p \frac{\partial L}{\partial u_2^p} = 0$$

$$(ep4) \quad \frac{\partial L}{\partial B_{12}^p} = -\xi(1 - \nu) + \lambda(2\nu - 1) \leq 0; \quad B_{12}^p \frac{\partial L}{\partial B_{12}^p} = 0$$

$$(ep5) \quad \frac{\partial L}{\partial B_{11}^p} = -\xi\nu - \lambda(2\nu - 1) \leq 0; \quad B_{11}^p \frac{\partial L}{\partial B_{11}^p} = 0$$

Condition (ep5) immediately implies that  $B_{11}^p = 0$ . Combining (ep3) and (ep4) we have that  $B_{12}^p > 0$  if and only if  $\nu > \frac{1}{2-\xi}$ , which also implies that  $u_2^p = 0$  since  $\lambda < (1 - \xi)$ . If  $\nu < \frac{1}{2-\xi}$ , then  $B_{12}^p = 0$ , and the no-signal contract is optimal. ■

## **Appendix 2: Proof of Proposition 2**

The principal maximizes the profit function

$$\xi(\nu(\theta_1 e_{11}^a - t_{11}^a - B_{11}^a) + (1 - \nu)(\theta_1 e_{12}^a - t_{12}^a - B_{12}^a)) + (1 - \xi)(e_2^a - t_2^a)$$

subject to

$$(IR-1) \quad \nu \left( t_{11}^a - \frac{(e_{11}^a)^2}{2} \right) + (1 - \nu) \left( t_{12}^a - \frac{(e_{12}^a)^2}{2} \right) \geq 0$$

$$(IR-2) \quad t_2^a - \frac{(e_2^a)^2}{2} = u_2^a \geq 0$$

$$(IC) \quad u_2^a \geq (1 - \nu) \left( t_{11}^a - \frac{(\theta_1 e_{11}^a)^2}{2} \right) + \nu \left( t_{12}^a - \frac{(\theta_1 e_{12}^a)^2}{2} \right)$$

$$(PIC) \quad \theta_1 e_{11}^a - t_{11}^a - B_{11}^a = \theta_1 e_{12}^a - t_{12}^a - B_{12}^a$$

and non-negativity conditions for  $e_{11}^a, e_{12}^a, e_2^a, B_{11}^a$ , and  $B_{12}^a$ .

First, as is typical, the (IR-1) is binding, using which, we can re-write the (IC).

Then, the principal's problem is to choose  $e_{11}^a, e_{12}^a, e_2^a, B_{11}^a, B_{12}^a$  and  $u_2^a$  to maximize

$$\xi \left( v \left( \theta_1 e_{11}^a - \frac{e_{11}^{a^2}}{2} - B_{11}^a \right) + (1-v) \left( \theta_1 e_{12}^a - \frac{e_{12}^{a^2}}{2} - B_{12}^a \right) \right) + (1-\xi) \left( e_2 - \frac{e_2^2}{2} - u_2^a \right)$$

subject to

$$(IC) \quad u_2^a \geq \left[ (v - (1-v)\theta_1^2) \frac{e_{11}^{a^2}}{2} - (v\theta_1^2 - (1-v)) \frac{e_{12}^{a^2}}{2} \right] - (2v-1)(\theta_1(e_{11}^a - e_{12}^a) + B_{12}^a - B_{11}^a)$$

and the non-negativity conditions on  $e_{11}^a, e_{12}^a, e_2^a, B_{11}^a, B_{12}^a$  and  $u_2^a$ , where the non-negativity condition on  $u_2^a$  has replaced (IR-2).

Denoting by  $\lambda$ , the non-negative multiplier on the (IC), the Kuhn-Tucker conditions for a maximum are the following:

$$(1) \quad \frac{\partial L}{\partial e_{11}^a} = \xi v(\theta_1 - e_{11}^a) + \lambda \theta_1(2v-1) - \lambda(v - (1-v)\theta_1^2)e_{11}^a \leq 0; \quad e_{11}^a \frac{\partial L}{\partial e_{11}^a} = 0$$

$$(2) \quad \frac{\partial L}{\partial e_{12}^a} = \xi(1-v)(\theta_1 - e_{12}^a) - \lambda \theta_1(2v-1) + \lambda(v\theta_1^2 - (1-v))e_{12}^a \leq 0; \quad e_{12}^a \frac{\partial L}{\partial e_{12}^a} = 0$$

$$(3) \quad \frac{\partial L}{\partial e_2^a} = (1-\xi)(1 - e_2^a) \leq 0; \quad e_2^a \frac{\partial L}{\partial e_2^a} = 0$$

$$(4) \quad \frac{\partial L}{\partial u_2^a} = -(1-\xi) + \lambda \leq 0; \quad u_2^a \frac{\partial L}{\partial u_2^a} = 0$$

$$(5) \quad \frac{\partial L}{\partial B_{12}^a} = -\xi(1-v) + \lambda(2v-1) \leq 0; \quad B_{12}^a \frac{\partial L}{\partial B_{12}^a} = 0$$

$$(6) \quad \frac{\partial L}{\partial B_{11}^a} = -\xi v - \lambda(2v-1) \leq 0; \quad B_{11}^a \frac{\partial L}{\partial B_{11}^a} = 0$$

plus the constraint and its complementary slackness condition. The second-order conditions include the following:<sup>34</sup>

$$(SOC) \quad \xi(1 - \nu) - \lambda(\nu\theta_1^2 - (1 - \nu)) > 0.$$

*Claim 1:*  $e_2^a = 1$ , the (IC) is binding,  $\theta_1 > e_{11}^a > e_{12}^a > 0$ ,  $B_{11}^a = B_{12}^a = 0$ , and the rent is

$$u_2^a = \left[ (\nu - (1 - \nu)\theta_1^2) \frac{e_{11}^{a^2}}{2} - (\nu\theta_1^2 - (1 - \nu)) \frac{e_{12}^{a^2}}{2} \right] - (2\nu - 1)(\theta_1(e_{11}^a - e_{12}^a))$$

*Proof:*

(a) From (3), we have  $e_2^a = 1$  and from (6), we have  $B_{11}^a = 0$ .

(b) From (1), we have  $e_{11}^a > 0$ , and

$$(E11) \quad e_{11}^a = \left\{ \frac{\xi\nu + \lambda(2\nu - 1)}{\xi\nu + \lambda(\nu - (1 - \nu)\theta_1^2)} \right\} \theta_1,$$

which implies  $e_{11}^a < \theta_1$  since  $\theta_1 < 1$ .

