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Does Flexibility Pay Off?

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Intensity-Based Permit Quotas and the Business Cycle:

Does Flexibility Pay Off?

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Abstract

 Tradable permit markets for carbon dioxide (CO₂) emissions respond to short-run fluctuations in economic activity. To provide stability, both price and quantity interventions have been proposed. This paper focuses on the relative performance of fixed versus intensity allowances in the presence of both productivity and energy price uncertainty. Both instruments achieve the same steady-state emissions reduction target of 20 percent, which is similar to the current policy proposals, and the regulator then chooses the allowance policy that has the lowest expected abatement cost. A standard real business cycle (RBC) model is used to solve for the expected abatement cost under both policies. Expected cost outcomes are compared using data from the U.S. economy as the baseline scenario. Unlike previous studies, this paper’s results show that, under a reasonable model calibration, fixed allowances outperform intensity allowances by a cost difference of as much as 30 percent.

JEL classifications: E32, Q54, Q58
Keywords: Cap-and-Trade, Emissions intensity target, Energy price uncertainty, Expected abatement cost

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1. Introduction

Cap-and-trade schemes have stood at the forefront of policy options aiming at curbing anthropocentric greenhouse gas (GHG) emissions in the fight against climate change. These schemes fix the quantity of total emissions through permit issuance, and then enable trading among permit holders with the goal of achieving the least overall cost of abatement. Most notably, the European Union Emissions Trading Scheme (EU ETS) launched its first phase in 2005 and is currently entering its third phase in 2013. Fixing the amount of emissions years in advance, however, leads to uncertainty about the ultimate cost of abatement. During a period of economic expansion the cost of abatement may become higher than expected, whereas recessions reduce the demand for polluting goods, which in turn means lower than expected cost of abatement.¹

To contain the range of ex post abatement costs, some countries have adopted intensity-based targeting, with the most popular target being GDP. In this case, rather than fixing the quantity of emissions, the regulator fixes the GDP share of emissions, that is, the emissions intensity in the economy.² The idea is to allow the quantity of emissions to move together with GDP changes, thus relaxing the constraint when the economy is growing and tightening when slowing down. In the context of a cap-and-trade scheme, emissions intensity targeting would essentially mean adjusting the permit allowance periodically based on the level of economic activity, therefore resulting in more predictability in the permit price signal.³

A measure of economic activity such as GDP does not, however, carry all the relevant information related to firms’ cost of abatement. There are other factors and variables that can directly influence the ex post abatement cost, one example being the price of energy inputs. Consequently, intensity targeting based on GDP indexation will not fully remove all the

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¹ The most recent economic downturn provides a case in point: the functioning of the EU ETS has been highly criticized, with emission allowances trading well below their intended price. This type of wide variability in permit prices can undermine the credibility of the permit system, and it may also discourage investment in cleaner technologies and hinder cost efficiency (Zhao, 2003; Baldursson and von der Fehr, 2004). Additionally, uncertainty stemming from the EU Commission’s “backloading” plan has further exacerbated concerns over the system’s overall credibility (Grubb, 2012; Kossoy and Guigon, 2012).

² For example, the Bush administration targeted an emission intensity reduction of 18 percent. China has pledged to decrease the carbon intensity of its economy by 40-45 percent from the 2005 baseline. Similarly, India is aiming at decreasing its own carbon intensity by 20-25 percent.

³ In practice, this would entail periodical quota adjustments that followed a mechanical rule based on the given intensity target (Herzog, Baumert, and Pershing, 2006). Commitment and credibility may become important issues in such a policy design. We leave these questions for future research.
uncertainty vis-à-vis the cost of abatement. The question then becomes, under what circumstances will intensity-based permit allowances dominate fixed allowances, or will they always dominate.

The purpose of this paper is to compare the expected cost of abatement between two alternative policy scenarios: fixed-permit allowance versus intensity-based permit allowance. We evaluate how energy price and productivity uncertainties affect the relative performance of these two policy instruments. Following Fischer and Springborn (2011) and Heutel (2012), we use a simple real business cycle (RBC) model to simulate fluctuations in economic activity and policy outcomes, but we add stochastic energy prices as a new element. In contrast to their work, our main focus is on the first two moments of the endogenous permit price variable, which in the RBC model is captured by the shadow price of the emission constraint. We then introduce a simple decision framework where the regulator prefers the policy instrument that has the lowest expected abatement cost.\(^4\)\(^5\) The difference between the abatement costs under the alternative policies is fully conveyed in the first two moments of the permit price variable.

As in Fischer and Springborn (2011), we impose an exogenous 20 percent emission reduction target from the policy-free steady state.\(^6\) Both policy instruments, therefore, achieve the same long-run emissions reduction goal. Intensity targeting, however, entails more stringent emissions intensity in the steady state than in the case of a fixed quantity target. The reason is that, with an intensity target, there is an incentive to produce more in order to gain more permits and thus relax the constraint. Consequently, the expected value of the permit price will be higher under intensity targeting but its variance will be lower than under a fixed allowance.

The comparison between fixing the periodic quantity of emissions versus fixing the periodic emissions intensity boils down to the relative magnitude of the mean and variance of the permit price variable, which itself is the carrier of information about the relative cost of abatement. We start with a baseline model calibration and evaluate which policy dominates in

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\(^4\) Environmental damages are assumed to be the same under both policies since we are framing the emissions reduction target in terms of steady state reduction. Under intensity targeting, the amount of CO\(_2\) emissions does fluctuate in the short run, but since CO\(_2\) is a long-lived stock pollutant these short-term variations are not of significant importance.

\(^5\) Our analysis does not examine distributional effects of different policy instruments. An outcome with lower expected abatement cost, however, increases the potential for compensating the sectors and households that have been burdened the most under the emissions regulation scheme.

\(^6\) The 20 percent reduction resembles the EU reduction target and the target level proposed in the Waxman-Markey and Kerry-Lieberman Bills (Fischer and Springborn, 2011).
the baseline scenario in terms of expected abatement cost. We then proceed to find the critical
levels for persistence and variance of the energy and productivity processes after which intensity
allowance begins to dominate fixed allowances. We use both first order and second order
approximations when linearizing the model around its steady state since the approximation
method has an impact on the magnitudes of the first two moments of the permit price variable.7

Our results extend the analysis in Fischer and Springborn (2011) and provide further
insight into the comparison between the policy instruments. Unlike their study, we find that fixed
allowances may actually outperform intensity allowances in terms of expected abatement cost,
and the approximation method matters.8 When using second order approximation, the energy
price and productivity uncertainties also influence the expected permit price level. Our results
show that this has a more pronounced impact on the expected abatement cost under intensity
allowances.

