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Top Incomes, Rising Inequality, and Welfare

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Tinbergen Institute Discussion Paper

Top Incomes, Rising Inequality, and Welfare

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Top Incomes, Rising Inequality, and Welfare*

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October 24, 2012

Abstract

This paper develops a general-equilibrium model of skill-biased technological change that approximates the observed shifts in the shares of wage and non-wage income going to the top decile of U.S. households since 1980. Under realistic assumptions, we find that all agents can benefit from the technology change, provided that the observed rise in redistributive transfers over this period is taken into account. We show that the increase in capital's share of total income and the presence of capital-entrepreneurial skill complementarity are two key features that help support the wages of ordinary workers as the new technology diffuses.

Keywords: *Income Inequality, Skill-biased Technological Change, Capital-skill Complementarity, Redistribution, Welfare.*

JEL Classification: E32, E44, H23, O33.

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1 Introduction

Income inequality in many industrial countries increased markedly over the past three decades. Most of the increase can be traced to gains made by those near the top of the income distribution. According to a recent study by the OECD (2011), "the highest 10% of earners have been leaving the middle earners behind more rapidly than the lowest earners have been drifting away from the middle." The study asserts that technological progress and a more integrated global economy have brought profound changes in the ways that firms produce and distribute goods and services, and that these changes have shifted production technologies in favor of highly-skilled individuals.

Rising inequality from top incomes is particularly evident in the U.S. economy. Autor, et al. (2006) show that since the mid-1980s, upper tail U.S. wage dispersion has increased significantly while lower tail wage dispersion has actually declined. The share of total pre-tax income including capital gains going to the top decile of U.S. households rose from 35% in 1980 to around 48% in 2010 (Piketty and Saez 2003, updated). The increase in the top decile income share was driven by shifts in both labor and capital incomes. Changes in capital gains and dividend income were the two largest contributors to the increase in the Gini coefficient from 1996 and 2006 according to a study by the Congressional Research Service (Hungerford 2011). Capital's share of total income in the U.S. economy increased from about 35% in 1980 to around 41% in 2010. Given that the distribution of wealth in the U.S. economy is highly skewed, the observed increase in capital's share of income would be expected to disproportionately benefit households near the top of the income distribution.[1] As a mitigating factor, transfer payments from the government and businesses to individuals increased from 10% of GDP in 1980 to around 15% in 2010. These transfers would be expected to disproportionately benefit households outside the top decile of the income distribution.

This paper examines the welfare consequences of a gradual shift in firms' production technologies that increases income inequality in a manner consistent with U.S. experience over the past three decades. The framework of our analysis is a general equilibrium model in which the top decile of households owns 100 percent of the productive capital stock—a setup that roughly approximates the highly skewed

[1] The top decile of U.S. households owns approximately 80 percent of financial wealth and about 70 percent of total wealth including real estate. See Wolff (2006), Table 4.2, p. 113.

distribution of U.S. financial wealth.[2] Unlike income inequality, the degree of wealth inequality in the U.S. economy has remained relatively steady over time. The consumption of the capital owners in the model is funded from wages and dividends while the consumption of the remaining agents, called workers, is funded from wages and redistributive government transfers. All agents supply labor endogenously to firms. Capital owners are interpreted as entrepreneurs whose labor input exhibits complementarity with the stock of physical capital. This effect, which we label as "capital-entrepreneurial skill complementarity" works in much the same way as the mechanism proposed by Krusell, et al. (2000), except that here the complementarity effect applies more narrowly to the labor supply of the top decile, as opposed to the broader population of college-educated workers. An empirical study by Lemieux (2006) provides support for our specification. Specifically, he finds that wage inequality among college-educated workers has increased significantly in recent decades. The study concludes (p. 199) that "changes in wage inequality are increasingly concentrated in the very top end of the wage distribution."

We show that the welfare effects of rising inequality in the model depend crucially on several features. These include: (1) the nature of capital owners' expectations (which affects perceptions of permanent income and the resulting investment/saving response), (2) the assumed paths for redistributive government transfers and capital's share of total income, and (3) the degree of complementarity between physical capital and entrepreneurial labor. Under realistic assumptions, we find that all agents can benefit from the technology change, provided that the observed rise in redistributive transfers over this period is taken into account. The increase in capital's share of total income and the presence of capital-entrepreneurial skill complementarity are two key features that help support the wages of ordinary workers as the new technology diffuses.

We model skill-biased technological change as a diffusion process that shifts the parameters of the representative firm's constant elasticity of substitution (CES) production function in a way that approximates observed movements in the shares of wage and non-wage income going to the top decile of U.S. households since 1980. Specifically, the share parameters for the three productive inputs (physical capital, entrepreneurial labor, and ordinary worker labor) are allowed to evolve according to

[2] Similar concentrated capital ownership models have been applied recently to asset pricing. See, for example, Danthine and Donaldsen (2008), Guvenen (2009), and Lansing (2011). Mankiw (2000) examines the implications of such a model for fiscal policy.

an S-shaped trajectory, consistent with empirical studies on the manner in which new innovations are adopted over time (Comin, et al. 2008). We calibrate the law of motion for the diffusion process to approximately match the average U.S. adoption rate for three important technology innovations, namely, personal computers, mobile telephones, and internet use. Coincident with the technology diffusion process, we allow redistributive government transfers from the top decile to the remainder of households to increase in a manner consistent with U.S. data.

Our approach to modeling skill-biased technological change is similar to the framework of Goldin and Katz (2007) who allow CES production function share parameters to shift over time as a way of capturing technology-induced changes in the demand for skilled versus unskilled labor. According to Acemoglu and Autor (2012), shifts in these parameters can also be interpreted as capturing "skill-replacing technical changes" that increase firms' demand for one type of skill at the expense of another.[3]

The introduction of any new technology naturally involves considerable uncertainty about its potential widespread use in the future. We therefore examine the role of expectations in shaping the transition paths of the endogenous variables and the resulting welfare effects. We first consider the case where capital owners have perfect foresight about the transition path.[4] While this information assumption may be viewed as extreme, it serves as a useful benchmark. Next, we examine the case where capital owners employ myopic (or random walk) expectations. Specifically, their forecasts for variables dated $t+1$ or later are given by the most recently observed value of the same variable. Such a forecast rule can be viewed as boundedly-rational because it economizes on the costs of collecting and processing information. Finally, we consider a formulation labeled "learning" in which the share of capital owners with knowledge about the laws of motion governing the transition increases gradually over time as the new technology is adopted.

The welfare outcomes for both types of agents are sensitive to the way that expectations are formed. Capital owners always benefit from the technology change but the size of their welfare gains depend on their degree of foresight. Their optimal investment response and the resulting path for their consumption depend crucially on whether they foresee the permanent shift in their income. Workers' welfare may

[3] Along somewhat similar lines, Ríos-Rull and Santaeulàlia-Llopis (2010) introduce "redistribution shocks" which take the form of stochastic variation in the share parameters of a Cobb-Douglas production function.

[4] Workers consume their wage income plus transfers each period, so they make no intertemporal decision.

either rise or fall, depending on the magnitude of the capital owners' investment response which in turn influences the equilibrium path of workers' wages. Under perfect foresight, welfare gains are highest for capital owners but workers suffer a welfare loss. In this case, capital owners immediately increase their consumption at the expense of investment because they foresee the large increase in their permanent income. The initial jump in their consumption yields a large welfare gain—in excess of 30% of per-period consumption for the baseline calibration. However, the resulting slowdown in capital accumulation lowers the paths of workers' wages and consumption relative to the model's no-change trend. As a result, workers suffer a welfare loss of 1.3% of per-period consumption in the baseline model under perfect foresight.

In the case of myopic expectations, capital owners do not foresee the large increase in their permanent income. Consequently, their consumption does not jump at the beginning of the transition, but rather increases gradually along with their current income. We view this scenario as more realistic than the perfect foresight regime. Similarly, investment increases gradually relative to the no-change trend which boosts capital accumulation and raises the paths of workers' wages and consumption. At the same time, redistributive government transfers are growing faster than GDP, as observed in the data. For the baseline model, the welfare gain for capital owners is about 9% of per-period consumption whereas workers now achieve a welfare *gain* of about 1.5%. The welfare results for the learning regime fall in between those for perfect foresight and myopic expectations. Similar to myopic expectations, the learning mechanism precludes an immediate jump in capital owners consumption at the beginning of the transition path. However, as more capital owners learn about the process governing their future income, their consumption starts increasing faster, eventually catching up to the perfect foresight trajectory. Under learning, capital owners' achieve a welfare gain of about 15% of per-period consumption whereas workers achieve a welfare gain of about 0.6%.

As part of the analysis, we consider how different categories of income contribute to the welfare effects of the transition. When the ratio of redistributive government transfers to GDP is held constant at the 1980 level of 10% (rather than increasing to 15% as in the data), capital owners enjoy a welfare gain of 16% of per-period consumption under myopic expectations versus a gain of 9% in the baseline scenario. Workers now suffer a small welfare loss of 0.15% versus a baseline gain of 1.5%. This experiment highlights the importance of the rising trend of redistributive transfers

in allowing workers to achieve a positive welfare gain in the baseline scenario. We also consider an experiment where capital's share of total income is held constant at its 1980 level while the share of wage income going to the top decile continues to rise in a manner consistent with the data. Both types of agents are made worse-off relative to the baseline scenario. Under myopic expectations, the capital owners' welfare gain is now only 1.1% versus a baseline gain of 9%. Workers suffer a welfare loss of 2.6% versus a baseline gain of 1.5%. Interestingly, both types of agents benefit from an increase in capital's share of total income even though capital ownership is concentrated in the hands of the top decile. As discussed further below, this result is due to the positive wage impacts of a technology-induced increase in the productivity of physical capital. The positive wage impacts are stronger in the presence of capital-entrepreneurial skill complementarity.

To gauge the influence of capital-entrepreneurial skill complementarity, we compare the baseline model to one with a standard Cobb-Douglas production function. In the Cobb-Douglas model, both types of labor exhibit the same (unitary) elasticity of substitution with physical capital. The share parameters of the Cobb-Douglas production function are assumed to shift over time in manner that matches the U.S. income distribution data. We find that both types of agents are considerably worse-off in the Cobb-Douglas world. For example, under myopic expectations, the capital owners' welfare gain is only 0.4% of per-period consumption versus a baseline gain of 9%. Workers now suffer a large welfare loss if 12.5% versus a baseline gain of 1.5%. The absence of capital-entrepreneurial skill complementarity means that a technology change which raises the productivity of physical capital now bestows less benefits on entrepreneurial labor, thus lowering the capital owner's wage path relative to the baseline model. The wage path of workers is also lowered, as dictated by the equilibrium conditions of the competitive labor market. Lower wage paths for both types of agents bring about lower labor supplies, which in turn slows the growth rate of aggregate output during the transition period. The upward shift in the top decile income share still allows the capital owner's consumption path to surpass the no-change trend, but the gains are much smaller than in the baseline model. But the worker's consumption path now drops below the no-change trend, leading to a large welfare loss. This experiment shows that capital-entrepreneurial skill complementarity is an important feature that not only benefits the suppliers of entrepreneurial labor; it can also deliver benefits to ordinary workers.

We also investigate the sensitivity of the welfare results to changes in the values of other key parameters, including the elasticities of intertemporal substitution for consumption and for labor supply, the subjective time discount factor, and the speed of technology diffusion. We show that each of these parameters can have a significant impact on welfare outcomes. Overall, we find that the range of possible welfare outcomes from skill-biased technological change is enormous, even in the relatively simple framework considered here with only two types of agents. These findings suggest that conclusions regarding the appropriate policy response to rising income equality can be strongly influenced by the details of any particular model.

