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The Autocatalytic Character of the Growth of Production Knowledge: What Role Does Human Labor Play?

by

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THE AUTOCATALYTIC CHARACTER OF THE GROWTH OF PRODUCTION KNOWLEDGE: WHAT ROLE DOES HUMAN LABOR PLAY?

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ABSTRACT

This paper analyzes how the qualitative change in human labor occurs in mutual dependence with the advancement of the epistemic base of technology. Historically, a recurrent pattern can be identified: humans learned to successively transfer labor qualities to machines. The subsequent release of parts of the workforce from performing this labor enabled them to spend this spare time in the search for further technical innovations, i.e., the generation and application of ever-more knowledge. A model examines the autocatalytic relationship between the production of commodities and knowledge. The driving forces of these processes and the mechanisms that limit them are analyzed.

Keywords: technological change – long-term economic development – production – productivity growth – labor market

JEL Codes: E23, J24, N30, O30, O40

1. Introduction

Given the historical evidence, continuous qualitative change driven by the expansion of human knowledge is what characterizes economic development. Showing how this historical nature of long-term economic phenomena can be dealt with theoretically is a major challenge for economics (see, e.g., Grossman and Helpman, 1994; Lawson, 1989; Mokyr, 1990; Rosenberg and Frischtak, 1983). In this context, the growth of knowledge and its prerequisites are central themes of a theory of economic change. This paper analyzes a crucial aspect of these developments: in the course of time, the increasing epistemic base of technology significantly affects the qualitative structure of human labor employed in production and *vice versa*: humans learned to transfer certain qualities of labor to machines. At the same time, the release of parts of the workforce from performing these kinds of work provided the basis for the generation and application of ever-more knowledge.

As will be shown in some detail, many of the issues raised here are addressed in the literature on the basis of empirical and historical studies. However, what is usually missing is a theoretical framework that allows researchers to analyze the influence of various factors, structures the empirical findings, and also guides further empirical research. We will argue that a theoretical formulation of the production of commodities and knowledge should be at the center of such a framework. In order to model these production processes, the following aspects are of central importance: first, qualitative differences of labor have to be explicitly considered. In most of macroeconomics, aggregate production theory, human capital theory, and growth theory, human labor is essentially considered to be homogeneous and problems concerning its aggregation are left unaccounted for (see, e.g., Becker, 1993; Grossman and Helpman, 1991; Romer, 1986; Solow, 1970; Schultz, 1974). In doing so, these approaches obscure much of the interesting historical interrelationship between technological advancement and complementary human work. Knowledge-driven qualitative changes – as opposed to merely quantitative developments – appear one of the fundamental determinants of economic development. For the purposes of such an analysis, the concept brought forward in this article will differentiate between different qualities of human skills and their role in production. Moreover, we will take into account the continuous substitution of these different kinds of labor and the implications for economic development. Some recurrent patterns in the long-term development of labor and technology are identified.

Second, technological progress has to be considered in the model. The origins of qualitative changes in human labor are ascribed to a growing epistemic base of technology and *vice versa*.¹ Technological progress is considered to be endogenous, meaning that it is the result of a knowledge production process that simultaneously influences other production processes. The production of technological progress in the form of innovations is examined extensively in the literature related to the innovativeness of firms (see, e.g., Nelson and Winter, 1982). A “knowledge production function” is used that describes the dependence of the innovation output on different factors, including the labor devoted to R&D (see, e.g., Griliches, 1979; Grossman and Helpman, 1991; Jaffe, 1989).

We will combine strands of literature that originate from economic history, production theory, and innovation economics to develop a model of the interrelation between technological progress and the use of different qualities of labor. The resulting model will be used to study in detail the positive feedback mechanism in the generation of knowledge. We will especially address those forces that cause self-augmentation and, thus, the acceleration of knowledge production, as well as those forces that potentially cause the slowing down of technological development.

The paper proceeds as follows: to carry out an analysis of knowledge-driven qualitative change in the realm of human labor, Section two presents selected historical material that exposes the changing role of the various types of human capabilities as labor inputs in long-term economic development.² In addition, the interplay of these inputs with the development of the epistemic base of technology will be shown. In Section three a theoretical model comprising two different kinds of production processes is developed. Section four contains some basic analyses of the dynamic features of the theoretical model. In Section five, the autocatalytic character of knowledge creation and its implications for economic development are addressed. Section six concludes.

¹ The driving forces of technological change at the micro level are not the explicit subject matter of this paper. For theoretical approaches in this direction see Rosenberg (1969), Sahal (1985), Saviotti and Metcalfe (1984), and Cordes (2005b). The term “epistemic base of technology” is borrowed from Mokyr (2002) and denotes the knowledge base of a technological regime.

² The historical examples in this section have been drawn from Cordes (2005a).

2. The changing composition of labor qualities in production

Reflecting on the historical development of the contribution of human labor to production processes, it is evident that its contributions have changed tremendously with respect to quality: before man attained the ability, i.e., the state of technical knowledge, to tap non-human energy sources on a large scale, human muscular power was both a important resource and a constraint on the growth of economic activity (see Weissmahr, 1992). Heavy physical work was the dominant type of human labor employed in early agrarian production. After man obtained, by means of creative mental effort, the knowledge to domesticate animals and to make use of the physical work these animals could provide, human physical labor was, for the first time, substituted by a non-anthropogenic energy source. By lowering the physical labor burden, accelerating work, and raising productivity this released human labor capacities from physical performances.³

In late medieval Europe, the watermill became one of the energetic foundations of contemporary society, where waterpower was used for a host of tasks (see White Jr., 1962, chap. III). Waterwheels were transformed from a device only used for grinding flour, into a ubiquitous source of energy utilized on rivers of every type. There were the first combinations of these non-human energy flows and non-human energy converters with sophisticated mechanical devices. As an example, cloth industry productivity increased dramatically after the invention of the spinning wheel and its combination with flywheels and the belt driven transmission of non-anthropogenic energy setting free great amounts of human physical labor (see Usher, 1954, p. 258ff). By the fourteenth century, Europeans had made extraordinary progress in their efforts to substitute water- and wind-power for physical labor in basic industries (see White Jr., 1962, p. 88).

There is a large amount of historical evidence from inquiries into the development of technology to substantiate the perpetual struggle of man against physical drudgery by means of technical progress (see, e.g., Mokyr, 1990; Cordes, 2005b). Contingent on a) their knowledge of the possibilities to ease or transfer certain sorts of labor to artificial devices (or, in the beginning, to animals), and b) the economic feasibility of these

³ Mammals were the major source for “industrial” power beyond human muscle power, for example, for turning grindstones, threshing, and operating water lifts (see Diamond, 1997, p. 355).

possibilities, man was capable of increasing production volume, of easing the labor burden, or of releasing humans from performing the corresponding tasks. For example, agricultural technology, such as steel breakers, plows, seed drills, reapers, mowers, and threshers, shifted from the use of rudimentary implements to reliance on increasingly sophisticated machinery (see David, 1975, p. 198; Day, 1967). By and by, these new production techniques greatly enhanced productivity and set free human labor, above all physical labor, from agricultural tasks.⁴

Original human work performances, in these cases, qualities of physical work, were relieved by or passed over to artificial devices and animals. Instead, mental work for supervising, attendance, guiding, and caring activities became necessary to a greater extent (see Witt, 1997). In addition, an increasing amount of mental skills were employed to create and implement the contemporary epistemic base of technology. At the same time, technological progress incorporating growing technical knowledge, freed human labor from many tasks, thereby providing the “mental capacities” for the generation and handling of a growing amount of production knowledge. Especially after the invention of the printing press in 1453 by Johannes Gutenberg, information handling abilities as aspects of mental labor qualities became important for a growing proportion of the population (see, e.g., Mokyr, 1990, p. 205; Eisenstein, 1997). Now it was possible for a literature of technology to emerge; technical knowledge became increasingly communicable and cumulative. It became easier to access and apply the epistemic base of technology and to creatively enhance it by recombining existing pieces of knowledge.

