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Private External Overborrowing in Undistorted Economies: Market Failure and Optimal Policy*

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Abstract

We show that an undistorted free market economy endowed with perfectly rational and informed agents overborrows, i.e., borrows beyond the efficient level, because of the agents' failure to internalize credit rationing resulting from sovereign risk. It follows that the elimination of the market imperfections previously identified in the literature as causes of market overborrowing, such as misinformation or moral hazard, will not cure private overborrowing. We then explore the possibilities of economic policy as a remedy and find optimal consumption and capital inflow taxes. Traditional capital control taxes are suboptimal and probably counterproductive, while a dual exchange rate and foreign debt quotas would be optimal.

1 Introduction

Overborrowing, i.e., borrowing beyond the optimal level, is frequently identified as the fundamental reason for the recurrent balance of payments crises and external credit rationing in developing countries. The various hypotheses that have been advanced to explain external overborrowing, both public and private, typically share one central policy implication: the ideal cure for overborrowing would be to eliminate market distortions, including misinformation, and subject external borrowing to the free market discipline. In contrast, in this paper we show that such free market discipline is no cure for overborrowing and that there is room for policy intervention even in an undistorted economy (in the absence of perfect access to international financial markets).

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The analysis of private overborrowing has so far emphasized a variety of microeconomic distortions, either caused by the public sector, such as implicit public financial guarantees or unsustainable trade liberalization, or due to imperfect information (see, for example, McKinnon and Pill 1997 and references therein). Whatever the relevance of the previous hypotheses of market distortions to explain private overborrowing, which is not analyzed in this paper, we show that the market mechanism itself also appears to be part of the problem. In other words, the free market benchmark case of the previous hypotheses turns out to be inefficient. From a policy viewpoint, the implication is that policy intervention would be required to cure private overborrowing even if all market distortions are eliminated, i.e., as a first-best policy. There would be a case for capital controls of some kind even after prices and information are set right.

In this paper, perfect access to international financial markets is precluded by sovereign risk, which is arguably the typical case in developing countries. The relevance of this assumption is clearly recognized by market practitioners: "Sovereign credit risk is always a key consideration in the assessment of the credit standing of banks and corporates. Sovereign risk comes into play because the unique, wide-ranging powers and resources of each national government affect the financial and operating environments of entities under its jurisdiction. Past experience has shown time and again that defaults by otherwise creditworthy borrowers can stem directly from a sovereign default. In the case of foreign currency debt, the sovereign has first claim on available foreign exchange, and it controls the ability of any resident to obtain funds to repay creditors." (Standard & Poor's 1997).

Also in theory, private risk in international credit goes beyond the traditional commercial risk, dependent on the debtor's solvency, and also encompasses sovereign risk, i.e., the risk that the country's welfare be maximized if debt is not repaid (see Fernandez-Arias 1996 for an analysis). Not only public debt, but also private debt is subject to sovereign risk because the government can be expected to block private debt repayments or acquiesce to their failure through legal means if such an action is perceived as welfare improving.[1]

At first glance, it appears implausible that the traditional "invisible hand" does not work. In fact, it is well known that the free market mechanism works efficiently in two key polar cases, namely, perfect access to external borrowing (i.e., access constrained only by the country's overall solvency), and no access to external borrowing (see, for example, Blanchard and Fischer 1989). In the first case, the free market in a small economy that takes international prices as given achieves efficient borrowing and an optimal consumption path. In the second case, since external borrowing is not feasible the free market consumption path delivers lower utility, but it is nevertheless an optimal consumption path for a closed economy, i.e., the market is constrained optimal.

However, we show that the market fails in the intermediate case of limited access to external borrowing due to sovereign risk because of its failure to inter-

[1] Ex-post collusion among creditors can also induce this kind of governmental intervention, as in Chile's 1982 debt crisis nationalizations (Diaz-Alejandro 1985).

nalize the costs of future credit rationing. Rational market agents realize that they will be unable to access the external financial market once the country is credit rationed and prudently adjust their borrowing plans in anticipation. But these individual borrowers fail to internalize their impact on the country's capacity to borrow, i.e., on the onset of credit rationing. The aggregate nature of this constraint offers an opportunity for them to free ride and borrow "before it's too late," which results in above-optimal borrowing levels, i.e., private overborrowing.

This market failure is different from the case analyzed in Harberger (1984) and Eaton and Gersovitz (1987), in which the market may fail because it is assumed that sovereign risk introduces a wedge between lenders' and borrowers' payoffs in default states (otherwise their expected net cost of capital would always be the risk-free rate, the lenders' perceived net benefit). In Harberger the wedge is due to differing expectations between lenders and borrowers, which is not consistent with this paper's rational expectations. In Eaton and Gersovitz the wedge is due to the counterfactual assumption that debt is actually repudiated and default penalties applied, rather than avoided through negotiated settlements. In fact, their market failure disappears if ex-post renegotiation is allowed to eliminate the costs of default responsible for the wedge between lenders' and borrowers' payoffs. In contrast, we set a model whose dynamics uncover that the market fails to internalize the deadweight cost of future credit rationing, irrespective of the efficiency of the repayment arrangements under default.[2]

Our analysis concerning private overborrowing under imperfect access fills the gap between two related questions previously addressed in the literature. First, this limited access case has been studied for a *command* economy (Detragiache 1992 for a pure neoclassical model; Sachs and Cohen 1986 and Isgut 1997 for economies with adjustment costs), where it has been found that the optimal borrowing program entails less borrowing than under perfect access, thus postponing credit rationing. Here we analyze this case for a *market* economy and find that borrowing exceeds that of the command economy, i.e., there is overborrowing, to the point that it may easily exceed borrowing under perfect access. Second, in a recent paper Atkeson and Rios-Rull (1996) studied the performance of a market economy under imperfect capital mobility due to *corporate* risk, as opposed to *sovereign* risk, and found that it works efficiently. In contrast, in this paper sovereign risk is relevant for private borrowing and leads to inefficiency.

We find that market overborrowing is generally inefficient and leads to a precipitous recession (and capital flight). A constant consumption tax can offset the market externality. However, capital control taxes tend to be dominated because they distort the intertemporal margins, and may easily be uniformly counterproductive. In contrast, in this model, dual exchange rates and policy interventions based on quantity constraints, such as the auctioning of external

[2]Furthermore, the no uncertainty assumption we will use in our model would eliminate by itself any market failure in these papers, because under certainty default occurs with zero probability.

borrowing quotas by the central bank, would be optimal mechanisms.

The plan for the remainder of the paper is as follows. The next section presents a simple, representative agent model of a small economy facing limited access to external borrowing in which borrowing is motivated by a transitory negative income shock or, equivalently, the anticipation of a permanent positive income shock. This model is used and extended in the following sections. Section 3 solves for the optimal borrowing program, which establishes the efficient benchmark. The market equilibrium is found in section 4, where the main market overborrowing result is established, and the corresponding policy issues, including capital controls and optimal policy, are discussed in section 5. Finally, section 6 extends the previous analysis by considering richer international financial markets. Concluding remarks follow.

2 The Basic Model

Consider a small open economy that consumes a single tradeable good. The economy is a price taker in both goods and financial world markets, which are assumed to be time invariant. To simplify, we abstract from uncertainty and imperfect information.

The economy is inhabited by a continuum of identical agents uniformly distributed in $[0,1]$ (indexed by $g \in [0,1]$). The representative agent has a time-separable utility function based on concave felicity functions:

$$U = \int_o^\infty e^{-\delta s} u(c_s) ds. \tag{1}$$

The world risk-free lending rate is r and external creditors are competitive. To simplify, we assume that impatience is not a motive for external borrowing:

$$\delta = r. \tag{2}$$

To consider the simplest possible environment, we assume that this is an *endowment* economy.[3] Every agent has an endowment which is temporarily lower:

$$\begin{aligned} y_t &= y(1-\alpha) \text{ , for } t < T \quad (\alpha > 0) \\ &= y \qquad\qquad\quad \text{for } t \geqq T. \end{aligned} \tag{3}$$

The motive for borrowing from abroad then derives from the desire to smooth consumption over the lifetime of the economy in anticipation of income recovery.[4]

[3] The incorporation of investment or intermediate inputs does not alter the qualitative results.

[4] The above simplifying assumptions about the rate of time discounting and the income process are not critical for the results. An alternative polar case in which borrowing is motivated by the desire to tilt consumption forward leads to qualitatively identical results. This case, available upon request, is obtained by assuming impatience ($\delta > r$) and constant income ($\alpha = 0$).

To simplify, we assume that the individual solvency of the market agents is not a relevant factor, i.e., commercial risk is zero in the relevant range. However, in this model, the individual credit risk of debt claims b_t^g with agent g also includes sovereign risk.[5] A welfare maximizing government cannot commit not to exercise its sovereign power and interfere with private debt payments to abroad for the same reasons that it cannot commit to honor its own debt obligations if default is welfare improving ex-post. In fact, market practice, both by lenders and rating agencies, fully agrees with this theoretical pricing framework of corporate risk.[6]

This paper assumes that private borrowing is constrained by a country credit ceiling resulting from sovereign risk, which is far below the countries' technical solvency because of the difficult enforceability of international payment contracts against the will of the sovereign of a debtor country (as in Eaton and Gersovitz 1981 and subsequent papers). There is an *aggregate credit ceiling* L, perfectly known by everybody, that the country as a whole faces in world markets[7]:

$$\int_0^1 b_t^g dg = B_t \leq L. \tag{4}$$

This simple setup is consistent with models developed by Bulow and Rogoff featuring a credit ceiling that results from a bargaining game in which creditors have a punishment technology at their disposal (see, for example, Bulow and Rogoff 1989). For example, if creditors can credibly threat the debtor with sanctions amounting to a loss flow of l for as long as debt obligations are not fulfilled, then, under the assumptions above, a continuous bargaining game á la Rubinstein yields a negotiated payment of $l/2$ and a corresponding credit ceiling $L = \frac{l}{2r}$.[8] In this model of private borrowing, this reduced form would similarly emerge from a government that interferes with private debt repayments to abroad, be it by blocking payments, allowing non-payments, or nationalizing debts, in order to maximize the utility of the domestic representative agent.

To eliminate all sources of misinformation and simplify the arguments, we further assume that aggregate borrowing, B_t, is observable.[9] Under these conditions, being an atom, each individual agent has perfect access to international borrowing at the riskless rate r as long as $B_t < L$, and has no access to international borrowing when $B_t \geq L$. Notice that if all agents are equally indebted

[5] This sharply differs from the model in Atkeson and Rios-Rull 1996, in which country risk is not a factor.

[6] For example, the International Finance Corporation (IFC) adds a country-specific macroeconomic spread, set in line with rates charged by international banks for loans to governments, to the project-specific spread. Similar pricing methodologies are followed by both official and commercial banks financing the private sector.

[7] Certainty is assumed only for simplicity. Our conjecture is that the qualitative results of the paper hold with generality for any borrowing market economy subject to sovereign risk, i.e., in which there is a positive probability that the credit ceiling is reached.

[8] This fixed credit ceiling formulation is also similar to the one assumed in Detragiache (1992) and Isgut (1997) to analyze optimal borrowing in a command economy.

[9] In this model this is a non-critical assumption, because without private information aggregate debt can be perfectly foreseen in equilibrium.

by a (density) amount b_t, then aggregate debt $B_t = b_t$. (If the credit ceiling were surpassed at some point, partial default payments would be renegotiated. In the absence of uncertainty, however, default does not occur and all lending is at the riskless rate.)