1.1 Related Literature

Much research has focused on the rising wage premium of skilled versus unskilled workers as an important driver of rising U.S. income inequality. The literature emphasizes the impact of skill-biased technological change which disproportionately benefits workers with a college education.[5] Heathcote, et al. (2010, 2011) focus on the welfare consequences of rising inequality that is driven by shifts in the relative wages of groups with different education levels. In contrast, our analysis focuses on the welfare consequences of rising inequality that is driven by gains in top incomes, i.e., the highest 10% of earners. We also take into account observed shifts in the distribution of both labor and capital incomes.

As an alternative to skill-biased technological change, Piketty, et al. (2011) argue that the dramatic rise in top incomes has been driven mainly by institutional changes which strengthened the bargaining power of top earners at the expense of lower earners. According to this theory, the shift in bargaining power has enabled rent-seeking top earners to successfully push their pay above their marginal product. Along these lines, Kumhof and Ranciere (2011) consider a model where rising income inequality (as measured by the income share of the top 5% of households) is driven by a decline in the bargaining power of workers. However, in reduced form, the worker's loss of bargaining power can be interpreted as roughly equivalent to a shift in the firm's production technology. Their analysis focuses on the link between rising inequality and a shock-induced financial crisis. In contrast, our aim is to gauge the

[5] A partial list of research in this area includes: Katz and Autor (1999), Krusell, et al. (2000), Acemoglu (2002), Agion (2002), Card and DiNardo (2002), Hornstein, et al. (2005), Goldin and Katz (2007, 2008), and Acemoglu and Autor (2012). For a recent overview of the literature, see Violante (2012).

welfare consequences of the observed three-decade rise in the U.S. top decile income share.

Our finding that all agents can achieve welfare gains in an economy with rising income inequality compliments the results of Heathcote, et al. (2010, 2011). As in our analysis, they obtain smaller welfare gains for agents who are myopic. This is because myopic agents in their model fail to anticipate the future rise in the college wage premium and thus do not invest in a college education. In our model, welfare gains are smaller for myopic capital owners because they fail to anticipate the future rise in their permanent income, and thus postpone consumption relative to the perfect foresight trajectory. However, the capital owners' myopia is actually beneficial for workers because it leads to faster capital accumulation which in turn boosts workers' wages and consumption.

In contrast to the structural model approach, empirical studies have mostly found large welfare losses from rising income inequality (Attanasio and Davis 1996, Krueger and Perri 2004). As a caveat, it must be noted that empirical data on shifts in relative wages may not give an accurate picture of the quantities that matter for household welfare, namely consumption and leisure. Krueger and Perri (2006) argue that the impact of rising income inequality on consumption inequality was partially mitigated by an increase in household borrowing to finance consumption at the lower end of the income distribution. Recently, however, Aguiar and Bils (2011) and Attanasio, et al. (2012) argue that consumption inequality, when properly measured, appears to mirror income inequality.

The remainder of the paper is organized as follows. Section 2 presents some stylized facts about the increase in income inequality in the U.S. economy over the past three decades. Section 3 describes the model. Section 4 describes our calibration procedure. Section 5 presents our quantitative results. Section 6 concludes. An appendix provides details on the model solution procedure and the welfare computation.

2 Stylized Facts

Figure 1 shows the evolution of the share of pre-tax income (including capital gains) going to the top decile of U.S. households, as documented by Piketty and Saez (2003, updated). The top decile income share rose from 35% in 1980 to around 48% in

Figure 1: The top decile income share increased from 35% in 1980 to around 48% in 2010. The trend was driven by shifts in the distribution of income from wage and non-wage sources. Capital's share of total income, as defined by the U.S. Bureau of Labor Statistics, increased from about 35% in 1980 to around 41% in 2010.

2010.[6] Income from wage and non-wage sources both contributed to the rise, but most of the trend is attributable to the rising share of wage income going to the top decile. It is worth noting, however, that the category of wages includes income derived from the exercise of employee stock options—a component that blurs the distinction between labor and capital incomes. Capital's share of total income, as defined by the U.S. Bureau of Labor Statistics, increased from about 35% in 1980 to around 41% in 2010.[7]

[6] Updated annual data through 2010 are available from Emmanuel Saez's website: http://elsa.berkeley.edu/~saez/. The trends in this figure and others are constructed using the Hodrick-Prescott filter with a smoothing parameter of 100.

[7] Capital's share is defined here as one minus labor's share where labor's share is obtained from www.bls.gov/data using series ID PRS85006173. The tabulated series is indexed to 100 in 1992 which corresponds to a labor share of 63.2%. For additional details, see Gomme and Rupert (2004).

Figure 2: Decomposition of top decile income share into wage and non-wage sources. Non-wage sources of income for the top decile (roughly in order of importance) include: entrepreneurial income, capital gains, dividends, interest income, and rents.

Figure 2 shows the decomposition of the top decile income share into its component parts. Non-wage sources of income for the top decile (roughly in order of importance) include: entrepreneurial income, capital gains, dividends, interest income, and rents.

Figure 3 plots transfer payments to individuals as a percentage of GDP from 1959 to 2010. These are payments from governments and businesses to individuals or nonprofit institutions serving individuals.[8] Examples include benefits from Old Age, Survivors, and Disability Insurance (OASDI), Medicare and Medicaid benefits, Supplemental Security Income, Family Assistance, Food Stamps, and Unemployment Insurance Compensation. The figure shows that the ratio of transfer payments to GDP increased from 10% of GDP in 1980 to around 15% in 2010.

[8] Data on transfer payments and GDP are from the Federal Reserve Bank of St. Louis' FRED data base. Payments from businesses accounted for only about 1% of total transfers in 2005. For a detailed description of the various transfer programs, see http://www.bea.gov/regional/pdf/spi2005/06%20Personal%20Current%20Transfer%20Receipts.pdf

Figure 3: Transfer payments from the government and businesses to individuals increased from 10% of GDP in 1980 to around 15% of GDP in 2010.

While some of the run-up in transfer payments in recent years appears to have been triggered by the government's response to the financial crisis of 2007-2009, it is also true that pre-tax income inequality, as measured by the top decile income share, continued to trend upward over this period. More generally, it seems reasonable to view the upward trend in transfer payments from 1980 to 2010 as a deliberate effort by the government to address the trend of rising pre-tax income inequality. In the model, we make the simplifying assumption that transfer payments represent a pure redistribution from the top decile to the remainder of households, accomplished via a lump-sum tax on capital owners administered by the government. We investigate the sensitivity of our results the assumed path for these transfers.

A basic assumption of our analysis is that the increase in U.S. pre-tax income inequality over the past three decades was driven by a slow moving technological change that made production processes more capital intensive and raised the wages of highly-skilled entrepreneurs in the top decile. As evidence of technological change, Figure 4 plots the U.S. adoption trajectories for three important technology innovations,

Figure 4: The diffusion path for information and communication technology in the U.S. economy can be approximated by the law of motion $\pi_t = \pi_{t-1} + \kappa\pi_{t-1}(1 - \pi_{t-1})$, with $\kappa = 0.25$.

namely, personal computers, mobile cellular telephones, and internet use—three series which measure the spread of information and communication technology (ICT).[9] All three series exhibit an S-shaped trajectory—a typical pattern for the manner in which new innovations are adopted over time (Comin, et al. 2008). Comparing Figure 4 to Figure 1 shows that the spread of ICT in the U.S. economy follows roughly the same trajectory as the rise in the top decile income share. While suggestive, this comovement does not prove causation running from ICT diffusion to income inequality. However, it is consistent with the mechanism of skill-biased technological change emphasized by many authors. There are other examples in history when major technological change was accompanied by a rise in income inequality. These include the

[9] Personal computer ownership data are from the NBER's Cross-country Historical Adoption of Technology (CHAT) data set available at http://www.nber.org/data/chat/. Data for years 2002 and 2003 are missing. Data on mobile celluar telephones and internet use are from the World Bank's infrastructure data set available at http://data.worldbank.org/indicator.

Industrial Revolution in Great Britain from 1760 to 1860 (Greenwood, 1999) and the U.S. economy during the 1920s (Atkinson, et al. 2011). Regarding the latter period, Nicholas (2008) argues that the 1920s was "a period of unprecedented technological advance."

To formalize the process of technology diffusion in the model, we employ the following nonlinear law of motion

$$\pi_t \;=\; \pi_{t-1} \;+\; \kappa \pi_{t-1} \left(1 - \pi_{t-1}\right), \tag{1}$$

where $\pi_t \in [0,1]$ represents the share of firms employing the new technology and $\kappa > 0$ governs the speed of diffusion. Starting from a small positive value, the law of motion implies $\pi_t \to 1$ as $t \to \infty$. Figure 4 plots the theoretical diffusion path with $\kappa = 0.25$ which is the calibration employed in our quantitative analysis. Starting at $\pi_0 = 0$ in 1980, we assume that 1% of firms unilaterally adopt the new technology at $t = 1$, corresponding to the year 1981. For $t > 1$, the theoretical diffusion path tracks roughly in between the observed diffusion paths for personal computers, mobile telephones, and internet use, reaching an adoption share of about 92% in 2010. The theoretical diffusion path takes about 18 years to move from a 10% adoption share to 90%. This result is close to the corresponding average period of 15 years estimated by Jovanovich and Lach (1997) for a wide variety of new product innovations.

3 Model

The model economy consists of workers, capital owners, competitive firms, and the government. There are n times more workers than capital owners, with the total number of capital owners normalized to one. Capital owners represent the top decile of households as measured by both wealth and income. Naturally, firms are owned by the capital owners. Both types of agents supply labor endogenously to firms. The government's only role is to redistribute income from capital owners to workers via a lump-sum tax and transfer scheme.

3.1 Workers

The individual workers' decision problem is to maximize

$$\widehat{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^w - \frac{D^w}{\gamma} H_t \left(\ell_t^w\right)^{\gamma}\right]^{1-\alpha} - 1}{1-\alpha}, \tag{2}$$

subject to the budget constraint

$$c_t^w \;=\; w_t^w \ell_t^w \;+\; T_t/n, \tag{3}$$

where the symbol $\widehat{E}_t$ represents the agent's subjective expectation conditional on information available at time t. Under rational expectations, $\widehat{E}_t$ corresponds to the mathematical expectation operator E_t evaluated using the laws of motion that govern the technology diffusion process. The parameter β is the subjective time discount factor, c_t^w is the individual worker's consumption, and ℓ_t^w is labor supply. Along the lines of Greenwood, et al. (1988), the disutility of non-leisure time is governed by the functional form $(D^w/\gamma^w)\; H_t\,(\ell_t^w)^\gamma$, where $D^w > 0$, and $\gamma > 1$. This specification implies that foregone leisure is adjusted to reflect trend growth according to $H_t = \exp(z_t)$, where z_t represents labor-augmenting technological progress, to be described more fully below. The labor disutility function may be interpreted as the reduced form of a more-elaborate specification that incorporates home production.[10] The elasticity of intertemporal substitution in labor supply is given by $1/(\gamma - 1)$. As $\gamma \to \infty$, the model reduces to one with fixed labor supply. The parameter α represents the inverse of the elasticity of intertemporal substitution (EIS) for the worker's composite consumption basket.

Workers are assumed to incur a transaction cost for saving or borrowing small amounts which prohibits their participation in financial markets. As a result, they simply consume their income each period, consisting of labor income $w_t^w \ell_t^w$ and a per-worker transfer payment T_t/n received from the government.

