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Collateral requirements and asset prices

Johannes Brumm
(University of Zurich)

Felix Kubler
(University of Zurich and Swiss Finance Institute)

Michael Grill
(Deutsche Bundesbank)

Karl Schmedders
(University of Zurich and Swiss Finance Institute)

Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

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Non-technical summary

Many financial securities derive their value not only from future cash flows but also from their ability to serve as collateral. This second source of value varies with macroeconomic condi-tions. In this paper, we investigate this collateral premium and its impact on security returns.

We examine a model with two agents facing collateral constraints for borrowing. The agents can borrow against positions in assets which differ only by their "collateralizability". The collateralizability of an asset determines the fraction of the asset that can be confiscated in case of default and depends on its physical and legal properties. We document that borrowing against collateral contributes substantially to the return volatility of the assets. In our calibration of the model there are two types of agents who differ with respect to their risk aversion. The agent with the low risk aversion is the natural buyer of risky assets, and leverages to finance these investments. The agent with the high risk aversion has a strong desire to insure against bad shocks and thus is a natural buyer of safe bonds. When the economy is hit by a negative shock, the collateral constraint forces the leveraged agent to reduce consumption and to sell risky assets to the risk-averse agent, leading to substantial changes in the wealth distribution, which in turn affect agents' portfolios and asset prices.

We further show that assets with different degrees of collateralizability which are otherwise identical exhibit substantially different return dynamics. In particular, the more collateralizable asset has both a smaller excess return and a smaller return volatility. However, the main economic mechanism leading to this result is straightforward; in response to negative shocks, the less risk-averse agent, holding both infinitely-lived assets and a large negative bond position against the collateralizable asset, must deleverage. She first sells the less col-lateralizable asset since it does not provide collateral value to her. In contrast, the less risk-averse agent holds on to the collateralizable asset as long as possible which leads only to a small drop in its price. As a consequence of this trading pattern, the less collateralizable asset has both a higher excess return and a higher return volatility than the more collateralizable asset.

Finally, we document that the prices of collateralizable assets contain a sizable collateral premium which depends strongly on the difference in the collateralizability between the assets and much less so on an asset's expected future cash flows. In fact, assets that never pay dividends can still have a positive price in equilibrium if they are much more collateralizable than other assets in the economy.

Nicht-technische Zusammenfassung

Der Wert vieler Finanzinstrumente hängt nicht nur von ihren künftigen Zahlungen ab, sondern auch davon, ob sie als Sicherheit verwendet werden können. Dieser zweite wertbestimmende Faktor variiert je nach gesamtwirtschaftlicher Entwicklung. In der vorliegenden Arbeit werden die Sicherheitenprämie und deren Auswirkungen auf die Renditen von Wertpapieren analysiert.

Hierbei wird ein Modell mit zwei Akteuren, die Kreditbeschränkungen unterliegen, zugrunde gelegt. Die Akteure können Kredite gegen Hinterlegung von Vermögenswerten aufnehmen, die sich lediglich durch ihre "Besicherungsfähigkeit" unterscheiden. Die Besicherungsfähigkeit eines Vermögenswerts gibt an, welcher Anteil des Vermögenswerts im Falle einer Insolvenz verwertet werden kann, und hängt von dessen physischen und rechtlichen Eigenschaften ab. Die Autoren kommen zu dem Ergebnis, dass eine besicherte Kreditaufnahme erheblich zur Renditevolatilität der Aktiva beiträgt. Zur der Kalibrierung des Modells werden zwei Akteure herangezogen, die sich im Hinblick auf ihre Risikoscheu unterscheiden. Bei dem Akteur mit geringerer Risikoscheu handelt es sich um einen typischen Käufer risikoreicher Aktiva, der sich verschuldet, um seine Investitionen zu finanzieren. Der Akteur mit höherer Risikoscheu ist bestrebt, sich gegen negative Schocks abzusichern, und erwirbt daher üblicherweise sichere Anleihen. Wird die Wirtschaft von einem negativen Schock getroffen, führen sicherheitenbezogene Kreditbeschränkungen dazu, dass der verschuldete Akteur seinen Konsum verringern und risikoreiche Vermögenswerte an den risikoscheueren Akteur veräußern muss. Hierdurch kommt es zu einer erheblichen Veränderung der Vermögensverteilung, was sich wiederum auf die Portfolios der Akteure und die Vermögenspreise auswirkt.

In der vorliegenden Arbeit wird darüber hinaus gezeigt, dass sich die Renditen unterschiedlich besicherungsfähiger, aber ansonsten gleicher Aktiva sehr verschieden entwickeln. So erzielen Aktiva mit höherer Besicherungsfähigkeit eine niedrigere Rendite, wobei deren Rendite aber auch weniger volatil ist. Diesem Ergebnis liegt ein einfacher wirtschaftlicher Mechanismus zugrunde: Als Reaktion auf einen negativen Schock muss der risikofreudigere Akteur, der sowohl Aktiva mit unbestimmter Laufzeit hält als auch umfangreiche, durch besicherungsfähige Vermögenswerte abgesicherte Kredite aufgenommen hat, seinen Fremdkapitalanteil abbauen. Er veräußert zunächst den Vermögenswert, der über geringeren Sicherheitenwert verfügt. Dagegen hält er den besicherungsfähigen Vermögenswert so lange wie möglich in seinem Portfolio, wodurch dessen Preis nur geringfügig zurückgeht. Im Vergleich zum besicherungsfähigeren Vermögenswert zeichnet sich die weniger besicherungsfähige Position daher sowohl durch eine höhere Rendite als auch durch eine stärkere Renditevolatilität aus.

Abschließend wird aufgezeigt, dass die Preise besicherungsfähiger Aktiva eine erhebliche Sicherheitenprämie enthalten, die im Wesentlichen von der jeweiligen Besicherungsfähigkeit der einzelnen Aktiva und deutlich weniger von den erwarteten künftigen Zahlungen abhängt. So können Aktiva, bei denen keine Dividende ausgeschüttet wird, trotzdem einen positiven Gleichgewichtspreis aufweisen, wenn sie sich viel besser als Sicherheiten verwenden lassen als andere Vermögenswerte.

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Johannes Brumm
University of Zurich

Michael Grill
Deutsche Bundesbank

Felix Kubler
University of Zurich
and Swiss Finance Institute

Karl Schmedders
University of Zurich
and Swiss Finance Institute

Abstract

Many assets derive their value not only from future cash flows but also from their ability to serve as collateral. In this paper, we investigate this collateral value and its impact on asset returns in an infinite-horizon general equilibrium model with heterogeneous agents facing collateral constraints for borrowing. We document that borrowing against collateral substantially increases the return volatility of long-lived assets. Moreover, otherwise identical assets with different degrees of collateralizability exhibit substantially different return dynamics because their prices contain a sizable collateral premium that varies over time. This premium can be positive even for assets that never pay dividends.

Keywords: Collateral constraints, collateral premium, endogenous margins, heterogeneous agents, leverage.

JEL Classification: D53, G11, G12.

*Johannes Brumm, University of Zurich, johannes.brumm@bf.uzh.ch; Michael Grill, Deutsche Bundesbank, michael.grill@bundesbank.de; Felix Kubler, University of Zurich, kubler@isb.uzh.ch; Karl Schmedders, University of Zurich, karl.schmedders@business.uzh.ch. We thank seminar audiences at various universities and conferences for their helpful comments. We are indebted to Klaus Adam, Peter Ove Christensen, Dirk Krueger, David Longworth, Jean-Charles Rochet, and Dimitri Vayanos for helpful comments. We are particularly grateful to our discussants Francisco Gomes, Marcel Rindisbacher, Ctirad Slavik, Cédric Tille, and Andrea Vedolin for detailed feedback on earlier versions of the paper. Felix Kubler and Karl Schmedders gratefully acknowledge financial support from the Swiss Finance Institute and NCCR-FINRISK. Johannes Brumm and Felix Kubler acknowledge support from the ERC. The views expressed in this paper are those of the authors and do not necessarily reflect the official position of the Deutsche Bundesbank.

1 Introduction

Many assets derive their value not only from future cash flows but also from their ability to serve as collateral. This second source of value varies with macroeconomic conditions. In this paper, we investigate this collateral premium and its impact on asset returns. We examine an infinite-horizon general equilibrium model with two heterogeneous agents facing collateral constraints for borrowing. The agents can borrow against positions in two infinitely-lived assets ("Lucas trees") which differ only by their "collateralizability". The collateralizability of an asset determines the fraction of the asset that can be confiscated in case of default and depends on its physical and legal properties. We document that borrowing against collateral contributes substantially to the return volatility of the two long-lived assets. In addition, we show that otherwise identical assets with different degrees of collateralizability exhibit substantially different return dynamics. In particular, the more collateralizable asset has both a smaller excess return and a smaller return volatility. Furthermore, the prices of collateralizable assets contain a sizable collateral premium which depends strongly on the difference in the collateralizability between the assets and not only on an asset's expected future cash flows. In fact, assets that never pay dividends can still have a positive price in equilibrium if they are much more collateralizable than other assets in the economy.

In our study, we assume that agents can default on debt at any time without any utility penalties or loss of reputation. Therefore, financial securities in zero net supply such as bonds are only traded if the promises associated with a sale of these securities are backed by collateral. Put differently, agents can only borrow, i.e. take short positions in bonds, if they hold an infinitely-lived asset (a Lucas tree) as collateral. An asset that can serve as collateral in the economy is not only characterized by its future (risky) dividend stream but also by the fraction of the asset that can be confiscated in case of default, the asset's collateralizability. Our analysis is centered on two infinitely-lived assets that differ in their collateralizability but have identical dividend streams. The model feature of differing degrees of collateralizability is motivated by the observation that some assets, e.g. houses, can be used as collateral very easily for loans with comparatively low interest rates, while other assets, e.g. stocks, can only be used as collateral for loans with high margin requirements and typically much higher interest rates; see Willen and Kubler (2006). Collateralizability depends on property rights, regulations, and also on how tangible the assets are. We document that exogenous differences in collateralizability lead to different endogenous margin requirements for borrowing against these assets, that is, how much agents can borrow against such risky assets. Following Geanakoplos (1997) we endogenize the margin requirements by introducing a menu of financial securities. All securities promise the same payoff but differ in their respective margin requirement. In equilibrium, only some of them are traded, thereby determining an endogenous margin requirement. This setup implies that for many bonds and many shocks, the face value of the debt falls below the value of the collateral. As a result, there is default in equilibrium. We assume that default is costly by introducing a real cost to the lender. In our calibration of the model, trade in defaultable bonds ceases to exist with moderate default costs.

We assume that there are two types of heterogeneous agents with Epstein-Zin utility. They have identical elasticities of substitution (IES) but differ with respect to their risk aversion (RA). The agent with the low risk aversion is the natural buyer of risky assets and leverages to finance these investments. The agent with the high risk aversion has a strong desire to insure against bad shocks and thus is a natural buyer of safe bonds. When the economy is hit by a negative shock, the collateral constraint forces the leveraged agent to reduce consumption and to sell risky assets to the risk-averse agent, leading to substantial changes in the wealth distribution, which in turn affect agents' portfolios and asset prices. In a standard calibration with normal business-cycle-sized shocks, the presence of collateral constraints has only small effects on asset volatility. To obtain a sizable market price of risk, we follow the specification in Barro and Ursua (2008) and introduce the possibility of 'disaster shocks'. In the presence of such shocks, introducing collateral constraints raises the volatility of asset prices by more than fifty percent and leads to a realistic market price of risk.

In the first step of our analysis, we examine a specification of the model in which only the first infinitely-lived asset can be used as collateral. Perhaps somewhat surprisingly at first, both the average excess return and the return volatility of the collateralizable asset are significantly smaller than the corresponding figures for the second infinitely-lived asset, which cannot be used as collateral. However, the main economic mechanism leading to this result is straightforward; in response to a disaster shock or several successive recession shocks, the less risk-averse agent, holding both infinitely-lived assets and much debt against the collateralizable asset, must de-leverage. She first sells the second asset since it does not provide collateral value to her. To induce the more risk-averse agent to buy this risky asset, it must offer a high risk premium, and thus its price must fall. In contrast, the less risk-averse agent holds on to the collateralizable asset as long as possible, which leads only to a small drop in its price. As a consequence of this trading pattern, the non-collateralizable asset has both a higher excess return and a higher return volatility than the collateralizable asset. This effect is amplified by the fact that in bad times the endogenously determined margin requirement for the first asset rises, forcing the less risk-averse agent to sell even more of the second asset.

In the second step of our analysis, we vary the collateralizability of the second infinitely-lived asset while maintaining the full collateralizability of the first asset. As the degree of collateralizability of the second asset increases, both its excess return and its return volatility relative to the first asset decrease. In essence, our analysis shows that a different degree of collateralizability among different long-lived assets contributes substantially to the differences in their return dynamics.

In the third and final part of the analysis, we show that a collateralizable asset can have a positive price even if it pays no dividends at all. Such an equilibrium exists as long as the other asset's collateralizability is limited—in our baseline calibration, as long as it is below fifty percent. In this case the positive collateral premium on the zero-dividend asset can be interpreted as a "bubble"; see, for example, Kocherlakota (1992, 2009) and Fostel and Geanakoplos (2012) for an interpretation along these lines. In our economy with two infinitely-lived assets, the presence of a bubble in the price of the zero-dividend asset leads to high return volatility of the other infinitely-lived asset, which pays dividends but is less collateralizable. If the bubble bursts, the price of this other asset drops discontinuously, but then recovers quickly and exhibits substantially lower volatility. In this sense the presence of bubbles leads to excess volatility in the prices of all assets.

There is a quickly growing literature in economics investigating the effects of collateralized borrowing on asset prices and economic activity. Numerous papers in this literature have formalized the idea that borrowing against collateral may give rise to cyclical fluctuations in real activity and can lead to increased volatility of asset prices. Among many others, Kiyotaki and Moore (1997) and more recently Brunnermeier and Sannikov (2013) consider production economies and show that collateral constraints can lead to an amplification of shocks and volatile real activity. Brunnermeier and Sannikov (2013) also explain that large economic shocks are necessary for collateral constraints to have a quantitatively nontrivial impact on asset price volatility. As a result, substantial deviations from the steady state occur in equilibrium and require global solution methods for the analysis of equilibria. They also confirm previous results from the literature that, in the absence of large shocks, establishing the quantitative importance of collateral requirements as a source of volatility has been a challenge; see, for example, Kocherlakota (2000) or Cordoba and Ripoll (2004).