Starting from an initial (gross) debt of b_0, the (gross) debt dynamics b_t implied by a consumption program c_t of the representative agent is:

$$\dot{b}_t = rb_t + c_t - y_t, \; b_0 \text{ given.} \tag{5}$$

Several remarks are in order regarding this standard identity. First, it assumes riskless lending, because otherwise competitive creditors would charge a risk premium over the risk-free rate. This assumption is justified in this model as long as the credit ceiling is not exceeded, which will not happen in equilibrium because there is no uncertainty.[10] Second, it holds irrespective of the maturity of the loans. Our preferred interpretation of the above condition is that debt takes the form of consoles, i.e., loans with infinite maturity that pay a constant stream of interest service. Otherwise, losing access to external borrowing means that new borrowing is limited to rolling over debt amortization so that the credit ceiling is not exceeded.[11]

Finally, the condition involves two simplifying assumptions that are relaxed in section 6, namely: a) more flexible debt instruments, such as future debt contracts and credit lines, are not available; and b) there is no available storage technology for the consumption good, either physically or financially (lending to abroad).

3 Optimal Borrowing

In the above setting, consider the central planner's problem of maximizing social welfare. Assuming that the utility of all identical agents is equally weighted, the central planner maximizes the utility of the representative agent:

$$\underset{\{c_t\}_0^\infty}{Max} \int_o^\infty e^{-\delta s} u(c_s) ds \tag{6}$$

$$\text{subject to } \dot{b}_t = rb_t + c_t - y_t \,, \quad b_o \text{ given.}$$

3.1 Perfect Access

The above maximization problem (plus the "no-Ponzi games" condition to rule out unsustainable debt paths) is valid under the standard assumption of perfect access to international financial markets, i.e. no binding credit ceiling. It is useful to solve it as a benchmark for the more interesting cases to come.

[10] Credit exceeding the ceiling implies a financial loss to all creditors (assuming an equal sharing seniority rule). Uncertainty is required to make such ex-post losses compatible with creditors' zero ex-ante profits.

[11] As we will see, without such refinancing commitment short-term debt would not be issued in equilibrium.

To solve the above problem, set up the Hamiltonian (in present values) and impose the first order conditions:

$$\begin{aligned} H_t &= e^{-\delta t}\left[(u(c_t)) + \lambda_t\left(rb_t + c_t - y_t\right)\right] && (7)\\ H_{c_t} &= 0 \Longrightarrow u'(c_t) = -\lambda_t && (8)\\ \frac{\dot{\lambda}}{\lambda} &= (\delta - r) = 0 \Longrightarrow c_t = c^* = \text{constant for all } t. \end{aligned}$$

From these conditions, we learn the well known result that the optimal path for consumption is constant throughout, which is accomplished by borrowing during the temporary negative income shock and servicing accumulated debt once income recovers. Since the representative agent individually faces the same problem, it will arrive at the same program and, as it is well known, the resulting market equilibrium is optimal.

The interpretation of this optimality condition of smoothing consumption over time is that any non-constant consumption path would imply a discrepancy between the marginal rate of intertemporal consumption substitution and the cost of such substitution through external borrowing. If consumption increases it would pay to advance borrowing forward; if consumption decreases it would pay to push borrowing into the future. Since both borrowing strategies are feasible, the optimal consumption path is flat. The implication is that a flat consumption schedule is a necessary condition for both optimality and market equilibrium within any period of time during which there is access to external borrowing.

How is the consumption level c^* determined? Here the "no-Ponzi games" condition eliminates unsustainable borrowing and imposes that the present value of consumption be equal to the present value of net income. This yields a measure of permanent net income:

$$c^* = y\left(1-\alpha\right) + y\alpha e^{-rT} - rb_0. \tag{9}$$

Finally, how large does debt get in this optimal program? The optimal borrowing program can be obtained by substituting optimal consumption into the debt dynamics condition and integrating (see Appendix A.1). Debt b_t^* grows exponentially at the rate r until income recovers at time T, staying constant thereafter. The maximum debt level b_T^* is:

$$b_T^* = b_0 + \alpha\frac{y}{r}(1 - e^{-rT}). \tag{10}$$

3.2 Imperfect Access

A credit ceiling L implies an additional constraint to the central planner's maximization problem:

$$b_t \leq L. \tag{11}$$

From the solution under perfect access, we learn that sovereign risk is relevant, i.e., this constraint is binding, iff:

$$L < b_0 + \alpha\frac{y}{r}(1 - e^{-rT}). \tag{12}$$

We assume the above condition to make the problem interesting: the borrowing headroom $(L - b_0)$ is not enough to accommodate accumulated unconstrained optimal borrowing. In a sense, this problem is akin to an exhaustible resource problem, in this case the pool of loanable funds.

The key to solving the above constrained planner's problem is to break it into two parts: before and after the credit ceiling L is reached at time τ. Until the credit ceiling is reached, the planner enjoys perfect access and follows the standard optimal borrowing rule, i.e., consumption is flat. The level of such consumption, and therefore welfare during this access period, depends on the time τ at which accumulated borrowing reaches the ceiling. After the credit ceiling is reached, the optimal feasible choice is to keep the stock of debt at the constrained level L, and therefore welfare (terminal welfare) depends only on τ. Adding up welfare during both periods, maximum welfare can then be expressed as a function of τ. Once the problem is concentrated on τ in this manner, the optimal choice of this parameter leads to the constrained optimal program.

The resulting optimal consumption and debt paths, denoted by c_t^P and b_t^P, can be characterized as follows (see figure 1):

Proposition 1 ***Optimal Program.*** *In the constrained central planner solution the economy hits the credit ceiling L exactly at T ($\tau = T$) and debt remains at the ceiling thereafter. The consumption level is constant up to T (at a level c_0^P) and is also constant from T onward (at a level c_T^P), in such a way that:*

$$b_t^P < b_t^*$$

$$c_0^P < c^* < c_T^P.$$

Proof: See Appendix A.1.

This constrained optimal program can be interpreted by noticing that the optimality condition that consumption be constant while there is access to external borrowing holds in this constrained optimum, until access is lost at time T. The upward consumption jump from c_0^P to c_T^P does not open an arbitrage opportunity because advancing borrowing to increase low consumption and smooth the jump is not feasible due to the credit ceiling (which implies that it would not be optimal not to hit the ceiling). After T, constant income allows for full smoothing of consumption without additional borrowing. Since all consumption programs are equal in present value, the constrained optimal consumption levels bracket unconstrained optimal consumption c^*.

The remaining issue to analyze is why it is optimal to hit the credit ceiling at T and not before. If it were the case that the ceiling is hit at $\tau < T$, then consumption would drop at τ as the borrowing conducted until then to support consumption cannot be longer sustained. Contrary to the consumption *jump* at T previously analyzed, this *drop* in consumption does open an arbitrage opportunity. In fact, it would be profitable (and feasible) to reduce borrowing before τ and use the borrowing headroom so created after τ, thus smoothing consumption.

In a sense, this constrained optimal program can be interpreted as the best way to stretch the borrowing headroom over time in order to avoid a drop in consumption. This conservative borrowing strategy implies that debt uniformly remains below the unconstrained optimal indebtedness path ($b_t^P < b_t^*$).

4 Market Borrowing

How does a decentralized market economy behave under the preceding assumptions?

It is useful to show right from the start that the market equilibrium differs from the optimal program just described. To show this, we check whether the representative agent would find it profitable to deviate from such optimal program. An individual agent facing the consumption jump from c_0^P to c_T^P would find it in her interest to unilaterally deviate from such program and borrow more before access is lost. In that way, she would smooth the consumption discontinuity by increasing low consumption through borrowing and decreasing high consumption when debt is serviced after T.[12]

This individual arbitrage opportunity was not available to the central planner because he could not increase borrowing in any measurable amount, i.e., an amount that would impact social welfare, without hitting the ceiling before time and suffering a recession, the cost of which would more than offset the benefit in terms of *social* welfare. But from the point of view of individuals maximizing their *individual* welfare, borrowing in significant amounts, i.e., borrowing in amounts that impact their own welfare, has no measurable adverse consequences to them.

Therefore, the market equilibrium differs from the optimal program.[13] Now that it has been established that the market mechanism does not replicate the central planner command when access to external borrowing is limited by sovereign risk, we move to actually find the market equilibrium, i.e., a feasible allocation from which no individual has an incentive to deviate.

The key to finding the market equilibrium is to observe that each agent rationally anticipates that at some point the economy as a whole, and therefore herself individually, will lose access to external borrowing. Since there is no uncertainty, the *particular instant* τ when the credit ceiling will be hit can be perfectly foreseen. Each representative agent takes this information into account when making borrowing decisions before τ, which generate an individually optimizing debt demand path b_t. Of course, in equilibrium τ is determined by the

[12] This thought experiment uses the fact that, within this analytical framework, each individual agent is an atom, and therefore, as far as she is concerned, has a negligible impact on the aggregate variables. A discrete model would lead to the same qualitative results but diluted by the partial internalization of this impact.

[13] Notice that the arguments supporting the suboptimality of the market equilibrium are quite general. We conjecture that the conclusion holds true with generality as long as the country credit ceiling is a binding constraint under some states of the world, i.e., as long as sovereign risk is relevant.

aggregation of these paths according to the market clearing condition $b_\tau = L$.[14]

Provisionally assuming that access is forever lost when the credit ceiling is reached at τ, the representative agent maximizes:[15]

$$\begin{aligned} & \underset{\{c_t\}_0^\infty}{Max} \int_o^\infty e^{-\delta s} u(c_s)\, ds \\ \text{subject to } \dot{b}_t &= r b_t + c_t - y_t\,, \quad b_o \text{ given} \\ b_t &\leq b_\tau \text{ if } t \geq \tau. \end{aligned} \tag{13}$$

As in the central planner's problem, this representative agent problem can be broken into two parts: before and after the aggregate credit ceiling is reached at time τ. Important features of the solution are also similar between the two problems. First, until the credit ceiling is reached, the agent enjoys perfect access and follows the standard market (and optimal) borrowing rules, i.e., consumption is also flat. Second, after the credit ceiling is reached, the optimal feasible choice is also to keep the stock of debt at the constrained level inherited at time τ, i.e., $b_t = b_\tau \ \forall t \geq \tau$.

The key difference between the two problems is that, contrary to the conditions for the central planner, for the individual agent the parameter τ is exogenous, i.e., when maximizing her welfare she can neglect the impact of her individual borrowing decisions on aggregate debt stocks and the timing of credit rationing.[16] Therefore, in the perception of the individual agent any level of borrowing is feasible up to time τ and no new borrowing is feasible after that time.[17] The resulting maximizing debt path b_t depends on the parameter τ, $b_t(\tau)$, which can again be found by breaking the problem into two periods and letting the agent decide the level of debt $b_\tau(\tau)$ she wants to carry into the no access period.

For market equilibrium, these demanded stocks of debt have to equal the credit ceiling. Provisionally focussing on symmetric equilibria, to be later justified, in equilibrium the aggregate market clearing condition $b_\tau = L$ holds. It is shown in the Appendix that the desired debt stock increases as credit rationing

[14] We will show that in this representative agent model without uncertainty the equilibrium has to be symmetrical.

[15] Market agents can also borrow domestically from each other. We will show that the domestic financial market can be ignored for the purpose of finding the equilibrium consumption allocation.

[16] Notice that the failure of the market regarding the timing of credit rationing, i.e., the crisis cost externality, can be captured only with a dynamic model. Traditional two-period models miss this externality.

[17] In particular, lower borrowing before τ does not enable the agent to borrow more after that time. Under the maintained assumption that aggregate debt is observable, an out-of-equilibrium borrowing shortfall resulting from a unilateral deviation would lead to a re-optimization revision on the part of all players (subgame perfection) and the consequent sharing of the unexpectedly available borrowing among all agents. As a result, the benefit to the deviating agent would be negligible. This result can be obtained with generality as long as agents are assumed to follow the weakly dominant strategy of continuing to demand finance after they expect to be rationed just in case a creditor provides it for whatever reason (trembling hand).

is postponed: $\frac{db_\tau(\tau)}{d\tau} > 0$. Therefore, the market clearing condition uniquely determines the parameter τ and the market equilibrium.

