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Consumer flexibility, data quality and targeted pricing

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Consumer Flexibility, Data Quality and Targeted Pricing*

Geza Sapi† Irina Suleymanova‡

November 2013

Abstract

We investigate how firms’ incentives to acquire customer data for targeted offers depend on its quality. A two-dimensional Hotelling model is proposed where consumers are heterogeneous both with respect to their locations and transportation cost parameters (flexibility). Firms have perfect data on the locations of consumers while data on their flexibility is imperfect. When consumers are relatively homogeneous in their flexibility, in equilibrium both firms acquire customer data regardless of its quality. This increases profits but harms consumers. When consumers are relatively differentiated in flexibility, data acquisition incentives depend on its quality. Only if the data is sufficiently precise, both firms acquire it and their profits decrease, while consumers are better-off. Our model has particular relevance for location-based marketing such as in mobile telephony, where firms have near-perfect information on the proximity of customers but may have imperfect knowledge of other consumer characteristics.

JEL-Classification: D43; L13; L15; O30.

Keywords: Price Discrimination, Customer Data, Market Segmentation.

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1 Introduction

The widespread use of smartphones with built-in GPS chips is responsible for a boom of location-based marketing where targeted advertisements and offers to consumers are related to their precise geographic position (see, for instance, Beard, 2011). Several mobile phone applications (such as 8Coupons and Mobiqupons) rely on the GPS devices to transmit real-time information on the physical location of a consumer and allow retailers to custom-target rebates respectively (see Wortham, 2009). However, location data is nowadays by far not the only type of customer information used in mobile marketing. Huge customer databases collected by mobile network operators provide an additional opportunity for targeted pricing and advertising. In the recent years mobile firms started selling this data to marketing firms, advertisers or other interested parties, raising concerns of privacy advocates (see, for instance, Tene and Polonetsky, 2012, and Tode, 2013a). For example, since 2012 Verizon Wireless, the largest network operator in the U.S., sells data on location, demographics (age, gender) and consumption habits of its customers (Panzarino, 2012). In 2013, AT&T, the American telecommunications giant, also announced its plans to sell data it obtained on its customers' smartphone usage (see Fitchard, 2013).\footnote{In some cases mobile operators do not directly sell customer data, they instead use it to design targeted offers on behalf of interested advertisers. For instance, in 2013 the three UK’s largest mobile network operators, Vodafone UK, Telefonica UK (O2) and EE, organized a joint venture Weve where they pooled their data on millions of customers (see Hawkes, 2013). Weve’s product “WeLocate” provides an opportunity for interested firms to advertise and send special offers depending on both a consumer’s location and demographics. The same year Barclaycard, a global payment business, launched for UK customers “bespoke offers,” where consumers can search for personalized offers (based on customer data such as spending history and demographics) on a wide range of products assessable both online and on mobile devices (see Winch, 2013). This allows consumers to search for the nearest savings suggestions based on their location.}

The ability of firms to convert their knowledge about customers into attractive offers depends crucially on data quality. It appears that in mobile marketing interested retailers can have near perfect information on customer locations. The acquisition of additional customer data allows firms to conclude on other dimensions of consumer preferences relevant in spatial competition, such as consumer flexibility.\footnote{For example, Factual, a mobile marketing firm, recently launched Geopulse Audience, a data platform that allows advertisers to deliver personalized offers to consumers based not only on their locations, but also on their income, which is estimated based on their geo-behavioral patterns (see http://www.factual.com/products/geopulse-audience). A user’s income can serve as a good proxy for her flexibility to switch between competing advertisers. Sense Networks is another mobile advertising company, which allows consumer targeting based on location and behavioral data. Information is available on age, income, education and ethnicity, all reasonable signals on consumer flexibility (see https://www.sensenetworks.com/audience-segments-and-results/).} Data on consumer flexibility is most likely to be imperfect. We consider a model of spatial competition à la Hotelling and focus on firms’ incentives to acquire
customer flexibility data (on consumer transportation cost parameters) for targeted offers depending on its quality.\textsuperscript{3,4} We show that there is a subtle relationship between customer data quality, heterogeneity of consumers and firms’ profits.\textsuperscript{5}

Our article contributes to the strand of literature on competitive price discrimination with demand-side asymmetries. In that case consumers can be classified into different groups depending on their preferences for a particular firm. The question most often analyzed in that strand resolves around how firms’ ability to discriminate based on consumer locations (brand preferences) influences prices and firms’ profits. Thisse and Vives (1988) were the first to show that firms end up in a prisoner’s dilemma such that every firm has a unilateral incentive to discriminate, while both firms are worse-off compared to the no-discrimination case.\textsuperscript{6}

In contrast to most articles in this strand we allow consumers to differ not only in their locations, but also in flexibility. In particular, we follow Jentzsch, Sapi and Suleymanova (2013) and consider an augmented version of the Hotelling model with consumer heterogeneity along two dimensions: locations and transportation cost parameters.\textsuperscript{7} While firms have perfect knowledge

\textsuperscript{3}The term “flexibility” captures the intuition that depending on whether transportation costs are high or low, consumers are less or more likely to buy from the farther firm, respectively. Consumers with high (low) transportation costs can be referred to as less (more) flexible.

\textsuperscript{4}“Geo-conquesting” is becoming an extremely popular strategy in mobile marketing, where a firm targets prospective customers when they are close to the competitor’s location (see, for instance, Tode, 2013b). In that case while designing its targeted offers a firm takes into account not only the distance of prospective consumers to its own location, but also to that of the rival, which calls for the analysis of firms’ targeting activities in a model of spatial competition.

\textsuperscript{5}Our analysis is also relevant for traditional coupon marketing, where firms may infer the distance to the shops based on consumers’ physical addresses, which are easy to get even from public sources. On the top of address data firms can acquire additional data on consumer preferences. For example, in Germany Deutsche Post sells household-level data on demographics, living situation, purchasing power and several other dimensions, which reveals much about consumer flexibility (see “Deutsche Post, Advertising by mail, Local Promotion” at http://www.deutschepost.de/dpag?tab=1&skin=hi&check=yes&lang=de_EN&xmlFile=link1017338_1010544).

\textsuperscript{6}A similar contribution is made in Shaffer and Zhang (1995) and Bester and Petrakis (1996). Other papers show that firms’ ability to discriminate based on consumer locations does not necessarily lead to a prisoner’s dilemma. For example, in Shaffer and Zhang (2000) firms may benefit from the ability to discriminate among the two consumer groups loyal to each of the firms if these groups are sufficiently heterogeneous in the strength of their loyalty. Chen, Narasimhan and Zhang (2001) show that when the targeting ability of one or both firms improves, but remains imperfect, firms’ profits may increase. In Shaffer and Zhang (2002) a firm with a stronger brand loyalty may benefit from firms’ ability to discriminate among individual consumers based on the strength of brand loyalty.

\textsuperscript{7}Borenstein (1985) also considers a model where consumer preferences are heterogeneous along different dimensions. His simulation results show that price discrimination based on transportation cost parameters is profitable. We provide analytical results, which support this conclusion and specify that consumers should be relatively homogeneous in flexibility. Armstrong (2006) shows that firms benefit from the ability to discriminate between the two consumer groups with high and low transportation costs both of which are heterogeneous in brand preferences. We show that this result does not hold when consumers are relatively differentiated in flexibility and firms have perfect data on consumer locations. Also, we allow firms to identify more than two flexibility segments depending on data quality. Liu and Shuai (2013) also consider a model with two-dimensional consumer heterogeneity.
of consumer locations, they may acquire data on consumer flexibility of the exogenously given quality, which allows to discriminate along that dimension too. We show that the profit effect of firms’ ability to discriminate based on consumer flexibility is driven by the type of the equilibrium strategy firms use on their turfs in the absence of flexibility data and the resulting balance between the competition and rent-extraction effects following data acquisition. When consumers are relatively homogeneous (differentiated) in flexibility, every firm follows a monopolization (market-sharing) strategy on its turf. The profit effect of the ability to discriminate based on consumer flexibility is positive (negative) in the former (latter) case. As a result, if consumers are relatively homogeneous, in equilibrium both firms acquire flexibility data, regardless of its quality. Data acquisition does not lead to welfare improvement and takes place solely at the expense of consumers. If, in contrast, consumers are relatively differentiated, both firms acquire flexibility data in equilibrium only if its quality is sufficiently high. Better data, however, drives firms into a prisoner’s dilemma, making them worse-off while social welfare and consumer surplus increase.

The article most closely related to ours is Liu and Serfes (2004), who develop a location model of oligopolistic third-degree price discrimination to study the incentives of firms to acquire data on consumer locations (brand preferences) depending on its quality. We extend the analysis of Liu and Serfes by adding another dimension of consumer heterogeneity, flexibility, and allow firms to get data on it. We believe that in reality price discrimination along the flexibility dimension plays an important role. Our modelling approach allows us to obtain new results on firms’ incentives to acquire customer data compared to Liu and Serfes. In Section 4 we provide a detailed comparison with Liu and Serfes.

Our article is also related to Corts (1998) who shows that best-response asymmetry is a necessary condition for third-degree price discrimination to have an unambiguous effect on equilibrium prices and profits. Corts, however, does not further specify under which conditions equilibrium prices and profits would decrease and when they would increase. In our model firms’ best-response functions are characterized by best-response asymmetry, such that for a given location on a firm’s turf that firm considers consumers with relatively high transportation costs to be its strong market, while the strong market of the rival are consumers with relatively low transportation costs. Our results show that when consumers are relatively homogeneous,

However, in their analysis the strength of consumer preferences is same among all consumers.
with the ability to discriminate based on consumer flexibility equilibrium prices (weakly) increase and firms’ profits get larger. In contrast, when consumers are relatively differentiated, the ability to discriminate based on consumer flexibility results in lower equilibrium prices and lower profits. Our analysis extends Corts’ results by specifying conditions under which best-response asymmetry yields lower or higher equilibrium prices and profits when firms can discriminate based on consumer flexibility.

A further article close to ours is Jenzsch, Sapi and Suleymanova (2013). The authors show that competitors’ incentives to share customer data among each other depend on its type and on how strongly consumers are heterogeneous in flexibility. We extend this work by showing that consumer heterogeneity in flexibility is also crucial for firms’ incentives to acquire customer data. Also, in our current analysis customer flexibility data can be imperfect, while it is always perfect in Jenzsch, Sapi and Suleymanova.

Our paper is organized as follows. In Section 2 we present the model. In Section 3 we state the results of the equilibrium analysis. Precisely, we derive firms’ equilibrium incentives to acquire customer flexibility data depending on its quality and consumer heterogeneity in flexibility. In Section 4 we compare our results with the closest article, Liu and Serfes (2004). Finally, in Section 5 we conclude.

2 The Model

There are two firms, \( A \) and \( B \), producing two brands of the same product at zero marginal cost and competing in prices. Firms are situated at the two ends of a Hotelling line of unit length with firm \( A \) being located at \( x_A = 0 \) and firm \( B \) at \( x_B = 1 \). There is a unit mass of consumers, each of whom is characterized by an address \( x \in [0, 1] \), which corresponds to her preference for the ideal product. If a consumer does not buy her ideal product she incurs linear transportation costs proportional to the distance to the firm. We follow Jentzsch, Sapi and Suleymanova (2013) and assume that additionally to their addresses consumers are also differentiated in transportation costs per unit distance, \( t \in [\bar{t}, \bar{t}] \), where \( \bar{t} > \bar{t} \geq 0 \). Each consumer is uniquely characterized by a pair \((x, t)\). In the following we say that consumers with addresses \( x < 1/2 \) (\( x > 1/2 \)) belong to the turf of firm \( A \) (\( B \)).