During the Industrial Revolution, machines that worked rapidly, regularly, precisely, and tirelessly, in combination with new non-anthropogenic sources of energy – in the first instance the steam engine – substituted human skill and physical effort on a large scale (Landes, 1969, p. 41ff). Sophisticated techniques in machine building, for example, provided equipment to perform sawing, boring, mortising, shaping, and riveting. This genre of machine tools facilitated the construction of highly specialized machinery for manufacturing firms and became progressively more automatic and

⁴ The development of harvesting methods proceeded from sickles, via scythes, to grain cradles, and ultimately to mechanical reapers, which could be combined with non-human sources of energy and which mainly require mental labor inputs (see David, 1975).

productive (see, e.g., Hunt, 1981). Non-creative mental work tasks, such as psychic attention, monitoring, reaction, and supervising of machinery, became more significant in the direct process of production. Simultaneously, as the epistemic base of production grew, these machines required new, highly specialized occupations and tasks, such as provided by inventors, engineers, and technicians, in their creation and implementation (see, e.g., Atack et al., 2004). Increasingly complex instructions and more sophisticated machinery required higher levels of mental skills in surveillance, implementation, and maintenance, while the complementary direct labor inputs to these devices were often frugal. Moreover, instead of craft methods for generating knowledge about production, a set based on scientific and engineering discourse was employed (see for a concrete example Vallas and Beck, 1996). The qualitative structure of employed labor had changed.

From the middle of the 19th century, scientific advances inspired a growing number of technologies in the Western World, so that the former became an important handmaiden of technological progress.⁵ The major technological innovations of this time were enabled by the increasing provision of creative mental labor qualities (Landes, 1969, p. 61; Mokyr, 1990, p. 167ff). At the same time, emerging societal knowledge systems had to organize and manage the growing amount of technical knowledge, i.e., taking both effects together, more and more labor was employed in the production and dissemination of knowledge. A systematic method of gaining knowledge and the drive for scientific insights became institutionalized.

With respect to labor qualities, throughout the twentieth century a rapid and persistent secular growth in the demand for more highly educated workers has been demonstrated. New technologies have been skill-biased with an acceleration in skill-bias beginning in the late 1970's (see, e.g., Berman and Machin, 1995; Chennells and Van Reenen, 1999; Manacorda and Petrongolo, 1999). The largest and most widespread change in the complementary relationship between labor and technology of this era was caused by the introduction of information technology. Hitherto, The spread of computer usage is strongly associated with an overall process of upskilling (see, e.g., Autor et al., 2001; Green et al., 2003).

⁵ Though, the origins of this scientific revolution lie in the Europe of the seventeenth century (see Mokyr, 1990, p. 73ff).

Modern production techniques are characterized by a combination of this technology and sophisticated mechanical devices. Industrial robots are gaining frugal cognitive capabilities, such as sensory perception and orientation in space, thus capable of performing many more tasks that involve aspects of physical work than in the past (see, e.g., Kalmbach and Kurz, 1992, p. 88). Productivity again increased tremendously, as did the amount of knowledge that is incorporated by these machines and that has to be generated and managed by humans providing mental work inputs. There has been a trend toward a higher ratio of information-processing to goods-handling employment (Castells and Yuko, 1994).⁶

Due to these advances in information manipulating technologies, in post-industrial production systems even many aspects of non-creative mental work are delegated to artificial devices. Computer programs execute general frugal problem-solving procedures that are applied to well-defined tasks (Autor et al., 1998). They substitute routine information processing, communication, and coordinating functions performed by clerks, cashiers, telephone operators, bank tellers, bookkeepers, etc. Work processes are restructured by allocating routine symbol processing tasks to computers while separating out tasks that still require human labor qualities. For example, the advancement of information processing techniques promoted the development of computer software as a substitute for much of the human control function in production. These qualities of labor could then be transferred to artificial devices raising productivity and freeing humans from these mental tasks.⁷ Information and communication technologies provide many opportunities to transfer even more elements of mental work involving the acquisition, maintenance, processing, and dissemination of information to electronic automata that permit a multiplication and acceleration of these work tasks. Automated machinery has started to replace man's hands and mind in production. Residual labor is likely to be transferred to work activities that involve

⁶ Zuboff (1988) termed this development the increasing "abstraction" of work. Agents now mainly work with symbolic representations of the production process.

⁷ What is more, modern object oriented CAD software is endowed with artificial intelligence tools to analyze various aspects of a design, such as consistency. For some design problems, such as devising integrated circuits, CAD software can automate the design process itself for a defined problem (see Norman, 1993, p. 95ff).

higher cognitive capabilities, especially tasks requiring higher problem solving skills and creativity.

In the course of a growing epistemic base of technology, gradually more knowledge is needed to handle the existing knowledge reflected, for example, in prolonged time periods dedicated to the acquisition of knowledge, resulting in highly specialized occupations and greater division of knowledge (Mokyr, 2002, p. 9). What is more, the increasing number and accelerating pace of introduction of products, processes, and services and the emergence of problems in the course of innovative activities, inevitably embody an augmented amount of creative mental work in production, itself fueling technological advancement.⁸ To enable these shifts in the qualitative structure of labor inputs, working time has to be set free in other realms of human physical or mental activity by means of productivity gains (Witt, 1997).

3. The model

Our model is based on three basic assumptions: first, we assume a central role of the attained level of production knowledge with respect to understanding changes in the importance of different qualities of labor. Within the scope of this model, we treat the economy on a macro-economic level and propose that one production function can be defined for commodity production. Second, given the historical evidence of the preceding section, a recurrent pattern in socio-economic development is identified: due to human technological creativity, man learned to successively transfer certain qualities of human labor to artificial devices, most of which can be combined with non-human sources of energy. On account of the growing feasibility of first transferring different forms of physical labor and then manifestations of mental work to automata, the role of human labor inputs changed, which is reflected in the complementary qualitative structure of employed human capabilities in production.

Third, we assume that technological progress, i.e., the accumulation of production knowledge, plays a crucial role. Above all, we assume that the extension of the epistemic base of technology can be modeled by a production function as well.

⁸ A technologically progressive and dynamic economy requires skills enabling an employee to adapt to change, to learn, and to introduce new technologies (Nelson and Phelps, 1966).

However, this production function contains another structure of labor qualities than the production function for commodities.

Let us begin with the discussion and set up the commodity production function. We start from a standard production function

$$Y_g = Y_g(K, L)$$

where the total output of goods Y_g depends on the inputs of capital, K , and labor, L . The function $Y_g(\cdot)$ is usually assumed to represent the technological frontier and, thus, technological advancement of the economy. We will explicitly include technological progress in our model so that we can discuss the relation between this production function and the growing epistemic base of technology in more detail. One specific technology implies that with a certain amount and quality of labor, as well as a certain amount of capital, a clearly defined good can be produced. Usually, it is assumed that many different technologies exist to produce a certain good, so that a continuous set of efficient technologies exists. Labor and capital can be substituted in such a situation because one can choose between different technologies. In reality there is no such continuous set of technologies available. Rather, in the long run, economic development can be considered as a sequence of switches between regimes, in which knowledge about the potential of possibilities to transfer certain sorts of labor to artificial devices and their economic feasibility is mirrored in a complementary structure of employed qualities of labor in production.

Due to this important interaction between technological development and the change of labor inputs, we introduce a more complex setting: as stated before, each technology i implies that a certain good can be produced through making use of a certain amount of labor and capital. Hence, for each technology i a fixed factor production function

$$Y_{g,i} = a_i \cdot \min(K, \beta_i L) \tag{1}$$

holds, where a_i denotes the number or value⁹ of goods that can be produced with one unit of capital and β_i denotes the amount of labor that is necessary to use the capital (usually machines) properly. Furthermore, we claim that the growing epistemic base of

⁹ The unit of a_i depends on whether the output of the production process is measured in numbers or in monetary units. For the approach taken here it does not matter how output is measured; the fundamental arguments remain the same.

technology, i.e., the knowledge about the transferability of certain kinds of human labor to artificial devices, crucially determines the complementary qualitative structure of labor employed in production. Simple substitutability between various forms of human labor, as well as between labor and capital, does not exist within a technological regime. We assume that labor input is qualitatively heterogeneous and keeps changing.

For example, to give some historical evidence: in the case of mechanical spinning, inventors sought to mimic human fingers. Replacing the finger by a machine turned out to be an innovation that changed spinning in particular, and production in general, forever (see Mokyr, 1999, p. 21). An originally human task was completely transferred to an artificial device, i.e., the role of human labor inputs changed fundamentally in this process. In another case, skilled machinists who operated a lathe and had to perform tasks involving demanding handicraft, have been replaced with automatic machines, whose working process was controlled by electromechanical devices and later on by computers that require several forms of higher mental work to be provided by the operator. On the one hand, these developments increased capital input and decreased the necessary overall amount of human labor inputs. On the other hand, they changed the qualitative structure of labor involved. As has been shown, in the history of mankind, several such replacements have taken place.