At the high end, as $\tau \to T$, the analysis of the optimal program at the beginning of this section suggests that debt demand exceeds the credit ceiling (and $\tau > T$ cannot be an equilibrium because in the high income period there is no reason to increase debt). At the low end, as $\tau \to 0$, desired new debt is negligible, because otherwise a consumption peak would result. In fact, the equilibrium is found at some intermediate point in time: $0 < \tau < T$ (see Appendix A.2).

The solution to the above constrained market equilibrium, in which consumption and debt paths will be denoted by c_t^M and b_t^M, can be characterized as follows (see figure 2):

Proposition 2 ***Market Equilibrium.*** *In the constrained market equilibrium the economy hits the credit ceiling L at τ, with $0 < \tau < T$, and loses access thereafter. The consumption level is constant up to τ at a level c_0^M (phase 1), drops to a recessionary level c_τ^M until T (phase 2), and then jumps to a level c_T^M from T onwards (phase 3), in such a way that:*

$$\begin{aligned} b_t^M &> b_t^P \text{ for } t < T \\ b_t^M &= b_t^P = L \text{ for } t \geq T \end{aligned}$$

$$y(1-\alpha) - rL = c_\tau^M < c_0^P < c_0^M < c_T^M = c_T^P = y - rL.$$

Proof: See Appendix A.2.

The two main features of this result have been already interpreted and commented upon. First, the market overborrows, i.e., $\tau < T$. To continue with the analogy of the exhaustible resource problem, the sharing of the pool of loanable funds is subject to the "(dynamic) commons problem", i.e., there is "overfishing" from the pool of loans. Second, after access is lost there is an abrupt recession, which is fully consistent with actual crisis experiences.

These two features of the solution are a stark difference between the suboptimal market equilibrium and the optimal program and, therefore, deviate from the market optimality result obtained in Atkeson and Rios-Rull (1996). In particular, our model implies a deep recession after access is lost, which is the one stylized fact of the Mexican 1994 crisis that their model fails to replicate.

During the boom, market borrowing is uniformly higher than optimal borrowing until access is lost ($b_t^M > b_t^P$), which leads to suboptimally high market consumption $c_0^M > c_0^P$. (It can be shown that until the recession market investment would be efficient and overborrowing would only finance overconsumption.) This consumption boom comes at the cost of a subsequent recession entailing suboptimally low market consumption $c_\tau^M < c_0^P$. These events, i.e., overborrowing, consumption boom, and recession, are perfectly foreseen by the representative agent but cannot be avoided due to a failure in the market mechanism.

Even though the representative agent does not internalize her effect on τ, the mere anticipation that she will be cut off from external credit at a given time and face crisis induces prudent private borrowing, because the burden of new debt would further exacerbate the cost of the recession. For this reason, it could be expected that the introduction of a binding credit ceiling and the corresponding knowledge of the prospects of crisis lead to slower market borrowing ($b_t^M < b_t^*$), qualitatively confirming the corresponding optimality result ($b_t^P < b_t^*$). However, this is not so. The free riding incentives leading to the borrowing race among market agents may dominate this effect and lead to an overall absolute acceleration in market borrowing when access is limited ($b_t^M > b_t^*$).[18]

Finally, we explicitly incorporate the existence of perfect domestic financial markets in which market agents can borrow and lend with no risk, which is of course an important feature of the undistorted economy being modeled. We first find the equilibrium conditions in the domestic financial market that support the above equilibrium allocation, in which market agents borrow only externally. We then show that, except for trivial variations, such market equilibrium is unique.

Let $i_t(m)$ be the equilibrium domestic interest rate for a loan extended at time t with maturity m. Let $c_0^{'}, c_\tau^{'}, c_T^{'}$ be the marginal utility of consumption in the three equilibrium phases, i.e., for consumption levels $c_0^M, c_\tau^M, and\ c_T^M$, respectively. Then, $c_T^{'} < c_0^{'} < c_\tau^{'}$.

Remark 1 ***Domestic Interest Rates in Market Equilibrium.*** *For domestic loans within the three market equilibrium phases, i.e., loans whose maturity does not extend beyond the consumption discontinuity points τ or T, the domestic interest rate is $i = r$. Otherwise:*

a) From phase 1 to phase 2, $i < r$ according to:

$$\frac{i_t(m)}{r} = \frac{(c_0^{'} - c_\tau^{'} e^{-rm})}{(c_0^{'} - c_\tau^{'} e^{-rm}) + (c_\tau^{'} - c_0^{'}) e^{-r(\tau - t)}} \tag{14}$$

b) From phase 2 to phase 3, $i > r$ according to:

$$\frac{i_t(m)}{r} = \frac{(c_\tau^{'} - c_T^{'} e^{-rm})}{(c_\tau^{'} - c_T^{'} e^{-rm}) + (c_T^{'} - c_\tau^{'}) e^{-r(T - t)}} \tag{15}$$

c) From phase 1 to phase 3, $i = r$.

Proof: See Appendix A2.

It is clear that within a phase, consumption is flat and the elimination of financial arbitrage is achieved at the discount rate r. Otherwise, additional borrowing taking place in relatively low consumption (phase 2 to phase 3) is

[18] Simulations upon request. Also, if borrowing is driven by impatience, free riding dominates if the elasticity of consumption substitution is larger than one.

especially desirable and, therefore, the interest rate exceeds r. Conversely, additional lending that can be recovered in relatively low consumption (phase 1 to phase 2) is especially desirable and, therefore, the interest rate is below r (and may be negative for short maturities).

The result that a domestic loan in phase 1 maturing in phase 3 carries an interest rate r can be interpreted along the same lines by observing that it is equivalent to a shorter loan up to phase 2 (cheap) and a second shorter loan up to phase 3 (expensive).[19] However, a deeper insight can be gained by observing that domestic loans in phase 1 maturing in phase 3 are perfect substitutes for external loans accessible in phase 1, and therefore carry the same terms. (Not so domestic loans maturing in phase 2, because the corresponding short-term external loans carry a valuable implicit rolling over option at rate r, which is costless to the external creditor because in this model rolling over is riskless but is costly to domestic creditors facing recession. This assurance is valuable to the debtor because in phase 2 aggregate demand for foreign capital exceeds aggregate supply, which is restricted to scheduled amortization.)[20]

The above interest rates are such that all agents are indifferent between borrowing and lending domestically at the market equilibrium described above, i.e., the marginal utility of consumption obtained from borrowing equals the integral of discounted consumption utility forgone by the corresponding debt service at the rate i. The entire interest rate structure can be derived accordingly, either directly or combining the above results. Therefore, they support the above equilibrium derived under the assumption of no domestic borrowing and lending between market agents.

Is this market equilibrium unique? Since agents are identical atoms they can always attain the payoff of any other agent by replicating her strategy. Therefore, in equilibrium all agents attain the same expected utility. At equilibrium prices, all (deterministic) consumption paths of agents with equal preferences are "parallel," i.e., they exhibit the same marginal rates of intertemporal substitution. Hence, as provisionally assumed, equilibrium is symmetric in the sense that all agents actually attain the same consumption path and, consequently, the same borrowing path.

Remark 2 ***Uniqueness.*** *The market equilibrium allocation of Proposition 1 is unique.*

Nevertheless, there are trivial asymmetric equilibria involving the same borrowing path but based on different amounts of external and domestic borrowing for different agents. In all these equilibria, aggregate external borrowing remains the same because aggregate domestic borrowing adds up to zero: agents

[19] This can be confirmed by linking the two loans using the formulas in the Proposition.

[20] Unless there is an understanding that the creditor has a preference for its debtor when it comes to rolling over, as opposed to treating her as one more anonymous agent seeking finance, the resulting refinancing risk would make short-term (phase 1 to phase 2) external lending too expensive and it would not be issued in equilibrium. Consequently, if the possibility that the implicit assurance of refinancing may fail is considered a "trembling hand" refinement, then in this model only long-term (phase 1 to phase 3) external finance would be issued in equilibrium.

who borrow more externally lend domestically in such a way that each agent's net borrowing is invariant. Therefore, the domestic financial system in this model is irrelevant for real market equilibrium as well as for net financial variables, including overall external borrowing. In a fundamental sense, the market equilibrium described above is unique.

5 Capital Controls and Optimal Policy

As shown, the free market economy produces an inefficient allocation because of the market failure to internalize sovereign risk. If country credit ceilings could be relaxed sufficiently to become irrelevant for the optimal borrowing program, then in this model, in the absence of market distortions, the market mechanism would be efficient. However, in the presence of such binding constraint there is room for policy intervention to interfere with the free market, e.g., capital controls. Nevertheless, market overborrowing does not automatically justify capital controls, even if optimally calibrated to impede external borrowing and free from implementation costs. Appropriate capital controls minimize distortionary side effects by specifically addressing the nature of the market failure (see Fernandez-Arias and Montiel 1996 for an overview). This section analyzes the kind of policies that match the market failure causing overborrowing in this model, first reviewing tax policy aimed at aligning prices and then exploring policies based on quantitative restrictions.

5.1 Taxes

In what follows we analyze the menu of taxes available to policy makers. We allow that tax rates be contingent on *aggregate* variables, so that policy makers confronting the dynamic problems associated with market overborrowing have at their disposal flexible tax schemes whose time profile depends on the state of the economy.[21] The only restriction placed on this menu of taxes is the implementability requirement that tax rates do not depend on *individual* variables, which requires the monitoring of individual allocations as opposed to macroeconomic outcomes. This requirement leaves out non-flat tax schedules on individual allocations, which are analyzed together with quantitative restrictions in the next subsection. However, it encompasses all of the flat taxes that are commonly used in practice for the purpose at hand. For example, traditional taxes on consumption and financial transactions take this form, whose rates depend on the state of the economy but are constant at a point in time. Harberger's optimal tax and Tobin-like turnover taxes also take this form.[22] The

[21] Time consistency should be a restriction if policy makers lack the ability to commit. However, commitment is not an issue because optimal policies in this model, on which this section focuses, are time consistent.

[22] An example is Chile's flat tax on borrowing irrespective of maturity, so that its incidence on the cost of capital r decreases with maturity. In this way, borrowings at different maturities are discriminated in favor of longer maturities. Since in this model maturity is irrelevant, such tax would induce an all-console external debt portfolio.

customary policy recommendation of (possibly contingent) reserve requirements on external borrowing implicitly amounts to a flat tax on external borrowing.

In what follows we explore the properties of the available menu of flat taxes to be applied when there is access to external borrowing. In this model the future evolution of aggregate variables is perfectly predictable and flat taxes can be represented without loss of generality by time-varying tax rates. We consider three alternative tax bases: consumption (at rate $x_c(t)$), external debt service (at rate $x_b(t)$), and external inflows (at rate $x_{\dot{b}}(t)$).[23]

The overall tax revenue is returned to agents in the form of identical lump-sum subsidies s(t). In equilibrium, the representative agent receives back an amount equal to her own tax payments:

$$s(t) = x_c(t)c_t + x_b(t)rb_t + x_{\dot{b}}(t)\,\dot{b}_t \tag{16}$$

It is worth noting that it is valid to represent a tax on (gross) inflows as a tax on net borrowing $\dot{b}$ because such inflow tax would give incentives to minimize their volume, thus leading to an all-console equilibrium in which gross inflow is also net borrowing. The same result would obtain if amortization payments are taxed, which implies that in this model principal payments are an irrelevant tax base. Therefore, the tax on debt service can be interpreted as a tax on interest payments (or on debt stock at the rate rx_b).