(c) (IC) is binding ( $\lambda > 0$ ):

Suppose  $\lambda = 0$ . Then (1), (2), (4) and (5) imply that  $e_{11}^a = e_{12}^a = e_1^{FB}$ ,  $B_{12}^a = u_2^a = 0$ . However, this would violate the (IC) since its right hand side is positive for  $\nu < 1$ . Therefore  $\lambda > 0$  whenever  $\nu < 1$  and the rent is

$$u_2^a = \left[ (\nu - (1 - \nu)\theta_1^2) \frac{e_{11}^{a^2}}{2} - (\nu\theta_1^2 - (1 - \nu)) \frac{e_{12}^{a^2}}{2} \right] - (2\nu - 1)(\theta_1(e_{11}^a - e_{12}^a) + B_{12}^a)$$

(d)  $B_{12}^a = 0$  and  $e_{12}^a > 0$ . It will suffice to show that  $\lambda < \frac{\xi(1 - \nu)}{(2\nu - 1)}$  since (5) then implies

$B_{12}^a = 0$ , and (2) then implies  $e_{12}^a > 0$ . Rewrite (2) as

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<sup>34</sup> It can be derived from the Bordered Hessian – see Appendix 5.



$$\theta_1 [\xi(1-\nu) - \lambda(2\nu-1)] - [\xi(1-\nu) - \lambda(\nu\theta_1^2 - (1-\nu))] e_{12}^a \leq 0; \quad e_{12}^a \frac{\partial L}{\partial e_{12}^a} = 0$$

Therefore, if  $\lambda = \frac{\xi(1-\nu)}{(2\nu-1)}$ , the condition (2) implies that  $e_{12}^a = 0$ , and we will show next that

the rent is negative, which is a contradiction. That will complete the proof since (5) implies that

$$\lambda \leq \frac{\xi(1-\nu)}{(2\nu-1)}.$$

Step 1: Use  $\lambda = \frac{\xi(1-\nu)}{(2\nu-1)}$  in (E11) to see that:

$$e_{11}^a = \left\{ \frac{\xi\nu + \left( \frac{\xi(1-\nu)}{(2\nu-1)} \right) (2\nu-1)}{\xi\nu + \left( \frac{\xi(1-\nu)}{(2\nu-1)} \right) (\nu - (1-\nu)\theta_1^2)} \right\} \theta_1.$$

$$\text{Step 2: } (\nu - (1-\nu)\theta_1^2) \left\{ \frac{\xi\nu + \left( \frac{\xi(1-\nu)}{(2\nu-1)} \right) (2\nu-1)}{\xi\nu + \left( \frac{\xi(1-\nu)}{(2\nu-1)} \right) (\nu - (1-\nu)\theta_1^2)} \right\} \theta_1 < 2\theta_1(2\nu-1) :$$

Proof:

$$\begin{aligned} (\nu - (1-\nu)\theta_1^2) \left\{ \xi\nu + \left( \frac{\xi(1-\nu)}{(2\nu-1)} \right) (2\nu-1) \right\} \theta_1 &< 2\theta_1(2\nu-1) \left\{ \xi\nu + \left( \frac{\xi(1-\nu)}{(2\nu-1)} \right) (\nu - (1-\nu)\theta_1^2) \right\} \\ &\Leftrightarrow (\nu - (1-\nu)\theta_1^2) < 2\{\nu(2\nu-1) + (1-\nu)(\nu - (1-\nu)\theta_1^2)\} \\ &\Leftrightarrow (2\nu-1)(\nu - (1-\nu)\theta_1^2) < 2\nu(2\nu-1) \\ &\Leftrightarrow -\nu - (1-\nu)\theta_1^2 < 0 \quad \square \end{aligned}$$

Next, using Step 1 for the first equality and Step 2 for the second inequality, we can write

$$(\nu - (1 - \nu)\theta_1^2)e_{11} = (\nu - (1 - \nu)\theta_1^2) \left\{ \frac{\xi\nu + \left(\frac{\xi(1-\nu)}{(2\nu-1)}\right)(2\nu-1)}{\xi\nu + \left(\frac{\xi(1-\nu)}{(2\nu-1)}\right)(\nu - (1-\nu)\theta_1^2)} \right\} \theta_1 < 2\theta_1(2\nu-1)$$

Therefore, when  $e_{12}^a = 0$ , the rent  $\left[ (\nu - (1 - \nu)\theta_1^2) \frac{e_{11}^{a^2}}{2} \right] - \theta_1(2\nu - 1)e_{11}^a - (2\nu - 1)B_{12}^a < 0$

which is a contradiction, and we have shown that  $\lambda < \frac{\xi(1-\nu)}{(2\nu-1)}$ . Hence that  $B_{12}^a = 0$ , and

$$e_{12}^a > 0.$$

Therefore, we can use (2) to characterize the optimal  $e_{12}$  by the following condition:

$$(E12) \quad e_{12}^a = \left\{ \frac{\xi(1-\nu) - \lambda(2\nu-1)}{\xi(1-\nu) - \lambda(\nu\theta_1^2 - (1-\nu))} \right\}.$$

(e) Using (E11) and (E12) it is straightforward to show that  $e_{11}^a > e_{12}^a$ . The steps are as follows:

$$\begin{aligned} & \left\{ \frac{\xi\nu + \lambda(2\nu-1)}{\xi\nu + \lambda(\nu - (1-\nu)\theta_1^2)} \right\} > \left\{ \frac{\xi(1-\nu) - \lambda(2\nu-1)}{\xi(1-\nu) - \lambda(\nu\theta_1^2 - (1-\nu))} \right\} \\ & \Leftrightarrow (\xi\nu + \lambda(2\nu-1))(\xi(1-\nu) - \lambda(\nu\theta_1^2 - (1-\nu))) \\ & \quad > (\xi\nu + \lambda(\nu - (1-\nu)\theta_1^2))(\xi(1-\nu) - \lambda(2\nu-1)) \\ & \Leftrightarrow \xi^2\nu(1-\nu) - \lambda\xi\nu(\nu\theta_1^2 - (1-\nu)) + \lambda\xi(1-\nu)(2\nu-1) - \lambda^2(2\nu-1)(\nu\theta_1^2 - (1-\nu)) \\ & \quad > \xi^2\nu(1-\nu) - \lambda\xi\nu(2\nu-1) + \lambda\xi(1-\nu)(\nu - (1-\nu)\theta_1^2) - \lambda^2(2\nu-1)(\nu - (1-\nu)\theta_1^2) \end{aligned}$$

$$\begin{aligned}
&\Leftrightarrow -\lambda\xi v^2\theta_1^2 - \lambda\xi(1-v)^2 + \lambda^2(2v-1)(1-v\theta_1^2) \\
&\quad > -\lambda\xi v^2 - \lambda\xi(1-v)^2\theta_1^2 + \lambda^2(2v-1)(1-v)\theta_1^2 \\
&\Leftrightarrow \lambda\xi v^2(1-\theta_1^2) - \lambda\xi(1-v)^2(1-\theta_1^2) + \lambda^2(2v-1)(1-\theta_1^2) > 0 \\
&\Leftrightarrow \left(\lambda\xi(v^2 - (1-v)^2) + \lambda^2(2v-1)\right)(1-\theta_1^2) > 0 \quad \square
\end{aligned}$$

*Claim 2:* The rent is strictly positive for  $v < \tilde{v}$ , where  $\frac{1}{2} < \tilde{v} < \frac{1}{2-\xi}$ , and it is zero otherwise.