We consider two versions of our model, depending on the level of consumer heterogeneity in flexibility measured by the ratio of the largest to the lowest transportation cost parameters,
In the first version $t = 0$, such that $\lim_{t \to 0} l = \infty$. We say that in this case consumers are \textit{relatively differentiated in flexibility}. In the second version $t > 0$ and $l \leq 2$ and consumers are \textit{relatively homogeneous in flexibility}. The two versions of our model represent two extreme cases regarding consumer heterogeneity in flexibility.

We assume that firms have perfect information on consumer addresses and can acquire data on consumer flexibility which is imperfect. In particular, we assume that firms can acquire an external dataset containing the flexibility characteristics of consumers in the market. If firms acquire this dataset, they can identify the transportation cost parameters of individual consumers, or of consumer groups, depending on the quality of data. In particular, the quality of data on consumer flexibility is measured by the parameter $k = 0, 1, 2, \ldots, \infty$. For a given $k$ firms can divide the interval $t \in [\bar{t}, \bar{t}]$ into $2^k$ segments and identify every consumer as belonging to one of those segments.\textsuperscript{8} Segment $m = 1, 2, \ldots, 2^k$ consists of consumers with the transportation cost parameters $t \in [\bar{t}^m(k); \bar{t}^m(k)]$, where $\bar{t}^m(k) = \bar{t} + (\bar{t} - \bar{t})(m - 1)/2^k$ and $\bar{t}^m(k) = \bar{t} + (\bar{t} - \bar{t})m/2^k$ denote the most and the least flexible consumers on segment $m$, respectively. For any $m$ we can compute the ratio of the largest to the lowest transportation cost parameters on that segment, $l^m(k) := \bar{t}^m(k)/\bar{t}^m(k)$. We say that consumers are \textit{relatively differentiated in flexibility on segment} $m$ if $\bar{t}^m(k) = 0$, such that $\lim_{\bar{t}^m(k) \to 0} l^m(k) = \infty$. Similarly, consumers are \textit{relatively homogeneous in flexibility on segment} $m$ if $\bar{t}^m(k) > 0$ and $l^m(k) \leq 2$. Note that segment $m = 1$ always contains the most flexible consumer (with $t = \bar{t}$).

When $k \to \infty$, firms have perfect data on the addresses and flexibility of all consumers in the market and, hence, can charge individual prices. In all other cases firms have to charge the same price to consumers belonging to one segment and having the same address. We denote the price of firm $i$ to consumers with address $x$ on segment $m$ when customer data is of quality $k$ as $p_{im}(x, k)$.

The utility of a consumer $(x, t)$ from buying at firm $i = \{A, B\}$ is

$$U_i(p_{im}(x, k), t, x) = v - t |x - x_i| - p_{im}(x, k).$$

(1)

In equation (1) $v > 0$ denotes the basic utility, which is high enough such that the market is

\textsuperscript{8}In the marketing science literature it is standard to model consumer heterogeneity along two dimensions: brand preferences and responsiveness to marketing variables, such as price and advertising. For example, based on a sample of weekly store-level data for ketchup, Besanko, Dubé and Gupta (2003) identify three flexibility segments characterized by different price elasticities: price-insensitive, moderately price-sensitive and very price-sensitive shoppers.
always covered in equilibrium. A consumer buys from a firm proposing the higher utility. We follow Thisse and Vives (1988) and assume that if a consumer is indifferent, she buys from the closer firm. If \( x = 1/2 \), then in the case of indifference a consumer buys from firm \( A \). The game unfolds as follows.

**Stage 1** (Customer data acquisition). Firms observe the exogenously given quality of customer flexibility data, \( k \), and decide independently from each other whether to acquire this data.

**Stage 2** (Competition). First, firms independently and simultaneously choose regular prices for each address \( x \). Subsequently the firm(s) with customer flexibility data issues (issue) discounts to consumers in different flexibility segments.

The timing of the competition stage is consistent with a large body of literature on competitive price discrimination where firms make their targeted offers after setting regular prices (e.g., Thisse and Vives, 1988; Shaffer and Zhang, 1995, 2002; Liu and Serfes, 2004, 2005). It reflects the observation that discounts issued to finer consumer groups can be changed easier than prices targeted at broader consumer groups. Moreover, if firms decide simultaneously on regular prices and discounts, Nash equilibrium in pure strategies does not always exist.

### 3 Equilibrium Analysis

We solve the game backwards and start with the competition stage where firms choose prices taking their decisions in the data acquisition stage as given. Two subgames can emerge in the second stage. In the **symmetric subgame** both firms hold customer flexibility data. In the **asymmetric subgame** only one firm holds data on consumer flexibility. We derive the equilibrium in each subgame and compare profits in different subgames to conclude about firms’ incentives to acquire customer flexibility data in the first stage. We denote the equilibrium profit of firm \( i = A, B \) as \( \Pi^{A,A}_i(k) (\Pi^{A,N_A}_i(k)) \) in the symmetric (asymmetric) subgame when data quality is given by \( k \).

---

9 Note that the timing in Stage 2 is equivalent to the following: \( i) \) in the subgame where both firms hold flexibility data, firms choose all the prices simultaneously, and \( ii) \) in the subgames where only one firm holds flexibility data, the firm without data chooses its prices first, and the other firm follows.

10 We observe in many markets that regular prices change less frequently than coupon discounts. For example, www.hutfans.com (retrieved on January 26, 2013) mentions that Pizza Hut discount coupons tend to “change often.”

11 If \( k = 0 \), this subgame is equivalent to neither of the firms holding customer flexibility data.
3.1 Symmetric subgame: Both firms hold data on consumer flexibility

When both firms hold data on consumer flexibility, they can identify each consumer as belonging to one of the flexibility segments and can charge different prices to different segments. As firms are symmetric, we only focus on the turf of firm A. Consider an address \( x < 1/2 \) on the turf of firm A and an arbitrary segment \( m \). Under prices \( p_{Am}(x, k) \) and \( p_{Bm}(x, k) \) the transportation cost parameter of the consumer indifferent between buying from firms A and B is

\[
\tilde{t}_m(x, k) = \frac{p_{Am}(x, k) - p_{Bm}(x, k)}{1 - 2x}, \text{ provided } \tilde{t}_m(x, k) \in \left[ t^m(k); t^m(k) \right].
\]

On segment \( m \) firm A serves consumers with high transportation cost parameters; i.e., those with \( t \geq \tilde{t}_m(x, k) \). Firm B at the same time attracts consumers with low transportation cost parameters (\( t < \tilde{t}_m(x, k) \)) Then for any address \( x \) and any segment \( m \) under data quality \( k \) firm A maximizes the expected profit

\[
E \left[ \Pi_{Am}(x, k) \middle| x < 1/2 \right] = p_{Am}(x, k) \Pr \left\{ t \geq \tilde{t}_m(x, k) \right\}
\]

by choosing the price function \( p_{Am}(x, k) \). Firm B maximizes the expected profit

\[
E \left[ \Pi_{Bm}(x, k) \middle| x < 1/2 \right] = p_{Bm}(x, k) \Pr \left\{ t < \tilde{t}_m(x, k) \right\}
\]

by choosing the price function \( p_{Bm}(x, k) \). The following proposition states equilibrium prices and profits depending on the quality of customer flexibility data.

**Proposition 1.** (Symmetric subgame.) Assume that both firms hold customer flexibility data of quality \( k \). Equilibrium prices, demand regions and profits depend on data quality and consumer heterogeneity in flexibility as given in Table 1.

**Proof.** See Appendix.

As we see from Table 1, in the symmetric subgame the equilibrium on the segment \( m = 1 \) depends on consumer heterogeneity in flexibility. If consumers are relatively homogeneous, firm A targets on its turf the most flexible consumers on \( m = 1 \) (with \( t = t \)) and serves all consumers there although the rival charges the price of zero. If consumers are relatively differentiated, firm A targets the less flexible consumers on \( m = 1 \) on its turf, and the more flexible consumers
Table 1: Equilibrium prices, demand regions on firm i’s turf and profits in the symmetric subgame.

<table>
<thead>
<tr>
<th>Consumer heterogeneity in flexibility</th>
<th>$p_{im}^*(x, k)$</th>
<th>$p_{jm}^*(x, k)$</th>
<th>Consumers served by firm $i$</th>
<th>$\Pi_{A}^{A,A}$ = $\Pi_{B}^{A,A}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relatively differentiated(*)</td>
<td>$m = 1: \frac{2</td>
<td>1-2x</td>
<td>}{3 \times 2^x}$</td>
<td>$m = 1: \frac{</td>
</tr>
<tr>
<td>Relatively homogeneous(**)</td>
<td>$t^m(k)</td>
<td>1-2x</td>
<td>$</td>
<td>0</td>
</tr>
</tbody>
</table>

\(^{(e)} t = 0; (^{*}) t > 0 \text{ and } l \leq 2\)

switch to the rival, which charges positive prices on firm A’s turf. On the segments $m \geq 2$ in both versions of our model we get the same equilibria, similar to those with $m = 1$ in the case of relatively homogeneous consumers.

To understand those differences, we should first note that in the version with relatively homogeneous consumers under any data quality consumers remain relatively homogeneous on any segment. Indeed, $l^m(k) \leq 2$ holds for any $k \geq 0$ and any $m \geq 1$. However, in the version with relatively differentiated consumers, consumers remain relatively differentiated only on the segment $m = 1$, while on all other segments they become relatively homogeneous. Indeed, for any $k \geq 1$ and any $m \geq 2$ we have $l^m(k) \leq 2$, while for any $k \geq 0$ we have $\lim_{l^m(k) \to 0} l^m(k) = \infty$ when $m = 1$. We will next show that depending on consumer heterogeneity on a given segment a firm follows a market-sharing or a monopolization strategy on its turf, which in turn determines the equilibrium.

**Equilibrium strategies: monopolization or market-sharing.** The difference in equilibria depending on consumer heterogeneity is driven by the type of the best-response function a firm follows on its turf. On any address on their turfs, firms follow the **monopolization strategy** on segments with relatively homogeneous consumers. In contrast, on segments on their turfs with relatively differentiated consumers, firms resort to the **market-sharing strategy.** To demonstrate this, we consider the turf of firm A. Consider first an arbitrary segment $m$ with relatively homogeneous consumers, where $l^m(k) \leq 2$ holds. The best-response function of firm A on segment $m$ on some address $x < 1/2$ takes the form:

$$p_{Am}(x, k, p_{Bm}| x < 1/2) = p_{Bm} + t^m(k)(1-2x) \text{ for any } p_{Bm}.$$  \hfill (2)
As Expression (2) shows, for any price of the rival, firm A optimally charges a relatively low price targeted at the most flexible consumer on segment \( m \) (with \( t = t^m(k) \)) to serve there all consumers. We say that firm A follows a monopolization strategy. With homogeneous consumers it suffices for firm A to slightly reduce the price targeted at the least flexible consumers to get all consumers on segment \( m \) with a given address. Although in equilibrium firm B has to charge the price of zero, it does not serve any consumers on a segment with relatively homogeneous consumers.