We are interested in the general structure of this process. Each time such a replacement takes place, one kind of labor input is replaced by another kind of labor and new capital. In analyzing this substitution process, it is sufficient to vicariously consider two kinds of labor that we call the prior labor input L_a and the posterior labor input L_p . The production function for a specific technology, i , is thus given by

$$Y_{g,i} = a_i \cdot \min(\kappa_i K, \beta_i L_a, \gamma_i L_p). \quad (2)$$

κ_i , β_i , and γ_i determine the amount of capital and prior and posterior labor inputs that are necessary to efficiently produce α_i units of goods under technology i . According to the assumption that prior labor input is replaced by posterior labor input and new capital, technological progress leads, in the grand total, to lower values of κ_i and γ_i and higher values of β_i . Hence, if Technology 2 is more advanced than Technology 1, $\kappa_2 \leq \kappa_1$, $\beta_2 \geq \beta_1$, and $\gamma_2 \leq \gamma_1$ can be expected to hold. Such a replacement of one labor input by another has a different impact than a simple increase of productivity, as one can see

from the analysis below. Usually, the changes in κ_i , β_i , and γ_i from Technology 1 to Technology 2 will be such that the overall productivity of labor increases:

$$\frac{1}{\beta_1} + \frac{1}{\gamma_1} > \frac{1}{\beta_2} + \frac{1}{\gamma_2}.$$

Finally, we address the growth of the epistemic base of production. Analytically, one can distinguish between three labor-relevant aspects related to the evolving epistemic base of technology: first, there are consequences of applying the epistemic base of technology in production, thus directly affecting the machine operators' working requirements. Second, the creation and implementation of this epistemic base also requires some complementary human labor inputs. Third, the management, i.e., the storing, processing, and disseminating, of the epistemic base of technology demands certain qualities of human labor. The latter two enter the knowledge production process. In the course of a growing amount of production knowledge, increasingly more human mental work effort is needed to acquire, to teach, and to generate knowledge. The possibility to furnish such a knowledge system with the necessary complementary forms of labor is a result of productivity gains in other fields of production, setting free human capacities.

We assume a knowledge production function, as it is common in the literature (see, e.g., Griliches, 1979; Jaffe, 1989). This means that the average amount of technological progress is given by

$$Y_k = A \cdot L^\lambda$$

where A represents the productivity in the knowledge production process and λ is a parameter that determines the elasticity with respect to the labor input that is found in the literature to be around 0.7 (see Jaffe, 1989), so that we assume $0 < \lambda < 1$. Other approaches examine additional input factors. However, since we are especially interested in the impact of labor inputs – and R&D is very labor intensive – other factors are ignored to keep the model simple.

Above, we argued that different kinds of activities, such as acquiring, teaching, and generating knowledge, are involved in the production of technological knowledge. However, to keep the model as simple as possible, we only introduce one additional kind of labor: L_r , involved in research and development, i.e., in increasing the epistemic

base of technology. As the other labor inputs, the amount of L_r is endogenously determined according to the rules below. The wage rate for this kind of labor is denoted by ω_r . The knowledge production function reads:

$$Y_k = A \cdot L_r^\lambda. \quad (3)$$

The knowledge that is produced by this function could be used to increase the productiveness of labor in the research process. However, for simplicity, we assume that the output of knowledge production is only used to improve the technology that is used in commodity production.

We argued above that a set of technologies exists denoted by $T(t)$ at each time t . Each innovation creates a new technology i which is added to this set. Two kinds of innovations have to be distinguished. First, there are innovations that only increase the efficiency of a given production process. This means that output is increased with no change in inputs. According to Equation (2) this would mean that the value of a_i is larger for the new technology than for the old one. If output is measured in monetary units and innovation improves product quality, the same mathematical outcome is obtained. The production of this kind of technological advancement is given by

$$Y_{k,e} = A \cdot L_{r,e}^\lambda. \quad (3a)$$

Second, an innovation might create a new technology that uses less and different labor and more capital as described above. Prior labor is replaced by capital (usually machines) and less posterior labor. This means that the new technology is characterized by a higher value of β_i and a lower value of κ_i and γ_i , thus indicating that less prior labor, along with more capital, and posterior labor are used to produce the same output. The rates of replacement of prior labor by capital, r_c , and posterior labor, r_p , are given by (i denotes the new technology while j denotes the old one)

$$r_c = \frac{\frac{1}{\kappa_i} - \frac{1}{\kappa_j}}{\frac{1}{\beta_j} - \frac{1}{\beta_i}} \quad (4)$$

and

$$r_p = \frac{\frac{1}{\gamma_i} - \frac{1}{\gamma_j}}{\frac{1}{\beta_j} - \frac{1}{\beta_i}} < 1. \quad (5)$$

Equation (4) implies that a decrease of the necessary input of prior labor by one unit causes an increase of the necessary capital input of r_c units. Equation (5) implies that a decrease of the necessary input of prior labor by one unit causes an increase of the necessary posterior labor input of r_p units. $r_p < 1$ results from the fact that the total amount of labor inputs decreases in this process. The production of this kind of technological advancement is given by

$$Y_{k,r} = A \cdot L_{r,r}^\lambda. \quad (3b)$$

In order to keep the model simple, we assume that these rates of replacement, r_c and r_p , are parameters that are constant in time. This means that the same share of prior labor is replaced by the same amount of capital and new labor each time a substitution process occurs.

Finally, we define the step-width of each technological improvement. In the case of an innovation that improves the efficiency of production, the step-width is given by

$$s_e = \frac{\alpha_i}{\alpha_j} > 1, \quad (6)$$

where j denotes the old technology and i denotes the new technology. In the case of an innovation that leads to the replacement of labor inputs, the step-width is given by

$$s_r = \frac{\beta_i}{\beta_j} > 1. \quad (7)$$

Again, for the simplicity of the model, we assume that s_e and s_r are constant in time. This implies that each innovation leads either to an increase of productivity by the same percentage or a replacement of the same share of the actually employed prior labor. Only with an infinite time horizon, the production costs might converge to zero and prior labor might be replaced completely. In the case labor input replacements, this means that each innovation leads to a smaller replacement than the innovation before. This reflects the fact that the first technologies developed lead to a replacement of prior labor in processes where this is very efficient.

Furthermore, constant values of s_e , s_r , r_c , and r_p imply that technology i can be defined by two numbers, n and m , in relation to an initial technology i_0 (defined by a_0 , κ_0 , β_0 , and γ_0). n denotes the number of innovations that have improved the efficiency of production while m denotes the number of innovations that have caused a replacement of labor inputs since the initial technology i_0 . The numbers n and m are assumed to be given by the total research output, $Y_{k,e}$ and $Y_{k,r}$, respectively, that has occurred so far. Hence, the available technologies can be depicted in a two-dimensional space (see Figure 1).

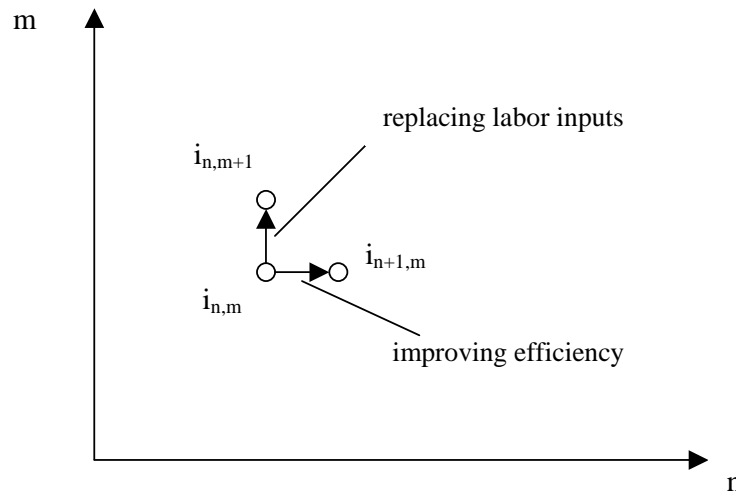


Figure 1: Technological space.

