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MEASURING AID FLOWS: A NEW APPROACH

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Abstract

The debate on the effectiveness of foreign aid has intensified in recent years as aid has come under increasing budgetary pressures in donor countries. Whatever the merits of the opposing arguments, the fundamental issue arises of whether conventionally-used measures of aid such as ODA, that lump together grants and loans, accurately reflect true aid flows. In this paper we analyze the methodological shortcomings of conventional aid measures, and propose a new valuation approach that measures official aid flows as the sum of grants and the grant equivalents of official loans. We show how this conceptually-superior aid measure can diverge significantly from the conventional aggregates and provide a quite different view on major aid trends. We include a companion data set with this paper to provide estimates of our new aid measure - Effective Development Assistance - for a set of 133 developing countries from 1975 to 1995.

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INTRODUCTION

Many poor aid-recipient countries view foreign aid as a critical ingredient in their development strategy, even though its development effectiveness remains in question among many economists. At the same time, the level and trends of foreign aid are increasingly becoming sensitive issues in donor countries’ budgetary discussions, with analysts observing increasing signs of “donor fatigue”. In particular, International Financial Institutions have expressed concerns regarding the level of overall development aid and the possible crowding out of poor traditional recipients by former socialist economies. Whatever the merits of these views, the key issue arises of whether the aid aggregates commonly used by policymakers and researchers in their assessments of development aid provide an accurate measure of true aid flows. In this paper we analyze the traditional methodology underlying these conventional measures and propose a new approach.

Official financial flows are traditionally classified as concessional on the basis of the OECD’s Official Development Assistance (ODA) classification, and aid flows are traditionally measured by the corresponding Net ODA statistic. For example, the World Bank’s Global Development Finance uses Net ODA information to analyze trends in aid flows over time as well as across recipients and donors. Despite its popularity, however, the methodology underlying Net ODA aggregates suffers from a number of shortcomings. Consequently, the analysis of aid flows needs to be revisited in the light of more satisfactory measures based on improved methodologies. In this paper we analyze the nature of the proposed improvements and illustrate them with a comparative assessment of the overall trends in aid flows to 133 developing economies.1

The remainder of the paper is structured as follows. Section 1 summarizes the shortcomings of conventional measures of aid, and Section 2 proposes a new approach to overcome them. The implementation of this new approach leads to an alternative aid measure which we label Effective Development Assistance (EDA), which is developed in Section 3. Section 4 illustrates the proposed method by re-assessing trends in aid flows on the basis of EDA, and comparing them with those implied by Net ODA. Finally, Section 5 offers some concluding remarks.

1. WHAT IS WRONG WITH CONVENTIONAL FOREIGN AID MEASURES?

Foreign aid is conventionally measured on the basis of the OECD’s ODA, a concept introduced in the early 1970s. ODA comprises official financial flows with a development purpose in the form of grants (inclusive of those tied to technical assistance) and highly concessional loans. Loans are defined as highly concessional when their grant element $C$ is at least 25 percent, as measured by a formula to be analyzed in depth in the next section. The leading measure of foreign aid flows is the so-called Net ODA, which is the net disbursement amount, i.e., disbursements minus amortization, of those flows classified as ODA.

Is Net ODA an appropriate measure of aid flows? Conceptually, international finance flowing to capital-scarce developing countries may involve efficiency gains even if the flows accrue on market terms as long as the funds are used appropriately. Such efficiency gains translate into net financial gains for the recipient countries. The main purpose of measuring foreign aid flows is to assess the portion of those gains that is due to a pure transfer of resources from donors to recipients through below-market, subsidized financial terms, i.e., to assess the donors’ net financial cost, rather than the (presumably larger) recipients’ benefit.

Net ODA, however, does not accurately measure the cost that donors incur in connection with their aid (especially debt) flows, and as a result the evolution of Net ODA over time, as well as across donors and recipients, likely provides a distorted picture of aid trends. This distortion is due to seven conceptual shortcomings of Net ODA that we detail below.

**Shortcomings of Net ODA: Gross and Net Flows**

1. *Under-estimation of the aid content due to netting out.* The financial cost involved in donors’ aid provided in a given year is a forward-looking concept reflected in the fractional value of the debt service claims acquired in exchange for up-front (gross) disbursements in that period, irrespective of the amortization of previously contracted debt obligations. Therefore, on this account, the net flow nature of Net ODA, i.e., disbursements minus amortization, underestimates the aid content of disbursed flows by netting out amortization payments. For example, a constant flow of identical highly concessional loans over time entails a continuous cost on the part of the donor but yields a zero Net ODA flow, since amortization payments exactly offset disbursements. In such case, Net ODA would completely fail to capture the aid content of flows.

Aside from this netting out involved in Net ODA, the rest of the shortcomings relate to the flow amounts classified as ODA:

**Shortcomings of ODA: Aggregation and Coverage**

Design shortcomings of ODA related to loan coverage and aggregation obscure the interpretation of this measure. The three main conceptual problems are:

2. *Over-representation of loans with high concessionality.* ODA includes the full face value of both grants and highly concessional loans without distinguishing between the two. However, concessional loans entail repayment obligations, and, therefore, the aid they involve, i.e., the net financial cost to donors, is only a fraction of their face value. The inclusion in ODA of the full face value of these loans overestimates their aid content. Only grants, that is to say pure unrequited transfers, should be accounted at full value.\(^2\)

3. *Under-representation of loans with low concessionality.* Under the ODA definition, non-concessional loans include loans on market terms as well as concessional loans with low degree of concessional. The aid content of the latter, i.e., the donors’ cost involved in these loans, is therefore not captured by ODA.

\(^2\)Assuming that the grant is not tied or subject to other financial quid pro quo.
4. **Coverage.** The inclusion in ODA of official technical assistance grants by their full value can be seen as another shortcoming. In this case, the donor benefits from payments received in return for the technical assistance supplied, and this may greatly reduce the donor’s net financial cost.\(^3\)

**Shortcomings of ODA: The Grant Element**

As noted above, ODA is based on a sharp distinction between concessional and non-concessional loans, drawing from their respective grant elements. Conceptually, the calculation of the grant element, i.e., the degree of concessionality, involves the computation of the expected present value of the stream of debt service obligations associated with the loan under consideration. To the extent that the discount rate utilized reflects the creditor’s opportunity cost, i.e., the return it could make on alternative investments of the same capital, this present value measures the economic value of debt service repayments and, on this account, the financial value of the loan. The grant element of the loan is the portion of the loan that, at a given time, is not expected to be repaid, i.e. the shortfall of the above-mentioned present value relative to the amount disbursed.

For the purposes of ODA, loans are classified as concessional if their grant element exceeds 25 percent, and as non-concessional (and hence ignored) otherwise. The grant elements are computed using some special assumptions, however: most importantly, loan interest rates (used to compute interest charges) are assumed to remain constant throughout the life of the loan, and a fixed 10 percent discount rate is utilized in all present value calculations. This methodology for computing grant elements contains a number of shortcomings, which may lead to loan misclassification and distortion of ODA figures across time, donors, and recipients:

5. **Discount Rates.** In order to reflect donors’ opportunity costs, the discount rates used for present value calculations should correspond to applicable market rates. The fixed 10 percent discount rate utilized in ODA fails that test on at least three important dimensions to which it should be sensitive, namely time, currency, and maturity:

a) **Time.** Discount rates should evolve over time with market conditions prevailing at the time the aid content of loans is estimated. For example, to measure the donors’ cost as seen at the time of loan disbursement, the market terms prevailing at that time should be used.

b) **Currency.** At any point in time, market rates, and therefore appropriate discount rates, are currency specific. The discount rate should follow the currency in which debt service is payable.

c) **Maturity.** At any point in time and for any given currency, market rates depend on the length of the repayment period according to the so-called yield curve. Therefore, the discount rates applied to the debt service stream should vary over the life of the loan according to the timing of service payments.