Consider now some segment \( m \) with relatively differentiated consumers, where 
\[
\lim_{t^m(k) \to 0} I^m(k) = \infty
\]
holds. The best-response function of firm A on segment \( m \) on some address \( x < 1/2 \) takes the form:
\[
p_{Am}(x, k, p_{Bm} | x < 1/2) = \begin{cases} 
  p_{Bm} & \text{if } p_{Bm} \geq t^m(k)(1 - 2x) \\
  \left[ p_{Bm} + I^m(k)(1 - 2x) \right] / 2 & \text{if } p_{Bm} < t^m(k)(1 - 2x).
\end{cases}
\]  
(3)  

As the most flexible consumer on segment \( m \) can switch brands costlessly (\( I^m(k) = 0 \)), in order for firm A to attract all consumers on segment \( m \) with a given address it has to charge a price that is at least as low as that of the rival. As the best-response function (3) shows, it is optimal for firm A to monopolize segment \( m \) for a given address only if the rival’s price is sufficiently high, with \( p_{Bm} \geq t^m(k)(1 - 2x) \). Otherwise, firm A prefers to let the rival gain the more flexible consumers on segment \( m \). We say that firm A follows a market-sharing strategy: Firm A is ready to share segment \( m \) with the rival when serving all consumers is too costly, which is the case if the rival charges a relatively low price. In equilibrium both firms charge positive prices on segment \( m \) on address \( x < 1/2 \). The less preferred firm B charges a relatively low price in order to attract consumers, which makes it unprofitable for firm A to monopolize segment \( m \) for any address on its own turf. As a result, for all addresses on its own turf firm A serves only the less flexible consumers on segment \( m \) while firm B attracts the more flexible ones.

### Rent-extraction and competition effects.

When \( k = 0 \), firms are not able to discriminate based on consumer flexibility. With relatively homogeneous consumers a firm follows a monopolization strategy on any address on its turf. In equilibrium every firm serves all consumers on its turf. With relatively differentiated consumers a firm follows a market-sharing strategy on any address on its turf. In equilibrium every firm serves only the less flexible consumers on its turf. Figure 1 shows equilibrium demand regions in both versions of our model for \( k = 0 \). How does
the equilibrium change when the quality of customer flexibility data improves and takes values \( k \geq 1 \)? In literature on competitive price discrimination one usually distinguishes between two effects. Data of better quality allows firms to potentially extract more rents from consumers. To this we refer as the rent-extraction effect. Data quality may also change the intensity of competition between the firms, to which we refer as competition effect.

Consider first the case with relatively homogeneous consumers. As we mentioned above, for any data quality consumers remain relatively homogeneous on every segment such that firms always resort to the monopolization strategy on any segment on their turfs. In equilibrium each firm targets the most flexible consumer on a given segment for any address on its own turf while the rival is forced to charge the price to zero. Then under any data quality each firm serves all consumers on its own turf. Figure 2 shows equilibrium demand regions in both versions of our model for \( k = 1 \). Improvements in customer data quality result in a situation, in which every consumer is charged a (weakly) higher price. This is because each consumer is now allocated to a segment in which the most flexible consumer is (weakly) less flexible than before. As a result firms’ profits unambiguously increase. Higher quality data improves the ability of firms to extract rents from their loyal consumers, while competition effect is absent as the rival always charges the price of zero on a firm’s turf. Figure 3 depicts a firm’s equilibrium profit as a function of customer data quality in the symmetric subgame in the two versions of our model. For the example we used the values \( \underline{t} = 1 \ (\bar{t} = 0) \) and \( \underline{t} = 2 \ (\bar{t} = 1) \) in the case of relatively homogeneous
(differentiated) consumers.

Consider now the case with relatively differentiated consumers. As we showed above, for any quality of customer data, consumers remain relatively differentiated only on the segment $m = 1$, while on all other segments they become relatively homogeneous. If $k = 0$, firms pursue a market-sharing strategy on their turfs. However, as $k$ increases by one step, on the segment $m = 2$ firms switch to the monopolization strategy while on the segment $m = 1$ they maintain the market-sharing strategy. As a result, on both segments on any address on its turf firm $A$ charges lower prices compared to the uniform price at $k = 0$. The reason is that on the segment $m = 1$ consumers become on average more flexible, and on the segment $m = 2$ firm $B$ responds aggressively to firm $A$’s monopolization strategy and decreases its price to zero. Lower prices on a firm’s turf result in lower equilibrium profits. However, when $k$ increases further, profits start to increase. To understand this result consider some $k \geq 1$. On (all) the segment(s) $m \geq 2$, where consumers are relatively homogeneous, a firm’s profits on its turf increase when $k$ gets larger by one step, along the logic explained earlier for the case of relatively homogeneous consumers. On the segment $m = 1$, where consumers are relatively differentiated, a firm’s profits decrease due to the logic described above for the case $k = 0$. As the segment $m = 1$ comprises (weakly) less than half of all consumers on a firm’s turf for any $k \geq 1$, the negative profit effect on that segment is outweighed by the positive profit effect on all the other segment(s), such that a firm’s profits on its turf increase. For the same reason, with an increase in $k$ each firm serves more consumers on its turf as for any $k \geq 1$ firms lose consumers on their turfs only on the segment $m = 1$ (compare Figures 1 and 2).

We can summarize our results as follows. When consumers are relatively homogeneous, each firm follows a monopolization strategy on its turf, such that the rival charges the price of zero there and competition is very intense. The improvement in data quality results then only in a positive rent-extraction effect, while competition cannot be intensified as the rival cannot go below the price of zero. When consumers are relatively differentiated, each firm follows a market-sharing strategy on its turf, where the rival charges positive prices and competition is not very intense. The improvement in data quality from $k = 0$ to $k = 1$ results in a negative competition effect, such that profits decrease. With a further improvement in data quality, the rent-extraction effect starts to dominate, and profits increase.
Figure 2: Equilibrium demand regions for $k = 1$. We used the values $\bar{t} = 1$ ($\bar{t} = 3/4$) and $t = 0$ ($t = 1/2$) in the case of relatively differentiated (homogenous) consumers.

Figure 3: Individual profits and the quality of customer flexibility data (symmetric subgame). We used the values $\bar{t} = 1$ ($t = 0$) and $\bar{t} = 2$ ($\bar{t} = 1$) in the case of relatively homogeneous (differentiated) consumers.
3.2 Asymmetric subgame: Only one firm holds data on consumer flexibility

We assume without loss of generality that firm $A$ acquires customer flexibility data while firm $B$ remains without. The latter must offer the same price to all consumers with a given address irrespectively of their flexibility. In contrast, firm $A$ can price-discriminate based on both consumer addresses and flexibility. The following proposition summarizes our results for the case of relatively homogeneous consumers.

**Proposition 2. (Asymmetric subgame with relatively homogenous consumers.)** Assume that consumers are relatively homogeneous in flexibility and only firm $A$ holds data on consumer flexibility. Equilibrium prices, demand regions and profits are as given in Table 2.

**Proof.** See Appendix.

For the intuition behind the equilibrium, consider first the turf of firm $A$. For any data quality consumers remain relatively homogeneous on any flexibility segment, where firm $A$ adopts a monopolization strategy. Firm $B$ cannot do better than charging a zero price on any address for any data quality, therefore firm $A$ does not face a negative competition effect when data quality improves. In the latter case every consumer is allocated to a segment in which the most flexible consumer is now (weakly) less flexible than before. As a result the profits of firm $A$ on its own turf increase with data quality improvement due to a positive rent-extraction effect. Firm $B$ does not earn any profits there.

Consider now the turf of firm $B$. Different from the symmetric subgame where in equilibrium each firm serves all consumers on its turf, in the asymmetric subgame the firm without flexibility data loses some consumers on its turf if consumers are weakly homogeneous; i.e., $l > 3/2$. To understand this result, assume that firm $B$ targets the most flexible consumers on its turf and charges $p_B(x, k) = \frac{t}{2} (2x - 1)$ for any $x > 1/2$. Does firm $B$ have an incentive to increase prices and lose some of the more flexible consumers (on the segment $m = 1$)? Compared to the symmetric subgame, firm $B$ loses less consumers in case of a price increase because firm $A$ responds with a higher price on the segment $m = 1$. Different from the symmetric subgame, this makes it profitable for firm $B$ to charge a price above $p_B(x, k) = \frac{t}{2} (2x - 1)$ and lose some consumers on the segment $m = 1$, provided consumers are weakly homogeneous ($l > 3/2$).

How does the equilibrium change with the improvement in data quality? If consumers are strongly homogeneous; i.e, $l \leq 3/2$, irrespectively of data quality firm $B$ targets the most flexible consumer on any address. This leaves no scope for firm $A$ to attract away any of its
Table 2: Equilibrium prices, demand regions and profits in the asymmetric subgame. The case of relatively homogenous consumers.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Equilibrium Price $p^*_A(x,k)$</th>
<th>Equilibrium Price $p^*_B(x,k)$</th>
</tr>
</thead>
</table>
| $l \leq \frac{1}{2}$ and $k \geq 0$ | $p^*_A(x,k) = \begin{cases} 
\frac{1+(l-1)(m-1)}{2^x} (1-2x) & \text{if } x \leq \frac{1}{2} \\
0 & \text{if } x > \frac{1}{2} 
\end{cases}$ | $p^*_B(x,k) = \begin{cases} 
0 & \text{if } x \leq \frac{1}{2} \\
\ell (2x-1) & \text{if } x > \frac{1}{2} 
\end{cases}$ |

Consumers served by firm $A$: $x \leq \frac{1}{2}$

$\Pi_A^{A,NA}(k) = \frac{1+(l-1)(m-1)}{8} - \frac{(l-1)^4}{2^{x+3}}$  
$\Pi_B^{A,NA}(k) = \frac{1}{4}$

$\frac{3}{2} < l \leq 2$ and $0 \leq k \leq \log_2 \frac{2(l-1)}{2l-3}$

Consumers served by firm $A$: $x \leq \frac{1}{2}$

$\Pi_A^{A,NA}(k) = \frac{12l^2-12(l+1)\ell}{64(l-1)} - \frac{(l-1)^4}{2^{x+3}}$  
$\Pi_B^{A,NA}(k) = \frac{(l-1)^2\ell}{32(l-1)}$

$\frac{3}{2} < l \leq 2$ and $k \geq \log_2 \frac{2(l-1)}{2l-3}$

Consumers served by firm $A$: $x \leq \frac{1}{2}$

$\Pi_A^{A,NA}(k) = \frac{1+(l-1)(m-1)}{2^x} (1-2x) \ell$  
$\Pi_B^{A,NA}(k) = \frac{1}{8} - \frac{(l-1)(2^x+1)}{2(x+4)} \ell$

$\frac{3}{2} < l \leq 2$ and $0 \leq k \leq \log_2 \frac{2(l-1)}{2l-3}$

Consumers served by firm $A$: $x \leq \frac{1}{2}$

$\Pi_A^{A,NA}(k) = \frac{1+(l-1)(m-1)}{8} - \frac{(l-1)^4}{2^{x+3}} \ell$  
$\Pi_B^{A,NA}(k) = \frac{1}{4}$

$\frac{3}{2} < l \leq 2$ and $k \geq \log_2 \frac{2(l-1)}{2l-3}$
loyal consumers, even with perfect customer data. Then both firms’ profits on firm B’s turf do not change with the improvement in data quality.

Consider now the case with weakly homogeneous consumers ($l > 3/2$). If data quality is low ($k < \log_2 [(l - 1) / (l - 3/2)]$), firm A has a limited ability to attract consumers on the turf of firm B. In that case firm B does not have to reduce its prices with data quality improvement to keep consumers on the segments $m \geq 2$. Then firms’ profits on the turf of firm B do not depend on $k$. However, when $k$ increases above $k = \log_2 [(l - 1) / (l - 3/2)]$, the improved ability of firm A to target consumers forces firm B to reduce its prices to avoid losing consumers on the segments $m \geq 2$. Although due to a decrease in firm B’s equilibrium prices it gains consumers on its own turf, its profits there decrease. The profits of firm A on the turf of firm B decrease as well as it has to reduce its prices on any address on the segment $m = 1$, while its market share gets also smaller. As the profits of firm A on the rival’s turf constitute only a small share of its total profits, the latter undoubtedly increase with the improvement in data quality.

