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Working Paper
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Research Papers in Economics, No. 5/13

Provided in Cooperation with:
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Suggested Citation: Lorenz, Normann (2013) : Adverse selection and risk adjustment under imperfect competition, Research Papers in Economics, No. 5/13, Fachbereich IV - Volkswirtschaftslehre, Universität Trier, Trier

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Adverse selection and risk adjustment under imperfect competition

Normann Lorenz
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Normann Lorenz*

September 14, 2013

Abstract

This paper analyzes the distortions of health insurers’ benefit packages due to adverse selection when there is imperfect competition. Within a discrete choice setting with two risk types, the following main results are derived: For intermediate levels of competition, the benefit packages of both risk types are distorted in the separating equilibrium. As the level of competition decreases, the distortion decreases for the low risk type, but increases for the high risk type; in addition, the number of insurers offering the benefit package for the low risk type increases. If the level of competition is low enough, a pooling equilibrium emerges, which generally differs from the Wilson-equilibrium. It is shown that these results have important implications for risk adjustment: For intermediate levels of competition, risk adjustment can be ineffective or even decrease welfare if it is not reasonably precise.

JEL-classification: I 18.

Keywords: Adverse selection, discrete choice, risk adjustment.

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1 Introduction

Adverse selection has long been recognized as a potentially serious problem for insurance markets in general, and health insurance markets in particular. If individuals differ in their expected medical cost, but health insurers are not allowed to charge an individual-specific premium, this creates incentives to distort the benefit package, so that the medical services offered are attractive for some individuals, but not for others. Several empirical studies have shown that these distortions exist and can be severe.

Theoretical studies analyzing these distortions have usually considered the case of perfect competition, see, e.g., the highly influential paper of Glazer and McGuire (2000). Health insurance markets may, however, not always be perfectly competitive. For the U.S., Dafny (2010) has demonstrated that in some markets, health insurers have a considerable degree of market power. For the European context, Schut et al. (2003) and Tamm et al. (2007) have shown that price elasticities of demand are low and that the number of individuals switching insurers is smaller than what would have to be expected in a perfectly competitive market. Some health insurance markets are rather imperfectly competitive.

This paper analyzes the interaction of these two phenomena – adverse selection and imperfect competition – with a special focus on the distortions of the benefit packages offered. The literature that explicitly considers this interaction for health insurance markets is rather small, and so far has only examined the following two settings: Either, all insurers offer one contract, and a pooling equilibrium is assumed, see, e.g., Frank et al. (2000). Or, a separating equilibrium is considered, where (for the case of two risk types) each insurer offers two contracts so that an incentive compatibility constraint is satisfied, see Olivella and Vera-Hernandez (2007).

The first type of analysis is restrictive in the sense that it rules out the sorting of individuals into different contracts by assumption; in equilibrium, all risk types receive the same contract.

The second class of models, on the other hand, implicitly assumes a strong asymmetry of demand responses: A new contract, yielding slightly higher utility for some individuals than the contract they currently hold, would attract all these individuals, if offered by the same insurer, but only a small share of them, if offered by a different insurer. For some health insurance settings, this is a very reasonable assumption and captures the behavior of the insured well. One example is a fee-for-service setting, where contracts differ mainly in the deductibles and coinsurance rates (and maybe the drug formularies). Insured will easily switch to a different contract of the same insurer if it yields higher utility, but – being not perfectly informed about whether other insurers reimburse bills as timely and at the same level of generosity – may hesitate to switch to another insurer if the benefit package itself is only slightly superior.

The setting we want to analyze is a different one, where each insurer offers only one contract, and contracts do not specify reimbursement rates but benefit packages of medical

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1See Cutler and Zeckhauser (2000) and Breyer et al. (2011).
3See also Cebul et al. (2011); for the Medigap market, see Maestas et al. (2009) and Starc (2013).
4See also Biglaiser and Ma (2003), Jack (2006) and Bijlsma et al. (2011).
services (possibly at different quality levels). We therefore consider insurers that are integrated to a certain degree. To be more concrete, the setting we have in mind is that an insurer’s physician network may either be small or large, comprising few or many specialists, but an insurer does not offer a choice of different physician networks. Also, an insurer either monitors utilization closely or not, but does not offer several contracts that differ in the level of utilization reviews. In a similar manner, this applies to disease management programs, that are either implemented for certain illnesses or not; pay-for-performance is yet another example, which may be difficult to conceive to be offered at different levels by one insurer.

However, our model also applies to a setting where each insurer can offer several contracts, but contracts are so different with respect to the benefit package (like the physician networks), that for the insured it does not make much of a difference whether two contracts are offered by the same insurer or two different insurers. We return to this interpretation of the model in the discussion section (see Section 5.4), but throughout the paper, we refer to the setting that each insurer offers just one contract.

To keep the model simple we consider the case of only two risk types. Also, to focus on the interaction of imperfect competition and adverse selection, we do not add heterogeneity in a second dimension, like risk aversion or preferences for the level of medical services conditional on being ill.

If there are two risk types, but each insurer offers only one contract, a meaningful model that is supposed to also capture a separating equilibrium must comprise more than two insurers. With more than two insurers, a Hotelling-model – often used to analyze imperfect competition – is not appropriate. We therefore consider a discrete choice model that imposes no restriction on the number of insurers and allows to endogenize whether a pooling or a separating equilibrium emerges. The discrete choice model has been extensively used for empirical analyses of health insurance choice; here it is used for a theoretical model of adverse selection.

For a very high level of competition, this discrete choice model replicates the results of a model under perfect competition, where an efficient benefit package is offered for the high risk type, and an inefficient one for the low risk type. With this model we then show that the distortions caused by adverse selection critically depend on the level of competition. In particular, the following main results are derived:

First, for intermediate levels of competition, not only the benefit package of the low risk type, but also the benefit package of the high risk type is distorted in a separating equilibrium. Therefore, the result of no distortion at the top does not hold in general under imperfect competition. This implies that even in a setting where there are indeed only two

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5See Cutler et al. (2008), Einav et al. (2010) and Bundorf et al. (2012).
6Olivella and Vera-Hernandez (2010) have analyzed a different extension of the Hotelling model, the spokes model of Chen and Riordan (2007). They show that when each insurer can offer two contracts, a pooling equilibrium does not exist; also, an equilibrium where each insurer offers only one contract (but contracts differ by insurer) does not exist either: At least one insurer offers both contracts so that the incentive compatibility constraint is satisfied.
risk types – e.g., being chronically ill or not – the more comprehensive benefit package can be a (severely) biased indicator of the efficient level of medical services.

Secondly, if the level of competition decreases, the distortion of the benefit package decreases for the low risk type, but increases for the high risk type. In addition, the number of insurers offering the contract for the low risk type increases, until a pooling equilibrium is reached. The pooling equilibrium, however, usually differs from the Wilson-equilibrium.

Thirdly, in the pooling equilibrium, welfare increases if competition becomes less intense; for the separating equilibrium, the reverse may hold (as is shown in an example), but the welfare effects of a decrease in competition are in general indeterminate.

We then show that the economic forces driving these results have important implications for risk adjustment: For intermediate levels of competition, a risk adjustment scheme that is imprecise and only partially compensates insurers for the cost differences of different risk types may be ineffective or even increase distortions; at such levels of competition, risk adjustment only increases welfare if the cost differences are reduced by a considerable amount. This contrasts with the case of either high or low levels of competition, where risk adjustment always increases welfare, even if transfers only compensate cost differences to a small degree. With these results we add to the small literature that analyzes the negative side effects of risk adjustment.8

To illustrate these results in a less abstract manner, we first provide a concrete example in the following Section 2. This example will demonstrate the effects in greater detail, and will make it easier to precisely state the properties of the equilibrium under imperfect competition, which are then shown to hold in general. We postpone the detailed outline of the remainder of the paper until the end of Section 2 after we introduced the basic model and discussed the example.

2 Basic model and example

We consider a setting as in Frank et al. (2000) where each individual may suffer from $S$ different illnesses. In case an illness $s$ is developed, utility changes by $v_s(m_s)$, where $m_s$ is the medical services (measured in monetary terms) provided by the insurer; $v_s(m_s)$ is increasing at a decreasing rate, i.e. $v'_s(m_s) > 0$ and $v''_s(m_s) < 0$. The individual has income $y$ and has to pay a premium $\tilde{R}$. Adopting the separability assumption of Frank et al. (2000), utility is given by

$$u = y - \tilde{R} + \sum_{s=1}^{S} p_s v_s(m_s),$$

where $p_s$ is the probability for illness $s$. The efficient level of medical services for each illness is implicitly defined by $v'_s(m^*_s) = 1$.

8See Brown et al. (2012) who show that for the U.S., the improvement of the risk adjustment scheme used for Medicaid has increased the incentive to enroll certain subgroups of individuals which are now even more ‘overpriced’ than before the reform; this increases health insurers’ wasteful expenditures to attract these individuals.
Insurers maximize profits by deciding which levels of medical services to offer and which premium to charge. We comment on why we do not consider the case where the premium is set by a regulator in the discussion section (see Section 5.6). For all illnesses \( s \) for which the probability \( p_s \) is identical across individuals, insurers will offer the efficient level of medical services (see Appendix A.1). Distortions only arise for those illnesses for which there is heterogeneity in risk. To keep the model as simple as possible, we analyze the case where probabilities differ for only one of the illnesses. Since insurers will then offer all the other medical services at the efficient level, we will skip these illnesses to simplify the notation, and write utility as

\[
 u = pv(m) - R.
\]

We consider, however, the full model to be that in addition to \( m \), insurers also offer these other medical services (at the efficient level), and charge a premium \( \tilde{R} \) that differs from \( R \) by the expected cost of these other illnesses. Any distortion of \( m \) that occurs should thus be considered to apply to a specific illness, like diabetes, rather than an overall level of medical services.\(^\text{10}\)

There are two risk types, \( L \) and \( H \), with \( p_L < p_H \); the share of \( L \)-types is \( \lambda \). Each insurer offers a contract \( c = \{m, R\} \). Under perfect competition, applying the equilibrium concept of Rothschild and Stiglitz (1976), it can be shown that if an equilibrium exists, \( H \)-types receive a contract with the efficient level of medical services at a fair premium,

\[
 c^H = \{m^H, R^H\} = \{m^*, p^Hm^*\},
\]

while \( L \)-types receive a contract with \( m^L < m^* \) at their fair premium so that \( H \)-types are indifferent between the two contracts.\(^\text{11}\)

\[
 c^L = \{m^L, R^L\} = \{m^L, p^Lm^L\}, \quad \text{with} \quad u^H(c^L) = u^H(c^H).
\]

Consider the following example with \( p_L = 0.2, p_H = 1, \lambda = 0.5 \) and \( v(m) = \ln(m) \), so that the efficient level of medical services is \( m^* = 1 \) and one of the risk types is chronically ill. Then

\[
 c^H = \{1, 1\} \quad \text{and} \quad c^L = \{0.398, 0.0797\}.
\]

A graphical depiction of these equilibrium contracts can be found in Figure 1, where \( A \) denotes the contract for the \( L \)-types, and \( B \) the contract for the \( H \)-types; \( p_L^H \) and \( p_H \) represent the zero profit lines for the two risk types.

If the pooling zero profit line \( p^{LH} \) crosses the indifference curve of the \( L \)-types that passes through contract \( A \) (denoted by \( I_{RS}^{LH} \)), the separating Rothschild-Stiglitz equilibrium does not exist.\(^\text{12}\) In this case, it is common to refer to the Wilson pooling equilibrium: the contract on the pooling zero profit line that maximizes the utility of the \( L \)-types.\(^\text{13}\)

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\(^9\)The main results regarding the distortions of the benefit packages would be identical, but the analysis of the welfare effects would be obscured by the adjustment of the premium that becomes necessary when the level of competition changes.

\(^{10}\)We prefer this interpretation of the model because it seems unusual to have one probability of consuming the whole benefit package of a health insurer.

\(^{11}\)See Zweifel et al. (2009, p. 264).

\(^{12}\)The probability \( p^{LH} \) is given by \( p^{LH} = \lambda p^L + (1 - \lambda)p^H \).

\(^{13}\)See Zweifel et al. (2009, p. 178).
Figure 1: Rothschild-Stiglitz-equilibrium and ‘Wilson’-contract under perfect competition

With $\lambda = 0.5$, the pooling zero profit line does not cross the $I_{RS}^-\text{-indifference curve}$ (see Figure 1), so the separating equilibrium does exist. Because there exists a pooling equilibrium for this example under imperfect competition, we nevertheless determine the contract on the pooling zero profit line that maximizes the utility of the $L$-types: it is $c^{W} = \{0.333, 0.2\}$, and is depicted as contract $C$ in Figure 1. We will denote this contract, as it may often not constitute the Wilson-equilibrium, simply as the ‘Wilson’-contract.

We now compare these results to the case of imperfect competition, which is captured with a discrete choice model. This model is explained in greater detail in Section 3; here we only state its main components and the results.