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\(^3\)The exclusion of private source financing from ODA coverage appears justified, however, because commercial lending contains no aid by definition.
6. **Variable Rate Loans.** In the case of variable rate loans, the construction of the future debt service stream requires a forecast of interest rate charges. This is especially important for floating rates linked to future market conditions (e.g., indexed to six-month LIBOR). ODA makes no attempt to predict these conditions and implicitly assumes that, like in the case of fixed-rate loans, variable rates will remain constant at their level at the time of disbursement.

7. **Credit Risk.** In the absence of credit risk, the ‘market rates’ mentioned above are risk-free rates. However, credit risk is implicitly an additional source of donor financial cost from an economic viewpoint -- as opposed to a contractual, legal perspective. As such, it should be incorporated in the calculation of the grant element, either through augmenting the discount rate or through the utilization of lower expected debt service projections, or both. This is admittedly a difficult task, as it would require the use of borrower-specific risk spreads and/or default probabilities. In any case, ODA makes no attempt to adjust for credit risk.

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### 2. A NEW APPROACH: EFFECTIVE DEVELOPMENT ASSISTANCE

On the whole, the methodological shortcomings of Net ODA just summarized underscore the need for an alternative approach to the measurement of aid flows. Our proposed approach is based on the grant equivalent of financial flows. We first elaborate on grant equivalents and then explain the methodology for computing the adjusted foreign aid measure, which we call Effective Development Assistance, or EDA. This section provides a brief overview of the key issues; the analytical details are described at length in the next section.

**Measuring the Aid Content of Financial Inflows: Grant Equivalent and Grant Element**

The grant equivalent of a financial inflow is the amount that, at the time of its commitment, is not expected to be repaid, i.e., the amount subsidized through below-market terms at the time of commitment. By definition, the grant equivalent of a pure grant is the amount of the grant itself. In contrast, the grant equivalent of a concessional loan is only part of the loan amount, and becomes negligible as loan terms approach market terms. In other words, the grant equivalent, \( G \), measures the shortfall between the loan amount disbursed, \( D \), and the present value of the associated expected debt service obligations, \( E \). Correspondingly, a loan can be interpreted as including two components: a loan component, equal to the debt equivalent \( E \), and a grant component, equal to the grant equivalent \( G \), so that a concessional loan disbursed by an amount \( D \) is equivalent to a non-concessional loan by an amount \( E \) and a pure grant by an amount \( G \):

\[
G = D - E
\]  

(2.1)

By definition, the grant element, \( g \), measures the grant equivalent as a proportion of the inflow disbursed. Therefore, in terms of the more familiar grant element:

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\(^4\)Conceptually, this present value \( E \) was first made operational and utilized to reduce bond exchanges under the Brady debt reduction program to comparable debt equivalent terms (see Claessens, Diwan, and Fernandez-Arias, 1992), and was later utilized to measure the debt equivalent of concessional debt stocks for the purpose of measuring country indebtedness (e.g., in the Heavily Indebted Poor Countries (HIPC) initiative).
\[
g = \frac{G}{D}
\]

In the extreme case of a pure grant, no service payments are called for (E=0), the grant equivalent is the grant itself (G=D) and the grant element (expressed as a percentage) is \( g = 100 \) percent. At the other extreme, in financially “fair” lending, debt service equals disbursements in present value (E=D) and the grant equivalent and grant element are zero. This is the case expected of competitive commercial lending and, at least approximately, is also the case of market-based, non-concessional official lending. However, official creditors providing “soft” loans, i.e., concessional lending, extend loans with relatively low debt service (E<D), whose grant equivalent and grant element are consequently positive.

We define EDA as the sum of the grant equivalents of all development flows disbursed in a given period, thus eliminating the distortion introduced by the deduction of amortization payments in Net ODA noted in shortcoming (1) above. This simple method of converting all inflows to a common denominator also eliminates the over- and under-estimation of the aid content of loans as measured by ODA, allowing for a correct aggregation of the aid included in grants and loans in terms of comparable units. It therefore solves the ODA shortcomings (2) and (3) above.

In order to address the coverage shortcoming (4), and following traditional World Bank methodology, grants tied to technical assistance are excluded from EDA. This is just a first approximation to the ideal procedure by which the grant equivalent of tied grants would be adjusted by deducting the benefit that the donor enjoys from the quid pro quo.

The grant content of a loan whose service due extends over T periods (e.g., years) can also be analyzed by focusing on each period \( t \) (ranging from 1 to T) in the life of the loan and comparing the contractually-determined interest rate \( i_t \) applicable to the period in question with the discount rate \( d_t \) employed to bring to present-value terms the service payments due in that period. If the interest rate coincides with the discount rate (\( i_t = d_t \)) in every period, the grant element and grant equivalent are null irrespective of the debt service time profile (e.g., the grace and maturity of the loan). If the interest rate is consistently below the discount rate (\( i_t < d_t \)), the grant element and grant equivalent are both positive. The larger the gap between the interest and discount rates, the larger the absolute value of the grant element. For any given gap between the two rates, the grant element becomes larger as debt service payments are stretched over a longer time period by backloading amortization payments (e.g. longer grace period and maturity), reaching its maximum in the case of a consol, i.e., a loan with an infinite grace period (see the details in Appendix B).

5 In practice, lags between commitment and disbursement times may result in small deviations from this benchmark. Likewise, price-smoothing practices of Multilateral Development Banks may lead to cyclical variation in grant elements: with loan interest rates determined as a moving-average of current and past market interest rates, loan rates tend to lag behind market rates, so that grant elements temporarily rise when market rates are rising and fall when market rates are falling.

6 The expressions in the text implicitly assume that loans are disbursed in a single installment, and need modification if this is not the case. The general formulas are presented in the next section.

7 However, in the data files made available with this paper, technical assistance is attached as a memo item for interested users.
One implication of the above discussion is that as long as the appropriate discount rate is below 10 percent, the grant element calculations underlying ODA which make use of the arbitrary 10 percent discount rate will lead to a systematic over-estimation of the grant element and grant equivalent of loans. Further, such overestimation is larger the longer the duration of the loan. Therefore, non-concessional loans would be incorrectly shown as concessional, and marginally concessional loans would be shown as highly concessional, especially if they are long-term.  

To avoid this distortion, our proposed method to compute grant elements and grant equivalents is based on market discount rates sensitive to currency, timing, and maturity, thus addressing shortcoming (5) of ODA as described in the previous section. In essence, this is achieved by extracting the discount rates from the yield curve for risk-free instruments prevailing at the time of disbursement. We do this separately for each currency under consideration. These time-, currency- and maturity-specific discount rates are used to bring future debt service to present-value terms or, in other words, to compute the debt equivalent $E$.

As part of this calculation, our proposed method involves the construction of the debt service payment stream, which is accomplished taking into account the amortization schedule and interest terms of each loan. In the (relatively frequent) case of variable-rate debt, this in turn requires a forecast of the interest rates applicable in future years, in order to address shortcoming (6) of ODA. As explained in detail below, we obtain such forecast making use of the yield curve of the relevant currency.

**Donors’ Effort and Expected Aid**

It should be noticed that the donors’ “effort”, that our improved aid measure tries to capture, refers to donors’ voluntary net financial costs, as opposed to the financial costs realized ex-post, which are partly determined by the realization of market rates over time. For that reason, donors’ expectations should refer to the conditions at the time at which financial commitments were made. Consequently, in this proposed approach expected interest rates are derived from the market conditions prevailing at the time of commitment, rather than those prevailing at a later date or realized ex-post.

Credit risk, i.e., the risk of default, is also a factor to consider in the measurement of donors’ expected financial losses and, consequently, grant elements. Traditional ODA measures ignore this factor, as it was pointed out in shortcoming (7). While we recognize that credit risk is a factor to be taken into account in the new approach being proposed, either by a downward adjustment of the expected debt service stream or by augmenting the discount rate to reflect a risky opportunity cost, we have chosen not to make adjustments on this account in our EDA calculations. Correspondingly, our EDA statistics are subject to the caveat that they reflect contractual aid, as opposed to aid inclusive of the anticipation of failures to comply with contractual payments.

