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The Impact of Uncertainty on the European Energy Market: A Scenario Aggregation Approach

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The Impact of Uncertainty on the European Energy Market: A Scenario Aggregation Approach

Abstract

We present a simple approach to transform a deterministic numerical model, where several agents simultaneously make decisions, into a stochastic model. This approach, which builds on scenario aggregation, a numerical method developed to solve decision problems under uncertainty, is used to build a large stochastic numerical equilibrium model of the Western European energy markets. We use the stochastic model to analyze the impact of economic and political uncertainty on the Western European energy markets. We demonstrate that the equilibria under uncertainty differ significantly from the deterministic outcomes.

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1 Introduction

Agents in the European energy market face considerable uncertainty. Recently, the region experienced a severe recession with declining energy demand. In the future, climate treaties may have substantial impact on the sector. Such abundant uncertainties can have huge consequences on investments in capacities. At the same time, if there is reluctance to invest in some technologies, for example, due to expectations about high future taxes on emissions of greenhouse gases, the market looks more promising for other technologies, like renewables. Thus to fully analyze the impact of uncertainty, we need to take into account the interdependence of different technologies, energy carriers and end users – this calls for a numerical equilibrium model. But it is not trivial to solve such a model when all agents face uncertainty. Thus it is not surprising that most analyses assume full certainty, or if uncertainty is analyzed, rely on simulations instead of examining agents optimizing under uncertainty.

In this paper we present a simple method to transform a numerical deterministic model where several agents simultaneously make decisions into a stochastic model. We then use this method to transform LIBEMOD, a numerical deterministic multi-market equilibrium model of the Western European energy markets (see Aune et al., 2008), into a stochastic model. The stochastic version of LIBEMOD is used to analyze the impact of uncertainty on the European energy markets.

In spirit, our approach to modeling uncertainty is similar to the discussion of uncertainty in Debreu's (1959, chapter 7) classic 'Theory of Value', where uncertainty is represented by a discrete event tree. In our terminology, each branch of Debreu's event tree is called a scenario. Hence, in our model uncertainty is represented by a set of scenarios. Each scenario is one possible future realization of the uncertainty.

The models presented in this paper have two periods. In period 1, some agents make decisions under uncertainty, typically to determine their future capacities through investments. In the beginning of period 2, the uncertainty is resolved and all agents learn the true state of the economy. Then all agents make decisions; producers determine how much to produce (given the predetermined capacities) and consumers determine how much to consume. For each realization of the uncertainty, that is, for each scenario, the model determines supply of, and demand for, all goods from all agents, and the corresponding vector of prices that clear all markets.

When we use the stochastic version of LIBEMOD to analyze the impact of uncertainty, we distinguish between two sources of uncertainty; either there is uncertainty about GDP growth and fossil fuel prices (henceforth referred to as economic uncertainty), or there is uncertainty about future climate policy (henceforth referred to as political uncertainty). The results indicate that uncertainty has a considerable impact on optimal investments. First, investment in electricity transmission is considerably higher when there is economic uncertainty than in the case of no uncertainty. Second, the composition of investment in electricity technologies differs significantly between the case of no uncertainty and the two uncertainty cases (economic uncertainty and political uncertainty). For example, optimal investment in wind power is much higher under economic uncertainty than in the case without uncertainty.[2]

We also compare Monte Carlo simulations to the true optimal solutions under uncertainty. The average of the Monte Carlo simulations is usually closer to the optimal outcome than the deterministic solution (no uncertainty). However, in some cases, and in particular for single countries and single technologies, the average of the Monte Carlo simulations may produce numbers that are far from the optimal ones.

Our paper demonstrates that it is possible to solve large computable equilibrium models with significant uncertainty in several variables. Our approach builds on elements of scenario aggregation, a numerical method developed to solve decision problems under uncertainty (Wets, 1989; Rockafellar and Wets 1991; Kall and Wallace 1994). Scenario aggregation, and more generally stochastic programming, examines a *single* optimizing agent under uncertainty. Choosing a planner as the optimizing agent, one can find the efficient outcomes of an economy (see e.g., Kolstad, 1996). Our contribution is to use scenario aggregation in order to analyze uncertainty – within numerical multi-market equilibrium models - when *many* optimizing agents make decisions simultaneously. For a given specification of scenarios, our method finds the

[2] There are definitely more possible sources of uncertainty, for example, the weather. To test the importance of weather uncertainty, in Brekke et al. (2013) we introduced uncertain precipitation and wind (number of hours it blows in a season) in Scandinavia. Using data for a period of 21 years to generate 21 scenarios, we found that the equilibrium under this type of uncertainty was only marginally different from the deterministic outcome, with the exception of a somewhat increased transmission capacity between Scandinavia and Continental Europe.

optimal solution under uncertainty.[3]

Scenario aggregation should not be mixed with scenario analysis. Under scenario aggregation, each scenario is assigned a probability and the sum of probabilities over the scenarios is one. This is in contrast to scenario analysis where scenarios are not assigned probabilities but are rather seen as some of the possible future states. In general, scenario analysis is a widely used, informal policy exercise, whereas scenario aggregation is a mathematical method for stochastic optimization. Further, scenario aggregation explicitly takes into account that some decisions are made *before* the uncertainty is revealed; scenario analysis makes no such distinction.

The LIBEMOD model is too large and complex to be suitable for a simple representation of the scenario aggregation method. In Section 2 we therefore present a much simpler model to illustrate the scenario aggregation method. Section 3 provides a short description of the numerical model LIBEMOD. In Section 4 we first describe the scenarios and then compare the equilibrium when the stochastic version of LIBEMOD is used to the equilibrium under no uncertainty and also to Monte Carlo simulations. Finally, Section 5 concludes.

2 Modeling uncertainty

Below we first consider a model with only one source of uncertainty. This model is so simple that it can easily be solved analytically. We first solve the model in the standard way and then use the scenario aggregation approach to solve the simple model. We then consider the most frequently used approach to analyze uncertainty, namely Monte Carlo simulations, and compare this approach to the scenario aggregation method. Finally, we present how to use scenario aggregation within a general set up.

[3] The present paper builds on stochastic programming. An alternative approach is dynamic programming (Stokey et al., 1989), in particular numerical solutions to variational inequalities, see, for example, the study of Haurie, Zaccour and Smeers (1990) on oligopolistic markets under uncertainty.