There are $n$ insurers $j$, each offering a contract $c^j = \{m^j, R^j\}$. Individuals’ utility is augmented by an insurer specific utility component $\varepsilon_{ij}$, that captures all the influences on the choice of an insurer that are independent of $m$ and $R$. The utility of individual $i$ when choosing insurer $j$ therefore is

$$u_i(m^j, R^j) = p^j v(m^j) - R^j + \varepsilon_{ij}.$$ 

Each individual chooses the insurer that offers the highest utility, taking into account not only $m^j$ and $R^j$, but also $\varepsilon_{ij}$. We assume $\varepsilon_{ij}$ to be i.i.d. extreme value with $\text{Var}(\varepsilon_{ij}) = \sigma^2 \frac{\pi^2}{6}$, but later show that the main results also hold for other distributional assumptions. The variance of $\varepsilon_{ij}$ is a measure of the degree of competition: If $\sigma$ is large, the additional utility component is important, so competition with respect to different benefit packages is low. If, on the other hand, $\sigma$ is small, $\varepsilon_{ij}$ only has a small influence on the decision of which insurer to choose, so competition is high. With $\sigma = 0$, the model encompasses the case of

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14Note that for the case of imperfect competition contracts are indexed by insurer, not risk type. In Section 3 it will become apparent why this is more appropriate.

15Note that it is common to state the variance of $\varepsilon_{ij}$ as a multiple of $\frac{\pi^2}{6}$ for the extreme value distribution, see Train (2009, p. 24).
Table 1: Example I with $v(m) = \ln(m)$, $p^L = 0.2$, $p^H = 1$, $\lambda = 0.5$, for different values of $\sigma$. The first row (RS) contains the Rothschild-Stiglitz-equilibrium, the last row (WI) the ‘Wilson’-contract.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>4 insurers</th>
<th>6 insurers</th>
<th>10 insurers</th>
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<tr>
<td></td>
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<td>$n^B$</td>
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<td>.25</td>
<td>pooling</td>
<td>.556</td>
<td>.556</td>
</tr>
<tr>
<td>WI</td>
<td>pooling</td>
<td>.333</td>
<td>.333</td>
</tr>
</tbody>
</table>

perfect competition. With $\sigma > 0$, the level of competition of course also increases in the total number of insurers, $n$.

Table I presents the equilibrium when there are 4, 6 or 10 insurers, for different values of $\sigma$; in addition, the Rothschild-Stiglitz equilibrium (RS) and the ‘Wilson’-contract (WI) can be found in the first and in the last row, respectively. The superscripts $A$ and $B$ indicate the ‘type’ of the insurer: Insurers of type $A$ offer the contract for the $L$-type individuals, insurers of type $B$ the contract for the $H$-type individuals.

Using this example we can now state in greater detail, what is then shown to hold in general in this imperfect competition setting.

**Separating equilibrium: Decrease in $n$**

A decrease in competition due to a smaller number of insurers increases $m^A$, the level of medical services offered for the $L$-types, so the distortion is reduced; see, e.g., the row for $\sigma = 0.08$, where $m^A$ increases from 0.334 (10 insurers) to 0.361 (4 insurers).

**Separating equilibrium: Increase in $\sigma$**

A decrease in competition due to a larger value of $\sigma$ has a different effect: $m^A$ first decreases and only later increases in $\sigma$. Note that for 4 insurers, the increase begins before the number

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16 Section 3.3 will make clear why do not use $L$ and $H$ to denote insurer type.

17 For low levels of $\sigma$ the increase in $m^A$ may not be seen; this is, however, only due to rounding.
of insurers offering the two different types of contracts ($n^A$ and $n^B$) changes; see, e.g., $m^A$ for $\sigma \leq 0.08$ for 4 insurers.

A decrease in competition due to a larger value of $\sigma$ also has an effect on $m^B$, the medical services offered for the $H$-types: At some point, $m^B$ decreases, and therefore deviates from the efficient level. Note that for the case of 10 insurers, $m^B$ is heavily distorted as $\sigma$ increases up to 0.18. The result of no distortion at the top clearly does not hold in general under imperfect competition.

Finally, a decrease in competition due to a larger value of $\sigma$ also has an effect on the number of insurers offering the two different contracts: As $\sigma$ increases, the number of insurers offering the contract for the $L$-types increases, until the pooling equilibrium is reached; see any of the three columns for $n^A$.

**Pooling equilibrium: Decrease in $n$ and increase in $\sigma$**

For the pooling equilibria we can observe that as competition decreases, the distortion is reduced: $m$ increases as either $n$ decreases (see, e.g., the row for $\sigma = 0.20$) or as $\sigma$ increases (see, e.g., the case of 4 insurers for $\sigma \geq 0.16$).

Note that in this example, in all the pooling equilibria $m$ is above the value of the ‘Wilson’-contract (see the last row). This is, however, not a general result. In Section 3.4 we present an example where $m$ is below, at or above the ‘Wilson’-contract, depending on the level of competition; there we also explain what determines which of the three cases occurs.

**Welfare**

We finally comment on welfare $W$, which is calculated as the sum of expected surplus generated by the consumption of $m$,

$$W = \sum_i S^i,$$

(1)

where for each risk type $i$, the expected surplus is given by

$$S^i = p^i v(m) - p^i m,$$

(2)

with $m$ being the level of medical services consumed by a particular risk type. Of course, the premium $R$ does not appear in (2), as it is only a transfer from the insured to the insurer.

For the pooling equilibrium, it is obvious from what we found for $m$ that welfare increases as competition decreases; see, e.g., the column $W$ for $\sigma \geq 0.16$ for 4 insurers, or the row for $\sigma = 0.20$.

For the separating equilibrium, the opposite holds: Welfare decreases as the level of competition decreases; see any of the three columns for $W$ with $\sigma$ below the value at which the pooling equilibrium arises. This result, however, may not hold in general, as the countervailing effects that are derived in the following section will show. Because of these countervailing effects, the change in welfare is also indeterminate when competition decreases due to a decrease in $n$. Compare, e.g., the rows for $\sigma = 0.04$ and $\sigma = 0.08$: in the first
case, welfare slightly increases as $n$ decreases from 10 to 4; in the second, welfare slightly decreases.

Comparison of the effect of $n$ and $\sigma$

For both the level of medical services $m$ and welfare $W$, the differences due to different levels of $\sigma$ are much larger than the differences due to different levels of $n$. What is important for a large effect is individuals’ responsiveness to different benefit packages, but not the number of insurers.[18]

In the following Section 3 we show that the results just discussed do hold in general. The section is organized in a way so that an intuitive understanding of the economic forces driving the results can be provided. Because the demand response is somewhat different than in a standard Hotelling-model, we will first analyze the case of one observable risk type at some length in Section 3.1; there we also give an explanation for why $\sigma$ has a larger effect on the results than $n$. We proceed with two observable risk types in Section 3.2, where we determine the number of insurers $n^A$ and $n^B$; as it turns out, the share of insurers of type $A$ generally differs from the share of $L$-types. We then consider the case that the risk type is unobservable: in Section 3.3 we derive the separating equilibrium, and show how it depends on $\sigma$ (Section 3.4) and the total number of insurers, $n$, (Section 3.5). The pooling equilibrium is discussed in Section 3.6. We comment on the welfare effects of a decrease in competition for both the separating and the pooling equilibrium in Section 3.7.

We then analyze the implications of these results for risk adjustment in Section 4. We first present an example where welfare decreases as the risk adjustment scheme becomes more precise (Section 4.1). We then explain under what conditions this decrease occurs by analyzing the separating equilibrium in Section 4.2 and the pooling equilibrium in Section 4.3.

Finally, several of the assumptions of the model are discussed in Section 5 and Section 6 concludes.

3 The discrete choice model

3.1 One risk type

There are $n$ insurers $j$, each offering a contract $c^j = \{m^j, R^j\}$. An individual $i$, choosing insurer $j$ receives utility

$$u_i(m^j, R^j) = pv(m^j) - R^j + \varepsilon_{ij},$$

(3)

where $\varepsilon_{ij}$ captures the utility component of choosing insurer $j$ that is independent of the benefit-premium-bundle. We denote the part that depends on the benefit-premium-bundle by

$$V^j = pv(m^j) - R^j.$$  

This also holds if $n$ is increased to a much larger number.
Individual $i$ will choose an insurer $k$, if $u_i^k$ yields the highest utility, i.e. if
\[ V^k + \varepsilon_{ik} > V^l + \varepsilon_{il} \quad \forall \ l \neq k. \]
Assuming that all $\varepsilon_{ij}$ are distributed i.i.d. extreme value with variance $\text{Var}(\varepsilon_{ij}) = \frac{\sigma^2 e^\frac{\sigma^2}{2}}{2}$, it follows that the probability of $i$ choosing insurer $k$ is\[ \text{Prob}(i \text{ chooses } k) = \frac{e^{V^k/\sigma}}{\sum_j e^{V_j/\sigma}}. \]

We denote this probability by $P^k$. Normalizing the mass of individuals to one, and assuming profit maximization, the objective of insurer $k$ is
\[ \max_{m^k, R^k} \pi^k = P^k \pi^k_i, \]
where $\pi^k_i = R^k - pm^k$ denotes insurer $k$’s profit per individual.

It will turn out much easier to derive the main results for the case of unobservable risk types if we reformulate the insurer’s objective in terms of $\{m, V\}$ instead of $\{m, R\}$. Graphically, in $m$-$R$-space, insurer $k$ chooses an indifference curve $I^{V^k}$ associated with the utility level $V^k$, and a level of medical services $m^k$ along this indifference curve.

Using (4) to substitute for $R^k$ in $\pi^k_i$, the insurers objective can be restated as
\[ \max_{m^k, V^k} \pi^k = P^k \pi^k_i = \frac{e^{V^k/\sigma}}{\sum_j e^{V_j/\sigma}} \left( pv(m^k) - V^k - pm^k \right). \] (5)

Before we derive the solution to this problem, note that the derivative of $P^k$ with respect to $V^k$ can be expressed in a simple way:
\[ \frac{\partial P^k}{\partial V^k} = \frac{P^k(1 - P^k)}{\sigma}. \] (6)

The FOCs of the insurer’s objective (5) are
\[ \frac{\partial \pi^k}{\partial m^k} = P^k \left[ pv'(m^k) - p \right] = 0 \] (7)
\[ \frac{\partial \pi^k}{\partial V^k} = \frac{P^k(1 - P^k)}{\sigma} \pi^k_i - P^k = 0. \] (8)

Condition (7) requires $v'(m^k) = 1$, so $m^k$ is chosen efficiently. This holds regardless of which benefit-premium-bundles are offered by the other insurers. It also holds irrespective of the utility level $V^k$ chosen by insurer $k$: along an indifference curve $I^{V^k}$, $m^k$ will always be set at the efficient level $m^*$. Therefore, all insurers will offer $m^*$.

\[ \text{See Train (2009, p. 40).} \]
Condition (8) shows the two countervailing effects of increasing $V^k$: The share of individuals choosing $k$ increases by $P^k(1 - P^k)^\frac{1}{\sigma}$; weighting by $\pi^k_i$ captures the additional profit. On the other hand, increasing $V^k$ (for a given $m^k$) implies reducing $R^k$ by the same amount, and therefore $\pi^k_i$; this applies to the share of individuals choosing $k$, $P^k$, capturing the loss in profit. For these two effects to cancel out, we have to have $\pi^k_i = \frac{\sigma}{1 - P^k}$. It can be shown that the only equilibrium is a symmetric one, where all insurers choose the same level of utility $V^j = \bar{V}$ $\forall j$. Since, in this case, $P^k = \frac{1}{n}$, in equilibrium profit per individual is
$$\pi^k_i = \frac{n}{n - 1} \sigma,$$
(9)
and total profit per insurer is
$$\pi^k = \frac{\sigma}{n - 1}.$$
(10)

As is to be expected, more competition leads to lower profits: both, profit per individual, $\pi^k_i$, and total profit per insurer, $\pi^k$, increase in $\sigma$ and decrease in $n$.

If $\sigma$ is small, offering a higher utility level yields a large increase in the share of individuals. This raises the incentive to offer a higher utility level (i.e. a lower premium), thereby reducing profits in equilibrium.

If $n$ is large, each insurer’s market share is small. Offering a higher utility level then attracts individuals from a large ‘external’ market share $1 - P^k$. This again raises the incentive to offer higher utility levels, lowering profits. We refer to this as the ‘more competition due to a larger external market share’-effect. This effect plays an important role when risk types are unobservable, and also when there is risk adjustment.

Note that this external market share $1 - P^k$ is confined to the interval $[0.5, 1]$. The effect of the total number of insurers on profits is therefore rather limited: Increasing this number from $n = 2$ to $n \to \infty$ only cuts profit per individual $\pi^k_i$ in half, see condition (9). In contrast, the effect of $\sigma$ on profit per individual is not bounded. In that sense, $\sigma$ can be considered to be the more important variable to capture large differences in the level of competition. This is what we found in the example in Section 2, where the differences in $m$ and $W$ are very small for different values of $n$ compared to different values of $\sigma$.

We will now present this solution graphically in somewhat greater detail than necessary for this basic model, because it facilitates the derivation of the results for the case of unobservable risk types.

As $P^k$ denotes the probability that an individual $i$ chooses insurer $k$, it can be considered a distribution function $P^k(V^k)$, with corresponding density $P^k(1 - P^k)^\frac{1}{\sigma}$, (see equation (6)). In equilibrium, when all the other insurers offer the same level of utility $\bar{V}$, we have
$$P^k = \frac{e^{\bar{V}^k}}{e^{\bar{V}^k} + (n - 1)e^{\bar{V}}}.$$
(11)

We can depict this distribution function by drawing a shaded area around the $\bar{V}$-indifference curve, which captures (the support of) the corresponding density, and where the darkness
of the shaded area is a measure of the level of that density, see Figure 2 again. As contract $A$ is above the shaded area, $P_k = 0$. Increasing utility $V^k$ than moves contract $A$ (along the line $m = m^*$) into the shaded area, which increases $P_k$ and decreases $\pi^k$. These two effects cancel out when contract $A$ lies on the $\tilde{I}^V$-indifference curve. Increasing $V^k$ even further then increases $P_k$ beyond $\frac{1}{n}$; as soon as contract $A$ is below the shaded area, $P_k = 1$. 