Figures 4a-4b: Individual profits and the quality of customer flexibility data (asymmetric subgame, homogenous consumers).

Figures 4a and 4b depict firms’ profits depending on data quality in the case of relatively homogenous consumers. On the left figure, 4a, consumers are strongly homogeneous ($l \leq 3/2$) and on the right figure, 4b, they are weakly homogeneous ($l > 3/2$). We can summarize our results as follows. Firm A, which holds customer flexibility data, follows a monopolization strategy on its turf, which leaves no scope for a negative competition effect as the rival always charges the price of zero. Customer data of a better quality allows firm A to extract more rents, such that its profits on the own turf increase in data quality, while firm B does not get any
profits there. On its own turf firm $B$ serves all consumers only when consumers are strongly homogeneous, in which case again the negative competition effect is absent, and the profits of firm $B$ do not change with data quality. Otherwise, firm $B$ loses some consumers on its turf and suffers from intensified competition when the rival gets data of a better quality, and its profits decrease in data quality.

We now turn to the case of relatively differentiated consumers. The following proposition describes the equilibrium in the asymmetric subgame.

**Proposition 3. (Asymmetric subgame with relatively differentiated consumers.)**
Assume that consumers are relatively differentiated in flexibility and only firm $A$ has data on consumer flexibility. Equilibrium prices, demand regions and profits are as given in Table 3.

**Proof.** See Appendix.

Although firm $A$ has informational advantage over the rival, it does not serve all consumers on its turf. This happens for the same reason as in the symmetric subgame. Since on the segment $m = 1$ consumers always remain relatively differentiated, firm $A$ follows a market-sharing strategy there and loses some of the more flexible consumers. Due to a negative competition effect the profits of firm $A$ on its turf decrease when its targeting ability improves.\(^{12}\) Precisely, anticipating the informational advantage of firm $A$, firm $B$ responds by reducing its uniform prices, which serve as an anchor for the discriminatory prices of firm $A$. The profits of firm $B$ on the rival’s turf decrease as well, because both its price and market share become smaller.

Due to its informational advantage, in the asymmetric subgame firm $A$ gains about half of the consumers on the turf of firm $B$, whereas in the symmetric subgame it serves consumers only on the segment $m = 1$. The profits of firm $A$ on the rival’s turf exhibit a $U$-shaped relationship in $k$: They first decrease and start to increase at $k = 1$. Firm $B$ reduces the uniform prices on its turf to protect its market shares, which constitutes a negative competition effect for firm $A$’s profits there. On the other hand, with better data firm $A$ can extract more rents from consumers it serves. The latter effect starts to dominate when data quality becomes good enough.\(^{13}\) Firm

\(^{12}\) Note that in the symmetric subgame the profits of firm $A$ on its turf exhibit a $U$-shaped relationship in $k$. In that case the strength of the negative competition effect for firm $A$’s profits is limited. As firm $B$ can discriminate, it reduces its price to zero on the segments with relatively homogeneous consumers ($m(k) \geq 2$). Then on those segments data improvement gives rise only to the rent-extraction effect, where the profits of firm $A$ increase. Competition is only intensified on the segment $m(k) = 1$, which provides a decreasing share of firm $A$’s profits on its turf when data quality improves.

\(^{13}\) Note that on the turf of firm $A$ its profits always decrease in $k$, such that the rent-extraction effect never
Table 3: Equilibrium prices, demand regions and profits in the asymmetric subgame. The case of relatively differentiated consumers.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$p^*_A(m, x, k) = \begin{cases} \frac{3(1-2x)t}{4} &amp; \text{if } \frac{x}{2} \leq \frac{1}{2} \ \frac{1}{2} (2x-1) t &amp; \text{if } x &gt; \frac{1}{2} \end{cases}$</th>
<th>$p^*_B(x) = \begin{cases} \frac{(1-2x)t}{2} &amp; \text{if } x \leq \frac{1}{2} \ \frac{1}{2} (2x-1) t &amp; \text{if } x &gt; \frac{1}{2} \end{cases}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 0$</td>
<td>Consumers served by firm $A$:</td>
<td>$\Pi_A^{A,NA}(0) = \frac{13t}{32}$ $\Pi_B^{A,NA}(0) = \frac{5t}{32}$</td>
</tr>
<tr>
<td></td>
<td>$x \leq \frac{1}{2}$ &amp; $t \geq \frac{t}{2k+2}$</td>
<td>$x \geq \frac{1}{2}$ &amp; $t &lt; \frac{t}{2k} \left( \frac{2^k}{2} + \frac{1}{2} \right)$</td>
</tr>
<tr>
<td></td>
<td>$x \geq \frac{1}{2}$ &amp; $t \geq \frac{t}{2k} \left( \frac{2^k}{2} - \frac{3}{2} \right)$</td>
<td>$x \geq \frac{1}{2}$ &amp; $0 \leq t &lt; \frac{t}{2k} \left( \frac{2^k}{2} - \frac{3}{2} \right)$</td>
</tr>
<tr>
<td>$k \geq 1$</td>
<td>Consumers served by firm $A$:</td>
<td></td>
</tr>
<tr>
<td>$p^<em>_A(m, x, k) = \begin{cases} \frac{3(1-2x)t}{2^{2k+1}} &amp; \text{if } \frac{x}{2} \leq \frac{1}{2} &amp; m = 1 \ t^m(k) \left( 1 - 2x \right) + p^</em>_B(x) &amp; \text{if } \frac{x}{2} \leq \frac{1}{2} &amp; m &gt; 1 \ p^*_B(x) - t^m(k) \left( 2x - 1 \right) &amp; \text{if } x &gt; \frac{1}{2} &amp; m &lt; 2^{k-1} \ \frac{5t}{2} (2x-1) &amp; \text{if } x &gt; \frac{1}{2} &amp; m = 2^{k-1} \ \frac{3t}{2} (2x-1) &amp; \text{if } x &gt; \frac{1}{2} &amp; m = 2^{k-1} + 1 \ 0 &amp; \text{if } x &gt; \frac{1}{2} &amp; m &gt; 2^{k-1} + 1 \end{cases}$</td>
<td>$p^*_B(x) = \begin{cases} \frac{7(1-2x)}{2^{2k+1}} &amp; \text{if } x \leq \frac{1}{2} \ \frac{7}{2} \left( 1 + \frac{1}{2^{2k+1}} \right) (2x - 1) &amp; \text{if } x &gt; \frac{1}{2} \end{cases}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x &lt; \frac{1}{2}$ &amp; $t \geq \frac{t}{2k+2}$</td>
<td>$x \geq \frac{1}{2}$ &amp; $\frac{t}{2} &lt; t &lt; \frac{t}{2k} \left( \frac{2^k}{2} + \frac{1}{2} \right)$</td>
</tr>
<tr>
<td></td>
<td>$x \geq \frac{1}{2}$ &amp; $0 \leq t &lt; \frac{t}{2k} \left( \frac{2^k}{2} - \frac{3}{2} \right)$</td>
<td>$x \geq \frac{1}{2}$ &amp; $0 \leq t &lt; \frac{t}{2k} \left( \frac{2^k}{2} - \frac{3}{2} \right)$</td>
</tr>
</tbody>
</table>

| $\Pi_A^{A,NA}(k) = \frac{t}{32} \left( 5 - \frac{1}{2^k} + \frac{7}{2^{2k+2}} \right)$ | $\Pi_B^{A,NA}(k) = \frac{t}{16} \left( 1 + \frac{1}{2^k} + \frac{1}{2^{2k+2}} \right)$ |
B’s profits earned on its own turf decrease monotonically with the improvement in data quality. It is forced into a downward spiral where it must charge lower prices while it still loses market shares.

Figure 5 shows the combined effect of data quality on firms’ profits on the two turfs, where we used $\tilde{t} = 1$. The profits of firm $B$ decrease monotonically with the improvement in data quality. The profits of firm $A$ in turn exhibit a $U$-shaped relationship in data quality, exactly as in the symmetric subgame (compare with Figure 3).

![Figure 5: Individual profits and the quality of customer flexibility data (asymmetric subgame, relatively differentiated consumers). We used the values $\tilde{t} = 0$ and $\tilde{t} = 1$.](image)

3.3 Acquisition of customer data

In this subsection we analyze firms’ incentives to acquire customer flexibility data in the first stage of the game and its welfare implications. In particular, we assume that firms can obtain data on the flexibility of all consumers in the market with an exogenously given precision $k \geq 1$. For simplicity, we assume that data can be acquired free of charge. Then data acquisition incentives are driven purely by profit considerations. The following proposition summarizes the main results, depending on consumer heterogeneity in flexibility.
Proposition 4. (Customer-data acquisition.) Firms’ decisions to acquire customer data depend on consumer heterogeneity in flexibility as follows.

i) If consumers are relatively homogeneous, for any \( k \geq 1 \) there is a unique Nash equilibrium (in dominant strategies) in Stage 1 where both firms acquire customer flexibility data. Both firms are strictly better-off compared to the case where none of the firms holds customer flexibility data.

ii) If consumers are relatively differentiated, there are two Nash equilibria where only one of the firms acquires customer data if \( k = 1 \). If \( k \geq 2 \) a prisoner’s dilemma emerges: There is a unique Nash equilibrium (in dominant strategies) where both firms acquire customer flexibility data, making them worse-off.

Proof. See Appendix.

With relatively homogeneous consumers in the absence of customer flexibility data every firm serves only consumers on its own turf and targets on each address the most flexible consumer. If one of the firms acquires customer data, it gains consumers on the rival’s turf (if consumers are weakly homogeneous, \( l > 3/2 \)) and extracts more rents from consumers on its own turf. The unilateral acquisition of customer data is then always profitable. The best response to the rival acquiring customer flexibility data is to do so as well. In that case a firm gains consumers on the own turf (if consumers are weakly homogeneous, \( l > 3/2 \)) and can extract more rents from them. In equilibrium both firms acquire data of any quality and their profits increase.

With relatively differentiated consumers every firm has a unilateral incentive to acquire customer data of any quality. Although firm \( B \) (the firm without data) responds to data acquisition by charging lower prices on both the rival’s and the own turf, unilateral data acquisition is still profitable because firm \( A \) gains consumers on both turfs due to its improved targeting ability. In contrast, acquiring data when the rival holds it is only profitable if data quality is sufficiently high, \( k \geq 2 \). On the one hand, data acquisition allows firm \( B \) to gain consumers on its own turf. On the other hand, on those segments on firm \( B \)’s turf where firm \( A \) loses consumers, it reduces its price to zero. Only when data quality is high enough, is this negative effect compensated by the improved ability of firm \( B \) to extract rents from the less flexible consumers on its turf (to whom firm \( A \) always charges the price of zero independently of whether firm \( B \) holds flexibility data). These data acquisition incentives result in equilibria such that both firms acquire flexibility data only when its quality is sufficiently high, \( k \geq 2 \). Otherwise, only one firm acquires
customer data in equilibrium.

In the next proposition we characterize the equilibrium profits, consumer surplus and social welfare under any data quality based on firms’ equilibrium decisions to acquire customer data in the first stage of the game.