The focus of this paper follows on from the work of Geanakoplos (1997), Aiyagari and Gertler (1999), Coen-Pirani (2005) and Rytchkov (2013), all of whom also concentrate on the effects of collateral requirements on asset price volatility. As in our paper, in the models of Geanakoplos and Aiyagari-Gertler asset prices may deviate substantially from the corresponding prices in economies with frictionless markets. However, these early studies largely focus on rather stylized examples. The results in our paper are in stark contrast to the findings of Coen-Pirani (2005) and of Rytchkov (2013). Both authors examine a setup that is similar to ours but both make crucial simplifying assumptions to solve the model. These simplifications lead to results that are opposite to those reported in this paper. Coen-Pirani (2005) also considers a discrete time Lucas style model with Epstein-Zin agents that differ in risk aversion but have identical IES. By further assuming that the common IES is equal to one and that all income stems from dividend payments only, he can show analytically that collateral constraints have no effect on stock return volatility. We find that this result changes dramatically for economies in which labor income finances a large part of aggregate consumption. In our economy, collateral constraints substantially increase return volatility even if the common IES is equal to one. Rytchkov (2013) considers a continuous-time model where two agents maximize expected CRRA utility and differ in their levels of risk aversion (and hence their IES). As in Coen-Pirani (2005), all consumption stems from dividend payments. Collateral requirements force the less risk-averse agent to hold less of the stock than he would otherwise and typically lead to a reduction of stock return volatility. Again, we show an opposite result for economics with substantial labor income.

It is theoretically well understood that margin requirements have important effects on an asset's expected returns; see, for example, Garleanu and Pedersen (2011) or to some extent already Detemple and Murthy (1997). Garleanu and Pedersen show that margin requirements are important determinants of expected returns and provide strong empirical evidence for the role of margin requirements as a determinant of expected returns. An important difference between Garleanu's and Pedersen's work and our paper is that they impose exogenous margin requirements while in our economy these are determined in equilibrium.

Other related works include Gromb and Vayanos (2002) on margin constraints and segmented markets, Brunnermeier and Pedersen (2009) on the amplification of shocks due to value-at-risk constraints, Chabakauri (2013) on equilibrium with two trees, margin and leverage constraints but no labor income, and Simsek (2013) on equilibrium with endogenous margins and heterogeneous beliefs.

To the best of our knowledge, there have been no quantitative studies of economic models in which households have the option of choosing from several different collateralizable assets; until now no study has examined how borrowing against one collateralizable asset affects the volatility of other collateralizable long-lived assets in the economy. The purpose of the present paper is to provide such a quantitative study. Assessing the quantitative impact of collateral constraints on asset returns also enables us to provide additional qualitative insights into the economic mechanisms resulting from collateralized borrowing. Similar to Brunnermeier and Sannikov (2013), we need sizable shocks to generate significant movements in the equilibrium wealth distribution and thereby volatile asset price dynamics.

The remainder of this paper is organized as follows. We introduce the economic model and its parameterization in Section 2. In Section 3 we discuss the relationship between collateralizability and asset prices and explain the central economic mechanism of the economic model. Section 4 focuses on the collateral premium and the case of a collateralizable asset with zero dividends but positive price. Section 5 concludes and discusses avenues for further research. In the Appendix we provide a sensitivity analysis.

2 The Economy

We examine an infinite-horizon exchange economy with infinitely-lived heterogeneous agents, long-lived assets and collateral constraints for short-term borrowing.[1] Section 2.1 describes the economic model and Section 2.2 discusses the calibration of our baseline economy.

2.1 The Model

Time is indexed by $t = 0, 1, 2, \ldots$. A time-homogeneous Markov chain of exogenous shocks (s_t) takes values in the finite set $\mathcal{S} = \{1, \ldots, S\}$. The $S \times S$ Markov transition matrix is denoted by π. We represent the evolution of time and shocks in the economy by a countably infinite event tree Σ. The root node of the tree represents the initial shock s_0. Each node of the tree, $\sigma \in \Sigma$, describes a finite history of shocks $\sigma = s^t = (s_0, s_1, \ldots, s_t)$ and is also called date-event. We use the symbols σ and s^t interchangeably. To indicate that $s^{t'}$ is a successor of s^t we write $s^{t'} \succ s^t$. We use the notation s^{-1} to refer to the initial conditions of the economy prior to $t = 0$.

At each date-event $\sigma \in \Sigma$ there is a single perishable consumption good. The economy is populated by H agents, $h \in \mathcal{H} = \{1, 2, \ldots, H\}$. Agent h receives an individual endowment in the consumption good, $e^h(\sigma) > 0$, at each node. In addition, at $t = 0$ the agent owns shares in long-lived assets ("Lucas trees"). We interpret these Lucas trees to be physical assets such as firms, machines, land or houses. There are A different such assets, $a \in \mathcal{A} = \{1, 2, \ldots, A\}$. At the beginning of period 0, each agent h owns initial holdings $\theta^h_a(s^{-1}) \geq 0$ of tree a. We normalize aggregate holdings in each Lucas tree, that is, $\sum_{h \in \mathcal{H}} \theta^h_a(s^{-1}) = 1$ for all $a \in \mathcal{A}$. At date-event σ, we denote agent h's (end-of-period)

[1]Simpler versions of this model have been examined by Kubler and Schmedders (2003) and Cao (2013).

holding of Lucas tree a by $\theta_a^h(\sigma)$ and the entire portfolio of tree holdings by the A-vector $\theta^h(\sigma)$.

The Lucas trees pay dividends $d_a(\sigma) \geq 0$ in units of the consumption good at all date-events. We denote aggregate endowments in the economy by

$$\bar{e}(\sigma) = \sum_{h \in \mathcal{H}} e^h(\sigma) + \sum_{a \in \mathcal{A}} d_a(\sigma).$$

Each agent has preferences over infinite consumption streams that can be represented by a recursive utility function, U^h, as in Epstein and Zin (1989),

$$U^h(c, s^t) = \left\{ \left[c^h(s^t)\right]^{\rho^h} + \beta \left[\sum_{s_{t+1}} \pi(s_{t+1}|s_t) \left(U^h(c, s^{t+1})\right)^{\alpha^h} \right]^{\frac{\rho^h}{\alpha^h}} \right\}^{\frac{1}{\rho^h}},$$

where $\frac{1}{1-\rho^h}$ is the intertemporal elasticity of substitution (IES) and $1 - \alpha^h$ is the relative risk aversion of the agent.

At each date-event, agents can engage in security trading. Agent h can buy $\theta_a^h(\sigma) \geq 0$ shares of tree a at node σ for a price $q_a(\sigma)$. Agents cannot assume short positions of the Lucas trees. Therefore, the agents make no promises of future payments when they trade shares of physical assets and thus there is no possibility of default when it comes to such positions.

In addition to the physical assets, there are J one-period financial securities, $j \in \mathcal{J} = \{1, 2, \ldots, J\}$, available for trade. We denote agent h's (end-of-period) portfolio of financial securities at date-event σ by the vector $\phi^h(\sigma) \in \mathbb{R}^J$ and denote the price of security j at this date-event by $p_j(\sigma)$. These assets are all one-period bonds in zero-net supply; they promise one unit of the consumption good in the subsequent period. Whenever an agent assumes a short position in a financial security j, $\phi_j^h(\sigma) < 0$, she promises a payment in the next period. Such promises must be backed by collateral.

2.1.1 Collateral and Default

At each node σ, we associate with each financial security $j \in \mathcal{J}$ a tree $a(j) \in \mathcal{A}$ and a collateral requirement $k_{a(j)}^j(\sigma) > 0$. If an agent sells short one unit of security j, then she is required to hold $k_{a(j)}^j(\sigma)$ units of tree $a(j)$ as collateral. If an asset a can be used as collateral for different financial securities, the agent is required to buy $k_{a(j)}^j(\sigma)$ shares (per unit) of each security $j \in \mathcal{J}_a$, where $\mathcal{J}_a \subset \mathcal{J}$ denotes the set of financial securities collateralized by the same tree a. The reader may be more familiar with the term "margin requirement" used in financial markets and the empirical literature. We relate margin and collateral requirements below. It is notationally simpler to write the model in terms of collateral requirements, $k_{a(j)}^j(\sigma)$.

Following Geanakoplos and Zame (2002), we assume that an agent can default on her earlier promises without declaring personal bankruptcy. In this case the agent does not incur any penalties but loses part of the collateral she had to put up. In turn, the buyer of the financial security receives a fraction δ of the collateral associated with the initial promise. We say a tree $a \in \mathcal{A}$ is fully collateralizable if $\delta_a = 1$. However, we also consider the case where for some a, $\delta_a < 1$. The interpretation is that the borrower can divert a fraction $(1 - \delta_a)$ of the asset, hiding it from the lender. This constraint is similar to the one in Gertler and Kiyotaki (2010).

Since there are no penalties for default, an agent who sold security j at date-event s^{t-1} defaults on her promise at a successor node s^t whenever the initial promise exceeds the current value of the fraction of collateral that is pledgable, that is, whenever

$$1 > k^j_{a(j)}(s^{t-1})\delta_{a(j)} \left(q_{a(j)}(s^t) + d_{a(j)}(s^t)\right).$$

Therefore, a borrower of security j at node s^t has to pay (per unit if debt)

$$f_j(s^t) = \min\left\{1, k^j_{a(j)}(s^{t-1})\delta_{a(j)} \left(q_{a(j)}(s^t) + d_{a(j)}(s^t)\right)\right\}.$$

Our model includes the possibility of costly default. This feature of the model is meant to capture costs of default such as legal cost or the physical deterioration of the collateral asset. For example, it is well known that housing properties in foreclosure deteriorate because of moral hazard, destruction, or simple neglect. We model such costs by assuming that part of the collateral value is lost and thus the payment received by the lender is smaller than the value of the borrower's collateral. Specifically, if $f_j(s^t) < 1$ then the loss is proportional to the difference between the face value of the debt and the value of collateral (up to the point of total loss of collateral), that is, the loss is

$$l_j(s^t) = \min\left\{\lambda\left(1 - k^j_{a(j)}(s^{t-1})\delta_{a(j)} \left(q_{a(j)}(s^t) + d_{a(j)}(s^t)\right)\right), f_j(s^t)\right\}$$

for some parameter $\lambda \geq 0$. The resulting payment to the lender of the loan in security j when $f_j(s^t) < 1$ is thus given by

$$r_j(s^t) = f_j(s^t) - l_j(s^t) = \max\left\{0, (1+\lambda)k^j_{a(j)}(s^{t-1})\delta_{a(j)}(q_{a(j)}(s^t) + d_{a(j)}(s^t)) - \lambda\right\}.$$

If $f_j(s^t) = 1$ then $r_j(s^t) = f_j(s^t) = 1$. This repayment function does not capture all costs associated with default. For example, it does not allow for fixed costs which are independent of how much the collateral value falls short of the repayment obligation. However, the advantage of our functional form is that the resulting model remains tractable since the repayment function is continuous in the value of the collateral.

2.1.2 Margin Requirements and Collateral

An agent must hold collateral worth at least $k^j_{a(j)}(s^t)q_{a(j)}(s^t)$ if she wants to sell one unit of bond j at price $p_j(s^t)$. The difference between the value of the collateral holding and the current value of the loan is the amount of capital an agent must put up to obtain the loan. The collateral requirement $k^j_{a(j)}(s^t)$ thus imposes a lower bound $m^j_{a(j)}(s^t)$ on this capital-to-value ratio,

$$m^j_{a(j)}(s^t) = \frac{k^j_{a(j)}(s^t)q_{a(j)}(s^t) - p_j(s^t)}{k^j_{a(j)}(s^t)q_{a(j)}(s^t)}. \tag{1}$$

Using language from financial markets, we call these bounds "margin requirements" throughout the remainder of the paper. Equation (1) provides the definition of the term "margin" according to Regulation T of the U.S. Federal Reserve Board.

To simplify the exposition of our model, we state agents' trading restrictions as well as the payoff functions of the bonds in terms of the collateral requirements $k^j_{a(j)}(s^t)$. However, only the margin requirements $m^j_{a(j)}(s^t)$ are usually mentioned on financial markets. Therefore, we report these margin requirements in our results sections below.

2.1.3 Default and Endogenous Margin Requirements

One of the contributions of this paper is to endogenize margin requirements in an infinite-horizon dynamic general equilibrium model. For this purpose, we follow Geanakoplos (1997) who suggests a simple and tractable method of endogenizing margin requirements. In principle, financial securities with any margin requirement could be traded in equilibrium. Only the scarcity of available collateral leads to equilibrium trade in only a small number of such securities.

To formalize this approach, recall that the S direct successors of a node s^t are denoted $(s^t, 1), \ldots, (s^t, S)$ and that $\mathcal{J}_a$ denotes the set of all bonds collateralized by the same tree a. We define endogenous margin requirements for bonds $j \in \mathcal{J}_a$ collateralized by the same tree $a \in \mathcal{A}$ as follows. For each shock next period, $s' \in \mathcal{S}$, there is a bond which satisfies

$$k^j_{a(j)}(s^t)\delta_{a(j)}\left(q_{a(j)}(s^t, s') + d_{a(j)}(s^t, s')\right) = 1.$$

This bond defaults precisely in those states in which the cum-dividend price of the tree is lower than in the state s'. The bond which defaults in all states but the one with the highest cum-dividend price is redundant because its return (net of costs of default) is identical to the tree return. The payoffs of the remaining $S - 1$ bonds and of tree a are independent. Therefore, the defaultable bonds have the potential to enhance risk-sharing opportunities. In the absence of costs of default, agents typically trade in these $S - 1$ bonds in equilibrium.[2]

The inclusion of costs of default makes defaultable bonds less attractive. In fact, in our calibration agents no longer trade bonds that may default if the costs of default are moderate. Then only a single bond collateralized by tree a is traded in equilibrium. We refer to this bond as the "risk-free" or "no-default" bond. This bond's collateral requirements are endogenously set to the lowest possible value that still ensures no default in the subsequent period. Formally, the resulting condition for the collateral requirement $k^1_{a(1)}(s^t)$ of this bond is

$$k^1_{a(1)}(s^t)\left(\min_{s^{t+1}\succ s^t} \delta_{a(1)}\left(q_{a(1)}(s^{t+1}) + d_{a(1)}(s^{t+1})\right)\right) = 1. \tag{2}$$

For $\delta_{a(1)} = 1$ this is the collateral requirements in Kiyotaki and Moore (1997) generalized to the stochastic case. We discuss realistic values for δ in some detail below.