Each technology, $i_{m,n}$, causes certain costs for the production of one unit of goods (given that the optimal amount of inputs is used):

$$c_{n,m} = \frac{1}{\alpha_0 \cdot s_e^n} \cdot \left[\frac{\rho}{\kappa_0} + \frac{\rho \cdot r_c}{\beta_0} + \frac{\omega_n}{\gamma_0} + \frac{\omega_n \cdot r_p}{\beta_0} + \frac{1}{\beta_0 \cdot s_r^m} \cdot (\omega_o - \rho \cdot r_c - \omega_n \cdot r_p) \right] \quad (11)$$

where ρ represents the costs of one unit of capital and ω_o and ω_n represent the wages for one unit of prior and posterior labor, respectively.

4. The dynamics of the model

Above, a model has been set up that describes commodity production, its changes by different technological developments, and the knowledge production process that causes these changes. This model enables us to calculate the optimal production process for each time period given the technologies available. However, we are interested in the interaction between the accumulation of production knowledge and changes in labor inputs. This means that we want to understand to what extent the change of the quality of labor inputs and technological development are interlinked and the kind of dynamics this mutual dependence causes.

We try to keep the analysis as simple as possible through focusing on the central factors and ignoring others. To this end, a number of assumptions are introduced.

ASSUMPTION 1: ρ , ω_o , ω_n and ω_r are constant in time.

Wages and capital costs definitely have an impact on technological development and the labor inputs in the production process and also change in time in reality. However, this should be discussed in a separate paper. Here, we intend to focus on the replacement of labor inputs caused by technological advancement that seems to be crucial in a long-term perspective. We also exclude from the modeling the precise actions of governments, meaning the collection of taxes and the financing of public research, and the capital market. Thus, we do not care about which wages are paid by whom and who consumes. We see the economy in total and only care about the distribution of the available labor to the different production processes:

ASSUMPTION 2: *The total amount of the labor input L , given by $L=L_a(t)+L_p(t)+L_r(t)$, is constant in time, while each labor input, $L_a(t)$, $L_p(t)$ and $L_r(t)$, varies endogenously in time.*

Assumption 2 implies that we consider an economy with a fixed population where the same share is always active in production processes, thus neglecting population growth and unemployment. Population growth could be represented by a scaling factor that would not alter the characteristics of the model, although it would, of course, increase the growth rates by the amount of population growth. The assumption of a constant share of working people is more problematic as it is related to the assumption of constant wages. It is assumed that all people who can work do work and that they solely

choose their occupation. This idealization seems to be inadequate for times when unemployment and wage differences are major issues. However, since we are taking a long-term perspective, ignoring these issues seems acceptable and simplifies the analysis significantly.

ASSUMPTION 3: In the production of commodities in each time period the type of technology that is used allows for the cheapest production of one unit of goods.

Assumption 3 implies that producing entities maximize their profits and are able to immediately switch to new technologies. Hence, neither costs nor time necessary for the adaptation of production sites to new technologies are considered here.

ASSUMPTION 4: The total output of production $Y(t)$ is used for consumption $C(t)$ and the production of capital goods:

$$C(t) = Y_g(t) - \delta \cdot K(t) \quad (12)$$

where δ denotes the depreciation rate of capital.

It is assumed that an increase of capital input in the production process from time t to time $t+1$ requires a production of the respective capital goods in the period before. Furthermore, a certain share of capital goods has to be replaced. Only the remaining part of production can be consumed. Again, for simplicity, we refrain from explicitly modeling savings and the capital market.

ASSUMPTION 5: Below a certain consumption level C_0 – the subsistence level – all output $Y_g(t)$ is used for consumption. Above this level the agents of the economy are willing to trade current consumption $C(t)$ for future consumption $C(t+1)$. We assume a two-generation model so that the valuation of consumption by a young person is given by

$$V_y(t) = C(t)^\phi + \psi_f \cdot C(t+1)^\phi$$

while the valuation by a old person is given by

$$V_o(t) = C(t)^\phi.$$

If there is a certain constant share of the population that reaches the second phase of life, the total evaluation is given by

$$V(t) = \psi_c \cdot C(t)^\phi + \psi_f \cdot C(t+1)^\phi. \quad (13)$$

This evaluation, $V(t)$, is maximized, where ϕ , ψ_c and ψ_f are parameters. In calculating the consumption in the next period, decision makers are assumed to expect that the same share of labor will be used for the same kinds of research. Hence, without knowing about the decisions that will be made in the next period, future consumption is assumed to increase proportionally with future output.

The first part of Assumption 5 applies to periods in human history when people struggle to satisfy their basic needs such as food, shelter and heat. During these periods, they are not very concerned with technological development to improve the future situation. A rise in agricultural production and the satisfaction of basic needs are crucial prerequisites to provide subsistence for a larger number of people not mainly engaged in agriculture, but, for example, innovative activities (see, e.g., White Jr., 1962, ch. II; Grigg, 1980). Including this in the model allows to examining the transition and the differences between ancient and modern societies. The second part of Assumption 5 models the behavior of the whole economy in the standard way of an overlapping generations model (see, e.g., Geanakoplos, 1991; Pecchenino and Pollard, 1997).

The assumptions made above are sufficient to solve the complete model and examine the resulting dynamics. This will be done in the following way: firstly, the amount of effort put into both types of innovative activity and the overall amount of research conducted will be calculated. Secondly, we show and how output, consumption, the qualitative structure of labor inputs, and the epistemic base of production will change over time. In this analysis it is assumed that a fundamental innovation occurs at time $t=0$ that allows, for the first time, to replace prior labor inputs with new capital and posterior labor inputs. This means that we assume the specific labor replacements we want to study will become feasible at time $t=0$. This implies that no posterior labor inputs and no capital inputs are used in production at $t=0$: $\gamma(0)=\infty$ and $\kappa(0)=\infty$.

According to the model, research can have two aims: it can either increase the efficiency of the production process or it can lead to new technologies that allow for the replacement of one kind of labor by another as well as new capital. We first address the question of how much labor is put into each of these two goals. For this purpose, the valuation $V(t)$ given in Equation (13) is maximized. First, we obtain a condition for the conduction of research to replace labor inputs:

LEMMA 1: *Let Assumptions 1-5 hold. Then, labor is put into research to replace prior labor inputs by posterior labor inputs and capital if and only if*

$$(1-r_p) \cdot \left(\frac{\alpha(t)}{1+\delta} - \frac{1}{\kappa(t)} \right) > r_c \cdot \left(\frac{1}{\gamma(t)} + \frac{1}{\beta(t)} \right) \quad (14)$$

and

$$\frac{\alpha(t) \cdot \beta(t) \cdot \gamma(t)}{\beta(t) + \gamma(t)} \cdot L \cdot \left(1 - \frac{\delta}{\alpha(t) \cdot \kappa(t)} \right) > C_0. \quad (15)$$

The proof of Lemma 1 is given in the appendix. Equation (15) represents the condition that as long as the economy produces below its subsistence level, no research is conducted. The factor before total labor input L on the left-hand side of Condition (15) represents labor productivity in production. The model implies that labor inputs would never be spent on research that would decrease productivity. As a consequence, labor productivity always increases or stays constant. Hence, two situations might occur. First, Condition (15) is satisfied at the beginning of the analyzed period of time ($t=0$). Then, Condition (15) is also satisfied at any time $t>0$. Second, Condition (15) is not satisfied at the beginning. Then, no research is conducted, $\alpha(t)$, $\beta(t)$, and $\gamma(t)$ will remain constant and Condition (15) will never be satisfied.

Condition (14) is more complex to be interpreted economically. It basically states that prior labor inputs have to be replaced by small amounts of posterior labor inputs (r_p small so that the left-hand side of (14) is sufficiently large) and by small amounts of capital (r_c small so that the right-hand side of (14) is sufficiently small).

A different situation is obtained for the conduction of research to make production processes more efficient:

LEMMA 2: *Let Assumptions 1-5 hold. Then, labor is put into research to increase the efficiency of production processes if and only if*

$$\frac{\alpha(t) \cdot \beta(t) \cdot \gamma(t)}{\beta(t) + \gamma(t)} \cdot L \cdot \left(1 - \frac{\delta}{\alpha(t) \cdot \kappa(t)} \right) > C_0.$$

The proof of Lemma 2 is given in the appendix. Lemma 2 implies that in the case of research increasing the efficiency of production processes, only the subsistence level matters. If the economy is able to produce more than needed for subsistence, this kind of research is conducted, at least, to some extent. The economic intuition behind this is

that an increase of production efficiency is always favorable while a replacement of labor inputs includes costs for capital and, thus, is not always favorable.