Proof:

(I) If the rent is positive, it is strictly decreasing with respect to  $v$ .

Recall that when rent is positive, (4) implies  $\lambda = (1 - \xi)$ . Using (1), (2) and the rent expression, we derive parts (a) – (e) to establish the signs of the individual terms as follows:

$$\begin{aligned}
\frac{du_2^a}{dv} &= \frac{\partial u_2^a}{\partial v} + \frac{\partial u_2^a}{\partial e_{11}^a} \frac{\partial e_{11}^a}{\partial v} + \frac{\partial u_2^a}{\partial e_{12}^a} \frac{\partial e_{12}^a}{\partial v}, \\
&= (-) + (+)(+) + (+)(-)
\end{aligned}$$

Part (f) then shows  $\frac{\partial u_2^a}{\partial e_{11}^a} \frac{\partial e_{11}^a}{\partial v} + \frac{\partial u_2^a}{\partial e_{12}^a} \frac{\partial e_{12}^a}{\partial v} < 0$ , using results established in parts (a) – (e).

$$\begin{aligned}
\text{(a)} \quad \frac{\partial u_2^a}{\partial v} &= -2\theta_1(e_{11}^a - e_{12}^a) + (1 + \theta_1^2)(e_{11}^{a^2} - e_{12}^{a^2})\frac{1}{2} < 0 \\
&\text{since } -2\theta_1 + (1 + \theta_1^2)(e_{11}^a + e_{12}^a)\frac{1}{2} < -2\theta_1 + (1 + \theta_1^2)\theta_1 < 0
\end{aligned}$$

Note: If  $v = \frac{1}{2}$ , then  $e_{11}^a = e_{12}^a$ , and  $\frac{\partial u_2^a}{\partial v} = 0$ .

$$\text{(b)} \quad \frac{\partial u_2^a}{\partial e_{11}^a} = -\theta_1(2v - 1) + (v - (1 - v)\theta_1^2)e_{11}^a > 0$$

$$\begin{aligned}
& \text{since } \left( \nu - (1-\nu)\theta_1^2 \right) \left\{ \frac{\xi\nu + (1-\xi)(2\nu-1)}{\xi\nu + (1-\xi)(\nu - (1-\nu)\theta_1^2)} \right\} \theta_1 > (2\nu-1)\theta_1 \\
& \Leftrightarrow \left( \nu - (1-\nu)\theta_1^2 \right) \left( \nu - (1-\nu) + \xi(1-\nu) \right) > \left( \nu - (1-\nu) \right) \left( \nu - (1-\nu)\theta_1^2 + \xi(1-\nu)\theta_1^2 \right) \\
& \Leftrightarrow \xi\nu(1-\nu) > \xi\nu(1-\nu)\theta_1^2
\end{aligned}$$

$$\begin{aligned}
\text{(c)} \quad \frac{\partial e_{11}^a}{\partial \nu} &= \frac{(\xi + 2(1-\xi))\theta_1 - (\xi + (1+\theta_1^2)(1-\xi))e_{11}^a}{\xi\nu + (1-\xi)(\nu - (1-\nu)\theta_1^2)} > 0 \\
&\text{given (SOC), } \theta_1 > e_{11}^a \text{ and } \xi + 2(1-\xi) > \xi + (1+\theta_1^2)(1-\xi)
\end{aligned}$$

$$\begin{aligned}
\text{(d)} \quad \frac{\partial u_2}{\partial e_{12}^a} &= \theta_1(2\nu-1) - (\nu\theta_1^2 - (1-\nu))e_{12}^a > 0 \\
&\text{since } \theta_1 > e_{12}^a \text{ and } (2\nu-1) > (\nu\theta_1^2 - (1-\nu))
\end{aligned}$$

$$\begin{aligned}
\text{(e)} \quad \frac{\partial e_{12}^a}{\partial \nu} &= \frac{(\xi + (1+\theta_1^2)(1-\xi))e_{12}^a - (\xi + 2(1-\xi))\theta_1}{\xi(1-\nu) - (1-\xi)(\nu\theta_1^2 - (1-\nu))} < 0 \\
&\text{since } \theta_1 > e_{12}^a \text{ and } \xi + 2(1-\xi) > \xi + (1+\theta_1^2)(1-\xi) \text{ and (SOC)}
\end{aligned}$$

(f) It remains to show  $\frac{\partial u_2^a}{\partial e_{11}^a} \frac{\partial e_{11}^a}{\partial \nu} + \frac{\partial u_2^a}{\partial e_{12}^a} \frac{\partial e_{12}^a}{\partial \nu} < 0$ . From the foregoing results, this is equivalent to showing that the following inequality holds, where each of the three terms in curly brackets on either side of the inequality has been proven to be (or is obviously) strictly positive:

$$\begin{aligned}
& \left\{ (\xi + 2(1-\xi))\theta_1 - (\xi + (1+\theta_1^2)(1-\xi))e_{12}^a \right\} \left\{ \xi\nu + (1-\xi)(\nu - (1-\nu)\theta_1^2) \right\} \\
& \left\{ \theta_1(2\nu-1) - (\nu\theta_1^2 - (1-\nu))e_{12}^a \right\} > \left\{ (\xi + 2(1-\xi))\theta_1 - (\xi + (1+\theta_1^2)(1-\xi))e_{11}^a \right\} \\
& \left\{ \xi(1-\nu) - (1-\xi)(\nu\theta_1^2 - (1-\nu)) \right\} \left\{ (\nu - (1-\nu)\theta_1^2)e_{11}^a - \theta_1(2\nu-1) \right\}
\end{aligned}$$