Allowing for less-than-perfect correlation between the index of choice, here GDP, and
abatement costs has been studied before in Jotzo and Pezzey (2007). They derive a country-
specific optimal indexation rule using a static model of global permit trading and an endogenous
emissions reduction target. Our novel contribution is to identify energy price volatility as the
main source of exogenous noise that reduces the contemporaneous correlation between GDP and
abatement costs, and then use an RBC modeling framework to find the critical levels of
persistence and shock variance in the productivity and energy processes.9 These critical values
can then be compared to the historical values found in different countries.

The recent experiences in the EU ETS shed some light on how energy prices changes
transmit to permit markets. In general, energy prices tend to have a greater impact on energy-
intensive industries (Grubb, 2012). Periods of high energy prices curb demand for fuels and
electricity, and as a result, demand for permits also decreases, thus lowering the price of permits.
On the other hand, “coal-gas price differential” has had a clear impact on permit prices in the EU
ETS. Increases in oil prices have a tendency to drive natural gas prices higher, which in turn
increases the marginal cost of abatement as switching to natural gas becomes more expensive.

7 When using first order approximation, certainty equivalence holds.
8 Fischer and Springborn (2011) use standard welfare metrics to compare policy outcomes, and their analysis is
based on first order approximation.
9 For example, Hamilton (1983) finds that an increase in oil prices typically leads by 3-4 quarters a period of slower
output growth. Rotemberg and Woodford (1996) estimate that a 10 percent exogenous increase in the price of oil has
been followed by an output decline of 2.5 percent 5-6 quarters after the price shock.
Hence, permit price response can be positive to increases in oil prices. In our RBC model setup, higher energy prices always reduce demand for permits. We introduce an exogenous energy price process following the specification and estimation results in Kim and Loungani (1992) and Dhawan and Jeske (2008).

The plan of the paper is as follows. The next section provides a short literature review over the ongoing debate about the relative merits of fixed versus intensity targets. Section 3 lays out the real business cycle model that we use to derive the moments of the permit price process. Section 4 presents the derivation of expected abatement cost which the regulator uses when choosing between policy instruments. Section 5 defines the functional forms and parameter values used in the baseline case. Section 6 presents the results, and the last section offers concluding remarks and policy recommendations.

2. Intensity Targets

During the past decade, there has been growing interest in the relative merits of intensity-based regulation versus more traditional price instruments and fixed quotas. Proponents of intensity targeting argue for flexibility and lesser abatement cost uncertainty, which in turn result in more investments in cleaner technology (Herzog, Baumert and Pershing, 2006). Critics worry about the ensuing uncertainty over emissions reduction targets as the cap is allowed to vary, for example, with ups and downs in GDP. Pizer (2005) contends that, while intensity targets make sense in the context of framing reduction goals for developing countries, but that intensity targets with annual adjustments to emissions cap should not be viewed as a solution to abatement cost uncertainty. Kolstad (2005), on the other hand, reasons that intensity-based flexible caps can have potential in reducing abatement cost uncertainty. This in turn may have the beneficial effect of making international agreements in multilateral emission reductions more likely.

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10. Intensity targeting as a potential greenhouse gas (GHG) mitigation policy began receiving more attention in the academic and policy circles in the aftermath of the Kyoto Protocol’s ratification. For a review, see Peterson (2008). Herzog, Baumert and Pershing (2006) provide a lucid discussion of emission intensity targets and how they can be embedded in an emissions trading system together with absolute emissions targets.

11. Permit banking and permit price collars have also been suggested as policy measures to reduce abatement cost uncertainties (Newell and Pizer, 2003; Newell, Pizer and Zhang, 2005). Permit banking enables transferring of current unused emissions allowances for future use, and it may also allow borrowing future allowances for current use. These transfers would provide flexibility in intertemporal permit use and potentially lower abatement costs. Permit price collars in effect set price floors or price ceilings, or both, to reduce cost uncertainty. For example, if permit price were to go above the price ceiling, the regulator would increase the number of permits in the
The cost advantage of emissions indexation hinges on the premise that GDP and emissions levels are correlated. Jotzo (2006) and Heutel (2012) provide empirical evidence supporting this assumption. Jotzo and Pezzey (2007) and Wing, Ellerman and Song (2008) examine whether current cap-and-trade schemes can be improved upon in a way that reduces the inherent uncertainties of abatement costs. Their studies find that indexation can deliver improvements in abatement outcomes both in terms of cost uncertainty and emissions levels. Correspondingly, Quirion (2005) and Newell and Pizer (2008) set out to determine the conditions under which intensity targeting ought to be preferred over fixed targets, or vice versa, by choosing the policy that maximizes the expected net benefits from abatement. Following the analytical framework in Weitzman (1974), they find that indexed quantities are likely to perform better when there is a stronger positive correlation between the index and abatement cost uncertainty, with relatively small index variance as well. Furthermore, in the case of GHG reductions, they find that countries with a strong correlation between output and emissions, combined with relatively low output variance, may prefer indexation over fixed quantities.

Fischer and Springborn (2011) and Heutel (2012) are the first studies to compare the performance of different emissions regulation instruments using an RBC model that simulates random economic fluctuations. Whereas the former imposes an exogenous 20 percent emissions reduction target from a no-policy steady state scenario, the latter focuses on analyzing the optimal policy under endogenous reduction targets. In both papers, the source of business cycle uncertainty stems from serially correlated productivity shocks. Heutel (2012) finds that optimal policy allows for relaxation of the cap during economic expansion, and conversely, tightening of the cap during downturns. He also finds that indexing emissions to GDP replicates the optimal policy with a good approximation. Fischer and Springborn (2011), on the other hand, show that fixed caps lead to the lowest level of variability in all variables, except in the shadow price of the constraint, i.e., the permit price. Under intensity targeting, the magnitudes of variance terms do not differ from the no-policy baseline scenario (business-as-usual). Since intensity targeting marketplace to keep the price at the ceiling level. Our current analysis does not incorporate these policy measures, but we intend to examine them in our future research.

12 Jotzo and Pezzey (2007) use a single-period, stochastic, globally integrated partial equilibrium model to analyze the advantage of more flexible intensity targeting over fixed targets. Their model allows for varying degrees of indexation and endogenously determined target levels. Wing, Ellerman, and Song (2008) apply two criteria to compare the relative performance of fixed and intensity based emissions targets: first, how well each instrument preserves the initial expectations of the amount of emissions reductions and costs involved, and second, how well each instrument minimizes the volatility of the same variables due to uncertainty in future GDP and emissions.
absorbs the effects stemming from productivity shocks, the resulting permit price remains constant in their model. They also find that intensity targets outperform fixed caps when comparing steady state values using standard welfare based metrics. Overall, however, these differences are small, and the authors conclude that when deciding between policy instruments, the regulator may want to focus on other metrics instead.