**Proposition 5. (Profit and welfare depending on data quality.)** Firms’ profits, consumer surplus and social welfare depend on the quality of customer flexibility data as follows.

i) If consumers are relatively homogeneous in flexibility, under any $k \geq 0$ the profits of firm $i = A, B$, consumer surplus and social welfare are given by 

\[ \Pi_i^*(k) = \frac{t}{4} + \left( \bar{t} - \frac{t}{2} \right) \left[ 1/8 - 1/ \left( 2^k \right) \right], \]

\[ CS(k) = v - 3(\bar{t} + \frac{t}{8})/8 + (\bar{t} - \frac{t}{2})/2^{k+2} \]

and 

\[ SW = v - (\bar{t} + \frac{t}{2})/8, \]

respectively.

ii) If consumers are relatively differentiated in flexibility, under $k = 0$ firms’ profits, consumer surplus and social welfare are 

\[ \Pi_A^* = \Pi_B^* = 5\bar{t}/36, \]

\[ CS = v - 31\bar{t}/72 \]

and 

\[ SW = v - 11\bar{t}/72, \]

respectively. If $k = 1$, then 

\[ \Pi_A^* = 79\bar{t}/512, \]

\[ \Pi_B^* = 27\bar{t}/256, \]

\[ CS = v - 417\bar{t}/1024 \]

and 

\[ SW = v - 15\bar{t}/1024. \]

If $k \geq 2$, the profits of firm $i = A, B$, consumer surplus and social welfare are given by 

\[ \Pi_i^*(k) = 5\bar{t}/ \left( 9 \times 2^{(1+k)} \right) + \bar{t} \left( 1/8 - 1/2^{(3+k)} \right), \]

\[ CS(k) = v - 11\bar{t}/ \left( 9 \times 2^{(2k+2)} \right) + \bar{t} \left( 1/2^{(k-1)} - 3 \right)/8 \]

and 

\[ SW(k) = v - 7 \left[ 1 + 1/ \left( 9 \times 2^{(2k-1)} \right) \right] /8, \]

respectively.

**Proof.** See Appendix.

When consumers are relatively homogeneous, both firms acquire customer flexibility data of any quality. With customer data every firm can extract more rents from consumers on its turf, while the negative competition effect is absent there as the rival charges the price of zero both with and without customer data. As a result, following data acquisition each firm enjoys higher profits, and the increase in profits is larger, the better is data quality. Both with and without customer data every firm follows a monopolization strategy on its turf and serves there all consumers, such that social welfare is always maximized. Higher profits following data acquisition then go hand in hand with a reduction in consumer surplus, and data of a better quality harms consumers more.

When consumers are relatively differentiated, in equilibrium only one of the firms acquires customer data if $k = 1$, and both firms acquire data if $k \geq 2$. In the former case the profits of firm $A$ (acquiring customer data) naturally increase. Firm $B$ is worse-off because it loses markets shares and charges lower prices on both turfs. The share of consumers buying from the preferred firm increases because firm $A$ gains more consumers on the own than on the rival’s turf, such that social welfare increases. Consumers are better-off because both the total transportation
costs and their payments to the firms decrease.

If data quality is relatively high, \( k \geq 2 \), firms end up in the prisoner’s dilemma and are worse-off with data acquisition. Intensified competition does not allow firms to make full use of customer data, and their profits decrease. However, firms’ profits decrease less if they acquire data of a better quality. Data acquisition is beneficial for social welfare as the distribution of consumers between the firms becomes more symmetric. If firms acquire perfect customer data, then social welfare is maximized. Consumers benefit from data acquisition, while the benefit is smaller the better is data quality.

4 Comparison with Liu and Serfes (2004)

Our model is closely related to Liu and Serfes (2004, in the following: LS), who apply spatial model to investigate firms’ incentives to acquire data on consumer locations (brand preferences) of various quality for third-degree price discrimination. With data of a better quality firms can identify consumers as belonging to finer location segments. According to the definitions introduced in our analysis, in LS consumers are relatively differentiated (in transportation costs), because there is a consumer who can switch brands costlessly (consumer with address \( x = 1/2 \)).14

This explains the similarity between the results of LS and ours in the version with relatively differentiated consumers. For instance, in LS in the symmetric subgame profits also exhibit a U-shaped relationship in data quality, \( k \). Precisely, at \( k = 1 \) each firm follows a market-sharing strategy on its turf because consumers are relatively differentiated there. Data improvement by one step leads to a decrease in firms’ profits due to a negative competition effect, as we showed in our analysis. At the next step profits start to increase, because the negative competition effect is absent on the segment where firm \( A \) (\( B \) sticks to a monopolization strategy, \( m = 1 \)

\[ 14 \text{For the definitions introduced in our analysis to be applicable to both the model of LS and ours, we have to state them in a more general way, related to consumer transportation costs. Let } d := t(1-2x). \text{ Then in our model, for any address } x < 1/2 \text{ on the turf of firm } A \text{ consumers on any segment } m \text{ with } t \in [t^m_1(k); t^m_2(k)] \text{ can be characterized by } d \in [d^m_1(k); d^m_2(k)], \text{ where } d^m_1(k) = t^m_1(k)(1-2x) \text{ and } d^m_2(k) = t^m_2(k)(1-2x). \text{ We say that consumers on segment } m \text{ are relatively homogeneous in transportation costs if } t^m_1(k)/d^m_1(k) \leq 2 \text{ and are relatively differentiated if } d^m_1(k) = 0, \text{ in which case } \lim_{d^m_1(k) \to 0} t^m_1(k)/d^m_1(k) = \infty. (\text{In a similar way, segments on the turf of firm } B \text{ can be characterized by } \bar{d} := t(2x - 1)). \text{ Applying those definitions to LS, we get that for any } k \geq 1 \text{ consumers are relatively differentiated in transportation costs on the segments } m(k) = 2^{k-1} \text{ and } m(k) = 2^{k-1} + 1, \text{ because these segments contain a consumer who can switch brands costlessly (with address } x = 1/2). \text{ Differently, for any } k \geq 2 \text{ in LS consumers on the segments } m \leq 2^{k-1} - 1 \text{ and } m \geq 2^{k-1} + 2 \text{ are relatively homogeneous. Our definitions do not apply only to the case } k = 0 \text{ in LS, where firms cannot distinguish between the turfs of each other.} \]
In the asymmetric subgame in LS the profits of firm $A$ (holding customer data) exhibit a $U$-shaped relationship in data quality, again exactly as in the version of our model with relatively differentiated consumers. However, in LS profits increase above the initial level when data quality becomes perfect, while in our model they never reach that level. This difference is driven by the differences in the customer data available to the firms in LS and in our model. In our setup each firm has perfect data on consumer addresses, such that in the asymmetric subgame firm $B$ prices very aggressively the loyal consumers of the rival, and its price to them approaches zero when the quality of flexibility data available to firm $A$ becomes perfect. In LS firm $B$ cannot distinguish among its own loyal consumers and those of the rival in the asymmetric subgame. It then charges a price which decreases in data quality, but approaches some positive value when data becomes perfect as firm $B$ aims to extract rents from its loyal consumers, such that the negative competition effect is not that strong.

As a result firms’ incentives to acquire customer data and their welfare implications are similar in LS and in the version of our model with relatively differentiated consumers. Precisely, both firms acquire customer data in equilibrium only when its quality is sufficiently high resulting in a prisoner’s dilemma for the firms and an increase in consumer surplus. However, different from LS, we get two asymmetric equilibria, where only one of the firms acquires customer data, when data quality is low. This is related to the difference in the underlying equilibrium strategies of the firms. In our model a firm always has a unilateral incentive to acquire customer data, while in LS this is the case only when data quality is sufficiently high.

As expected, in the version with relatively homogeneous consumers our results are very different from those of LS. In that case both firms acquire data of any quality in equilibrium, because data acquisition gives rise only to the positive rent-extraction effect, while the negative competition effect is absent. The acquisition of customer flexibility data reduces consumer surplus in that case while it leaves social welfare unchanged. In summary, compared to LS, we identify more scope for profitable customer data acquisition, which at the same time harms consumers.
5 Conclusions

In this article we analyze firms’ incentives to acquire data on consumer characteristics of the exogenously given quality and its ensuring effects on competition intensity and welfare, when customer data can be used for targeted pricing. This article makes two main contributions. First, we propose a model of price discrimination that applies particularly well to markets where firms’ attempts to acquire customer data have attracted public debate, such as the use of location-based targeting on mobile devices. In contrast to typical spatial models, in our setup consumer locations are known to firms, but their information on consumer transportation cost parameters (flexibility) is imperfect. Second, we show that data acquisition incentives and the resulting market outcomes crucially depend on how strongly consumers differ in flexibility. These differences are driven by the balance of the competition and rent-extraction effects resulting from firms’ ability to better target consumers with additional data. If consumers are relatively homogenous with respect to their flexibility, competition is strong when firms have no data on flexibility with no room to further intensify as firms acquire data. In this case data is used solely for rent extraction, harming consumers. However, if consumers are relatively heterogenous in flexibility, the competition effect induced by additional data is very strong. Both firms acquire data only if it is of high quality and end up in the prisoner’s dilemma, while both the social welfare and consumer surplus increase. Consumers benefit more when data quality is low. Our results suggest that there is scope for welfare increasing privacy regulation that would restrict the acquisition or use of customer data in location-based marketing. However, such a policy needs to be nuanced, for example by being limited to cases where firms can obtain particularly detailed information about consumer characteristics.
6 Appendix

Proof of Proposition 1. We start with the version of our model with relatively differentiated consumers. In the following claim we state the equilibrium for \( k \geq 2 \).

Claim 1. Assume that consumers are relatively differentiated in flexibility and \( k \geq 2 \). On the turf of firm \( i = A, B \) prices vary depending on segment. On the segment \( m = 1 \) equilibrium prices are \( p^*_A(x, k) = 2\bar{t}(1 - 2x)/(3 \times 2^k) \) and \( p^*_B(x, k) = \bar{t}(1 - 2x)/(3 \times 2^k) \), where firm \( i \) serves consumers with \( t \geq \bar{t}/(3 \times 2^k) \). On the segments \( 2 \leq m \leq 2^k \) equilibrium prices are \( p^*_A(x, k) = \bar{t}(m - 1)(1 - 2x)/2^k \) and \( p^*_B(x, k) = 0 \), where firm \( i \) serves all consumers. Firm \( i \) realizes the profit \( \Pi^A_i(k) = 5\bar{t}/(9 \times 2^{2(1+k)}) + \bar{t}(1/8 - 1/2^{3+k}) \).

Proof of Claim 1. As firms are symmetric, we only consider the turf of firm \( A \) and some \( x < 1/2 \).