2.1.4 Financial Markets Equilibrium with Collateral

We are now in the position to formally define the concept of a financial markets equilibrium. It is helpful to define the terms $[\phi^h_j]^+ = \max(0, \phi^h_j)$ and $[\phi^h_j]^- = \min(0, \phi^h_j)$. We

[2] The arguments in Araujo et al. (2012) show that adding additional bonds with other collateral requirements (also only using tree a as collateral) do not change the equilibrium allocation. In the presence of $S - 1$ bonds as specified above, any bond with an intermediate collateral requirement can be replicated by holding a portfolio of the described bonds and tree a using the same amount of collateral.

denote equilibrium values of a variable x by $\bar{x}$.

Definition 1 *A financial markets equilibrium for an economy with initial shock* s_0 *and initial tree holdings* $(\theta^h(s^{-1}))_{h\in\mathcal{H}}$ *is a collection of agents' portfolio holdings and consumption allocations as well as security prices, payouts of financial securities to lender and borrower, and collateral requirements for all one-period financial securities* $j \in \mathcal{J}$

$$\left(\left(\bar{\theta}^h(\sigma), \bar{\phi}^h(\sigma), \bar{c}^h(\sigma)\right)_{h\in\mathcal{H}}; (\bar{q}_a(\sigma))_{a\in\mathcal{A}}, (\bar{p}_j(\sigma))_{j\in\mathcal{J}}; \left(\bar{r}_j(\sigma), \bar{f}_j(\sigma)\right)_{j\in\mathcal{J}}; \left(\bar{k}^j_{a(j)}(\sigma)\right)_{j\in\mathcal{J}} \right)_{\sigma\in\Sigma}$$

satisfying the following conditions:

(1) Markets clear:

$$\sum_{h\in\mathcal{H}} \bar{\theta}^h(\sigma) = 1 \quad \text{and} \quad \sum_{h\in\mathcal{H}} \bar{\phi}^h(\sigma) = 0 \quad \text{for all } \sigma \in \Sigma.$$

(2) For each agent h*, the choices* $\left(\bar{\theta}^h(\sigma), \bar{\phi}^h(\sigma), \bar{c}^h(\sigma)\right)$ *solve the agent's utility maximization problem,*

$$\begin{aligned} \max_{\theta\geq 0,\phi,c\geq 0} U_h(c) \quad & \text{s.t.} \quad \text{for all } s^t \in \Sigma \\ c(s^t) \quad = \quad & e^h(s^t) + \sum_{j\in\mathcal{J}} \left([\phi_j(s^{t-1})]^+ \bar{r}_j(s^t) + [\phi_j(s^{t-1})]^- \bar{f}_j(s^t)\right) + \\ & \theta^h(s^{t-1}) \cdot \left(\bar{q}(s^t) + d(s^t)\right) - \theta^h(s^t) \cdot \bar{q}(s^t) - \phi^h(s^t) \cdot \bar{p}(s^t) \\ 0 \quad \leq \quad & \theta^h_a(s^t) + \sum_{j\in\mathcal{J}_a} \bar{k}^j_a(s^t)[\phi^h_j(s^t)]^-, \quad \text{for all } a \in \mathcal{A}. \end{aligned}$$

(3) For all s^t *and for each* $a \in \mathcal{A}$*, there exists for each state* $s' \in \mathcal{S}$ *a financial security* j *such that* $a = a(j)$ *and*

$$\bar{k}^j_a(s^t)\delta_a \left(\bar{q}_a(s^t, s') + d_a(s^t, s')\right) = 1.$$

(4) The payoffs of the financial securities are given by

$$\bar{f}_j(s^t) = \min\left\{1, k^j_{a(j)}(s^{t-1})\delta_{a(j)} \left(q_{a(j)}(s^t) + d_{a(j)}(s^t)\right)\right\}$$

and

$$\bar{r}_j(s^t) = \begin{cases} \max\left\{0, (1+\lambda)k^j_{a(j)}(s^{t-1})\delta_{a(j)}(q_{a(j)}(s^t) + d_{a(j)}(s^t)) - \lambda\right\} & \text{if } \bar{f}_j(s^t) < 1 \\ 1 & \text{if } \bar{f}_j(s^t) = 1. \end{cases}$$

To approximate equilibrium numerically, we use the algorithm developed in Brumm and Grill (2010). In Appendix B, we describe the computations and the numerical error analysis in detail. For the interpretation of the results it is useful to understand the recursive formulation of the model. The natural endogenous state space of this economy consists of all agents' beginning-of-period financial wealth as a fraction of total financial wealth (i.e. value of the trees cum dividends) in the economy. That is, we keep track of the current shock s_t and of agents' wealth shares, which in the case of no default are

$$\omega^h(s^t) = \frac{\sum_{j\in\mathcal{J}}\left([\phi_j^h(s^{t-1})]^+ r_j(s^t) + [\phi_j^h(s^{t-1})]^- f_j(s^t)\right) + \theta^h(s^{t-1})\cdot(q(s^t)+d(s^t))}{\sum_{a\in\mathcal{A}}(q_a(s^t)+d_a(s^t))}.$$

In our calibration we assume that shocks are i.i.d. and that these shocks only affect the aggregate growth rate. In this case, policy and pricing functions are independent of the exogenous shock, and thus depend on the wealth distribution only, and our results can easily be interpreted in terms of these functions.

We emphasize that the endogenous state variable in our model is time-stationary despite the heterogeneity of agents' levels of risk aversion. Both agents "survive" in the long run since the collateral and short-sale constraints prohibit the agents from assuming more and more debt over time.

2.2 The Baseline Economy

We consider a growth economy with stochastic growth rates. The aggregate endowment at date-event s^t grows at the stochastic rate $g(s_{t+1})$ which (if no costs of default are incurred) only depends on the new shock $s_{t+1}\in\mathcal{S}$. Thus, if either $\lambda=0$ or $f_j(s_{t+1})=1$ for all $j\in\mathcal{J}$, then

$$\frac{\bar{e}(s^{t+1})}{\bar{e}(s^t)} = g(s_{t+1})$$

for all date-events $s^t\in\Sigma$. If there is default in s_{t+1}, then the endowment $\bar{e}(s_{t+1})$ is reduced by the costs of default and the growth rate is reduced accordingly.

There are $S=4$ exogenous shocks. We declare the first of them, $s=1$, to be a "disaster". We calibrate the disaster shock based on data from Barro and Ursua (2008). A disaster is defined as a drop in aggregate consumption of more than 15%, which has a probability of 2.2% and an average size of 28% (see Table 10 in Barro and Ursua (2008)). Following Barro (2009), we choose transition probabilities such that the four exogenous shocks are i.i.d. The non-disaster shocks, $s=2,3,4$, are then calibrated such that the average growth rate (including disasters) is 2 percent and the standard deviation of the growth rate (excluding disasters) matches the data on typical business cycle fluctuations which have a standard deviation of about 2 percent. Table 1 provides the resulting growth rates and the probability distribution for the four exogenous shocks of the economy. Because of their respective size, we call the four shocks: disaster, recession, normal times, and boom.

Shock s	1 (disaster)	2 (recession)	3 (normal times)	4 (boom)
$g(s)$	0.72	0.967	1.027	1.087
$\pi(s)$	0.022	0.054	0.870	0.054

Table 1: Growth Rates and Probabilities of Exogenous Shocks

In our results sections below we report that collateral requirements have quantitatively strong effects on equilibrium prices. Obviously, the question arises as to what portion of these effects is due to the large magnitude of the disaster shock. We address this issue in the discussion of our results. In addition, Appendix A examines the equilibrium effects of collateral requirements for an economy with much less severe bad shocks.

To simplify the analysis we assume that all Lucas trees pay dividends that are proportional to aggregate endowments, that is, $d_a(s^t) = \mathfrak{d}_a \bar{e}(s^t)$, $\mathfrak{d}_a \geq 0$ for all trees $a = 1, ..., A$. In our baseline calibration we assume that the dividend share in the economy is 15 percent, i.e. $\sum_a \mathfrak{d}_a = 0.15$. However, in Section 3 we show how our results change as $\mathfrak{d}_1$ varies from zero to 50% while maintaining $\mathfrak{d}_2$ constant at 7.5%. While the dividend share of stocks traded on national exchanges is much smaller than 15 percent, the Lucas trees in our models also represent real-estate, long-term bonds and other tangible assets.

Recall that the agents have recursive utility functions (Epstein and Zin (1989)) with parameters ρ^h and α^h where $\frac{1}{1-\rho^h}$ is the intertemporal elasticity of substitution (IES) and $1-\alpha^h$ is the relative risk aversion of the agent. As in Barro and Ursua (2008) we assume that both agents have an identical IES of 2. However we assume that the two agents differ in their risk aversions. In the baseline case we set the risk aversion of agent 1 to 0.5, i.e. $\alpha^1 = 0.5$ and the risk aversion of agent 2 to 7, i.e. $\alpha^2 = -6$. We set $\beta = 0.977$; this value ensures that in the baseline economy the average risk-free rate is one percent.

Each agent h receives a fixed share of aggregate endowments as individual endowments. We abstract from idiosyncratic income shocks because it is difficult to disentangle idiosyncratic and aggregate shocks for a model with only two types of agents. We assume that agent 1 receives 10 percent of all individual endowments, and that agent 2 receives the remaining 90 percent of all individual endowments. That is, the first agent's labor income is given by

$$e^1(s^t) = 0.1(\bar{e}(s^t) - \sum_{a \in \mathcal{A}} d_a(s^t)) = 0.085\bar{e}(s^t).$$

The labor income of agent 2 is $e^2(s^t) = 0.765\bar{e}(s^t)$. Although agent 1 receives only a relatively small fraction of aggregate income, her risk aversion is much smaller and induces her to hold a substantial fraction of financial wealth in equilibrium. Therefore, the labor income share of agent 1 is chosen to roughly match the percentage of agents in the US population that hold substantial amounts of stocks outside of retirement accounts. Vissing-Jorgensen and Attanasio (2003) report that about 20 percent of the US population holds stocks, but many of these households have only small stock investments; see Poterba et al. (1995). As a compromise, we choose 10 percent for the income share of agent 1. Despite this small labor income share, agent 1 is in equilibrium, on average, much wealthier than agent 2. This equilibrium feature is consistent with the U.S. data reported by Guvenen (2009) that stockholders own about 80 percent of all net worth (including housing).

In our baseline calibration, we assume positive costs of default of 5%. In the sensitivity analysis in Appendix A, we show that our quantitative results are rather insensitive to this assumption. Lower costs lead to only very little variation in the results. Higher costs of default lead to identical results, since agents do not trade defaultable bonds for costs of default exceeding 4%. Recall from the description in Section 2.1 that the costs are proportional to the difference between the face value of the bond and the value of the underlying collateral. Therefore, a proportional cost of 5 percent means a much smaller cost as a fraction of the underlying collateral. Campbell et al. (2011) find an average “foreclosure discount” of 27 percent for foreclosures in Massachusetts from 1988 until 2008. This discount is measured as a percentage of the total value of the house. As a percentage of the difference between the house value and face value of the debt this figure would be substantially larger. A value of $\lambda = 0.05$, therefore, seems certainly realistic and is, if anything, too small when we compare it to figures from the U.S. housing market. It is difficult to assess costs of default in securities markets. In these markets, agents cannot legally default on individual contracts. However, as Fortune (2000) writes, “Customers typically find reasons to dispute their liability, and while the requirement of binding arbitration of disputes tilts the scales in favor of brokers, it does not always avoid expensive litigation, nor does it always lead to successful recovery. This suggests that margin loans, while legally recourse loans, might be in a limbo, somewhere between recourse and non-recourse.” To simplify the analysis, we assume that costs of default are identical across the different Lucas trees.

For some of our analysis, we vary the collateralizability parameters of the two trees, δ_1 and δ_2. These parameters are exogenously given in our economy. There are varies reasons for the collateralizability to vary across assets. First, some assets can be partly diverted by borrowers in case of default. For example, owners of small businesses may steal part of their equipment in the case of bankruptcy. Therefore, lenders will only let them borrow up to the non-divertible part of their assets. In a similar vain, financial intermediaries managing assets on behalf of agents may also divert funds. If we interpret the Lucas trees as pools of individual assets (e.g. stocks or mortgages) that are managed by intermediaries, this assumption also implies that investors can only borrow up to a fraction of the value of these trees. Assuming that borrowing is limited by the ability of bankers to divert funds is quite common in the literature; see e.g. Gertler and Kiyotaki (2010). Finally, the collateralizability of an asset may also depend on regulation. An example is Federal Reserve Board Regulation T, which sets minimum margins for stocks bought on margin in the United States. This and other aspects of margin regulation are explicitly modeled in Brumm et al. (2013). In contrast, the present paper is agnostic about the factors determining the collateralizability of an asset. Instead, we investigate the consequences of differences in the collateralizability across assets.

3 Collateral and Asset Returns

The purpose of this first results section is to explore the impact of collateral constraints on the first and second moments of asset returns in the infinite-horizon collateral-constrained economy. We assume that the first long-lived asset is fully collateralizable while the second one is not, so $1 = \delta_1 > \delta_2$. In the first part of our analysis, we assume that the second asset cannot be used as collateral, i.e. $\delta_2 = 0$. This extreme assumption enables us to provide a clear economic intuition for the mechanisms driving asset returns when agents face collateral constraints. Subsequently, we show that the obtained insights are robust for intermediate values $0 < \delta_2 < 1$ of the collateralizability parameter of the second tree.

3.1 Only One Tree Can Be Used as Collateral

In our initial analysis, we examine our baseline economy, (*CC: Collateral Constraints*) with the calibration described in Section 2.2. Both assets (Lucas trees) pay identical dividends, $\mathfrak{d}_1 = \mathfrak{d}_2 = 7.5\%$. For ease of reference, we call the first, fully collateralizable, tree the "marginable" asset. The agents cannot borrow against the second tree, which we call the "non-marginable" asset.

We now document that the interplay of disaster risk, heterogeneous risk aversion, and collateral constraints generates large return volatilities for the two assets. In order to understand the economic mechanisms leading to large asset return volatility and to evaluate the quantitative effects of scarce collateral in our baseline economy, we benchmark the results against those for three well-known and much simpler economic models. The first benchmark model demonstrates that disaster risk without heterogeneous risk aversion does not deliver high return volatility. The other two benchmark models show that disaster risk with heterogeneous risk aversion also fails to deliver high volatility if borrowing constraints are either too tight or too loose. The first benchmark model, *B0: Single Agent*, assumes that both agents have not only identical IES of 2 but also identical relative risk aversion of 7. This model is comparable to the representative agent model in Barro (2009) who assumes the same value for the IES but slightly lower risk aversion. The second benchmark model, *B1: No borrowing*, assumes that the two agents have different risk aversions and that they cannot borrow, that is, neither tree is collateralizable. The third benchmark model, *B2: Unconstrained*, is an economy in which agents can use their entire endowment as collateral. This model is equivalent to a model with natural borrowing constraints (and without short-sale constraints on the trees). The common discount factor in all four models is $\beta = 0.977$. For this value, the average risk-free interest rate in *CC* is about 1 percent, which matches the annual real interest rate in U.S. data. This value of the discount factor leads to negative interest rates for models *B0* and *B1*. However, for a different value of the discount factor, $\beta = 0.95$, the interest rate in the models *B0* and *B1* is about one percent and the equity risk premium and the standard deviations of returns remain virtually unchanged. It is impossible to choose β high enough to drive the real rate in *B2* down to one percent.