3. Real Business Cycle Model

The basic RBC model presented in this section follows closely the presentation in Fischer and Springborn (2011). The representative consumer derives utility from consumption, $C_t$, and leisure, $h_t$, given by a standard utility function $U(C_t, h_t)$, and sells labor input, $L_t$, to a firm in a competitive labor market. By normalizing the total time endowment to one, we can write labor allocation as $L_t = 1 - h_t$. The representative firm uses capital, $K_t$, labor, $L_t$, and an intermediate polluting energy input, $e_t$, to produce output, $Y_t$. The economy’s total output (GDP) is defined as

$$Y_t = z_t F(K_t, L_t, e_t)$$

It is a product of a stochastic productivity term, $z_t$, which follows a stationary process with $E(z_t) = 1$, and a deterministic production function $F(K_t, L_t, e_t)$. Notice that we are abstracting from economic growth in our model.

Capital accumulates according to the equation

$$K_{t+1} = I_t + (1 - \delta)K_t$$

where the parameter $\delta$ is the rate of capital depreciation, and $I_t$ denotes investment. Following Kim and Loungani (1992) and Finn (2000), we assume that the firm takes the price of the polluting energy input as given. The economy’s resource constraint can therefore be written as

$$Y_t \geq C_t + I_t + p_t e_t$$

where $p_t$ is a stationary energy price process with $E(p_t) = 1$. The way the energy input enters the resource constraint in equation (3) can be interpreted to mean that the economy imports all of

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13 But the ordering is reversed when they account for the transition phase with elevated level of consumption under fixed caps.

14 Instead of a welfare comparison, we compare the expected cost of abatement.

15 We use specific functional forms in later sections.
its energy input from the world market and hence is small enough to have no influence on the price (Kim and Loungani, 1992).

Emissions, $M_t$, are proportional to the use of energy inputs and to minimize notation the units of emissions are chosen so that one unit of energy input emits one unit of emissions. Henceforth, variable $M_t$ denotes both energy use and emissions. To reduce the amount of emissions in the economy, the regulator imposes an emissions constraint:

$$M_t \leq T(Y_t)$$

where $T(Y_t)$ is called the allowance function that takes different forms under different policies.\(^{16}\) We focus on two cap-and-trade policies: i) intensity targeting with an adjustable allowance, and ii) fixed allowance. With fixed emissions allowance, $\bar{M}$, the allowance function is simply

$$T(Y_t) = \bar{M}$$

(5)

With intensity targeting, the regulator adjusts the amount of available permits based on the realized output in the current period. The emissions allowance function is hence defined as

$$T(Y_t) = sY_t$$

(6)

where $s$ denotes the allowed output share of emissions, or the emissions intensity.\(^{17}\) The regulator chooses $\bar{M}$ and $s$, depending on the policy, to achieve any given emissions reduction target.\(^{18}\)

Assuming competitive labor and capital markets, the social planner solves the following infinite horizon utility maximization problem:

$$\max_{C_{t+1},L_t} E_t \sum_{\tau=0}^{\infty} \beta^\tau U(C_t, 1 - L_t)$$

subject to the resource constraint (3) and the constraint on emissions (4). The parameter $\beta$ is the discount factor defined as $\beta = 1/(1 + r)$ where $r$ is the discount rate, and $E_t$ denotes the

\(^{16}\) The only way to reduce emissions here is through the reduction of polluting energy input used in the economy. This feature neglects the possibility of improving energy efficiency and abatement technology. Our model does, however, allow for substitution between the energy input and labor and capital. Notice that these inputs are not perfect substitutes. We discuss the roles of energy intensity and fuel mix in more detail in the model specification section.

\(^{17}\) We could also have alternative specifications for the intensity based allowance function. For instance, the regulator could choose the current period’s permit allowance in the previous period given his expectation of the level of output: $T(Y_t) = s E_{t-1}(Y_t)$.

\(^{18}\) This target level is also related to the environmental damages since the use of polluting input incurs damages. In this paper, we use an exogenous target of 20 percent reduction in steady state emissions.
expectation operator given the information available at time $t$. Define $\hat{\mu}_t = \mu_t / \lambda_t$ where $\mu_t$ is the Lagrangian multiplier of constraint (4), and $\lambda_t$ of constraint (3). The system of first order conditions for the social planner’s problem can then be written as:

$$

tF_L(K_t, L_t, M_t)(1 + \hat{\mu}_t T_y) = -\frac{U_L}{U_C} \\
tF_K(K_t, L_t, M_t)(1 + \hat{\mu}_t T_y) = \beta^{-1} E_t \left( \frac{U_{C,t}}{U_{C,t+1}} + \delta - 1 \right) \\
tF_M(K_t, L_t, M_t)(1 + \hat{\mu}_t T_y) = p_t + \hat{\mu}_t \\
tF(K_t, L_t, M_t) = K_{t+1} - (1 - \delta) K_t + C_t + p_t M_t \\
M_t = T(Y_t)
$$

(8)

where functions with subscripts denote the derivative of the function with respect to the variable in the subscript. The above system of nonlinear equations characterizes the equilibrium relationships that have to hold in optimum in every period, $t$. Assuming the existence of a steady state in system (8), it is implicitly defined for each variable by the following system:

$$
F_L(K, L, M)(1 + \hat{\mu} T_y) = -\frac{U_L}{U_C} \\
F_K(K, L, M)(1 + \hat{\mu} T_y) = \beta^{-1} \delta \\
F_M(K, L, M)(1 + \hat{\mu} T_y) = 1 + \hat{\mu} \\
F(K, L, M) = \delta K + C + M \\
M = T(Y)
$$

(9)

where all the variables take steady state values (e.g., $K_{t+1} = K_t = K$). Notice that the only difference between the system in (8) and the corresponding conditions in Fischer and Springborn (2011) is the introduction of random energy prices, $p_t$. Since the expected value of $p_t$ is one, the steady state conditions are identical with their study.