We start with the segment \( m = 1 \). Maximization of firm \( A \)'s profits yields the best-response function

\[
p_{A1}(x, k, p_{B1}(x, k)) = \begin{cases} 
 p_{B1}(x, k) & \text{if } p_{B1}(x, k) \geq \frac{\bar{t}(1-2x)}{2^k} \\
 \frac{p_{B1}(x, k)}{2} + \frac{\bar{t}(1-2x)}{2^{k+1}} & \text{if } p_{B1}(x, k) < \frac{\bar{t}(1-2x)}{2^k}.
\end{cases}
\]

Maximization of firm \( B \)'s profits yields the best-response function

\[
p_{B1}(x, k, p_{A1}(x, k)) = \begin{cases} 
 p_{A1}(x, k) - \frac{\bar{t}(1-2x)}{2^k} & \text{if } p_{A1}(x, k) \geq \frac{2\bar{t}(1-2x)}{2^k} \\
 \frac{p_{A1}(x, k)}{2} & \text{if } p_{A1}(x, k) < \frac{2\bar{t}(1-2x)}{2^k}.
\end{cases}
\]

These best-response functions yield the equilibrium prices \( p^*_A(x, k) = 2\bar{t}(1 - 2x)/(3 \times 2^k) \) and \( p^*_B(x, k) = \bar{t}(1 - 2x)/(3 \times 2^k) \). Firm \( A \) serves consumers with \( t \geq \bar{t}/(3 \times 2^k) \). On the segments \( 2 \leq m \leq 2^k \) the best-response function of firm \( A \) takes the form

\[
p_{Am}(x, k, p_{Bm}(x, k)) = p_{Bm}(x, k) + \frac{m\bar{t}(1-2x)}{2^{k+1}},
\]

such that firm \( A \) serves all consumers for any \( p_{Bm}(x, k) \) there. In equilibrium it must be that \( p_{Bm}(x, k) = 0 \). Indeed, assume that \( p_{Bm}(x, k) > 0 \). Then given the price of firm \( A \) (4), firm \( B \) has an incentive to decrease slightly its price to gain some consumers. Hence, we get \( p^*_Am(x, k) = (m-1)\bar{t}(1 - 2x)/2^k \) and \( p^*_Bm(x, k) = 0 \). Firm \( A \) serves all consumers on segments
$2 \leq m \leq 2^k$. The profits of firm $A$ are computed as

$$
\Pi_A^{A,i}(k) = \int_0^{1/2} \int_0^{2\bar{t}/2^k} f_t 2\bar{t}(1-2x) dx dt + \int_0^{1/2} \int_0^{\bar{t}/(3 \times 2^k)} f_t \bar{t}(2x-1) dx dt
$$

$$
+ \sum_{(a,b) = \left(\frac{\bar{t}}{2k}, \frac{2\bar{t}}{2k}\right), \left(\frac{2\bar{t}}{2k}, \frac{3\bar{t}}{2k}\right), \ldots, \left(\frac{(2k-1)\bar{t}}{2k}, \frac{2k\bar{t}}{2k}\right)} f_t a (1-2x) dx dt
$$

$$
= \frac{5\bar{t}}{9 \times 2^{2(1+k)}} + \frac{1}{4\bar{t}} \left[ \frac{\bar{t}}{2k} \times \frac{1}{2^k} + \left( \frac{2\bar{t}}{2k} + \cdots + \frac{(2k-1)\bar{t}}{2k} \times \frac{1}{2^k} \right) \right]
$$

$$
= \frac{5\bar{t}}{9 \times 2^{2(1+k)}} + \frac{1}{4\bar{t}} \left( \frac{\bar{t}}{2^k} \right)^2 \sum_{n=1,\ldots,2^k-1} n = \frac{5\bar{t}}{9 \times 2^{2(1+k)}} + \bar{t} \left( \frac{1}{8} - \frac{1}{2^{3+k}} \right)
$$

This completes the proof of the Claim.

A similar analysis as in Claim 1 can be conducted for the cases $k = 0$ and $k = 1$. If $k = 0$, then on the turf of firm $i$ firms charge prices $p_{i1}^*(x,k) = 2\bar{t}|1-2x|/3$ and $p_{i2}^*(x,k) = 7|1-2x|/3$. Firm $i$ serves consumers with $t \geq \bar{t}/3$ on its own turf and consumers with $t < \bar{t}/3$ on the competitor’s turf and realizes the profit $\Pi_i^{A,i}(0) = 5\bar{t}/36$. If $k = 1$, then on the turf of firm $i$ firms charge prices $p_{i1}^*(x,k) = \bar{t}|1-2x|/2$ and $p_{i2}^*(x,k) = 0$ on segment $m = 2$ and prices $p_{i1}^*(x,k) = \bar{t}|1-2x|/3$ and $p_{i2}^*(x,k) = \bar{t}|1-2x|/6$ on segment $m = 1$. Firm $i$ serves all consumers on segment $m = 2$, consumers with $t \geq \bar{t}/6$ on segment $m = 1$ and realizes the profit $\Pi_i^{A,i}(1) = 7\bar{t}/72$. Note finally that these equilibrium results can be derived from Claim 1 through setting $k = 0$ and $k = 1$, respectively.

We now turn to the version of our model with relatively homogeneous consumers. In the following claim we state the equilibrium for $k \geq 2$.

Claim 2. Assume that consumers are relatively homogeneous in flexibility and $k \geq 2$. In equilibrium on the turf of firm $i$ on the segment $m = 1, \ldots, 2^k$ firms charge prices $p_{i1}^*(x,k) = 0$ and $p_{im}^*(x,k) = [t + (\bar{t} - t)(m-1)/2^k]|1-2x|$, where firm $i$ serves all consumers. Profits are

$$
\Pi_i^{A,i}(k) = \frac{t}{4} \left[ 1 + (\bar{t} - t) \left[ 1/8 - 1/ (2^{3+k}) \right] \right]
$$

Proof of Claim 2. We only consider the turf of firm $A$. Maximization of firm $A$’ profits on segment $m$ on some address $x < 1/2$ yields the best-response function:

$$
p_{Am}(x,k, p_{Bm}(x,k)) = p_{Bm}(x,k) + \left[ t + \frac{(\bar{t} - t)(m-1)}{2^k} \right] (1-2x), \quad (5)
$$
such that firm $A$ serves all consumers for any $p_{Bm}(x,k)$. In equilibrium it must be that $p^*_{Bm}(x,k) = 0$. Assume that $p_{Bm}(x,k) > 0$. Then given the price of firm $A$ (5), firm $B$ has an incentive to decrease slightly its price to gain some consumers. Hence, we have $p^*_{Am}(x,k) = \left[ t + (\bar{t} - t) (m-1)/2^k \right] (1-2x)$ and $p^*_{Bm}(x,k) = 0$. Firm $A$’s profit is computed as

$$
\Pi^A_A(k) = \sum_{(a,b) = (\ell + \frac{\tau - t}{2^k}, \ldots, \ell + \frac{2(\tau - t)}{2^k})} \int_0^{1/2} \int f_{l,x} a (1-2x) dx dt
$$

$$
= \frac{1}{2^{2+k}} \sum_{a = \ell + \frac{\tau - t}{2^k}, \ldots, \ell + \frac{2(\tau - t)}{2^k}} a = \frac{t}{4} + \left( \bar{t} - t \right) \left( 1 - \frac{1}{3 + k} \right).
$$

This completes the proof of the Claim.

A similar analysis as in Claim 2 can be conducted for the cases $k = 0$ and $k = 1$. If $k = 0$, then on the turf of firm $i$ firms charge prices $p^*_{im}(x,0) = t|1-2x|$ and $p^*_{jm}(x,0) = 0$, where firm $i$ serves all consumers. The profit of firm $i$ is $\Pi^i_{i-A}(0) = t/4$. If $k = 1$, then on the turf of firm $i$ on the segment $m = 1$ firms charge prices $p^*_{i1}(x,1) = \bar{t} |1-2x|$ and $p^*_{j1}(x,1) = 0$. On the segment $m = 2$ equilibrium prices are $p^*_{i2}(x,1) = \left( \bar{t} + \frac{t}{2} \right) |1-2x|/2$ and $p^*_{j2}(x,1) = 0$. Firm $i$ serves all consumers on its turf and realizes the profit $\Pi^i_{i-A}(1) = (\bar{t} + 3t)/16$. Note finally that these equilibrium results can be derived from Claim 2 through setting $k = 0$ and $k = 1$, respectively. Q.E.D.

**Proof of Proposition 2.** Consider first the turf of firm $A$ and some $x < 1/2$. Given $p_B(x,k)$ firm $A$ maximizes its profit on every segment separately. The best-response function of firm $A$ on segment $m$ is

$$
p_{Am}(x,k,p_B(x,k)) = p_B(x,k) + t^m(k)(1-2x),
$$

such that for any $p_B(x,k)$ firm $A$ serves all consumers. Firm $B$ cannot do better than charging $p^*_B(x,k) = 0$, which yields $p^*_Am(x,k) = t^m(k)(1-2x)$. On address $x < 1/2$ firm $A$ realizes the profit

$$
\Pi_A(x,k) = \int_\ell^{\bar{t}} \left[ f_t \left( t + \frac{\bar{t} - t}{2^k} \right) + \ldots + \left( t + \frac{(\bar{t} - t)(2^k - 1)}{2^k} \right) \right] \frac{(\bar{t} - t)}{2^k} dt = \frac{t + (\bar{t} - t)(2^k - 1)}{2^{k+1}},
$$

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such that the total profit on its turf is
\[
\Pi_A(x < 1/2, k) = \frac{t + \bar{t}}{8} - \frac{\bar{t} - t}{2^{k+3}}, \tag{6}
\]
and the profit of firm B on A’s turf is zero.

Consider now the turf of firm B and some \( x > 1/2 \). Given \( p_B(x, k) \) firm A maximizes its profit on every segment separately. The best-response function of firm A on segment \( m \) is

\[
p_{Am}(x, k, p_B(x, k)) = \begin{cases} 
0 & \text{if } p_B(x, k) \leq e \\
\frac{p_B(x, k) - t^m(k)(2x-1)}{2} & \text{if } e < p_B(x, k) < f \\
p_B(x, k) - t^m(k)(2x - 1) & \text{if } p_B(x, k) \geq f,
\end{cases}
\]

where \( e = t^m(k)(2x - 1) \) and \( f = \left[2t^m(k) - t^m(k)\right](2x - 1) \). We have to find now the optimal price of firm B given firm A’s best-response function. Note that for any \( m \geq 2 \) and \( k \geq 1 \) it holds that \( 2t^{m-1}(k) - t^{m-1}(k) = t^m(k) \). Assume further that \( k \geq 1 \). The profit function of firm B for some \( x > 1/2 \) then takes the form

\[
\Pi_B(x, k, p_B(x, k)) = \begin{cases} 
p_B(x, k) & \text{if } p_B(x, k) < a \\
\frac{2t - t}{2} - \frac{p_B(x, k)}{2(2x-1)} & \text{if } a \leq p_B(x, k) \leq b \\
\bar{t} + \frac{t - \bar{t}}{2^{k+1}} - \frac{p_B(x, k)}{2x-1} & \text{if } b \leq p_B(x, k) < c \\
\bar{t} + \frac{t - \bar{t}}{2^{k+1}} - \frac{p_B(x, k)}{2x-1} & \text{if } c \leq p_B(x, k) < d \\
0 & \text{if } p_B(x, k) \geq d,
\end{cases}
\]

where \( a = \bar{t}(2x-1) \), \( b = \left[\bar{t} + (\bar{t} - t)/2^k\right](2x-1) \), \( c = \bar{t}(2x-1) \) and \( d = \left[\bar{t} + (\bar{t} - t)/2^k\right](2x-1) \). It is straightforward to show that \( \Pi_B(x, k, p_B(x, k)) \) decreases on \( b \leq p_B(x, k) \leq d \). Depending on \( l \) and \( k \) three possibilities emerge. First, if \( l < 3/2 \), then \( \Pi_B(x, k, p_B(x, k)) \) increases on \( p_B(x, k) < a \) and decreases on \( a \leq p_B(x, k) < d \), such that \( p_B^*(x, k) = \bar{t}(2x - 1) \). For any \( x > 1/2 \) firm B serves all consumers. Second, if \( l > 3/2 \) and \( k < \log_2 [(l - 1)/(l - 3/2)] \), then \( \Pi_B(x, k, p_B(x, k)) \) increases on \( p_B(x, k) < (2\bar{t} - \bar{t})(2x - 1)/2 \) and decreases on \( (2\bar{t} - \bar{t})(2x - 1)/2 \leq p_B(x, k) < d \), such that \( p_B^*(x, k) = (2\bar{t} - \bar{t})(2x - 1)/2 \). For any \( x > 1/2 \) firm B serves consumers with \( t \geq (2\bar{t} + \bar{t})/4 \). Third, if \( l > 3/2 \) and \( k \geq \log_2 [(l - 1)/(l - 3/2)] \), then \( \Pi_B(x, k, p_B(x, k)) \) increases on \( p_B(x, k) < b \) and decreases on \( b \leq p_B(x, k) \leq d \), such that \( p_B^*(x, k) = [t + (\bar{t} - t)/2^k](2x - 1) \). For any \( x > 1/2 \) firm B serves consumers with \( t \leq t + (\bar{t} - \bar{t})/2^{k+1} \).
We now compute firms’ profits on the turf of firm $B$. Consider first $l \leq 3/2$. Firm $A$ serves no consumers on the turf of firm $B$, the profit of firm $B$ is computed as $\Pi_B^{1,NA}(k) = \int_{1/2}^1 \int_{l_k}^{T} [f_{t,x}t(2x-1)] \, dx \, dt = t/4$. Consider now $l > 3/2$. Assume that $k < \log_2 [(l-1)/(l-3/2)]$.