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8 This explains the sometimes large ODA-based grant element of commitments of private creditors to developing countries, as reported in the World Bank’s *Global Development Finance*.
The case has been made that multilateral lending, and perhaps official lending in general, carries a negligible risk of default. If official lenders perceive that to be the case, then the previous caveat is not relevant. This case is supported by the fact that open defaults on official lending are rare. However, debt reductions and condonations of various kinds are common, especially of bilateral debts. The view that default risk is negligible is consistent with the interpretation that this kind of debt relief is additional voluntary aid.\(^9\) Nevertheless, an alternative interpretation is that this debt relief is the manifestation of contract breaching on the part of the debtor. Furthermore, some analysts view credit risk on official lending higher than it appears by virtue of being hidden by new lending at higher concessional terms to sustain full service of all debts. In the absence of an accepted methodology, we chose to neglect credit risk in the proposed approach at this time.\(^10\)

3. **EMPirical IMPLEMENTATION**

We next summarize the procedure followed to construct our proposed measure of Effective Development Assistance (EDA) defined above. We proceed in two stages: first we describe the data used, and then we spell out in more detail the application of the methodological framework introduced in the previous section.

3.1 **Data**

Construction of effective development assistance along the lines described in section 2 requires two basic pieces of information: first, detailed data on the volume and features of official financial flows -- specifically, the characteristics of the disbursement and repayment schedules, interest charges and other fees of official loans -- and, second, a suitable set of discount rates that properly reflect opportunity costs.

*Financial flow data*

As noted earlier, official financial flows consist of loans and grants, with the latter sometimes defined to include technical assistance (TA). Figure 3.1 presents the historical distribution of official flows among these three categories. As discussed in the previous section, the inclusion of technical assistance in aid aggregates is highly questionable, due to its *quid pro quo* nature. For this reason, we restrict the focus to official flows exclusive of TA.

Since we have defined EDA as the sum of official grants plus the grant equivalent of official development-oriented loans, measuring EDA involves the computation of the grant equivalents of official loans. This is inherently a loan-by-loan task requiring information on grace, maturity, interest,

\(^9\) Because we adopt an *ex-ante* perspective, our EDA calculations do not consider debt relief, be it in the form of debt forgiveness or debt rescheduling, as new aid. Debt relief, or the accumulation of arrears for that matter, essentially amounts to an *ex-post* modification of the debt service schedule that involves *involuntary* aid, thus outside the scope of EDA. Notice that conventional Net ODA accounts for debt relief by recording a fictitious disbursement matched with an equally fictitious loan repayment, with no effect on Net ODA totals.

\(^10\) The assessment of the credit risk of official debt is mired with a number of technical difficulties (see Demirguc-Kunt and Fernandez-Arias 1992 for details). It is clear that such assessment requires an analysis of the conditions of each recipient country over time. Perhaps less obvious, credit risk assessments should also discriminate among official creditors.
disbursement profiles, repayment schedules, and any other contractual provision relevant to the expected cash flow streams associated with each official loan included in the analysis.

Such highly-detailed information was obtained from the World Bank’s Debtor Reporting System (DRS) C the main database underlying the World Bank’s Global Development Finance. The DRS, established in 1952, is debtor-based C i.e., the data is compiled from reports provided by borrowing countries. Its coverage is quite broad: it includes all bilateral, multilateral and private creditors, and all aid-receiving members of the World Bank.12

The DRS is very similar to the OECD’s Creditor Reporting System (CRS), which provides the basis for the OECD’s Geographical Distribution of Financial Flows to Aid Recipients. There are some differences, however, between the two systems. The CRS encompasses some 180 recipients, including a considerable number of dependent territories and colonies that are excluded from the DRS. Apart from these sub-sovereign entities, the recipient coverage of both systems is nearly identical, with the main difference being the exclusion from the DRS system of a handful of countries that have graduated from the World Bank’s lending programs.13 Also, in contrast with the debtor-based DRS, the CRS collects information directly from the creditors that it covers C members of the OECD Development Assistance Committee (DAC) and multilateral lending agencies. As described in more detail below, this may lead to some discrepancies between the two sources.

In practice, our choice of the DRS as the basic source of loan data imposes some limitations on the time dimension of the analysis. Although the DRS contains information on loans contracted as far back as the mid 50s, loan coverage is somewhat limited prior to 1970 C the year when the system underwent a major upgrade and expansion. Thus, we confine our sample to all official loans contracted in or after 1970 which amount to some 40,000 loans.14 Because of disbursement delays, these loans account for the bulk of official loan disbursements only starting in 1975; hence the aggregates reported below span the period 1975-95.

Lastly, we also exclude from our coverage the 1995 Tequila loans to both Mexico and Argentina. These special loans carried hard terms that would translate into (sharply) negative grant elements in our EDA calculations. Since disbursed amounts were substantial (on the order of $26

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11 Upon founding of the DRS in 1952, the system collected data only on public and publicly guaranteed debt. In 1970, the system was expanded to cover private non-guaranteed debt.

12 Almost all developing countries are members of the World Bank. Current membership, including developed countries, is 181.

13 See Appendix A for a complete listing of the countries included in our analysis.

14 However, it is important to note that our database lacks loan-by-loan information on IMF flows. These consist of standard IMF loans, which are typically excluded from conventional aid aggregates, and highly-concessional loans (such as SAF and ESAF), which are not. Further, the terms attached to the former loan category display considerable variation over time and across loan types, while those on the latter category do not. This means that an approximate calculation of loan grant elements based on average (rather than loan-specific) IMF terms, which are available, should be reasonably accurate for the concessional-window loans, and much less so for the non-concessional ones. In view of these facts, we opted for retaining the concessional-window IMF flows in our calculations, and discarding the non-concessional window loans.

15 In practice, this choice of starting period becomes almost unavoidable given the very limited availability of interest rate data for non-dollar currencies prior to 1970, which would prevent construction of appropriate discount rates even if complete loan data were available.
billion), their inclusion would result in significant changes in 1995 EDA measures. We chose not to include Tequila loans in our coverage because this kind of financial rescue package is atypical in our sample, both in its financial terms and its volume, and would distort the aid trends we analyze in this paper.  

Having defined our loan universe, an important step before embarking on the calculation of grant elements is to verify the consistency of the DRS loan data with the CRS data underlying the conventionally-used Net ODA. This is important because we will later compare our EDA with the OECD's Net ODA, and we need to ensure that divergences between the two aid measures are not just due to discrepancies between their respective data sources. To verify this, we used data from the DRS to construct a DRS-based aid aggregate comparable to Net ODA. One problem in doing so is the stated purpose of official loans: ODA includes only those concessional loans with a development-related objective, while DRS includes all concessional loans regardless of their purpose. To follow the ODA definition, we opted for removing from our DRS-based loan data those loans with the clearest non-developmental purpose \(C\) military and defense-related loans.

A second problem has to do with the treatment of debt forgiveness. As noted earlier, the OECD accounts for debt forgiveness by recording a fictitious grant matched with an equally fictitious loan repayment. In contrast, the DRS system does neither. Thus, to ensure comparability between OECD ODA and DRS-based ODA, we deducted debt forgiveness from the grant totals when adding grant flows to DRS loan flows to construct our DRS-based ODA.