For the purpose of this study, the properties of the variable $\hat{\mu}_t$, the shadow value of the emissions constraint, are of primary interest. This variable is the equilibrium permit price in a competitive permit market. Notice that it has been written in terms of output, that is, it is the real price of a permit. Our next goal is to log-linearize the above system (8) around its steady state given in (9), and to solve for the equations of motion that characterize the movement of the endogenous and state variables in the proximity of that steady state (e.g., Uhlig, 1995). For

19 By specifying functional forms for $U(C, h)$ and $F(K, L, M)$, we can explicitly solve for each steady state value. We do this in our simulation section.
20 In reality, the permit markets may not be competitive since some market participants may have market power. Since market structure is assumed to be the same under both policies, this may not have an effect on our abatement cost comparisons. We leave the determination of the effect of market structure for future research.
example, the logarithm of the permit price variable, $\log \mu_t$, is written as a linear function of the exogenous variables $\log p_t$ and $\log z_t$, and the predetermined variable $\log K_t$. Before solving the above RBC model under the two alternative allowance policies, we need to answer the question of how the regulator determines which permit system is better. To do this, we build a decision framework based on the abatement cost minimization problem. As we will see, information about the difference in abatement cost between the two policies is captured by the first two moments of the permit price variable, $\mu_t$.

4. Abatement Cost Comparison

Suppose that the regulator’s goal is to achieve a given emissions reduction target with the smallest expected abatement cost which is measured as the cost of deviating from the business-as-usual (BAU) scenario. The reduction target is exogenously given, for example, as a result of an international agreement or domestic legislation. Due to the global nature of GHG stock, any benefit from the emissions reduction is fully exogenous from the perspective of the regulator, and furthermore, these benefits are the same under both policy instruments.

Following the standard practice in the literature (Weitzman, 1974; Newell and Pizer, 2003; Newell and Pizer, 2008; Fell, MacKenzie, and Pizer, 2012), we use a quadratic approximation for the ex-post emissions input cost function around the expected BAU emissions level, $E_0(M_t^{BAU})$. This can be written as

$$C(M_t; \theta_t) = \theta_t(M_t - \bar{M}) + \frac{b}{2}(M_t - \bar{M})^2$$  \hspace{1cm} (10)

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21 Detailed analytical examples and derivations are available from the authors upon request.
22 We could use the above RBC modeling framework to compare the welfare difference between the two policies in terms of consumption. The welfare differences are, however, small (Fischer and Springborn, 2011). It is more likely that the regulator is interested in knowing the abatement cost difference rather than the welfare difference.
23 We define this in more detail below.
24 In the next section, we define the reduction target in terms of the policy-free steady state.
25 This corresponds to a setting such as the Kyoto Protocol where the participants have agreed to reduce their emissions by a certain percentage amount from a given baseline year within the next 10 to 20 years. Most of the recent unilateral pledges from, e.g., China, Mexico and Australia, similarly target a certain amount of reduction within the next 20 years. Since only the global atmospheric concentration of CO$_2$ is what matters, the decision of any single country should be irrelevant to the overall global benefits.
26 Alternatively, we could just assume that any benefits under the two policies are equivalent, constant, and fully known. Since carbon dioxide is a slowly decaying stock pollutant, the annual variation in emissions around its target level during the regulation period has no impact on final benefits.
27 This can be thought of as the energy input cost function, but since we are using emissions and energy interchangeably, we retain the emissions cost function terminology.
where we have defined $\bar{M} \equiv E_0(M_{t}^{BAU})$ to reduce notational clutter. Randomness in emissions cost is captured by the mean zero random variable $\theta_t$ with $E(\theta_t^2) = \sigma^2$. Parameter $b > 0$ together with $\theta_t$ determines the curvature properties of the cost function.$^{28}$ Figure 1 depicts the expected (ex ante) form of (10), whereas Figure 2 shows multiple realizations of (10) for different values of $\theta_t$.

4.1 No Policy Scenario (BAU)

The ex-post marginal cost of emissions is defined as

$$-C'(M_{t}^{BAU}; \theta_t) = -\theta_t - b(M_{t}^{BAU} - \bar{M}) \quad (11)$$

In the absence of regulation, the optimal emissions level, $M_{t}^{BAU}$, is given by

$$M_{t}^{BAU} = \bar{M} - \frac{1}{b} \theta_t \quad (12)$$

where we have used the first order condition $-C'(M_t; \theta_t) = 0$ to derive (12). In Figure 2, the points where the ex post cost curves attain their minima correspond to $M_{t}^{BAU}$ for different realizations of $\theta_t$. Any deviation from point $M_{t}^{BAU}$ would be suboptimal. We define the cost of abatement as the inefficiency cost resulting from choosing some emissions level $M_{t}'$ such that $M_{t}' < M_{t}^{BAU}$. The abatement cost can be written as

$$AC(M_{t}', M_{t}^{BAU}; \theta_t) = C(M_{t}'; \theta_t) - C(M_{t}^{BAU}; \theta_t) \quad (13)$$

Here, the BAU level, $M_{t}^{BAU}$, acts as the reference level.

4.2 Abatement Cost under Permit Allowances

Any allowance policy put in place will in general make the ex post emissions level, $M_t$, deviate from the cost minimizing outcome: $M_t \leq M_{t}^{BAU}$. Notice that under fixed allowance, $M_t = \bar{M}$, whereas $M_{t}^{BAU}$ can fluctuate periodically, but under intensity allowances, both $M_t$ and $M_{t}^{BAU}$ fluctuate periodically. Suppose now that the regulator uses emissions allowances to restrict the amount of emissions in the economy, $M_t < \bar{M}^{BAU}$, and enables permit trading between firms. Assuming that the permit market is competitive and efficient, the representative firm chooses its level of emissions to equate the marginal cost of emissions to the market permit price, $\hat{\mu}_t$.

$^{28}$ We assume that parameter $b$ has a value that is not too small, thus guaranteeing an interior solution to the cost minimization problem.
\[ \hat{\mu}_t = -C'_t(M_t; \theta_t) \]  

(14)

Using (14), we can find the optimal emissions level, \( M_t^* \), under either allowance scheme:

\[ M_t^* = \bar{M} - \frac{\theta_t + \hat{\mu}_t}{b} \]  

(15)

The ex post amount of abatement is defined as

\[ M_t^{BAU} - M_t^* = \frac{1}{b} \hat{\mu}_t \]  

(16)

Notice that this amount is always positive as long as the permit price is positive as well. We restrict \( \hat{\mu}_t \) to only take positive values in our RBC model.\(^{29}\) Using (16), we can now rewrite the abatement cost function in (13) as

\[ AC(M_t^*, M_t^{BAU}; \theta_t) = \frac{1}{2b} \hat{\mu}_t^2 \]  

(17)

The expected cost of abatement then becomes

\[ E_0[AC(M_t^*, M_t^{BAU}; \theta_t)] = \frac{1}{2b} [E_0(\hat{\mu}_t)^2 + Var(\hat{\mu}_t)] \]  

(18)

The above expression is written only in terms of the expectation and the variance of the permit market price, \( \hat{\mu}_t \), and parameter \( b \).