Of course, the analysis conducted here is of little interest if the economy cannot conduct research because the subsistence level cannot be reached or if the economy does not conduct research to replace labor inputs. Thus, we are only interested in a time at which Conditions (14) and (15) are satisfied, at least, at the beginning ($t=0$). In order to examine the dynamics of such an economy, the amount of labor that is put into the two kinds of research has to be calculated. The result is given in the Appendix by the Equations (A9), (A10), and (A12). We will discuss the most important features in the following. All proofs are given in the appendix.

We first study an economy that is well above its subsistence level. This means that the use of available labor that maximizes the valuation of the situation, according to Assumption 5, leads to a labor input in production that satisfies Condition (A11). Hence, Equations (A9) and (A10) give the use of labor inputs.

THEOREM 1a: *Let Conditions (14) and (15) hold for the optimal assignment of labor inputs. Then, $L_{r,e}(t)$ decreases indefinitely with time.*

Theorem 1a states that after the fundamental innovation occurred that allows for the replacement of labor inputs and although effort is put into both kinds of research at the beginning, with time research becomes increasingly concentrated on the improvement of production efficiency that continues for ever, while the replacement of certain labor inputs comes to an end. In order to enhance research on the replacement of labor inputs, a new fundamental innovation is necessary.

THEOREM 1b: *Let Conditions (14) and (15) hold for the optimal assignment of labor inputs. Then, the share of labor, $L_p(t)$, put into production in the form of posterior labor inputs and the share of labor, $L_{r,e}(t)$, put into research on efficiency converge to positive values while the share of labor, $L_a(t)$, put into production in the form of prior labor inputs decreases to zero.*

Theorem 1b states that in the long run a fixed share of the available labor force is put into research that is completely focused on the increase of the efficiency of production processes, while all prior labor inputs have been replaced by posterior – meaning upskilled – labor inputs. However, this replacement may take very long since a decreasing amount of labor is put into research on this replacement. Furthermore, these

results are obtained under the assumption that wages are constant and there is no problem in upskilling all labor inputs.

THEOREM 1c: *Let Conditions (14) and (15) hold for the optimal assignment of labor input. Then, the growth rate of the economy, given by $(Y(t+1)-Y(t))/Y(t)$, starts from a value determined by the initial state,*

$$g(0) = \frac{Y(t+1) - Y(t)}{Y(t)} \Big|_{t=0} = \frac{s_e^{Y_{r,e}(0)} \cdot s_r^{Y_{r,r}(0)}}{1 + r_p \cdot (s_r^{Y_{r,r}(0)} - 1)} - 1. \quad (16)$$

and converges toward a lower basic value g_b given by

$$g(\infty) = \frac{Y(t+1) - Y(t)}{Y(t)} \Big|_{t \rightarrow \infty} = s_e^{Y_{r,e}(\infty)} - 1. \quad (17)$$

A general comparison of the initial growth rate, $g(0)$, and the final growth rate, $g(\infty)$, is impossible because it depends on too many parameters in a complex way. However, assuming r_p to be small – meaning that much prior labor inputs are replaced by little posterior labor inputs – makes things easier. It causes the values of $Y_{r,e}(0)$ and $Y_{r,r}(0)$ to be similar (see Equation (A10)). Hence, the initial growth rate is much higher than the final growth rate. In other words, a fundamental innovation that allows for the replacement of much prior labor inputs by small amounts of posterior labor inputs gives the economy a push, so that after this innovation a higher growth rate can be observed. With time, the growth rate converges back to the growth rate observed before this fundamental innovation.

A simulated example of such a development is given in Figure 2.

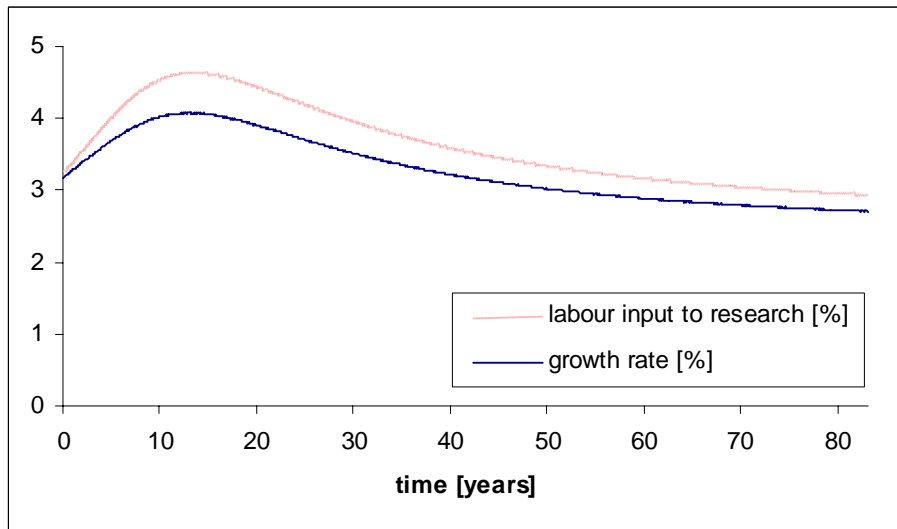


Figure 2: Simulated dynamics of the labor input $L_{r,r}(t)$ and the growth rate, $g(t)$, for one exemplary set of parameters ($\alpha(0)=1$, $\beta(0)=1$, $\gamma(0)=10,000,000$, $\kappa(0)=10,000,000$, $\lambda=0.7$, $\delta=0.003$, $r_c=0.3$, $r_p=0.8$, $se=1.005$, $sr=1.05$, $\phi=0.7$, $\psi_c=1.7$, $\psi_f=0.7$, $A=0.0014$, $L=1000$, $C_0=0$ and people living 60 years).

A completely different situation is obtained if an economy is just above the subsistence level.

THEOREM 1d: *Let the value of $L_g(t)$ that results from Equations (A9) and (A10) be to small to foster consumption above the subsistence level, although Condition (15) is satisfied, and let the initial efficiency of production $\alpha(0)$ be very small:*

$$\alpha(0) < \frac{r_c \cdot (1 + \delta)}{\beta(0) \cdot (1 - r_p)}. \tag{18}$$

Then, at the beginning ($t=0$) only research to improve the efficiency of production is conducted. The replacement of labor inputs takes place after this initial phase.

Theorem 1d states that a certain level of economic development, modeled as a certain efficiency of production, has to be attained before labor inputs are replaced. A society that produces just above the subsistence level with a low efficiency is not able to replace labor inputs even if they would be technologically feasible. This hinders growth as the following theorem shows.

THEOREM 1e: *Let the value of $L_g(t)$ that results from Equations (A9) and (A10) be to small to foster consumption above the subsistence level, although Condition (15) is*

satisfied, and let the initial efficiency of production $\alpha(0)$ be very small (see Condition (18)). Then, the growth of the economy starts from a value below $g(\infty)$. It increases with time. Two situations might occur: The growth rate might increase above $g(\infty)$ and then converge toward $g(\infty)$ from above or it might never reach $g(\infty)$ and converge to this value from below.

Theorem 1e implies that an economy just above the subsistence level is able to grow only very slowly. It takes some time before the small labor input that can be assigned to research causes a significant improvement of productivity. Only then, more labor can be used for research and growth can speed up. An example of the resulting dynamics is given in Figure 3.