2.1 A simple example

Consider an economy with uncertainty in demand only. The model has one representative producer and one representative consumer. The producer has to decide on energy production capacity in period 1, that is, before observing the realization of the stochastic variable. The consumer observes the realization of the stochastic variable (in period 2) before deciding how much energy to buy. Finally, because there is no cost of production and the producer is assumed to be a price taker, the producer will, in period 2, use his entire capacity in energy production.

2.1.1 The standard solution

<u>Supply</u>

There is one representative producer who installs a capacity K at a unit cost c, and maximizes expected profits:

$$\max E\left[p(\theta)K - cK\right]$$

where the price of the product, p, is assumed to depend on the uncertain weather θ. The first-order condition is

$$Ep = c. \tag{1}$$

Because the producer does not know θ before making a decision about the capacity, expected price has to equal marginal cost.

<u>Demand</u>

We assume a quasi-linear utility in two goods; one good, z, termed comfort, and another, y, representing all other goods:

$$u(z, y) = \sqrt{z} + y.$$

Comfort depends on energy consumption x and the weather θ:

$$z = \theta^2 x\,.$$

Normalizing the price of other goods to 1, the budget condition becomes $px + y = m,$ where m is the income of the consumer. Utility can now be written as

$$\theta\sqrt{x} + (m - px).$$

Maximizing utility yields the first-order condition

$$2p\sqrt{x} = \theta.$$

<u>Equilibrium</u>

Because the model is very simple, we can easily derive the equilibrium solution. In equilibrium, demand has to equal supply, which equals capacity, thus $x = K$. The first-order conditions for the producer and the consumer, along with the equilibrium condition, provide the following system of equations:

$$\begin{aligned} 2p\sqrt{x} &= \theta \\ Ep &= c \\ x &= K \end{aligned}$$

with the solution

$$K = \left(\frac{E\theta}{2c}\right)^2.$$

2.1.2 The scenario aggregation approach

We now analyze the same model using scenarios to describe the uncertainty. Define a set of scenarios $s \in S$. In general, a scenario determines all uncertain variables, but in our simple case there is only one uncertain variable, namely the weather. The "value" of the weather depends on the scenario, which we formalize by writing θ_s. A scenario *s* is realized with a probability q_s, and obviously $\sum_{s \in S} q_s = 1$.

Let all decision variables depend on the scenario. Thus, we index both consumption and capacity by *s*; x_s and K_s. Hence, an agent now has one decision variable for each scenario. In the

model above we assumed that the consumer knew θ_s before deciding on x_s (in period 2). Thus, the choice of consumption will depend on the state of the economy, that is, which scenario that has materialized. Hence, for two different scenarios s and s' we will in general have $x_s \neq x_{s'}$. For the producer, however, θ_s is *not known* when K_s is chosen (in period 1). Therefore, the producer cannot choose different capacities in different scenarios, that is, we impose the requirement $K_s = K$ for all $s \in S$. The producer maximizes expect profits subject to the latter restriction:

$$\max \sum_{s \in S} q_s \left[p_s K_s - c K_s \right]$$
$$\text{s.t. } K_s = K \text{ for all } s \in S.$$

The first-order conditions are:[4]

$$q_s p_s = q_s c + \mu_s \tag{2}$$

where μ_s is the Lagrange multiplier of the restriction $K_s = K$ for all $s \in S$. Adding all first-order conditions in (2) yields

$$Ep = c + \sum_{s \in S} \mu_s .$$

From the first-order condition (1) we know that $Ep = c$, and hence

$$\sum_{s \in S} \mu_s = 0. \tag{3}$$

Note that if we define $\tilde{\mu}_s = \frac{\mu_s}{q_s}$, the first-order condition (2) simplifies to

$$p_s = c + \tilde{\mu}_s \text{ for all } s \in S .$$

Moreover, condition (3) now becomes

$$E\tilde{\mu} = 0$$

[4] In LIBEMOD, these are actually complementary slackness conditions due to the non-negativity constraint $K \geq 0$.

which is the only equation where the probabilities appear.

We now collect the first-order conditions and the constraints:

$$
\begin{aligned}
2p_s\sqrt{x_s} &= \theta_s \text{ for all } s \in S \\
p_s &= c + \tilde{\mu}_s \text{ for all } s \in S \\
x_s &= K_s \text{ for all } s \in S \\
K_s &= K \text{ for all } s \in S \\
E\tilde{\mu} &= 0.
\end{aligned}
$$

We see that relative to the deterministic case, uncertainty only amounts to i) a small modification in the second first-order condition (the term $\tilde{\mu}_s$), and ii) the additional constraints $K_s = K$ and $E\tilde{\mu} = 0$. The first four equations apply to each scenario, so with n scenarios the number of equations equals $4n$. Because the first-order conditions are either (de facto) unchanged or only slightly modified relative to the deterministic case, the scenario aggregation approach requires only a modest change in computable equilibrium models like Libemod. However, the number of equations increases from 3 (under certainty) to $4n+1$ (under uncertainty).

2.1.3 Monte Carlo simulations

Consider the following system of equations:

$$
\begin{aligned}
2p_s\sqrt{x_s} &= \theta_s \text{ for all } s \in S \\
p_s &= c \text{ for all } s \in S \\
x_s &= K_s \text{ for all } s \in S.
\end{aligned}
$$

The solution of these equations corresponds to a Monte Carlo simulation of the original model where we obtain one value for each endogenous variable p_s, x_s, K_s for each scenario *s*, that is, for each realization of θ_s. In particular, the capacity K_s will in general differ between the scenarios. Monte Carlo simulations thus simply ignore the fact that producers in the economy do not know which scenario that will materialize. Put differently: Under Monte Carlo simulations, the solution is found under the false assumption that producers consider the future as certain –

which scenario that for sure will materialize differs between the simulations.

Comparing the Monte Carlo approach and the assumption that agents take the uncertainty into account when making decisions, we note some major differences. In the Monte Carlo simulations, $p_s = c$ in all scenarios. Thus there is no variation in the price, but production and capacity will be different in each scenario. On the other hand, with stochastic optimizing agents, capacity (K), which is determined *before* the producer knows which scenario that will materialize, and production, which is equal to capacity, do not differ between the scenarios, whereas the price differs between the scenarios: $p_s = \theta_s / (2\sqrt{K})$.