The worker's first-order conditions with respect to c_t^w and ℓ_t^w are given by

$$\left[c_t^w - \frac{D^w}{\gamma} H_t\,(\ell_t^w)^\gamma\right]^{-\alpha} \;=\; \lambda_t^w, \tag{4}$$

$$D^w H_t\,(\ell_t^w)^{\gamma-1}\left[c_t^w - \frac{D^w}{\gamma}\,H_t\,(\ell_t^w)^\gamma\right]^{-\alpha} \;=\; \lambda_t^w w_t^w, \tag{5}$$

where λ_t^w is the Lagrange multiplier associated with the budget constraint (3). Since the worker makes no intertemporal decision, the subjective expectation operator $\widehat{E}_t$ does not appear in the first-order conditions. The first-order conditions imply the following labor supply equation

$$\ell_t^w \;=\; \left(\frac{w_t^w}{D^w H_t}\right)^{\frac{1}{\gamma-1}}. \tag{6}$$

[10] The linearity in H_t ensures that agents' time allocations are stationary along the model's balanced growth path. See Greenwood, Rogerson, and Wright (1995, p. 161).

3.2 Capital Owners

Capital owners represent the top decile of earners. Their decision problem is to maximize

$$\widehat{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^c - \frac{D^c}{\gamma} H_t \left(\ell_t^c\right)^\gamma\right]^{1-\alpha} - 1}{1-\alpha}, \tag{7}$$

subject to the budget constraint

$$c_t^c \;+\; p_t s_{t+1} \;=\; w_t^c \, \ell_t^c \;+\; (p_t + d_t)\, s_t \;-\; T_t, \tag{8}$$

where c_t^c is the individual capital owner's consumption and ℓ_t^c is labor supply. For simplicity, we assume that the functional form of the utility function and the preference parameters β, γ, and α are the same for both capital owners and workers. Capital owners earn labor income in the amount $w_t^c \, \ell_t^c$ and may invest in shares of the firm's equity in the amount s_{t+1} at the ex-dividend price p_t. Shares owned in the previous period yield a dividend d_t.[11]

Equity shares are assumed to exist in unit net supply. Market clearing therefore implies $s_t = 1$ for all t. In equilibrium, the capital owner's budget constraint becomes $c_t^c = w_t^c \, \ell^c + d_t - T_t$, which shows that the capital owner's consumption is funded from wage income and dividends, after subtracting a lump-sum tax levied by the government.

The capital owner's first-order conditions with respect to c_t^c, ℓ_t^c, and s_{t+1} are given by

$$\left[c_t^c - \frac{D^c}{\gamma} H_t \left(\ell_t^c\right)^\gamma\right]^{-\alpha} \;=\; \lambda_t^c, \tag{9}$$

$$D^c H_t \left(\ell_t^c\right)^{\gamma-1} \left[c_t^c - \frac{D^c}{\gamma} H_t \left(\ell_t^c\right)^\gamma\right]^{-\alpha} \;=\; \lambda_t^c w_t^c, \tag{10}$$

$$p_t \;=\; \widehat{E}_t \, \beta \, \frac{\lambda_{t+1}^c}{\lambda_t^c} \left(p_{t+1} + d_{t+1}\right), \tag{11}$$

where λ_t^c is the Lagrange multiplier associated with the budget constraint (8). The capital owner's labor supply equation is given by

$$\ell_t^c \;=\; \left(\frac{w_t^c}{D^c H_t}\right)^{\frac{1}{\gamma-1}}. \tag{12}$$

[11] The capital owner's decision problem can be represented in different ways. We employ this particular decentralization because it shows the link between the firm's equity price and investment.

As $D^c \to \infty$ we have $\ell_t^c \to 0$ such that only the workers supply labor. This case corresponds to a standard framework for considering optimal redistributive capital taxation (Judd 1985, Lansing 1999, and Krusell 2002).

3.3 Firms

Competitive firms are owned by the capital owners who we interpret as entrepreneurs. Firms produce output according to the technology

$$y_t = A\left\{\theta_t \left[(1-\rho_t)\, k_t^{\psi_k} + \rho_t \left[\exp(z_t)\, \ell_t^c\right]^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}} + (1-\theta_t)\left[\exp(z_t)\, n\, \ell_t^w\right]^{\psi_\ell}\right\}^{\frac{1}{\psi_\ell}} \quad (13)$$

$$\text{where} \quad \psi_k \equiv \frac{\sigma_k - 1}{\sigma_k}, \qquad \psi_\ell \equiv \frac{\sigma_\ell - 1}{\sigma_\ell},$$

$$z_t = z_{t-1} + \mu, \quad (14)$$

$$\theta_t = \theta_0 \exp\left[\delta_\theta\left(\pi_t - \pi_0\right)\right], \quad (15)$$

$$\rho_t = \rho_0 \exp\left[\delta_\rho\left(\pi_t - \pi_0\right)\right], \quad (16)$$

$$\pi_t = \pi_{t-1} + \kappa \pi_{t-1}\left(1 - \pi_{t-1}\right), \quad (17)$$

with z_0, θ_0, ρ_0, and π_0 given. The symbol k_t is the firm's stock of physical capital and z_t is a labor-augmenting technology process that evolves as a random walk with drift. The drift parameter μ determines the trend growth rate of output. We abstract from stochastic variation in trend growth because we wish to focus on the dynamics that arise from shifts in the income shares, as opposed to ordinary business cycle fluctuations. The parameter ψ_k depends on the elasticity of substitution between physical capital and entrepreneurial labor, denoted by σ_k. The parameter ψ_ℓ depends on the elasticity of substitution between entrepreneurial inputs and workers' labor, denoted by σ_ℓ. When $\sigma_\ell > \sigma_k$, the production function exhibits what we call "capital-entrepreneurial skill complementarity." This means that entrepreneurial labor is more complementary to physical capital than ordinary workers' labor. In other words, the capital owners' entrepreneurial skills are more closely coupled to the physical assets of the firm than are workers' skills.

Motivated by the technology diffusion process shown in Figure 4, our production specification is intended to capture the emergence of unique business skills tied to the spread of ICT. Examples would be the skills associated with setting up and operating a technology company such as Microsoft, Apple, Amazon, Ebay, Oracle,

Google, etc. This type of skills yielded significant monetary rewards (mainly in the form of valuable stock options) to the founders and early employees who conceived and executed the firms' original business strategies. The entrepreneurial skills we have in mind are much more concentrated than the broader college education-based skills emphasized by Krusell, et al. (2000), Goldin and Katz (2007, 2008), Heathcote, et al. (2010, 2011) and others. Our setup is motivated by empirical evidence which shows that the observed trends in U.S. income inequality over the past three decades were driven mainly by gains in top incomes, as documented in various ways by Autor, et al. (2006), Lemieux (2006), Atkinson, et al. (2011), and OECD (2011).

When $\sigma_k = \sigma_\ell = 1$ (or $\psi_k = \psi_\ell = 0$), we recover the standard Cobb-Douglas production technology which does not exhibit capital-entrepreneurial skill complementarity. When $\sigma_k \to 0$ and $\sigma_\ell \to 0$ (or $\psi_k \to -\infty$ and $\psi_\ell \to -\infty$), the production technology takes a Leontief form such that capital and both types of labor become perfect compliments. When $\sigma_k \to \infty$ and $\sigma_\ell \to \infty$ (or $\psi_k \to 1$ and $\psi_\ell \to 1$), capital and both types of labor become perfect substitutes.

The OECD (2011) argues that technological progress and globalization have shifted firms' production technologies in favor of highly-skilled workers, yielding these workers higher rewards from labor at the expense of others who lack these unique skills. We capture this idea by assuming that the representative firm's production technology (13) shifts over time, as governed by equations (15) through (17). Specifically, the diffusion process shifts the income share parameters θ_t and ρ_t along an S-shaped trajectory as the new technology is gradually adopted by firms. The state variable π_t can be interpreted as the share of firms employing the new technology. Our setup can also be viewed as capturing a process whereby old firms using obsolete technology die off over time and are replaced by new firms using the latest technology. Along these lines, Hobijn and Jovanovic (2001, p. 1219) argue that "major technological change—like the IT [information technology] revolution—destroys old firms. It does so by making machines, workers, and managers obsolete."

Goldin and Katz (2007) develop an analytical framework that allows CES production function share parameters to shift over time as a way of capturing skill-biased technological change. Our setup can be interpreted in the same way. To see this, we

can rewrite the production function (13) as follows

$$y_t \;=\; A \exp(z_t) \left\{ \left[\theta_t^{\frac{\psi_k}{\psi_\ell}} (1-\rho_t)\, k_{n,t}^{\psi_k} + \theta_t^{\frac{\psi_k}{\psi_\ell}} \rho_t\, (\ell_t^c)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} + (1-\theta_t)\,(n\,\ell_t^w)^{\psi_\ell} \right\}^{\frac{1}{\psi_\ell}}, \tag{18}$$

where we define $k_{n,t} \equiv k_t / \exp(z_t)$ as the normalized capital stock (a stationary variable). In the above formulation, shifts in z_t represent "neutral" technology changes that affect output generally, whereas shifts in θ_t or ρ_t represent "biased" technology changes that affect the relative demand for the different productive inputs. Equation (18) also shows that the quantitative impact of a given shift in either θ_t or ρ_t on input demand will depend on the values the substitution elasticity parameters σ_k and σ_ℓ which govern the values of ψ_k and ψ_ℓ.

Equation (17) has two steady states at $\pi_t = 0$ and $\pi_t = 1$. At the initial steady state, we have $\theta_t = \theta_0$ and $\rho_t = \rho_0$. At date $t = 1$, corresponding to the year 1981, we assume that 1% of firms unilaterally adopt the new technology (or, alternatively, that 1% of existing firms die and are replaced by new firms using the new technology). Given this initial impulse, the diffusion law of motion implies $\pi_t \to 1$ as $t \to \infty$. The response parameters δ_θ and δ_ρ govern the degree to which the technology diffusion shifts the production function parameters θ_t and ρ_t, which in turn govern the shares of wage and non-wage incomes going to the top decile of households. When $\delta_\theta = \delta_\rho = 0$, the model economy grows along the "no-change trend," such that the top decile income share does not increase over time, but instead remains constant at the level observed in 1980.

Resources devoted to investment augment the stock of physical capital according to the law of motion

$$k_{t+1} \;=\; B\, k_t^{1-\lambda}\, i_t^{\lambda}, \tag{19}$$

with k_0 given. The parameter $\lambda \in (0, 1]$ is the elasticity of new capital with respect to new investment. When $\lambda < 1$, equation (19) reflects the presence of capital adjustment costs.[12]

Under the assumption that the labor market is competitive, firms take wages as given and choose sequences of $n\,\ell_{t+j}^w$, ℓ_{t+j}^c, and k_{t+1+j} to maximize the following

[12] Equation (19) can be interpreted as a power-function approximation of the following specification employed by Jermann (1998): $k_{t+1} = k_t\,[1 - \delta + \psi_0\,(i_t\,/k_t)^{\psi_1}\,]$.

discounted stream of expected dividends:

$$\widehat{E}_0 \sum_{j=0}^{\infty} M^c_{t+j} \underbrace{\left[y_{t+j} - w^w_{t+j}\, n\, \ell^w_{t+j} - w^c_{t+j}\, \ell^c_{t+j} - i_{t+j} \right]}_{d_{t+j}}, \tag{20}$$

subject to the production function (13) and the law of motion for capital (19). Firms act in the best interests of their owners such that dividends in period $t+j$ are discounted using the capital owner's stochastic discount factor $M^c_{t+j} \equiv \beta^j \lambda^c_{t+j}/\lambda^c_t$, where λ^c_t is given by equation (9).