Solving for borrowing $\dot{b}$, the new law of motion generalizing the budget constraint (5) is:

$$\dot{b}_t = f(b_t, c_t) = \frac{(1+x_b(t))}{(1-x_{\dot{b}}(t))}rb_t + \frac{(1+x_c(t))}{(1-x_{\dot{b}}(t))}c_t - \frac{y(t)}{(1-x_{\dot{b}}(t))} - \frac{s(t)}{(1-x_{\dot{b}}(t))} \tag{17}$$

The corresponding first order conditions while access is available are as follows:

$$\begin{aligned} H &= e^{-\delta t}\left[(u(c_t)) + \lambda_t f(b_t, c_t)\right] \\ H_{c_t} &= 0 \Longrightarrow u'(c_t) = -\lambda_t f_{c_t} = -\lambda_t \frac{(1+x_c(t))}{(1-x_{\dot{b}}(t))} \\ \frac{\dot{\lambda}_t}{\lambda_t} &= \delta - f_{b_t} = \delta\left(\frac{x_{\dot{b}}(t) - x_b(t)}{1-x_{\dot{b}}(t)}\right) \end{aligned} \tag{18}$$

Optimal borrowing requires that the consumption path be flat (Proposition 1), which implies that $u'(c_t)$ is constant over time. This necessary optimality condition is obviously not sufficient; in fact, it also holds for the suboptimal market equilibrium. Our strategy to find optimal taxes is to first determine the class of taxes for which this necessary condition holds and then analyze how

[23] An income tax would be irrelevant in this model because income is exogenous. If investment is incorporated in the model, the market would efficiently allocate investment by equating its marginal return and the cost of capital and a tax on income or investment would be distortionary. In all cases, income or investment taxes are not optimal.

to calibrate taxes within that class to alter market equilibrium until it reaches the optimal allocation. Combining the FOCs above, the necessary optimality condition can be expressed as (see Appendix A.3):

$$(1 + x_c)(rx_b + rx_b - \dot{x}_b) = \dot{x}_c (1 - x_b) \tag{19}$$

It is important to note at this point that this consumption optimality condition also holds in an economy with investment. Concerning taxation on investment or income, since market investment is efficient, zero taxation of investment and income is optimal. More generally, investment is optimal when any tax on investment is offset by a subsidy to income at the same rate. Since all the tax experiments contemplated in what follows obey such sufficient optimality condition, the policy conclusions in this section apply with generality.

For the most part, we concentrate on the properties of each tax separately, keeping the other two taxes at a zero level.[24] We first examine the implications of such experiment in terms of the above condition to find candidates for optimal taxes, whose performance is then analyzed. The conclusions for the three taxes under consideration are summarized in the following propositions:

Proposition 3 ***Optimal Consumption Tax.*** *Optimal borrowing can be supported as a market equilibrium with a consumption tax during the low income period $t<T$ at a constant rate: $x_c(t) = x^* = \frac{u'(c_0^P)}{u'(c_T^P)} - 1 > 0$.*[25]

Proof: When only a consumption tax is applied (19) is verified only if $\dot{x}_c = 0$, i.e., the consumption tax rate is constant while access lasts. Any such consumption tax is intertemporally neutral and leads to a flat consumption path. The appendix shows that, under market access, the market equilibrium consumption level decreases as the consumption tax rate increases, until it reaches the optimal path at x^*.

Proposition 4 ***Suboptimality of Taxes on Debt Service.*** *Optimal borrowing cannot be supported as a market equilibrium with a tax on external debt service.*

Proof: When only a tax on external debt service is applied (19) is not verified because the LHS is $rx_b \neq 0$.

The reason why a tax on debt service fails can be traced to the FOCs in (26). Such tax introduces an intertemporal distortion in the borrowing path by penalizing the marginal cost of capital, from r to $r+x_b(t)>r$, leading to $\dot{\lambda}>0$. As a result, overborrowing diminishes but at the cost of an inefficient upward-sloping consumption path.

Proposition 5 ***Optimal Capital Inflow Tax.*** *Optimal borrowing can be supported as a market equilibrium with an external inflow tax during the low income period $t<T$ whose tax rate* increases *over time: $x_b(t) = f^* exp(rt) = \frac{x^*}{1+x^*} exp(-r(T-t)) > 0$.*

[24] The analysis of tax packages can done similarly.

[25] More generally, x_c^* is the surtax above the permanent consumption tax.

Proof: When a tax on external borrowing is applied (19) is verified only if $(\dot{x}_b - rx_b) = 0$, which holds for $x_b(t) = f\exp(rt)$, i.e., the capital inflow tax rate grows exponentially at the rate r. Notice that this optimality condition can be interpreted as an intertemporal neutrality with respect to the cost of capital, because it requires that the capital inflow tax rate increases over time in such a way that its present value, f, remains constant. The appendix shows that, under market access, the market equilibrium consumption level decreases as the capital inflows tax time profile increases with the parameter f, until it reaches the optimal path at $f^* = \frac{x^*}{1+x^*}exp(-rT)$.

Remark 3 ***Inadequacy of a Constant Capital Inflow Tax.*** *A capital inflow tax at a constant rate $x_b > 0$ is suboptimal and may be uniformly welfare reducing.*

It is easy to check in equation (17) that a traditional capital inflow tax at rate f is equivalent to a combination of a consumption tax at rate f/(1-f) and a debt service tax at rate rf/(1-f). While both components reduce borrowing, the capital inflow tax is necessarily suboptimal because it introduces the intertemporal distortion implicit in the tax on debt service.[26] Even if imperfect, it could perhaps be argued that some level of capital inflow tax of this kind is bound to improve welfare. However this is not necessarily the case: under a number of reasonable scenarios, the second best policy is zero tax on inflows.[27]

Since tax proceeds are redistributed in lump sums, the above optimal taxes can be easily replicated by tax packages leading to expressions in equation (17) that are equivalent from an incentive perspective. A simple tax package that replicates a consumption tax at the rate x consists of a capital inflow tax at the rate x/(1+x) and a debt service subsidy at the same rate, i.e., a tax at rate -x/(1+x). This scheme can be interpreted as a dual exchange floating rate regime, in which the goods exchange rate contains a premium of x% relative to the financial exchange rate, i.e., the home currency is *more* depreciated in the goods market by a margin x:

Proposition 6 ***Optimal Dual Exchange Rate.*** *Optimal borrowing can be supported as a market equilibrium with a dual floating exchange rate during the low income period $t<T$ such that the home currency price of foreign exchange in the goods market contains a premium x^* relative to that in the financial market.*

Proof. In such a dual exchange rate regime with margin x, a unit of capital inflow delivers only a fraction 1/(1+x) of a unit of consumption, i.e., a fraction x/(1+x) is "taxed away." Similarly, a unit of debt service requires only 1/(1+x) of consumption, so that it receives a subsidy x/(1+x). Such scheme is, from an incentive point of view, equivalent to a consumption tax at the rate x, which is optimal at the rate x^*.

[26] Notice that this intertemporal insight in capital control policy requires a dynamic model; a two-period model would wrongly suggest that all tax bases are equivalent.

[27] Simulations available upon request. This can easily be shown analytically if the instantaneous utility function is logarithmic.

The optimal dual exchange rate can be interpreted as an alternative way to tax consumption. In fact, capital inflows finance debt service and net imports, i.e., consumption in excess of domestic production, and therefore any tax on capital inflows is implicitly a tax on both current account items. By excepting debt service with an offsetting subsidy that recovers the full foreign exchange value of the inflow, and since production is given, a selective tax on consumption is obtained. (Imported goods are taxed and exported goods are subsidized at the same rate; in this model imports and exports are determined only up to their net amount.) Like before, this policy is also optimal in an economy with investment because the corresponding tax on investment is offset by the implicit subsidy to domestic production.

The optimality of a simple Pigouvian consumption tax shown in proposition 3 implies that the market distortion to be addressed can be thought of as a consumption cost externality. It suggests that it may be helpful to think of market overborrowing as resulting from market overconsumption, i.e., a consumption boom, rather than the other way around. (The absence of overinvestment in a model with investment further reinforces this interpretation.) It follows that policy can be best thought in terms of dampening consumption booms, rather than controlling capital coming from abroad.

5.2 Quantitative Restrictions

In this model, and as suggested by the previous analysis of capital controls, it would make sense for capital controls to focus on eliminating the free riding problem caused by the aggregate nature of the credit ceiling, rather than simply making borrowing more expensive through taxes. One way to achieve this result is by allocating the credit ceiling L among market agents in the form of "foreign debt quotas", which entitle each agent g to borrow up to an accumulated amount equal to her quota q^g:

$$\int_0^1 q^g dg = L \tag{20}$$

This strategy of granting permits is akin to assigning consistent property rights to the exhaustible resource subject to the "commons problem". We continue to assume perfect information, so that these foreign exchange permits are credible commitments on the part of the Central Bank. Then the new market agent problem would be:

$$\begin{aligned} & \underset{\{c_t\}_0^\infty}{Max} \int_o^\infty e^{-\delta s} u(c_s) ds \\ \text{subject to } \dot{b}_t &= r b_t + c_t - y_t \,, \quad b_o \text{ given} \\ b_t &\leq q^g \end{aligned} \tag{21}$$

Once each market agent owns her permit, the free market would sort out the ex-post efficient equilibrium, i.e., intertemporally efficient consumption paths. To see this, notice that the above market agent's maximization problem is formally

equivalent to that of the central planner for a ceiling equal to q^g (Proposition 1): The elimination of sovereign risk as the binding constraint for individual financing eliminates the free-rider problem and would lead to flat consumption during the entire low-income period. Therefore, each agent attains an efficient path given her individual credit ceiling q^g, i.e., the market equilibrium would be efficient given that set of constraints.

The above efficiency result confirms the optimality result in Atkeson and Rios-Rull (1997) if the ceilings q^g are re-interpreted as individual solvency constraints. However, contrary to this solvency interpretation, in this model, if permits are tradeable, the market may improve upon the previous allocation by re-allocating permits through trading. It is easy to check that, in equilibrium, there is no meaningful trade between constrained agents ($q^g < b_T^*$) because they place the same marginal value on borrowing permits, but unconstrained agents ($q^g \geq b_T^*$) sell their excess permits to constrained agents to their benefit. As a result, the market always arrives at an equilibrium in which, in the aggregate, the consumption path is fully efficient, i.e., aggregate market consumption coincides with the optimal consumption path of Proposition 1 (because the flat consumption path in the low-income period is supported by the full aggregate utilization of the overall credit ceiling L).

Nevertheless, while a quota system eliminates individual intertemporal inefficiency and quota tradeability takes care of Pareto improvements, which in this model implies the optimality of the aggregate consumption path, social welfare is not maximized. The reason is that in this representative agent framework deviations from identical individual consumption paths entail suboptimality. The following quota-based capital controls induces the free market to achieve the optimum:

Proposition 7 ***Optimal Debt Quotas.*** *The allocation of debt quotas in equal parts, i.e., $q^g = L \ \forall g$, attain the optimal program of Proposition 1.*

Proof. As shown in previous sections, in this representative agent framework, optimality requires symmetric allocations. With this symmetric quota assignment, the representative agent's problem becomes the central planner's problem (Proposition 1) and the market attains the optimal program (with no ex-post trading of quotas taking place).