More fundamentally, according to economic theory uncertainty about future conditions change the *behavior* of agents (compared with the case of no uncertainty). This is captured by the scenario aggregation method as producers maximize expected profits, but it is not captured by Monte Carlo simulations. For each Monte Carlo simulation, a realization of the stochastic variables is drawn from a probability distribution. This realization amounts to specific parameter values that are used to find the equilibrium in a *deterministic* model. By simulating *n* times, one finds *n* equilibria, all obtained from the same deterministic model. Needless to say, the realizations (parameter values) will in general differ between each of the *n* runs, but agents neglect uncertainty simply because the model is deterministic.

2.2 Scenario aggregation – the general case

We now turn to the general case of several sources of uncertainty and several decisions to be taken under uncertainty. In order to simplify, we neglect equilibrium aspects and consider the maximization problem of a single agent, here a producer deciding which capacities that maximize expected profits. This is a stochastic optimization problem that may be written on the form

$$\max_K \left[\sum_{s \in S} f(K, \xi_s) q_s \right].$$

Here $f(K, \xi)$ is a value function, K is a vector of decision variables, ξ_s is a realization of a vector of uncertain variables, and s is a scenario index, $s \in S = \{1, 2, ..., n\}$. Finally, q_s is the probability that scenario s will materialize. Assuming that we have no information that rules out

any scenario, we include all possible scenarios in the set S.

A core idea of the method of scenario aggregation is to rewrite this problem by using one vector of decision variables for each scenario:

$$\max_{K_1, K_2, \ldots} \left[\sum_{s \in S} f(K_s, \xi_s) q_s \right]$$

subject to $K_s = K_n$ for all $s \in S$, $s \neq n$, where K_s is the vector of decision variables in scenario s. With this reformulation, the Lagrange function is of the form

$$\begin{aligned} L &= \sum_{s \in S} f(K_s, \xi_s) q_s + \sum_{s=1}^{n-1} \lambda_s (K_s - K_n) \\ &= \sum_{s \in S} \left[f(K_s, \xi_s) q_s + \lambda_s K_s \right] = \sum_{s \in S} L_s \end{aligned}$$

where $\lambda_n = -\sum_{s \neq n} \lambda_s$ and $L_s = q_s f(K_s, \xi_s) + \lambda_s K_s$ can be seen as the Lagrange function for scenario *s*. The first-order conditions for the maximization problem are

$$\begin{aligned} \nabla_{x_s} L_s &= 0 \quad \text{for all } s \in S \\ \sum_{s \in S} \lambda_s &= 0 \\ K_s &= K_n \quad \text{for all } s \neq n. \end{aligned}$$

Like in 2.1.2 the normalization

$$\tilde{\lambda}_s = \frac{\lambda_s}{q_s}$$

allows us to rewrite the conditions as

$$\begin{aligned} \nabla_{x_s} f(K_s, \xi_s) + \tilde{\lambda}_s &= 0 \quad \text{for all } s \in S \\ E\tilde{\lambda} &= 0 \\ K_s &= K_n \quad \text{for all } s \neq n. \end{aligned}$$

We thus find a similar modification of the first-order conditions in the general case.

3 Libemod

We now describe LIBEMOD – the numerical equilibrium model that will be used to study decisions under uncertainty. LIBEMOD allows for a detailed study of the energy markets in Western Europe, taking into account factors like inter-fuel competition, technological differences in electricity supply, transport of energy through gas pipelines/electricity lines and investment in the energy industry.

The core of LIBEMOD is a set of markets for seven energy goods: electricity, natural gas, oil, steam coal, coking coal, lignite and biomass. All energy goods are produced and consumed in each country in Western Europe. Natural gas and electricity are traded in competitive Western European markets using gas pipelines and electricity transmission lines that connect the model countries. There are competitive world markets for coking coal, steam coal and oil, but only domestic (competitive) markets for lignite and biomass. While fuels are traded in annual markets, there are seasonal (summer vs. winter) and time-of-day markets for electricity.

In each country in Western Europe (model country), electricity can (with several exceptions) be produced by a number of technologies: steam coal power, lignite power, gas power, oil power, reservoir hydropower (including run-of-river and pondage), pumped storage hydropower, nuclear power, waste power, biomass power and other renewables power (primarily wind power). Each electricity producer maximizes profit. Installed and maintained electricity capacity can be used to produce electricity or is sold as reserve capacity to a domestic system operator. There are a number of costs related to production of electricity. First, for fuel based plants there are costs directly related to combustion of fuels. These costs depend on plant efficiency, which in the model differs across countries, technologies and plants. Second, there are other inputs that are assumed to vary proportionally with production, and third, there are maintenance costs for electricity production capacity. Finally, there are start-up and ramping-up costs if the capacity used in one time period differs from the capacity used in the next time period.

In LIBEMOD there is a distinction between power plants that existed in the data year of the model (which we refer to as old plants) and new power plants. For the first group, there is increasing marginal costs along the merit order supply curve for each type of technology in each model country. Also new reservoir hydro and new wind power have increasing marginal costs,

reflecting scarcity of favorable sites. However, each type of *new* thermal power plants has constant returns to scale cost functions, that is, efficiency does not vary between plants using the same type of technology.

Electricity producers face some technical constraints, e.g., maintained capacity should not exceed installed (or invested) capacity, and plants need some time for technical maintenance. In addition, there are technologically specific constraints. For example, for reservoir hydro, in each season total availability of water, that is, the amount of water at the end of the previous season plus water inflow in the present season, must equal total use of water, that is, water used to produce electricity plus water saved for the next season. Moreover, water filling at the end of the season cannot exceed the reservoir capacity.

In each model country, there is demand for all types of energy from three groups of end users; the household segment (including service and the public sector), the industry segment and the transport sector. In addition, there is intermediate demand for fuels from fuel-based electricity producers. Demand from each end-user group (in each model country) is derived from a nested multi-good multi-period constant elasticity of substitution (CES) utility function. Domestic transport and distribution costs for electricity and natural gas differ across countries and user groups, but are otherwise fixed (with no capacity constraints).

There are several versions of LIBEMOD. These differ with respect to the data year (1996 vs. 2000), market structure (competitive markets vs. imperfect competition), time horizon (short run vs. long run), number of periods over the day (two vs. six), and heterogeneity in supply of, and demand for, coking coal and steam coal. In this study we use a version of LIBEMOD with competitive markets, long-run horizon (that is, there are investments in electricity production capacity, in international transmission capacity for natural gas and in international transmission capacity for electricity), two periods over the 24 hour cycle (day and night) and a simple modelling of coking coal and steam coal. The data year is 2000. The model determines all energy prices and all energy quantities invested, produced, traded and consumed in each sector in each model country. The model also determines all prices and quantities traded in the world markets, and emissions of CO_2 by model country and sector, se Aune *et al.* (2008) for a detailed documentation of LIBEMOD and Golombek *et al.* (2011) for an application of the LIBEMOD model to analyse the potential of CCS power plant technologies.