The firm's first-order conditions with respect to $n\,\ell^w_t$, ℓ^c_t, and k_{t+1} are given by:

$$w^w_t \;=\; (1 - s^c_t)\; y_t / (n\,\ell^w_t)\,, \tag{21}$$

$$w^c_t \;=\; \left(s^c_t - s^k_t\right)\; y_t/\ell^c_t \tag{22}$$

$$i_t/\lambda \;=\; E_t\, M^c_{t+1}\, [\, s^k_{t+1}\, y_{t+1} - i_{t+1} \;+\; i_{t+1}/\lambda\,], \tag{23}$$

where s^c_t represents the share of pre-tax income going to capital owners and s^k_t represents capital's share of total income. The share of pre-tax income going to workers is $1 - s^c_t$, while labor's share of total income is $1 - s^k_t$. The share of pre-tax income going to entrepreneurial labor is $s^c_t - s^k_t$.

Equations (21) and (22) show that each type of labor is paid its marginal product. Comparing the firm's intertemporal first-order condition (23) to the equity pricing equation (11) shows that the ex-dividend price of an equity share is given by $p_t = i_t/\lambda$. The equity share is a claim to a perpetual stream of dividends $d_{t+1} = s^k_{t+1}\, y_{t+1} - i_{t+1}$ starting in period $t+1$.[13] The model's adjustment cost specification (19) implies a direct link between the equity price and investment, consistent with a standard Tobin's q framework. This feature is also consistent with the observed low-frequency comovement between the S&P 500 stock price index and business investment in recent decades, as documented by Lansing (2012).

[13] After taking the derivitive of the profit function (20) with respect to k_{t+1}, we have multiplied both sides of the resulting first-order condition by k_{t+1}, which is known at time t.

Given the form of the production function (13), we have

$$s_t^c \;=\; \frac{\partial y_t}{\partial \ell_t^c}\frac{\ell_t^c}{y_t} + \frac{\partial y_t}{\partial k_t}\frac{k_t}{y_t} \;=\; \frac{\theta_t \left[(1-\rho_t)\, k_{n,t}^{\psi_k} + \rho_t\, (\ell_t^c)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}}}{\theta_t \left[(1-\rho_t)\, k_{n,t}^{\psi_k} + \rho_t\, (\ell_t^c)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}} \;+\; (1-\theta_t)\,(n\,\ell_t^w)^{\psi_\ell}} \tag{24}$$

$$s_t^k \;=\; \frac{\partial y_t}{\partial k_t}\frac{k_t}{y_t} \;=\; \frac{\theta_t\,(1-\rho_t)\left[(1-\rho_t)\, k_{n,t}^{\psi_k} + \rho_t\, (\ell_t^c)^{\psi_k}\right]^{\frac{\psi_\ell-\psi_k}{\psi_k}} k_{n,t}^{\psi_k}}{\theta_t \left[(1-\rho_t)\, k_{n,t}^{\psi_k} + \rho_t\, (\ell_t^c)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}} \;+\; (1-\theta_t)\,(n\,\ell_t^w)^{\psi_\ell}}, \tag{25}$$

where $k_{n,t} \equiv k_t/\exp(z_t)$. In the Cobb-Douglas case when $\psi_k = \psi_\ell = 0$, the above equations simplify to $s_t^c = \theta_t$ and $s_t^k = \theta_t\,(1-\rho_t)$.

3.4 Government

The government redistributes income from capital owners to workers by means of a lump-sum tax and transfer scheme. We abstract from distortionary taxation given that most of the revenue collected by distortionary taxes in the U.S. economy is used for either direct government purchases of goods and services or debt service—two features which are absent from our model. Moreover, in the case of the OASDI program, transfers are financed by a tax on income up to a given threshold, so there is no marginal tax distortion for income earned above the threshold.

We assume that the ratio of aggregate transfer payments to output in the model is governed by the following law of motion:

$$\tau_t \;\equiv\; T_t/y_t \;=\; \tau_0 \exp\left[\delta_\tau\,(\pi_t - \pi_0)\right], \tag{26}$$

where τ_t represents the lump-sum tax rate and τ_0 is given. We link τ_t to the technology adoption share π_t as a way of capturing the rising trend of U.S. transfer payments relative to GDP plotted earlier in Figure 3. The underlying assumption is that the rapid growth in various types of means-tested transfers and income security programs from 1980 to 2010 reflects a deliberate effort by the government to try to offset the trend of rising pre-tax income inequality. The response parameter δ_τ governs the path of transfers during the transition period. Along the economy's no-change trend, we have $\delta_\tau = 0$ such that the ratio of transfers to GDP remains constant.

3.5 Expectations

Following Heathcote, et al. (2010), we consider different assumptions about the way in which agents form expectations about future variables that will affect their permanent income. Here, only firms and capital owners make forecasts about future variables; workers simply consume their wage income plus transfers each period. In the appendix, we show that the firm's intertemporal first-order condition (23) can be written in terms of stationary variables as follows:

$$\mathrm{f}\left(x_t,\, \ell_t^c,\, n\,\ell_t^w,\, k_{n,t},\, \pi_t\right) \;=\; \widehat{E}_t\, \mathrm{h}\left(x_{t+1},\, \ell_{t+1}^c,\, n\,\ell_{t+1}^w,\, k_{n,t+1},\, \pi_{t+1}\right), \tag{27}$$

where $x_t \equiv i_t/y_t$ is the investment-output ratio and $k_{n,t} \equiv k_t/\exp\left(z_t\right)$ is the normalized capital stock.

To establish a benchmark, we first consider the standard case of rational expectations where agents are assumed to know the laws of motion governing the evolution of future variables. In our setting, rational expectations corresponds to perfect foresight because the laws of motion that govern trend growth and the diffusion of new technology abstract from stochastic variation. Under perfect foresight, we drop the subjective expectation operator $\widehat{E}_t$ in equation (27), thus yielding a set of deterministic nonlinear difference equations that can be solved numerically, as described in the appendix.

The notion that agents have perfect foresight about the process governing their future income is obviously an extreme assumption. This is especially true in our setting, where the economy is undergoing a never-before-seen shift in technology that significantly alters firms' production processes. At the other end of the information spectrum, we might assume that agents are myopic, i.e., their forecast about a future variable is given by the most recently-observed value of the same variable. This type of forecast rule is optimal when the variable in question evolves as a random walk. But even if this is not the case, a random walk forecast can be viewed as boundedly-rational because it economizes on the costs of collecting and processing information. As noted by Nerlove (1983, p. 1255): "Purposeful economic agents have incentives to eliminate errors up to a point justified by the costs of obtaining the information necessary to do so...The most readily available and least costly information about the future value of a variable is its past value." To implement myopic expectations in equation (27), we assume $\widehat{E}_t\, \mathrm{h}\left(t+1\right) = \mathrm{h}\left(t-1\right)$, which implies that agents do not

observe the realized value $\mathrm{h}(t)$ at the time they construct their forecast.[14]

According to Heathcote et al. (2010, p. 717) "Myopic beliefs and perfect foresight represent polar extreme models for expectations, and presumably the truth lies somewhere in between the two." Along these lines, we consider an intermediate case labeled "learning" in which the share of firms and capital owners with knowledge about the future transition path increases gradually over time as the new technology is adopted. Put differently, we assume that entrepreneurial agents who adopt the new technology acquire knowledge about its speed of diffusion and its implications for their future income. To implement learning in equation (27), we assume $\widehat{E}_t \mathrm{h}(t+1) = \omega_t \mathrm{h}(t+1) + (1-\omega_t)\mathrm{h}(t-1)$, where ω_t represents the fraction of entrepreneurial agents with knowledge about the laws of motion governing the transition path. Intuitively, one might expect the fraction of knowledgeable agents to start at zero and then increase gradually over time, eventually reaching unity when the new technology has been fully adopted. We can achieve such a trajectory very simply by linking the fraction of knowledgeable agents to the diffusion process itself, i.e., by imposing $\omega_t = \pi_t$.

It should be noted that the learning regime can be interpreted as imposing an even higher level of sophistication on the part of knowledgeable capital owners. Not only do the knowledgeable capital owners need to understand the dynamics of the exogenous technology diffusion process, but now they also need to understand the influence of the remaining myopic capital owners on the future transition path of the economy. For this reason, one could argue that myopic expectations regime is the most plausible setup, given the assumed one-time shift in the production technology.

4 Model Calibration

Table 1 summarizes our choice of parameter values for the baseline model. Some parameters are set to achieve target values for steady-state variables while others are set to commonly-used values in the literature.

[14] Alternativelty, we could assume $\widehat{E}_t \mathrm{h}(t+1) = \mathrm{h}(t)$ which would allow for simultaneity in the observed and expected values of the forecast variables. For our setting, the solution turns out to be nearly identical to the case where $\widehat{E}_t \mathrm{h}(t+1) = \mathrm{h}(t-1)$. This result may not hold for others settings, however. See, for example, Lettau and Van Zandt (2003).

Table 1: Baseline Model Parameter Values

Parameter	Value	Description/Target
n	9	Capital owners = top income decile.
μ	0.02	Per capita trend growth = 2%.
α	2	EIS = $1/\alpha = 0.5$.
β	0.964	Equity return = 8%.
γ	3	Labor supply elasticity = 0.5.
D^w	0.65	Initial worker labor supply $\ell^w = 1$.
D^c	5.54	Initial relative wage $w^c/w^w = 2$.
σ_k	0.4	Empirical estimates.
σ_ℓ	1.0	Empirical estimates.
A	0.816	Match Cobb-Douglas initial steady state.
B	1.273	Initial steady-state $k/y = 2.6 \times 0.8$.
λ	0.088	Initial steady-state $i/y = 0.21 \times 0.8$.
κ	0.25	Match ICT diffusion path for U.S. economy.
π_0	0	Initial steady state $\pi = 0$.
θ_0	0.350	Initial steady-state $s^c = 0.35$.
ρ_0	0.001	Initial steady-state $s^k = 0.35 \times 0.8 = 0.28$.
τ_0	0.100	Initial steady-state transfers/GDP = 10%.
δ_θ	0.336	Final steady-state $s^c = 0.49$.
δ_ρ	0.685	Final steady-state $s^k = 0.41 \times 0.8 = 0.328$.
δ_τ	0.405	Final steady-state transfers/GDP = 15%.

The time period in the model is one year. The number of workers per capital owner is $n = 9$ so that capital owners represent the top decile of households. In the model, capital owners possess 100% of the physical capital wealth, whereas the top decile of U.S. households owns approximately 80% of financial wealth. Our setup implies a Gini coefficient for physical capital wealth of 0.90. The Gini coefficient for financial wealth in U.S. data has ranged between 0.89 and 0.93 over the period 1983 to 2001.[15]

The parameter $\mu = 0.02$ implies a per capita trend growth rate of 2%, consistent with the long-run U.S. average. The value $\alpha = 2$ implies an EIS of $1/\alpha = 0.5$ for the composite consumption basket of each agent—a typical value.[16] In the sensitivity analysis, we also consider the values $1/\alpha = 1$ and $1/\alpha = 0.33$. Given the baseline values for α and μ, we choose β such that the steady-state net equity return is $r^s = \beta^{-1} \exp(\alpha\,\mu) - 1 = 8\%$, consistent with the long-run real return on the S&P 500 stock price index.

[15] See Wolff (2006), Table 4.2, p. 113.
[16] See, for example, Mendoza (2010).