Figure 3.2 plots the nominal differences between DRS-based ODA constructed in this way and conventional OECD ODA. In general, the differences are quite small. Mean differences and mean absolute differences between both measures, computed for each recipient country over the period 1975-1995, cluster around zero for most countries. The average across countries of the mean differences is just $2.4 million. By contrast, mean absolute differences are much larger -- their cross-country average is $29 million. This suggests timing mismatches between creditor and debtor reporting, which cancel out in the average differences but not in the mean absolute differences. Indeed, mean absolute differences of three-year moving averages are substantially lower. Relative to the recipient economy's size (in terms of GNP), the mean discrepancy is less than 1 percent of the recipient's GNP for 125 of the 133 countries in the sample. The mean difference in the total sample is -.07 percent of GNP, with a standard deviation of .77 percent.

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16 However, with the onset of financial crises in emerging markets in 1997-98, there appears to be an institutionalization of this kind of lending, both in volume (rescue and emergency lending packages will approach $200 billion in the period) and in the hardness of the terms. Future revisions of EDA methodology will need to incorporate these new developments in some way.

17 This purpose group includes some 350 loans.

18 Since we are comparing ODA totals, we adjusted for ODA debt forgiveness only, rather than for all forgiveness, as defined by the OECD. The main difference between the two is the forgiveness of two large non-ODA loans to Egypt in 1991 and 1992.

19 We also examined the correlation between the OECD and the DRS-based ODA measures. The correlation was computed on the loan component only, since both measures share the same grant component. On a nominal basis, the sample correlation coefficient is .89, and in three-year moving averages, the sample correlation coefficient is .93.
The conclusion from these consistency checks is that despite the differences in lender coverage and reporting methodology, the World Bank and the OECD’s databases are actually quite similar for most countries in our sample. Having comparable underlying loan data indicates that any major divergences between our EDA and the OECD’s Net ODA should be driven mainly by differences in methodology, which is precisely the focus of our subsequent analysis.

**Interest Rate Data**

The other key ingredient for the computation of the grant element is a set of appropriate rates to discount the expected cash flow streams associated with each loan. As discussed in the previous section, the conventional approach underlying ODA uses an arbitrary rate of 10 percent for all discounting, a practice that completely ignores market conditions and loan features relevant to the opportunity costs that discount rates ought to capture. Proper valuation should employ instead discount rates that reflect the time, currency and maturity characteristics of the relevant cash flow. Such rates therefore need to vary along all these dimensions.

To this end, we use as discount rates the market yields on government securities of various maturities denominated in the six major lending currencies: U.S. dollar, yen, deutsche mark, French franc, British pound and Swiss franc. Together (by themselves and as part of currency baskets) these six currencies account for 88.8 percent of the total volume of loan disbursements in our loan sample. We take the market yields on government securities of a given maturity denominated in these currencies as a measure of the risk-free rates of return on debt of the corresponding maturity denominated in each respective currency.

From the 1980s on, the bulk of our interest rate data are collected from Bloomberg’s bond indexes. The indexes track the composite price and yield of a basket of government bonds considered to be benchmark issues in each currency. For each currency and maturity, we use the annual averages of all available monthly data. A major shortcoming of this source, however, is the very limited availability of data prior to 1980 for all currencies other than the US dollar. To complete the time series, we resorted to other sources—mostly central bank bulletins and the IMF’s International Financial Statistics.

With the raw annualized rates, we constructed yield curves for each of the six currencies and each year of the period 1970-1995. Where yields are unavailable for a given maturity, we interpolate across maturities using the observed rates for each year; hence, the interpolation derives the term structure of interest rates implied by the observed benchmark rates. We experimented with both linear and logarithmic interpolation. The latter involves fitting a log-linear curve across the observed rates, while the former involves fitting piece-wise straight lines across the same observations. Figure 3.3 plots the matrix of our interpolated rates for the US dollar.

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20 Following the IMF’s practice, where rates on T bills or other short term instruments are not available, 3-month deposit rates are used instead. See Appendix A for a complete listing of available rates and their sources.
In the end, we opted for the logarithmically-interpolated curves for several reasons. First, linear interpolation is problematic in cases where the rates at the long end of the yield curve are missing. The reason is that the linear method would essentially imply a flat term structure beyond the last available short rate, or alternatively, dictate that the tail of the curve adopt the same slope of its shorter end, both highly questionable assumptions. The logarithmic approach offered more flexibility in dealing with this problem. Second, as will become clear later, we utilize the yield curves to project future short-term interest rates, which serve as the basis of interest service on floating-rate loans. Since these future short rates are derived from the slope of the yield curve, they take on extreme values from abrupt changes in slope that occur around observed rates on linearly-interpolated yield curves.

Lastly, to discount the flows associated with loans denominated in multiple currencies, we constructed yield curves for various currency baskets including the Special Drawing Rights (SDR), European Currency Unit (ECU), and the lending baskets of major multilateral institutions (IBRD and IDB). Currency weights in each basket are applied to the discount rates available in our six currencies to build yield curves for each basket. Underlying weights for the baskets were constructed from publicly-available sources. In particular, the ECU, the SDR, the IBRD and the IDB baskets are collected from the respective institutions.

### 3.2 Computing Effective Development Assistance

Using the World Bank’s DRS loan data and the currency-specific yield curves summarized above, adjusted grant elements and grant equivalents were computed for all official (i.e., bilateral and multilateral) non-military loans in the DRS database contracted from 1970 onward. The actual calculations take into consideration the features of each individual loan as specified in the loan contract. Thus, they allow for multi-year disbursement periods, commitment fees and one-time charges, various amortization profiles, variable interest rates and so on. Formally, the general expression used to compute the grant element of a given loan is:

\[
g = \frac{PV(D_t^\delta) - \left[ PV(A_t^T) + PV(I_t^T) + PV(C_t^T) \right]}{PV(D_t^\delta)}
\]  

(3.1)

However, all the calculations in the paper were performed with both sets of rates. The differences between the respective results were significant for some loans, but negligible for the broad aggregates presented below.

Historical interest rate data were not available for all underlying currencies in some baskets. In such cases, we construct the full-basket rates using only the available currencies and rescaling their weights to add up to 100 percent.

For loans denominated in currencies other than the major six (and the baskets just mentioned), we adopt the following procedure. We construct generic baskets whose currency composition reflects the general currency profile of multilateral and bilateral loans. We derive the basket weights each year from bilateral and multilateral disbursements made in each currency in that year (scaled to exclude currencies with unavailable rates), and use the weights to aggregate the discount rates for each currency. The resulting basket rates represent the weighted average opportunity costs of bilateral and multilateral lending made during the year, and the associated discount rate is used for loans denominated in currencies for which market interest rates are unavailable.
where \( \{D\} \) is a sequence of disbursements, \( \{A\} \) is a sequence of principal amortization payments, \( \{I\} \) is a sequence of interest payments, \( \{C\} \) is a sequence of other charges such as commitment fees, and \( PV \) denotes the present value. The time indices 0, δ, γ, and \( T \) respectively denote the date of first disbursement, the number of periods between first and last disbursement, the grace period, and the maturity of the loan. To keep the calculations manageable, annual periods were used; this obviously represents an approximation in the (relatively frequent) case of loans specifying semi-annual service payments. Notice that the term in square brackets in the numerator of (3.1) above is just the present value of debt service obligations \( C \); i.e., the debt equivalent \( E \) introduced in the preceding section.

Present values of disbursement and service payment streams are computed using the discount factors derived from the currency-specific yield curves just described. Hence, for any sequence \( \{X\} \),

\[
PV(\{X_t\}_{t=0}^T) = \sum_{t=0}^{T} \frac{X_t}{(1 + R_{0,t})^t}
\]  

(3.2)

where \( R_{0,t} \) is the (annualized) interest rate on \( t \)-year maturity instruments prevailing at time 0; it is also currency-specific, an issue which for notational simplicity is ignored here. Hence, in accordance with the ex-ante perspective adopted in this analysis, the interest rates used for discounting are those prevailing at the time of loan agreement, rather than those effectively observed ex-post.\(^24\)

Some remarks on the construction of the disbursement and loan service streams may be useful. As with discount rates, the different components of loan service were constructed on the basis of the ex-ante contractual information \( C \) and not from the interest and amortization payments actually made ex-post by the borrower. Regarding the amortization schedule, the arrangement most frequently found in the data is that of equal repayments following a grace period. However, a wide variety of amortization profiles exist in the data, ranging from annuity-based amortization schedules to “balloon” principal repayments (whereby the loan is amortized in full in one single payment at maturity).