Table 2 reports three aggregate statistics for each of the four economies. Throughout the paper we measure asset return volatility by the average return standard deviation over repeated long simulations of the economy (see Appendix B for a description of the simulation procedure). We also report average risk-free interest rates (RFR) and excess returns (ER) from repeated long simulations. While we calibrate the discount factor to match an RFR of one percent in the baseline economy *CC*, it is not the aim of this paper to match additional real world asset pricing moments. Instead, we want to investigate how much collateral constraints can contribute to explaining the high return volatility that risky long-lived assets exhibit.[3] Clearly, there are other factors, which we do not model, that also contribute to this explanation, e.g. uninsurable idiosyncratic risk and dividends that are more volatile than aggregate endowments. Therefore, it is expected that the return volatility we find in the model is lower than the volatility that many long-lived assets exhibit in the real world. Having said this, let us turn to the results we obtain for the various model specifications considered.

In our first benchmark model, *B0: Single agent*, the equity premium is realistically large but the return volatility is significantly smaller than in U.S. data. A comparison to the second benchmark model *B1: No borrowing* reveals only small changes in the asset

[3] Estimated values for the standard deviation of the real annual returns of S&P 500 stocks are 19.3% (Guvenen, 2009) and 15% (Lettau & Uhlig, 2002).

Model	STD Returns	RFR	ER
B0: Single agent	5.1	-1.8	6.2
B1: No borrowing	5.0	-1.8	5.2
B2: Unconstrained	5.0	3.2	0.3
CC: Collateral constraints	7.8	1.0	2.3

Table 2: Aggregate Moments for the Baseline Economy and Three Benchmarks (all figures in percent)

return moments even though the two agents are now heterogeneous. In the long run, agent 1, who is much less risk averse than agent 2, holds the two (identical) Lucas trees. Agent 2 consumes only her labor income. The interest rate is determined by (the Euler equation of) agent 2 since she would like to save in the risk-free bond. Agent 1 would like to borrow at this low rate, but the no-borrowing constraint prevents her from doing so. There is essentially no movement in the distribution of financial wealth over time and price-dividend ratios are (approximately) constant.[4] Only the equity premium decreases by a non-trivial amount compared to the representative agent model *B0*. It stays relatively large, however, because agent 1 requires high average returns to be induced to save enough to afford both Lucas trees. In the third benchmark model, *B2: Unconstrained*, the less risk-averse agent 1 holds both trees during the vast majority of time periods.[5] A bad shock to the economy leads to a wealth shift from agent 2 to agent 1 and a decrease of the two tree prices. However, these effects are very small. Thus the resulting return volatility in model *B2* is only barely larger than the volatility in *B1* (in fact, the first two significant digits are identical). In model *B2* the risk-free rate is large and the equity premium is very low. Despite the presence of disaster shocks, the market price of risk is low because risk is borne almost entirely by agent 1 who has very low risk aversion. The high interest rate now reflects the fact that agent 1 has a desire to leverage. Despite being leveraged, agent 1 can easily buffer bad shocks with additional borrowing; thus, bad shocks only lead to small movements in the price of the two trees.

Compared to all three benchmark models, the collateral-constrained economy generates a return volatility that is more than 50 percent larger. This result critically relies on the interplay of disaster risk, heterogeneous risk aversion, and collateral constraints. The first benchmark model, *B0: Single agent*, demonstrates that disaster risk without heterogeneous risk aversion does not deliver high return volatility. Surprisingly, disaster risk with heterogeneous risk aversion also fails to deliver high volatility if borrowing constraints are either too tight, *B1: No borrowing*, or too loose, *B2: Unconstrained*. Instead only all three model features combined generate high return volatility. To understand

[4]In the case of a constant price-dividend ratio, the aggregate return volatility is only slightly higher than the volatility of aggregate endowment growth which is 4.92%. The reason is that in this case asset returns are $\left(p(s^{t+1}) + d(s^{t+1})\right)/p(s^t) = g(s_{t+1})\left(p(s^t) + d(s^t)\right)/p(s^t) = g(s_{t+1})\left(1 + d(s^t)/p(s^t)\right)$.

[5]Contrary to all other economies considered in this paper, the benchmark model *B2* is not time-stationary. In the long run, only the much less risk-averse agent 1 survives while the second agent's consumption approaches zero. Therefore, the market price of risk and the resulting asset return moments are driven by the marginal utility of agent 1. Conditional return moments derived from long-run simulations starting from different points in the state space are virtually identical.

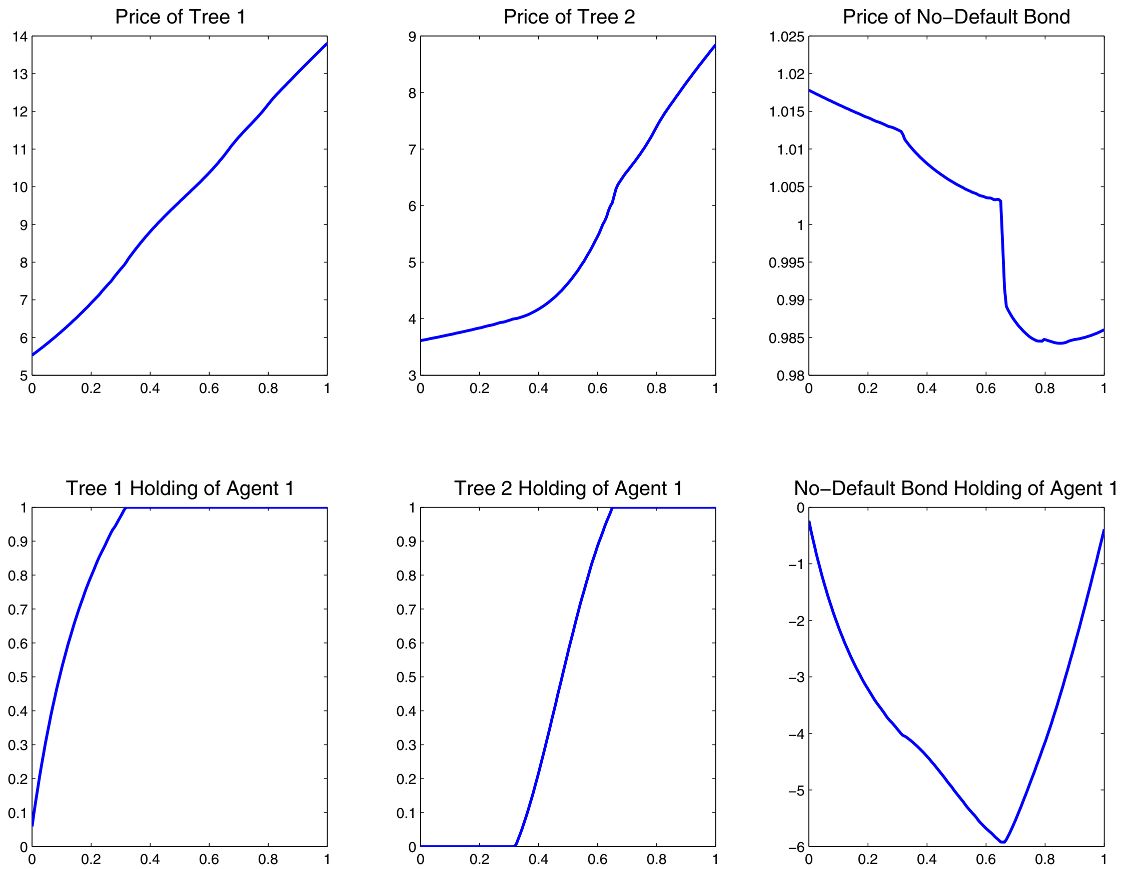

Figure 1: Price and Policy Functions (as a function of the first agent's wealth share, w^1)

how their interaction creates the high return volatility in the model CC, we examine the policy functions that describe equilibrium asset prices and agents' portfolio choices as a function of the wealth distribution in the economy. Actual asset prices in the growth economy vary across shocks and grow over time. However, as shocks are i.i.d., we obtain time- and shock-independent prices as soon as we normalize actual prices by aggregate consumption. Similarly, the (portfolio) policy functions are independent of the shocks.

Figure 1 displays (normalized) asset prices and portfolio choices as a function of agent 1's wealth share, w^1, for our baseline economy CC. The three graphs in the bottom row of Figure 1 show the asset positions of the much less risk-averse agent 1 as a function of her beginning-of-period financial wealth share, w^1. As the first agent's wealth share increases, she purchases an increasing share of the marginable asset (tree 1) by increasing her borrowing in the no-default bond. Only after she has bought the entire marginable asset does she start buying the non-marginable tree 2. Along the way, she increases her leverage by increasing the short position in the risk-free bond which is collateralized by tree 1, the marginable asset. When her wealth share becomes sufficiently large enough to hold both trees, she can afford to hold smaller short positions in the bond. For very high wealth shares, agent 1 is not fully leveraged. Not surprisingly, the agents' portfolio positions depend very strongly on the wealth distribution in the economy. More surprisingly, the three graphs in the top row of the figure show that asset prices also vary substantially with the wealth distribution. If the more risk-averse agent 2 holds all financial wealth in the economy, so $w^1 = 0$, the price of tree 1, the marginable asset, is below 6 and the price of tree 2, the non-marginable asset, is below 4. If, to the contrary, agent 1 holds all financial wealth, $w^1 = 1$, the price of the marginable asset is almost 14 and the price of the non-marginable asset is almost 9. Put differently, the prices of both trees fluctuate significantly whenever substantial wealth shifts occur between the two agents.

Observe that the more risk-averse agent 2 does not hold any long-lived assets in the economy for values of the endogenous state variable w^1 exceeding 0.63. Along the simulation path, w^1 in fact does exceed this threshold most of the time. So our model with collateral constraints naturally generates portfolios similar to those of the literature on endogenous limited stock market participation. The explanation for this phenomenon is straightforward. Recall that agents' labor income is perfectly correlated with the aggregate endowment and the dividends. Because of her high risk aversion, agent 2 would like to hedge her labor income risk by shorting the long-lived assets, particularly when she is relatively poor and the asset prices are high (due to the high demand from the much less risk-averse agent 1). However, the short-sale constraints preclude her from doing so. Thus, the constraints are binding and she does not hold long-lived assets in equilibrium.

The price and policy functions in Figure 1 highlight the importance of the wealth distribution in the collateral-constrained economy. To obtain an economic intuition for the behavior of asset prices and portfolio choices over time, particularly in reaction to good and bad shocks, we, therefore, need to examine the response of the wealth distribution to the exogenous shocks. Figure 2 shows how the wealth distribution and the endogenous margin requirement for borrowing against tree 1 vary as the economy experiences growth shocks of different sizes. In this sample path, a disaster shock (shock 1) occurs in periods 20 and 62; recessions (shock 2) happen in periods 10, 11, 50, 60, and 61; booms (shock 4) occur in periods 5, 40, 41, 90 and 95; finally, there is "normal" growth (shock 3) in all other periods.

The upper graph in Figure 2 shows that even during a snapshot of a long-run simulation of the collateral-constrained economy, the wealth distribution varies substantially. We stress again that the endogenous state variable, agent 1's wealth share w^1, is time-stationary in the collateral-constrained economy, since the collateral and short-sale constraints prohibit the agents from taking on increasingly large debt positions. Therefore, both agents survive in the long run. The wealth share of the first agent varies between 0.36 and 0.76. Disaster shocks (shock 1) reduce the agent's wealth share drastically, while recessions (shock 2) have a milder negative impact on her share of wealth. Booms (shock 4) lead to an increase of her wealth share. Of course, all these effects are exactly reversed for the second agent. In light of the normalized asset price functions in Figure 1, the large variation in the wealth distribution leads to considerable volatility of the normalized tree prices.

To appreciate the price impact of the wealth share volatility displayed in the upper graph of Figure 2, we briefly reflect upon normalized asset prices in the benchmark models *B0: Single agent* and *B1: No borrowing*. When a bad shock occurs, both the current dividend and the expected net present value of all future dividends of a tree decrease. As a result the price of the tree drops, but in the absence of further effects, the normalized price would remain the same, because we consider i.i.d. shocks to the growth rate and dividends are proportional to aggregate endowments. In model *B0*, the normalized price is constant, in model *B1* the same is approximately true. In both models, asset price volatility is entirely driven by the exogenous growth shocks. In stark contrast,

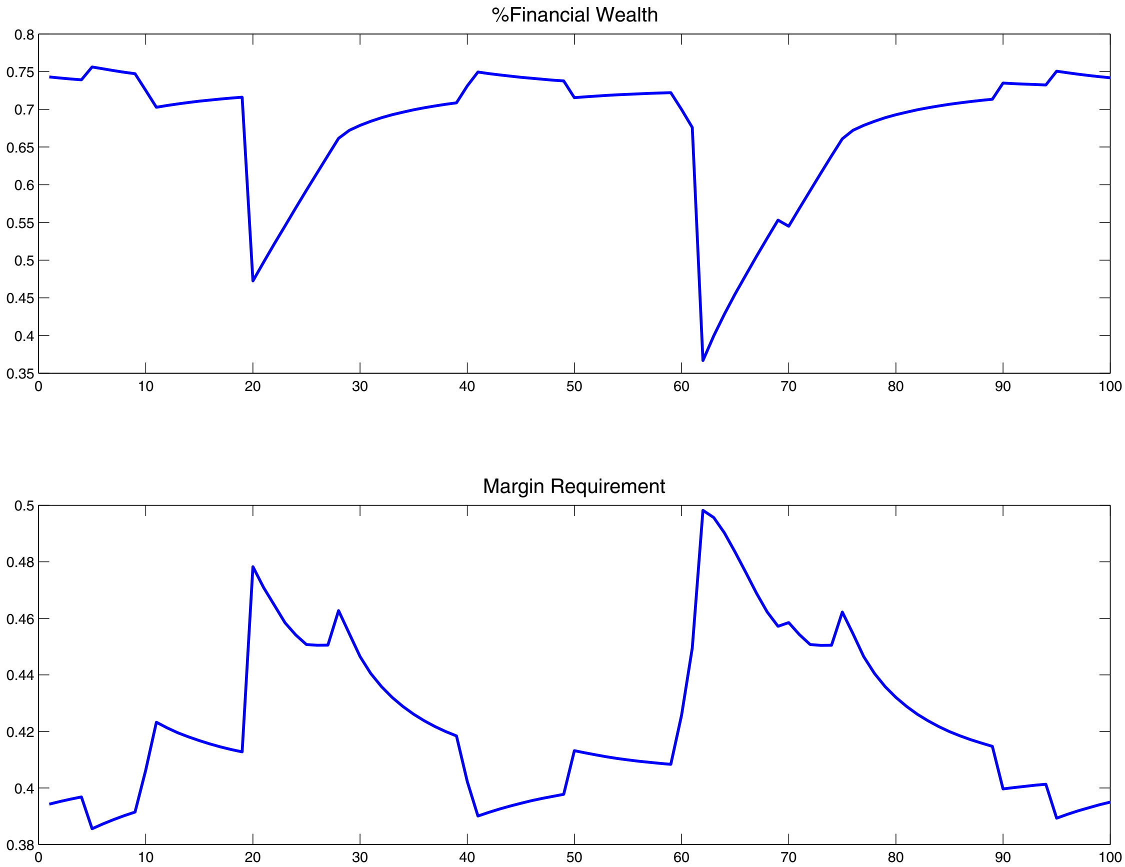

Figure 2: Snapshot from a simulation of the model

the collateral-constrained economy, CC, exhibits significant volatility of normalized tree prices which greatly contribute to the overall volatility of the non-normalized tree prices as reported in Table 2.