Note that as insurer $k$ moves along the $I^V_k$-indifference curve, $P_k$ does not change, regardless of whether $I^V_k$ is above, within or below the shaded area. This is because the distance between $I^V_k$ and $\tilde{I}^V$ in the $R$-direction is the same for all levels of $m$. The $I^V_k$-indifference curve is therefore also an iso-$P^k$-curve.

We can now discuss the effects of an increase of $\sigma$: First, the iso-profit line associated with the equilibrium contract is shifted upwards. Secondly, it is straightforward to show that the distribution function $P^k$ as stated in (11) decreases for $V^k < \tilde{V}$, and increases for $V^k > \tilde{V}$; it also becomes less steep at $V^k = \tilde{V}$, so the density decreases around $\tilde{V}$ (see Figure 7 in Appendix A.2 for a graphical representation of $P^k$). As $\sigma$ increases, the distribution function is spread out (over a wider range), which can be depicted in Figure 2 by drawing a wider shaded area around the indifference curve $\tilde{I}^V$.

20 As a technical detail, note that for $n = 2$, the maximum of this density is at $V^k = \tilde{V}$, but for $n > 2$, it is at $V^k > \tilde{V}$. Therefore the ‘center’ of the shaded area is at the $\tilde{I}^V$-indifference curve for $n = 2$, and somewhat below it for $n > 2$. To simplify the exposition in the graphs, we will always draw the center of the shaded area at $\tilde{V}$.

21 Of course, technically, $P^k > 0 \forall V^k$, see (11), but above the shaded area, both $P^k$ and the density $P^k(1 - P^k) \frac{1}{\sigma}$ are almost equal to zero.
Finally note that $P^k$ depends on the utility levels offered by the other insurers only via the aggregate $\sum_{j \neq k} e^{V^j}$. It is easy to show that an increase in this aggregate only shifts the distribution function to the right (in a $V^k$-$P^k$-diagram like in Figure 7(a)), but does not change its shape; of course, then also the density is only shifted to the right (in a $V^k$-density-diagram like in Figure 7(b)). If, e.g., all insurers except $k$ increase the utility level they offer by some $\Delta V$, then $I^V$ and the shaded area in Figure 2 are shifted downwards, but the width of the shaded area does not change. Insurer $k$ then has to increase $V^k$ by the same $\Delta V$ to keep $P^k$ unaltered. The same applies if the total number of insurers is increased: in equilibrium, this increases the aggregate $\sum_{j \neq k} e^{V^j}$, shifting $I^V$ and the shaded area downwards, but keeping the width of the shaded area constant, again.

### 3.2 Two observable risk types

When there are two observable risk types $L$ and $H$, with $p^L < p^H$, insurers will offer different contracts. We denote the insurers offering contracts for the $L$-types as insurers of type $A$, and insurers offering contracts for the $H$-types as insurers of type $B$. The number of insurers is $n^A$ and $n^B$ respectively, with $n^A + n^B = n$.

It follows immediately from what we derived for the case of one observable risk type that, in equilibrium, all insurers will offer the efficient level of medical services $m^A = m^B = m^*$, but premiums will differ according to risk type. As insurers can decide whether to be of type $A$ or type $B$, in equilibrium $\pi^A = \pi^B$ has to hold. Taking into account that the share of $L$-types is $\lambda$ and using (10), total profits per insurer are

$$\pi^A = \lambda \frac{\sigma}{n^A - 1} \quad \text{and} \quad \pi^B = (1 - \lambda) \frac{\sigma}{n^B - 1}. \quad (12)$$

Solving for $\pi^A = \pi^B$, it follows that

$$n^A = \lambda n + (1 - 2\lambda) \quad \text{and} \quad n^B = (1 - \lambda) n - (1 - 2\lambda). \quad (13)$$

As can be seen, the share of insurers of type $A$ equals the share of $L$-types only for $\lambda = \frac{1}{2}$. For $\lambda < \frac{1}{2}$, we have $n^A > \lambda n$. This is for the following reason: With $\lambda < \frac{1}{2}$, there will be fewer insurers of type $A$ than of type $B$, $(n^A < n^B)$, so the market served by insurers of type $A$ will be less competitive. This causes profit per individual to be higher in the smaller market ($\pi^A > \pi^B$), which induces a somewhat higher number of insurers to become of type $A$ than given by $\lambda n$.

The same reasoning applies for $\lambda > \frac{1}{2}$. However, as $\lambda \in [0, 1]$, we have $-1 < (1 - 2\lambda) < 1$, so there will be at most one more insurer in the smaller market than given by the share of the respective risk type.

Of course, $n^A$ and $n^B$ have to be integer numbers, so that the expressions given in (13) are only an approximation to the true value. As it is not important for the derivation of our main results, we do not elaborate on a formula that indicates whether $n^A$ as given by (13) has to be rounded up or off. We do state, however, that the requirement of $n^A$ and $n^B$ to be integer can, for some parameter settings, cause an equilibrium not to exist: For some value of $n^A$
and $n^B$, it may be profitable for an insurer of type $B$ to enter the market for the $L$-types and become an insurer of type $A$; but after the new ‘equilibrium’ has been attained, where $\pi_i^A$ is decreased and $\pi_i^B$ increased, the same insurer may then find it profitable to become of type $B$ again. We comment on this problem of the existence of an equilibrium in the discussion section (see Section 5.5).

Note that for $n^A = n^B$, (i.e. $\lambda = \frac{1}{2}$), the iso profit lines associated with the contracts offered by the two types of insurers start at the same point on the ordinate (at $\frac{n^A}{n^A-1}\sigma = \frac{n^B}{n^B-1}\sigma$). This is not the case for $n^A \neq n^B$. However, to simplify the exposition in the graphs, we always depict the case where both iso profit lines start at the same point, but all results that are derived hold for the general case of $n^A \neq n^B$.

As is apparent from condition (13), $n^A$ and $n^B$ do not depend on $\sigma$, the level of competition. This will, however, change for the case of unobservable risk types, to which we now turn. We begin with the separating equilibrium.

### 3.3 Two unobservable risk types: the separating equilibrium

In this section, we derive the separating equilibrium when the risk type is unobservable. Under perfect competition, for the separating equilibrium to exist, the share of $L$-types must be below a critical level (Rothschild and Stiglitz 1976). The same applies for this discrete choice model if the level of competition is high. Then the shaded areas around the indifference curves are very narrow and the argument for the non-existence of an equilibrium is the same as under perfect competition: If the share of $L$-types is too large, the ‘separating equilibrium’ can be destroyed by offering a contract that would be chosen by both risk types and yield a higher profit than either of the two contracts in the ‘separating equilibrium’. On the other hand, a pooling equilibrium can be destroyed by offering a contract that is chosen only (or primarily) by the $L$-types. However, as we saw in the example in Section 2 (where the separating equilibrium does exist), a pooling equilibrium emerges if the level of competition is low enough. As we show in the following, this result does hold in general. In fact, the pooling equilibrium always emerges if the level of competition is low enough, irrespective of whether the separating equilibrium under perfect competition does exist or not.

Therefore, in this section we assume $\sigma$ and $\lambda$ to be small enough, so that the separating equilibrium does exist. We begin with the case of a very small level of $\sigma$ so that the contract designated to the $L$-types will yield a negative profit when chosen by an $H$-type. The effects of an increase in $\sigma$ are then derived in the following Section 3.4. There we also discuss the case that the separating equilibrium does not exist because a pooling equilibrium emerges.

If the risk type is unobservable, a contract offered by insurer $A$ may be chosen by both risk types (and likewise for insurer $B$)\footnote{In the following we will often use the term ‘insurer $A$’ instead of ‘one of the insurers of type $A$.’}. The utility level associated with such a contract $\{m^A, R^A\}$ depends on the risk type according to

$$V_{L}^A = p^L v(m^A) - R^A \quad \text{and} \quad V_{H}^A = p^H v(m^A) - R^A.$$
We formulate the objective of insurer $A$ in terms of $V_L^A$ and $m^A$, and express $V_H^A$ as

$$V_H^A = V_L^A + (p^H - p^L)v(m^A).$$

The utility levels associated with the contract offered by insurer $B$ are defined equivalently. As insurer $B$ offers a contract for the $H$-types, we formulate its objective in terms of $V_H^B$.

As each contract may be chosen by both risk types, four probabilities (or market shares) have to be distinguished: We denote by $P_L^A$ the probability that an $L$-type chooses insurer $A$; it is given by

$$P_L^A = \frac{e^{V_L^A}}{e^{V_L^A} + \sum_{j \neq A} e^{V_L^j}}. \quad (14)$$

The remaining probabilities (and market shares), $P_H^A$, $P_L^B$ and $P_H^B$, are defined accordingly.

Note that $P_H^A$ depends on $V_L^A$ and $m^A$ according to

$$P_H^A = \frac{e^{V_L^A}}{e^{V_L^A} + \sum_{j \neq A} e^{V_L^j}} = \frac{e^{V_L^A + (p^H - p^L)v(m^A)}}{e^{V_L^A + (p^H - p^L)v(m^A)} + \sum_{j \neq A} e^{V_L^j}}. \quad (15)$$

In equilibrium, when all insurers of type $A$ offer the same contract for the $L$-types, and all insurers of type $B$ offer the same contract for the $H$-types, we have

$$P_L^A = \frac{e^{V_L^A}}{n_A e^{V_L^A} + n_B e^{V_B^A}}, \quad (15)$$

and equivalently for $P_H^A$, $P_L^B$ and $P_H^B$. Finally, we have to define profit per individual of a specific type in terms of $V$ and $m$. For insurer $A$ we have

$$\pi_L^A = p^L v(m^A) - V_L^A - p^L m^A \quad \text{and} \quad \pi_H^A = p^L v(m^A) - V_L^A - p^H m^A. \quad (16)$$

Using these definitions, insurer $A$’s objective reads as

$$\max_{V_L^A, m^A} \pi^A = \lambda \pi_L^A + (1 - \lambda) \pi_H^A, \quad (17)$$

with FOCs

$$\frac{\partial \pi^A}{\partial V_L^A} = \lambda \left[ \frac{P_L^A(1 - P_A^A)}{\sigma} \pi_L^A - P_L^A \right] + (1 - \lambda) \left[ \frac{P_H^A(1 - P_A^A)}{\sigma} \pi_H^A - P_H^A \right] = 0 \quad (18)$$

$$\frac{\partial \pi^A}{\partial m^A} = \lambda P_L^A \left[ p^L v'(m^A) - p^L \right] + (1 - \lambda) P_H^A \left[ p^L v'(m^A) - p^H \right] \quad (19)$$

$$+ (1 - \lambda) P_H^A \frac{1 - P_H^A}{\sigma} \left( p^H - p^L \right) v'(m^A) \pi_H^A = 0.$$

The FOCs of insurer $B$ are

$$\frac{\partial \pi^B}{\partial V_H^B} = \lambda \left[ \frac{P_L^B(1 - P_B^B)}{\sigma} \pi_L^B - P_L^B \right] + (1 - \lambda) \left[ \frac{P_H^B(1 - P_B^B)}{\sigma} \pi_H^B - P_H^B \right] = 0 \quad (20)$$

$$\frac{\partial \pi^B}{\partial m^B} = \lambda P_L^B \left[ p^H v'(m^B) - p^H \right] - (1 - \lambda) P_L^B \frac{1 - P_B^B}{\sigma} \left( p^H - p^L \right) v'(m^B) \pi_L^B \quad (21)$$

$$+ (1 - \lambda) P_H^B \left[ p^H v'(m^B) - p^H \right] = 0.$$
As insurers will decide whether to be of type A or type B, we finally have to have \( \pi^A = \pi^B \), i.e.,

\[
\lambda P_L^A \pi_L^A + (1 - \lambda) P_H^A \pi_H^A = \lambda P_L^B \pi_L^B + (1 - \lambda) P_H^B \pi_H^B.
\]

Without a specific utility function, the equilibrium contracts can of course not be determined explicitly from the four FOCs (18)-(21) and the profit equality condition (22). Nevertheless, all the properties of the equilibrium that have been illustrated in the example in Section 2 can be derived. To do so, it will be helpful to also present the main effects and results graphically.

Figure 3: Separating equilibrium with two unobservable risk types. Contracts B and A₃ are offered. The case \( n^A = n^B \) (i.e. \( \lambda = 0.5 \)) is depicted.

With unobservable risk types and perfect competition, in Figure 3 the equilibrium consists of contract B, chosen by the H-types, and contract A₁, chosen by the L-types. However, as the shaded area of the \( IV_H \)-indifference curve shows, in this case, insurer A would find a considerable share of H-types choosing contract A₁. Therefore, contract A₁ has to be shifted outside the shaded area. Assume, that it is shifted (along the iso-\( \pi_L \)-line) to A₂, where (almost) none of the H-types choose this contract. But then insurer A could move its contract along the \( IV_A \)-indifference curve to the right: This would leave the number of L-types choosing this insurer unaffected (see the definition of \( P_A^L \) in (14)), but increase profits per L-type, \( \pi_L^A \). This is because the slope of the \( IV_A \)-indifference curve is larger than the slope of the iso-\( \pi_L \)-lines for all contracts with \( m^A < m^A \). It would also increase the number of the H-types choosing this insurer; however, since the density \( P_H^A(1 - P_H^A) \frac{1}{\sigma} \approx 0 \) at contract A₂, in the beginning this effect is of second order. There is a third effect when moving along \( IV_A \): Depending on whether the slope of the \( IV_A \)-indifference curve is smaller or larger than the slope of the iso profit lines for the H-types, \( p_H \), this will increase or decrease profit per H-type, \( \pi_H^A \).