In turn, interest charges are typically accrued at a contractually-determined rate on the loan’s outstanding balance. While a time-invariant interest rate is by far the most common arrangement, in a large number of cases the loan contract sets interest rates that change over the life of the loan, either in a pre-specified manner or following a reference rate such as LIBOR. For the calculations, interest charges were generally computed as

\[
I_t = i_t \left( \sum_{r=0}^{t-1} (D_r - A_r) \right) \quad \text{for } 0 < t \leq T
\]  

(3.3)

\(^{24}\) For simplicity, in the text it is implicitly assumed that the dates of loan agreement and first disbursement are the same. While this is indeed correct for the majority of loans, it is not invariably the case -- the first disbursement often takes place one or more years after loan agreement. The discount rates used in the calculations in such cases were those observed at the time of agreement. Experiments were also made using instead the rates prevailing at the time of first disbursement. This of course affected the estimated grant elements of specific loans, but had a virtually negligible impact on the aggregate annual grant equivalents accruing to each recipient country.
where \( i_t \) is the interest rate contractually specified for period \( t \), and the term in large brackets is the loan’s outstanding principal as of the end of period \( t-1 \). The determination if \( i_t \) for the case of loans with variable interest rates deserves comment. For those loans with interest rate linked to some short-term market rate (e.g., 6-month LIBOR), a forecast of the latter was constructed using the anticipated one-period interest rate derived from the yield curve described earlier, adjusted for a margin calibrated from the historical data. Thus, the contractual interest rate was projected as:

\[
i_t = b_{0,t} + s \quad \text{where} \quad b_{0,t} = \left[ \frac{(1 + R_{0,t})^{t+1}}{(1 + R_{0,t})^t} - 1 \right] (1 + \mu)
\]

where \( b_{0,t} \) is the projected short-term market rate (e.g., LIBOR); \( s \) is a contractual spread, and the (proportional) margin \( \mu \) was computed through a univariate regression of the short-term market rate on the one-year rate from the yield curve.  

A different type of variable interest-rate arrangement concerns the loans granted by some multilateral institutions since the early 1980s, whose interest rates are determined as a spread over the lender’s average (as opposed to marginal) cost of borrowing. This can be viewed as a weighted sum of current and lagged interest rates on various currencies, and therefore follows market trends with a substantial lag. Projection into the future of the cost of funds is further complicated by the fact that the weights of the various currencies in the lender’s borrowing basket can, and do, change over time (albeit gradually). To simplify the calculations, however, anticipated changes in the currency composition of the borrowing basket were ruled out. Thus, the cost of funds relevant to each loan was determined by aggregating the yield curves of the various currencies making up the lender’s borrowing basket, using as weights their shares in the basket on the year of the loan’s first disbursement.

Finally, loans from major multilateral institutions typically carry commitment charges on the undisbursed principal, that accrue periodically (commonly starting 60 days after commitment). Hence, commitment fees were computed as:

\[
C_t = c \cdot \left( \sum_{t=0}^{\delta} D_t - \sum_{t=0}^{t-1} D_t \right) = c \left( \sum_{t=\delta}^{\delta} D_t \right), \quad 1 \leq t \leq \delta
\]

The term in brackets captures the undisbursed loan amount as of the end of period \( t-1 \), and \( c \) is the contractually-fixed rate at which commitment fees accrue.

In contrast with service payments, the contractual information usually does not specify a disbursement schedule. Thus, actual disbursements had to be used instead. This means that in our discussion we use the term “loan principal” to refer to the total amount effectively disbursed, which might differ from the contractually-agreed amount. Note also that it is not uncommon for actual disbursements to lag behind the originally planned schedule, and this could lead to an

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25 As a robustness check, all calculations were repeated using an additive, rather than multiplicative, margin \( \mu \). The impact on the estimated grant elements was negligible.
underestimation of the grant element relative to its true *ex-ante* value (i.e., as viewed at the time of agreement).  

Having obtained grant elements as just described, the grant equivalent of each disbursement is computed by imputing to the disbursement amount the loan’s overall grant element:

\[ G_t = g \cdot D_t \; ; \quad 0 \leq t \leq \delta \]  

(3.6)

It follows from (3.1) above that, in present value terms, \( PV(G_t) = PV(D_t) - \{PV(A_t) + PV(I_t) + PV(C_t)\} \). In words, the loan’s overall grant equivalent (given by the discounted sum of the grant equivalents of the different disbursements) equals the present value of the disbursement stream minus the present value of the loan’s debt service stream.

The above expressions are quite general and allow for a broad variety of contractual arrangements regarding amortization schedules, interest payments and other charges. In general, however, they do not yield a closed-form expression for the grant element, which is only available under additional assumptions. Specifically, Appendix B shows that if (i) the loan is disbursed in full at the time of commitment; (ii) the interest rate is fixed at \( i \); (iii) the amortization profile involves equal payments after a grace period \( \gamma \); (iv) the discount rate \( R \) is constant; and (v) commitment fees and other charges are ignored, then (3.1) simplifies to:

\[ g = \left(1 - \frac{i}{R}\right) \cdot \left(1 - \frac{1}{(1 + R)^T} - \frac{1}{(1 + R)^T R \cdot (T - \gamma)}\right) \]  

(3.7)

Conventional grant element calculations are based on (3.7) setting \( R \) equal to 10 percent.

In reality, of course, the assumptions just listed do not hold, and the “approximate” grant element obtained from (3.7) could differ greatly from the correct one obtained from (3.1). As will be shown below, however, in practice the main source of discrepancy turns out to be the assumption of a constant discount rate; the other assumptions have a comparatively more modest impact on the grant element calculation.

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26 In addition, comprehensive disbursement information is available only in US dollars. This might introduce another distortion in the computed grant component of loans denominated in other currencies: even if their disbursements proceed on schedule, the time profile of disbursements *expressed in dollars* could be altered *ex-post* due to unanticipated exchange rate changes during the disbursement period.
4 RE-ASSESSING TRENDS IN AID FLOWS

The equations outlined in the preceding section were numerically evaluated for each one of a total of over 40,000 loans\(^{27}\) in the DRS database (of which some 24,000 were from bilateral lenders, and the rest from multilateral lenders).\(^{28}\) All results are aggregated and summarized next, focusing first on the broad aggregates and turning then to their breakdown by recipient and donor.

4.1 Measuring grant elements

The improved methodology just described has a major impact on the estimated aid content of official loans. Figure 4.1 plots the average grant element of official loans as obtained from three different approaches. First, the conventional (i.e., the OECD’s) approach, using the 10-percent discount rate and imposing restrictive assumptions on the disbursement, interest and amortization schedules (as needed to arrive at the simplified expression (3.7) described earlier). Second, the adjusted methodology proposed above, imposing no restrictions on loans’ disbursement, interest and amortization schedules, and using the year-, currency- and maturity-specific discount rates constructed in the previous section. Third, for the purpose of comparison, the graph also presents an intermediate approach allowing for unrestricted disbursement and service schedules but still using the 10-percent discount rate. In each case, the annual average of the grant elements of individual loans is constructed using as weights the share of each loan in the year’s total disbursements.