The upper graph in Figure 2 depicts that agent 1 holds just above 70% of total wealth after a sequence of normal growth shocks (shock 3). Figure 1 shows that agent 1 is close to being fully leveraged for such values of the endogenous state variable. She holds both the marginable and the non-marginable asset entirely and borrows almost as much against the marginable asset as the collateral constraint permits. If in this situation a disaster shock (shock 1) occurs, agent 1 must make a net payment to the lender, agent 2. A net payment requires her to either reduce her consumption or sell some of her assets. As a response to the disastrous shock she does, in fact, both. The policy functions in the bottom graph of Figure 1 reveal that agent 1 always first sells the non-marginable asset, tree 2. She holds on to the entire marginable asset, tree 1, because this tree has an additional value to the agent since it serves as collateral. In fact, in our calibration, agent 1 only sells a portion of the marginable asset after selling off the entire non-marginable asset. During the snapshot of the sample path in Figure 2 she never has to do so. The policy functions in Figure 1 show that she would eventually have to sell portions of the marginable asset after two or more consecutive disaster shocks. But the probability of such a shock sequence is extremely low. In contrast, agent 1 sells the non-marginable asset much more often. She always does in disasters (shock 1). She also does so in recessions, if her wealth share was not too large before the recession.

Agent 2 is much more risk averse than agent 1, and so is only willing to hold a share of a tree if its expected returns are high, that is, if its price is low. As a result, the price of the second tree plummets as soon as agent 1 starts selling it to agent 2. In this situation, an additional effect emerges; since agent 1 holds less financial wealth than before and is fully leveraged, it is more likely than before that she will eventually need to sell portions of the marginable asset, tree 1. Thus, its price also decreases. At the same time the collateral requirement k^1 shoots up in anticipation of a possible further significant price decrease in the next period - see (2)- and so the margin requirement jumps as well. The impact on the margin requirement, shown in the bottom graph of Figure 2, implies that in order to hold on to her position in the marginable asset, tree 1, agent 1 has to sell even more of tree 2, depressing its price even further. Table 3 reports the movements in the margin requirements. We observe that margin requirements vary substantially over time.

Shock s	1 (disaster)	2 (recession)	3 (normal times)	4 (boom)
Average margin of tree 1	46.7	43.7	42.2	40.9

Table 3: Endogenous Margins across Growth Shocks

The table shows that they are, on average, highly counter-cyclical. In booms, agent 1 has easy access to credit and can borrow large amounts against her holdings in the marginable asset. In recessions, margin requirements increase substantially, making it impossible for agent 1 to simply "roll over" her debt.

Table 4 reports moments of the two trees' returns. Observe that the two trees exhibit substantially different returns despite paying identical dividends. The standard deviation of returns of the non-marginable asset is much higher than that of the marginable asset. The same is true for the excess return. In light of our description of the economic

	STD Returns	Excess Returns
Tree 1 (marginable)	7.3	1.9
Tree 2 (non-marginable)	8.7	2.6

Table 4: Moments of Trees' Returns When Only Tree 1 is Collateralizable (all entries in percent)

mechanisms driving the asset prices in the collateral-constrained economy, we can offer a straightforward explanation for the return moments. In normal times, the much less risk-averse agent 1 holds both long-lived assets. In response to a disaster shock or a series of recession shocks, she first starts selling the non-marginable asset; only after she has reduced her position in that asset to zero does she sell portions of the marginable asset. To induce the much more risk averse agent 2 to buy a risky asset, its price must drop considerably. Along an equilibrium sample path, agent 1 sells portions of the non-marginable asset much more often than any part of the marginable asset. As a result, the non-marginable asset exhibits a larger price and return volatility than the marginable asset. This difference in the return standard deviations provides a reason for the difference between the excess returns: being more volatile, the non-marginable asset, tree 2, must provide higher excess returns to be held in equilibrium. A second source for the higher excess return of the non-marginable asset is its collateral value. Whenever agent 1 is fully leveraged, the value of the marginable asset exceeds next period's (with agent 1's state prices) discounted payoffs since it provides collateral value for agent 1. Since both trees pay identical dividends, an agent can only be induced to hold the less attractive tree 2 if it pays a higher expected return.

This concludes our explanation for the different first and second moments of the two tree returns in our baseline economy. In the remainder of this section we report results from a series of variations of the baseline economy to deepen our understanding of the economic mechanisms.

3.2 Availability of Collateral

We want to examine the robustness of the asset return results that we have obtained thus far. For this purpose we take a closer look at the role of the total magnitude of collateral available in the economy. We document that the availability of collateral strongly affects both the aggregate market volatility and the differences between the individual return volatilities.

Until now we have assumed a dividend share of 15 percent of aggregate endowment and that both Lucas trees pay identical dividends, $\mathfrak{d}_1 = \mathfrak{d}_2 = 7.5\%$. Figure 3 shows how the asset return volatilities change as the dividend share of the marginable asset changes while the dividend share of the non-marginable asset remains constant at 7.5 percent.

The figure illustrates that the described qualitative effects are robust with respect to changes in the amount of collateral available in the economy. Even if 20 percent of aggregate consumption can be used as collateral, i.e. $\mathfrak{d}_1 = 0.2$, the collateral-constrained economy continues to deliver substantial excess volatility compared to the benchmark models. Moreover, the return volatility of the non-marginable asset still substantially exceeds that of the marginable asset, tree 1.

The return volatility of the marginable asset has an interior maximum at a dividend share of about 10 percent. If the marginable asset pays very little dividends, $\mathfrak{d}_1 < 0.05$, then it constitutes only little collateral. Thus, agent 1 cannot borrow much and is rarely if ever forced to sell this asset despite being fully leveraged. However, she frequently must sell the non-marginable asset which, therefore, exhibits a large return volatility. An increase of the dividend share of the marginable asset enables the much less risk-averse agent 1 to borrow more. Therefore, she can sustain larger asset holdings in the face of bad shocks, causing the return volatility of the non-marginable asset to decrease. At the same time, after disaster shocks or long recessions she now has to unwind her increased debt positions more often and even sell part of the marginable asset. Therefore, the return volatility of this asset increases initially as long as $\mathfrak{d}_1 \leq 0.1$. The decreasing return volatility of the non-marginable asset and the increasing return volatility of the marginable asset lead to an aggregate volatility that is essentially flat for small values of $\mathfrak{d}_1$. The return volatility of the non-marginable asset continues to decrease as the amount of available collateral in the economy continues to grow, particularly for values of $\mathfrak{d}_1 > 0.15$. As more collateral becomes available, the collateral constraint holds less frequently and fire sales in tree 2 become less frequent. At the same time, agent 1 becomes

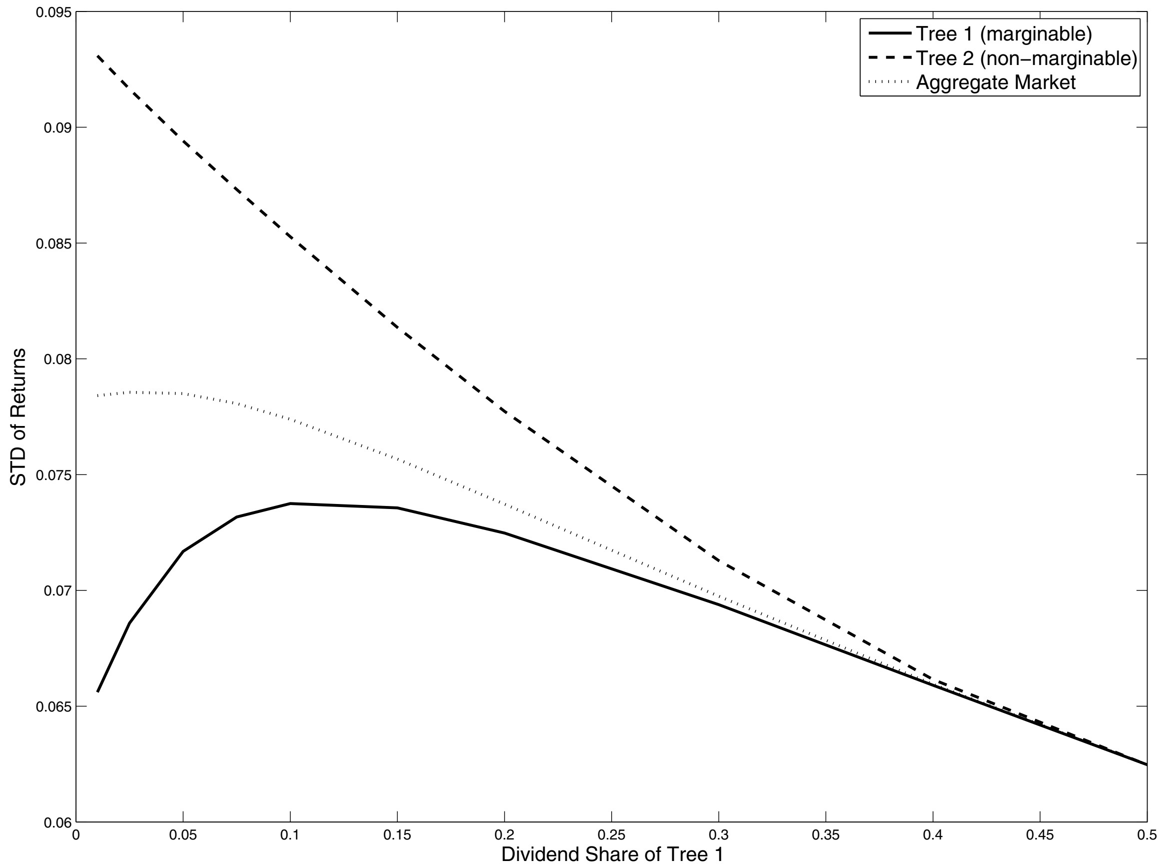

Figure 3: Volatility and the Amount of Available Collateral

less likely to have to sell any portion of the marginable asset. Therefore, both individual asset return volatilities and the aggregate market volatility decrease. Finally, when the dividends of the marginable asset become a large fraction of the aggregate endowment, the collateral constraint is essentially never binding. Agent 1 holds both trees in equilibrium almost all the time. The fact that tree 1 is marginable and tree 2 is not no longer affects asset prices. Thus, the difference between the return volatilities of the marginable and the non-marginable asset disappears.

In sum, we have observed that the economic mechanism that we detected in our baseline economy is robust to changes in the magnitude of the collateral available in the economy. In Appendix A we provide a further sensitivity analysis. We examine in detail how robust our results are with respect to the specification of preferences and aggregate shocks. They are very robust qualitatively, yet in order for quantitatively large effects the presence of disaster shocks and of substantial differences in risk aversion among the two agents is necessary.

3.3 Collateralizability of the Second Asset

Recall that the collateralizability parameter δ_2 determines the fraction of the second tree that can be confiscated if an agent defaults on a loan that was secured with this tree as collateral. Until now we have assumed that the second tree is non-marginable, $\delta_2 = 0$. We now relax this assumption and vary this parameter as well. The labor endowment shares of the two agents and the dividend shares of the two trees remain the same as in the baseline economy; the much less risk-averse agent 1 receives 10% and the more risk-averse agent 2 receives 90% of the labor endowment. The two trees have identical dividend shares of 7.5% of the aggregate endowment in the economy. The first tree remains fully collateralizable.

Table 5 reports the relative differences in the standard deviations of the two tree returns and in the two excess returns. In addition, the table reports the correlation between the margin requirements of the two assets. The first row of results, i.e. for

δ_2	STD2/STD1 − 1	ER2/ER1 − 1	Margin Correlation
0	19	37	0
20	17	30	4
40	13	22	56
60	8	13	89
80	3	5	99
100	0	0	100

Table 5: Collateralizability and Moments (all figures in percent)

$\delta_2 = 0$, corresponds to the results reported for the baseline economy in Table 4. If the second asset is not collateralizable, then its excess return and return volatility exceed the corresponding values of the marginable asset, tree 1, by 37% and 19%, respectively. As we would expect, both relative differences decrease monotonically in the collateral parameter δ_2 of the second asset. The larger the value of δ_2, the more the less risk-averse agent 1 can borrow and the less often she needs to sell this asset after bad shocks. The collateral feature makes the asset more attractive to agent 1, and so its excess returns and return volatility (relative to the first asset) decrease. In the extreme case, when the second asset is fully collateralizable, $\delta_2 = 1$, both assets are identical and thus offer the same returns.

The difference in the collateralizability of the two assets results in different price dynamics. These price differences in turn lead to different endogenous margin requirements for the two assets. If the second asset is not collateralizable, then its margin requirement is constant at 100 percent and therefore uncorrelated with the margin requirement of the marginable asset, tree 1. As the last column of Table 5 shows, when the collateral parameter δ_2 increases, the margin requirement for the second asset becomes positively correlated with that of the first one. The correlation coefficient among the two margin requirements is monotonically increasing in δ_2. However, it is well below 100% for a wide range of values of δ_2. Of course, when the assets are identical, $\delta_2 = 1$, then the same is true for their margin requirements.