Insurer A will therefore move along the \( IV_A \)-indifference curve until these three effects cancel out, which will be at a contract as indicated by A₃ in Figure 3.

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23In this case, the iso profit lines would of course start at the origin, as \( \sigma \to 0 \).

24The shaded area represents the density of the distribution function \( P_H^A(V_H^A) = P_A^H(V_A^A, m^A) \).
In equilibrium, a small share of $H$-types chooses contract $A$; this contrasts to the contract offered by insurer $B$: As contract $B$ is far away from the shaded area that can be drawn around the $I^{V_L}$-indifference curve, none of the $L$-types choose contract $B$.\footnote{The shaded area represents the density of $P^B_L(V^B_L) = P^B_L(V^B_H, m^B)$.} As there is no interference of the $L$-types, contract $B$ is at the efficient level, as in the case of perfect competition.

**Results 1.** In the separating equilibrium, if $\sigma$ is small, then only the benefit package for the $L$-types is distorted: $m^A < m^*$ and $m^B = m^*$. A small share of the $H$-types chooses the contract designated for the $L$-types, but none of the $L$-types choose the contract designated for the $H$-types: $P^A_H > 0$ and $P^B_L = 0$.

In the remainder of this section we show how these results are reflected in the FOCs (18)-(21). First, since $P^B_L = 0$ (as none of the $L$-types choose contract $B$), condition (21) simplifies to $v'(m^B) = 1$, (i.e., $m^B = m^*$), and from condition (20) it follows that

$$\pi^B_H = \frac{P^B_H}{1 - P^B_H} \sigma.$$  

For insurer $A$, dividing (19) by $\lambda P^A_L p^A$, the FOC with respect to $m^A$ can be rewritten as

$$v'(m^A) - 1 + \frac{1 - \lambda P^A_H}{\lambda P^A_L} \left[ v'(m^A) - \frac{1 - \lambda P^A_H}{\lambda P^A_L} \frac{P^H + p^L}{\sigma} \right] + \frac{1 - \lambda P^A_H}{\lambda P^A_L} \frac{1}{\sigma} \frac{P^A - p^L}{p^L} \pi^A_H v'(m^A) = 0. \tag{23}$$

If $P^A_H$ was equal to zero, this condition would simplify to $v'(m^A) - 1 = 0$, so we would have $m^A = m^*$. This, together with the lower premium, would induce at least some of the $H$-types to choose insurer $A$, a contradiction to $P^A_H = 0$, so $P^A_H > 0$. With $P^A_H > 0$, the following distortionary effects can be identified: First, because of the term in brackets, we have to have $v'(m^A) > 1$, as $\frac{P^H}{p^L} > 1$. Note that if at $m^A$, the $I^{V^A_L}$-indifference curve is less steep than the iso profit line for the $H$-types, ($p^L v'(m^A) < p^H$), the bracket is negative, capturing the third effect stated above. Secondly, since $\pi^A_H$ is negative, the last summand of (23) is negative, which is an additional effect that requires $v'(m^A) > 1$. Therefore, $v'(m^A) > 1$ and $m^A < m^*$.

We now turn to condition (18), the FOC with respect to $V^A_L$: With $P^A_H = 0$, this condition would simplify to $\pi^A_L = \frac{P^A_L}{1 - P^A_L} \lambda p^A$, the FOC if risk types were observable (condition (8) derived in Section 3.1). However, with $P^A_H > 0$ the second bracket is negative because $\pi^A_H < 0$; therefore, $\pi^A_L$ has to be larger than for the case of $P^A_H = 0$: Increasing $V^A_L$ not only reduces profits for $L$-types by $P^A_L$ (and profits for $H$-types by $P^A_H$), but has the additional effect of increasing the share of $H$-types by $P^A_H (1 - P^A_H) \frac{1}{\sigma}$, which yield negative profit per individual. This reduces the incentive to offer higher utility $V^A_L$, which increases $\pi^A_L$, so the equilibrium contract $A_3$ is above the iso-profit-line as shown in Figure 3.

However, if $\sigma$ is very small, $P^A_H$ will be very close to zero, and $A_3$ will only be slightly above the iso-profit-line $\pi^A_L$ as shown in Figure 3. As $\pi^A_L$ is almost not affected by the very low share of high risks, the number of insurers of type $A$ and type $B$, $n^A$ and $n^B$, will then not be different from the case when risk types are observable. This, however, changes as $\sigma$ increases.
3.4 The dependence of the separating equilibrium on $\sigma$

So far, the equilibrium under imperfect competition looks rather similar to the case of perfect competition. We will now show that this only holds for high levels of competition. In this section, we analyze the effects of a decrease in competition due to an increase in $\sigma$; we discuss a decrease of competition due to a decrease of $n$ in the following Section 3.5.

In Section 3.1 it was shown that an increase in $\sigma$ increases profits, as insurers reduce the utility levels they offer by increasing the premium; this shifts the iso profit line associated with the equilibrium upwards. The same applies if risk types are unobservable. However, the increase in premiums alone does not yet constitute the new equilibrium because of the following additional effects:

Effect on $m^A$ and $P^A_H$

First, as $\sigma$ increases, the shaded area around the $I^V_B$-indifference curve becomes wider. This creates an incentive to decrease $m^A$. As the distribution function $P^A_H$ increases in $\sigma$ for all values of $V^A_H < V^B_H$, if (after the increase in $\sigma$) $m^A$ did not change, more $H$-types would choose insurer $A$. To avoid being chosen by these $H$-types, insurer $A$ reduces $m^A$. This first effect can be seen in condition (23) by considering the terms up to and including the bracket as a weighted average: As the weight of the second summand goes up, $v'(m^A)$ has to be increased.

Secondly, there is the countervailing effect that as premiums increase, insuring an additional $H$-type now causes a smaller loss; this creates an incentive to increase $m^A$. The aggregate of these two effects on $m^A$ is indeterminate, but in Appendix A.3 it is shown that the share of $H$-types choosing insurer $A$, $P^A_H$, unambiguously increases. Here, we only give a brief intuitive explanation: If, by the first effect, $m^A$ was reduced to a level so that $P^A_H$ was the same as before the increase in $\sigma$, there would then be an incentive to increase $m^A$ (and thereby $P^A_H$) for three different reasons: First, if $P^A_H$ is at the same level as before, but $v'(m^A)$ has been increased, condition (23) is not satisfied anymore, so $v'(m^A)$ has to be decreased. Secondly, if $P^A_H$ is at the same level as before, the density $P^A_H(1 - P^A_H)\frac{1}{\sigma}$ now is lower (due to the larger value of $\sigma$); moving along the $I^V_B$-indifference curve does not attract as many $H$-types as before. In condition (23), this effect is found by $\sigma$ in the denominator in the last term. Thirdly, as premiums have been increased, $\pi^A_H$ is increased, so attracting an additional $H$-type causes a smaller loss than before. In (23) this effect can again be found in the last term, where $\pi^A_H$ increases.

After the increase of $\sigma$, more $H$-types will choose the contract offered by insurers of type $A$. Whether $m^A$ increases or decreases is indeterminate for a general utility function $v(\cdot)$. It depends on whether the effect of the wider shaded area around the $I^V_B$-indifference curve or the three countervailing effects dominate. Initially, as $\sigma$ is very small, and $\pi^A_H$ is far below zero, the effect of an increase in $\sigma$ on $\pi^A_H$ is small in relative terms, which makes it likely that $m^A$ decreases, as was the case in our example in Section 2.

Effect on $n^A$

We now turn to the question of why the number of insurers offering contract $A$ increases: In Appendix A.4 it is shown that as $\sigma$ increases, profits increase faster for type-$A$ insurers.
than for type-B insurers, so that at some point it will be profitable for one of the type-B insurers to switch and to become a type-A insurer. Here again, we only state an intuitive explanation: If \( \sigma \) increases, \( P^A_H \) increases, so the number of individuals choosing any of the type-B insurers decreases. This is the first effect reducing (the increase of) total profits of type-B insurers. In addition, as the number of individuals choosing type-A insurers increases, for the type-B insurers there is the 'more competition due to a larger external market share'-effect, which, as we saw in Section 3.1, decreases profits per individual. Due to these two effects, \( \pi^B \) increases at a lower rate than \( \pi^A \).

As \( \pi^A \) increases faster than \( \pi^B \), \( n^B \) would have to decrease continuously in \( \sigma \). However, as \( n^A \) and \( n^B \) have to be integer numbers, there will only be a switch of an insurer of type \( B \) to become of type \( A \) if the difference between \( \pi^A \) and \( \pi^B \) is large enough. This is why in the example in Section 2, \( n^A \) is constant in the first rows of Table 1 and does not increase in \( \sigma \).

**Effect on \( m^B \)**

We finally discuss why for an intermediate level of competition, contract \( B \) is distorted. As \( \sigma \) increases, the shaded areas around both indifference curves get wider. At some level of \( \sigma \), the shaded area around the indifference curve of the \( L \)-types, \( I^V_L \), becomes so wide that it 'reaches' contract \( B \), so that a small share of \( L \)-types chooses contract \( B \). It will then be profitable for insurer \( B \) to move along the \( I^V_B \)-indifference curve and reduce \( m^B \) (see Figure 4 where only the shaded area around the \( I^V_B \)-indifference curve is drawn). On the one hand, this reduces profits per \( H \)-type, \( \pi^B_H \), but at (or close to) the efficient level of \( m \), this effect will be of second order. On the other hand, it increases the share of the \( L \)-types, (as the iso-\( P^B_L \)-curves have a lower slope than the \( I^V_B \)-indifference curve), thereby increasing profits.

\[ R \]

\[ m \]

\[ m^* \]

\[ m^A \]

\[ m^B \]

\[ B_0 \]

\[ B_1 \]

\[ I^V_L \]

\[ I^V_B \]

\[ I^V_H \]

\[ Figure 4: \text{Separating equilibrium with two unobservable risk types; } \sigma \text{ large: Contract } B \text{ distorted from } B_0 \text{ to } B_1. \]

\[ ^{26}\]Of course, technically speaking, \( m^B \) is always distorted, as \( P^B_L \) is always larger than zero. However, for low levels of \( \sigma \), \( P^B_L \) is so close to zero, that the distortion of \( m^B \) is negligible. In our example, with \( \sigma = 0.01 \), \( P^B_L \) is on the order of \( 10^{-30} \).
Comparing the effects of a decrease in competition on $m^A$ and $m^B$, we see that as $\sigma$ increases, $m^A$ changes even for low values of $\sigma$, while the effect on $m^B$ only arises above some threshold level of $\sigma$, at which the shaded area ‘reaches’ contract $B$.

**Results 2.** In the separating equilibrium, if $\sigma$ is at an intermediate level, then both benefit packages are distorted: $m^A < m^B < m^*$. A small share of both risk types chooses the contract designated for the other risk type: $P^A_H > 0$ and $P^B_L > 0$. The number of insurers offering the contract designated for the $L$-types increases in $\sigma$.

In the remainder of this section, we illustrate these effects with an example. Table 2 presents the example of Section 2 again, now with the additional variables $P^A_L$, $P^A_H$, $P^B_L$, $P^B_H$ and profits, for the case of 10 insurers. As $\sigma$ increases from 0.01 to 0.08, $P^B_L$ remains zero, so (almost) none of the $L$-types choose insurer $B$; on the other hand, $P^A_H$ increases to 0.0172, so that for $\sigma = 0.08$, 8.62% of the $H$-types choose one of the insurers of type $A$ (see the column $\sum P^A_H$). For $\sigma = 0.13$, already about 25% of the $H$-types choose an insurer of type $A$, while still less than 1% of the $L$-types choose an insurer of type $B$ (since $\sum P^A_L > 0.99$).

<table>
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<th>$n^B$</th>
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<th>$m^B$</th>
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<th>$P^B_L$</th>
<th>$P^B_H$</th>
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The second to last and last column show total profit per insurer of type $A$ and type $B$, divided by $\sigma$. Dividing by $\sigma$ makes it easier to see that profits increase faster for insurers of type $A$ than for type $B$: While $\pi^A/\sigma$ increases in $\sigma$, $\pi^B/\sigma$ decreases, so that at some point, it becomes profitable for one of the insurers of type $B$ to switch and to become an insurer of type $A$. Of course, after the switch, profits per insurer are larger for insurers of type $B$ than for type $A$ (see the row for $\sigma = 0.10$). As profits increase faster for type $A$ than for type $B$, this quickly reverses, so that for $\sigma = 0.13$, profits for type $A$ are already larger than for type $B$ again. If $\sigma$ increases further, then the next insurer of type $B$ will switch and become an insurer of type $A$.

Note that we can have $n^B = 1$ at some high level of $\sigma$, see Table 1 in Section 2. Even with only one insurer of type $B$, there is still enough competition to keep the premium $R^B$ down because insurer $B$ would lose too many individuals to the type-$A$ insurers if $R^B$ was increased.

\footnote{For $\sigma = 0.09$ an equilibrium does not exist.}
3.5 The dependence of the separating equilibrium on \( n \)

We now discuss how the equilibrium depends on \( n \), the total number of insurers. As \( n \) increases, \( n^A \) and \( n^B \) increase proportionally for \( \lambda = \frac{1}{2} \), and almost proportionally for \( \lambda \neq \frac{1}{2} \). Accordingly, all market shares decrease (about) proportionally, which leaves condition (23) unchanged. Also, there is no widening of the shaded areas around the indifference curves. Recall, that as the aggregate \( \sum_{j \neq k} e^{\frac{V_{j}}{\sigma}} \) changes, (which is the case as \( n \) increases), this only shifts the distribution function, but does not change its shape.