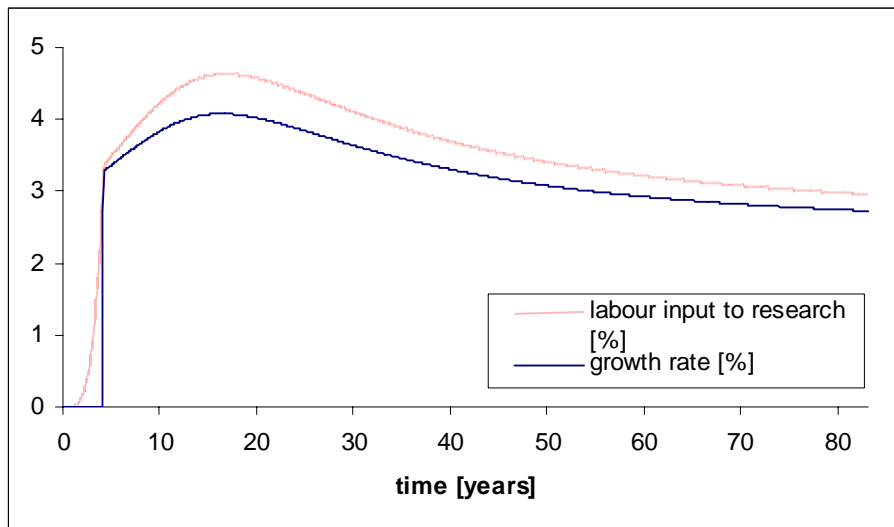


Figure 3: Simulated dynamics of the labor input $L_{r,r}(t)$ and the growth rate, $g(t)$, for one exemplary set of parameters ($\alpha(0)=1$, $\beta(0)=1$, $\gamma(0)=10,000,000$, $\kappa(0)=10,000,000$, $\lambda=0.7$, $\delta=0.003$, $r_c=0.3$, $r_p=0.8$, $s_e=1.005$, $s_r=1.05$, $\phi=0.7$, $\psi_c=1.7$, $\psi_f=0.7$, $A=0.0014$, $L=1000$, $C_0=999.9999$ and people living 60 years).

5. Some implications of the autocatalytic character of knowledge production

The accumulation of production knowledge is decisive for the pace of economic development. New production knowledge enables a reduction in working time for the same output and sets free spare time and effort for the systematic search for, and application of, further innovations in tools, appliances, and energy uses again raising

productivity (see Witt, 2004). Moreover, a growing amount of working time can then be spent for the acquisition of more and more specialized knowledge and skills resulting in an augmented division of labor essentially enabling increasing returns.

A prerequisite for these self-augmenting or autocatalytic economic developments are innovative activities within the economy that facilitate a successive transfer of human labor qualities to machines combined with non-human energy sources setting free human labor time that can be used for cognitive work such as research and development. According to Theorem 1d it is necessary for the replacement of labor inputs to occur that production capacities are expanded quite above the supply of the subsistence level. In the beginning, some excess labor is necessary to improve the efficiency of production. This in turn makes the replacement of labor inputs possible. It has been shown that all this works at an increasing speed, at least for some time. These first steps toward releasing parts of the population from spending the bulk of their time performing physical work have enabled humankind to generate and apply ever-more knowledge. Of course, the released working time could, and has, also be spent on leisure or unnecessary consumption that delays a potential growth of the economy. If it is spent on knowledge creation, a kind of autocatalytic cycle results that causes an increasing growth rate as is predicted by Theorem 1e. The growing epistemic base of production creates the prerequisites for its own further extension.

On the one hand, such a development increases capital input and decreases human labor input. On the other hand, it changes the quality of labor involved. The prior kind of labor might be completely replaced by the posterior kind of labor and capital as predicted by Theorem 1b. As has been shown above, in the history of mankind several such replacements have taken place. Further implications of these processes can be shown: the possibility to ease or replace a certain quality of human labor with mechanical appliances or electronic automata permitted a multiplication and acceleration of tasks formerly done by humans. The resulting productivity gains triggered periods of strong economic growth. However, according to Theorem 1a the magnitude of research on a particular replacement process of one kind of labor by another kind of labor slowly disappears with time. As a consequence, the growth rate converges to the long-term rate given by Equation (17) in Theorem 1c. However, each time a replacement of one kind of labor input becomes feasible, the growth rate might increase for a certain period of time before it falls back to the long-term value again.

Given the historical evidence, the dynamics of capitalist economies over long periods experience significant long-term variations in their aggregate performance (see, e.g., Kuznets, 1940; Rosenberg and Frischtak, 1983). The successively growing knowledge about the transferability of certain qualities of human labor to machines and the corresponding technological innovations contribute to an understanding of the generative forces underlying long cycles in economic growth that exhibit the effects of the above-mentioned autocatalytic development. Freeman and Perez (1988) have suggested some characteristics of successive long waves of economic growth: first, they identify the “early mechanization wave” of the 1770s to 1840s. In that period, the attained technical knowledge allowed for the development of many sophisticated mechanical devices employed, for example, in the domestic industries (Cerman and Ogilvie, 1996; Goldin and Katz, 1998). These contrivances enabled the substitution of demanding handicraft for frugal psychomotor skills by fulfilling their tasks almost automatically. Second, during their “steam power and railway wave” of the 1840s to 1890s machines in combination with the new non-anthropogenic sources of energy substituted for human skill and physical effort on a large scale (Landes, 1969, p. 41ff). Hence, progress in transportation technologies set free further human labor.

Third, Freeman and Perez identify the “electrical and heavy engineering wave” of the 1890s to 1940s. Electromechanical devices, for instance, started to enable the replacement of more complex performances involving demanding handicraft by (semi-) automatic machines. During a fourth “Fordist mass production wave” from the 1940s to 1980s production became more routinized and undifferentiated labor was employed to perform tasks on extensive machinery that was easy to operate, while enabling massive gains in productivity (see, e.g., Mokyr, 2002, p. 111). A last “information and communication wave” from the 1980s entailed advances in information manipulating technologies that made possible the delegation of facets of human non-creative mental labor to artificial devices associated with an acceleration and multiplication of these tasks.

In the course of this long wave development, the amount of knowledge embodied by machinery, products, and human agents increased continuously: man accumulated knowledge about the transferability of work to artificial devices combined with non-human energy sources or about improving the efficiency of existing production processes. Resulting productivity gains set free labor resources that can be devoted to

the handling of the increasing stock of societal knowledge, mainly enabled by an emerging educational infrastructure, and that can be employed in a growing institutionalized private and public R&D sector adding ever-new pieces of technical knowledge. These are the ingredients of an endogenous autocatalytic economic development manifesting itself in an accelerating pace of technological progress during these periods.

There is another insight gained from the analysis in this paper: most of economic growth theory and aggregate production theory include the possibility that skilled labor is a more or less perfect substitute for unskilled labor and that both are substitutable with capital. In an equilibrium framework, the skill-biased technological change over the past 60 years and the unskilled biased technological progress during the late eighteenth and early nineteenth centuries, as exemplary cases, are explained by referring to reversible price and market size effects (see, e.g., Acemoglu, 2002). In contrast, according to the approach brought forward in this paper, the growing epistemic base of technology, i.e., the knowledge about the transferability of certain kinds of human labor to artificial devices, crucially determines the complementary qualitative structure of labor employed in production. Simple substitutability between various forms of human labor and between labor and capital does not exist within a technological regime.

6. Conclusions

This paper has delivered an analysis of knowledge-driven qualitative change in the realm of human labor in mutual dependence with the development of the epistemic base of technology. With the help of selected historical material, the role of various types of human capabilities in long-term economic development has been presented along with the interplay of these inputs with technological advancement. Historically, an outstanding recurrent pattern of economic development has been identified: humans found ways to successively transfer qualities of labor to artificial devices that can be combined with non-human energy sources, a process that started with tools and ended up with automata that are even capable of adopting cognitive tasks from man.

In the long run, economic development may be stylized as a sequence of switches between regimes in which the knowledge about the potential and the economic feasibility of possibilities to transfer certain sorts of human labor to artificial devices is

mirrored in a characteristic complementary structure of employed labor qualities in production. The socio-economic consequences of this expression of technological creativity were manifold: the possibility of easing or replacing a certain quality of human labor with mechanical appliances and electronic automata permitted a multiplication and acceleration of respective tasks setting free human labor that could then be employed for the generation of new production knowledge and the handling of the growing amount of existing knowledge.

New production knowledge enabled a reduction in working time necessary for the same amount of output setting free spare time, cognitive capacities, and human effort for the systematic search for, and application of, further innovations in tools, appliances, and energy uses again raising productivity. A model comprising a production function for commodities and a knowledge production function has shown that innovative activities within the economy that facilitate a successive transfer of human labor qualities to machines can trigger periods of self-augmenting or autocatalytic economic developments. Releasing human agents from performing certain work has enabled humankind to generate and apply ever-more production knowledge, which is decisive for the pace of economic growth.