It is interesting to notice that in this case a quantitative control on capital in terms of maximum stocks of debt tend to outperform the continuous application of tax-based capital controls. But the primacy of quantity over price as a policy dimension in this case is largely illusory. While the above debt quota scheme has a market-based flavor that simplifies implementation relative to quantity restrictions on flows through borrowing permits, it still requires debt monitoring at the *individual* level which enormously complicates implementation. In fact, non-flat taxes with the same implementation requirements can replicate the outcome of this optimal quota system.[28] Furthermore, while in this simple

[28] For example, no tax is assessed until the quota is reached and a prohibitive tax is assessed otherwise.

representative agent setting the quota allocation mechanism, e.g., central assignment or market auction, and whether or not the quotas are personalized or can be traded is irrelevant because identical agents will bid identically and will not trade their identical quotas, more complex settings such as economies with (privately known) agent heterogeneity pose obvious additional implementation difficulties.[29]

Finally, we explore the possibility of devising a scheme based on quantitative restrictions on aggregate borrowing, which could be deemed implementable for the same reasons that flat taxes were considered so. Such scheme would have the central bank limiting aggregate borrowing according to the optimal borrowing path, which is in the spirit of Soros' recent proposal in Financial Times (1997) of monitoring and controlling aggregate debt in order to prevent crises.

The consistent application of these quantitative restrictions would ensure that access will not be lost, i.e., $\tau = T$. Is the anticipation of such orderly outcome enough to remove the market's tendency for overborrowing? The answer is no. Under this scenario in which access is not lost before time, the representative agent still prefers to borrow more than optimal borrowing $b^P(t)$, for example the unconstrained borrowing level $b^*(t)$. Therefore, the aggregate quota needs to be rationed among agents and the implementation issue of the allocation of individual quotas reappears. The issue of how to coordinate individual agents in a market economy to conform to the socially optimal program, left out as a detail in Soros' proposal, continues to be a key issue of economic policy.[30]

6 Extending the Model

In this section we generalize the simple model by considering various extensions to the set of financial instruments available for international transactions and the corresponding definition of the credit ceiling to encompass them.

We find that the optimal program (Proposition 1) is not affected by these extensions to the external environment because the new instruments can be replicated by the traditional loans in the simple model. However, the free market equilibrium (Proposition 2) is altered by the availability of new instruments. Despite the fact that the choice set remains the same in the sense that new instruments do not span additional feasible space in purely financial terms, market agents do alter their choices. As a result, market overborrowing deepens.

[29] With heterogeneity, it is clear that a quota system would benefit from allowing the ex-post trading of the quotas to eliminate Pareto inefficiencies. Furthermore, the design of an auction mechanism to elicit the agents' preferences and provide a good starting point for trading would be relevant (and complicated by its multiple-unit nature).

[30] Soros in Financial Times (1997): "...There are many issues to be resolved. The most important is the link between the borrowing countries and the borrowers within those countries.[...] the details could be worked out."

6.1 Completing Borrowing Markets

An important extension to borrowing markets is the existence of a future debt market, in which new debt to be disbursed at arbitrary points in time is committed in advance. The credit ceiling should be correspondingly extended to encompass debt committed to be incurred in the future, in such a way that the implied debt path generated over time does not exceed the credit ceiling at any point.[31]

The existence of future debt markets separate the timing of consumption and borrowing decisions, in contrast to the simpler model in which the existence of only spot markets (and the absence of a storage technology) implied that borrowing translated into current consumption. In this model, any feasible set of future debt contracts can be replicated by a set of spot loans contracted over time, so the optimal consumption program does not change. However, the constrained market equilibrium is altered rather dramatically as a result of the expansion of the choice set faced by individual agents:

Remark 4 ***Overborrowing with Complete Borrowing Markets.*** *Complete borrowing markets lead the free market to extreme overborrowing: the credit ceiling is instantly reached and rationed among agents.*

Proof. See appendix.

To see this, notice that, with future debt markets, market agents could immediately contract, at time zero, the unrestricted optimal borrowing schedule supporting consumption c^* they would follow if they had permanent access to credit (at the riskless rate), which is not feasible in the aggregate. In fact, market agents would anticipate such outcome and know that there will be unsatisfied borrowing demand to profit from by on-lending the excess external debt they can secure through the domestic financial system, in a way similar to the ex-post trading of excess quotas discussed in the previous section. As a result, in equilibrium, individual demands are unbounded.[32]

6.2 Completing Lending Markets: International Reserves and Capital Flight

If there is access to lending abroad, then the debt dynamics equation needs to be modified to include assets abroad, to be denoted by a_t, whose yield is i:

[31] Future contracts of the traditional loan instruments utilized in the simple model suffice to span all the space generated by more complex debt contracts, i.e., fair debt contracts with arbitrary cash flow profiles. In this perfect foresight model, credit lines are equivalent to future contracts.

[32] How inefficient is this extreme overborrowing cannot be fully assessed in this walrasian market-clearing framework. A full account of the inefficiency associated with this corner solution requires the detailed specification of how the credit ceiling is rationed among market agents to eliminate excess demands, which is a game in itself of a non-walrasian nature. Apart from the possibility of an intertemporally suboptimal aggregate consumption path, there are two new potential sources of inefficiency: i) asymmetric rationing among market agents leading to asymmetric equilibrium consumption paths; and ii) uncertainty about rationing outcomes among risk-averse agents.

$$\dot{b}_t - \dot{a}_t = rb_t - ia_t + c_t - y_t, b_0 \text{ given}, a_0 = 0 \tag{22}$$

where for simplicity initial assets are assumed to be zero. Under perfect access to lending abroad the yield on lending abroad is $i = r$ and keeping international assets as reserves is costless. Otherwise, if $i < r$, keeping international reserves entails a carrying cost. Nevertheless, as long as assets yield a positive return, this extension permits us to drop the assumption of no physical storage technology, which in this model without investment would be dominated by financial storage in the form of assets abroad.

Allowing domestic agents to lend abroad opens up a number of interesting issues and possibilities. In a world of certainty, as in the models in this paper, the traditional portfolio motivation for two-way financial flows does not apply and there would be no financial reason for a (net) debtor country to lend abroad. Therefore the optimal program is not altered by the ability to lend abroad, i.e., the central planner does not benefit from holding international reserves. However, for individual agents, lending abroad may be a way of securing future access to funds by selling assets previously acquired once borrowing becomes unfeasible. In the presence of sovereign risk, this additional motivation may justify two-way flows and alter the market equilibrium. This mechanism provides a novel reason for capital flight unrelated to heightened risks to domestic capital investment, namely that, in countries with limited access to external credit, borrowing cum capital flight improves future access to credit (in the absence of future markets).

Lending abroad provides market agents with an instrument to secure future financing after access to borrowing is lost (in a way similar to future markets or credit lines) and, not surprisingly, the market overborrows even more:

Remark 5 ***Overborrowing with Lending Markets.*** *Access to lending abroad leads to more market overborrowing, in which the final push is incurred to finance massive capital flight (and then credit becomes rationed). If carrying costs are small, access to lending leads the free market to extreme overborrowing and capital flight: the credit ceiling is instantly reached, rationed among agents, and lent abroad.*

Capital flight is useful as a reserve to be used to alleviate recession after access to borrowing is lost. If lending abroad is costless, borrowing cum lending abroad is costless and, therefore, the stock of assets abroad is equivalent to a (costless) credit line. Consequently, market equilibrium coincides with that under complete borrowing markets, as market agents attempt to secure future consumption with the instruments available.[33] It is easy to check that this corner solution still applies with small carrying costs.

With substantial carrying costs, however, reserves created with flight capital are used faster, and are consequently smaller than available credit. In that case,

[33] For the same reasons, evaluating the degree of inefficiency of this extreme overborrowing requires knowing how the credit ceiling is rationed among agents and cannot be fully assessed in this competitive equilibrium framework.

gradual borrowing to support flat consumption is topped by a discrete jump in borrowing cum capital flight, provoking access to be lost in a way that is foreseen and yet abrupt.[34] Flight capital is then gradually repatriated and used to supplement income during the subsequent recession, which is longer but shallower than otherwise. The resulting consumption path declines at the rate $-(r-i)$, the gap between the rate of impatience, r, and the opportunity cost of capital, i.[35] Figure 3 compares the market and the optimal allocation for a case in which lending abroad is feasible at a carrying cost (see Appendix for details).

A final remark is in order. So far we have maintained the assumption that the credit ceiling L is on gross debt. A new key issue when the country may have financial assets abroad is how to extend this definition of the credit ceiling. The answer to this question depends on the extent to which assets abroad contribute to a country's creditworthiness, i.e., their impact on the credibility of the country's repayment commitment. If assets do not increase willingness to pay, then the maintained assumption of a ceiling on gross debt is applicable. If they do, however, assets abroad would be an offset to gross debt for the purpose of the credit ceiling.[36]

For simplicity, we will now focus on the alternative polar case in which the credit ceiling refers to net debt (full offset). It is easy to check that in this case access to lending abroad does not alter the market equilibrium by reinterpreting b_t as *net* debt, as opposed to *gross* debt. In the absence of carrying costs, gross assets and liabilities yielding the unique net debt equilibrium path are indeterminate; with carrying costs, lending abroad is strictly dominated and the zero-asset equilibrium of the previous models is restored:

Remark 6 ***Net Debt Ceiling.*** *If the debt ceiling is on net debt, access to lending abroad does not alter the market equilibrium.*

Why is it that lending abroad does not serve as a valuable store of funds to market agents facing future rationing and recession? The reason is that in this case the selling of assets cannot be effectively used for consumption, because the satisfaction of the credit ceiling in its net version requires that an asset reduction be compensated by an equivalent reduction in liabilities, leaving nothing for additional consumption. It follows that the equilibrium just described requires that such offsetting reduction in liabilities can be enforced by creditors, the feasibility of which is not clear. The implication is that if it is not enforceable

[34] This foreseen collapse is remindful of Krugman's speculative attack model. In this model, what is istantaneously exhausted is not foreign reserves, but rather the remaining credit line with the rest of the world.

[35] This formulation of individual self-financing out of the stock of flight capital during the recession provides an alternative interpretation of the model in Atkeson and Rios-Rull (1996), in which the interest rate would be i and the individual credit ceilings would be the stock of flight capital. While market overborrowing is inefficient overall, the market allocation during the recession is ex-post efficient given initial conditions.

[36] Characteristics such as the sizeability of these assets would be relevant in a model explaining the determination of the credit ceiling. Bulow and Rogoff (1989) have argued for a small partial offset in the context of a bargaining model, in which the existence of international reserves increases the value of the sanctions available to creditors to some extent.

against the will of the sovereign, then the credit ceiling cannot be in the form of net debt.[37]

7 Concluding Remarks

As it is well known, optimal borrowing with perfect access to international financial markets entails full consumption smoothing over time. Such an unconstrained program is arrived at by both a central planner and a decentralized market economy: the free market equilibrium in an undistorted economy is optimal.

However, the anticipation that sovereign power will be applied ex-post to support non-payment to foreign creditors when the government finds it in the country's interest limits access ex-ante to both public and private borrowers, which may render infeasible the above unconstrained program. Under such limited access to international financial markets due to sovereign risk, overall external borrowing is like an exhaustible resource whose optimal utilization requires careful rationing by the central planner. In fact, optimal borrowing spreads out capacity to borrow over time, i.e., slows down borrowing relative to unconstrained borrowing (Proposition 1). The question we ask is: does the free market also achieve the same outcome in this case? In other words, is the free market still optimal in the presence of sovereign risk?

We find that, in equilibrium, the free market overborrows, i.e., borrows more heavily than optimally, and, therefore, the access to international financial markets is lost before time (Proposition 2). Overborrowing finances a consumption boom (investment remains optimal), possibly followed by capital flight, and leads to a perfectly foreseen, but yet unavoidable, loss of credit access and recession. This market outcome obtains under rather ideal conditions of market perfection: the domestic economy is undistorted, information is complete, and agents are fully rational. Furthermore, due to the assumption of no uncertainty, default does not occur.

The reason for market overborrowing in this model is that individual agents attempt to secure financing before external access is lost because they fail to internalize the cost of credit rationing associated with sovereign risk, which depends on *aggregate* indebtedness. In other words, external debt is like an exhaustible resource subject to the *(dynamic) commons problem.* We conjecture that this free market overborrowing result obtains with generality (as long as external market borrowing is subject to sovereign risk).