The only effect of an increase of \( n \) therefore is on profits (in particular on \( \pi^A_H \)): As \( n \) increases, profits per individual go down. This increases the loss caused by an \( H \)-type, so the incentive to avoid the \( H \)-types increases; see condition (23) where \( m^A \) has to be decreased when \( \pi^A_H \) decreases. Therefore, \( m^A \) decreases in \( n \).

Results 3. In the separating equilibrium, the distortion of the benefit package of the low risk type increases in the total number of insurers: \( \frac{\partial m^A}{\partial n} < 0 \).

3.6 The pooling equilibrium

As has been shown in Section 3.4 when \( \sigma \) increases, the number of type-\( A \) insurers increases. At some point, all insurers will be of type \( A \) and a symmetric equilibrium occurs.\(^\text{28}\)

Using the fact that in this case \( n^B = 0 \) and \( P^A_L = P^A_H = \frac{1}{\pi^A} \), where \( n^A = n \), condition (18), the FOC with respect to \( V^A_L \), simplifies to

\[
\lambda \pi^A_L + (1 - \lambda) \pi^A_H = \frac{n\sigma}{n - 1}.
\]

(24)

Solving this equation for \( R^A \) and substituting in the FOC with respect to \( m^A \), we have

\[
\left[ 1 - \frac{\lambda(1 - \lambda)(p^H - p^L)^2}{\frac{n\sigma}{n - 1} m^A} \right] v'(m^A) = 1.
\]

(25)

Because the fraction in (25) is positive, it is immediately apparent that \( v'(m^A) > 1 \), so that \( m^A \) is distorted downward. As is to be expected, the distortion increases in the difference \( p^H - p^L \). Also, it decreases in \( \sigma \) and increases in \( n \): The distortion in the symmetric equilibrium is less severe if the market is less competitive.

We can now show whether \( m \) is below, at, or above the ‘Wilson’-contract that is characterized by maximizing the utility of the \( L \)-type on the pooling iso-profit line, where, formally, \( m \) satisfies \( p^L v'(m^W) = \overline{p} \). Using conditions (24) and (25) it is straightforward to show that this results in \( \pi^A_H = 0 \), see Appendix A.5. Of course, if profits for the \( H \)-types are zero, \( H \)-types do not play a role when choosing the optimal contract on the iso-profit line, so insurers will maximize the utility of the \( L \)-types (to have as many \( L \)-types as possible). If \( \pi_H < 0 \), it will be profitable to reduce \( m \) along the iso profit line: this will only have

\(^{28}\)We denote insurers to be of type \( A \) in the pooling equilibrium, because the level of medical services offered in the pooling equilibrium is closer to \( m^A \) than \( m^B \) offered in the separating equilibrium; see Table 1 in Section 2 and Table 3 in this section.
Table 3: Example II: $p_L = 0.5$, $p_H = 1$, $\lambda = 0.5$. 

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>4 insurers</th>
<th>$m^A_A$</th>
<th>$m^B_B$</th>
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</table>

a second order effect on the utility of $L$, but a first order effect of reducing the number of $H$-types. If, on the other hand, $\pi_H > 0$, then having more $H$-types increases profits, so insurers will raise $m$ above $m^W$. Table 3 presents an example where the pooling equilibrium is below, at or above the ‘Wilson’-contract, depending on the level of $\sigma$.

**Results 4.** In the pooling equilibrium, the distortion increases in the level of competition: $\frac{\partial m^A_A}{\partial n} < 0$ and $\frac{\partial m^A_A}{\partial \sigma} > 0$. The pooling equilibrium only coincides with the ‘Wilson’-contract if profit per $H$-type is zero: $m^A_A \leq m^W$ for $\pi_H^A \leq 0$.

From a technical perspective, this result shows that in a Rothschild-Stiglitz model under imperfect competition, where a pooling equilibrium is not imposed by assumption, a pooling equilibrium in pure strategies can exist. It is therefore possible to rationalize the pooling equilibrium without imposing Wilson-foresight, a concept that has been criticized by Rothschild and Stiglitz (1997).

Newhouse (1996) had already identified a different reason for a pooling equilibrium to exist, fixed costs of setting up a new contract: If trying to attract the $L$-types with a new contract causes high costs, the symmetric equilibrium is stable.

Here, the argument is somewhat similar, but the costs are of a different kind: Offering a contract between the indifference curves of the two risk types would, under perfect competition, only attract the $L$-types and thereby destroy the symmetric equilibrium. Here, if $\sigma$ is large, a contract close to the symmetric equilibrium attracts both $L$- and $H$-types, where, due to the large influence of the utility component $\varepsilon_{ij}$ that is independent of the benefit-premium-bundle, the relative share of the $L$-types in this new contract is not much larger than the share of $L$-types in the symmetric equilibrium. To only attract the $L$-types, the new contract would have to be far away from the symmetric equilibrium, where the shaded areas of the two indifference curves do not overlap. But if this contract is far away, it would be below the iso-profit line for the $L$-types and thereby not provide a higher profit than the contract in the symmetric equilibrium.
3.7 Welfare effects of a decrease in competition

From what has been derived in the previous section it follows that the welfare effects of a decrease in competition for the separating equilibrium are ambiguous, while for the pooling equilibrium, welfare increases.

For the separating equilibrium we found that an increase in $\sigma$ creates countervailing effects for $m^A$: it may either increase or decrease in $\sigma$. The additional effects we identified, however, clearly decrease welfare: First, $P^A_H$ increases, so that more individuals choose the benefit package with the higher distortion; secondly, $n^A$ increases, so again the number of individuals choosing the benefit package with the higher distortion increases. Thirdly, at some point $m^B$ is distorted. Therefore, if in addition to these three effects, also $m^A$ decreases, welfare unambiguously decreases; if not, the welfare effects are indeterminate. However, in the large number of examples where we derived the equilibrium for a specific utility function explicitly, welfare in the separating equilibrium always decreased.

The welfare effects of a decrease in competition due to a decrease in the total number of insurers are indeterminate as well. It was shown that as $n$ decreases, $m^A$ increases, because the loss associated with the $H$-types decreases as competition decreases. But for the same reason $P^A_H$ increases, creating a countervailing effect on welfare. In addition, because $n^A$ and $n^B$ have to be integer, there will be both up- and downward jumps of the relative share $\frac{n^A}{n}$ as $n$ decreases; (there is, e.g., both an increase and a decrease in either sequence $\frac{5}{5} \rightarrow \frac{5}{4} \rightarrow \frac{4}{4}$ or $\frac{5}{5} \rightarrow \frac{4}{5} \rightarrow \frac{4}{4}$ as $n$ decreases from 10 to 8).

For the pooling equilibrium, on the other hand, as competition decreases, welfare increases unambiguously, see condition (25). This holds for both the increase in $\sigma$ and the decrease in $n$.

To sum up: Should policy makers try to increase competition in health insurance markets? Regarding the distortions of the benefit packages, and from a welfare perspective, there is a definitive answer only for some levels of competition and it is negative: If competition is low, so that a pooling equilibrium emerges, more competition decreases welfare. If competition is high enough, so that a separating equilibrium emerges, there is no definitive answer.

4 Implications for risk adjustment

We now discuss the implications of the results derived so far for risk adjustment. In particular, we show that the welfare effects of introducing or improving a risk adjustment scheme (RAS) critically depend on the level of competition: For low and high levels of competition, a RAS that becomes more precise unambiguously increases welfare. However, for intermediate levels of competition, welfare may initially remain constant or even decrease as the RAS is improved.

We will not model explicitly which risk adjusters are used in the RAS, or which econometric method is applied to estimate the payments. What is important for our model is that whenever a RAS becomes more precise, it reduces the cost difference between the two risk types.
to a larger extent. A RAS can be improved by, e.g., using more and more risk adjusters, like hospital stays, or diagnostic information; a regulator may also apply the formula for optimal risk adjustment developed by Glazer and McGuire (2000). In all cases, the cost difference between risk types will be reduced, and with a perfect RAS, this cost difference is eliminated completely.

We will model the RAS in the easiest way possible: Each insurer receives a payment of $RA^H$ for an $H$-type, and has to pay $RA^L$ for an $L$-type. For the RAS to break even, we have to have

$$\lambda RA^L = (1 - \lambda)RA^H.$$ 

Setting $RA^H$ to some level $RA^H$, this requires $RA^L = \frac{1-\lambda}{\lambda} RA$. In this way, the RAS can be expressed with only one parameter, $RA$. As $RA$ increases, the RAS becomes more precise.

We will first present an example to show how the welfare effects of increasing $RA$ depend on the level of competition. We then explain why for intermediate levels of competition welfare may decrease in $RA$ by analyzing the effects of the RAS-payments for the separating equilibrium in Section 4.2, and for the pooling equilibrium in Section 4.3.

### 4.1 Example

We present the same example as before, with $v(m) = \ln(m)$, $p^L = 0.2$, $p^H = 1$ and $\lambda = 0.5$, and show the impact on welfare by increasing $RA = RA^L = RA^H$ from 0 to 0.4, at which level the cost difference between the $L$-type and the $H$-type is eliminated. Results are shown for 10 and 20 insurers (see Figure 5(a) and (b) respectively), for different levels of competition: $\sigma = 0.01$ (very competitive), $\sigma = 0.10$, $\sigma = 0.12$ and $\sigma = 0.14$ (intermediate levels of competition), and for the lowest level of $\sigma$ for which the pooling equilibrium emerges: $\sigma = 0.19$ for 10 insurers, and $\sigma = 0.20$ for 20 insurers.

The equilibrium values for the level of medical services $m^A$ and $m^B$ and the number of insurers $n^A$ and $n^B$ for one of the cases ($n = 20$ and $\sigma = 0.12$) can be found in Table 5 in Appendix A.6. Here, we only plot the equilibrium levels of welfare as a function of $RA$ for these five different values of $\sigma$. The highest level of welfare for this example is 0.6, which occurs when all individuals receive $m^* = 1$.

As can be seen, for $\sigma = 0.01$ and $\sigma = 0.19$, welfare increases monotonously in $RA$. However, for intermediate levels of competition, welfare stays about constant or even decreases as long as $RA$ is below the threshold level, at which the pooling equilibrium is reached; only above this level, welfare increases monotonously in $RA$. For the case of 20

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29 The Excel-files for the other cases are available from the author upon request.
30 For $\sigma = 0.01$, there is a small decrease in welfare for some high level of $RA$; this is because at this level of $RA$ there is a switch from the separating to the pooling equilibrium.
31 Note that for these intermediate levels of competition, there is usually one level of $RA$ for which an equilibrium does not exist: As we already mentioned in Section 3.2, for one of the candidate equilibria $(m^A, V^A, m^B, V^B)$, one of the insurers of type $B$ has an incentive to become an insurer of type $A$; in the candidate equilibrium for these new levels of $n^A$ and $n^B$, an insurer of type $A$ then has an incentive to become an insurer of type $B$. In Figure 5, we plot the higher of the two levels of welfare of the two candidate equilibria to present the case where the RAS is more successful in improving welfare.
Figure 5: Example III with $p^L = 0.2$, $p^H = 1$, $\lambda = 0.5$ and different levels of $\sigma$. Welfare $W$ is depicted as a function of $RA$, with $RA$ increasing from 0 to 0.40.

(a) 10 insurers

(b) 20 insurers

For intermediate levels of competition welfare initially does not increase in $RA$ because the RAS-payments not only reduce a distortion (by increasing $m_A$), but also introduce or exacerbate two other distortions: As we show in the following section, the share of $H$-types choosing the benefit package designated for the $L$-types increases in $RA$; in addition, the distortion of the benefit package for the $H$-types becomes more severe ($m_B$ decreases).

This contrasts with the case of either a low or a high level of competition, where these additional distortions do not occur or are so small that they are negligible; for these levels of competition, welfare unambiguously increases in $RA$.

4.2 Risk adjustment in the separating equilibrium

Taking into account the payments of the RAS, type specific profits for insurer $A$ are now

$$
\pi^A_L = p^L v(m^A) - V^A_L - \frac{1 - \lambda}{\lambda} RA - p^L m^A
$$

$$
\pi^A_H = p^L v(m^A) - V^A_L + RA - p^H m^A.
$$

The FOCs for insurer $A$’s objective are therefore identical to (18) and (19), but with $\pi^A_L$ and $\pi^A_H$ now defined by (26) and (27). The same applies to insurer $B$.

For insurer $B$, from the FOC with respect to $m^B$ it follows that for low values of $\sigma$ (so that $P^B_L = 0$), we have $v'(m^B) = 1$, as before. From the FOC with respect to $V^B_H$, we have $\pi^B_H = \frac{m^B}{n-1} \sigma$, again as before. If $RA$ is increased, so that insurer $B$ receives a larger subsidy for each $H$-type, premiums are reduced (and utility $V^B_H$ increased) by the same amount, so that $\pi^B_H$ stays constant. For insurer $B$, we can therefore depict an increase in $RA$ by a decrease in $R^B$ of equal size: In Figure 6, the contract offered is shifted from $B_0$...
to $B_1$; accordingly, there is a downward shift of the corresponding iso profit line and the indifference curve.