In this paper, LIBEMOD is changed from a deterministic model into a stochastic model following the methodology outlined in Section 2, that is, (i) all decision variables depend on the scenario, and (ii) for each variable that has to be determined *before* the agent knows which scenario that will materialize, here investments in the energy industry, we impose the requirement that the agent has to choose the same value for all scenarios. Once the agents know the scenario, all the remaining variables are determined in a standard way by identifying the vector of prices that clear all markets. Note that for each realization of a scenario there will be one vector of market clearing prices, see the discussion above. In solving the model we find all the S price vectors simultaneously with the investment levels in period 1.

4 Simulation results

4.1 Scenarios

We use the stochastic version of LIBEMOD to study investment decisions under uncertainty: We assume that agents make investment decisions in the year 2000 (period 1) and that in the beginning of 2030 (period 2) agents learn which scenario that has materialized. Moreover, in the beginning of 2030 the new capacities are available. We consider two types of uncertainty; economic uncertainty and political uncertainty, see Table 1.

[Table 1 about here]

In the first case, oil and coal prices in 2030, as well as GDP in all model countries in 2030, are uncertain at the time of investment, that is, in the year 2000. Because prices are endogenous in LIBEMOD, uncertain oil and coal prices are simulated by linear shifts in the supply functions of oil and coal.

At the time of investment we assume that investors correctly foresee the development between 2000 and 2010, which we set equal to the true development in oil and coal supply and GDP growth rates. Hence, the capital stocks in 2010 are optimal given the information about the

true development until 2010. The development between 2010 and 2030 is, however, uncertain for investors in 2000. This is handled by constructing a set of possible future states and assigning a probability to each state.

There is no obvious way to specify the future states. In general, the definition of states and the corresponding probabilities should reflect the information set of the agents when they make the investment decision, that is, in the year 2000. Obviously, the set of information should reflect the history until 2000, but it may also contain probability distributions of future events. In retrospect, the period after 2000 is characterized by the US shale gas revolution and a rise in intermittent power in Europe, particularly in Germany. We believe, however, that even to most energy experts these two events have come as a big surprise; they were hardly contained in the information set in 2000. Therefore, it easily becomes speculative to include probability distributions of future events in the information set in 2000. We therefore limit the information set to reflect history; it does not to contain probability distributions of future events.

We now describe how we have constructed the scenarios based on historic observations. We use annual data for the period 1970 to 2010 for changes in coal and oil supply and GDP growth rates and group these into four 10-years periods. We assume that the development from 2010 to 2020 is characterized by one of these four 10-years periods. Similarly, the development from 2020 to 2030 is also characterized by one the four 10-years period, which may, by coincidence, be the same as the 10-years period from 2010 to 2020. This gives us 4^2 states, and we assume that each state has the same probability. However, in our model the sequence of two 10-years periods between 2010 and 2030 is of no importance; the state reached if 2010-20 is characterized by the development in the 70'ies and 2020-30 is characterized by the development in the 90'ies is the same state as if 2010-20 is characterized by the development in the 90'ies and 2020-30 is characterized by the development in the 70'ies. We therefore have 10 unique possible states in 2030, and each of these is termed a scenario. Four of these scenarios have a probability of 1/16, whereas the probability of the other six scenarios, for example the one where the development between 2010 and 2030 mimics the 70'ies and 90'ies, have a probability of 2/16.

To sum up, in period 1 agents know that there are 10 scenarios, and they also know the probability of each scenario. In addition, for each scenario they correctly predict the equilibrium prices if this scenario is materialized (rational expectations).

Turning to political uncertainty, the general idea is to introduce uncertainty wrt. future climate policy: agents do not know which climate policy that will be imposed in the future. To simplify, we assume that there is a probability of 50 percent that one specific carbon policy will be implemented, and there is a 50 per chance that no carbon policy will be imposed. Hence, under political uncertainty there are two scenarios, that is, two possible future states, each with a probability of 50 per cent.

We assume that the carbon policy that may be implemented in the future is characterized by a uniform tax on CO_2 emissions that is imposed on all agents in the model countries. We set this tax equal to \$90 per ton CO_2, which, according to IEA (2008), will be sufficient to stabilize global GHG concentrations in the atmosphere at 550 ppm. This is by many, among others IEA (2008), considered as the most likely scenario. Thus agents know that in the future, here 2030 (period 2), there will either be no climate policy, or a uniform tax of \$90 per ton CO_2 will be imposed on all emissions. Agents also know that the probability of each scenario is 50 percent. This information is taken into account in period 1 (here 2000) when investments in the Western European energy industry are determined.

In examining the importance of uncertainty, we compare the equilibrium of the stochastic LIBEMOD model with the equilibrium of the deterministic LIBEMOD model. In doing so, the stochastic exogenous variables are replaced by their expected values. Under political uncertainty this is simply \$45 per ton CO_2. We also compare the equilibrium of the stochastic LIBEMOD model with the output from Monte Carlo simulations.

The model is solved on a state of the art Intel-based application server with the GAMS (Brooke et al., 1998) modeling language and the Path (Ferris and Munson, 1998) solver. For efficiency, the variables are initialized with their 2000 calibration values when solving the 2030 equilibrium under no uncertainty. This solution is then used as the starting point for each of the independent Monte Carlo equilibria, which further provides initialization for the equilibrium under uncertainty.