The four panels of Figure 4.1 respectively show the average grant element of all official loans, all multilateral loans, multilateral concessional-window loans, and bilateral loans. They bring out three important facts. First, the conventional method leads to a systematic overestimation of the average grant element, as shown by the fact that the adjusted grant element is almost invariably lower than the conventional one; the exception are the early 1980s in the case of bilateral and multilateral concessional loans. Second, the gap between the grant element implied by the two approaches shows substantial variation over time: it declined during the late 1970s, became almost negligible in the early 1980s, and has widened dramatically since 1985. In fact, after that date the conventionally-measured grant element has shown an upward trend, while the adjusted grant element has plunged particularly in the case of bilateral loans, whose concessionality fell to historical lows in the 1990s. Third, the overestimation of the grant element by the conventional method appears to be due to its assumption of a 10-percent discount rate. As the figures show, if all other restrictive assumptions underlying the conventional method are removed, but the 10 percent discount rate is retained, the resulting grant element is closer to the adjusted grant element, but the improvement is indeed rather minimal.

The main lesson from these facts is that the use of an arbitrary 10-percent discount rate clearly distorts the conventionally-measured grant elements. In the sample, there is a positive bias

\(^{27}\) Of this total, some 50 loans had to be discarded because their contractual information regarding interest, amortization or both was incomplete and did not allow calculation of the corresponding grant element.

\(^{28}\) For concessional IMF loans, the loan-by-loan information is not available, as noted above. Hence we used a simplified approach: we plugged into (3.7) the average terms of concessional IMF disbursements in each given year, using as disbursement figure the corresponding annual total, and allowing the discount rate to vary over time.
that varies considerably over time. A natural question is: how does this arbitrary 10-percent rate differ from the “right” one? Conceptually, as argued above, the discount rate should be not only time-varying, but also currency- and maturity-specific -- precisely the approach underlying the calculations of the adjusted grant elements above. For the purpose of illustration, however, it is possible to construct an ideal “average” discount rate, which is the one implied by the loan-by-loan calculations of the adjusted grant elements. One simple way to do this is to solve the conventional formula (3.7) for the discount rate that would yield the average adjusted grant element just presented in the above graphs, for an ideal loan whose features (grace, maturity and interest rate) match those of the average loan in the sample. Notice that, by doing this, all the other subtleties found in the contractual loan information (multi-year disbursements, time-varying interest rates, irregular amortization schedules, various loan charges...) are ignored.

Following this procedure for every sample year, a time-series of implicit discount rates can be constructed. The results are presented in Figure 4.2, which, for the purpose of illustration, also plots the interest rate on 10-year U.S. government securities. The figure shows that the implicit discount rate comes close to the horizontal 10-percent line only during the first half of the 1980s (and the former even exceeds the latter in 1982). In the remaining years, the implicit rate is well below the 10 percent benchmark, and especially so in the early 1970s and the 1990s. This, of course, is just the underlying reason for the overstatement of the conventionally-measured grant elements in those years, as described above. Finally, it is important to note that the time pattern of the implicit discount rate is governed by the changing conditions in world financial markets. As the figure also shows, the swings of the former track those in U.S. long-term interest rate, although with considerably reduced magnitude -- reflecting the (increasing) role of non-dollar-denominated official loans in overall lending flows.

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It is worth noting that the weights used in each year to construct the average loan features (i.e., the average grant element, grace, maturity and interest rate) for this calculation from those of the individual loans are given by the face value of each loan committed in the year in question. This follows logically from the use in the calculations of equation (3.7), that assumes full loan disbursement at the time of commitment. However, it differs from the weighting scheme used in Figure 4.1 which, as noted in the text, is disbursement-based rather than commitment-based.
4.2 Reassessing time trends in aid

So far the discussion has focused on the concessionality of official loans. What are the implications for measured aid flows? To answer this question, the first step is to assess the aid volume implicit in official loans according to their adjusted grant elements; we shall refer to such aid volume as the effective development assistance of official loans. This can be constructed by summing their adjusted grant equivalents, computed according to equation (3.6) above. The results are shown in Figure 4.3, which also presents the grant equivalents derived from the conventional method as well as those arising from the intermediate method that uses the general expression (3.1) with a 10-percent discount rate. To facilitate comparability across years, the resulting totals have been converted to 1995 U.S. dollars.

The graph again reveals a stark contrast between the conventional and adjusted figures after 1985: while the conventional grant equivalent has risen steadily to a record-high 20 billion by the end of the sample period, the adjusted grant equivalent has changed little over the last decade, and at present amounts to some 8 billion. In other words, in recent years the conventional grant equivalent overestimates the aid content of official loans by 150 percent! Finally, like with the grant elements, the chart again confirms the relatively secondary role of factors other than the discount rate in explaining the discrepancy between conventional and adjusted grant equivalents.

Conventionally, however, the aid content of official loans is measured by the net disbursement of ODA loans. Figure 4.4 compares such conventional measure with the EDA of official loans just described. Unlike Net ODA, EDA is based on gross flows, and hence it is not surprising that the levels of the two measures should display persistent differences. However, the pattern is indeed very similar to those of the preceding figures: ODA overstates systematically the aid content of official loans and, except for the early 1980s, the overstatement is quantitatively very large. Moreover, because of the diverging pattern of both measures since 1985 -- rising Net ODA loans and declining EDA -- the former at present exceeds the latter by around 100 percent.

The above discussion has been confined to official loans. What is its relevance for total aid? Adding official grants to the adjusted grant equivalent of official loans, one can construct a measure of total EDA, analogous to ODA. Since grants have, by definition, a 100 percent grant equivalent, any discrepancy between ODA and EDA is just due to their different measurement of the aid content of official loans. As just shown, such discrepancy is quite considerable; relative to total ODA or EDA, however, it is somewhat lessened by the large (and increasing) magnitude of official grants documented earlier. This is illustrated by Figure 4.5, which compares Net ODA and total EDA over the last two decades. As with official loans, it is apparent that since 1985 Net ODA considerably overstates aid flows -- by as much as 25-30 percent in recent years.

30 As noted earlier, the loan database used here does not allow construction of a satisfactory DRS-based ODA measure prior to 1975, and hence the chart begins in that year.
Figure 4.3 Grant Equivalent of Official Loans

- Using adjusted grant elements
- Using conventional grant elements
- Using 10% discount rate

Figure 4.4 Official Loans: EDA vs Net ODA

- EDA
- Net ODA

Figure 4.5 Net ODA vs Total EDA

- Effective Development Assistance
- Net Official Development Assistance
4.3 EDA vs. ODA by recipient

Because EDA is based on a consistent measure of the concessionality of financial flows, it allows meaningful comparisons of aid flows across donors and recipients. This is not the case with comparisons of ODA flows, however, which can be misleading for a number of reasons: 1) because of the potentially large differences in the grant/loan composition of flows from different donors or to different recipients, 2) because of the potentially different degree of concessionality of the loans involved in those flows, or 3) because of differences in the time profile of lending flows impacting the netting out of amortization in ODA calculations.

Indeed, there are systematic differences in the concessionality of official flows across both recipients and donors. Figure 4.6 focuses on the recipient side, grouping aid-receiving countries into six geographical regions, according to the World Bank’s definition. The figure presents the average grant element of official loans to each of the regions over the period 1990-95, computed according to both the conventional method and the adjusted method introduced earlier.

As the figure shows, both methods reveal a considerable variation of grant elements across regions. Conventional grant elements range from 18 percent in Europe and Central Asia to 49 percent in Sub-Saharan Africa. In turn, adjusted grant elements range from virtually zero in Europe and Central Asia, as well as Latin America (for practical purposes, official loans to these two regions are not concessional), to 47 percent in Sub-Saharan Africa. Interestingly, the regional rankings by grant element are similar under both measures. However, the extent of divergence between them varies considerably across regions -- from just over 2 percentage points in Sub-Saharan Africa, to 12 in South Asia and some 20 percentage points in the other developing regions.