This inequality can be seen to hold term-by-term:

$$\begin{aligned}
\text{(i)} \quad & \left\{ (\xi + 2(1-\xi))\theta_1 - (\xi + (1+\theta_1^2)(1-\xi))e_{12}^a \right\} > \left\{ (\xi + 2(1-\xi))\theta_1 - (\xi + (1+\theta_1^2)(1-\xi))e_{11}^a \right\} \\
& \text{because } e_{11}^a > e_{12}^a
\end{aligned}$$

$$(ii) \quad \left\{ \xi \nu + (1-\xi)(\nu - (1-\nu)\theta_1^2) \right\} > \left\{ \xi(1-\nu) - (1-\xi)(\nu\theta_1^2 - (1-\nu)) \right\}$$

given  $\xi(2\nu - 1) > 0 > -(1-\xi)(2\nu - 1)(1 + \theta_1^2)$

$$(iii) \quad \left\{ \theta_1(2\nu - 1) - (\nu\theta_1^2 - (1-\nu))e_{12}^a \right\} > \left\{ (\nu - (1-\nu)\theta_1^2)e_{11}^a - \theta_1(2\nu - 1) \right\}$$

$$\Leftrightarrow 2(2\nu - 1) > (\nu - (1-\nu)\theta_1^2) \left( \frac{\nu - (1-\nu)(1-\xi)}{\nu - (1-\nu)(1-\xi)\theta_1^2} \right) + (\nu\theta_1^2 - (1-\nu)) \left( \frac{(1-\nu) - \nu(1-\xi)}{(1-\nu) - \nu(1-\xi)\theta_1^2} \right)$$

$$\Leftrightarrow \xi\nu(1-\nu)(2\nu - 1)(1 - \theta_1^2)(1 + (1-\xi)\theta_1^2) > 0 \text{ (steps omitted).}$$

(II) The cutoff  $\tilde{\nu} \in \left(\frac{1}{2}, \frac{1}{2-\xi}\right)$ .

The rent is strictly positive for  $\nu = \frac{1}{2}$  since the no-signal contract is offered in this case. If rent is positive,  $\lambda = (1 - \xi)$ , and replacing  $\lambda$  in (E11) and (E12), those effort expressions are continuous for  $\nu \in \left(\frac{1}{2}, \frac{1}{2-\xi}\right)$ . Therefore the expression  $u_2^a$ , calculated using (E11) and (E12), is also continuous for  $\nu \in \left(\frac{1}{2}, \frac{1}{2-\xi}\right)$ , and it is strictly negative when  $\nu = \frac{1}{2-\xi}$ . Defining the cutoff by  $u_2^a(\tilde{\nu}) \equiv 0$ , we have just proved that  $\tilde{\nu} \in \left(\frac{1}{2}, \frac{1}{2-\xi}\right)$ .

(III) Rent is zero for  $\nu \in \left[\tilde{\nu}, \frac{1}{2-\xi}\right]$ .

Assume to the contrary that  $u_2^a > 0$ . Then  $\lambda = (1 - \xi)$ , and we can replace  $\lambda$  in (E11) and (E12) and evaluate rent to show that  $u_2^a < 0$  (based on the arguments in (II)), which is a contradiction.

(IV) Rent is zero for  $\nu \in \left[\frac{1}{2-\xi}, 1\right]$ .

Suppose to the contrary that the rent is strictly positive but that  $\nu > \frac{1}{2-\xi}$ . A strictly positive rent implies  $\lambda = 1 - \xi$ , and  $\lambda < \frac{\xi(1-\nu)}{2\nu-1}$  was proved as part of Claim 1 (d). Therefore,  $1 - \xi < \frac{\xi(1-\nu)}{2\nu-1}$ , which is equivalent to  $\nu < \frac{1}{2-\xi}$ , and we have a contradiction which proves claim 2.  $\square$

This completes the proof of the first two parts of Proposition 2.

Finally, to prove part (iii) of Proposition 2, note that the ex post signal problem of Proposition 1 is nested in the ex ante signal problem in Proposition 2 since it can be obtained by setting  $e_{11}^a = e_{12}^a = e_1^a$ . Since the solution to the ex ante signal case, stated in Proposition 2, does not involve  $e_{11}^a = e_{12}^a$ , the principal's payoff must be higher than the ex post signal case. ■

### **Appendix 3.1: Min scale and optimality of burning money**

We consider only  $e_0 < e_1^{SB}$ , and it is known that both  $e_{11}$  and  $e_2$  always exceed  $e_1^{SB}$ .

Adding the following constraint to the problem of proposition 2:

(MS)  $e_{12} \geq e_0$ , we obtain the following Lagrangian:

The Lagrangian for this problem is

$$\begin{aligned} \mathcal{L} = & \xi \left( \nu \left( \theta_1 e_{11} - e_{11}^2 / 2 - B_{11} \right) + (1 - \nu) \left( \theta_1 e_{12} - e_{12}^2 / 2 - B_{12} \right) \right) + (1 - \xi) \left( e_2 - e_2^2 / 2 - u_2 \right) \\ & + \lambda_1 \left[ u_2 - \left( \nu - (1 - \nu) \theta_1^2 \right) \left( e_{11}^2 / 2 \right) + \left( \nu \theta_1^2 - (1 - \nu) \right) \left( e_{12}^2 / 2 \right) + (2\nu - 1) \left( \theta_1 (e_{11} - e_{12}) - B_{11} + B_{12} \right) \right] \\ & + \lambda_2 (e_{12} - e_0) \end{aligned}$$

The first order conditions are:

$$(1b) \quad \frac{\partial \mathcal{L}}{\partial e_{11}} = \xi \nu (\theta_1 - e_{11}) + \lambda_1 \theta_1 (2\nu - 1) - \lambda_1 (\nu - (1 - \nu) \theta_1^2) e_{11} \leq 0; \quad e_{11} \frac{\partial \mathcal{L}}{\partial e_{11}} = 0$$

$$(2b) \quad \frac{\partial \mathcal{L}}{\partial e_{12}} = \xi (1 - \nu) (\theta_1 - e_{12}) + \lambda_1 \left\{ (\nu \theta_1^2 - (1 - \nu)) e_{12} - (2\nu - 1) \theta_1 \right\} + \lambda_2 \leq 0; \quad e_{12} \frac{\partial \mathcal{L}}{\partial e_{12}} = 0$$

$$(3b) \quad \frac{\partial \mathcal{L}}{\partial e_2} = (1 - \xi) (1 - e_2) \leq 0; \quad e_2 \frac{\partial \mathcal{L}}{\partial e_2} = 0$$

$$(4b) \quad \frac{\partial \mathcal{L}}{\partial u_2} = -(1 - \xi) + \lambda_1 \leq 0; \quad u_2 \frac{\partial \mathcal{L}}{\partial u_2} = 0$$

$$(5b) \quad \frac{\partial \mathcal{L}}{\partial B_{12}} = -\xi(1-\nu) + \lambda_1(2\nu-1) \leq 0; \quad B_{12} \frac{\partial \mathcal{L}}{\partial B_{12}} = 0$$

$$(6b) \quad \frac{\partial \mathcal{L}}{\partial B_{11}} = -\xi\nu - \lambda_1(2\nu-1) \leq 0; \quad B_{11} \frac{\partial \mathcal{L}}{\partial B_{11}} = 0$$

As in the proof of Proposition 2, (6b) immediately implies  $B_{11} = 0$ .