We choose $\gamma = 3$ to achieve an intertemporal elasticity of substitution in labor supply of $(\gamma - 1)^{-1} = 0.5$, consistent with the range of estimates obtained by Eissa (1996) and Mulligan (1999), among others. In the sensitivity analysis, we also examine the effects of a more-elastic labor supply with $(\gamma - 1)^{-1} = 1.5$. We choose the labor supply disutility parameter D^w in order to normalize $\ell^w = 1$ at the initial steady state. Given this value, we choose D^c to achieve a target relative wage at the initial steady state of $w^c/w^w = 2$. For comparison, Heathcote, et al. (2010, p. 686) report a male college wage premium of about 1.4 in 1980, whereas Gottschalk and Danziger (2005, p. 238) report a male wage ratio of about 4 when comparing the top decile to the bottom decile. The wage ratio w^c/w^w in our model compares the top decile to the remainder of households, so we would expect it to fall somewhere in between the values reported by the two studies, but likely closer to the value reported by Heathcote, et al. (2010).

The baseline values for the production function curvature parameters σ_k and σ_ℓ strike a balance between various empirical estimates. Using data on the observed wage premium of college-educated workers in the U.S. economy from 1963 to 1992, Krusell, et al. (2000, p. 1041) estimate a substitution elasticity of 0.67 between equipment capital and skilled labor. They estimate a substitution elasticity of 1.67 between equipment capital and unskilled labor. There is also a large literature that estimates the elasticity of substitution between aggregate physical capital and aggregate labor, without distinguishing between skilled versus unskilled labor. In a review of this literature, Chirinko (2008) concludes that the evidence suggests a range of 0.4 to 0.6 for the aggregate capital-labor substitution elasticity. The capital-entrepreneurial skill complementarity effect considered here applies to the top decile which is a more exclusive group than the pool of college-educated workers. Workers comprise nine-tenths of the population in our model, and thus represent a broader group than the pool of unskilled (non-college) workers. Based on this reasoning, we set $\sigma_k = 0.4$ and $\sigma_\ell = 1$, which imply that both types of labor in our model exhibit stronger complementarity to physical capital than the college versus non-college workers considered by Krusell, et al. (2000). In the sensitivity analysis, we consider different combinations of values for σ_k and σ_ℓ, including the Cobb-Douglas case when $\sigma_k = \sigma_\ell = 1$.

We normalize the production function parameter A to unity in the Cobb-Douglas case. When $\sigma_k \neq 1$ or $\sigma_\ell \neq 1$, we choose the value of A to maintain the same initial steady-state value of k_n as in the Cobb-Douglas model. In this way, changes in either

σ_k or σ_ℓ identify a family of CES production functions that are distinguished only by the elasticity parameters, and not by their initial steady-state allocations.[17] The parameter B in the capital law of motion (19) is chosen to be consistent with the long-run average capital-output ratio in the U.S. economy. The average ratio from annual data is about 2.6, but this figure includes all physical capital whereas the top decile of U.S. households owns about 80% of financial wealth. We therefore apply a scale factor of 0.8 to the U.S. capital-output ratio to arrive at a target capital-output ratio of 2.08 for the model. The parameter λ in the capital law of motion (19) is chosen to be consistent with the U.S. average investment-output ratio of about 0.21 (including business investment and purchases of consumer durables). We again apply a scale factor of 0.8 to the U.S. ratio to arrive at a target investment-output ratio of 0.168 for the model.

The initial share parameter $\theta_0 = 0.35$ is chosen to match the 35% income share of the top decile of U.S. households in 1980, as plotted earlier in Figure 1. Similarly, we choose ρ_0 to match capital's share of total income in the U.S. economy in 1980, also plotted in Figure 1. Similar to the other capital-related parameters, we apply a scale factor of 0.8 to the 1980 capital income share of 0.35, resulting in an initial steady-state capital share in the model of 0.28. The technology diffusion speed is set to $\kappa = 0.25$, as noted earlier in the discussion of Figure 4 and implies that technology adoption is about 92% complete after three decades. Given θ_0, ρ_0 and κ, we choose δ_θ and δ_ρ to achieve target values for the top decile income share s^c and the capital share s^k at the final steady state. The target values at the final steady state are slightly above the (scaled) end-of-sample values plotted in Figure 1. Finally, we choose $\tau_0 = 0.10$ to match the 10% ratio of U.S. transfers to GDP in 1980, as shown in Figure 3. Based on the trend plotted in Figure 3, we choose δ_τ to achieve a target ratio of 15% at the final steady state.

5 Quantitative Results

In this section, we examine the quantitative implications of the model via numerical simulations. We first consider the baseline model's dynamic response to shifting income shares under different expectation regimes. Next, we examine the implications of departing from the baseline assumptions regarding the path for redistributive

[17] Klump and Saam (2008) emphasize that such a normalization procedure is necessary to avoid "arbitrary and inconsistent results" when comparing CES production models with different parameterizations.

government transfers, the path for capital's share of total income, and the degree of capital-entrepreneurial skill complementarity. Finally, we consider the welfare consequences of rising income inequality and its sensitivity to different model specifications and parameter values. Details regarding the model solution procedure and the welfare computation are contained in the appendix.

5.1 Dynamic Response to Shifting Income Shares: Baseline Model

Figure 5 plots the transition paths for selected model variables starting from the initial steady state with $\pi_0 = 0$. At date $t = 1$, we assume that 1% of firms unilaterally adopt the new technology. For $t > 1$, the technology diffusion process is governed by equations (15) through (17). For each variable, we plot the equilibrium trajectory for three different expectation regimes: perfect foresight (solid blue line), myopic expectations (dashed red line), and learning (dash-dotted green line).

The top left panel of Figure 5 plots the transition path for the top decile income share s_t^c. By design, the model path roughly approximates the U.S. top decile income share shown earlier in Figure 1. The model path starts at 35% and then increases to about 48% at $t = 30$, corresponding to the year 2010. Our baseline calibration with $\sigma_\ell = 1$ implies $\psi_\ell = 0$ such that $s_t^c = \theta_t$ from equation (24). Since θ_t follows an exogenous law of motion, expectations do not influence the trajectory of s_t^c, unlike the other variables in the figure. Capital's share of total income s_t^k (top right panel) starts from an initial steady state of 28% and eventually reaches a final steady state of 32.8%. In between, the trajectory is governed by equation (25) which depends on the endogenous variables $k_{n,t}$ and ℓ_t^c even when $\psi_\ell = 0$. Under all three expectation regimes, the transition path for s_t^k exhibits some overshooting such that value at $t = 30$ is somewhat above the final steady state value.

The role of expectations is most clearly illustrated in the middle left panel of Figure 5, which plots the equilibrium investment-output ratio i_t/y_t. Under perfect foresight, the investment-output ratio drops sharply at $t = 1$. This is because capital owners foresee the large increase in their permanent income over the future transition period. As a result, they immediately increase their consumption at the expense of investment. While such dynamics do not seem very plausible, it must be remembered that our model abstracts from stochastic shocks which would introduce a precautionary saving motive, thus limiting the sharp drop in the investment-output ratio.[18]

[18] Our closed economy model also abstracts from foreign capital inflows. Such inflows could finance an increase in domestic investment even if there were a sharp drop in domestic saving.

Figure 5: Under perfect foresight, the investment-output ratio drops sharply at $t = 1$ because capital owners forsee the increase in their permanent income. The drop in investment slows capital accumulation, thereby hindering the growth of wages and total income relative to the model with either myopic expectations or learning.

Under myopic expectations, capital owners do not foresee the increase in their permanent income. Consequently, their consumption at $t = 1$ does not jump (investment at $t = 1$ does not fall), but rather the capital owner's consumption and investment both increase gradually along with current income. Under learning, the trajectories for all variables initially mimic those under myopic expectations, but the paths eventually catch-up and merge with the perfect foresight trajectories.

The middle right panel of Figure 5 plots the evolution of the capital stock expressed as a percent deviation from the no-change trend (which holds income shares constant at their initial levels). The capital stock increases fastest under myopic expectations due to the higher investment trajectory, which boosts capital accumulation. In contrast, the perfect foresight path for the capital stock initially drops

below the no-change trend due to the sharp drop in the investment-output ratio at $t = 1$. Later, however, the rising marginal product of capital from the technology diffusion process (as summarized by the shifts in θ_t and ρ_t) stimulates an increase in investment which allows the capital stock to surpass the no-change trend.

The bottom panels in Figure 5 plot the agents' total income after taxes and transfers, again expressed as percent deviations from the no-change trend. These two panels provide insight into the welfare effects to be discussed later. In the bottom left panel, the capital owner's total income increases fastest under myopic expectations and slowest under perfect foresight. This is due to the faster rate of capital accumulation under myopic expectations which contributes to faster wage growth for capital owners. But workers also receive wage benefits from faster capital accumulation. The bottom right panel shows that the worker's total income is highest under myopic expectations and lowest under perfect foresight. For workers, more income translates directly into more consumption, which in turn contributes to higher welfare. For capital owners, more income under myopic expectations translates into more investment, thus postponing consumption and reducing welfare relative to the perfect foresight case. Hence, as we shall see, myopia is harmful for capital owners' welfare but beneficial for workers' welfare.

Figure 6 plots the paths of some additional model variables as percent deviations from the no-change trend. The top left panel shows the immediate jump in the capital owner's consumption that occurs under perfect foresight. This is the flip-side to the sharp drop in the investment-output ratio shown in Figure 5. The immediate jump in the capital owner's consumption hinders capital accumulation, which lowers the wage paths for both capital owners and workers, as shown in the two middle panels. The top right panel of the figure shows that myopic expectations deliver the most favorable consumption path for workers, again because faster capital accumulation boosts wages relative to the other two expectation regimes. Notice that the path for the worker's consumption in Figure 6 is identical to the path for the worker's total income (including transfers) shown in Figure 5. The worker's consumption under myopic expectations initially declines relative to the no-change trend as the technology shift relentlessly shrinks the pre-tax income share of workers. Eventually, however, when $t \gtrsim 30$, recovering wages for workers (from capital accumulation) together with rising transfer payments from the government lead to an increase in the worker's consumption relative to the no-change trend. As a result, the myopic

Figure 6: The capital owner's consumption jumps immediately at $t = 1$ under perfect foresight. This hinders capital accumulation and lowers the wage trajectories for both capital owners and workers. The myopic expectations regime delivers the most favorable consumption trajectory for workers because faster capital accumulation boosts wages relative to the other two expectation regimes. The transition paths for labor hours mimic the patterns for wages.

expectations regime can deliver welfare gains to workers.

To better understand the behavior of wages during the transition, we can combine the firm's first-order conditions (21) and (22) with the labor supply equations (6) and (12) to obtain the following equilibrium relationship

$$\begin{aligned} w_t^w &= w_t^c \left[\frac{1 - s_t^c}{s_t^c - s_t^k} \right] \frac{\ell_t^c}{n\, \ell_t^w}, \\ &= w_t^c \left[\frac{1 - s_t^c}{s_t^c - s_t^k} \right]^{\frac{\gamma-1}{\gamma}} \left(\frac{D^w}{D^c} \right)^{\frac{1}{\gamma}}, \end{aligned} \tag{28}$$

which is a similar to the standard skill premium equation estimated by numerous

empirical studies.[19] The term in square brackets summarizes the effects of "skill-biased" or "skill-replacing" changes in technology. Changes in the ratio $\ell_t^c / (n\, \ell_t^w)$ capture shifts the relative supplies of the two types of labor.

Equation (28) shows that the worker's wage w_t^w is influenced by several variables. An increase in the capital owner's wage w_t^c (due to technology diffusion or ordinary trend growth) will serve to increase the worker's wage. In contrast, an increase in the top decile income share s_t^c or an increase in the wage income share of the top decile $s_t^c - s_t^k$ will both serve to decrease the worker's wage. All else equal, an increase in capital's share of total income s_t^k will serve to increase the worker's wage. The strength of these various opposing effects depends strongly on the degree of capital-entrepreneurial skill complementarity. In the baseline model with $\sigma_k < \sigma_\ell$, capital owners enjoy a large increase in w_t^c as the technology diffusion increases the productivity of both capital and entrepreneurial labor which are tightly coupled when $\sigma_k = 0.4$. The increase in w_t^c helps to offset the upward shifts in s_t^c and $s_t^c - s_t^k$ such that the equilibrium path for w_t^w is higher than otherwise. As evidence, the middle panels of Figure 6 show that the largest increase in w_t^c occurs under myopic expectations, which also delivers the most favorable path for w_t^w.