We also find that the availability of a more flexible and complete set of instruments for external borrowing and lending leads to more market overborrowing, to the point that it degenerates into an immediate loss of access and rationing equilibrium if external financial markets are sufficiently complete and frictionless. More flexible instruments are like "grease in the wheels" of the

[37] The intermediate case of partially offsetting assets yields results qualitatively similar to the gross debt ceiling case because assets retain liquidity value. However, the enforceability of the credit ceiling remains a problem.

market mechanism, while market overborrowing calls for "sand in the wheels", to paraphrase James Tobin.

The finding that an undistorted market economy overborrows does not imply that distortions in the domestic economy previously identified in the literature as causes of market overborrowing, such as moral hazard due to implicit public guarantees of foreign investments or inadequate disclosure of financial information, are not relevant. What it does imply is that, contrary to what it is usually assumed, the free market is not an adequate efficiency benchmark to guide the analysis of market overborrowing: even if feasible, the removal of all distortions in the domestic market does not cure overborrowing and there is room for additional policy action.

Finally, we analyze policies to rectify the market. In our model, the market failure that induces the free market to overborrow can be offset by a constant consumption tax applied during the borrowing period (proposition 3).

Capital controls, in contrast, may or may not be suitable to achieve optimal borrowing or even welfare improvements. For example, a constant capital inflow tax (as well as any taxation of debt service) distorts intertemporal market efficiency and may be uniformly welfare reducing at all rates (proposition 4). However, optimality can be attained with the following alternative forms of capital controls. First, the taxation of capital inflows at an increasing rate whose present value remains constant (propositions 5). Second, the establishment of a dual exchange rate in which the foreign exchange in the financial market sells at a discount (proposition 6). Finally, the assignment of individual external debt quotas (*property rights* over the common exhaustible resource). This quantitative constraint is able to eliminate the market failure by credibly securing external finance to market agents (proposition 7).

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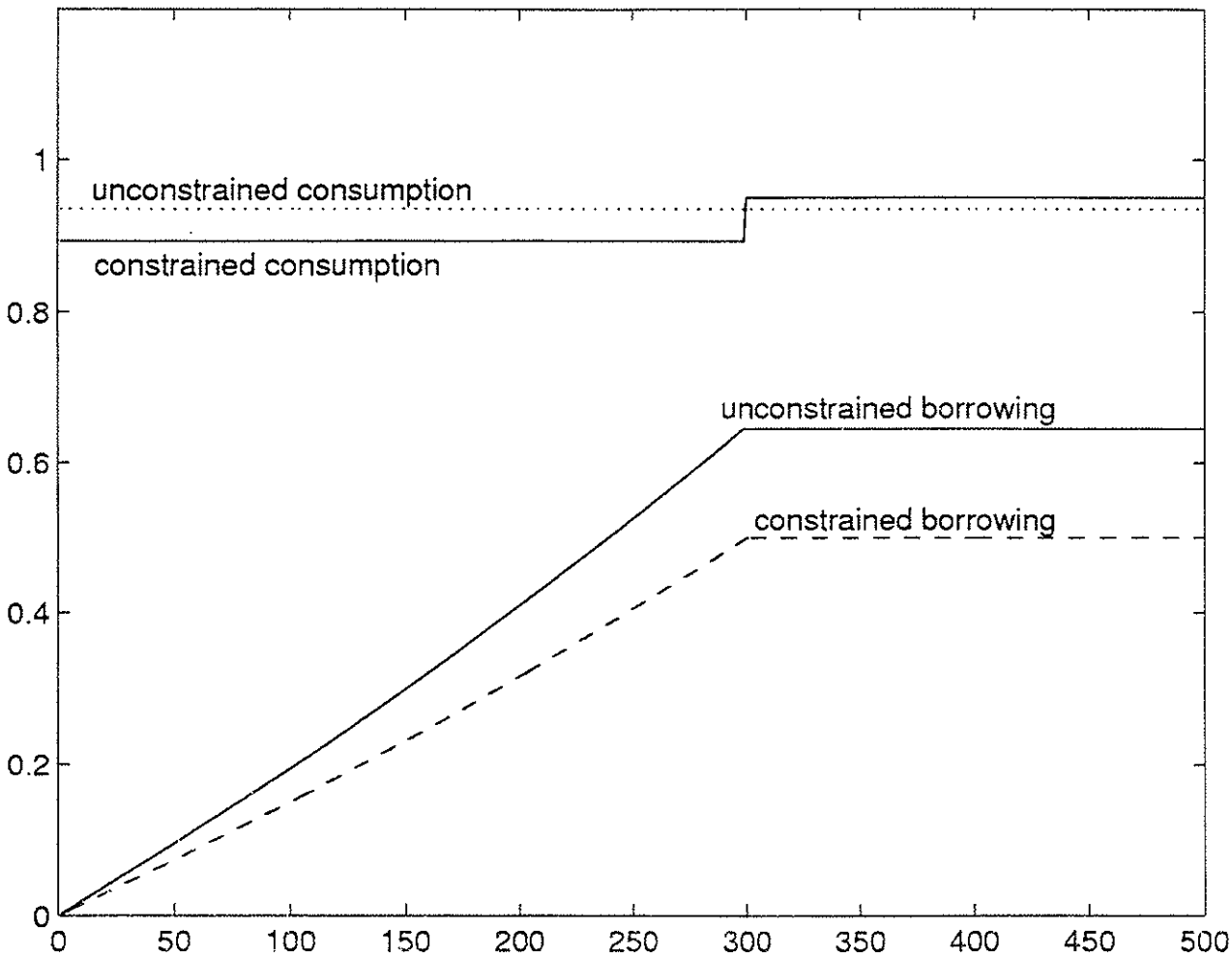
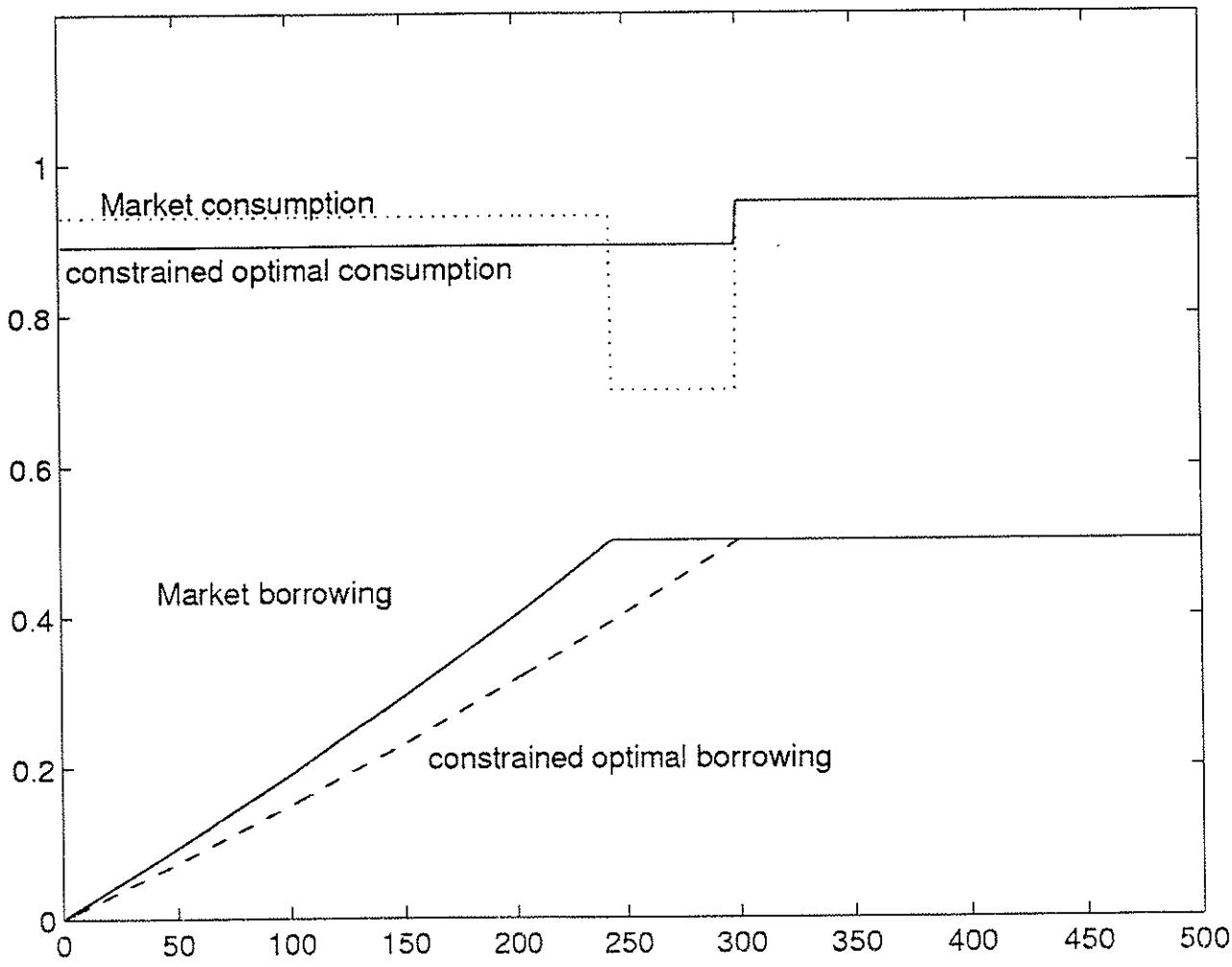
Figure 1: Optimal borrowing



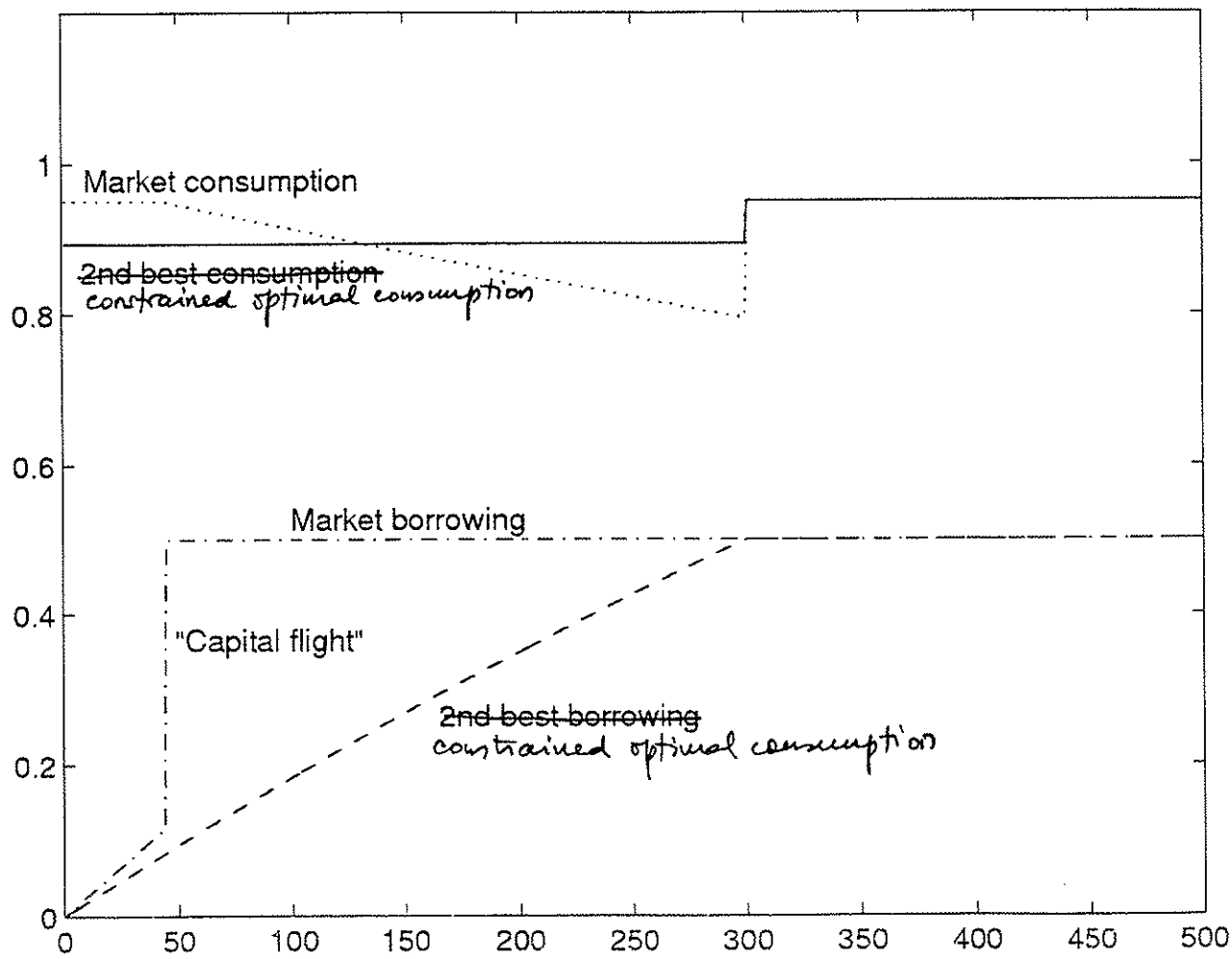
Parameters: L=.5;T=3;y=1;alpha=.25;r=.1;b0=0