There is an opposite effect on the premium of insurers of type $A$: as $RA$ increases, this, c.p., increases the premium $R^A$ (and reduces $V^L_A$) by $RA^L = \frac{1}{\lambda} RA$, shifting the iso-profit-line upwards. Similar to the case of an increase of $\sigma$ in Section 3.4 this does not yet constitute the new equilibrium; there will also be an effect on $m^A$.

Figure 6: Equilibrium without and with (imprecise) risk adjustment; the case $RA^L = RA^H$ (i.e. $\lambda = 0.5$) is depicted.

As can be seen from Figure 6 due to the downward shift of the $I^V_H$-indifference curve (to $I^\tilde{V}_H$), and the upward shift of the iso-profit-line of insurer $A$, offering a contract with the same level of $m^A$ reduces the share of $H$-types choosing contract $A$. This also follows immediately from the definition of $P^A_H$, $P^A_H = \frac{V^A_H}{n^A e^{\bar{v}^A} + n^B e^{\bar{v}^B}}$, which decreases as $V^A_H$ decreases and $V^B_H$ increases.

This decrease in $P^A_H$ creates an incentive to increase $m^A$, which can also be seen from the FOC with respect to $m^A$: $v'(m^A) - 1 + \frac{1 - \lambda P^A_H}{\lambda P^A_L} \left[ v'(m^A) - \frac{p_H}{p^L} \right] + \frac{1 - \lambda}{\lambda} \frac{P^A_H}{P^A_L} \left[ 1 - P^A_H \right] \frac{p_H - p^L}{p^L} \pi^A_H v'(m^A) = 0$. (23)

As $P^A_H$ is reduced, $m^A$ has to be increased, so that (23) is satisfied again. In addition, due to the RAS-payment for the $H$-types, $\pi^A_H$ is increased, which – in an equivalent manner as for the case of an increase in $\sigma$ in Section 3.4 – creates a second incentive to increase $m^A$. As $m^A$ unambiguously increases, this, c.p., leads to an increase in welfare.

The effect on $P^A_H$, however, is ambiguous: Assume that $m^A$ is increased to a level so that $P^A_H$ is the same as before. At that point, it is not clear whether there is an incentive to
increase $m^A$ (and thereby $P_{HA}^A$) even further or not. On the one hand, $\pi_{HA}^A$ is increased, but on the other hand, $v'(m^A)$ has already been decreased, so for a general utility function, it is indeterminate whether $P_{HA}^A$ is positive or negative. As the effect of $RA$ on $\pi_{HA}^A$ is linear, while the effect on $v'$ is decreasing, it is likely that $P_{HA}^A$ increases in $RA$, if $RA$ is large. In the large number of examples where we derived the equilibrium for a particular utility function explicitly, $P_{HA}^A$ always increased in $RA$ even from the beginning ($RA = 0$).

The increase in $P_{HA}^A$, if it occurs, captures the first effect that reduces welfare: Each $H$-type choosing contract $A$ instead of contract $B$ induces a loss of welfare, because $m^A < m^B$. In addition, if $P_{HA}^A$ increases, we have the same effects on profits as already described in Section 3.4. Due to the loss of individuals, competition among insurers of type $B$ increases, which reduces profits per individual; together with the smaller market share, profit per insurer of type $B$ decreases. At some point, a type-$B$ insurer will switch and become a type-$A$ insurer. This is the second negative effect on welfare: Each insurer that switches to become an insurer of type $A$ incurs a welfare loss, as all its insured receive $m^A$ instead of $m^B$.

There is a third negative effect on welfare that occurs regardless of whether $P_{HA}^A$ increases or not: We saw that as $RA$ increases, this shifts the $IV_{HA}^B$-indifference curve downwards, and the $IV_{HA}^A$-indifference curve upwards. This will, in similar manner as described in Section 3.4, lead to a distortion of $m^B$ below the efficient level, as soon as the shaded area around the $IV_{HA}^A$-indifference curve ‘reaches’ contract $B$.

Therefore, in addition to the welfare increasing effect of an increase of $m^A$, a RAS that becomes more precise may create these three countervailing effects: a decrease of $P_{HA}^B$, a decrease of $n^B$, and, at some point, a decrease of $m^B$ below the efficient level. Whether these three effects are significant, or only reduce the effectiveness of the improvement of the RAS, of course depends on the specific utility function.

It also depends on the level of $\sigma$: If $\sigma$ is small, the shaded area around the $IV_{HA}^B$-indifference curve will be small. In this case, the density $P_{HA}^B (1 - P_{HA}^A)^{\frac{1}{\sigma}}$ will already be large when $P_{HA}^A$ is still small, so for small values of $\sigma$ the first countervailing effect is greatly reduced. As $P_{HA}^A$ is small, the difference in profits $\pi^A - \pi^B$ is small (see Appendix A.4), so that none of the insurers of type $B$ switches to become of type $A$; then the second countervailing effect does not exist. Thirdly, if $\sigma$ is small, the shaded area around the $IV_{HA}^A$-indifference curve will be narrow, so it will not ‘reach’ contract $B$ until $RA$ is large; for small and intermediate levels of $RA$, the third countervailing effect does not exist either. Therefore, if $\sigma$ is small enough, welfare increases as a RAS becomes more precise unless $RA$ is close to the level at which the cost difference is eliminated. This exception, however, does not seem to be important, because a RAS will usually not be perfect and only eliminate the cost difference between risk types to a certain degree.

32 Of course, when this insurer switches and becomes a type-$A$ insurer, a large share of the $H$-types of this insurer will choose another insurer of type $B$; but those with a high preference for this particular insurer (high $\varepsilon_{ij}$) will stay with this insurer, causing the welfare loss.
4.3 Risk adjustment in the symmetric equilibrium

For the symmetric equilibrium, the FOC with respect to $m^A$ simplifies to

$$\left[1 - \frac{(1 - \lambda)(p^H - p^L)[\lambda(p^H - p^L)m^A - RA]}{n\cdot \overline{p}}\right] v'(m^A) = 1. \quad (28)$$

With $RA = 0$, i.e. without risk adjustment, we have condition (25) from Section 3.4. As $RA$ increases, the fraction in (28) decreases, so $m^A$ increases. With $RA = \lambda(p^H - p^L)m^*$, the distortion is eliminated. For $\lambda = \frac{1}{2}$, as soon as $RA$ equals half the difference in expected costs between the two risk types, the cost difference vanishes; this is because $RA$ both has to be paid by the insurer for an $L$-type, and is paid to the insurer for an $H$-type.

Therefore, for the symmetric equilibrium, an increase in $RA$ unambiguously decreases the distortion and increases welfare.

5 Discussion

In this section, we discuss several of the assumptions of our model and how they may affect the results that have been derived.

5.1 Two risk types

One of the assumptions of the model is that there are only two risk types. This contrasts with the assumption of a continuous distribution of risk types underlying most of the recent empirical papers that estimate the extent of adverse selection in health insurance markets due to inefficient pricing of a given set of contracts, see, e.g., Einav, Finkelstein, and Cullen (2010) or Bundorf et al. (2012). However, in these papers the distribution function basically refers to total health care expenditures; it is not a distribution function of the probability of contracting a specific illness. While individuals may easily be able to distinguish a large number of risk types regarding total health expenditures, so that a continuous distribution is a valid assumption, it is difficult to imagine that individuals can distinguish just as many risk types for the probability of contracting a specific illness, say, diabetes. Here, it may indeed be the case that individuals only distinguish whether they currently suffer from this illness or not, and that all individuals who are not (chronically) ill hold quite similar beliefs about the (small) probability of contracting this illness during the next period. For our setting, where $m$ denotes the level of medical services for a particular disease and not the overall generosity of the benefit package, assuming two risk types may therefore be more appropriate than assuming a continuous distribution.

33 For an early example, see Cutler and Reber (1998).
5.2 Distributional assumption for $\varepsilon_{ij}$

The model has been explicitly solved only under the assumption that $\varepsilon_{ij}$ is i.i.d. extreme value, but we think that the results that were derived also hold for different distributional assumptions. As the main effects have also been explained graphically, the results should be similar as long as the distributional assumption leads to shaded areas around the indifference curves that represent a unimodal density.

We determined the equilibrium under various other distributional assumptions for $\varepsilon_{ij}$ than the extreme value distribution for a large number of examples and always found the results to be very similar.

Table 4 presents the equilibrium values of the example of Section 2 for three distributional assumptions of $\varepsilon_{ij}$ other than the extreme value: the normal, the triangular and the uniform distribution. Even with a uniform distribution for $\varepsilon_{ij}$, the density represented by the shaded area is unimodal; (e.g., this density would be triangular for $n = 2$).

Table 4: Example I with $v(m) = \ln(m)$, $p_L = 0.2$, $p_H = 1$, $\lambda = 0.5$, $n = 10$ for different distributional assumptions

| $n^A$ | $n^B$ | extreme value | | normal | | triangular | | uniform |
|-------|-------|---------------|-------|-------|-------|-------|-------|
|       |       | $\sigma$ | $m^A$ | $m^B$ | $\sigma$ | $m^A$ | $m^B$ | $\sigma$ | $m^A$ | $m^B$ |
| 5     | 5     | .01  | .377 | 1.00  | .01  | .384 | 1.00  | .01  | .386 | 1.00  | .01  | .387 | 1.00  |
| 5     | 5     | .02  | .364 | 1.00  | .02  | .373 | 1.00  | .02  | .376 | 1.00  | .02  | .377 | 1.00  |
| 5     | 5     | .04  | .346 | 1.00  | .04  | .358 | 1.00  | .04  | .360 | 1.00  | .04  | .363 | 1.00  |
| 5     | 5     | .06  | .337 | 1.00  | .06  | .346 | 1.00  | .06  | .349 | 1.00  | .06  | .351 | 1.00  |
| 5     | 5     | .08  | .334 | .998  | .08  | .340 | 1.00  | .08  | .340 | 1.00  | .08  | .343 | 1.00  |
| 6     | 4     | .10  | .324 | .994  | .14  | .330 | 1.00  | .15  | .332 | 1.00  | .21  | .351 | .996  |
| 7     | 3     | .15  | .362 | .940  | .18  | .355 | .994  | .19  | .361 | .999  | .28  | .363 | .981  |
| 8     | 2     | .17  | .395 | .884  | .20  | .380 | .982  | .21  | .380 | .997  | .33  | .389 | .954  |
| 9     | 1     | .18  | .418 | .847  | .22  | .403 | .973  | .23  | .406 | .993  | .38  | .406 | .940  |
| pooling | | .19  | .442 | .442  | .23  | .421 | .421  | .24  | .416 | .416  | .42  | .423 | .423  |

For low values of $\sigma$ (see the upper part of Table 4 with $n^A = n^B = 5$), the differences are very small: For all four distributions, $m^A$ decreases in $\sigma$, while $m^B$ remains at the efficient level. Also $P^A_{H}$, the share of $H$-types choosing one of the insurers of type $A$, is very similar for all four distributions.

As $\sigma$ increases so that $n^A$ increases, two differences emerge: First, the levels of $\sigma$ at which the jumps of $n^A$ occur are not identical for the four distributions, see the lower part of Table 4 where always the smallest value of $\sigma$ after an increase in $n^A$ is presented. E.g., the lowest level of $\sigma$ so that $n^A = 6$ is 0.10 for the extreme value distribution; it is somewhat higher at 0.14 and 0.15 for the normal and the triangular distribution, and considerably higher for the uniform distribution at 0.21. However, this difference does not seem to be important.

34 The Gauss code is available from the author upon request.
35 Note that for all four distributions, the variance is given as $\text{Var}(\varepsilon_{ij}) = \sigma^2 \pi^2 / 6$. 29
Secondly, the distortion of $m^B$ is much smaller for the other three distributions than for the extreme value distribution. This is because for a given level of $\sigma$, the shaded area around the indifference curves is widest for the extreme value distribution; as this distribution has fatter tails, the shaded area around the $I^V_A$-indifference curve reaches contract $B$ for a lower level of $\sigma$ than is the case for the other distributions. In technical terms, the (excess) kurtosis is largest for the extreme value distribution: $k^{ev} = 2.4$; it is considerably smaller for the normal ($k^n = 0.0$), the triangular ($k^{tr} = -0.6$) and the uniform distribution ($k^u = -1.2$). The higher the kurtosis, the higher the distortion of $m^B$ (for a given level of $\sigma$).

On the other hand, the levels of $m^A$ are very similar for the four distributions, as is the level of $m$ when the pooling equilibrium is reached. Also, for each of the four distributions, welfare decreases in $\sigma$ for the separating equilibrium, and increases in $\sigma$ for the pooling equilibrium.

### 5.3 Multinomial Logit vs. Nested Logit

At first glance, it may appear as if for an individual who chooses a type-$A$ insurer, another type-$A$ insurer is a closer substitute than a type-$B$ insurer, so that a nested logit model may seem more appropriate than the simple multinomial logit we considered.

From the perspective of an econometrician, this is certainly true because if he observes $m^A$ to be the same among all type-$A$ insurers, this may indicate that there are also some unobserved factors that are more alike among type-$A$ insurers than between type-$A$ and type-$B$ insurers. The econometrician will simply test whether a nested logit model is more appropriate than the multinomial logit. According to the result of this test, he will then infer whether there are some unobserved factors or not.

Here, however, we want to explicitly analyze the effects that arise due to the differences in the benefit packages. Assuming, in addition, that there are also some unobserved factors which are equal among the type-$A$ insurers, i.e. assuming some non i.i.d.-error term structure, then would only obscure the effects we are interested in.