From (18), we can see that the regulator prefers the allowance policy that has the smallest sum of the permit price variance and the squared mean permit price. One interpretation for this rule is that the regulator chooses the instrument that minimizes the mean squared error of abatement cost. This “error” can be thought of arising from having an emissions reduction target put in place. In the absence of emissions regulation, the permit price would naturally be invariantly zero under both policies, and therefore, the mean squared error would be at minimum. Because of the reduction target, however, the regulator chooses the instrument that yields the smallest mean squared error.\(^{30}\) As we will see, the expected permit price under intensity allowance is always higher than with fixed allowance, but the variance is lower under

\(^{29}\) Positive permit price values can be justified by the notion of option value. Since in the steady state the amount of emissions reduction is always below the BAU level, the permit prices retain positive value in the short run.

\(^{30}\) Quadratic utility function yields a decision rule that minimizes mean squared error.
intensity allowance. Hence, what matters is the relative magnitude of the first two moments of the permit variable under each policy.

4.3 Cost Comparison

The difference in the expected abatement cost between intensity and fixed allowances is defined as

$$\Delta_{F-I} \equiv E_0 \left( AC_F \left( M_{t,F}^*, M_{t,BAU}^* ; \theta_t \right) \right) - E_0 \left( AC_I \left( M_{t,I}^*, M_{t,BAU}^* ; \theta_t \right) \right)$$

(19)

where subscript $I$ stands for intensity allowance and subscript $F$ for fixed allowance. Substituting (18) into (19) and assuming that all the model parameters are invariant between different policies, that is, only the amount of abatement may differ, we finally have

$$\Delta_{F-I} \equiv \frac{1}{2b} \left[ E_0 (\hat{\mu}_t^F)^2 + Var(\hat{\mu}_t^F) - E_0 (\hat{\mu}_t^I)^2 - Var(\hat{\mu}_t^I) \right]$$

(20)

If the expression in (20) is positive, then intensity allowance is preferred, whereas if the expression negative, then fixed allowance is preferred. To empirically assess which policy is superior, we would need to have estimates for the mean and the variance of the permit price under both policies. Since there are no data on intensity allowances, we resort to solving for these moments using the above RBC model.31

5. Model Specification

In selecting functional forms, we strictly follow Fischer and Springborn (2011) and Kim and Loungani (1992). The representative consumer’s utility function takes the following logarithmic form:

$$U(C_t, L_t) = \log C_t + \omega \log (1 - L_t)$$

(21)

where parameter $\omega$ is chosen so that the labor allocation decision matches the one found in the data. The production function specification is the standard constant returns to scale Cobb-Douglas (CD) production function:

$$F(K_t, L_t, M_t) = K_t^\alpha M_t^\nu L_t^{1-\alpha-\gamma}$$

(22)

31 Notice that we do not model the transitioning phase between the policy-free steady state and the resulting new steady state. We focus on comparing the expected costs when the policies are fully in place.
The parameter \( \gamma \) denotes the fossil fuel energy intensity of the economy.\(^{32}\) The productivity variable, \( z_t \), has an expected value of one and it follows a stationary AR-1 process:

\[
\log z_t = \eta \log z_{t-1} + \varepsilon_{z,t}
\]

The stochastic term, \( \varepsilon_{z,t} \), has a mean of zero and is normally distributed, \( \varepsilon_{z,t} \sim N(0, \sigma_z^2) \). The energy price variable, \( p_t \), follows an exogenous ARMA(1,1) process as in Kim and Loungani (1992) and Dhawan and Jeske (2008):

\[
\log p_t = \pi_1 \log p_{t-1} + \varepsilon_{p,t} + \pi_2 \varepsilon_{p,t-1}
\]

The expected price of energy input is one and \( \varepsilon_{p,t} \sim N(0, \sigma_p^2) \).

In order to derive numerical results, we use parameter values from previous studies to calibrate any remaining parameters. For the energy price process, we use the same set of parameter values as Dhawan and Jeske (2008). They use a GDP deflator to deflate the energy price index using quarterly data from 1970 to 2005, and then use these data to estimate an ARMA process. The AR(1) term, \( \eta \), takes the standard value that is frequently used in other RBC studies as well (e.g., Fischer and Springborn 2011). Finally, the parameters in the CD production function are the same as in Fischer and Springborn (2011) and apply only to the U.S. economy.\(^{33}\) Table 1 collects the above and all the remaining parameter values used in the baseline simulation.

---

\(^{32}\) Herzog, Baumert, and Pershing (2006) define an economy’s carbon intensity as follows:

\[
\frac{CO_2}{GDP} = \frac{Energy}{GDP} \times \frac{CO_2}{Energy}
\]

The first term on the right-hand side captures the economy’s energy intensity and the second term captures the economy’s fuel mix. Our current model specification in effect assumes that energy intensity is defined in terms of fossil fuels (parameter \( \gamma \)) and hence the fuel mix is simply equal to one. For example, parameter \( \gamma \) would be smaller in an economy that produced most of its energy with nuclear power than in an economy that was more reliant on fossil fuels.

\(^{33}\) Generally speaking, lower values of \( \gamma \), ceteris paribus, decrease the expected cost of abatement since the economy is then less reliant on the fossil fuel energy input. Our current baseline parameterization applies only to the U.S. case, but it will be an interesting future extension to examine how more general specification of energy intensities and fuel mixes affects our simulation results.
6. Results and Discussion

6.1 Results

In this section, we derive the first two moments of the permit price variable under the two competing policies and then compute the difference in expected abatement cost. Before doing so, we need to determine the steady-state level of emissions in a policy-free scenario, $M_{NP}$. We then impose a 20 percent reduction target from this steady-state emissions level. In the case of fixed allowance, the resulting allowance function can be written as:

$$T(Y_t) = \bar{M} = 0.8M_{NP}$$

(25)

In the case of intensity allowance, we have to additionally solve for the intensity target $s^*$ that guarantees the same 20 percent emissions reduction from the policy-free steady state. The allowance function becomes:

$$T(Y_t) = s^* Y_I(s^*) = 0.8M_{NP}$$

(26)

where $Y_I(s^*)$ denotes the resulting steady state level of output under the intensity target. Notice that $Y_I(s^*)$ depends on $s^*$.