The bottom panels of Figure 6 show that the transition paths for labor hours mimic the patterns for wages. This is a direct consequence of the labor supply equations (6) and (12) which show that movements in ℓ_t^w and ℓ_t^c are directly proportional to movements in w_t^w and w_t^c, respectively. The increase in labor hours for capital owners, together with the increase in the productivity of the two entrepreneurial inputs (k_t and ℓ_t^c) is more than enough to offset the decline in the worker labor hours. As a result, aggregate output eventually surpasses the no-change trend under all expectation regimes (top left panel of Figure 10). The higher level of aggregate output boosts the amount of redistributive transfers received by workers each period since transfers are computed as a fraction of GDP.

5.2 Departures from the Baseline Model

We now consider three experiments that depart from the baseline model.[20] The results will prove helpful for understanding the welfare effects to be discussed later. The

[19] See, for example, Goldin and Katz (2007, p. 7) and Acemoglu and Autor (2012, p. 434).

[20] Whenever a parameter value is changed from the baseline value shown in Table 1, we recalibrate the remaining parameters, where applicable, to achieve the same empirical targets as the baseline model.

Figure 7: When the ratio of redistributive transfers to GDP is held constant at its initial level, wage paths are lower than the baseline paths under perfect foresight but higher than the baseline paths under myopic expectations. Holding capital's share of income constant at its initial level lowers the wage paths of both types of agents relative to the baseline paths. The results for the Cobb-Douglas model are qualitatively similar to those for holding s_t^k constant, but the quantitative effects on the wage paths are now much larger.

first experiment imposes $\delta_\tau = 0$ in equation (26) such that the ratio of redistributive government transfers to GDP remains constant at the 1980 level of $T_t/y_t = 10\%$, rather than increasing to 15% as in the data. The second experiment holds capital's share of total income constant at the initial calibrated level of $s_0^k = 0.35 \times 0.8 = 0.28$, rather than increasing to a final share of $0.41 \times 0.8 = 0.328$.[21] The third experiment imposes $\sigma_k = \sigma_\ell = 1$ in equation (13) to recover a standard Cobb-Douglas production function which omits the feature of capital-entrepreneurial skill complementarity.

[21] For this experiment, the target top decile income share at the final steady state is adjusted downward from the baseline value of $s^c = 0.49$ to $s^c = 0.442$ in order to maintain the same absolute change in the top decile wage income share as in the baseline model. We then solve for a sequence of values for ρ_t from $t = 1$ to $t = 1500$ such that $s_t^k = s_0^k$ while θ_t is governed by equation (12) using the re-calibrated value $\delta_\theta = 0.234$.

Figure 7 shows how each experiment influences the path of wages, as expressed in percent deviations from the no-change trend. Figures 8 and 9 show the effects on the actual consumption trajectories of capital owners and workers. Figure 10 shows the effects on aggregate output.[22]

In the baseline model, the capital owner's consumption rises faster than the no-change trend under all expectation regimes (top left panel of Figure 8).

Figure 8: Holding transfers to GDP constant boosts the capital owner's consumption trajectory relative to the baseline model. The opposite is true when either capital's share of total income is held constant or when the production function is Cobb-Douglas. In all cases, however, the capital owner's consumption trajectory surpasses the no-change trend.

The worker's consumption in the baseline model initially falls below the no-change trend as the top decile income share shifts upward in favor of capital owners (top left panel of Figure 9). But under myopic expectations, the worker's consumption later starts catching up and can even surpass the no-change trend as rising wages (from

[22] For clarity, we omit the learning regime plots in Figures 7 through 10 because these always track in between the plots for the other two expectation regimes.

capital accumulation) and rising transfer payments from the government increase the worker's total income.

Figure 9: Under myopic expectations, the worker's consumption trajectory can surpass the no-change trend for $t \gtrsim 35$ in the baseline model and when transfers to GDP are held constant. However, the worker's consumption trajectory remains below the no-change trend when capital's share of total income is held constant or when the production function is Cobb-Douglas.

Under perfect foresight, aggregate output in the baseline model initially experiences a slowdown relative to the no-change trend, but growth later accelerates to allow output to surpass the no-change trend for $t > 25$ (top left panel of Figure 10). This type of trajectory is consistent with the narratives emphasized by Hornstein and Krusell (1996) and Greenwood and Yörükoğlu (1997) whereby a skill-biased technology improvement initially leads to a measured slowdown in total factor productivity. The empirical evidence on the links between income inequality and growth remains inconclusive. In a recent cross-country study, Berg and Ostry (2011) find that higher levels of income inequality are often (but not always) associated with shorter growth spells, such that higher inequality tends to reduce an economy's average long-run

Figure 10: In the baseline model, aggregate output surpasses the no-change trend during the transition as the technology shift increases the productivity of physical capital and entrepreneurial labor. A similar results obtains when holding transfers to GDP constant. But aggregate output grows slower than the no change trend when capital's share of total income is held constant or when the production function is Cobb-Douglas.

growth rate. Figure 10 shows that, depending on assumptions, our model can produce simulations in which rising income inequality is associated with either faster or slower output growth in comparison to the no-change trend.

Effect of Redistributive Government Transfers

Under perfect foresight, holding T_t/y_t constant lowers the wage paths for both types of agents relative to the baseline paths (top panels of Figure 7). In contrast, the wage paths for both types of agents are *raised* relative to the baseline paths under myopic expectations (bottom panels of Figure 7). Holding T_t/y_t constant leads to a larger initial jump in the capital owner's consumption under perfect foresight because the agent foresees that future lump-sum tax rates will not be increasing, thus

implying higher permanent income relative to the baseline model. While beneficial for the welfare of capital owners, the larger initial jump in consumption slows capital accumulation which depresses the wage paths of both types of agents relative to the baseline model. In the case of myopic expectations, holding T_t/y_t constant allows the capital owner's consumption and investment to both increase faster than the baseline paths because after-tax income is now higher in each period. The resulting boost in capital accumulation raises the wage paths of both types of agents relative to the baseline wage paths. In the long-run, the ratio of lump-sum transfers to GDP has no effect on the marginal products of labor so the wage paths eventually converge to the baseline paths, regardless of the expectation regime.

Under perfect foresight, holding T_t/y_t constant leads to a larger initial jump in the capital owner's consumption (top right panel of Figure 8). The larger initial jump is detrimental to the worker's wage and consumption paths. But under myopic expectations, the higher after-tax income for capital owners induces higher investment and hence a higher wage path for workers relative to the baseline model. Consequently, the worker's consumption path can still catch up and surpass the no-change trend, despite the constant transfer ratio (top right panel of Figure 9). Aggregate output surpasses the no-change trend under both expectations regimes (top right panel of Figure 10).

Effect of Capital's Share of Total Income

Figure 7 shows that holding s_t^k constant lowers the wage paths for both types of agents relative to the baseline paths, regardless of the expectation regime. The capital owner's wage path continues to significantly exceed the no-change trend (i.e., the percent deviation remains in positive territory) but the worker's wage path now drops below the no-change trend and stays there—representing a permanent downward level shift. This experiment shows that both types of agents derive wage benefits from a rise in capital's share of total income even though capital ownership is concentrated in the hands of the top decile. The intuition for this result is straightforward. Since factor markets are competitive, any increase in s_t^k reflects an increase in the productivity of physical capital. In the presence of capital-entrepreneurial skill complementarity, a more productive capital stock also raises the marginal product of entrepreneurial labor, thus bestowing wage benefits on capital owners. The equilibrium conditions of the labor market, as summarized by equation (28), imply that workers can also receive wage benefits, since the marginal products of both types of labor are positively

linked along the model's balanced growth path.

In Figures 8 and 9, we see that holding s_t^k constant leads to less-favorable consumption trajectories for both types of agents relative to the baseline model. This result is due to the less-favorable income paths for both types of agents. The capital owner's consumption trajectory still exceeds the no-change trend (bottom left panel of Figure 8) but the worker's consumption trajectory now drops below the this trend and remains there (bottom left panel of Figure 9). Recall that in the baseline model, the worker's consumption trajectory was able to eventually surpass the no-change trend, particularly under myopic expectations. The bottom left panel of Figure 10 shows that aggregate output grows slower than the no-change trend when s_t^k is held constant. This is because the technology change now omits an important feature that serves to increase the productivity of both physical capital and entrepreneurial labor (which are strong compliments in production).

Effect of Capital-Entrepreneurial Skill Complementarity

The Cobb-Douglas experiment can be viewed as a more extreme version of the previous experiment that holds s_t^k constant. The absence of capital-entrepreneurial skill complementarity means that a technology change which raises the productivity of physical capital yields lower wage paths than otherwise for both types of agents. Figure 7 shows that the wage paths in the Cobb-Douglas model are significantly lower than the baseline paths, regardless of the expectation regime. Although w_t^c continues to exceed the no-change trend, the magnitude of the increase is now much smaller than in the baseline model. The behavior of the worker's wage can once again be understood from the labor market equilibrium relationship (28). The smaller net increase in w_t^c over the transition means that the dynamics of w_t^w now tend to be dominated by shifts in the income shares s_t^c and $s_t^c - s_t^k$, which transfer resources away from workers. Accordingly, the permanent shifts in the income shares now push w_t^w well below the no-change trend.

The lower wage path for workers reduces their labor supply by enough to keep aggregate output well-below the no-change trend (bottom right panel of Figure 10). Lower output during the transition implies lower transfer payments for workers since transfers are computed as a fraction of aggregate output. Consequently, the worker's total income takes a hit from two sides: lower wages and a lower level of transfers than otherwise, resulting in a severe drop in consumption relative to both the baseline model and the no-change trend (bottom right panel of Figure 9).

The capital owner's consumption trajectory still exceeds the no-change trend, but the gains are much smaller than in the baseline model (bottom right panel of Figure 8). Although capital owners receive a lower wage path relative to the baseline model, the effect on their consumption trajectory is mitigated by a lower level of lump sum taxes that must be paid to the government each period.

5.3 Welfare Analysis

Table 2 summarizes the welfare effects of rising income inequality for a variety of different model specifications. Welfare effects are measured by the constant percentage amount by which the agent's composite consumption basket in the no-change economy must be adjusted upward or downward each period to make lifetime utility equal to that obtained in the transition economy. Going from left to right in the table, the three expectation regimes postulate successively higher degrees of knowledge about the economy's future transition path on the part of capital owners. The boxed entries in the table represent the best welfare outcome for each type of agent in a given expectation regime.