How do these diverging assessments of loan concessionality across recipient regions impact on measured regional aid flows? Figure 4.7 presents the ratio of Net ODA to total EDA in recent years for each of the six developing regions above; the underlying flows are expressed in 1995 US dollars. The figure illustrates eloquently how the degree of distortion embodied in conventional aid measures can vary across aid recipients. For East Asia and the Pacific, and Latin America and the Caribbean, the contrast between both Net ODA and total EDA is dramatic: during 1990-95, Net ODA to these two regions exceeded effective assistance by around 60 percent. At the other end, for Sub-Saharan Africa and South Asia (who receive mostly grants), the divergence is much smaller – around 10 percent in both cases.

4.4 EDA vs. ODA by donor

Like with recipients, EDA also permits consistent comparisons of aid flows across donors. By way of example, we conclude this section with a brief look at the comparative performance of bilateral donors.

Conceptually, it should be clear from the earlier discussion that, since conventional aid measures typically overstate the grant elements of concessional loans, such measures will tend to exaggerate the contribution of donors whose disbursements take the form mostly of loans. Further, the overstatement will be more severe for donors lending in low-yield currencies, because the interest
rate on their loans will tend to be below the arbitrary 10 percent discount rate used in conventional grant element calculations. The opposite will happen with donors giving mostly grants, or concessional loans in high-yield currencies – conventional measures will tend to overstate their contribution to a more limited extent.

Figure 4.8 ranks DAC donors according to their respective volume of aid relative to GNP, using both Net ODA and EDA, for the periods 1981-85, 1986-90 and 1991-95. Two qualifications are in order. First, the figures underlying the rankings only include donors’ direct aid to developing countries and therefore exclude their contributions to multilateral institutions. Second, because we lack detailed debt forgiveness data by donor prior to 1991, the figures for 1989 and 1990 underlying the rankings are not adjusted for ODA debt forgiveness – i.e., donors’ grant totals (and hence total aid) in those years might include forgiveness of ODA debt.  

With these caveats, the figure reveals interesting contrasts between the ODA and EDA-based donor rankings. Most remarkable is the fact that a few donors -- notably Japan, but also France and Spain (the latter after 1981-1985) -- rank systematically higher under ODA than under EDA. This likely reflects the high reliance of these donors on loans rather than grants and, at least in the case of Japan, also the fact that yen interest rates have remained well below the 10 percent benchmark. Correspondingly, other donors find themselves in the opposite situation, ranking higher under EDA than ODA. This is the case of Canada (especially in 1986-90) and, more recently, the Netherlands as well.

31 Prior to 1989 ODA debt forgiveness is negligible.
Figure 4.6 Average Grant Element by Recipient Region 1990-95

![Average Grant Element by Recipient Region 1990-95](image)

Using adjusted grant elements
Using conventional grant elements

Europe and Central Asia
Latin America and Caribbean
Middle East and North Africa
East Asia and Pacific
South Asia
Sub-Saharan Africa

Figure 4.7 Net ODA Relative to EDA by Recipient Region, (1990-95 Totals)

![Net ODA Relative to EDA by Recipient Region, (1990-95 Totals)](image)

Figure 4.8 Ranking of Bilateral Aid by Donor: ODA/GNP vs EDA/GNP

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1. Adjusted for ODA debt forgiveness
5. CONCLUDING REMARKS

Conventional aid aggregates such as the OECD’s Net ODA provide a distorted measure of true aid flows. The reason for this distortion is the weak methodological foundation of such aggregates. They lump together the net increase in loans, which entail future interest and repayment obligations, with grants, that do not; further, they include certain loans at full face value and totally exclude others; finally, the selection of which loans to include is based on a calculation of their grant elements that, among other simplifications, makes use of an arbitrary discount rate set at 10 percent.

This paper has proposed a new approach that corrects most of the major methodological shortcomings underlying conventional aid measures. The approach is based on the use of the overall grant equivalent of official disbursements as the key measure of aid, resulting in a new aggregate that we label Effective Development Assistance (EDA). The use of grant equivalents as the basic aid measure has the virtue of allowing comparability between aid flows implicit in grants as well as loans with varying degrees of concessionality. Most importantly, EDA is based on adjusted grant equivalents constructed using discount rates sensitive to market conditions along several dimensions – timing, currency and maturity of the loans, unlike the conventional grant elements based on the arbitrary 10 percent discount rate.

The paper has implemented empirically this new approach using detailed data on some 40,000 official loans from the World Bank’s DRS database virtually all the official loans during the period 1975-1995 to the 133 developing countries included in the DRS system. The numerical results, which are available to all interested researchers, underscore several important facts. First, the conventional approach to grant element calculation has led to a systematic overestimation of the concessionality of official loans. Second, this overestimation has increased significantly since the mid-1980s. After that date, conventionally-computed grant elements have shown a rising trend, while the conceptually superior adjusted grant elements show, if anything, the opposite time pattern. Grant equivalents obtained by both methods reveal the same contrasting pattern. Third, as a consequence of these results, Net ODA increasingly overstates the true aid content of official flows, as more accurately measured by EDA even though the large divergence between both approaches in their assessment of the aid content of official loans is somewhat muted by the rising importance of grants over loans in total official flows.

Since conventional aid measures typically overstate the grant elements of concessional loans, they tend to exaggerate comparatively more the aid flows to those recipients getting mostly loans. This is so especially if the loans are given in low-yield currencies, because their interest rate will tend to be further below the arbitrary 10 percent discount rate used in conventional grant element calculations. The same happens with donors: the aid contribution of those giving mostly grants (and loans in high-yield currencies) will be understated relative to the others. The paper presents numerical findings confirming these intuitive facts. This in turn implies that conventional aid measures can be very misleading for purposes of comparison across recipients and/or donors.

While we believe that these results are highly relevant to the current policy debate on aid, some caveats remain. First, our approach to the measurement of aid adopts an ex-ante perspective in order to capture donors’ effort, and it would be interesting to compare our results with those
obtained from an \textit{ex-post} approach – based on actual rather than contractual debt service flows. Second, and perhaps more important, while our analysis addresses most of the key limitations of conventional aid measures, it still does not incorporate default risk in the calculations of loan concessionality. Doing so would basically require construction of the risk spreads applicable to each borrower each year for each creditor class. However, in the absence of markets for official debt to price directly such risks, this would be a rather problematic task, well beyond the scope of this paper.
Appendix A

This appendix lists the countries included in our analysis and the sources of our raw discount rate data.

I. INCLUDED RECIPIENT COUNTRIES

Recipient countries included in our analysis are listed by region in the box below:

<table>
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<tr>
<th>East Asia and Pacific</th>
<th>Europe and Central Asia</th>
<th>Latin America and the Caribbean</th>
<th>Middle East and North Africa</th>
<th>Sub-Saharan Africa</th>
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Total Included: 133
II. SOURCES OF DISCOUNT RATE DATA\textsuperscript{32}

\textit{Bloomberg:} The bulk of our data after the mid-80s is collected from Bloomberg. Bloomberg maintains indexes on benchmark treasuries in all six major lending currencies. These Bloomberg “generic” treasury indexes track the composite price and yield of a basket of bonds considered to be benchmark issues in each market. Price of a “generic” bond, note or bill of a specified maturity is an average of a number of market maker bid-side quotes updated hourly, several times throughout the day, or at closing, depending on the market.

For consistency, all monthly data available in Bloomberg are collected and used. Annual rates are calculated by taking simple arithmetic averages of monthly yields. The resulting data is comprehensive for the 90s for all currencies. A major shortcoming of these Bloomberg rates is the lack of historical data for all currencies except the US dollar. To complete the series, we collected data from other sources - mostly central bank bulletins and the IMF’s International Financial Statistics.

Listed below are sources from which we collected historical yields and interest rates. Following the IMF’s practice, where rates on T bills or other short term instruments are not available, 3 month deposit rates are used instead. Note that yields on Bloomberg generic government bills, notes or bonds are simply referred to as “Bloomberg.”

\textbf{US RATES}

3 month T bills  \quad IMF, International Financial Statistics, discount on new issues of three month T bills, annual data are averaged weekly rates.