To achieve the same 20 percent reduction, the intensity target $s^*$ has to be set stricter than what the corresponding steady state intensity is under a fixed quota (Fischer and Springborn, 2011). This occurs because the intensity-based allowance induces the representative firm to produce more in order to receive more permits, hence the need for a stricter intensity in the steady state. This effect can be seen, for example, by comparing the first order conditions under the alternative policies:

$$z_t F_M (K_t, L_t, M_t) = p_t + \hat{\mu}_t$$

$$z_t F_M (K_t, L_t, M_t) = \frac{1}{(1 + \hat{\mu}_t s)}(p_t + \hat{\mu}_t)$$

(27)

The last line reveals how the intensity allowance in effect reduces the marginal cost of using the polluting input in comparison to the fixed allowance policy. By inspecting the first order conditions for intensity allowances, we can see that the “mark-up” term $1/(1 + \hat{\mu}_t s)$ enters all of the optimality conditions.34

34 Notice that the equilibrium permit price can be negative in (27). By using logarithmic transformation when solving our RBC model, we are able to circumvent this problem. In effect, we restrict each variable to take only
Table 2 presents steady states of each variable under three different scenarios: no policy (BAU), intensity allowance, and fixed allowance. These results are similar to the ones in Fischer and Springborn (2011) since in steady state, the energy price variable takes value one and this corresponds to the specification in their study. Both policies achieve the same 20 percent steady state reduction in the polluting good, which can be verified by comparing the values of $M$. Under intensity allowances, the permit price variable has a higher steady state value, and also higher levels of output, consumption, and capital in the steady state. Emissions intensity, $s$, is lower (stricter) in the case of the intensity allowance.

Next we solve for the equations of motion using a first order Taylor approximation around the steady state. The equations of motion for the state and endogenous variables take the following vector forms:

$$
k_{t+1} = P_i k_t + Q_i w_t
$$

$$
v_t = R_i k_t + S_i w_t
$$

where

$$w_{t+1} = J_i w_t + N_i \psi_{t+1}$$

for $i = F, I$. All the variables are written in terms of deviations from the steady state. Given the specifications for the productivity and energy price processes, the two vectors $w_t, \psi_t$ are defined as:

$$w_t = (\log z_{t-1}, \log p_{t-1}, \epsilon_{p,t-1})'
$$

$$\psi_t = (\epsilon_{z,t}, \epsilon_{p,t})'
$$

Table 3 presents solutions to the equations of motion in (28) for two variables: permit price, $\hat{\mu}_t$, and the only state variable, $K_t$, both in logarithms. As can be seen from the equation for the permit price variable, intensity allowance fully absorbs the random variation coming from the productivity shock, and also any effect coming from the state variable, $k_t$. Energy price shocks are transmitted to the permit price under both policies but the coefficients differ. The permit price variable under intensity allowances is less responsive to energy price fluctuations, positive values, and therefore, the emissions constraint always binds with positive shadow value. This can be justified by noting that permits have a positive option value due to the steady state reduction target.

35 The main difference between the steady state values in our study and theirs is that we have calibrated the parameter $\omega$ in the utility function so that the steady state labor allocation is about 1/3 of total time endowment.

36 We use Dynare 4.3.3 (Adjemian et al., 2013) to derive these and all subsequent numerical results.
which can be seen by comparing the coefficients. The constant terms denote the steady states in logs. In Figure 3, we graph an example of a permit price process realization using the equations of motion in Table 3.

To gain further insight, the equation of motion for the permit price variable can be written as

\[
\log \hat{\mu}_t = \bar{a}_0 + \bar{a}_1 k_t + \bar{a}_2 \log z_{t-1} + \bar{a}_3 \log p_{t-1} + \bar{a}_4 \varepsilon_{p,t-1} + \bar{a}_5 \varepsilon_{z,t} + \bar{a}_6 \varepsilon_{p,t} \tag{30}
\]

where the coefficients correspond to the solutions in Table 3. Notice that all the variables on the right-hand side are normally distributed. Hence, \( \log \hat{\mu}_t \) has a normal distribution with its mean at the steady state value, \( a_0 \).\(^{37} \) Consequently, \( \hat{\mu}_t \) has a log-normal distribution. Once we solve for the first two moments of \( \log \hat{\mu}_t \) using equation (30), we can then derive the corresponding moments in levels using the following relationships:

\[
\log E(\hat{\mu}_t) = E(\log \hat{\mu}_t) - \frac{1}{2} Var(\log \hat{\mu}_t)
\]

\[
Var(\hat{\mu}_t) = \left[ \exp \left( Var(\log \hat{\mu}_t) \right) - 1 \right] * \exp \left( 2E(\log \hat{\mu}_t) + Var(\log \hat{\mu}_t) \right) \tag{31}
\]

Tables 4 and 5 present the means and variances of the permit price variables under the two policy instruments. We use both the first and the second order Taylor approximation around the steady state. Table 4 reports these values in logarithmic scale, and Table 5 reports the values in levels after applying the mappings in (31). We furthermore compute the difference in expected abatement cost in percentage terms, hence cancelling out the parameter \( b \) in (20):

\[
\frac{\Delta F - I}{\Delta F} * 100\% = \left[ \frac{E_0(\hat{\mu}_F^F)^2 + Var(\hat{\mu}_F^{I}) - E_0(\hat{\mu}_I^F)^2 - Var(\hat{\mu}_I^{I})}{E_0(\hat{\mu}_F^F)^2 + Var(\hat{\mu}_F^{F})} \right] * 100\% \tag{32}
\]

We use equation (32) to compare the allowance policies and make policy recommendations. The negative percentage value in (32) means that fixed allowance entails lower abatement cost by the corresponding percentage amount, and positive value means that intensity allowance is more cost efficient again by that percentage amount. In Table 4, we have not computed (32), since when squaring a negative log value the ranking is reversed and thus the abatement cost comparison becomes meaningless. As Table 4 shows, the second order approximation method yields smaller

\(^{37} \) The mean coincides with the steady state only when using a first order Taylor approximation.
mean values for both policy instruments. Variance terms are, however, the same with both approximation methods.

Based on the results in Table 5, we conclude that fixed allowances dominate the intensity based allowance system. Fixed allowance has an 8.76 percent lower expected abatement cost with first order approximation and almost 30 percent lower with second order approximation. The intuitive explanation for these results is that the mean permit price is always higher under the intensity allowance and the reduction in permit variance is not enough to compensate for the weight given to the higher mean value in (32). Since the sign of the expression in (32) depends on the relative magnitudes of the first and the second moments of the permit price variable, it is evident that the approximation method will affect the results. When using the second order approximation the mean of the permit price does not coincide with the steady state value due to the presence of cross derivatives. In Table 5, mean values with second order approximation are again smaller than with first order approximation. Interestingly, however, permit price variance is now higher under intensity allowance when using second order approximation. This result is somewhat surprising as we would expect intensity allowance to always have lower variance. By inspecting Table 4, we can see that in the logarithmic form, the ranking is as to be expected with both approximations. Hence the reversal of the ranking is due to the mapping in (31). First order approximation preserves the ranking based on variance even after applying (31).