4.2 Economic uncertainty

The importance of uncertainty

Below we discuss the outcome when there are three sources of uncertainty; future oil prices, future coal prices and future GDP levels. There is an important difference between the impact of uncertain fossil fuel prices and the impact of uncertain GDPs. The impact of uncertain GDP will differ across countries because some countries will experience high growth rates whereas other countries will experience low growth rates. In countries with high growth rates, demand for energy will increase significantly, which, *cet. par.*, tends to increase domestic energy prices, thereby providing an incentive for other countries to export energy to these countries. This suggests that international transmission capacity is of great importance. In contrast, the impact of uncertain fossil fuel prices may not differ that much between countries because under perfect competition all countries will face the same producer prices of oil and coal.[5]

In the deterministic case, that is, the stochastic exogenous variables are replaced by their expected values, 5 GW is invested in international electricity transmission capacity between the model countries, whereas under uncertainty (scenario aggregation) the level of investment is 16 GW, that is, three times higher, see Table 2. Hence, under uncertainty it is more profitable to invest in flexibility between countries. This may reflect differences in growth rates between countries: In the future (2030), a country *A* may experience high demand for electricity because of high growth rates. Rather than investing a lot in power plants so that future domestic demand for sure can be met by domestic production, electricity can partly be imported from the neighboring country *B* through existing and new transmission capacities if it turns out that future demand (in 2030) is low in country *B*. These capacities can alternatively be used to transport

[5] Although countries will face the same producer prices of oil and coal, the impact of uncertain producer prices may still differ somewhat between countries. First, the change in end-user prices of oil and coal will differ between countries because costs of domestic transport and distribution, and also taxes, differ across countries. Second, the market shares of oil and coal differ between countries, both in end-user demand and in electricity generation: if coal power has a large market share and the price of coal turns out to be high, production of coal power may decrease significantly. This tends to increase the domestic price of electricity, thereby providing an incentive for increased production from other domestic electricity technologies. This effect will, however, be dampened through international trade of electricity. The effect through international trade resembles the effect of differences in GDP growth rates, but it may be much lower in magnitude because it is a derived (indirect) effect.

electricity from *A* to *B* if it turns out that demand (in 2030) is high in country *B*, but low in country *A*.

Alternatively, future production of electricity may be high in country *A* but low in country *B* because of structural differences in electricity production capacities. If, for example, country *A* invests in coal power whereas country *B* invests in oil power, and the price of coal (in 2030) turns out to be low whereas the price of oil turns out to be high, then country *A* may use its entire new coal power capacity whereas oil power production in country *B* may not be profitable because oil is too expensive. In such a case, it would have been profitable (in 2000) to build a new transmission line between country *A* and *B*, which can now (in 2030) be used to export electricity from country *A* to country *B*. Likewise, if it turns out that the price of coal (in 2030) is high whereas the price of oil is low, a new transmission line can be used to transport electricity from *B* to *A*. As suggested in footnote 4, it seems reasonable that this effect is smaller than the effect caused by growth rate differentials.

[Table 2 about here]

As seen from Table 2, the difference in investment in international gas transmission capacity between the case of uncertainty and the deterministic outcome is marginal (2 percent). Whereas electricity transmission provides flexibility because electricity can be transported one way or the other, natural gas is mainly transported one way, that is, exported from the big extractors. Which scenario that materializes may have significant impact on the export magnitudes, but not much impact on which countries that are exporters of natural gas. Thus, for natural gas two-way flexibility is not a big issue.

In the deterministic case, total investment in electricity production capacity is 365 GW, whereas under uncertainty investment in electricity production capacity is slightly lower; 358 GW. The distribution of investment by technology differs, however, between the two cases: In the deterministic case, investment in coal power is 304 GW, which is 55 GW higher than under uncertainty. Under certainty investors know for sure the profitability of new coal power plants, and undertake all projects with non-negative profitability. Under uncertainty, investors know cost of investment, but cost of operating a new coal power plant is uncertain because the future price

of coal is uncertain. Thus, (part of the) new coal power capacity will not be used if the input price turns out to be too high. This explains why investment in coal power is lower under uncertainty than in the deterministic case.

As seen from Table 2, both in the deterministic case and under uncertainty there is no investment in oil power. In the deterministic case, investment in oil power capacity is simply not profitable. Under uncertainty, there is a chance that the oil price will turn out to be so low that investment in oil power will be profitable. However, at the point in time where investment has to take place (2000) this probability is too low to ensure a positive expected profit. Hence, also under uncertainty there will be no investment in oil power.

Because investment in coal power is higher in the deterministic case than under uncertainty (see discussion above), and demand for electricity in the deterministic case does not differ much from expected demand under uncertainty, there is room for additional investment in electricity production capacity under uncertainty relative to the deterministic case. Table 2 shows that under uncertainty, investment in hydro power is only marginally higher than in the deterministic case, investment in gas power is around 50 percent higher than in the deterministic case, and investment in wind power is roughly 200 percent higher than in the deterministic case. In general, investment in different electricity technologies depends on their long-run marginal cost of production. In LIBEMOD these are increasing in the equilibrium quantities; either because it is assumed that the best locations are taken first (reservoir hydro and wind), or because there are increasing costs of providing more of the input (natural gas and biomass). Our results reflect that reservoir hydro has a much stepper long-run marginal cost curve than wind power.

In the deterministic case, the entire new electricity production capacity will for sure be used in at least one time period. In contrast, under uncertainty part of the new production capacity may not be used at all; if, for example, the coal price turns out to be very high, it may not be profitable to run any of the new coal power plants.

Because fuel prices differ between scenarios, also production of electricity will differ between scenarios; total production of electricity varies under uncertainty between 3721 TWh and 4809 TWh, and has an expected value of 4571 TWh. Note that in the deterministic case, total electricity production is 4851 TWh, that is, slightly above the highest level of production under uncertainty. From the discussion above we know that total investment in electricity production capacity is almost identical in the two cases. Hence, the difference in total production reflects

higher rates of capacity utilization in the deterministic case. Because average electricity production under uncertainty is lower than electricity production in the deterministic case, and the demand curve is downward sloping, the average producer price of electricity under uncertainty is higher than under no uncertainty (64 USD/MWh vs. 55 USD/MWh), see Table 2.

Monte Carlo simulations

We now turn to the Monte Carlo simulations. As explained above, in the Monte Carlo simulations there is no uncertainty: agents know for sure in 2000 (period 1) which scenario that will materialize in 2030 (period 2). Hence, if agents know that there will be high growth rates, they will tend to investment more; if they know the price of coal will be low, they will tend to invest in coal power; and if they know the price of oil will be low, they will tend to invest in oil power – all solutions are tailor-made.

Under Monte Carlo simulations investment in coal power varies between 54 GW and 367 GW, see Table 2. The weighted average of the Monte Carlo simulations is 250 GW (see Table 2),[6] which is almost identical to investment in coal power under uncertainty (249 GW). For some of the other electricity technologies the (weighted) average of the Monte Carlo simulations does not, however, provide a good estimate of investment under uncertainty: For gas power, the difference is around 30 percent, whereas for wind power the difference is roughly 10 percent.