The profit of firm $B$ is computed as

$$\Pi_B^{1,NA}(k) = \int_{1/2}^1 \int_{l_k}^{T} \left[ f_{t,x} \frac{(2l-t)(2x-1)}{2} \right] \, dx \, dt = \frac{(2l-1)^2 t}{32 (l-1)}.$$ 

The profit of firm $A$ is computed as

$$\Pi_A(x > 1/2, k) = \int_{1/2}^1 \int_{l_k}^{T} \left[ f_{t,x} \frac{(2l-3t)(2x-1)}{4} \right] \, dx \, dt = \frac{(2l-3)^2 t}{64 (l-1)}. \quad (7)$$

Summing up the profits (6) and (7) we get the total profit of firm $A$

$$\Pi_A^{1,NA}(k) = \left( \frac{12l^2 - 12l + 1}{64 (l-1)} \right) t - \frac{(l-1) t}{2^{k+3}}.$$ 

Assume finally that $k > \log_2 [(l-1)/(l-3/2)]$. The profit of firm $B$ is computed as

$$\Pi_B^{1,NA}(k) = \int_{1/2}^1 \int_{l_k}^{T} \left[ f_{t,x} \left( t + \frac{7-t}{2^k} \right) (2x-1) \right] \, dx \, dt = \left( 1 + \frac{t-l}{2^k} \right) \left( 1 - \frac{l-1}{2^{k+1}} \right) \frac{t}{4}. \quad (8)$$

The profit of firm $A$ is computed as

$$\Pi_A(x > 1/2, k) = \int_{1/2}^1 \int_{l_k}^{T} \left[ f_{t,x} \frac{(k-t)(2x-1)}{2^{k+1}} \right] \, dx \, dt = \frac{(l-1) t}{2^{2k+4}}. \quad (8)$$

Summing up the profits (6) and (8) we get

$$\Pi_A^{1,NA}(k) = t \left[ \frac{1 + l}{8} - \frac{(l-1)(2^{k+1} - 1)}{2^{2k+4}} \right].$$

We finally note the above derived equilibrium for $k \geq 1$ describes also the equilibrium at $k = 0$.

Q.E.D.

**Proof of Proposition 3.** Consider first the turf of firm $A$. Given $p_B(x, k)$ the best-response
function of firm $A$ on $m = 1$ is

$$p_{A1}(x, k, p_B(x, k)) = \begin{cases} p_B(x, k) & \text{if } p_B(x, k) \geq \frac{\tilde{t}(1-2x)}{2^k} \\ \frac{p_B(x, k)}{2} + \frac{\tilde{t}(1-2x)}{2^{k+1}} & \text{if } p_B(x, k) < \frac{\tilde{t}(1-2x)}{2^k}. \end{cases}$$

The best-response function of firm $A$ on $m \geq 2$ is $p_{Am}(x, k, p_B(x, k)) = p_B(x, k) + \ell^m(k) (1 - 2x)$, such that firm $A$ serves all consumers on $m$. Assume that $p_B(x, k) < \tilde{t}(1 - 2x)/2^k$, in which case firm $B$ serves consumers with $t \leq \tilde{t}/2^{k+1} - p_B(x, k)/[2(1 - 2x)]$ on $m = 1$. Maximization of firm $B$'s expected profit yields $p^*_B(x, k) = \tilde{t}(1 - 2x)/2^{k+1}$ and $p^*_{A1}(x, k) = 3\tilde{t}(1 - 2x)/2^{k+2}$.

Firm $A$ serves consumers with $t \geq \tilde{t}/2^{k+2}$. The profit of firm $B$ on the turf of firm $A$ is

$$\Pi_B(x < 1/2, k) = \int_0^{1/2} \left[ f_{x,t} \frac{\tilde{t} - p_B(x, k)}{2^{k+1}} \frac{\tilde{t}(1-2x)}{2^{k+1}} \right] dx = \frac{\tilde{t}}{2^{2k+2}}.$$

The profit of firm $A$ on its own turf is

$$\Pi_A(x < 1/2, k) = \int_0^{1/2} \left[ f_{x,t} \frac{\tilde{t}}{2^k} \left( \frac{\tilde{t}(2^k - 1)}{2^{k+1}} + \frac{\tilde{t}}{2^k} + \frac{2\tilde{t}}{2^k} + \ldots + \frac{\tilde{t}(2^k - 1)}{2^k} \right) (1 - 2x) \right] dx + \int_0^{1/2} \left[ f_{x,t} \left( \frac{3}{4} \right)^2 \left( \frac{\tilde{t}}{2^k} \right)^2 (1 - 2x) \right] dx = \frac{\tilde{t}(2^k - 1)}{2^{k+3}} + \frac{\tilde{t}(2^{k+3} + 1)}{2^{k+6}}.$$

Consider now the turf of firm $B$ and some $x > 1/2$. On some segment $m$ the best-response function of firm $A$ takes the form:

$$p_{Am}(x, k, p_B(x, k)) = \begin{cases} 0 & \text{if } \frac{p_B(x, k)}{(2x-1)} \leq \ell^m(k) \\ \frac{p_B(x, k)}{2} - \ell^m(k)(2x-1) & \text{if } \ell^m(k) < \frac{p_B(x, k)}{(2x-1)} < 2\ell^m(k) - \ell^m(k) \\ p_B(x, k) - \ell^m(k)(2x-1) & \text{if } \frac{p_B(x, k)}{(2x-1)} \geq 2\ell^m(k) - \ell^m(k). \end{cases}$$

The profit of firm $B$ on some $x > 1/2$ is

$$\Pi_B(x, k, p_B(x, k)) = \begin{cases} \left[ \frac{\tilde{t} - p_B(x, k)}{2(2x-1)} \right] p_B(x, k) & \text{if } 0 \leq p_B(x, k) < a \\ \left[ \tilde{t} + \frac{p_B(x, k)}{2x-1} \right] p_B(x, k) & \text{if } a \leq p_B(x, k) < b \\ \left[ \tilde{t} + \frac{p_B(x, k)}{2(2x-1)} \right] p_B(x, k) & \text{if } b \leq p_B(x, k) < c \\ 0 & \text{if } p_B(x, k) \geq c, \end{cases}$$

where $a = \tilde{t}(2x - 1)/2^k$, $b = \tilde{t}(2x - 1)$, $c = (\tilde{t} + \tilde{t}/2^k)(2x - 1)$. Note that $\Pi_B(x, k, p_B(x, k))$ increases on $0 \leq p_B(x, k) < a$, decreases on $b \leq p_B(x, k) < c$ and gets its maximum at $p_B^*(x, k) =$
\((\bar{t}/2 + \bar{t}/2^{k+2}) (2x - 1)\) on \(a \leq p_B(x, k) < b\). For any \(k \geq 1\) it holds \(p_B^*(x, k)/(2x - 1) \in [\bar{t}/2, \bar{t}/2 + \bar{t}/2^k]\). Assume further that 
\(k = 1\). On \(m = 2^{k-1} + 1\) firm \(B\) serves consumers with 
\(t > \bar{t}/2 + \bar{t}/2^{k+3}\), on \(m = 2^{k-1}\) firm \(B\) serves consumers with 
\(t \geq \bar{t}/2 - 3\bar{t}/2^{k+3}\). If \(k \geq 2\), then 
firm \(B\) serves all consumers on \(m \geq 2^{k-1} + 2\) and firm \(A\) serves all consumers on \(m \leq 2^{k-1} - 1\).

The profit of firm \(B\) on its turf is

\[
\Pi_B(x > 1/2, k) = \int_{1/2}^{1} f_{x,t} \left( \frac{\bar{t}}{2} + \frac{\bar{t}}{2^{k+2}} \right)^2 \, dx = \frac{\bar{t}}{16} \left( 1 + \frac{1}{2^k} + \frac{1}{2^{2k+2}} \right). \tag{11}
\]

Summing up (9) and (11) we get the profit of firm \(B\) as

\[
\Pi_B^{A,NA}(k) = \frac{\bar{t}}{16} \left( 1 + \frac{1}{2^k} + \frac{3}{2^{2k+2}} \right), \text{ for any } k \geq 1.
\]

We now compute the equilibrium prices of firm \(A\) on the turf of firm \(B\). We get \(p_A^*(x, k) = 5\bar{t} (2x - 1)/2^{k+3}\) if \(m = 2^{k-1}\) and \(p_A^*(x, k) = \bar{t} (2x - 1)/2^{k+3}\) if \(m = 2^{k-1} + 1\). If \(k \geq 2\), then 
on \(m \leq 2^{k-1} - 1\) firm \(A\) charges \(p_A^*(x, k) = p_B^*(x, k) - \bar{t}^m(k) (2x - 1)\). The profit of firm \(A\) on 
the rival’s turf is

\[
\Pi_A(x > 1/2, k) = \int_{1/2}^{1} \left[ f_{x,t} (2x - 1) \left( \left( \frac{\bar{t}}{2^{k+3}} \right)^2 + \left( \frac{5\bar{t}}{2^{k+3}} \right)^2 \right) \right] \, dx 
+ \int_{1/2}^{1} \left[ f_{x,t} (2x - 1) \left( \frac{\bar{t}}{2^k} \left( \frac{\bar{t}}{2^{k+2}} \right) \left( 2^{k-1} - 1 \right) - \frac{\bar{t}}{2^k} \left( 1 + 2 + \ldots + 2^{k-1} - 1 \right) \right) \right] \, dx
= \frac{\bar{t} \left( 2^{2k+2} - 2^{k+2} + 5 \right)}{2^{2k+7}}. \tag{12}
\]

Summing up the profits of firm \(A\) on the two turfs, (10) and (12), we get

\[
\Pi_A^{A,NA}(k) = \frac{\bar{t} \left( 5 \times 2^{2k+2} - 2^{k+2} + 7 \right)}{2^{2k+7}}, \text{ for any } k \geq 1.
\]