From (1b), we have

$$(E11b) \quad e_{11} = \left\{ \frac{\xi\nu + \lambda_1(2\nu-1)}{\xi\nu + \lambda_1(\nu - (1-\nu)\theta_1^2)} \right\} \theta_1.$$

From (2b), we have

$$(E12b) \quad e_{12} = \frac{[\xi(1-\nu) - \lambda_1(2\nu-1)]\theta_1 + \lambda_2}{\xi(1-\nu) - \lambda_1(\nu\theta_1^2 - (1-\nu))} = e_0,$$

where  $\lambda_2 = 0$  if  $e_{12} > e_0$ .

We will show that the rent is increasing with respect to  $e_{12}$ , which implies that we could have a case where the rent might be strictly positive for  $e_0$  sufficiently large. Below, we will give an example of such a case and derive the critical level of  $e_0$  for which the rent is strictly positive

when  $\nu = \frac{1}{2-\xi}$  if  $B_{12} = 0$ . This will imply that  $B_{12}$  is strictly positive as soon as  $\nu > \frac{1}{2-\xi}$ . Note

that  $B_{12} > 0$  is feasible by (4b) and (5b) in this case.

**Claim:** Suppose that  $u_2 > 0$  and  $e_{12} = e_0$  when  $\nu = \frac{1}{2-\xi}$ , then  $B_{12} > 0$  for some  $\nu > \frac{1}{2-\xi}$ .

(a) Rent is increasing with respect to both efforts. This can be seen from the right hand side of (IC):

$$(IC) \quad u_2 \geq (\nu - (1-\nu)\theta_1^2)(e_{11}^2/2) - (\nu\theta_1^2 - (1-\nu))(e_{12}^2/2) - (2\nu-1)(\theta_1(e_{11} - e_{12}) + B_{12})$$

Rent is increasing with respect to  $e_{11}$  as long as  $e_{11} > \left\{ \frac{2\nu-1}{\nu-(1-\nu)\theta_1^2} \right\} \theta_1$ , which can be verified as

follows: 
$$\frac{\xi\nu + \lambda_1(2\nu-1)}{\xi\nu + \lambda_1(\nu-(1-\nu)\theta_1^2)} > \frac{2\nu-1}{\nu-(1-\nu)\theta_1^2} \Leftrightarrow 1 > \theta_1^2.$$

Rent can be seen to increase with respect to  $e_{12}$  since  $e_{12} < \theta_1$  and  $2\nu-1 > \nu\theta_1^2 - (1-\nu)$ .

(b) Given  $u_2 > 0$ , i.e.,  $\lambda_1 = (1-\xi)$ , and  $e_{12} = e_0$ , consider a small increase in  $\nu$  from  $\nu = \frac{1}{2-\xi}$ :

From (E11b), we have

$$\frac{de_{11}}{d\nu} = \frac{\partial e_{11}}{\partial \lambda_1} \frac{\partial \lambda_1}{\partial \nu} + \frac{\partial e_{11}}{\partial \nu}$$

It can be checked that  $\frac{\partial e_{11}}{\partial \lambda_1} < 0$ . To see that  $\frac{\partial e_{11}}{\partial \nu} > 0$  rewrite (E11b) as

$$e_{11} = \left\{ \frac{A\nu - \lambda_1}{B\nu - \lambda_1\theta_1^2} \right\} \theta_1$$

where  $A = (\xi + 2\lambda_1)$ , and  $B = \xi + (1 + \theta_1^2)\lambda_1$ . Since  $A > B$ , it can be readily checked that

$$\frac{\partial e_{11}}{\partial \nu} > 0.$$

Together, (4b) and (5b) imply that rent is zero and  $\lambda_1 < (1-\xi)$  if  $\nu > \frac{1}{2-\xi}$ . Thus *both* rent and  $\lambda_1$

decrease with  $\nu$  (i.e.,  $\frac{\partial \lambda_1}{\partial \nu} < 0$ ). Putting these together, we have that  $e_{11}$  increases with  $\nu$  while rent decreases with  $\nu$ . Since  $e_{12}$  cannot decrease, given  $e_{12} \geq e_0$ , it must be that  $B_{12} > 0$ . This proves the claim.  $\square$



To complete the proof of the proposition, we will show that there is a value for  $e_0$  such that, for

$$\nu = \frac{1}{2-\xi}, u_2 = \left(\nu - (1-\nu)\theta_1^2\right)\left(e_{11}^2/2\right) - \left(\nu\theta_1^2 - (1-\nu)\right)\left(e_0^2/2\right) - (2\nu-1)(e_{11}-e_0)\theta_1 > 0.^{35}$$

We are looking for  $e_0^*$  such that rent is zero when  $e_{12} = e_0^*$  and  $\nu = 1/(2-\xi)$ . Then, if  $e_0 > e_0^*$ , it is optimal to burn money ( $B_{12} > 0$ ) for  $\nu = \varepsilon + 1/(2-\xi)$ , for  $\varepsilon > 0$  sufficiently small. Then  $e_0^*$  is obtained as follows:

With  $\nu = 1/(2-\xi)$  and  $\lambda_1 = (1-\xi)$ , (1b) implies  $e_{11} = \left\{ \frac{\xi(2-\xi)}{1-(1-\xi)^2\theta_1^2} \right\} \theta_1$ . Feeding this into the right hand side of (IC), the rent is as follows:

$$\begin{aligned} R^*(e_0) = & \left(\frac{1}{2}\right) \left( \left( \frac{1}{2-\xi} \right) - \left( \frac{1-\xi}{2-\xi} \right) \theta_1^2 \right) \left\{ \frac{\xi(2-\xi)}{1-(1-\xi)^2\theta_1^2} \right\}^2 \theta_1^2 - \left( \frac{\xi}{2-\xi} \right) \left\{ \frac{\xi(2-\xi)}{1-(1-\xi)^2\theta_1^2} \right\} \theta_1^2 \\ & + \left( \frac{1}{2} \right) \left( \left( \frac{1-\xi}{2-\xi} \right) - \left( \frac{1}{2-\xi} \right) \theta_1^2 \right) (e_0)^2 + \left( \frac{\xi}{2-\xi} \right) \theta_1 e_0 \end{aligned}$$

Finally,  $e_0^*$  is the solution to  $R^*(e_0^*) = 0$ .