In sum, we observe that the results for economies in which the second asset is partially collateralizable qualitatively support our insights from the baseline economy in which the second asset is not marginable.

4 Collateral Premia and Bubbles

In the previous section we have shown that assets with identical cash flows but different collateralizability can have substantially different expected returns. The asset that is easier to use as collateral has a higher price because it derives part of its value from being collateralizable. For a more in-depth investigation of the collateral value, we formally define the (relative) *collateral premium* at a node s^t as

$$CP(s^t) = \frac{p_1(s^t) - p_2(s^t)\frac{\mathfrak{d}_1}{\mathfrak{d}_2}}{p_1(s^t) + p_2(s^t)}.$$

To understand the motivation for this definition, note that we maintain the assumption of collinear cash flows. Therefore, if the two infinitely-lived assets were equally collateralizable, the equilibrium prices would satisfy $p_1(s^t) = \frac{\mathfrak{d}_1}{\mathfrak{d}_2} p_2(s^t)$ for all s^t. We can, therefore, interpret the collateral premium, $CP(s^t)$, as the premium investors are paying for the additional degree of collateralizability of the first infinitely-lived asset. This premium is measured as a fraction of the entire asset market, which is particularly convenient for our analysis in Sections 4.1 and 4.2, where we consider an asset that pays no dividends yet has positive value due to its collateralizability. Before we turn to this case, we discuss the collateral premium for the case of two infinitely-lived assets with identical dividends; see Section 3.

When we report collateral premia, we report the average collateral premium over repeated long simulations of the model. Clearly, the collateral premium varies as the economy experiences different shocks. It is indeed countercyclical. This fact becomes apparent in Table 6, which reports the average collateral premium conditional on the realized growth shock for our benchmark economy *CC: Collateral Constraints*. To understand this result, recall that in bad times the price of the collateralizable asset, tree 1, falls less dramatically than the price of the non-marginable asset, tree 2. This fact directly implies that the collateral premium, as defined above, is higher for bad shocks.

Shock s	1 (disaster)	2 (recession)	3 (normal times)	4 (boom)
Collateral Premium	33.9	27.8	27.2	26.8

Table 6: Collateral Premium of Asset 1 Conditional on the Shock (all figures in percent)

δ_2	0	20	40	60	80	100
Collateral Premium	27.4	22.3	16.4	10.1	3.8	0.0

Table 7: Collateral Premium of Asset 1 as a Function of the Collateralizability of Asset 2 (all figures in percent)

The collateral premium in the price of the first asset also depends on the collateralizability of the other infinitely-lived asset. In particular, the collateral premium decreases as the second asset becomes more and more collateralizable. Table 7 reports the collateral premium for the specifications considered in Section 3.3. For the extreme case $\delta_2 = 0$ the price difference between the two infinitely-lived assets is, on average, 27.4 percent of the value of the entire asset market. As long as the collateralizability of the second asset is relatively low, the collateral premium remains substantial.

Another interesting question is how the collateral premium of the first asset depends on its dividend share. The answer is not obvious. On the one hand, we would expect scarce collateral to imply a high collateral premium. But, on the other hand, as the dividends of this first asset decrease its price should decrease as well, and, in fact, we may expect the collateral premium (measured again as a fraction of the entire asset market value) to decrease as well. Surprisingly, however, the price of the first asset does not converge to zero as its dividends tend to zero! Indeed, we next show that a marginable asset that does not pay dividends may have a positive price in equilibrium. Kocherlakota (1992) and Fostel and Geanakoplos (2012) observe a similar phenomenon and refer to it as a "bubble". Subsequently we examine how this collateral premium responds to changes of the asset's dividend share as well as to changes in the collateralizability of the other infinitely-lived asset.

4.1 An Equilibrium with a Bubble

Recall from the analysis of our baseline economy that the price of the marginable asset, tree 1, is substantially higher than the price of the non-marginable asset, tree 2. When the dividends of the marginable asset decrease while those of the non-marginable asset remain constant, then this price difference obviously decreases. Surprisingly, if $\mathfrak{d}_1 = 0$, then the economy possesses an equilibrium in which the price of the marginable asset remains positive. In line with the afore-mentioned previous literature we refer to this effect, which is due to a sizable collateral premium, as a "bubble". While we do not want to join the debate on the "correct" or "appropriate" definition of a bubble in this paper, we believe that it is important for our understanding of collateral economies to investigate this phenomenon in some detail. How can an asset that never pays any dividends have a positive price in a competitive equilibrium in which another asset paying positive dividends in each period has a finite price? And what is the quantitative effect of this phenomenon on the other asset's price?

To understand this phenomenon, observe that the real average interest rate in this economy is about 0.8 percent while the average growth rate is 2.0 percent. The less risk-averse agent 1 has, therefore, a great desire to hold as much of the dividend-paying non-marginable asset as possible. For this purpose, she holds a fully leveraged position in the marginable asset, tree 1, borrows as much as possible at the risk-free rate, and invests these funds to buy the second asset. In normal times and booms, the agent rolls over her debt every period and receives a positive return from her investment strategy. But in periods with a bad shock, the agent earns a negative return and needs to reduce her consumption to maintain her position. Thus, this trading strategy is not a Ponzi-scheme but rather a risky equilibrium strategy. In fact, the strategy's average return over long simulations of the economy is about 4.8 percent. The high risk premium is justified since agent 2, because of her high risk aversion, is willing to lend money even at very low interest rates to insure herself against bad growth shocks. Table 8 provides a glimpse of the asset returns in this equilibrium. The table shows that in the equilibrium

	STD	ER	agg STD	agg ER	RFR	STD *B1*
Tree 1 (marginable with $\mathfrak{d}_1 = 0\%$)	5.0	1.2	7.4	2.3	0.8	5.0
Tree 2 (non-marg. with $\mathfrak{d}_2 = 15\%$)	8.9	3.0				

Table 8: Asset Returns with a Marginable Asset Paying No Dividends (all entries in percent)

with a bubble, the productive second asset has very high return volatility and high excess returns. The risk-free rate is lower than in the baseline case but not by much. The key observation is that, although expected returns for the marginable tree are relatively low, they still exceed the risk-free rate, on average. Most notably, the volatility of the returns of the marginable asset is very low.

While the theoretical mechanism leading to a "bubble" is similar to the one described in Kocherlakota (1992), our model has various interesting aspects to add to the story. Most importantly, we find that the price of the marginable asset, tree 1, is positive even though there is a more productive asset available for trade in the economy. Clearly, it is its collateralizability that gives the marginable asset its value. Naturally, we may ask how this value depends on the scarcity of collateral in the economy. Another obvious question to ask is how the magnitude of the premium depends on the asset's dividends. We address these issues next.

4.2 Analysis of the Collateral Premium

We have seen that the collateral premium of an asset decreases as the other asset becomes more collateralizable. While an asset with zero cash flows can have a positive price due to its collateral premium, this will depend on the collateralizability of the second asset. We consider an economy in which the second asset pays 15 percent of aggregate consumption as dividends, $\mathfrak{d}_2 = 0.15$. Figure 4 shows the collateral premium, CP, as a function of the collateralizability parameter δ_2 of the second asset. The dashed line in the figure shows CP for an economy in which the first asset pays no dividends; the solid line shows CP for an economy in which the first asset pays dividends of 5 percent of aggregate consumption.

If the first asset pays no dividends, then CP provides us with an indication of the size of the bubble. When the second asset is not collateralizable, then the first asset is the only source of collateral. The bubble and CP are of maximal size. The larger the portion of the second asset that is collateralizable, the more collateral is available in the economy and the relatively less important is the collateralizability of the first asset. Naturally, its price and CP decrease. Once about half of the second asset becomes collateralizable, the price of the first asset drops to zero. In such an economy, the less risk-averse agent 1 has sufficient collateral available by holding the entire second asset. It is perhaps somewhat surprising that the bubble is quite robust to the second tree being somewhat collateralizable. This feature is an indication of the first agent's great desire to borrow at a relatively low interest rate. Even if 40 percent of the second tree can be used as collateral, this is still not enough collateral for agent 1. She is willing to pay a positive price for the collateralizability of tree 1 even though it never pays any dividends.

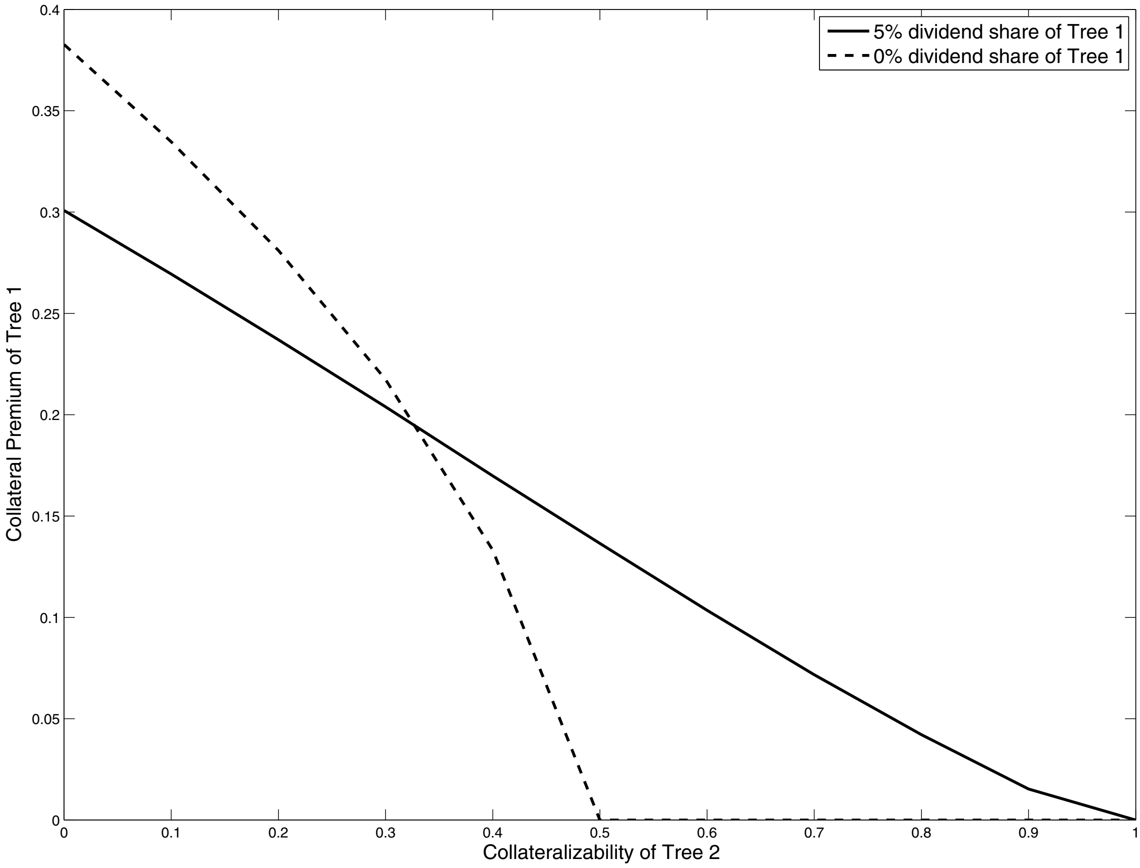

Figure 4: Collateral Premium of Tree 1 as a Function of δ_2

For economies in which the first, fully marginable, asset pays dividends of 5 percent of aggregate consumption, the collateral premium (solid line in Figure 4) remains positive even when the second asset becomes highly collateralizable. As for the case of zero dividends, the collateral premium is monotonically decreasing in the collateralizability of the second tree. Perhaps somewhat surprisingly, for small values of the collateral parameter δ_2 of the second asset, the collateral premium is higher when the first asset pays no dividends than when it pays dividends. To investigate this issue in more detail, we vary the size of the dividends of tree 1, while holding dividends of tree 2 fixed at 15 percent of aggregate consumption. Figure 5 shows how the collateral premium varies as a function of the first asset's dividends.

The solid line in the figure depicts the collateral premium of the first asset when tree 2 is not collateralizable. The premium is monotonically decreasing in the dividends of the first asset. The more collateral becomes available in the economy, the smaller is its value premium. However, the premium remains quite large even when the dividend share of this asset is 20% of aggregate endowments since collateral remains scarce. This result is consistent with our previous observations; for example, Figure 3 shows that for dividend shares of 20% excess volatility is also still significant.

The dashed line in Figure 5 depicts the collateral premium of the first asset when 25% of tree 2 are collateralizable. The collateral premium is significantly smaller than for the case of a non-marginable second asset. For very small dividends of the first asset, the collateral premium is increasing; subsequently it is decreasing. We can explain this

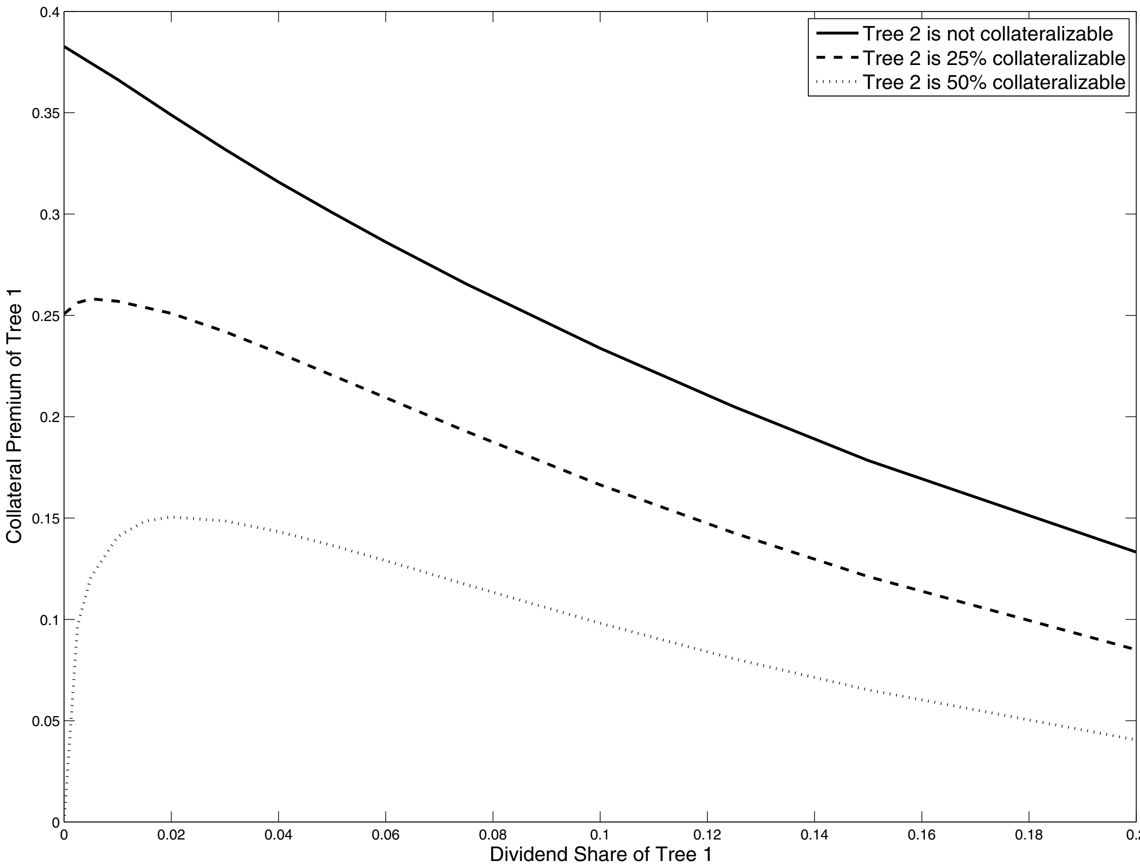

Figure 5: Collateral Premium of Tree 1 and Its Dividend Share

non-monotonicity if we examine an economy in which the collateralizability parameter of the second asset becomes even larger, $\delta_2 = 0.5$. The collateral premium in this economy is given by the dotted line. If the first asset pays no dividends, then its price is zero, that is, there is no bubble. While collateral is scarce, sufficient collateral is available in the economy to prohibit bubbles. When the dividend share of the first asset becomes positive and increases, then, initially, the collateral premium rises dramatically. It quickly peaks and then decreases monotonically as in the other two cases. Since the first asset is fully collateralizable and the second one is not, the premium remains positive. And once again, the more collateral becomes available, the smaller the collateral premium.