Table 2: Welfare Effects of Rising Income Inequality

	Myopic Expectations		Learning		Perfect Foresight	
Model Specification	Capital Owners	Workers	Capital Owners	Workers	Capital Owners	Workers
Baseline	9.09	1.51	14.9	0.56	31.7	−1.28
Constant T_t/y_t	$\boxed{16.2}$	−0.15	$\boxed{25.3}$	−2.38	$\boxed{66.2}$	−6.78
Constant s_t^k	1.13	−2.58	2.93	−3.08	8.18	−3.88
Cobb-Douglas	0.37	−12.5	2.62	−12.9	9.01	−13.6
$\sigma_k = 0.8$, $\sigma_\ell = 1$	2.62	−9.08	5.80	−9.64	14.9	−10.6
$\sigma_k = 0.4$, $\sigma_\ell = 1.4$	8.23	0.78	13.4	−0.05	26.5	−1.47
$1/\alpha = 1$	14.4	1.05	17.5	0.62	23.1	−0.01
$1/\alpha = 0.33$	6.48	1.67	12.5	0.52	36.8	−2.16
$(\gamma - 1)^{-1} = 1.5$	4.45	0.92	6.74	0.63	11.9	$\boxed{0.04}$
$\kappa = 0.20$	7.24	1.21	11.5	0.50	27.7	−1.32
$\beta = 0.982$	13.3	$\boxed{2.14}$	23.0	$\boxed{0.68}$	40.0	−1.06

Notes: Baseline model uses $\sigma_k = 0.4$ and $\sigma_\ell = 1$. Cobb-Douglas model uses $\sigma_k = \sigma_\ell = 1$. Welfare effects are measured by the percentage change in the per-period consumption basket to make the agent indifferent between the no-change economy (which holds income shares constant) and the transition economy. Boxed entries represent the best welfare outcome for each type of agent in a given expectation regime.

All of the various model specifications in Table 2 deliver positive welfare gains for the capital owners. The gains increase monotonically from left to right along with capital owners' knowledge about the future transition path. Conversely, the welfare outcomes for workers decline monotonically from left to right. At the extreme right under perfect foresight, the welfare outcomes for workers are almost always negative. The sole exception is when both types of agents have a more elastic labor supply, i.e., when $(\gamma - 1)^{-1} = 1.5$. This case is discussed in more detail below.

For the baseline model, the welfare gains for capital owners range from 9% under myopic expectations to about 32% under perfect foresight. The huge gain for capital owners under perfect foresight derives from the initial consumption jump at $t = 1$. Workers achieve a welfare gain of 1.5% under myopic expectations but suffer a welfare *loss* of about 1.3% under perfect foresight. The workers' loss under perfect foresight derives from the negative wage impacts induced by slower capital accumulation when the investment-output ratio drops sharply at $t = 1$. The welfare results under learning fall in between those for the other two expectation regimes. In the baseline learning regime, workers still manage to achieve a welfare gain of 0.5% while the welfare gain for capital owners is now 12.5%.

As expected, holding T_t/y_t constant is beneficial for capital owners but detrimental to workers. In the absence of a rising ratio of redistributive transfers to GDP, the workers always suffer a welfare loss that ranges from -0.15% under myopic expectations to -6.8% under perfect foresight. The boxed entries show that this particular model specification delivers the most favorable welfare outcomes for capital owners, regardless of the expectation regime. Interestingly, however, this specification does not deliver the worst welfare outcome for workers. Holding T_t/y_t constant boosts the after-tax income of capital owners which leads to higher investment than otherwise. The resulting faster rate of capital accumulation delivers wage benefits to workers which help to mitigate the loss of some transfer payments. Recall that the workers' consumption trajectory can still surpass the no-change trend even when transfer-to-GDP ratio is held constant at 10% (top right panel of Figure 9).

As noted previously, the Cobb-Douglas experiment can be viewed as a more extreme version of the experiment that holds s_t^k constant. Table 2 shows that both of these experiments deliver less favorable welfare outcomes in each cell when compared to the baseline model. This result is due to the less favorable wage paths obtained in these experiments, as shown earlier in Figure 7. The less favorable wage paths

reduce agents' labor supply relative to the baseline model, leading to slower growth in aggregate output during the transition (bottom panels of Figure 10). Of all the different specifications reported in Table 2, the Cobb-Douglas model delivers the worst welfare outcomes for workers, regardless of the expectation regime. This result is striking, particularly since Cobb-Douglas production functions are commonly used in the theoretical and empirical literature on income inequality. Our results show that the use of a Cobb-Douglas specification can lead to a downward bias when gauging the welfare consequences of shifting income shares.

We also experimented with changing either σ_k and σ_ℓ individually. When $\sigma_k = 0.8$ (with σ_ℓ maintained at the baseline value of 1), the degree of capital-entrepreneurial skill complementarity is weaker than in the baseline model but stronger than in the Cobb-Douglas model. Table 2 shows that this experiment delivers better welfare outcomes than the Cobb-Douglas model, but both types of agents are still worse-off relative to the baseline model which has $\sigma_k = 0.4$. When $\sigma_\ell = 1.4$ (with σ_k maintained at the baseline value of 0.4), both types of agents are again worse-off relative to the baseline model, but the decline in welfare outcomes is less severe than in the previous experiment with $\sigma_k = 0.8$. Hence, in the presence of a technological change that makes physical capital more productive, both types of agents will benefit if either type's labor supply becomes more complementary with physical capital.

Variations in the parameter α affect the EIS for the agents' composite consumption baskets. Recall that the baseline EIS for both types of agents is $1/\alpha = 0.5$. We experimented with setting $1/\alpha = 1$ or $1/\alpha = 0.33$, which allow for a higher or lower EIS than the baseline model. For capital owners, the EIS governs the relative size of the income and substitution effects of the technology change which, in turn, pin down the optimal split between consumption and investment along the transition path. Under perfect foresight, an EIS closer to unity implies a weaker income effect which implies a smaller jump in the capital owner's consumption at $t = 1$. This situation lowers the capital owner's welfare relative to the baseline model, but benefits the worker's welfare. However, under myopic expectations and learning, an EIS closer to unity implies a stronger income effect because capital owners now react to current income. A stronger income effect raises the capital owner's consumption trajectory relative to the baseline model. This is beneficial for the capital owner's welfare but since capital accumulation is now slower, the welfare of workers declines relative to the baseline model. All of these effects are reversed when the EIS is further away

from unity than the baseline value. For both types of agents, the EIS also influences the lifetime utility evaluation of a given consumption trajectory. But this effect is of second-order importance when compared to effect of the EIS on the level and slope of the consumption trajectory itself.

Our baseline calibration assumed a labor supply elasticity of $(\gamma - 1)^{-1} = 0.5$ for both types of agents. Keane and Rogerson (2012) argue that the intertemporal elasticity of substitution for labor supply at the macro level is in the range of 1 to 2. Consistent with this view, we set $(\gamma - 1)^{-1} = 1.5$. The results of this experiment are mixed. Capital owners are made worse-off relative to the baseline model under all three expectation regimes. Workers are made worse-off under myopic expectations, but their welfare outcomes are improved under learning and perfect foresight. In the case of capital owners, a more-elastic labor supply moderates the increase in their equilibrium wage path, since an increase in the price of their labor now brings forth more supply. This effect, together with the associated reduction in leisure time, moderates their welfare gains in comparison to the baseline model. Workers benefit from a higher aggregate labor supply because it raises the level of aggregate output and hence transfers. Recall, however, that the technology change causes the workers' own labor supply to initially decline relative to the no-change trend, particularly under learning or perfect foresight (bottom right panel of Figure 6). The decrease in their own labor supply results in more leisure time which, all else equal, is beneficial for their welfare. Relative to the baseline model, the positive effects on workers' welfare outweigh the negative effects under learning and perfect foresight. Table 2 shows that the calibration with $(\gamma - 1)^{-1} = 1.5$ delivers positive welfare gains for workers under all three expectation regimes.

The second-to-last row of Table 2 shows the effects of a slower diffusion speed for new technology. When $\kappa = 0.20$, the diffusion process is only 71% complete by the year 2010 versus 92% in the baseline model. The movement from a 10% adoption share to 90% now takes 22 years versus 18 years in the baseline model. Both capital owners and workers are made worse off by the slower diffusion speed, with the effect on capital owners being more pronounced. This experiment shows that more-rapid technological change can yield benefits to all agents, even when the technology change is biased in favor of highly-skilled workers.

The last row of Table 2 shows the effect of assuming that both types of agents are more patient. When $\beta = 0.982$, the steady-state net equity return is 6% versus

8% in the baseline model. As with the EIS for consumption, a change in β has a first-order effect on the level and slope of the agents' consumption trajectories and a second-order effect on the lifetime utility evaluation of a given consumption trajectory. A higher value for β improves the welfare outcomes for both types of agents relative to the baseline model. In the case of capital owners, increased patience yields more investment which, in turn, boosts the wage paths of both types of agents via faster capital accumulation. In the case of workers, a higher wage path allows more consumption than otherwise. In addition, the recovery in the worker's consumption trajectory that occurs later in the transition (top left panel of Figure 9) is now given more weight when computing lifetime utility.

Overall, we find that the range of possible welfare outcomes for both types of agents is enormous. The range of results presented in Table 2 might be viewed as something akin to a confidence interval for the potential welfare effects of rising U.S. income inequality over the past three decades. The welfare gains for capital owners range from a low of 0.37% (Cobb-Douglas, myopic expectations) to a high of 66.2% (constant T_t/y_t, perfect foresight). The welfare outcomes for workers range from a low of -13.6% (Cobb-Douglas, perfect foresight) to a high of 2.62% ($\beta = 0.982$, myopic expectations). We acknowledge that some of the model specifications are more relevant than others for comparison with the U.S. experience. In particular, the perfect foresight regime could be viewed as implausible while the specifications that hold either T_t/y_t or s_t^k constant are counterfactual. It should also be noted that the welfare outcomes for both types of agents would be scaled downward if we had assumed that redistributive transfers were financed by a distortionary tax on capital owners' income. Nevertheless, the main point to be taken away from Table 2 is that the welfare consequences of rising income inequality are highly uncertain, even in the relatively simple framework considered here with only types of agents. This finding would likely extend to more complex model environments that include the basic elements observed in the data, namely, rising income inequality and a stable distribution of financial wealth.

6 Conclusion

The U.S. economy experienced a profound upward shift in the share of income going to the top decile of households over the past three decades. The evidence suggests that some form of skill-biased technological change played an important role in this

trend. We developed a model of skill-biased technological change in which the share parameters of a CES production function shift over time, similar to the framework of Goldin and Katz (2008). But in contrast to much of the literature in this area, our approach focused on a technology-induced shift in the demand for entrepreneurial labor, representing top incomes, as opposed to the broader pool of college-educated labor. Empirical evidence shows that even among college-educated workers, the income gains of the highest earners are the primary driving force for rising U.S. income inequality.

Our analysis shows that the top decile of agents in the model always benefit from the technology change, but their degree of foresight influences the size of their welfare gains. Workers outside the top decile can also benefit when three elements are in place, namely, a rising ratio of redistributive transfers to GDP, an increase in capital's share of total income, and a strong degree of complementarity between physical capital and entrepreneurial labor. If any one of these three elements are absent from the model, then workers suffer a welfare loss from the technology change.

Two important caveats of our findings are in order. First, our framework does not allow us to say anything about changes in income inequality among agents in the lower nine-tenths of the U.S. income distribution. This group encompasses individuals with a wide range of skills and education levels. The empirical evidence shows that income inequality within this broad group has also increased markedly over the past three decades. A framework with more than two types of agents is needed to study the consequences of such developments. Second, we abstracted from endogenous human capital investment which could help spread the benefits of skill-biased technological change to agents who fall outside the top decile. Still, the inclusion of such features would not eliminate the large fundamental uncertainty that surrounds the welfare implications of rising income inequality. Overall, our findings suggest that caution is warranted when formulating potential policy responses to rising U.S. income equality.