Regarding the IIA assumption that is implied by the multinomial logit model, the famous red bus-blue bus problem does not occur in our setting, because we explicitly model two different risk types. Consider, e.g., the case of $\lambda = 0.5$ and four insurers: With $\sigma$ small enough, two insurers will be of type $A$, each covering half of the $L$-types, and a small share of the $H$-types, say 1% (i.e. 0.5% of the entire market); the other two insurers will be of type $B$, each covering about half of the $H$-types. Each insurer of type $A$ will therefore cover 25.5% of the entire market, and each insurer of type $B$ 24.5%. If we now add two more insurers of type $A$, these four type-$A$ insurers will not cover two thirds of the entire market, as in the red bus-blue bus example. Instead, all $L$-types are evenly distributed among the four type-$A$ insurers; in addition, the third and fourth type-$A$ insurer will cover about the same share of $H$-types as the first and the second type-$A$ insurer (1% of the $H$-types, or 0.5% of the entire market). Therefore, each insurer of type $A$ will cover about $\frac{1}{4} \cdot 50\% + 0.5\% = 13\%$, and the aggregate market share of all type-$A$ insurers will only increase from 51% to about 52%.

---

5.4 Insurers offering more than one contract

The results have been derived for a setting where each insurer offers one contract. A different interpretation of our model would be that each insurer offered several contracts, but that the demand responses regarding two contracts if offered by two different insurers are about the same as if offered by one insurer.

This would require that not all individuals choose the optimal contract (with respect to the benefit-premium-bundle) among all contracts offered by a particular insurer. For the following reasons, this may indeed be the case: First, not all insured will immediately become aware of when one of the contracts offered by their insurer is changed. Then, there may be fixed costs of switching to a new contract, like filling out an application form, even if it is from the same insurer. Insured may also find it difficult to understand a new contract, regardless of whether it is offered by their insurer or another insurer, and hesitate to switch; see Handel and Kolstad (2013) for these information problems. Also, Sinaiko and Hirth (2011) have shown that some individuals choose a strictly dominated contract even in a situation where they actively have to choose a new contract (so that switching costs do not play a role). Some individuals seem to make mistakes when choosing their health insurance contracts, or, put differently, there exists an error term $\varepsilon_{ij}$ which for some individuals reverses the utility ordering of contracts.

Of course, if insurers could offer several contracts, they would have to take into account the effect of changing one of their contracts on the profit they earn on the other contracts; if $n$ is small, these additional effects may not be negligible and have an impact on the equilibrium.

5.5 Existence of equilibrium

In our model, an equilibrium may not exist for two different reasons. The first reason – a share of $L$-types that is too high – is identical to the model by Rothschild and Stiglitz (1976) and has already been discussed at the beginning of Section 3.3.

The second reason is that $n^A$ and $n^B$ have to be integer numbers. We think that this is a much smaller problem than the nonexistence of equilibrium due to the first reason. Since we explicitly consider the case of imperfect competition, it does not seem likely that an insurer would indeed switch back and forth between being a type-$A$ and type-$B$ insurer. There will certainly be some costs associated with such a switch, so these switches will not occur very often. Note that a much smaller level of transaction costs than assumed by Newhouse (1996) would suffice to stabilize the equilibrium, as in our setting each insurer covers only a share of the $L$-types, while in the setting analyzed by Newhouse (1996), an insurer offering a new contract would be chosen by all the $L$-types (and none of the $H$-types), creating a much larger profit than in our setting.

5.6 Premium set by regulator

We stated in the introduction that we preferred to formulate the model in $m$-$R$-space, and not in $m_1$-$m_2$-space with $R$ set by a regulator, as was the setting of Glazer and McGuire.
We did this for the following reason: As has been shown in Section 3.1, profits increase in $\sigma$. Therefore, a regulator would have to increase $R$ as $\sigma$ increases; if not, welfare would be decreased, as insurers would lower the level of medical services they offer. As we saw in Section 3.4 for the case of unobservable risk types, profits for the two types of insurers increase at a different rate. Therefore, it is not clear at which rate the regulator would have to increase $R$ as $\sigma$ increases: at the rate of insurer $A$, insurer $B$, or some weighted average, and if so, which? To not obscure the welfare effects by an increase in $R$ that could always be considered arbitrary in some sense, the model was presented in $m$-$R$-space. However, all results regarding the distortions of the benefit packages are easily transferred into $m_1$-$m_2$-space. There, a distortion always consists of a too low level of $m_1$ (if $s = 1$ is the illness for which there is heterogeneity in risk) and a too high level of $m_2$, see Glazer and McGuire (2000). The shaded areas would then have to be drawn around the indifference curves in $m_1$-$m_2$-space, but the arguments for the different effects would be the same.

5.7 Total number of insurers fixed

We assumed the total number of insurers to be fixed at $n$. This number could easily be endogenized by considering fixed cost of setting up a new health insurance. Since we examine insurers that are integrated to a certain degree, these fixed cost are probably substantial. However, as we saw in Section 3.1, the main variable to capture different degrees of competition is not $n$, but $\sigma$.

6 Conclusion

We have analyzed the interaction of imperfect competition and adverse selection in health insurance markets. Within a discrete choice setting which endogenizes whether a separating or a pooling equilibrium emerges, the following main results have been derived: First, in a separating equilibrium, for intermediate levels of competition, both benefit packages are distorted. As competition decreases, the distortion decreases for the low risk type, but increases for the high risk type. As the level of competition decreases, the number of insurers offering the contract for the low risk type increases, until a pooling equilibrium is reached. The pooling equilibrium may be below, at, or above the ‘Wilson’-contract.

Our model complements a number of very recent empirical studies which analyze adverse selection in health insurance markets with a focus on inefficient pricing of a given set of benefit packages. These studies have found that the welfare loss caused by inefficient pricing is surprisingly low, see Einav, Finkelstein, and Cullen (2010), Bundorf et al. (2012) and Handel (2013). However, as explicitly stated by Einav, Finkelstein, and Levin (2010), the welfare loss due to an inefficient set of benefit packages may be much larger than the welfare loss due to inefficient pricing.

Our model focuses on these inefficiencies caused by the distortions of the benefit packages. We identified that for intermediate levels of competition, the benefit package of the high risk type will be distorted in a separating equilibrium. The more generous benefit package may
therefore not be an unbiased indicator of the first best level of medical services. Whether this distortion exists and is of economic importance in real health insurance markets, could – as a first step – be tested using the following prediction of our model: In those markets that are less competitive, the relative number of insurers offering the more generous benefit package should be smaller. Such an empirical test should be performed in future research, to determine the validity of the theoretical model that was presented here.

We also determined the implications of imperfect competition on the effectiveness of a risk adjustment scheme. For intermediate levels of competition we identified three welfare decreasing effects that can occur if a RAS that is imprecise is only improved to a small degree. If these effects are of economic importance, it is even more important for a regulator to use a RAS that reduces the cost differences between risk types to a large degree, so that one can be confident that the RAS creates the positive welfare effects it is implemented for.
A Appendix

A.1 Proof that all insurers offer $m^*$ if there is no heterogeneity in risk

An insurer not offering $m^*_s$ for some illness $s$ could always increase its profit by changing $m_s$: If $m_s < m^*_s$, then increasing $m_s$ by some small $dm_s$, and increasing $R$ by $dR = p_s v'(m_s)dm_s$ will leave the utility of all individuals constant, so the group of individuals choosing this insurer does not change; however, profits increase by $dR = p_s v'(m_s)dm_s - p_s m_s > 0$, because $v'(m_s) > 1$ for $m_s < m^*_s$. By the same argument, if $m_s > m^*_s$, profits are increased by reducing $m_s$ and $R$.

A.2 Graph of the distribution function $P^k$

![Graph](image_url)

Figure 7: Distribution function $P^k(V^k)$ and density function $P^k(1 - P^k)$ with $n = 2$ and $\tilde{V} = 1$ for different values of $\sigma$

A.3 Proof that $P^A_H$ increases in $\sigma$

In this section it is shown that $P^A_H$ increases in $\sigma$. To do so, we will first consider a small, noninfinitesimal increase in $\sigma$ by $\Delta \sigma > 0$, which allows to depict some of the effects graphically; we can then let $\Delta \sigma$ become arbitrarily small ($\Delta \sigma \to 0$).

$A \sim$ is used to indicate all variables after the increase of $\sigma$, so, e.g., $\tilde{\sigma} = \sigma + \Delta \sigma$.

We denote by $S^A_L$ and $S^B_H$ the surplus generated by $m^A$ and $m^B$ for the respective risk type, i.e.

$$S^A_L = p^L v(m^A) - p^L m^A \quad \text{and} \quad S^B_H = p^H v(m^B) - p^H m^B.$$  \hspace{1cm} (29)

We begin with insurer $B$. Using the FOC with respect to $V^B_H$,

$$\pi^B_H = \frac{\sigma}{1 - P^B_H}.$$
\[ \pi_B^H = S_B^H - V_B^H, \]

we have
\[ V_B^H = S_B^H - \frac{\sigma}{1 - P_B^H}. \]  \hspace{1cm} (30)

Since for low levels of \( \sigma \), \( m^B = m^* \), and therefore does not depend on \( \sigma \), we have
\[ \tilde{S}_B^H = S_B^H, \] so \( \Delta S_B^H = 0. \)

Under the assumption that \( P_B^H \) does not change, \( \Delta V_B^H \) is given by
\[ \Delta V_B^H = -\frac{\Delta \sigma}{1 - P_B^H}. \] \hspace{1cm} (31)

This decrease of \( V_B^H \) is depicted in Figure 8 by the movement of insurer B’s contract from \( B_0 \) to \( B_1 \).

We now turn to insurer A. We first rewrite the two FOCs with respect to \( V_A^L \) and \( m^A \):
\[ \lambda \left[ \frac{P_A^L(1 - P_A^H)}{\sigma} \pi_A^L - P_A^L \right] + (1 - \lambda) \left[ \frac{P_A^L(1 - P_A^H)}{\sigma} \pi_A^H - P_A^H \right] = 0 \] \hspace{1cm} (32)

\[ \left[ \lambda p_L^A P_A^L + (1 - \lambda)p_L^L P_A^A + (1 - \lambda)(p_H - p_L)^P_A^H (1 - P_A^H) \frac{\pi_A^H}{\sigma} \right] v'(m^A) = \lambda p_L^A P_A^L + (1 - \lambda)p_H^H P_A^H. \] \hspace{1cm} (33)

Condition (33) can be considered as implicitly defining a function \( m^A(V_A^L) \), which determines for each level of \( V_A^L \) the optimal level of \( m^A \). Likewise, condition (32) implicitly defines a function \( V_A^L(m^A) \). The loci of these two curves of course pass through \( A_0 \), the contract offered by insurer A before the increase of \( \sigma \).

With contract \( A_0 \), insurer A will have a certain share of H-types, \( P_A^H \). The set of all the benefit-premium-bundles with which insurer A attracts this share of H-types constitutes the iso-\( P_A^H \)-curve; it has the same shape as the \( I^V_B \)-indifference curve, shifted upwards; see Figure 8.

As \( V_B^H \) is reduced when \( \sigma \) is increased, this shifts the \( I^V_B \)-curve upwards, and with it the iso-\( P_A^H \)-curve. Note that the distance between the two iso-\( P_A^H \)-curves is larger than the distance between the two \( I^V_B \)-curves; this is, because as \( \sigma \) increases, the shaded area around the indifference curves becomes wider; see also Figure 7(a).

If insurer A still offered contract \( A_0 \) after \( \sigma \) has been increased, then \( P_A^H \) would increase. To have the same share as before the increase in \( \sigma \), insurer A would have to offer a contract on the new iso-\( P_A^H \)-curve, which is denoted by \( P_A^H(\tilde{\sigma}) \) in Figure 8.

It is now argued that the new contract chosen by insurer A will be to the right of this new iso-\( P_A^H \)-curve, so that in equilibrium \( P_A^H \) increases. To do so, it will be shown that the locus of the function \( \tilde{m}^A(V_A^L) \), implicitly defined by (33) with \( \sigma \) increased, is partly to the right of the new iso-\( P_A^H \)-curve, and that the new contract is exactly on this part of \( \tilde{m}^A(V_A^L) \).
Consider first, that insurer $A$ offers contract $A_1$, which is on the same iso-$\pi^A_H$-line as contract $A_0$. With $A_1$, in (33) all variables except for $m^A$ and $\sigma$ are at the same level as before. Because $\sigma$ has been increased, which increases the bracket, and because $m^A$ has been reduced, which increases $v'(m^A)$, the left hand side of condition (33) is now larger than the right hand side; therefore, $m^A$ has to be increased, which increases $P^A_H$.

Consider now, instead, contract $A_2$, which has been chosen so that $\tilde{\pi}^A_H(\tilde{\sigma}) = \pi^A_H(\sigma)$. At $A_2$, the bracket on the LHS of (33) attains the same value as before the increase of $\sigma$. At all points on the new iso-$P^A_H$-curve above $A_2$, the bracket is larger than before. In addition, for $\tilde{m}^A < m^A$, we have $v'(\tilde{m}^A) > v'(m^A)$. It can therefore be concluded that for all points on the new iso-$P^A_H$-curve between $A_2$ and $A_3$, the LHS of (33) is larger than the RHS, so $m^A$ has to be increased, which increases $P^A_H$. Condition (33) could only be satisfied for a point below $A_2$, or above $A_3$.

If such a point below $A_2$ or above $A_3$ did not exist, the locus of the function $\tilde{m}^A(\tilde{V}^A_L)$ would always be to the right of the new iso-$P^A_H$-curve; in this case, it follows immediately, that $P^A_H$ is increased. We therefore now consider the case that these points do exist.