Figure 2: Market borrowing



Parameters: L=.5;T=3;y=1;alpha=.25;r=.1;b0=0

Figure 3: Costly international reserves



Parameters: L=.5;T=3;y=1;alpha=.25;r=0.10;b0=0;i=.03

1 Appendix

1.1 The constrained central planner's solution

As argued in the text, the constrained problem:

$$\underset{\{c_t\}_0^\infty}{Max} \int_o^\infty e^{-\delta s} u(c_s) ds \tag{P1}$$

$$\begin{aligned} \text{subject to } \dot{b}_t &= r b_t + c_t - y_t \ , \quad b_o \text{ given} \\ \text{and } b_t &\leq L \end{aligned}$$

can be restated as follows:

$$\begin{aligned} V &= \underset{\{c_t\}_0^T}{Max} \int_0^T e^{-\delta s} u(c_s) ds + e^{-\delta T} \frac{u\left(y - r\,\bar{b}_T\right)}{\delta} \\ \text{s.t. } \dot{b}_t &= r b_t + c_t - y_t \ , b_0 \text{ given} \\ b_t &\leq L \ \forall t. \end{aligned} \tag{24}$$

To solve it, write down the Lagrangian:

$$L = H_t + e^{-\delta t}\theta(b_t - L) = e^{-\delta t}\left[u(c_t) + \lambda_t\left(r b_t + c_t - y_t\right) + \theta(b_t - l)\right]. \tag{25}$$

The first order conditions (FOC) are as follows:

Optimization w.r.t. the control variable:

$$H_{c_t} = 0 \Longrightarrow u'(c_t) = -\lambda_t \tag{26}$$

complementary slackness:

$$L - b_t \geqq 0 \ , \ \theta \geqq 0 \ , \ \theta(L - b_t) = 0 \tag{27}$$

costate dynamics:

$$\frac{\dot{\lambda}}{\lambda} = (\delta - r) + \frac{\theta}{\lambda} r = \frac{\theta}{\lambda} r \quad \text{if} \quad \delta = r. \tag{28}$$

Therefore, when the economy is unconstrained, consumption is constant. If c is constant, then $\dot{b}_t = r b_t + c_0 - y$ can be integrated between 0 and X (the instant in which the debt ceiling is hit) as follows (note that X cannot be greater than T, because that would contradict the constant consumption path which is optimal from T onward):

$$b_X e^{-rX} - b_0 = \frac{(c_0 - y(1-\alpha))}{r}(1 - e^{-rX}) \tag{29}$$

and imposing that at X, $b_X = L$ one obtains the one-to-one relationship between the constant rate of consumption c and the instant in which the limit is hit, X:

$$c_0(x) = y(1-\alpha) + \frac{r\left(Le^{-rX} - b_0\right)}{(1 - e^{-rX})}. \tag{30}$$

Now, suppose X<T. Then, by the requirement that $\dot{b}_X \leq 0$, we have $c(x) \leq (y(1-\alpha) - rl)$. We can then argue that $b_T = L$ is optimal. This can be seen from the comparison with the central planner's solution. Compared to that case, here the economy enjoys higher consumption after T (since l is less then $\bar{b}_T$ because we are assuming that the ceiling is binding) and lower

consumption before T. Therefore the central planner would like to borrow as much as possible at T, implying $b_T = l$. But then the optimal X solves the following problem:

$$\begin{aligned} & \max_X \int_0^X e^{-\delta s} u(c_s) ds + \int_X^T e^{-\delta s} u(c_s) ds + V(L) \\ \text{subject to } \dot{b}_t &= r b_t + c_t - y_t, \; b_0 \; given \\ b_X &= L \\ b_T &= L. \end{aligned} \tag{31}$$

From the previous analysis, we can simplify the problem as follows:

$$\begin{aligned} & \max_X \frac{1}{\delta} u(c_0(X))(1 - e^{-\delta X}) + \int_X^T e^{-\delta s} u(c_s) ds + V(L) \\ \text{subject to } \dot{b}_t &= r b_t + c_t - y_t, \; b_0 \; given \\ b_X &= L \\ b_T &= L. \end{aligned} \tag{32}$$

Note that between X and T, the social planner would like to borrow more, rather than less, since she sees higher incomes in the future. But the constraint is binding: hence we have:

$$c_t = y(1-\alpha) - rl \quad \forall t \in [X, T]. \tag{33}$$

Finally, therefore, we can express the above problem as:

$$\max_{X \in [0,T]} \frac{1}{\delta} u(c_0(X))(1 - e^{-\delta X}) + \frac{1}{\delta} u\left(y(1-\alpha) - rl\right)(e^{-\delta X} - e^{-\delta T}) + V(L) \tag{34}$$

where as long as L>b_0, as we assume,

$$c_0(X) = y(1-\alpha) + r(Le^{-rX} - b_0)/(1 - e^{-rX}). \tag{35}$$

Therefore the problem has the same solution as:

$$\max_{X \in [0,T]} \frac{\left(1 - e^{-\delta X}\right)}{\left(1 - e^{-\delta T}\right)} \frac{1}{\delta} u(c_0(X)) + \frac{\left(e^{-\delta X} - e^{-\delta T}\right)}{\left(1 - e^{-\delta T}\right)} \frac{1}{\delta} u\left(y(1-\alpha) - rL\right) + V(L) = \tag{36}$$

$$= \nu(X) \frac{1}{\delta} u(c_0(X)) + [1 - \nu(X)] \frac{1}{\delta} u\left(y(1-\alpha) - rL\right) + V(L).$$

Since c(X)>(y(1–α)-rL), the optimum is to set $\nu = 1 \Rightarrow x^* = T$. Therefore the *central planner's solution* is:

$$\begin{aligned} c_t &= y(1-\alpha) + r \frac{\left(Le^{-rT} - b_0\right)}{1 - e^{-rT}} \equiv c_0^P \text{ for } \quad 0 \leq t < T \\ &= (y - rL) \equiv c_T^P \qquad \text{for t} \geq T \end{aligned} \tag{37}$$

with:

$$\begin{aligned} c^* - c_0^P &= y(1-\alpha) + y\alpha e^{-rT} - rb_0 - y(1-\alpha) - r \frac{(Le^{-rT} - b_0)}{1 - e^{-rT}} = \\ &= \frac{e^{-rT}}{1 - e^{-rT}} (\alpha y (1 - e^{-rT}) - (L - b_0)) > 0 \end{aligned}$$

by the assumption on $(L - b_0)$ which ensures that the limit is binding. It is easy to check that, under the same assumption:

$$c_T^P - c^* > 0 \tag{38}$$

$$b_t^P - b_t^* < 0. \tag{39}$$

1.2 The constrained decentralized economy

1.2.1 The equilibrium consumption path

First, notice that in any equilibrium, the instants of time in which the economy is constrained is a set of the type I=$[\tau, \infty]$. That is, in equilibrium, it cannot be the case that the economy first hits the ceiling, then lowers its foreign debt so that there is τ' and τ'' (with $\tau' > \tau''$) such that $b_{\tau'} < b_{\tau''}(= L)$ because the structure of the endowment is such that it is individually optimal always to increase debt, whenever feasible, rather then decreasing it.

The problem then is the following:

$$\underset{\{c_t\}_0^\infty}{Max} \int_0^\infty e^{-\delta s} u(c_s) ds \tag{P2}$$

$$\begin{aligned} \text{subject to } \dot{b}_t &= rb_t + c_t - y_t,\ b_\tau \textit{ given} \\ \dot{b}_t &\leq 0 \text{ if } b_t = b_\tau. \end{aligned}$$

As argued in the test, this problem is equivalent to:

$$\underset{\{c_t\}_0^\tau}{Max} \int_0^\tau e^{-\delta s} u(c_s) ds + V(b_\tau, \tau)$$

$$\begin{aligned} \text{subject to } \dot{b}_t &= rb_t + c_t - y_t,\ b_0 \textit{ given} \\ & \quad b_\tau \text{ free,} \end{aligned} \tag{40}$$

where:

$$V(b_\tau, \tau) = \frac{1}{\delta} u\left(y(1-\alpha) - rb_\tau\right)\left(e^{-\delta\tau} - e^{-\delta T}\right) + \frac{1}{\delta} u\left(y - rb_\tau\right) e^{-\delta T}. \tag{41}$$

The Hamiltonian for this problem is:

$$H_t = e^{-\delta t}\left[u(c_t) + \lambda_t\left(rb_t + c_t - y_t\right)\right]. \tag{42}$$

Consumption is constant between 0 and τ because of the equality between r and δ. For the choice of b_τ the relevant condition is:

$$-\lambda_\tau + \frac{\partial V}{\partial b_\tau} = 0$$

$$\Rightarrow e^{-\delta t} u'(c_0(\tau, b_\tau)) - u'(y(1-\alpha) - rb_\tau)(e^{-\delta\tau} - e^{-\delta T}) - u'(y - rb_\tau)e^{-\delta T} = 0 \tag{43}$$

where, analogously to eq.(35),

$$c(b, \tau) = y(1-\alpha) + r(b_\tau e^{-r\tau} - b_0)/(1 - e^{-r\tau}). \tag{44}$$

Imposing the equilibrium condition $b_{\tau^*} = L$ determines τ^*:

$$u'(c_0(\tau^*, L)) = u'(y(1-\alpha) - rL)(1 - e^{-\delta(T-\tau^*)}) + u'(y - rL)(e^{-\delta(T-\tau^*)}) \tag{45}$$

Using the implicit function theorem, one can check in eq.(43) that $\frac{\partial b_\tau}{\partial \tau} > 0$. Therefore the equilibrium τ^* is unique. Note also that from eq.(45) we can immediately argue that the solution to the decentralized economy's problem will be different from the central planner's. For, suppose to the contrary that $b_{\tau^*} = L$ and $\tau^* = T$ (these values are the equilibrium ones for the centrally planned economy). Then $c_0 = (y - rl)$, but this is too high: with this level of consumption, $b_T > L$, contrarily to the assumption.

Another way to make the same point is to consider that if people expect the constraint to be hit at T, then they really don't feel any constraint (after all, even in the unconstrained setting they increased their borrowing only up to T). But then they would like to borrow the unconstrained first-best level, which is by assumption not feasible. The limit has to be hit before T.