6.2 Critical Values

Next we examine how the expression in (32) changes as we vary the parameter values that define the persistence and variance of the exogenous stochastic processes. We change each parameter in turn and keep the remaining parameters at their baseline values given in Table 1. We also continue applying the mapping in (31). Figure 4 shows how the expression in (32) changes, with both first and second order approximations, as the standard deviation of the energy price shock, $\sigma_p$, increases. The gap between fixed and intensity allowance diminishes, and when using the first order approximation, there is a switch point at 0.035. As the standard deviation increases further outside the range shown in the figure, intensity allowance starts to dominate with the second order approximation as well. The explanation for these findings is that, as $\sigma_p$ increases, the permit price variance increases much more under the fixed allowance than under the intensity

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38 The standard deviation estimate in Dhawan and Jeske (2008) is 0.031.
allowance, thus ultimately overturning the advantage of having a lower mean permit price under the fixed allowance.

Figures 5 and 6 tell similar stories vis-à-vis varying the energy price ARMA parameters $\pi_1$ and $\pi_2$. In effect, increasing these persistence terms increases the energy price variance, which ultimately results in the regulator preferring the intensity allowance over the fixed allowance. In Figures 5 and 6, the cutoff points, when using the first order approximation, are slightly above 0.98 for the AR part, $\pi_1$, and around 0.64 for the MA part, $\pi_2$. Figures 7 and 8 show the effect of changing the parameter values defining the productivity process. The higher standard deviation, $\sigma_z$, and persistence, $\eta$, reduce the gap between the policies, and the closer we get to the productivity process being a pure random walk the more preferred the intensity allowance becomes. The cutoff point for $\sigma_z$ is around 0.01, and for $\eta$ around 0.975 when using the first order approximation.

7. Conclusions
Recent studies comparing intensity-based emissions targeting and fixed emissions quotas have usually come out in favor of the former. It is true that flexibility in allowance allocation does reduce abatement cost uncertainty, but this comes at the cost of a more stringent intensity level to achieve the same reduction target. Our analysis weighs both of these aspects to determine which allowance policy is preferred. The above results show that when using a reasonable model calibration, fixed allowances outperform intensity allowances. Depending on the approximation method, the cost difference in percentage terms can be quite significant. We find that with second order approximation the abatement cost difference is 30 percent in favor of fixed allowances. With first order approximation, this gap is almost 9 percent in favor of fixed allowances. Our simulation results also show that as the economic environment becomes more uncertain, intensity allowances begin to dominate fixed allowances. Hence, in some cases where an economy faces large energy price and productivity uncertainties, intensity allowances may ultimately become the preferred policy option.

Intensity-based targeting, however, constitutes only one policy option among many current policy proposals that aim at reducing the abatement cost uncertainties of emissions permit schemes. Other such policies include carbon taxes, permit banking, permit price collars, and carbon offsets. Our future research aims at extending the above analytical framework to
incorporate this richer set of policy alternatives. Other possible extensions include examination of interaction effects between existing distortionary policies and permit trading. In a more nuanced general equilibrium analysis with other distortionary taxes, substitution possibilities may render environmental regulation such as carbon taxes and tradable permits less cost effective (Goulder et al., 1999). By decreasing other distortionary taxes, such as labor taxes, through recycling of the revenue from environmental regulation the government may be able to reduce the cost of abatement.

Our current analysis focuses on comparing the possible abatement cost outcomes under the same stringency target of 20 percent emissions reduction. The degree of stringency may, however, have interesting consequences. On the one hand, a more stringent target translates into a higher expected cost of abatement, and on the other hand, it also means higher variability in the permit price processes. Our future plan is to examine which of these effects dominates and whether the degree of stringency has an effect on the cost comparison outcomes. In addition, a more nuanced description and modeling of energy intensity and fuel mix possibilities may provide important insights. This would also enable the examination of other policies and their interaction with the economy-wide emissions cap. For example, energy efficiency standards and renewable mandates are likely to lower the energy intensity parameter $\gamma$ in the production function, hence resulting in a lower cost of abatement in terms of lost output. Pre-existing green energy subsidies funded through lump sum transfers, on the other hand, may increase the relative price of fossil fuels inputs. Even when assuming that green subsidies are equivalent under both intensity and fixed allowance schemes, the presence of that policy may have an interesting effect on the cost comparison results. The reason is that permit prices responds differently to changes in fossil fuel prices under the two policies. These topics, however, warrant further research.
References


Figure 1. Ex Ante Abatement Cost Curve

\[ \mathcal{M}_t \]

\[ \mathcal{M}^{BAU} \]

\[ E_0 \left( C(M_t, M_t^{BAU}) \right) \]

Figure 2. Ex Post Abatement Cost Curve

\[ \mathcal{M}^{BAU} \]
Figure 1. Permit Price Simulation
Figure 2. Abatement Cost Difference, Energy Price Shock Standard Deviation

1ST ORDER TAYLOR APPROXIMATION

2ND ORDER TAYLOR APPROXIMATION

x-axis: standard deviation, $\sigma_p$
y-axis: %-difference between fixed and intensity based allowance. Negative value means that fixed allowances are preferred.
Figure 3. Abatement Cost Difference, Energy Price AR(1) Persistence Term

1ST ORDER TAYLOR APPROXIMATION

2ND ORDER TAYLOR APPROXIMATION

x-axis: AR(1) term, $\pi_t$

y-axis: %-%difference between fixed and intensity based allowance. Negative value means that fixed allowances are preferred.
Figure 4. Abatement Cost Difference, Energy Price MA(1) Persistence Term

\begin{figure}
\centering
\begin{tikzpicture}
\begin{axis}[
    title={1ST ORDER TAYLOR APPROXIMATION},
    xlabel={MA(1) term},
    ylabel={%},
    xmin=0.25, xmax=0.7,
    ymin=-20, ymax=5,
    xtick={0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7},
    ytick={-20, -15, -10, -5, 0, 5},
    grid=both,
]
\addplot[blue, thick] coordinates {
    (0.25, -15)
    (0.3, -10)
    (0.35, -5)
    (0.4, 0)
    (0.45, 5)
    (0.5, 10)
    (0.55, 15)
    (0.6, 20)
    (0.65, 25)
    (0.7, 30)
};
\end{axis}
\end{tikzpicture}
\begin{tikzpicture}
\begin{axis}[
    title={2ND ORDER TAYLOR APPROXIMATION},
    xlabel={MA(1) term},
    ylabel={%},
    xmin=0.25, xmax=0.7,
    ymin=-35, ymax=-20,
    xtick={0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7},
    ytick={-35, -30, -25, -20},
    grid=both,
]
\addplot[blue, thick] coordinates {
    (0.25, -30)
    (0.3, -25)
    (0.35, -20)
    (0.4, -15)
    (0.45, -10)
    (0.5, -5)
    (0.55, 0)
    (0.6, 5)
    (0.65, 10)
    (0.7, 15)
};
\end{axis}
\end{tikzpicture}
\end{figure}