For an example where burning money occurs in equilibrium, let  $e_0 = \frac{\xi\theta_1}{1-(1-\xi)^2\theta_1^2} > e_0^*$ . Notice

this  $e_0$  is less than in the standard no-signal contract because  $(1-\xi)$  is squared in the denominator. Feeding in for  $\nu = \frac{1}{2-\xi}$ ,  $e_{11}$ , and  $e_0$ , and performing straightforward cancelations, we are left with the following inequality to verify that the rent is positive:

$$\begin{aligned} (2-\xi) \left[ (2-\xi)(1-\theta_1^2(1-\xi)) - 2(1-\theta_1^2(1-\xi)^2) \right] & > \theta_1^2 - (1-\xi) - 2(1-\theta_1^2(1-\xi)^2) \\ \Leftrightarrow \left[ (2-\xi)(1-\theta_1^2(1-\xi)) - 2(1-\theta_1^2(1-\xi)^2) \right] & > \theta_1^2 - (1-\xi) - 2(1-\theta_1^2(1-\xi)^2) \\ \Leftrightarrow (2-\xi)(1-\theta_1^2(1-\xi)) & > \theta_1^2 - (1-\xi) \end{aligned}$$

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<sup>35</sup> Note that  $e_{11}$  is continuous at  $\nu = \frac{1}{2-\xi}$ , since the values of  $\lambda_1$  implied by (4) and (5) are equal.

$$\Leftrightarrow 1 - \theta_1^2(1 - \xi) > \theta_1^2 - (1 - \xi)$$

Finally,  $1 > \theta_1^2$ , and  $\theta_1^2(1 - \xi) < (1 - \xi)$ . ■

### **Appendix 3.2: The uncertainty of the timing of the private signal**

Suppose, the principal receives at most one signal. Denote by  $\alpha, \beta$ , and  $\gamma$ , the probabilities that the principal receives the signal ex ante, ex post or not all, respectively. The contract is now  $\{(e_{11}^a, t_{11}^a), (e_{12}^a, t_{12}^a, B_{12}^a), (e_1^p, t_{11}^p, t_{12}^p, B_{12}^p), (t_1^0, B^0), (e_2, t_2)\}$ , where the super-script  $a$  refers to the ex ante signal, super-script  $p$  to the ex post case, and 0 to the case when no signal is received. Anticipating equilibrium values, we have assumed that the signal is ignored after the agent claims  $\theta_2$ , and that no money is burned when the signal corroborates the agent's claim.

We start with the ex post ( $PIC^p$ ). Provided that the principal has not already reported a signal ex ante, the ex post PIC is

$$(PIC^p) \quad t_{11}^p = t_{12}^p + B_{12}^p = t_1^0 + B^0,$$

where the first equality ensures the principal does not gain by lying about which ex post signal she has received, and the second equality ensures that the principal will not gain by claiming to have obtained a signal when she has not.

The ex ante PIC must ensure the principal does not gain by lying about which signal she has received ex ante ( $\theta_1 e_{11}^a - t_{11}^a = \theta_1 e_{12}^a - t_{12}^a - B_{12}^a$ ) and also whether she has received the signal ex ante or not:

$$\begin{aligned} \theta_1 e_{11}^a - t_{11}^a &= \theta_1 e_{12}^a - t_{12}^a - B_{12}^a \\ &= \theta_1 e_1^p - \frac{\beta}{\beta + \gamma} \{ \nu t_{11}^p + (1 - \nu)(t_{12}^p + B_{12}^p) \} - \frac{\gamma}{\beta + \gamma} \{ t_1^0 + B^0 \} \end{aligned}$$

Using ( $PIC^p$ ), we can re-write the ex ante PIC as

$$(PIC^a) \quad \theta_1 e_{11}^a - t_{11}^a = \theta_1 e_{12}^a - t_{12}^a - B_{12}^a = \theta_1 e_1^p - t_{11}^p.$$

The incentive constraint is

$$(IC) \quad u_2 \geq \alpha \left\{ (1 - \nu) \left( t_{11}^a - \frac{(\theta_1 e_{11}^a)^2}{2} \right) + \nu \left( t_{12}^a - \frac{(\theta_1 e_{12}^a)^2}{2} \right) \right\} \\ + \beta \{ (1 - \nu) t_{11}^p + \nu t_{12}^p \} + \gamma t_1^0 - (\beta + \gamma) \frac{(\theta_1 e_1^p)^2}{2}$$

The (IR2) requires that  $u_2 \geq 0$ , and

$$(IR1) \quad \alpha \left\{ \nu \left( t_{11}^a - \frac{(e_{11}^a)^2}{2} \right) + (1 - \nu) \left( t_{12}^a - \frac{(e_{12}^a)^2}{2} \right) \right\} + \beta \{ \nu t_{11}^p + (1 - \nu) t_{12}^p \} + \gamma t_1^0 - \\ (\beta + \gamma) \frac{(e_1^p)^2}{2} \geq 0.$$

The principal's problem is to choose the contract to maximize

$$\xi \{ \alpha ( \nu (\theta_1 e_{11}^a - t_{11}^a) + (1 - \nu) (\theta_1 e_{12}^a - t_{12}^a - B_{12}^a) ) + (\beta + \gamma) \theta_1 e_1^p - \beta ( \nu t_{11}^p + (1 - \nu) (t_{12}^p + \\ B_{12}^p) ) - \gamma (t_1^0 + B^0) \} + (1 - \xi) \{ e_2 - \frac{(e_2)^2}{2} - u_2 \},$$

Subject to

$$(PIC^a), (PIC^p), (IR1), (IC), \text{ and } u_2 \geq 0.$$

(i) We will first prove that the principal will not be able to use the signal if it arrives ex post and  $\nu \leq \frac{1}{2 - \xi}$ .