1.2.2 The equilibrium interest rates

To obtain the results in Remark 1 in the text, we use the knowledge of the proposed consumption path in equilibrium to compute the interest rates that will support it. Let $i_t(m)$ be the equilibrium domestic interest rate for a loan extended at time t with maturity m. Let $c_0^{'}, c_\tau^{'}, c_T^{'}$ be the marginal utility of consumption in the three equilibrium phases, i.e., for consumption levels c_0^M, c_τ^M, *and* c_T^M, respectively. Then, $c_T^{'} < c_0^{'} < c_\tau^{'}$. The general strategy consists of imposing indifference at the margin between borrowing one unit of the good at time t with the obligation to pay an interest rate of $i_t(m)$ up to (t+m), when the principal is repaid. For example, consider a loan issued at time t ($< \tau^*$) to be fully repaid at time t+m (with $\tau^* < t+m < T$): in the terminology of Remark 1, we are looking at a loan issued in phase 1 to be fully repaid in phase 2. In this case we impose:

$$e^{-rt}u^{'}(c_t) = \int_t^{\tau+m} i_t(m)e^{-rs}u^{'}(c_s)ds + e^{-r(t+m)}u^{'}(c_{t+m}). \tag{46}$$

Using the notation introduced above, and the results from the previous subsection:

$$e^{-rt}c_0^{'} = \frac{i_t(m)}{r}\left[c_0^{'}(e^{-rt} - e^{-r\tau}) + c_\tau^{'}(e^{-r\tau} - e^{-r(t+m)})\right] + e^{-r(t+m)}c_\tau^{'}$$

$$\Rightarrow \frac{i_t(m)}{r} = \frac{c_0^{'} - e^{-rm}c_\tau^{'}}{\left[c_0^{'}(1 - e^{-r(\tau^*-t)}) + c_\tau^{'}(e^{-r(\tau^*-t)} - e^{-rm})\right]} \tag{47}$$

as claimed in the text. In an analogous way, one can obtain the result in part sub b) of Remark 1. Let us now look at the trade-off in the decision to issue a loan in phase 1 to be fully repaid in phase 3. For indifference we impose the same condition as above, which in this case specializes into:

$$\begin{aligned} e^{-rt}c_0^{'} &= \frac{i_t(m)}{r}\left[c_0^{'}(e^{-rt} - e^{-r\tau}) + c_\tau^{'}(e^{-r\tau} - e^{-rT}) + c_T^{'}(e^{-rT} - e^{-r(t+m)})\right] \\ &\quad + e^{-r(t+m)}c_T^{'} \end{aligned} \tag{48}$$

from which it can be seen that $i_t(m) = r$ if:

$$e^{-r\tau}c_0^{'} = c_\tau^{'}(e^{-r\tau} - e^{-rT}) + c_T^{'}(e^{-rT} - e^{-r(t+m)}) + e^{-r(t+m)}c_T^{'} \tag{49}$$

or, equivalently,

$$(c_\tau^{'} - c_0^{'})e^{-r\tau} = (c_\tau^{'} - c_T^{'})e^{-rT} \tag{50}$$

which can be readily checked to hold, from the equation defining τ^* (eq.(45) above).

1.3 Optimal taxes

In this section, we show how to construct optimal tax schedules, that is tax schedules which support the second best allocation as a market equilibrium. As pointed out in the text, the necessary condition that has to be satisfied so as to ensure that a constant consumption level is optimal for the individual agent is equation (19), which is reported below for convenience:

$$(1 + x_c)(rx_b + rx_b - \dot{x}_b) = \dot{x}_c\,(1 - x_b) \tag{51}$$

It is evident that the only candidate tax bases are consumption (in the form of a constant tax, $x_c(t) = x$ for t<T and $x_c(t) = 0$ for $t \geqq T$) and capital inflows (with an exponential tax schedule: $x_b(t) = f \exp rt$). We analize both cases in turn.

1.3.1 Consumption taxes

With consumption as a tax base, the relevant flow budget constraint is:

$$\dot{b}_t = rb_t + (1 + x_c(t))c_t - y_t - s_t.$$

A constant consumption and a constant tax rate x imply that:

$$b(t)\exp(-rt) - b_0 = \frac{1}{r}(1-\exp(-rt))\left[(1+x)\,c - y(1-\alpha)\right] - \int_0^t s(z)dz \tag{52}$$

which, in turn, implies the link between the constant level of consumption and the amount of debt at T that the consumer needs to support that consumption path:

$$c(b_T) = \frac{1}{(1+x)}\left\{y(1-\alpha) + \frac{r}{(1-\exp(-rT)}\left[(b_T\exp(-rT) - b_0) + \int_0^t s(z)dz\right]\right\} \tag{53}$$

(Note that the path for subsidies is taken as given by the agent). Now, when choosing the optimal amount of debt, knowing that access will be lost at T, the agent maximizes the indirect utility:

$$U(b_T) = \frac{[1-\exp(-rT)]}{r}u(c(b_T)) + \frac{\exp(-rT)}{r}u(y - rb_T) \tag{54}$$

Taking the derivative and imposing it to equal zero when $b_T = L$, we get:

$$\frac{\partial U(b_T)}{\partial b_T} = 0 \Rightarrow \frac{1}{(1+x)}u'(c_0^P)\exp(-rT) = u'(c_T^P)\exp(-rT) \tag{55}$$

that is,

$$x^* = \frac{u'(c_0^P)}{u'(c_T^P)} - 1 \tag{56}$$

is the optimal consumption tax rate. (Since $c_0^P < c_T^P$, x^* is positive. Note also that in eq. (52), after substituting the equilibrium values for the rebates, b=L implies that $c=c_0^P$).

1.3.2 Taxes on inflows

With a tax on inflows the relevant budget constraint becomes:

$$(1 - x_{\dot{b}}(t))\,\dot{b} = rb_t + c_t - y_t - s_t \tag{57}$$

Imposing constant income and consumption and the hypotesized structure for the tax schedule ($x_{\dot{b}}(t) = f\exp rt$), by virtue of integration one gets:

$$\begin{aligned}
\left(\dot{b} - rb_t - f\exp(rt)\,\dot{b}\right)\exp(-rt) &= [(c-y) - s_t]\exp(-rt) \\
\frac{\partial}{\partial t}(b\exp(-rt)) - \dot{b}\,f &= [(c-y) - s_t]\exp(-rt) \\
b_T\exp(-rT) - b_0 - f(b_T - b_0) &= \frac{1}{r}(1-\exp(-rT))(c-y) - \int_0^T s(z)dz
\end{aligned}$$

which is the analogue of eq.(52). Following the same steps as above, the first order condition for the optimal choice of b_T now reads:

$$\frac{u'(c_0)}{u'(c_T)} = \frac{1}{1 - f\exp(rT)} \tag{58}$$

which tells us that by increasing f, we can increase the optimal difference between c_0 and c_T. To ensure that $c_0 = c_0^P$, so that the optimal demand for debt at T equals the ceiling L, we can use:

$$f^* = (1 - \frac{u'(c_T^P)}{u'(c_0^P)})\exp(-rT) \tag{59}$$

Again, we can notice that, since $c_0^P < c_T^P$, f^* is positive. Note also the relation $f^* = \frac{x^*}{1+x^*}\exp(-rT)$.

1.4 Proof of remark 4

First, unbounded demand at time zero is a market equilibrium, because it implies that the debt market will never reopen and, if so, each market agent should attempt to secure as much excess external finance as possible. Second, this market equilibrium is unique. To see this, consider by contradiction that there is an equilibrium in which debt markets remain open after t=0. Then agents who at time 0 did not secure enough debt to sustain the consumption level c^* (which exist because otherwise the credit ceiling would have been exceeded) would benefit from securing such amount, which upsets the candidate equilibrium.

1.5 The decentralized economy with the carrying cost of reserves

In this paragraph, we will limit ourselves to indicating the necessary steps to a complete solution of the decentralized economy model in the presence of a spread.[38]

The consumer takes τ as given and solves:

$$\max_{\{c\}_0^\infty,\{b\}_0^\infty,\{R\}_0^\infty} \int_0^\infty e^{-\delta t} u(c_t)dt$$

$$\begin{aligned} s.t.\ \dot{b}_t - \dot{R}_t &= rb_t + c_t - iR_t - y_t \\ 0 &\leqq b_t \leqq b_\tau \text{ , for all t} \\ R_t &\geqq 0 \quad \text{for all } t. \end{aligned} \tag{60}$$

As argued in the text, the observations that allow to characterize the market equilibrium are as follows. In equilibrium:

1) No foreign asset is hold before access is lost: all the reserves (if any) are demanded instantaneously at τ.

2) The path for consumption cannot be discontinuous. In particular it cannot have any downward jumps at τ, because if it did, consumers could increase their utility by consuming a little less just before τ and investing a little bit more abroad, so as to better smooth consumption over time.

3) Consumption is constant before access is lost, because $\delta = r$. Also, consumption is constant from T onward at the level (y-rL), where L is the per capita credit ceiling.[39]

Taking τ as given and taking b_τ and R (the amount of debt and of assets on foreign residents) as already determined, and using the observations in the main text, it is clear that the path of consumption between τ and T is determined by solving:

$$\max_{\{c\}_\tau^T} \int_\tau^T e^{-\delta(t-\tau)} u(c_t)dt$$

$$\begin{aligned} s.t.\ \dot{R}_t &= iR_t + y - rb_\tau - c_t \text{ , } R_\tau = R \\ R_t &\geqslant 0 \quad \text{for all } \tau < t < T. \end{aligned} \tag{61}$$

[38] The model can be solved numerically in a relatively easy way if one assumes, for example, the CRRA functional form for the utility function:

$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$.

Also, assuming the presence of zero-coupon bonds greatly facilitates the analysis, because, as explained in the text, it implies that the constant level of consumption before the ceiling is hit equals the level of consumption after income recovers, at T in our model.

[39] Furthermore, zero-coupon bonds guarantee that consumption is initially constant at this same level.

This problem of course has as solution a decreasing path for consumption, since the interest rate is less than the time preference factor.

From this we get the consumption in phase 2, call it c_{2t}, as a function of R and b_τ and τ (and of course t):

$$c_{2t} = f_2(R, b_\tau, \tau, t). \tag{62}$$

Going back to phase 1, we know that as long as the economy is unconstrained, the interest rate is r and therefore consumption is constant. The level of consumption, which we will call c_1, is equal to the consumption after the income recovers:

$$c_1 = y - rL \tag{63}$$

Imposing the continuity of the consumption path we get an expression for consumption after access is lost:

$$c(t) = c_1 e^{-(r-i)(t-\tau)}, \tau \leqslant t \leqslant T \tag{64}$$

So now we can compute, as a function of τ, the amount of reserves which are necessary to support the consumption path between τ and T (call this amount $R(\tau)$). Then one can determine the equilibrium τ^* by imposing that the accumulated debt up to τ^* (to finance a constant consumption of y-rL) plus the demand for reserves at τ^*, $R(\tau^*)$, be equal to the present value as of τ^* of the credit ceiling, that is: we get a relationship between R and b_τ and τ:

$$b(\tau^*) + R(\tau^*) = Le^{-r(T-\tau^*)} \tag{65}$$

(The above assumes the existence of zero-coupon bonds redeemable at T, which will be demanded in equilibrium as they don't require the payment of interests when income is particularly low, i.e. between τ^* and T).

In figure 3 we report the results of the numerical solution of the model with costly international reserves under the simplifying assumption of logarithmic utility and existence of zero coupon bond debt instruments on the world market. The parameters used for this simulation are: L=.5; T=3; y=1; alpha=.25; r=0.10; i=.03; b0=0. The utility levels attained are Um=-38.7790 for the decentralized economy and Ucp=-36.2259 for the central planner's solution.