4.3 Bursting Bubbles and Aggregate Volatility

In the final step of our analysis, we revisit the economy with a bubble equilibrium. We have documented above that an economy in which the marginable asset pays no dividends possesses an equilibrium in which the price of the marginable asset remains positive. Such an economy has a second equilibrium in which the price of the marginable asset is zero and, therefore, collateralized borrowing is impossible. In the economic literature, an unanticipated switch from the bubble equilibrium to the non-bubble equilibrium has been interpreted as a "bursting" of the price bubble; see, for example Kocherlakota (2009).

Figure 6 shows a simulated path of asset prices both for an economy with a persisting bubble (solid lines) and an economy in which the bubble suddenly bursts in period 1

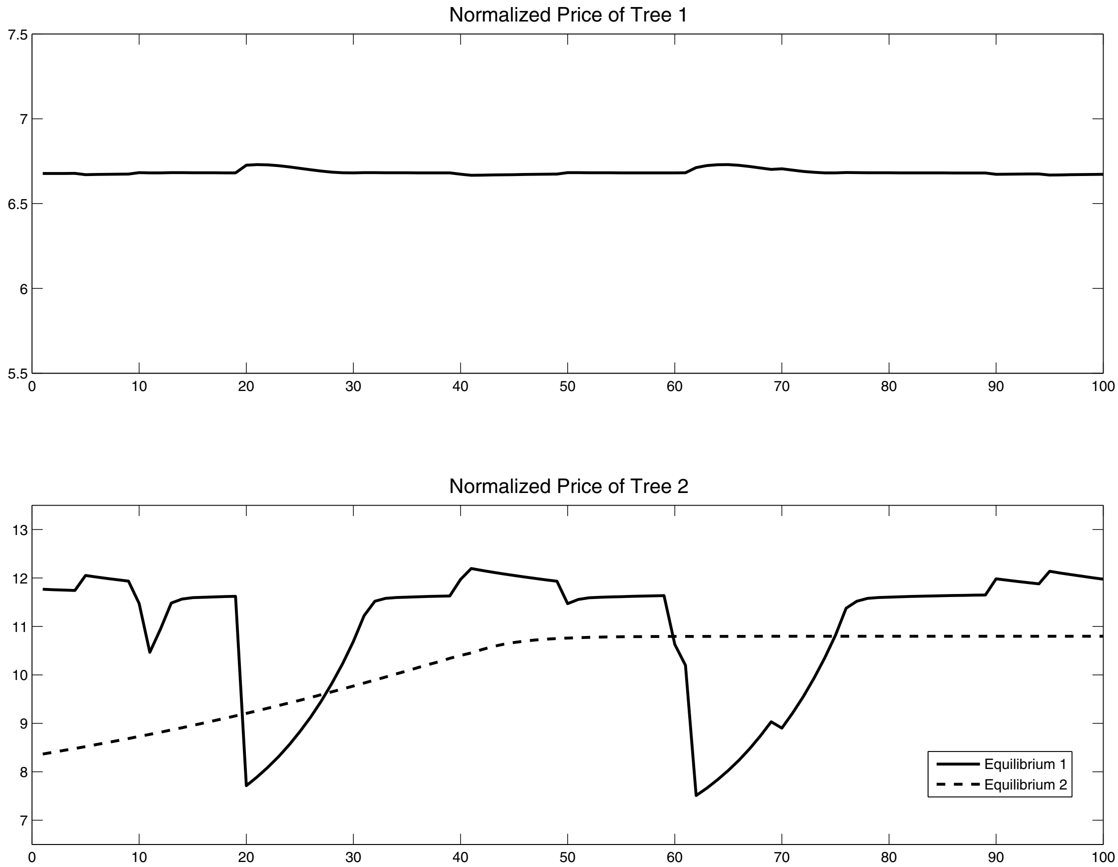

Figure 6: Simulation Paths of Equilibrium 1 with Bubble and Equilibrium 2 After Bubble Bursts

(dashed line). In the displayed simulation, the economy with the price bubble has experienced several normal growth shocks. Therefore, the normalized price of the non-marginable but dividend-paying asset, tree 2, is large. This price drops discontinuously in a dramatic fashion, when the price bubble in the marginable asset bursts. In this moment, the collateral-constrained economy turns into the benchmark model *B1: No borrowing*. During a rather slow recovery, taking about 45 periods in the displayed simulation, the normalized price of the second asset converges to its long-run value, which is constant in *B1*; see Section 3.1.

The upper graph in Figure 6 shows that the price of the marginable asset, tree 1, displays little volatility in the bubble equilibrium. In our opinion, this fact makes the interpretation of the positive price of this tree as a "bubble" somewhat less appealing. On the contrary: the price of the non-marginable asset exhibits significant volatility in the bubble equilibrium. In good (bad) times the price is significantly larger (smaller) in the bubble equilibrium than in the benchmark model *B1*. This shows that in general equilibrium a bubble in one asset may lead to higher volatility of returns for the other asset.

5 Conclusion

We summarize the results of this paper and provide a critical review of the necessary assumptions and the resulting limitations of our analysis.

5.1 Summary

In this paper we have analyzed collateral constraints and endogenous margin requirements in the context of an infinite-horizon pure exchange general equilibrium model with two heterogeneous agents and two infinitely-lived assets. We have documented that margin requirements play a quantitatively important role for the prices of the long-lived assets. This fact is true even for assets that cannot be used as collateral. In fact, perhaps somewhat surprisingly, we have shown that the presence of collateral constraints has a larger effect on both the excess return and return volatility of the non-collateralizable asset than on the corresponding values of the collateralizable asset. Moreover, we have observed that this phenomenon is rather robust to the amount of available collateral and the degrees of collateralizability. Furthermore, in the collateral-constrained economy, agents may value the collateralizability of an asset so much that it has a positive equilibrium price even if it never pays dividends.

5.2 Discussion of Assumptions and Limitations

Our analysis of exogenous collateral constraints and endogenous margin requirements rests on a number of assumptions. The most critical restrictions are the short-sale constraints on the infinitely-lived assets and some details of the model parametrization. We now critically review these limitations.

For our economic model to be numerically tractable, we have to limit the possible trades that agents can enter into. In our economy, short-sales of the long-lived assets are not permitted. Clearly, this assumption is not satisfied in practice. Investors can enter short positions in the stock market and secure such short sales by holding bonds as collateral. Allowing for such "reversed" portfolios of long bond and short stock positions appears important but would render the model (at least currently) intractable. We believe that an important area of future research in financial economics will be the examination of bond-secured short positions in long-lived financial assets or other financial securities. At the same time, we believe that, despite this limitation, our analysis in this paper contributes to our understanding of collateral constraints and endogenous margin requirements.

As in any quantitative study, our numerical results hinge on the parametrization of the economy. Our parametrization exhibits at least two special features; the two agents have quite different levels of risk aversion, and the exogenous shock process allows for very bad shocks to the growth rate of labor income and dividends. Both features are clearly important for the model to generate sizable effects of the collateral constraints on the excess returns and the return volatility of the infinitely-lived assets. However, the sensitivity analysis in the appendix shows that the qualitative properties of the quantitative results are surprisingly robust. In addition, we believe that historical (anecdotal) evidence suggests that collateral constraints are particularly important during recessions and even depression-like periods. In "normal" times, however, they have a more limited impact. Therefore, we argue that it appears to be quite sensible to examine exogenous collateral constraints and endogenous margin requirements in the context of very heterogeneous risk attitudes and large economic shocks to the economy.

In summary, for technical reasons we must impose short-sale constraints on the infinitely-lived assets; for sizable numerical results we must assume large differences in risk-aversion and allow for the possibility of disaster states. These assumptions influence the quantitative results. However, the described qualitative effects of collateral constraints and margin requirements are likely to be present in models far beyond the scope of ours.

Appendix

A Sensitivity Analysis and Extensions

As in any quantitative study, the results of our analysis depend on the parametrization of the economy. In this section, we first discuss how our results change with other preference parameters. Then we highlight the important role of the disaster shocks for our quantitative results, yet we also present model specifications with much less severe disaster shocks which nevertheless exhibit strong quantitative effects of collateral constraints. Next we report results for a varying distribution of labor income. And finally, we show that changing the size of costs of default does not substantially change our results.

A.1 Preferences

As a robustness check for the results in our baseline model (with the first asset being fully collateralizable and the second asset of identical size which cannot be used as collateral) from Section 3, we consider different specifications for the IES, risk aversion and the discount factor, β. Obviously, changes in the IES and in the risk aversion have effects on the risk-free rate. For these cases, we choose β so that the risk-free rate is equal to 1.0%. Recall that in our baseline model, we took $(IES, RA, \beta) = ((2, 2), (0.5, 7), (0.977, 0.977))$. Table 9 reports asset pricing moments for various different combinations of these parameters. For each of these specifications, we also report the standard deviation of returns for the benchmark case *B1: No bonds*, which is (almost) the same for all specifications, as normalized prices are constant in *B1* and thus returns are driven by exogenous shocks only.

(P1) shows the results for our benchmark. In (P2), where risk aversion of agent 2 is slightly smaller, the volatility of returns is almost unchanged. The marginable asset, tree 1, becomes a little more volatile, whereas volatility of the non-marginable asset decreases. In (P3) where risk aversion of agent 2 is slightly larger, the difference in volatility is higher than in the baseline economy, but market volatility is almost unchanged. (P4) and (P5) show that higher risk aversion of agent 1 does decrease overall volatility significantly but the difference between volatility of the two infinitely-lived assets remains almost unchanged. Finally, (P6) and (P7) show that our results are sensitive with respect to the intertemporal elasticity of substitution, but the qualitative insights remain intact. For an IES of 1.5 instead of 2 the results only change slightly; for an IES below 1, the quantitative

$(IES^1, IES^2), (RA^1, RA^2), (\beta^1, \beta^2)$	ER (1, 2, agg.)	STD (1, 2, agg.)	STD in B1
(P1): (2,2),(0.5,7),(0.977,0.977)	1.9, 2.6, 2.2	7.3, 8.7, 7.8	5.0
(P2): (2,2),(0.5,6),(0.983,0.983)	1.6, 2.0. 1.8	7.5, 8.4, 7.8	5.0
(P3): (2,2),(0.5,8),(0.969,0.969)	2.3, 3.4, 2.6	7.2, 9.2, 7.9	5.0
(P4): (2,2),(0.1,7),(0.977,0.977)	1.9, 2.6, 2.1	7.6, 9.0, 8.1	5.0
(P5): (2,2),(1,7),(0.976,0.976)	2.0, 2.7, 2.2	6.9, 8.3, 7.4	5.0
(P6): (0.75,0.75),(0.5,7),(0.984,0.984)	2.0, 2.8, 2.2	6.5, 6.8, 6.5	5.0
(P7): (1.5,1.5),(0.5,7),(0.979,0.979)	1.9, 2.6, 2.2	7.2, 8.5, 7.6	5.0

Table 9: Sensitivity Analysis for Preferences (all reported figures in percent)

results become weaker but the qualitative insights remain again valid.

A.2 Disaster Shocks

As explained above, it is clear that without disaster shocks, our model cannot produce realistic asset pricing moments. The specification (E2) in Table 10 shows this clearly. It shows moments of asset returns for a calibration where there is no disaster shock and probabilities of shocks 2-4 are scaled up. In this calibration, it is impossible to match a risk-free rate of 1 percent. We keep β unchanged, resulting in an unrealistically high risk-free rate of over 3 percent. Equity premia and return volatility are negligible. It is worth observing, however, that borrowing on collateral still increases volatility by 10 percent relative to the benchmarks. Even in this calibration, collateral constraints lead to movements in the wealth distribution which translate into movements in the asset prices. It just turns out that these effects are very small. Given this, it is interesting to see how the moments of asset returns behave quantitatively with respect to the size and probability of the disaster shock. In Table 10 we vary both the probability and the size of the disaster shocks, leaving preferences as in our benchmark calibration.