- $x$-axis: MA(1) term, $\pi_2$; the range is chosen around the value reported in Dhawal and Jeske (2005).
- $y$-axis: %-difference between fixed and intensity based allowance. Negative value means that fixed allowances are preferred.
Figure 5. Abatement Cost Difference, Productivity AR(1) Persistence Term

$x$-axis: AR(1) term, $\eta$. We do not compute the value for $\eta = 1$. The closer we get to the productivity process being a random walk, the more preferred intensity allowances become using both first and second approximations.

$y$-axis: %-difference between fixed and intensity based allowance. Negative value means that fixed allowances are preferred.
Figure 6. Abatement Cost Difference, Productivity Shock Standard Deviation

Table 1. Baseline Calibration

<table>
<thead>
<tr>
<th></th>
<th>β</th>
<th>δ</th>
<th>ω</th>
<th>η</th>
<th>σ_2^2</th>
<th>σ_p^2</th>
<th>π₁</th>
<th>π₂</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>0.990</td>
<td>0.025</td>
<td>1.72</td>
<td>0.950</td>
<td>0.007^2</td>
<td>0.031^2</td>
<td>0.975</td>
<td>0.422</td>
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<table>
<thead>
<tr>
<th></th>
<th>γ</th>
<th>α</th>
<th>1−γ−α</th>
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<tbody>
<tr>
<td>0.090</td>
<td>0.330</td>
<td>0.580</td>
<td></td>
</tr>
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Table 2. Steady State Values in Levels

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\mu}$</th>
<th>$K$</th>
<th>$C$</th>
<th>$M$</th>
<th>$L$</th>
<th>$Y$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Policy, BAU</td>
<td>0</td>
<td>7.714</td>
<td>0.554</td>
<td>0.074</td>
<td>0.333</td>
<td>0.821</td>
<td>0.090</td>
</tr>
<tr>
<td>Fixed Allowance</td>
<td>0.198</td>
<td>7.392</td>
<td>0.542</td>
<td>0.059</td>
<td>0.328</td>
<td>0.786</td>
<td>0.075</td>
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<tr>
<td>Intensity Allowance</td>
<td>0.246</td>
<td>7.689</td>
<td>0.552</td>
<td>0.059</td>
<td>0.333</td>
<td>0.803</td>
<td>0.074</td>
</tr>
</tbody>
</table>

Table 3. Equations of Motion

\[
\begin{bmatrix}
\log K_{t+1}^F \\
\log K_{t+1}^L
\end{bmatrix} =
\begin{bmatrix}
2.000 \\
2.040
\end{bmatrix} +
\begin{bmatrix}
0.952 & 0.106 & -0.0003 & -0.0001 \\
0.954 & 0.117 & -0.0076 & -0.0033
\end{bmatrix} k_t
+ \begin{bmatrix}
0.112 & 0.003 \\
0.124 & -0.006
\end{bmatrix} \psi_t
+ \begin{bmatrix}
0.106 & 0.117 & -0.0003 & -0.0001 \\
0.117 & -0.0076 & -0.0033 & -0.0076
\end{bmatrix} w_t
\]

\[
\begin{bmatrix}
\log \hat{\mu}_{t+1}^F \\
\log \hat{\mu}_{t+1}^L
\end{bmatrix} =
\begin{bmatrix}
-1.620 \\
-1.403
\end{bmatrix} +
\begin{bmatrix}
1.075 & 7.798 & -4.721 & -2.041 \\
0 & 0 & -4.357 & -1.884
\end{bmatrix} k_t
+ \begin{bmatrix}
8.208 & -4.757 \\
0 & -4.467
\end{bmatrix} \psi_t
+ \begin{bmatrix}
1.075 & 7.798 & -4.721 & -2.041 \\
0 & 0 & -4.357 & -1.884
\end{bmatrix} w_t
\]

Table 4. Baseline Results in Logs

<table>
<thead>
<tr>
<th></th>
<th>First order approximation</th>
<th>Second order approximation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_0(\hat{\mu}_t^F)$</td>
<td>-1.620</td>
<td>-2.186</td>
</tr>
<tr>
<td>$E_0(\hat{\mu}_t^L)$</td>
<td>-1.403</td>
<td>-1.880</td>
</tr>
<tr>
<td>$\text{Var}(\hat{\mu}_t^F)$</td>
<td>0.957</td>
<td>0.957</td>
</tr>
<tr>
<td>$\text{Var}(\hat{\mu}_t^L)$</td>
<td>0.782</td>
<td>0.782</td>
</tr>
<tr>
<td>$\frac{\Delta F-I}{\Delta F} \times 100%$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: The results are in logarithmic scale. Notice that we cannot determine which instrument has a lower abatement cost because taking a square of a negative number results in reversing the ranking between higher and lower expected permit price.
### Table 5. Baseline Results in Levels

<table>
<thead>
<tr>
<th></th>
<th>First order approximation</th>
<th>Second order approximation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_0(\hat{\mu}_F)$</td>
<td>0.319</td>
<td>0.181</td>
</tr>
<tr>
<td>$E_0(\hat{\mu}_I)$</td>
<td>0.364</td>
<td>0.226</td>
</tr>
<tr>
<td>$Var(\hat{\mu}_F)$</td>
<td>0.164</td>
<td>0.053</td>
</tr>
<tr>
<td>$Var(\hat{\mu}_I)$</td>
<td>0.157</td>
<td>0.060</td>
</tr>
<tr>
<td>$\frac{\Delta F - I}{\Delta F} \times 100%$</td>
<td>-8.76</td>
<td>-30.0</td>
</tr>
</tbody>
</table>

*Note: Baseline calibration. Mean and variance of permit price variable under intensity ($I$) and fixed ($F$) allowance are reported in levels. Negative value for expected abatement cost difference means that fixed allowance has lower expected abatement cost.*