In the Monte Carlo simulations, there is investment in oil power in scenario 5 only; this scenario is characterized by a very low oil price. Here the level of investment is as high as 211 GW. The Monte Carlo average of oil power investment (13 GW) therefore exceeds the optimal level under uncertainty (zero). However, the average of the Monte Carlo simulations overshoots oil power investment by almost the same magnitude as it undershoots gas power and wind power investments. Therefore, the Monte Carlo average of total investment in electricity production capacity differs by only one percent from the optimal solution under uncertainty. The difference in total production of electricity is somewhat higher (3 percent), which reflects that under Monte Carlo simulations investors know which scenario that will materialize. In contrast, under uncertainty the rate of capacity utilization depends on which scenario that will materialize, see the discussion above.

[6] We use the scenario probabilities as weights.

Finally, in scenario 5 there is high investment in oil power in a few countries only. In these countries it is profitable to invest in international electricity transmission capacity in order to export part of the domestic oil power production. This is the main reason why average Monte Carlo investment in electricity transmission lines is about 20 percent higher than the solution under uncertainty, see Table 2.

4.3 Political uncertainty

In this case we assume that there is one source of uncertainty, namely whether the future carbon tax will be zero or 90 USD/tCO2. Because the uncertainty is whether a tax will be implemented or not, there are two scenarios. The case Monte Carlo *T* shows the effect when it is known in 2000 that there will be a tax in the future (2030), while Monte Carlo *N* represents the case where it is known in 2000 that there will be no tax in the future (2030). The stochastic solution shows the impact of a tax that is lingering as a possibility until it might be imposed in the future. As above, we will also consider the deterministic case and the (weighted) average of the Monte Carlo simulations as possible approximations to the stochastic solution.

In general, a carbon tax enhances the competitive position of non-fossil fuel electricity technologies and weakens the competitive position of fossil fuel electricity technologies. Because emissions of CO2 per unit of energy (measured in toe) is larger for coal than for oil, and larger for oil and than for natural gas, coal will typically be the big loser if a carbon tax is imposed, whereas natural gas comes in an intermediate position: it improves its position relative to other fossil fuels, but weakens its position relative to non-fossil fuels. The net effect is therefore ambiguous, and may depend on a number of factors like the market share of natural gas in end-user demand and in electricity generation, and also on the carbon tax rate itself; if the tax is increased marginally from a low level, say zero, or marginally from a high level, say 90 USD/tCO2, the effect on use of natural gas may be very different.

Uncertainty vs. no uncertainty

Future fossil fuel prices are unknown in 2000 also in the present case of an uncertain carbon tax. The materialization of the uncertainty will not differ across countries – all countries will face the same producer prices of fossil fuels – but the impact of high (or low) producer prices of fossil fuels will differ somewhat across counties. First, fossil fuel end-user prices will differ across countries because of country differences in costs of transport/distribution and taxes. Second, the impact of changes in end-user prices differs across countries because of differences in end-user demand parameters (the CES utility functions). In addition, the market share of fossil fuels, as well as the composition of fossil fuels, differ across countries both in end-user demand and in electricity generation. These effects resemble the impact of uncertain oil and coal prices under economic uncertainty, see above. However, under economic uncertainty both the future level of demand, as well as differences in demand across countries, were important factors to understand why investments differ between the case of uncertainty and the deterministic outcome. We therefore expect the differences in investment to be much lower in the case of political uncertainty than under economic uncertainty.

[Table 3 about here]

Table 3 shows investment in electricity technologies under the four different cases; Deterministic (a carbon tax of 45 USD/tCO2 will for sure be imposed), Stochastic (uncertain carbon tax), Monte Carlo *N* (a carbon tax will for sure *N*ot be imposed) and Monte Carlo *T* (a carbon *T*ax of 90 USD/tCO2 will for sure be imposed). As seen from Table 3, under uncertainty investment in coal power is lower than in the deterministic outcome (144 GW vs. 120 GW). For other technologies, the difference is smaller: 105 GW vs. 111GW for wind power and 72 GW vs. 66 GW for gas power. Hence, investment under uncertainty differs somewhat from investment in the deterministic outcome. For total investment in power capacity, the difference is around 10 percent (360 GW under uncertainty vs. 336 GW under no uncertainty).

Table 3 also shows that under uncertainty, investment in international electricity transport capacity is only marginally higher than in the deterministic outcome. The difference in investment in international gas transmission capacity is opposite and larger; 136 mtoe under uncertainty vs. 146 mtoe under no uncertainty. These rankings reflect that natural gas transport capacity is typically used to transport gas from large gas suppliers to gas importing countries, that is, in "one direction" only, whereas electricity transport capacity may be used in "both directions" under uncertainty – which direction that will be used depends on which scenario that materializes. In addition, under uncertainty there is a tendency in large natural-gas extracting countries to set up natural gas power plants, whereas in the deterministic case these countries export a higher share of the natural gas. This explains why there is more investment in natural gas transport capacity under certainty than under uncertainty.

Table 4 shows production of electricity (in 2030) by technology. Under uncertainty investment is determined before the future tax is known, and hence investment cannot respond to the actual future tax. However, electricity production in 2030 will adjust to the tax: As seen in Table 4, for old fossil fuel technologies production is more than twice as high when a tax is not imposed (727 TWh, Stochastic *N*) than when it is imposed (264 TWh, Stochastic *T*).

As seen from Table 4 (production of electricity), the deterministic outcome is typically between Stochastic *N* and Stochastic *T*; the only exception is for new renewable electricity where the production capacity is higher in the deterministic outcome than under uncertainty, see Table 3. Note, however, that for production in old fossil fuel plants, the deterministic outcome is slightly below the highest value under uncertainty (Stochastic *N*), whereas for production in new fossil fuel plants, the deterministic outcome is slightly above the lowest value under uncertainty (Stochastic *T*). Hence, the deterministic outcome is not just a constant average of Stochastic *N* and *T*; its level relative to Stochastic *N* and *T* varies by fuel and country.

Table 5 provides information on use of natural gas and steam coal (in 2030) by sectors (households, industry and power generation). Use of natural gas does not differ much between Stochastic *N* and *T*, and the average of Stochastic *N* and *T* is close to natural gas use under certainty. For use of coal, the average of Stochastic *N* and *T* is somewhat (13 mtoe) lower than the deterministic outcome, but now there is a significant difference between Stochastic *N* and *T* . Primarily, this difference reflects that with a high carbon tax (Stochastic *T*) most of the old coal power plants are not profitable to operate, whereas if there is no carbon tax (Stochastic *N*) then

the entire old capacity of coal power plants is profitable to run. In addition, investment and production in new coal power plants is higher in Stochastic *N* than in Stochastic *T*.