We now derive the equilibrium on the turf of firm \(B\) for \(k = 0\). The best-response function 
of firm \(A\) is

\[
p_A(x, p_B(x, 0)) = \begin{cases} 
p_B(x, 0) & \text{if } p_B(x, 0) \leq 2\bar{t} (2x - 1) \\
p_B(x, 0) - \bar{t}(2x - 1) & \text{if } p_B(x, 0) > 2\bar{t} (2x - 1).
\end{cases}
\]

Assume that \(p_B(x, 0) \leq 2\bar{t} (2x - 1)\) for any \(x > 1/2\), such that firm \(B\) serves consumers with \(t \geq \ldots\)
\(p_B(x, 0)/[2(2x - 1)]\). Maximization of firm \(B\)’s profit yields \(p^*_B(x, 0) = \bar{t}(2x - 1) < 2\bar{t}(2x - 1)\) and \(p^*_A(x, 0) = \bar{t}(2x - 1)/2\). Firm \(B\) serves consumers with \(t \geq \bar{t}/2\). On the turf of firm \(B\) firms realize profits \(\Pi_A(x > 1/2, 0) = \bar{t}/16\) and \(\Pi_B(x > 1/2, 0) = \bar{t}/8\). Summing up these profits with (10) and (9) for \(k = 0\) we get \(\Pi^{A,N,A}_A(0) = 13\bar{t}/64\) and \(\Pi^{A,N,A}_B(0) = 5\bar{t}/32\), respectively. \(Q.E.D.\)

**Proof of Proposition 4.** We prove first part \(i)\) of the proposition. If a firm unilaterally acquires customer data, it keeps all consumers on its turf and extracts more rents from them, it also gains consumers on the rival’s turf (if \(l > 3/2\)), such that the unilateral acquisition of data is always profitable. Assume now that firm \(A\) has customer data. We show that firm \(B\) always has an incentive to acquire data. When both firms hold data of quality \(k \geq 1\), firm \(B\) realizes the profit

\[
\Pi^{A,A}_B(k) = \bar{t}/4 + (\bar{t} - \bar{t}) \left[ 1/8 - 1/2^{3+k} \right],
\]

as stated in Proposition 1. Assume first that \(l \leq 3/2\), in which case the profit of firm \(B\) is

\[
\Pi^{A,N,A}_B(k) = \bar{t}/4
\]

if only firm \(A\) holds data, as stated in Proposition 2. Comparing the profits (13) and (14) we get

\[
\Pi^{A,A}_B(k) - \Pi^{A,N,A}_B(k) = \frac{(\bar{t} - \bar{t})}{8} \left[ 1 - \frac{1}{2^k} \right] > 0, \text{ for any } k \geq 1,
\]

such that firm \(B\) has an incentive to acquire data when the rival holds it if \(l \leq 3/2\). Assume now that \(l > 3/2\) and \(k < \log_2 [(l - 1)/(l - 3/2)]\), in which case the profit of firm \(B\) is

\[
\Pi^{A,N,A}_B(k) = \frac{(2\bar{t} - \bar{t})^2}{32 (\bar{t} - \bar{t})}
\]

if only the rival holds data, as stated in Proposition 2. Comparison of the profits (13) and (15) yields

\[
\frac{\Pi^{A,A}_B(k) - \Pi^{A,N,A}_B(k)}{\bar{t}} = -\frac{(3 - 2l)^2}{32 (l - 1)} + \frac{(l - 1)}{8} \left( 1 - \frac{1}{2^k} \right) \geq \frac{(2\bar{t}^2 - 8\bar{t} + 7)}{32 (l - 1)}. \tag{16}
\]

The inequality in (16) follows from \(1/2^k \leq 1/2\) for any \(k \geq 1\). Note that \(2\bar{t}^2 - 8\bar{t} + 7 < 0\) for any \(3/2 < l \leq 2\), such that \(\Pi^{A,A}_B(k) > \Pi^{A,N,A}_B(k)\), and firm \(B\) has an incentive to acquire data when the rival holds it if \(l > 3/2\) and \(k < \log_2 [(l - 1)/(l - 3/2)]\) hold. Assume finally that \(l > 3/2\)
and $k \geq \log_2 [(l - 1)/(l - 3/2)]$, in which case the profit of firm $B$ is

$$\Pi_B^{A,NA}(k) = \frac{l}{4} \left[ 1 + \frac{(l - 1)}{2^k} \right] \left[ 1 - \frac{1}{2^{k+1}} \right]$$ (17)

if only the rival holds data, as stated in Proposition 2. Comparison of the profits (13) and (17) yields

$$\Pi_B^{A,A}(k) - \Pi_B^{A,NA}(k) = \frac{t}{8} \left[ (l - 1) \left( 1 - \frac{3}{2^k} + \frac{1}{2^{2k}} \right) + \frac{1}{2^k} \right].$$ (18)

The expression in (18), $1 - 3/2^k + 1/2^{2k}$, can be either positive or negative. If $1 - 3/2^k + 1/2^{2k} \geq 0$, then $\Pi_B^{A,A}(k) > \Pi_B^{A,NA}(k)$ for any $3/2 < l \leq 2$. Assume that $1 - 3/2^k + 1/2^{2k} < 0$. Then for any $3/2 < l \leq 2$ it holds that

$$\Pi_B^{A,A}(k) - \Pi_B^{A,NA}(k) \geq \frac{t}{8} \left( 1 - \frac{2}{2^k} + \frac{1}{2^{2k}} \right),$$ (19)

where we derived the RHS of (19) by plugging $l = 2$ into the RHS of (18). The derivative of the RHS of (19) with respect to $k$ is positive for any $k \geq 1$, hence, $\Pi_B^{A,A}(1) - \Pi_B^{A,NA}(1) = t/32$. It follows that firm $B$ has an incentive to acquire data when the rival holds it and $l > 3/2$ and $k \geq \log_2 [(l - 1)/(l - 3/2)]$ hold. Hence, firm $B$ has always an incentive to acquire data when the rival holds it. We conclude that there is the unique equilibrium (in dominant strategies) for any $k \geq 1$, where both firms acquire data.

We now prove part $ii)$ of the proposition. Assume that firm $B$ does not hold customer data. We analyze the incentives of firm $A$ to acquire data. If firm $A$ does not acquire data, its profit is

$$\Pi_A^{A,A}(0) = 57/36,$$ (20)

as stated in Proposition 1. If firm $A$ acquires data, its profit is

$$\Pi_A^{A,NA}(k) = \frac{l}{5} \left[ 5 \times 2^{2k+2} - 2^{k+2} + 7 \right] / 2^{2k+7},$$ (21)

as stated in Proposition 3. The comparison of the profits (21) and (20) yields

$$\frac{\left[ \Pi_A^{A,NA}(k) - \Pi_A^{A,A}(0) \right]}{l} \times 9 \times 2^{2k+7} = 5 \times 2^{2k+2} - 9 \times 2^{k+2} + 63.$$ (22)

Taking derivative of the RHS of (22) with respect to $k$ we get $2^{k+2} \ln 2 \left( 10 \times 2^k - 9 \right) > 0$ for
any \( k \geq 0 \). Hence, for any \( k \geq 1 \) it holds that \( \Pi_A^{A,N,A}(k) > \Pi_A^{A,A}(0) \), and a firm always has a unilateral incentive to acquire customer data.

Assume now that firm \( A \) holds customer data. We analyze whether firm \( B \) also has an incentive to acquire data. If it does not acquire, its profit is

\[
\Pi_B^{A,N,A}(k) = \bar{t} \left( 1 + 1/2^k + 3/2^{2k+2} \right) / 16, \tag{23}
\]
as shown in Proposition 3, while if it acquires data its profit is

\[
\Pi_B^{A,A}(k) = 5\bar{t} / \left( 9 \times 2^{2(1+k)} \right) + \bar{t} \left( 1/8 - 1/2^{3+k} \right), \tag{24}
\]
as stated in Proposition 1. The comparison of the profits (24) and (23) yields

\[
\frac{\Pi_B^{A,A}(k) - \Pi_B^{A,N,A}(k)}{\bar{t}} \times 9 \times 2^{2k+6} = 36 \times 2^{2k} - 108 \times 2^{k} + 53. \tag{25}
\]
Taking derivative of the RHS of (25) with respect to \( k \) we get \( 9 \times 2^{k+2} \ln 2 \left( 2^{k+1} - 3 \right) \), which is negative if \( k = 0 \) and positive if \( k \geq 1 \). Evaluating the RHS of (25) at \( k = 1 \) we get \(-19\), and at \( k = 2 \) we get \(187\). Hence, if \( k = 1 \), then a firm does not acquire data when the rival holds it, and acquires it if \( k \geq 2 \).

We conclude that if \( k = 1 \), then there are two Nash equilibria where only one of the firms acquires data. If \( k \geq 2 \), there is a unique Nash equilibrium (in dominant strategies) where both firms acquire data. \textit{Q.E.D.}

\textbf{Proof of Proposition 5.} We prove first part \( i) \) of the proposition. For any \( k \geq 0 \) in equilibrium every firm acquires customer data and serves all consumers on its turf. Then social welfare can be computed as

\[
SW^{A,A}(k) = v - 2 \int_{0}^{1/2} \int_{0}^{\bar{t}} [f_{x,t}t] dt dx = v - \frac{\bar{t} + t}{8}. \]

Consumer surplus can be computed by subtracting profits (stated in Proposition 1) from social welfare, which yields

\[
CS^{A,A}(k) = v - \frac{3}{8} \left( \bar{t} + t \right) + \frac{\bar{t} - t}{2^{k+2}}. \]
We turn now to part \( ii \) of the proposition. If \( k = 0 \), then social welfare is computed as

\[
SW^{A,A}(0) = v - 2 \int_0^{1/2} \int_0^{\frac{\pi}{2}} [f_{t,x} t] dt dx - 2 \int_0^{1/2} \int_0^{\frac{\pi}{2}} [f_{t,x} t (1 - x)] dt dx = v - \frac{11 \pi}{72}.
\]

Consumer surplus can be computed through subtracting profits (stated in Proposition 1) from social welfare: \( CS^{A,A}(0) = v - 31 \pi / 72 \). If \( k = 1 \), then only one firm acquires customer data in equilibrium, such that \( \Pi_A^{A,NA}(1) = 79 \pi / 512 \) and \( \Pi_B^{A,NA}(1) = 27 \pi / 256 \). Social welfare is computed as

\[
SW^{A,NA}(1) = v - \int_0^{1/2} \int_0^{\frac{\pi}{4}} [f_{t,x} t (1 - x)] dt dx - \int_0^{1/2} \int_0^{\frac{\pi}{4}} [f_{t,x} t] dt dx - \int_0^{1/2} \int_0^{\frac{\pi}{4}} [f_{t,x} t (1 - x)] dt dx.
\]

Subtracting profits from social welfare we get \( CS^{A,NA}(1) = v - 417 \pi / 1024 \). If \( k \geq 2 \), then both firms acquire customer data in equilibrium. Social welfare is computed as

\[
SW^{A,A}(k) = v - 2 \int_0^{1/2} \int_0^{\frac{\pi}{2}} [f_{t,x} t] dt dx - 2 \int_0^{1/2} \int_0^{\frac{\pi}{2}} [f_{t,x} t (1 - x)] dt dx
= v - \frac{\pi}{8} \left( 1 + \frac{1}{9 \times 2^{k-1}} \right).
\]

Subtracting profits (stated in Proposition 1) from social welfare we get consumer surplus

\[
CS^{A,A}(k) = v - \frac{11 \pi}{9 \times 2^{k+2}} + \frac{\pi}{8} \left( \frac{1}{2^{k-1}} - 3 \right).
\]

Q.E.D.
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