Using the (binding) (IR1) and (IC), and the two (PIC) constraints, we can rewrite the rent as:

$$u_2 = \left\{ \alpha \left\{ \nu \frac{(e_{11}^a)^2}{2} + (1 - \nu) \frac{(e_{12}^a)^2}{2} \right\} + (\beta + \gamma) \frac{(e_1^p)^2}{2} \right\} - \left\{ (1 - \nu) \frac{(\theta_1 e_{11}^a)^2}{2} + \nu \frac{(\theta_1 e_{12}^a)^2}{2} \right\} - \\ (\beta + \gamma) \frac{(\theta_1 e_1^p)^2}{2} \left\{ - \{ \alpha (2\nu - 1) (B_{12}^a + \theta_1 (e_{11}^a - e_{12}^a)) + \beta (2\nu - 1) B_{12}^p \} \right\}.$$

Similarly, we can use (IR1) to replace the transfers in the objective function:

$$\xi\left\{\alpha\left(v\left(\theta_1 e_{11}^a - \frac{(e_{11}^a)^2}{2}\right) + (1-v)\left(\theta_1 e_{12}^a - \frac{(e_{12}^a)^2}{2} - B_{12}^a\right)\right) + (\beta + \gamma)\left(\theta_1 e_1^p - \frac{(e_1^p)^2}{2}\right) - \beta(1-v)B_{12}^p - \gamma B^0\right\} + (1-\xi)\left\{e_2 - \frac{(e_2)^2}{2} - u_2\right\},$$

Finally, using the rent derived above, it can be seen that  $B_{12}^p > 0$  only if

$$-\xi\beta(1-v) + (1-\xi)\beta(2v-1) \geq 0, \text{ which is equivalent to } v \geq \frac{1}{2-\xi}.$$

(ii) We next construct an example with a feasible contract, in which  $e_{11}^a > e_{12}^a$ :

$$t_{11}^p = t_{12}^p = t_1^0 = t_1^{NS} = \frac{(e_1^{NS})^2}{2}; e_1^p = e_1^{NS}; B_{12} = B^0 = 0; e_2 = e_2^{FB}; t_{11}^a = \theta_1 e_{11}^a - \pi_1^{NS}, t_{12}^a = \theta_1 e_{12}^a - \pi_1^{NS}, \text{ where } \pi_1^{NS} \equiv \theta_1 e_1^{NS} - t_1^{NS}.$$

Check that it satisfies the  $(PIC^p)$  and  $(PIC^a)$  constraints, and choose  $t_2$  to satisfy  $(IC)$  as an equality. Pick the following values for  $e_{11}^a = 0.192474$ ,  $e_{12}^a = 0.1924695$ , and the parameters,  $v = 0.6$ ,  $\xi = 0.5$ ,  $\theta_1 = 0.36$ .

Given those values, we can verify that  $(IR1)$  is satisfied, and that  $u_2 \geq 0$ , which implies that  $(IR2)$  is also satisfied.

Thus, all the constraints are satisfied by our constructed example.

(iii) We prove next that the constructed contract improves on the no-signal contract.

First, we prove that the comparison between the two contracts boils down to a comparison of two rents only. Second, we use our numerical example to compute the rents and show that the constructed contract is more profitable than the no-signal contract.

(a) Take the objective function after using  $(PIC^a)$ ,  $(PIC^p)$ , and the binding  $(IR)$  and  $(IC)$ :

$$\xi\left\{\alpha\left(v(\theta_1 e_{11}^a - \theta_1 e_{11}^a + \pi_1^{NS}) + (1-v)(\theta_1 e_{12}^a - \theta_1 e_{12}^a + \pi_1^{NS})\right) + (\beta + \gamma)(\theta_1 e_1^p - t_1^{NS})\right\} + (1 - \xi)\left\{e_2 - \frac{(e_2)^2}{2} - u_2\right\}$$

Replace the values for the transfers from our constructed example to rewrite the objective function:

$$\xi \pi_1^{NS} + (1 - \xi) \left( e_2^{FB} - \frac{(e_2^{FB})^2}{2} - u_2 \right)$$

The objective function in the no signal contract can be written as

$$\xi \pi_1^{NS} + (1 - \xi) \left( e_2^{FB} - \frac{(e_2^{FB})^2}{2} - u_2^{NS} \right), \text{ where } u_2^{NS} \text{ is defined as the rent in the no signal case.}$$

Therefore when comparing our constructed contract with the no signal contract, we just need to compare the rents. We find using our example that the rent goes down from 0.0161222 to 0.0161219.

Therefore, our constructed contract which uses the signal to rescale efforts ex ante results in a lower rent for the agent and therefore a higher profit for the principal. Note that our constructed contract is not the optimal contract and therefore we underestimate the additional profit obtained from using the signal ex ante.

#### **Appendix 4.1: Ex post public information with collusion**

The principal's problem is to maximize

$$\xi \left( \theta_1 e_1^p - \nu t_{11}^p - (1 - \nu)(t_{12}^p + W^p) \right) + (1 - \xi)(e_2^p - t_2^p)$$

subject to

$$(IR-1) \quad \nu t_{11}^p + (1 - \nu)t_{12}^p - \frac{(e_1^p)^2}{2} \geq 0$$

$$(IR-2) \quad t_2^p - \frac{(e_2^p)^2}{2} = u_2^p \geq 0$$

$$(IC) \quad u_2^p \geq (1 - \nu)t_{11}^p + \nu t_{12}^p - \frac{(\theta_1 e_1^p)^2}{2}$$

$$(CIC^p) \quad W^p \geq t_{11}^p - t_{12}^p$$

and non-negativity conditions for  $e_1^p, e_2^p$ , and  $W^p$ .

## **Appendix 4.2: Ex ante public information with collusion**

The principal's problem is to maximize

$$\xi \left( v(\theta_1 e_{11}^a - t_{11}^a) + (1 - v)(\theta_1 e_{12}^a - t_{12}^a - W) \right) + (1 - \xi) \left( e_2^a - \frac{(e_2^a)^2}{2} - u_2^a \right)$$

subject to

$$(IR) \quad v \left( t_{11}^a - \frac{(e_{11}^a)^2}{2} \right) + (1 - v) \left( t_{12}^a - \frac{(e_{12}^a)^2}{2} \right) \geq 0$$

$$(IC) \quad u_2^a \geq (1 - v) \left( t_{11}^a - \frac{(\theta_1 e_{11}^a)^2}{2} \right) + v \left( t_{12}^a - \frac{(\theta_1 e_{12}^a)^2}{2} \right)$$

and <sup>36</sup>

$$(CIC) \quad W^a + t_{12}^a - \frac{(\theta_1 e_{12}^a)^2}{2} \geq t_{11}^a - \frac{(\theta_1 e_{11}^a)^2}{2}.$$

The problem can be simplified before solving. (IR) and (CIC) always bind. Solving these constraints simultaneously for the transfers and feeding the solutions into the objective function and the right hand side of (IC), the problem becomes the following:

Maximize, with respect to efforts,  $u_2^a$ , and  $W^a$ ,

$$\xi \left( v \left( \theta_1 e_{11}^a - \frac{(\theta_1 e_{11}^a)^2}{2} \right) + (1 - v) \left( \theta_1 e_{12}^a - \frac{(\theta_1 e_{12}^a)^2}{2} - W^a \right) \right) + (1 - \xi) \left( e_2^a - \frac{(e_2^a)^2}{2} - u_2^a \right)$$

subject to

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<sup>36</sup> The analogous constraint for a truthful low type is satisfied when (CIC) is satisfied, strictly whenever  $e_{11}^a > e_{12}^a$ . However, when  $e_{11}^a > e_{12}^a$  and (CIC) is satisfied, we also need to be concerned with a truthful low type helping the certifier to forge evidence of high type in exchange for a share of  $W^a$ . For the present problem, we will see that  $e_{11}^a = e_{12}^a$  at the optimum, so the issue does not arise.

$$(IC) \quad u_2^a \geq (1 - \theta_1^2) \left( v \frac{e_{11}^{a^2}}{2} + (1 - v) \frac{e_{12}^{a^2}}{2} \right) - (2v - 1)W^a,$$

and the non-negativity conditions on  $e_{11}^a, e_{12}^a, e_2^a, W^a$  and  $u_2^a$ .

Denoting by  $\lambda$ , the non-negative multiplier for the (IC) constraints, the Kuhn-Tucker conditions for a maximum are the following:

$$(c1) \quad \xi v(\theta_1 - e_{11}^a) - \lambda v(1 - \theta_1^2)e_{11}^a \leq 0; \quad e_{11}^a \frac{\partial L}{\partial e_{11}^a} = 0$$

$$(c2) \quad \frac{\partial L}{\partial e_{12}^a} = \xi(1 - v)(\theta_1 - e_{12}^a) - \lambda(1 - v)(1 - \theta_1^2)e_{12}^a \leq 0; \quad e_{12}^a \frac{\partial L}{\partial e_{12}^a} = 0$$

$$(c3) \quad \frac{\partial L}{\partial e_2^a} = (1 - \xi)(1 - e_2^a) \leq 0; \quad e_2^a \frac{\partial L}{\partial e_2^a} = 0$$

$$(c4) \quad \frac{\partial L}{\partial u_2^a} = -(1 - \xi) + \lambda \leq 0; \quad u_2^a \frac{\partial L}{\partial u_2^a} = 0$$

$$(c5) \quad \frac{\partial L}{\partial W^a} = -\xi(1 - v) + \lambda(2v - 1) \leq 0; \quad W^a \frac{\partial L}{\partial W^a} = 0$$

$$(c6) \quad \frac{\partial L}{\partial \lambda} = u_2^a - (1 - \theta_1^2) \left( v \frac{e_{11}^{a^2}}{2} + (1 - v) \frac{e_{12}^{a^2}}{2} \right) + (2v - 1)W^a \geq 0; \quad \lambda \frac{\partial L}{\partial \lambda} = 0$$

(a) From (c1) and (c2), we can characterize interior solutions for the two efforts,

$e_{11}^a = e_{12}^a = \frac{\xi \theta_1}{\xi + \lambda(1 - \theta_1^2)} > 0$ . Therefore, (CIC) requires  $W^a + t_{12}^a = t_{11}^a$ , hence (PIC) is satisfied under any solution to the present problem.

(b) If  $v < 1$ , then  $\lambda > 0$ . To see this, assume  $\lambda = 0$ . From (a),  $e_{11}^a = e_{12}^a = \theta_1$ ; from (c4)  $u_2^a = 0$ . Then (c6) requires  $(2v - 1)W^a \geq (1 - \theta_1^2) \frac{\theta_1^2}{2}$ . But then (c5) is violated unless  $v = 1$ .

(c) (i) If  $v < \frac{1}{2 - \xi}$ , then the certifier is not used and the no-signal contract is optimal with  $u_2^a > 0$  and  $W^a = 0$ ; (ii) if  $1 > v > \frac{1}{2 - \xi}$ , then the certifier's signal is used, with  $u_2^a = 0$  and  $W^a > 0$ .

*Proof:* Since  $\lambda > 0$ , condition (c6) requires  $u_2^a + (2\nu - 1)W^a = \frac{(1-\theta_1^2)}{2} \left( \frac{\xi\theta_1}{\xi+\lambda(1-\theta_1^2)} \right)^2 > 0$ , which implies that  $u_2^a$  and  $W^a$  cannot both be zero.

(i) Suppose  $\nu < \frac{1}{2-\xi}$ . If  $u_2^a > 0$ , then (c4) implies  $\lambda = (1 - \xi)$ . Then, from (c5),  $W^a = 0$  since

$$\nu < \frac{1}{2-\xi} \Leftrightarrow -\xi(1-\nu) + (1-\xi)(2\nu-1) < 0. \text{ If } u_2^a = 0, \text{ then } W^a > 0 \text{ by (c6); hence (c5)}$$

implies  $\lambda = \frac{\xi(1-\nu)}{2\nu-1} > (1 - \xi)$  since  $\nu < \frac{1}{2-\xi}$  which contradicts (c4).

(ii) Suppose  $1 > \nu > \frac{1}{2-\xi}$ . If  $W^a > 0$ , then, from (c5),  $\lambda = \frac{\xi(1-\nu)}{2\nu-1}$ . Then, from (c4),  $u_2^a = 0$ , since

$$\nu > \frac{1}{2-\xi} \Leftrightarrow (1-\xi) \frac{\xi(1-\nu)}{2\nu-1}. \text{ If } u_2^a > 0, \text{ then (c4) implies } \lambda = (1-\xi) > \frac{\xi(1-\nu)}{2\nu-1}, \text{ which contradicts (c5).} \quad \blacksquare$$

## **Appendix 5: Second order conditions:**

For the Kuhn-Tucker conditions to be satisfied for a maximum, the border-preserving principal minors of the bordered Hessian must alternate in sign. In particular, for the problem presented in appendix 2, the principal minor involving  $e_{12}^a$ ,  $u_2^a$ , and  $\lambda$  must be strictly positive:

$$\begin{vmatrix} \frac{\partial^2 \mathcal{L}}{\partial e_{12}^{a^2}} & 0 & \frac{\partial^2 \mathcal{L}}{\partial e_{12}^a \partial \lambda} \\ 0 & 0 & 1 \\ \frac{\partial^2 \mathcal{L}}{\partial e_{12}^a \partial \lambda} & 1 & 0 \end{vmatrix} = \frac{\partial^2 \mathcal{L}}{\partial e_{12}^{a^2}} (-1) = \xi(1-\nu) - \lambda(\nu\theta_1^2 - (1-\nu)) > 0$$



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