[Table 4 about here]

[Table 5 about here]

Monte Carlo simulations

As mentioned above, the Monte Carlo *T* scenario represents the effect of a 90 USD/tCO2 tax in 2030 which was credibly announced in 2000. Similarly, Monte Carlo *N* represents the case when it is credibly announced in 2000 that there will be no tax in 2030. Stochastic *T* and *N* represent the cases with Tax or No tax in 2030, but at the time of investment in 2000 it is unknown whether there will be a tax in 2030.

Surprisingly, total investment in electricity production capacity does not differ between Monte Carlo *N* and *T*, see Table 3. This reflects two counteracting factors: On the one hand, in Monte Carlo *T* production of electricity (in 2030) in fossil fuel plants that existed in 2000 is roughly half of the production level in Monte Carlo *N* – this simply mirrors the difference in the carbon tax between Monte Carlo *N* and *T* (no carbon tax vs. 90 USD/tCO2). A small production of electricity in old fossil fuel plants in Monte Carlo *T* tends to increase the price of electricity, thereby providing an incentive to expand the electricity production capacity. On the other hand, the high carbon tax in Monte Carlo *T* will lower demand, thereby making it less profitable to invest. By chance, the net effect of these two factors is zero when Monte Carlo *N* is compared to Monte Carlo *T*; in both cases investment amounts to 365 GW.

Note that the composition of investment in electric production capacity is very sensitive: If it is announced in 2000 that there will be no future tax (Monte Carlo *N*), new renewable electricity capacity amounts to 31 GW compared to 260 GW when it is (credibly) announced in 2000 that there will be a future tax (Monte Carlo *T*). For new fossil fuel electricity capacity, the effect is opposite: In Monte Carlo *N*, investment in fossil fuel electricity capacity amounts to 334

GW (304 GW coal power), while investment is only 105 GW (0 GW coal power) in Monte Carlo *T*. Thus there are huge differences in investment in coal power when a tax is (credibly) announced versus when it is (credibly) announced that there will be no future tax. This is also the case for wind power: 9 GW investment in Monte Carlo *N* versus 210 GW investment in Monte Carlo *T*.

We now turn to how the average of the Monte Carlo simulations and the deterministic outcome approximate the stochastic solution. First, under uncertainty, that is, when investors do not know whether there will be a tax or not in the future, there can be no difference in investment between the tax and no tax scenario. In this case, investment in coal power, gas power and wind power are close to the Monte Carlo *averages*, and total investment under uncertainty is only two percent lower than the Monte Carlo average, see Table 3. Hence, averages of the Monte Carlo simulations provide good estimates of investment under uncertainty.

Second, if there is uncertainty about the carbon tax and it turns out (in 2030) that no tax is imposed (Stochastic *N* in Table 4), then total production of electricity is roughly 10 percent lower than in Monte Carlo *N*. In particular, production from new fossil fuel technologies is 1703 TWh in Stochastic *N*, compared to 2632 TWh if it was (credibly) announced in 2000 that there will be no future tax (Monte Carlo *N*). The reason is that the composition of electricity capacity is significantly different in the two cases: In Monte Carlo *N*, most of the new capacity is fossil fuel based, which has a maximum rate of utilization of 90 percent. In Stochastic *N*, around one third of the new capacity is found in wind power, which has a maximum rate of utilization below 40 percent.

Similarly, if there is uncertainty about the carbon tax and it turns out that a 90 USD/tCO2 tax is imposed (Stochastic *T* in Table 4), then total production of electricity is roughly 5 percent higher than in Monte Carlo *T*. In particular, the difference in electricity production from new fossil fuels technologies is 1461 TWh (Stochastic *T*) versus 825 TWh (Monte Carlo *T).* Again, these differences reflect the composition of new electricity production capacity: in Monte Carlo *T*, there is no investment in coal power, but substantial investment in wind power (which has a low rate of utilization).

Because average electricity production under uncertainty is almost 5 percent below the Monte Carlo average, whereas it is almost 10 percent above the production level in the

deterministic outcome, the Monte Carlo average provides a better estimate of the outcome under uncertainty than the deterministic outcome does. This reflects the benefit of using the scenarios in stead of applying the expected value when one falsely assume there is no uncertainty. As seen from Table 4, among the five outcomes (Deterministic, Stochastic *N/T*, Monte Carlo *N/T*) electricity production is highest under Monte Carlo *N* and lowest under Monte Carlo *T*. Therefore, the producer price of electricity is lowest under Monte Carlo *N* (46 USD/MWh) and highest under Monte Carlo *T* (89 USD/MWh), but as seen from Table 3, the average producer price of electricity under uncertainty does not differ much from the Monte Carlo average.

Turning to investment in international electricity transmission capacity, this type of investment is lower under uncertainty than the average of the Monte Carlo simulations, see Table 3. The high Monte Carlo average reflects substantial investment in Monte Carlo *T*, where numerous wind power stations are set up, particularly in Scandinavia, accompanied by investment in international electricity transmission.

Table 5 provides information on use of natural gas and steam coal (in 2030) by sectors (households, industry and power generation) in the five cases. If it turns out that no tax is imposed (Stochastic *N)*, then use of natural gas in power generation is almost twice as high than under Monte Carlo *N*. If the tax turns out to be 90 USD/tCO2 (Stochastic *T*), then use of natural gas in power generation is roughly 25 percent lower than under Monte Carlo *T* . Thus, it makes a significant difference whether the policy is announced or not, especially in the power generation sector. However, total use of natural gas under uncertainty is almost equal to the Monte Carlo average.

For steam coal, the differences are much larger. An announced tax will almost wipe out use of steam coal; total use is only 37 mtoe in Monte Carlo *T* compared to 190 mtoe in Stochastic *T*. Similarly, an announcement that there will be no future tax would boost the use of steam coal to 562 mtoe (Monte Carlo *N*) compared to the more modest 318 mtoe in Stochastic *N*. A credibly announced policy will thus have a much stronger effect on steam coal use than a policy under which it remains uncertain until 2030 whether the tax will be implemented. Note though that total use of steam coal is almost twice as high in Stochastic *N* than in Stochastic *T* - there is scope for a significant adjustment of steam coal use even when investment cannot respond to the actual tax.

5 Conclusions

The stochastic scenario method can be used to solve numerical equilibrium models with several agents simultaneously maximizing their payoff under uncertainty. The method has been implemented in a stochastic version of LIBEMOD, a numerical multi-market model of the European energy market. We find that replacing the uncertainty with the expected value, which in the paper was referred to as the deterministic outcome, leads to large deviations from the optimal solution under uncertainty. The weighted average of the Monte Carlo simulations often approximate the optimal solution better, at least for aggregate numbers like total production of electricity.