Appendix

Proof of Lemma 1:

In order to examine whether research to replace labor inputs is conducted, the gain in valuation by assigning labor to this kind of research has to be calculated and compared to the alternative uses of labor. Hence, we calculate the change in valuation for each of the three possible uses of labor: research on replacing labor inputs $L_{r,r}$, research on increasing the efficiency of production $L_{r,e}$, and producing L_g . The valuation that is considered while deciding about the use of the available labor is given by Assumption 5. An increase of $L_{r,r}(t)$ has no impact on the production output $Y(t)$ and thus on the consumption $C(t)$, but it influences the technology available for production in the next period and thus $Y(t+1)$ and $C(t+1)$. The research output $Y_{r,r}(t)$ at time t influences the values of $\beta(t+1)$, $\gamma(t+1)$ and $\kappa(t+1)$. Thus,

$$\frac{dV(t)}{dL_{r,r}(t)} = \frac{dV(t)}{dC(t+1)} \cdot \left[\frac{dC(t+1)}{d\beta(t+1)} \cdot \frac{d\beta(t+1)}{dL_{r,r}(t)} + \frac{dC(t+1)}{d\gamma(t+1)} \cdot \frac{d\gamma(t+1)}{dL_{r,r}(t)} + \frac{dC(t+1)}{d\kappa(t+1)} \cdot \frac{d\kappa(t+1)}{dL_{r,r}(t)} \right].$$

The first derivative on the right-hand side can be calculated from Equation (13). In order to calculate the second derivative, the following considerations are helpful: Assumption 3 implies that $\beta(t)L_a(t)=\gamma(t)L_p(t)=\kappa(t)K(t)$ holds at any time. The total amount of labor used for the production of goods is denoted by $L_g(t) = L_a(t)+L_p(t)$. Hence, the total production of goods can be written as

$$Y_g(t) = \frac{\alpha(t) \cdot \beta(t) \cdot \gamma(t)}{\beta(t) + \gamma(t)} \cdot L_g(t). \quad (A1)$$

Furthermore, $\alpha(t)\kappa(t)K(t)=Y_g(t)$ holds. Inserting this into Equation (12) we obtain

$$C(t) = \frac{\alpha(t) \cdot \beta(t) \cdot \gamma(t)}{\beta(t) + \gamma(t)} \cdot L_g \cdot \left(1 - \frac{\delta}{\alpha(t) \cdot \kappa(t)} \right). \quad (A2)$$

With the help of Equation (A2) the second, fourth and sixth derivative on the right-hand side of the above equation can be calculated. Finally, Assumption 1 implies that always the technology with the highest value of n and m is applied and all research aims to improve this best technology further. Thus,

$$\beta(t+1) = \beta(t) \cdot s_r^{A \cdot L_{r,r}(t)^2}. \quad (A3)$$

Inserting Equations (4) and (5) leads to

$$\gamma(t+1) = \frac{1}{\frac{r_p}{\beta(t)} \cdot \left(1 - s_r^{-A \cdot L_{r,r}(t)^2} \right) + \frac{1}{\gamma(t)}} \quad (A4)$$

and

$$\kappa(t+1) = \frac{1}{\frac{r_c}{\beta(t)} \cdot \left(1 - s_r^{-A \cdot L_{r,r}(t)^2} \right) + \frac{1}{\kappa(t)}}. \quad (A5)$$

The Equations (A3), (A4) and (A5) can be used to calculate the remaining derivatives in the above Equation. It results

$$\frac{dV(t)}{dL_{r,r}(t)} = \frac{\phi \cdot \psi_f \cdot C(t+1)^\phi \cdot \ln s_r \cdot A \cdot \lambda \cdot (1+\delta)}{L_{r,r}(t)^{1-\lambda} \cdot \alpha(t+1) \cdot \beta(t+1)} \cdot \left(\frac{\left[\frac{\alpha(t+1)}{1+\delta} - \frac{1}{\kappa(t+1)} \right] \cdot (1-r_p)}{\frac{1}{\gamma(t+1)} + \frac{1}{\beta(t+1)}} - r_c \right). \quad (A6)$$

For $L_{r,r}(t) \rightarrow 0$ the right-hand side of Equation (A6) goes to infinity if the term in the bracket is positive. Otherwise, the right-hand side goes to minus infinity.

In the case of changing labor input $L_{r,e}(t)$ to research for the sake of increasing production efficiency, things are simpler because the resulting technological developments only change $\alpha(t+1)$. Thus,

$$\frac{dV(t)}{dL_{r,e}(t)} = \frac{dV(t)}{dC(t+1)} \cdot \frac{dC(t+1)}{d\alpha(t+1)} \cdot \frac{d\alpha(t+1)}{dL_{r,e}(t)}.$$

The calculation can be done as above and we obtain

$$\frac{dV(t)}{dL_{r,e}(t)} = \frac{\phi \cdot \psi_f \cdot C(t+1)^\phi \cdot \ln s_e \cdot A \cdot \lambda}{L_{r,e}(t)^{1-\lambda}}. \quad (A7)$$

The gain in valuation by using more labor for research into production efficiency always, independent of the parameter values, increases to infinity for $L_{r,e}(t) \rightarrow 0$.

Finally, changes in the labor inputs $L_g(t)$ into the production of goods have an impact on the consumption in this and the next period, $C(t)$ and $C(t+1)$. Thus, we obtain for labor inputs to good production:

$$\frac{dV(t)}{dL_g(t)} = \frac{dV(t)}{dC(t)} \cdot \frac{dC(t)}{dL_g(t)} + \frac{dV(t)}{dC(t+1)} \cdot \frac{dC(t+1)}{dL_g(t+1)}$$

whereby, according to Assumption 2 and Assumption 5, $L_g(t)=L_g(t+1)$ holds. The deviations can be directly calculated from Equations (13) and (A1) and we obtain:

$$\frac{dV(t)}{dL_g(t)} = \frac{\phi \cdot \psi_c \cdot C(t)^\phi}{L_g(t)} + \frac{\phi \cdot \psi_f \cdot C(t+1)^\phi}{L_g(t)}. \quad (A8)$$

Hence, as long as $\alpha(t)\kappa(t) > 1+\delta$ holds, using more labor for the production of goods increases valuation. The increase goes to infinity for $L_g(t) \rightarrow 0$. If the use of the available labor L is done such that $V(t)$ is maximized (see Assumption 5) and the subsistence level is reached, then all activities that are conducted with some labor input have to lead to the same marginal valuation:

$$\frac{dV(t)}{dL_g(t)} = \frac{dV(t)}{dL_{r,r}(t)} = \frac{dV(t)}{dL_{r,e}(t)}. \quad (\text{A9})$$

In this case always some labor input is assigned to research to increase the efficiency of production, $L_{r,e}(t) > 0$, because the right-hand side of Equation (A7) increases to infinity for $L_{r,e}(t) \rightarrow 0$. The same holds for the labor input to research for the replacement of labor inputs only if

$$(1 - r_p) \cdot \left(\frac{\alpha(t)}{1 + \delta} - \frac{1}{\kappa(t)} \right) > r_c \cdot \left(\frac{1}{\gamma(t)} + \frac{1}{\beta(t)} \right). \quad (\text{A10})$$

Hence, Condition (A10) is necessary for research into the replacement of labor inputs to be conducted. Furthermore, Assumption 5 states that research is only conducted if the subsistence level can be provided. If all labor L is put into production, the amount that is consumed is given by Equation (A2) with L instead of $L_g(t)$:

$$C(t) = \frac{\alpha(t) \cdot \beta(t) \cdot \gamma(t)}{\beta(t) + \gamma(t)} \cdot L \cdot \left(1 - \frac{\delta}{\alpha(t) \cdot \kappa(t)} \right). \quad \text{q.e.d.}$$

Proof of Lemma 2:

The proof of Lemma 1 contains already the result that some labor is put into research to increase the efficiency of production if the subsistence level can be guaranteed.

q.e.d.