6 month T bills  \quad 1983-1996: Bloomberg, generic 6 month T bills.

2 year treasury  \quad 1977-1996: Bloomberg.


\textbf{UK RATES}

3 month T bills  \quad 1970-1996: IMF, International Financial Statistics (as reported by Bank of England, \textit{Quarterly Bulletin}). Rates represent tender rates at which 91 day T bills are issued, calculated from amount of discount. Monthly data are weighted averages of Friday data. Annual figures are averaged monthly rates.


5 year UK Gilt  \quad 1970 to 1991: Annual Abstract of Statistics, Central Statistical Office (CSO) of Great Britain. Rates are par (gross redemption) yields calculated from yield-maturity curves fitted mathematically. Until 1979, annual rates are averages of Wednesday yields. From 1980 on, the averages are of all observations (usually 3 per week); and from 1982 forward, the figures are averages of working days. See Bank of England Quarterly Bulletin for descriptions of actual methods of calculation. Note that all CSO rates are calculated in the same fashion for the various maturities.


\textsuperscript{32} Discount rate data is available upon request.

**JAPANESE RATES**

3 month deposit rate IMF, International Financial Statistics. From June 1992, rates are average interest rate for the last week of the month on unregulated three-month time deposits. Prior to that, rates are end of month guideline rates on three month time-deposits set by the Bank of Japan. Annual rates are averaged monthly rates.
10 year 1970 to 1986: IMF, G7 database. Data collected on a monthly basis. Before (Nov. '84) end-of-month; thereafter, average of over-the-counter 10-year government bonds, with longest residual maturity; from January 6, 1992, the yield refers to benchmark 10-year government bond. Annual figures are averaged monthly rates.

**GERMAN RATES**

2 year 1977 to 1989: *Statistische Beihefte ze den Monatsberichten der Deutschen Bundesbank, or Kapitalmarkt-statistik after 1990*, Deutsche Bundesbank (Statistical Supplements to the Monthly Reports of the Deutsche Bundesbank, or Capital Market Statistics after 1990). Rates are yields on fully taxed public bonds outstanding with remaining maturity of over one up to two years. Annual rates are averaged monthly rates. Rates from this source reported below are referred to as Deutsche Bundesbank.
3 year 1977 to 1989: Deutsche Bundesbank, remaining maturity of over 2 up to 3 years.
4 year  1977 to 1989: Deutsche Bundesbank, remaining maturity of over 3 up to 4 years.  
5 year  1977 to 1989: Deutsche Bundesbank, remaining maturity of over 4 up to 5 years.  
6 year  1977 to 1992: Deutsche Bundesbank, remaining maturity of over 5 up to 6 years.  
7 year  1977 to 1989: Deutsche Bundesbank, remaining maturity of over 6 up to 7 years.  
8 year  1977 to 1992: Deutsche Bundesbank, remaining maturity of over 7 up to 8 years.  
9 year  1977 to 1992: Deutsche Bundesbank, remaining maturity of over 8 up to 9 years.  

**FRENCH RATES**

10 year 1970 to 1989: IMF, G7 Database. Rates are long-term (7-10 yr.) govt. bond yields (Emprunts d'Etat a long terme TME).  

**SWISS RATES**

2 year 1994-1995: Bloomberg
3 year 1994-1996: Bloomberg
5 year 1990-1996: Bloomberg
7 year 1970 to 1984: IMF, IFS, weighted average yield to maturity (if below par) of ten government bonds with at least five years to maturity, callable. Annual yields are computed from daily averages of prices.  
Appendix B

This appendix shows how the methodological approach used in the paper to compute the adjusted grant elements contains as a special case the conventionally-used grant element formula.

The starting point is provided by equations (3.1)-(3.3) in the text, to which the following simplifying assumptions are added:

**Assumption 1:** The loan is disbursed in full at the time of commitment

**Assumption 2:** The principal is repaid in $T-\gamma$ equal amortization payments, starting in year $\gamma+1$ and ending in year $T$ (where $T > \gamma \geq 0$)

**Assumption 3:** The interest rate is fixed at $i$ throughout the life of the loan

**Assumption 4:** The discount rate is fixed at $R$

**Assumption 5:** The loan involves no charges other than interest and amortization

The first assumption implies that the present value of the disbursement stream is just $PV(D_t) = D_0$, where $t=0$ is the date of commitment. In turn, Assumption 2 implies that amortization payments are of the form

$$A_t = \frac{D_0}{(T-\gamma)} \quad \text{for } \gamma < t \leq T$$

and zero otherwise. Hence, using Assumption 4, the present value of the amortization stream equals:

$$PV(A_t) = \left[ \frac{D_0}{T-\gamma} \right] \sum_{t=\gamma+1}^{T} \frac{1}{(1+R)^t} = \left[ \frac{D_0}{T-\gamma} \right] \cdot \frac{1}{R} \cdot \left( \frac{1}{(1+R)^T} - \frac{1}{(1+R)^\gamma} \right)$$

Using (A.1) and Assumption 3, the sequence of interest charges can be constructed as follows. First, the loan’s outstanding principal, equal to the cumulative sum of disbursements minus the cumulative sum of amortization payments, is given by:

$$D_0 \quad \text{for } 0 \leq t \leq \gamma;$$

$$D_0 - (t-\gamma) \left[ \frac{D_0}{T-\gamma} \right] \quad \text{for } \gamma < t \leq T$$

and interest charges therefore follow the path

$$I_t = i \cdot D_0 \quad \text{for } 0 < t \leq \gamma + 1$$

$$= i \cdot D_0 \cdot \left[ \frac{T-t+1}{T-\gamma} \right] \quad \text{for } \gamma + 1 < t \leq T$$

Thus, the present value of the stream of interest payments is:
\[ PV\{\{I_t\}\} = \sum_{t=1}^{\gamma+1} \frac{i \cdot D_0}{(1 + R)^t} + \sum_{t=\gamma+2}^{T} \frac{i \cdot D_0 \left( \frac{T - t + 1}{T - \gamma} \right)}{(1 + R)^t} \]  

(A.5)

where the two summation signs refer to the interest charges incurred during and after the grace period respectively (the second sum only applies if \( T - \gamma > 1 \), i.e., whenever the loan is not amortized in full in a single payment). Tedious manipulations permit simplifying this expression to:

\[ PV\{\{I_t\}\} = \frac{i \cdot D_0}{R} \left[ 1 + \frac{1}{T - \gamma} \left( \frac{1}{(1 + R)^\gamma} - \frac{1}{(1 + R)^T} \right) \right] \]  

(A.6)

Replacing (A.2) and (A.6) into the grant element equation (3.1) in the text,

\[ g = \frac{D_0}{D_0} \left[ \frac{D_0}{T - \gamma} \cdot \frac{1}{R} \left( \frac{1}{(1 + R)^\gamma} - \frac{1}{(1 + R)^T} \right) + \frac{i \cdot D_0}{R} \left[ 1 + \frac{1}{T - \gamma} \left( \frac{1}{(1 + R)^\gamma} - \frac{1}{(1 + R)^T} \right) \right] \right] \]

This can be easily simplified to:

\[ g = \left( 1 - \frac{i}{R} \right) \left( \frac{1}{(1 + R)^\gamma} - \frac{1}{(1 + R)^T} \right) \]

which is equation (3.7) in the text. This simplified expression allows some useful insights. First, the sign of the grant element depends only on the relation between the interest rate and the discount rate. In particular, if the discount rate equals the interest rate, then the grant element is zero regardless of loan grace and maturity (notice, however, that this would not be the case if the loan entailed any service fees in addition to interest charges). Second, the lower the interest rate \( i \) for a given discount rate \( R \), the larger the grant element. Third, for a given configuration of interest and discount rates, the absolute value of the grant element rises with the grace and maturity periods, and reaches a maximum (equal to \( 1 - i/R \)) when both grace and maturity approach infinity -- i.e., in the case of a consol.
References