In the case of political uncertainty, the method allows us to study the effect of indecisive policy makers: We find that a carbon tax of 90 USD/tCO2 has a much stronger effect if it is credibly announced early (Monte Carlo *T*) compared to the case in which investments are determined before it is known whether a future tax of 90 USD/tCO2 will be imposed or not.

The current version of LIBEMOD is not dynamic. However, by requiring that investments are identical across scenarios while energy consumption is scenario specific, we impose a structure where investments are decided before the uncertainty is revealed while use of energy is decided after the uncertainty is revealed. Our approach can be extended to dynamic multi-period models with learning. The information available in different periods would then be represented by partitions of the set of scenarios; the decision makers in a period do not yet know the exact scenario that will materialize in the future, only which subset the true scenario will belong to. Typically, decisions made in the first period will have to be the same in all scenarios, while decision in a later period will be the same within a subset of scenarios, but different across subsets. In the last period the exact scenario will be known. Learning is represented by the gradually finer partition of the set of scenarios.

We can also account for risk aversion, either by assuming that investment is decided by the firms' owners who are diversified in the financial market, or that investment is decided by risk-averse managers. In the first case, probabilities are replaced by weights derived from the prices of Arrow securities, that is, contingent claims that pay 1 $ if a particular scenario materializes, see Arrow (1964). After replacing probabilities with normalized prices of contingent claims, all agents will behave as if they were risk neutral. Risk aversion will be reflected in the

prices for contingent claims. This approach is similar to the use of equivalent martingale measures in finance (Harrison and Kreps, 1979; Duffie, 1996). With risk averse managers, a similar approach can be used, but in this case the scenario-weights will be firm specific, and thus some modest changes in the first-order conditions would be required.[7]

[7] More details on how to account for risk aversion and to extend the method to cover dynamic models can be obtained from the authors upon request.

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TABLES

Table 1 Sources of uncertainty. Investment in 2000.

A	Economic uncertainty. 2030 oil prices, coal prices and GDP levels are uncertain.*
B	Political uncertainty. A carbon policy may be imposed in 2030.

* Uncertain oil and coal prices are simulated by linear shifts in supply functions.

Table 2. Investments in transmission capacity and electricity production capacity in 2000 and equilibrium producer prices in 2030. Uncertain oil and coal prices, and GDPs in 2030 (uncertainty source A).

	New transmission capacity (GW or mtoe)		New power capacity (GW)							Producer prices (USD/MWh or USD/mtoe)	
	Electricity	Gas	Total	Hydro power	Gas power	Coal power	Oil power	Bio power	Wind power	Electricity	Gas
Deterministic	5	157	365	9	30	304	-	13	9	55	249
Stochastic*	16	154	358	11	49	250	-	17	31	64	297
Monte Carlo§	19	157	354	10	37	250	13	15	28	61	276
MC 1	*6*	*164*	*432*	*13*	*45*	*262*	*-*	*23*	*89*	*79*	*383*
MC 2	*3*	*173*	*405*	*10*	*27*	*330*	*-*	*16*	*22*	*61*	*285*
MC 3	*4*	*173*	*417*	*10*	*15*	*367*	*-*	*14*	*12*	*57*	*260*
MC 4	*8*	*156*	*323*	*13*	*71*	*138*	*-*	*22*	*79*	*78*	*363*
MC 5	*146*	*173*	*370*	*8*	*5*	*137*	*211*	*9*	*1*	*48*	*204*
MC 6	*25*	*165*	*377*	*8*	*10*	*348*	*-*	*10*	*1*	*49*	*211*
MC 7	*3*	*136*	*291*	*10*	*55*	*189*	*-*	*16*	*22*	*61*	*273*
MC 8	*32*	*162*	*398*	*7*	*3*	*379*	*-*	*9*	*0*	*47*	*202*
MC 9	*3*	*130*	*304*	*10*	*51*	*216*	*-*	*14*	*13*	*58*	*260*
MC 10	*26*	*150*	*238*	*12*	*90*	*54*	*-*	*21*	*61*	*74*	*336*

* Producer prices are weighted average of the ten scenario equilibrium prices.

§ Weighted average of ten Monte Carlo scenario values.

Table 3. Investments in transmission capacity and electricity production capacity in 2000 and equilibrium producer prices in 2030. Uncertain CO2 tax rate in 2030 (uncertainty source B).

	New transmission capacity (GW or mtoe)		New power capacity (GW)							Producer prices (USD/MWh or USD/mtoe)	
	Electricity	Gas	Total	Hydro power	Gas power	Coal power	Oil power	Bio power	Wind power	Electricity	Gas
Deterministic	19	146	336	14	66	120	-	25	111	70	250
Stochastic*	20	136	360	14	72	144	-	25	105	69	244
Monte Carlo§	30	165	365	13	67	152	-	23	110	68	244
MC N	*5*	*157*	*365*	*9*	*30*	*304*	-	*13*	*9*	*46*	*208*
MC T	*55*	*174*	*365*	*17*	*105*	*0*	-	*33*	*210*	*89*	*279*

* Producer prices are weighted average of the two scenario equilibrium prices.

§ Weigted average of two Monte Carlo scenario values.

Table 4 Production of electricity (TWh) in 2030. Uncertain CO2 tax rate in 2030 (uncertainty source B).

	Total	Old hydro	Old fossil fuel	Old renewable	Old nuclear	New hydro	New fossil fuel	New renewable
Deterministic	4126	432	717	91	836	43	1472	535
Stochastic *N*	4346	432	727	87	836	42	1703	518
Stochastic *T*	3644	432	264	91	836	42	1461	518
MC *N*	4851	432	697	92	836	29	2632	134
MC *T*	3467	432	363	79	836	52	825	880

Table 5 Use of energy (mtoe) in 2030. Uncertain CO2 tax rate in 2030 (uncertainty source B).

	Households	Industry	Power generation	Total
Natural gas				
Deterministic	173	108	120	401
Stochastic *N*	178	110	121	409
Stochastic *T*	158	100	131	389
MC *N*	192	131	68	391
MC *T*	150	90	166	406
Steam coal				
Deterministic	3	11	252	267
Stochastic *N*	4	19	294	318
Stochastic *T*	3	9	179	190
MC *N*	4	18	541	562
MC *T*	3	9	26	37