Use of available labor inputs:

If the resulting consumption level is above the subsistence level, the use of the total labor L is given by Condition (A9). Inserting Equations (A7) and (A8), we obtain

$$L_g(t) = \left[\frac{\psi_c \cdot \left(\frac{\alpha(t) \cdot \beta(t) \cdot \gamma(t)}{\beta(t) + \gamma(t)} \right)^\phi \cdot \left(1 - \frac{1 + \delta}{\alpha(t) \cdot \kappa(t)} \right)^\phi}{\psi_f \cdot \left(\frac{\alpha(t+1) \cdot \beta(t+1) \cdot \gamma(t+1)}{\beta(t+1) + \gamma(t+1)} \right)^\phi \cdot \left(1 - \frac{1 + \delta}{\alpha(t+1) \cdot \kappa(t+1)} \right)^\phi} + 1 \right] \cdot \frac{L_{r,e}(t)^{1-\lambda}}{\ln s_e \cdot A \cdot \lambda}, \quad (\text{A11})$$

and inserting Equations (A6) and (A7), we obtain

$$L_{r,r}(t) = \frac{(1+\delta)^{\frac{1}{1-\lambda}} \cdot L_{r,e}(t)}{\alpha(t+1)^{\frac{1}{1-\lambda}} \cdot \beta(t+1)^{\frac{1}{1-\lambda}}} \cdot \left(\frac{\left[\frac{\alpha(t+1)}{1+\delta} - \frac{1}{\kappa(t+1)} \right] \cdot (1-r_p)}{\frac{1}{\gamma(t+1)} + \frac{1}{\beta(t+1)}} - r_c \right)^{\frac{1}{1-\lambda}}. \quad (A12)$$

If the resulting amount of labor that is used in production, $L_g(t)$ according to (A11) satisfies

$$\frac{\alpha(t) \cdot \beta(t) \cdot \gamma(t)}{\beta(t) + \gamma(t)} \cdot L_g(t) \cdot \left(1 - \frac{\delta}{\alpha(t) \cdot \kappa(t)} \right) > C_0, \quad (A13)$$

then $L_{r,e}(t)$ can be calculated by inserting Equations (A11) and (A12) into $L=L_g(t)+L_{r,r}(t)+L_{r,e}(t)$. Otherwise,

$$L_g(t) = \frac{C_0 \cdot (\beta(t) + \gamma(t))}{\alpha(t) \cdot \beta(t) \cdot \gamma(t) \cdot \left(1 - \frac{\delta}{\alpha(t) \cdot \kappa(t)} \right)}, \quad (A14)$$

and the remaining labor, $L-L_g(t)$, is distributed between $L_{r,r}(t)$ and $L_{r,e}(t)$ according to Equation (A12).

Proof of Theorem 1a:

Let us assume that Theorem 1a is wrong and $L_{r,r}(t)$ never falls below a certain level that is strictly greater than zero. Then, $\beta(t)$ increases unboundedly and thus toward infinity. Equation (A12) can be rewritten, given $\gamma(0)=\infty$ and $\kappa(0)=\infty$, as

$$L_{r,r}(t) = L_{r,e}(t) \cdot \left(\frac{\left[1 - \frac{r_c \cdot (1+\delta)}{\alpha(t+1)} \cdot \left(\frac{1}{\beta(0)} - \frac{1}{\beta(t+1)} \right) \right] \cdot (1-r_p)}{r_p \cdot \left(\frac{\beta(t+1)}{\beta(0)} - 1 \right) + 1} - \frac{r_c \cdot (1+\delta)}{\alpha(t+1) \cdot \beta(t+1)} \right)^{\frac{1}{1-\lambda}}. \quad (A15)$$

Thus,

$$\lim_{\beta(t) \rightarrow \infty} L_{r,r}(t) = 0, \quad (A16)$$

which contradicts the assumption that Theorem 1a is wrong and a stable share of labor is put into research for technologies to replace labor inputs. q.e.d.

Proof of Theorem 1b:

In the proof of Lemma 1 we have found that $L_{r,e}(t) > 0$ always holds. Thus, $\alpha(t)$ increases with time unboundedly. Thus, $\alpha(t) \rightarrow \infty$ holds for $t \rightarrow \infty$. Furthermore, Equation (A16) implies $\beta(t+1) = \beta(t)$, $\gamma(t+1) = \gamma(t)$ and $\kappa(t+1) = \kappa(t)$. Thus, we obtain from Equation (A11):

$$\lim_{t \rightarrow \infty} L_g(t) = \frac{\left[\frac{\psi_c}{s_e^{A \cdot L_{r,e}(t)^\lambda}} + \psi_f \right] \cdot L_{r,e}(t)^{1-\lambda}}{\psi_f \cdot \ln s_e \cdot A \cdot \lambda}. \quad (\text{A17})$$

Hence, we obtain for $L_{r,e}(t)$ in case of $t \rightarrow \infty$ the implicit equation:

$$L_{r,e}(t) + \frac{\left[\frac{\psi_c}{s_e^{A \cdot L_{r,e}(t)^\lambda}} + \psi_f \right] \cdot L_{r,e}(t)^{1-\lambda}}{\psi_f \cdot \ln s_e \cdot A \cdot \lambda} = L. \quad (\text{A18})$$

The labor, $L_g(t)$ that is put into production can be calculated by Equation (A17) and the prior and posterior labor inputs are given by

$$L_a(t) = \frac{\gamma(t) \cdot L_g(t)}{\beta(t) + \gamma(t)} \quad \text{and} \quad L_p(t) = \frac{\beta(t) \cdot L_g(t)}{\beta(t) + \gamma(t)}.$$

According to Equation (A14), labor is put into research for replacing labor inputs as long as $\beta < \infty$, although this input decreases with time. Hence, $\beta \rightarrow \infty$ for $t \rightarrow \infty$. As a consequence, $L_a(t) = 0$ and $L_p(t) = L_g(t)$ for $t \rightarrow \infty$. q.e.d.

Proof of Theorem 1c:

Production output is given by Equation (A1). Assuming a stable labor input to production, we obtain

$$\frac{Y_g(t+1)}{Y_g(t)} = \frac{\frac{\alpha(t+1) \cdot \beta(t+1) \cdot \gamma(t+1)}{\beta(t+1) + \gamma(t+1)}}{\frac{\alpha(t) \cdot \beta(t) \cdot \gamma(t)}{\beta(t) + \gamma(t)}}.$$

Inserting Equations (6), (7) and (A4) leads to

$$\frac{Y_g(t+1)}{Y_g(t)} = \frac{s_e^{Y_{r,e}(t)} \cdot s_r^{Y_{r,r}(t)} \cdot [\beta(t) + \gamma(t)]}{\gamma(t) + r_r \cdot \gamma(t) \cdot [s_r^{Y_{r,r}(t)} - 1] + \beta(t) \cdot s_r^{Y_{r,r}(t)}}. \quad (\text{A19})$$

For $t=0$ the value $\gamma(t)$ is infinity, so that Equation (A19) becomes Equation (16). For $t \rightarrow \infty$ the value $\beta(t)$ becomes infinity, so that Equation (A19) becomes Equation (17).

q.e.d.

Proof of Theorem 1d:

The first part of this theorem follows directly from Lemma 1. Condition (18) contradicts Condition (14). Since Condition (15) is satisfied, research into the increase of efficiency in production is conducted, so that $\alpha(t)$ increases unboundedly with time. At the same time, since no research to replace labor inputs is conducted, $\beta(t)$, $\gamma(t)$ and $\kappa(t)$ remain the same. Hence, there is a time at which Equation (14) becomes satisfied and research to replace labor inputs starts according to Lemma 1.

q.e.d.

Proof of Theorem 1e:

Since no research to replace labor inputs is conducted at time $t=0$ according to Theorem 1d, the initial growth rate is given by (see Equation (16))

$$g(0) = s_e^{Y_{r,e}(0)}. \tag{A20}$$

and Equation (A11) becomes

$$L_g(t) = \frac{\left[\frac{\psi_c}{s_e^{A \cdot L_{r,e}(t)^\lambda}} + \psi_f \right] \cdot L_{r,e}(t)^{1-\lambda}}{\psi_f \cdot \ln s_e \cdot A \cdot \lambda}. \tag{A21}$$

The labor input to production that would result from Equation (A21) does not satisfy Condition (A13) according to the assumptions in Theorem 1e. Thus, the labor input to production increases to the value given by Equation (A14). This means that $L_{r,e}(0)$ decreases compared to the value that would result from Equation (A21). We have stated above that $\alpha(t)$ increases unboundedly with time, so that Condition (A13) becomes satisfied in the long run. Furthermore, Equation (A16) holds, so that $L_{r,e}(\infty)$ is given by Equation (A21). Thus, $L_{r,e}(0) < L_{r,e}(\infty)$. As a consequence, $Y_{r,e}(0) < Y_{r,e}(\infty)$. Using Equations (17) and (A20) we obtain $g(0) < g(\infty)$. It is also obvious that $g(t)$ increases at the beginning while production increases and thus more labor inputs can be used for research. However whether $g(t)$ becomes larger than $g(\infty)$ at some point in time cannot be stated in general.

q.e.d.

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