$\pi(s)$, $g(s)$, (β^1, β^2)	RFR	ER (1, 2, agg.)	STD (1, 2, agg.)	STD in B1
(E1): 2.2%, 0.72, (0.977,0.977)	1.0	1.9, 2.6, 2.2	7.3, 8.7, 7.8	5.0
(E2): 0.0%, n/a , (0.977,0.977)	3.2	0.2, 0.1, 0.1	1.9, 2.5, 2.2	2.0
(E3): 1.1%, 0.72, (0.977,0.977)	1.9	0.9, 1.6, 1.2	4.9, 6.3, 5.4	3.8
(E4): 2.2%, 0.86, (0.977,0.977)	2.8	0.2, 0.6, 0.4	3.1, 4.6, 3.7	3.2

Table 10: Sensitivity Analysis for Disaster Shocks (all reported figures in percent)

The specification (E1) denotes the moments for our baseline economy. In contrast, in (E3) we reduce the probability of the disaster shock from 2.2 to 1.1 percent (scaling up the probabilities of shocks 2–4 appropriately). As a result, the standard deviation decreases substantially both in the benchmark without borrowing, *B1*, and in the equilibrium with

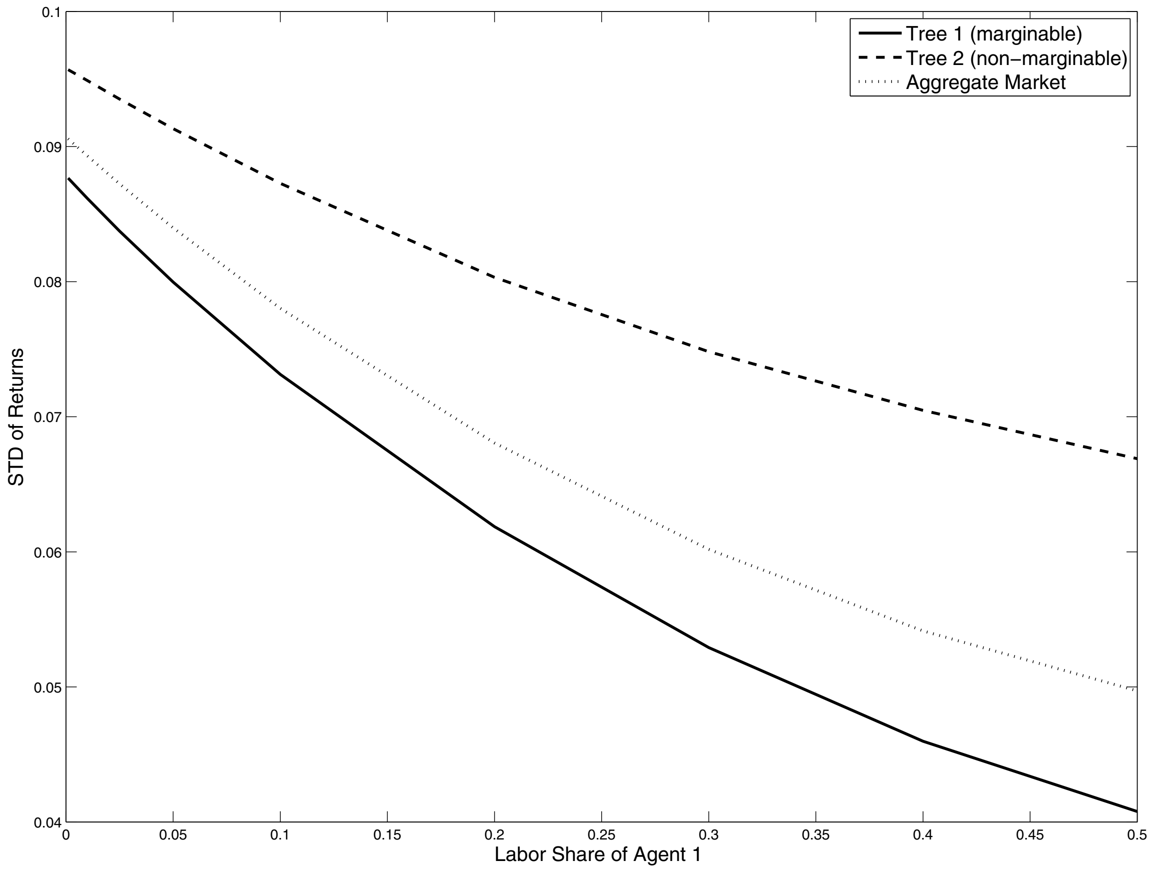

Figure 7: Volatility and the income distribution

borrowing on margin. However borrowing on margin still increases market volatility by 42 percent relative to *B1*, and it is still true that the volatility of asset 1 is much higher than the volatility of asset 2. In (E4) we consider the experiment where the disaster shock stays as likely as in the main example, but becomes less disastrous, that is, in this shock the economy only shrinks by 14 percent as opposed to 28 percent. This leads to a very large decrease in volatility. Moreover, borrowing on margin only increases market volatility by around 15 percent relative to the benchmark *B1*. Note that it is still the case, however, that volatility of tree 2 is substantially higher than the volatility of asset 1 (relative to overall volatility even more so than in our baseline case). It is clear that the size of the disaster shocks plays a crucial role for market volatility, much more so than the probability of their occurrence. This is partly due to the fact that with our costs of default, a larger disaster shock translates more or less directly into large equilibrium margin requirements, making it more likely that agent 1 becomes constrained in equilibrium.

A.3 The Distribution of Labor Income

In our baseline calibration, we assume that agent 1 receives 10 percent and agent 2 the remaining 90 percent of aggregate labor income. Figure 7 shows how the return volatilities of the two trees depend on the portion of the aggregate labor income going to agent 1. In the figure, we plot the volatility of the two tree returns and the volatility of the aggregate market as a function of the labor share of agent 1.

All three return volatilities decrease substantially with the relative size of the labor income of the less risk-averse agent 1. In the extreme case of agent 1 having zero labor income but holding a positive fraction of financial wealth, the aggregate return volatility exceeds 9 percent and is more than 15 percent larger than in the baseline economy; see Table 2. This result is perfectly consistent with our economic intuition. Whenever a bad shock (shocks 1 or 2) occurs, agent 1 cannot use any labor income to buffer the shock but instead has to sell off assets in order to fully cover her losses. Such a sale leads to price decreases for both assets. As before, she always sells the non-marginable asset before the marginable asset. The aggregate return volatility remains substantial even if the labor income share of agent 1 is 20 percent. This share roughly matches the fraction of all active stock-market participants in the US, even including those agents that own only small amounts of assets and who were therefore excluded in our baseline calibration. As the labor share of agent 1 increases to 50 percent, the excess volatility disappears. (In fact, market volatility is now slightly lower than in the benchmark models *B0*, *B1*, and *B2*, since the average risk aversion in the economy is much lower than in those three models). With such a high income share, agent 1 earns sufficient labor income to maintain her asset holdings in both trees even if a series of bad shocks occurs.

Perhaps somewhat surprisingly, the difference between the return volatilities of the two trees is increasing in the labor share of agent 1. While the return volatility of both trees decreases, the volatility of the non-marginable asset, tree 2, does so less than the volatility of the collateralizable asset, tree 1. As the labor share of the much less risk-averse agent 1 increases, she needs to sell the marginable asset much less often in response to a bad shock. While she also sells the non-marginable asset fewer times than before, she still frequently must sell a portion of it. Since tree sales by agent 1 are always accompanied by significant price drops, both return volatilities decrease in the first agent's wealth share, but the volatility of the non-marginable asset does so less.

A.4 Costs of Default

In our baseline calibration, we assume positive costs of default of 5%, which already implies that there is no trade in defaultable bonds. If costs of default are lower than that, then defaultable bonds are traded in equilibrium. Table 11 reports the trading volume in the various bonds. The reported trading volume is the average absolute bond holding of agent 1 (which is the same as that of agent 2) over the simulation paths. In brackets the maximum absolute bond holding is reported. In the absence of costs of default the trading volume is nonzero for all bonds. However, it is striking that for defaultable bonds the maximum trading volume is orders of magnitude higher than the average. This is because these bonds are traded only in very bad times when agent 1 is so poor that she cannot afford to use the no-default bond for borrowing. If costs of default are increased from zero to just one percent trade in the 2-default bond (which defaults in disasters and recessions) stops. At three percent costs of default trade in the bond that defaults only in disasters almost ceases to exist, and at five percent costs of default no trade in defaultable bonds is left. Concerning all the moments of asset prices that we are interested in throughout this paper, there is practically no difference across specifications of the costs of default. Thus, all our results are robust with respect to the size of costs of default.

	$\lambda = 0\%$	$\lambda = 1\%$	$\lambda = 2\%$	$\lambda = 3\%$	$\lambda = 5\%$
No-default bond	529(594)	522(586)	524(588)	528(592)	528(592)
1-default bond	2.2(310)	1.6(194)	0.7(75)	0.0(0.2)	0.0(0.0)
2-default bond	0.4(209)	0.0(0.0)	0.0(0.0)	0.0(0.0)	0.0(0.0)

Table 11: The Effect of Costs of Default on Average (Maximum) Bond Trading Volume (all figures in percent)

B Details on Computations

B.1 Time Iteration Algorithm

The algorithm used to solve all versions of the model is based on Brumm and Grill (2010). Equilibrium policy functions are computed by iterating on the per-period equilibrium conditions, which are transformed into a system of equations. We use KNITRO to solve this system of equations for each grid point. Policy functions are approximated by piecewise linear functions. By using fractions of financial wealth as the endogenous state variables, the dimension of the state space is equal to the number of agents minus one. Hence with two agents, the model has an endogenous state space of one dimension only. This makes computations much easier than in Brumm and Grill (2010), where two- and three-dimensional problems are solved. In particular, in one dimension reasonable accuracy may be achieved without adapting the grid to the kinks. For the reported results we used 160 or 640 grid points depending on the complexity of the version of the model, which results in average (relative) Euler errors with order of magnitude 10^{-4}, while maximal errors are about ten times higher. If the number of grid points is increased to a few thousands, then Euler errors fall about one order of magnitude. However, the considered moments only change by about 0.1 percent. Hence, using 160 or 640 points provides a solution which is precise enough for our purposes. Compared to other models the ratio of Euler errors to the number of grid points used might seem large. However, note that due to the number of assets and inequality constraints our model is numerically much harder to handle than standard models. For example, in the version with one tree and three bonds, seven assets are needed (as long and short positions in bonds have to the treated as separate assets) and we have to impose seven inequality constraints per agent.

B.2 Simulations

The moments reported in the paper are averages of 50 different simulations with a length of 10,000 periods each (of which the first 100 are dropped). This is enough to let the law of large numbers do its job even for the rare disasters.

B.3 Equilibrium conditions

For simplicity, we state the equilibrium equations as we implemented them in Matlab for economies with a single tree and a single bond. For our computation of financial markets equilibria we normalize all variables by the aggregate endowment $\bar{e}$. To simplify the notation, we drop the dependence on the date-event s^t and, in an abuse of notation, denote the normalized parameters and variables by e_t, d_t and c_t, q_t, p_t, r_t, f_t, respectively. Similarly, we normalize both the objective function and the budget constraint of agents' utility maximization problem. The resulting maximization problem is then as follows (index h is dropped).

$$\begin{array}{rrl} \max \quad u_t(c_t) = & & \left\{(c_t)^\rho + \beta\left[E\left(u_{t+1}g_{t+1}\right)^\alpha\right]^{\frac{\rho}{\alpha}}\right\}^{\frac{1}{\rho}} \\ \text{s.t.} \quad & 0 = & c_t + \phi_t p_t + \theta_t q_t - e_t - [\phi_{t-1}]^+ \dfrac{r_t}{g_t} + [\phi_{t-1}]^- \dfrac{f_t}{g_t} - \theta_{t-1}\left(q_t + d_t\right) \\ & 0 \leq & \theta_t + k_t[\phi_{t-1}]^-, \quad 0 \leq [\phi_{t-1}]^+, \quad 0 \leq [\phi_{t-1}]^-, \end{array}$$

The latter two inequalities are imposed because, for the computations, we treat the long and short position in the bond, $[\phi_{t-1}]^+$ and $[\phi_{t-1}]^-$, as separate assets.

Let λ_t denote the Lagrange multiplier on the budget constraint. The first-order condition with respect to c_t is as follows,

$$0 = (u_t)^{1-\rho}(c_t)^{\rho-1} - \lambda_t.$$

Next we state the first-order condition with respect to c_{t+1}.

$$0 = \beta u_t^{1-\rho}\left[E\left(u_{t+1}g_{t+1}\right)^\alpha\right]^{\frac{\rho-\alpha}{\alpha}}\left(u_{t+1}g_{t+1}\right)^{\alpha-1} g_{t+1}(u_{t+1})^{1-\rho}(c_{t+1})^{\rho-1} - \lambda_{t+1}.$$

Below we need the ratio of the Lagrange multipliers,

$$\frac{\lambda_{t+1}}{\lambda_t} = \beta\left[E\left(u_{t+1}g_{t+1}\right)^\alpha\right]^{\frac{\rho-\alpha}{\alpha}}(u_{t+1})^{\alpha-\rho}(g_{t+1})^\alpha\left(\frac{c_{t+1}}{c_t}\right)^{\rho-1}$$

Let μ_t denote the multiplier for the collateral constraint and let $\hat{\mu}_t = \frac{\mu_t}{\lambda_t}$. We divide the first-order condition with respect to θ_t,

$$0 = -\lambda_t q_t + \mu_t + E\left(\lambda_{t+1}\left(q_{t+1} + d_{t+1}\right)\right)$$

by λ_t and obtain the equation

$$0 = -q_t + \hat{\mu}_t + \beta\left[E\left(u_{t+1}g_{t+1}\right)^\alpha\right]^{\frac{\rho-\alpha}{\alpha}} E\left((u_{t+1})^{\alpha-\rho}(g_{t+1})^\alpha\left(\frac{c_{t+1}}{c_t}\right)^{\rho-1}(q_{t+1} + d_{t+1})\right)$$

Similarly, the first-order conditions for $[\phi_{t-1}]^+$ and $[\phi_{t-1}]^-$ are as follows,

$$\begin{array}{rcl} 0 & = & -p_t + \nu^+ + \beta\left[E\left(u_{t+1}g_{t+1}\right)^\alpha\right]^{\frac{\rho-\alpha}{\alpha}} E\left((u_{t+1})^{\alpha-\rho}(g_{t+1})^\alpha\left(\dfrac{c_{t+1}}{c_t}\right)^{\rho-1}\left(\dfrac{r_{t+1}}{g_{t+1}}\right)\right) \\ 0 & = & -p_t + \hat{\mu}_t k_t + \nu^- + \beta\left[E\left(u_{t+1}g_{t+1}\right)^\alpha\right]^{\frac{\rho-\alpha}{\alpha}} E\left((u_{t+1})^{\alpha-\rho}(g_{t+1})^\alpha\left(\dfrac{c_{t+1}}{c_t}\right)^{\rho-1}\left(\dfrac{f_{t+1}}{g_{t+1}}\right)\right), \end{array}$$

where ν^+ and ν^- denote the multipliers on $0 \leq [\phi_{t-1}]^+$ and $0 \leq [\phi_{t-1}]^-$.

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