Assume first, condition (33) is satisfied for a point below $A_2$. At such a point, we would have $\tilde{\pi}^A_H < \frac{\pi^A_H}{\sigma}$. Condition (32) then requires $\tilde{\pi}^A_H > \frac{\pi^A_L}{\sigma}$, which implies $\tilde{\pi}^A_H > \pi^A_L$. However, for all points below $A_2$, we have $\tilde{\pi}^A_L < \pi^A_L$. Therefore, at such a point below $A_2$, $V^A_L$ is too

\footnote{Note that below $A_2$, the bracket is smaller and $v'$ is larger than before the increase of $\sigma$; above $A_3$ the reverse holds. Therefore, (33) could indeed be satisfied below $A_2$ and above $A_3$.}
high, and has to be reduced.

Assume now, that condition (33) is satisfied for a contract \( A_4 \) above \( A_3 \), see Figure 8. At \( A_4, \Delta m^A > 0 \) und \( \Delta V^L_4 < 0 \). Using \( V^L_4 = V^H_4 - (p^H - p^L)v(m^A) \), we have

\[
\Delta V^L_4 = \Delta V^A_4 - (p^H - p^L)v'(\hat{m}^A)\Delta m^A,
\]

for some \( \hat{m}^A \in [m^A, m^A + \Delta m^A] \). Since \( P^A_4 \) can be rewritten as

\[
P^A_4 = e^{\frac{V^H_4}{\sigma}} = \frac{1}{n^A + n^B e^{\frac{V^B_4 - V^A_4}{\sigma}}},
\]

for \( P^A_4 \) to be identical for both levels of \( \sigma \), we have to have

\[
\frac{V^B_4 - V^A_4}{\sigma} = \frac{\tilde{V}^B_4 - \tilde{V}^A_4}{\sigma},
\]

where

\[
\frac{\tilde{V}^B_4 - \tilde{V}^A_4}{\sigma} = \frac{V^B_4 - V^A_4}{\sigma} + \Delta \sigma.
\]

Solving for \( \Delta V^A_4 \) yields

\[
\Delta V^A_4 = \Delta V^B_4 - (V^B_4 - V^A_4)\frac{\Delta \sigma}{\sigma}.
\]

Using condition (35), \( \frac{V^B_4 - V^A_4}{\sigma} \) can be expressed in terms of \( P^A_4 \) as,

\[
\frac{V^B_4 - V^A_4}{\sigma} = \ln \left( \frac{1}{n^B P^A_4} - \frac{n^A}{n^B} \right).
\]

Substituting in condition (38) yields

\[
\Delta V^A_4 = -\frac{\Delta \sigma}{1 - P^B_4} - \ln \left( \frac{1}{n^B P^A_4} - \frac{n^A}{n^B} \right) \Delta \sigma,
\]

so that for \( \Delta V^L_4 \) we have

\[
\Delta V^L_4 = -\frac{\Delta \sigma}{1 - P^B_4} - \ln \left( \frac{1}{n^B P^A_4} - \frac{n^A}{n^B} \right) \Delta \sigma - (p^H - p^L)v'(\hat{m}^A)\Delta m^A.
\]

We now rewrite condition (32) as

\[
\left[ \lambda P^A_4 (1 - P^L_4) (1 - \lambda)P^A_4 (1 - P^A_4) \right] (S^A - V^L_4)
\]

\[
-(1 - \lambda)P^A_4 (1 - P^A_4)(p^H - p^L)m^A - [\lambda P^A_4 (1 - \lambda)P^A_4] \sigma = 0.
\]

Note that contract \( A_4 \) has to be below \( A_5 \), the contract associated with the efficient level of care: If both \( A_5 \) and \( B_1 \) were offered, almost all \( L \)-types would choose \( B_1 \).
Denote by \(F(\sigma)\) the LHS of (42) evaluated at \(\sigma\), and likewise for \(F(\bar{\sigma})\). If, at \(A_4\), \(F(\bar{\sigma}) > 0\), \(\bar{\pi}_L^A\) is too large and has to be reduced, i.e. \(\bar{V}_L^A\) has to be increased. Since \(F(\sigma) = 0\), \(\bar{V}_L^A\) has to be increased if \(F(\bar{\sigma}) - F(\sigma) > 0\). This difference is given by

\[
\begin{align*}
\left[\lambda P_L^A(1 - P_L^A) + (1 - \lambda)P_H^A(1 - P_H^A)\right] (\Delta S_L^A - \Delta V_L^A) \\
-(1 - \lambda)P_H^A(1 - P_H^A)(p^H - p^L)\Delta m^A - [\lambda P_L^A + (1 - \lambda)P_H^A]\Delta \sigma = 0,
\end{align*}
\]

with

\[
\Delta S_L^A = p^L[v'(\hat{m}^A) - 1]|\Delta m^A,
\]

where \(\hat{m}^A\) is defined as above. Substituting (41) and (44) in (43), we have

\[
\begin{align*}
\left[\lambda P_L^A(1 - P_L^A) + (1 - \lambda)P_H^A(1 - P_H^A)\right] \left[p^L(v'(\hat{m}^A) - 1)\Delta m^A + \frac{\Delta \sigma}{1 - P_H^A}\right. \\
+. \ln \left(\frac{1}{n_B P_H^A} - \frac{n_A^A}{n_B^A}\right) \Delta \sigma + (p^H - p^L)v'(\hat{m}^A)\Delta m^A \right]
\end{align*}
\]

\[
- (1 - \lambda)P_H^A(1 - P_H^A)(p^H - p^L)\Delta m^A - [\lambda P_L^A + (1 - \lambda)P_H^A]\Delta \sigma.
\]

Since \(v' > 1\), expression (45) is larger than

\[
\begin{align*}
\left[\lambda P_L^A(1 - P_L^A) + (1 - \lambda)P_H^A(1 - P_H^A)\right] \left[\frac{\Delta \sigma}{1 - P_H^A} + \ln \left(\frac{1}{n_B P_H^A} - \frac{n_A^A}{n_B^A}\right) \Delta \sigma\right] \right.
\end{align*}
\]

\[
- [\lambda P_L^A + (1 - \lambda)P_H^A]\Delta \sigma.
\]

Solving \(n_A^A P_H^A + n_B^A P_H^A = 1\) for \(P_H^B\) and substituting in (46), expression (46) is positive if

\[
\begin{align*}
\left[\lambda P_L^A(1 - P_L^A) + (1 - \lambda)P_H^A(1 - P_H^A)\right] \left[1 + \left(1 - \frac{1}{n_B} + \frac{n_A^A}{n_B P_H^A}\right) \ln \left(\frac{1}{n_B P_H^A} - \frac{n_A^A}{n_B^A}\right)\right] \\
- [\lambda P_L^A + (1 - \lambda)P_H^A] \left(1 - \frac{1}{n_B} + \frac{n_A^A}{n_B P_H^A}\right) > 0.
\end{align*}
\]

As can be shown numerically, this condition is always satisfied for any values of \(P_H^A\), \(P_L^A\), \(\lambda\), \(n_A^A\) and \(n_B^A\) as long as \(P_H^B < 0.6 P_H^A\) and \(\lambda > 0.08\). Unless the share of \(L\)-types is very low, this condition is therefore satisfied for all reasonable values of \(P_H^A\).

If there exists a point \(A_4\) above \(A_3\), so that (33) is satisfied, condition (32) is violated in a way, so that \(\bar{V}_L^A\) has to be increased. Therefore, the crossing of the two curves \(\hat{m}^A(\hat{V}_L^A)\) and \(\bar{V}_L^A(\hat{m}^A)\) occurs to the right of the new iso-\(P_H^A\)-curve, so \(\bar{P}_L^A(\hat{\sigma}) > \bar{P}_L^A(\sigma)\).

### A.4 Proof that \(n_A^A\) increases in \(\sigma\)

In this section it is shown that as \(\sigma\) increases, the difference in profits \(\pi^A - \pi^B\) at some point becomes large enough, so that it is profitable for a type-\(A\) insuror to become a type-\(B\) insuror. To do so, it is shown that \(\frac{\pi^B}{\sigma}\) decreases (with a lower bound of zero), while \(\frac{\pi^A}{\sigma}\) does not fall below the level when \(P_H^A = 0\).
For \( \pi^B \), we have
\[
\frac{\pi^B}{\sigma} = (1 - \lambda)P^B_H \frac{\pi^B}{\sigma}, \tag{47}
\]
Solving the FOC
\[
(1 - \lambda)P^B_H \left[ (1 - P^B_H) \frac{\pi^B}{\sigma} - 1 \right] = 0
\]
for \( \frac{\pi^B}{\sigma} \) and substituting in (47), we have
\[
\frac{\pi^B}{\sigma} = (1 - \lambda) \frac{P^B_H}{1 - P^B_H}, \tag{48}
\]
so \( \frac{\pi^B}{\sigma} \) decreases as \( P^B_H \) decreases, with a lower bound of zero, i.e.
\[
\frac{\pi^B}{\sigma} \bigg|_{P^B_H \to 0} \to 0. \tag{49}
\]
For insurer \( A \), using \( \pi^A_H = \pi^A_L - (p^H - p^L)m^A \), we have
\[
\frac{\pi^A}{\sigma} = \left[ \lambda P^A_L + (1 - \lambda)P^A_H \right] \frac{\pi^A}{\sigma} - (1 - \lambda)P^A_H (p^H - p^L)m^A. \tag{50}
\]
Solving
\[
\frac{\partial \pi^A}{\partial V^A_L} = \left[ \lambda P^A_L (1 - P^A_H) \right] \frac{\pi^A}{\sigma} + (1 - \lambda) \frac{P^A_H (1 - P^A_H)}{\sigma} \frac{\pi^A}{\sigma} \tag{51}
\]
for \( \frac{\pi^A}{\sigma} \), and substituting in (50) yields
\[
\frac{\pi^A}{\sigma} = \frac{(1 - \lambda)P^A_L (1 - P^A_H)}{\lambda P^A_L (1 - P^A_H) + (1 - \lambda)P^A_H (1 - P^A_H)} \frac{\pi^A}{\sigma} + \frac{(1 - \lambda) \lambda (P^H - P^L)P^A_L P^A_H (P^A_L - P^A_H) m^A}{\lambda P^A_L (1 - P^A_H) + (1 - \lambda)P^A_H (1 - P^A_H)} \frac{\pi^A}{\sigma}. \tag{52}
\]
Now, compare expression (52) with \( \frac{\pi^A}{\sigma} \) for \( P^A_H \to 0 \), (i.e. for \( \sigma \to 0 \)), where
\[
\frac{\pi^A}{\sigma} \bigg|_{P^A_H \to 0} \to \lambda \frac{P^A_H}{1 - P^A_H}. \tag{53}
\]
Note that in this case we have \( \frac{\pi^A}{\sigma} \to \lambda \frac{P^A_H}{1 - n^A} \), where \( n^A \) and \( n^B \) is set so that \( \pi^A = \pi^B \).

It is straightforward to show that for the first fraction of (52),
\[
\frac{(1 - \lambda)P^A_L (1 - P^A_H)}{\lambda P^A_L (1 - P^A_H) + (1 - \lambda)P^A_H (1 - P^A_H)} \frac{\pi^A}{\sigma} >= \frac{\lambda P^A_L}{1 - P^A_H}. \tag{54}
\]
The second fraction of (52) is positive since \( P^A_L > P^A_H \). It can be concluded that \( \frac{\pi^A}{\sigma} \bigg|_{P^A_H > 0} \) is bounded from below by \( \lambda \frac{P^A_L}{1 - P^A_H} > 0 \), see (53), while \( \frac{\pi^B}{\sigma} \) decreases in \( P^B_H \), approaching zero as \( P^B_H \to 0 \), see (49). Therefore, if \( P^B_H \) is small enough, \( \pi^A - \pi^B \) is large enough, so that it is profitable for one of the type-\( B \) insurers to become a type-\( A \) insurer.
### A.5 Comparison of the pooling equilibrium and the ‘Wilson’-contract

Solving condition (24)

\[ R^A - \bar{p} m^A = \frac{n\sigma}{n-1} \]  \hspace{1cm} (55)

for \( R^A \) and substituting in \( \pi_H^A \) yields

\[ \pi_H^A = R^A - p^H m^A = \frac{n\sigma}{n-1} - (p^H - \bar{p}) m^A. \] \hspace{1cm} (56)

Substituting the condition for the ‘Wilson’-contract, \( v'(m^W) = \frac{\bar{p}}{p^L} \), in (25), we have

\[ \left[ 1 - \frac{\lambda(1-\lambda)(p^H - p^L)^2}{n-1} m^A \right] \frac{\bar{p}}{p^L} = 1. \] \hspace{1cm} (57)

Solving for \( m^A \),

\[ m^A = \frac{(\bar{p} - p^L)\frac{n\sigma}{n-1}}{\lambda(1-\lambda)(p^H - p^L)^2}, \] \hspace{1cm} (58)

and substituting in (56) then yields \( \pi_H^A = 0 \). Therefore the pooling equilibrium coincides with the ‘Wilson’-contract for \( \pi_H^A = 0 \).
A.6 Example with risk adjustment

Table 5: Example II with $v(m) = \ln(m)$, $p^L = 0.2$, $p^H = 1$, $\lambda = 0.5$, $n = 20$ and risk adjustment. $\sum S_H$ denotes the sum of expected surplus for the $H$-types, $\sum S_L$ the sum of expected surplus for the $L$-types, with welfare $W$ the weighted average of these two sums: $W = \lambda \sum S_L + (1 - \lambda) \sum S_H$. For the pooling equilibrium, all insurers are denoted as being of type $A$.

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