A Appendix: Model Solution

A.1 First-Order Conditions in Stationary Variables

Combining the agents' labor supply equations (6) and (12) with the firm's labor demand equations (21) and (22) yields the following pair of nonlinear equations that pin down the values of $n\,\ell_t^w$ and ℓ_t^c as functions of the two state variables $k_{n,t} \equiv k_t/\exp(z_t)$ and π_t :

$$n\,\ell_t^w \quad = \quad \left[\frac{A(1-\theta_t)\,n^{\gamma-1}}{D^w \left\{ \theta_t \left[(1-\rho_t) k_{n,t}^{\psi_k} + \rho_t \left(\ell_t^c\right)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} \; + \; (1-\theta_t)(n\,\ell_t^w)^{\psi_\ell} \right\}^{\frac{\psi_\ell - 1}{\psi_\ell}}} \right]^{\frac{1}{\gamma-\psi_\ell}}, \tag{A.1}$$

$$\ell_t^c \quad = \quad \left[\frac{A\,\theta_t \rho_t \left[(1-\rho_t) k_{n,t}^{\psi_k} + \rho_t \left(\ell_t^c\right)^{\psi_k} \right]^{\frac{\psi_\ell - \psi_k}{\psi_k}}}{D^c \left\{ \theta_t \left[(1-\rho_t) k_{n,t}^{\psi_k} + \rho_t \left(\ell_t^c\right)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} \; + \; (1-\theta_t)(n\,\ell_t^w)^{\psi_\ell} \right\}^{\frac{\psi_\ell - 1}{\psi_\ell}}} \right]^{\frac{1}{\gamma-\psi_k}}, \tag{A.2}$$

where we have made use of $H_t = \exp(z_t)$ and the expressions for the income share variables s_t^c and s_t^k given by equations (24) and (25). Recall that θ_t and ρ_t are functions of the state variable π_t, as given by equations (15) and (16).

To facilitate a numerical solution, the firm's intertemporal first-order condition (23) can be rewritten in terms of stationary variables. Dividing both sides of equation (23) by y_t and defining the firm's intertemporal decision variable as the investment-output ratio $x_t \equiv i_t/y_t$ yields

$$x_t \quad = \quad \widehat{E}_t\, \beta \left[\frac{\lambda_{t+1}^c}{\lambda_t^c} \frac{y_{t+1}}{y_t} \right] \left[\lambda s_{t+1}^k \; + (1-\lambda)\; x_{t+1} \right], \tag{A.3}$$

where we have substituted in $M_{t+1}^c \equiv \beta \lambda_{t+1}^c / \lambda_t^c$.

From the capital owner's first-order condition (9), we have

$$\frac{\lambda_{t+1}^c}{\lambda_t^c} \frac{y_{t+1}}{y_t} \quad = \quad \left[\frac{c_{t+1}^c/y_{t+1} - \frac{D^c}{\gamma} H_{t+1} \left(\ell_{t+1}^c\right)^{\gamma} / y_{t+1}}{c_t^c/y_t - \frac{D^c}{\gamma} H_t \left(\ell_t^c\right)^{\gamma} / y_t} \right]^{-\alpha} \left[\frac{y_{t+1}}{y_t} \right]^{1-\alpha}. \tag{A.4}$$

The above equation can be further transformed by substituting in the following expressions that derive from the capital owner's budget constraint (8), the capital

owner's labor supply equation (12), and the production function (13):

$$c_t^c/y_t \;=\; s_t^c - x_t - \tau_t, \tag{A.5}$$

$$\frac{D^c}{\gamma}\, H_t\, (\ell_t^c)^{\gamma}\, /y_t \;=\; \frac{1}{\gamma}\, w_t^c \ell_t^c/y_t \;=\; \frac{1}{\gamma}\, \left(s_t^c - s_t^k\right), \tag{A.6}$$

$$\frac{y_{t+1}}{y_t} \;=\; \exp\left(\mu\right) \left\{ \frac{\theta_{t+1}\, \left[\left(1-\rho_t\right) k_{n,t+1}^{\psi_k} + \rho_{t+1}\, \left(\ell_{t+1}^c\right)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}} + \left(1-\theta_{t+1}\right)\left(n\, \ell_{t+1}^w\right)^{\psi_\ell}}{\theta_t\, \left[\left(1-\rho_t\right) k_{n,t}^{\psi_k} + \rho_t\, \left(\ell_t^c\right)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}} + \left(1-\theta_t\right)\left(n\, \ell_t^w\right)^{\psi_\ell}} \right\}^{\frac{1}{\psi_\ell}}, \tag{A.7}$$

where $\tau_t \equiv T_t/y_t$ is the lump-sum tax rate and we have made use of $z_{t+1} - z_t = \mu$. The tax rate is a function of the state variable π_t, as given by equation (26). The stationary endogenous variables $n\, \ell_t^w$, ℓ_t^c, s_t^c, and s_t^k, are governed by equations (A.1), (A.2), (24) and (25), respectively.

The upshot of all this is that the firm's intertemporal first order condition (A.3) can now be written as the following nonlinear stochastic difference equation involving only stationary variables:

$$\frac{x_t \left\{ \theta_t\, \left[\left(1-\rho_t\right) k_{n,t}^{\psi_k} + \rho_t\, \left(\ell_t^c\right)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}} + \left(1-\theta_t\right)\left(n\, \ell_t^w\right)^{\psi_\ell} \right\}^{\frac{1-\alpha}{\psi_\ell}}}{\left[\frac{\gamma-1}{\gamma}\, s_t^c + \frac{1}{\gamma}\, s_t^k - x_t - \tau_t\right]^{\alpha}} \;=$$

$$\beta'\, \widehat{E}_t\, \frac{\left\{ \theta_{t+1}\, \left[\left(1-\rho_{t+1}\right) k_{n,t+1}^{\psi_k} + \rho_{t+1}\, \left(\ell_{t+1}^c\right)^{\psi_k}\right]^{\frac{\psi_\ell}{\psi_k}} + \left(1-\theta_{t+1}\right)\left(n\, \ell_{t+1}^w\right)^{\psi_\ell} \right\}^{\frac{1-\alpha}{\psi_\ell}}}{\left[\frac{\gamma-1}{\gamma}\, s_{t+1}^c + \frac{1}{\gamma}\, s_{t+1}^k - x_{t+1} - \tau_{t+1}\right]^{\alpha}} \tag{A.8}$$

where $\beta' \equiv \beta \exp\left[\left(1-\alpha\right)\mu\right]$ and we have collected variables dated $t+1$ on the right side. The object to be forecasted involves three future decision variables x_{t+1}, ℓ_{t+1}^c, and $n\, \ell_{t+1}^w$ and two future state variables $k_{n,t+1}$ and π_{t+1}. Since the law of motion for π_{t+1} is exogenous, the only remaining element needed for a solution is the endogenous law of motion for $k_{n,t+1}$, which is derived next.

Starting from the definitional relationship $k_{n,t+1} \equiv k_{t+1}/\exp\left(z_{t+1}\right)$, we have

$$\begin{aligned} k_{n,t+1} \;&=\; k_{n,t} \exp\left(-z_{t+1} + z_t\right) \frac{k_{t+1}}{k_t}, \\ &=\; k_{n,t} \exp\left(-\mu\right) B \left[\frac{i_t}{y_t}\frac{y_t}{k_t}\right]^{\lambda}, \end{aligned} \tag{A.9}$$

where we have substituted in the laws of motion for z_{t+1} and k_{t+1}. From the production function (13), we have

$$\frac{y_t}{k_t} \;=\; \frac{A}{k_{n.t}} \left\{ \theta_t \left[(1-\rho_t)\, k_{n,t}^{\psi_k} + \rho_t \, (\ell_t^c)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} \;+\; (1-\theta_t)\, (n\, \ell_t^w)^{\psi_\ell} \right\}^{\frac{1}{\psi_\ell}} . \tag{A.10}$$

Substituting equation (A.10) into (A.9) together with $x_t \equiv i_t / y_t$ yields the following law of motion for the normalized capital stock:

$$k_{n,t+1} \;=\; A^\lambda B \, \exp(-\mu) \; k_{n,t}^{1-\lambda} \, x_t^\lambda \left\{ \theta_t \left[(1-\rho_t)\, k_{n,t}^{\psi_k} + \rho_t \, (\ell_t^c)^{\psi_k} \right]^{\frac{\psi_\ell}{\psi_k}} \;+\; (1-\theta_t)\, (n\, \ell_t^w)^{\psi_\ell} \right\}^{\frac{\lambda}{\psi_\ell}} . \tag{A.11}$$

A.2 Perfect Foresight

Under perfect foresight, the transformed intertemporal first-order condition (A.8) becomes

$$\mathrm{f}\left(x_t,\, \ell_t^c,\, n\, \ell_t^w,\, k_{n,t},\, \pi_t\right) \;=\; \mathrm{h}\left(x_{t+1},\, \ell_{t+1}^c,\, n\, \ell_{t+1}^w,\, k_{n,t+1},\, \pi_{t+1}\right), \tag{A.12}$$

where we have eliminated s_t^c, s_t^k, s_{t+1}^c, s_{t+1}^k using equations (24) and (25). The decision variables $n\, \ell_t^w$ and ℓ_t^c must satisfy equations (A.1) and (A.2) each period.

Two approximate solutions of the model can be obtained by log-linearizing equations (A.1), (A.2), (A.8), and (A.11) around each of the two steady states corresponding to $\pi_t = 0$ and $\pi_t = 1$. We use the π_t-weighted average from the two sets of log-linear decision rules to construct an initial conjectured sequence of values for the nonlinear function $\mathrm{h}(\cdot)$ from $t = 0$ (the initial steady state) to $t = 1500$ (the final steady state). At each time t, the conjectured value for $\mathrm{h}(t+1)$ is substituted into the right side of equation (A.12). Given $\mathrm{h}(t+1)$ equations (A.1), (A.2) and (A.12) can be solved simultaneously for x_t, ℓ_t^c, and $n\, \ell_t^w$ using a nonlinear equation solver. The resulting values are used to compute $k_{n,t+1}$ from equation (A.11) with π_{t+1} given by the exogenous law of motion (17). This procedure is repeated each time period, yielding a new conjectured sequence for $\mathrm{h}(\cdot)$ from $t = 0$ to $t = 1500$. The perfect foresight solution is obtained when the conjectured sequence for $\mathrm{h}(\cdot)$ does not change (to an accuracy of 0.0001) from one simulation to the next. In practice, convergence is obtained after about 70 simulations.

A.3 Myopic Expectations

Under myopic expectations, we assume $\widehat{E}_t \, \mathrm{h}\,(t+1) \;=\; \mathrm{h}\,(t-1)$. The transformed intertemporal first-order condition (A.8) becomes

$$\mathrm{f}\left(x_t,\, \ell_t^c,\, n\, \ell_t^w,\, k_{n,t},\, \pi_t\right) \;=\; \mathrm{h}\left(x_{t-1},\, \ell_{t-1}^c,\, n\, \ell_{t-1}^w,\, k_{n,t-1},\, \pi_{t-1}\right). \tag{A.13}$$

At each date t, equations (A.1), (A.2) and (A.13) can be solved simultaneously for x_t, ℓ_t^c, and $n\,\ell_t^w$ using a nonlinear equation solver. The resulting values are used to compute $k_{n,t+1}$ from equation (A.11) with π_{t+1} computed using the exogenous law of motion (17).

A.4 Learning

Under learning, we assume $\widehat{E}_t\,\mathrm{h}\,(t+1) = \omega_t\,\mathrm{h}\,(t+1) + (1-\omega_t)\;\mathrm{h}\,(t-1)$, where $\omega_t = \pi_t$. The transformed intertemporal first-order condition (A.8) becomes

$$\begin{aligned}\mathrm{f}\,(x_t,\,\ell_t^c,\,n\,\ell_t^w,\,k_{n,t},\,\pi_t) \quad &= \quad \pi_t\,\mathrm{h}\left(x_{t+1},\,\ell_{t+1}^c,\,n\,\ell_{t+1}^w,\,k_{n,t+1},\,\pi_{t+1}\right) \\ &\quad + \;(1-\pi_t)\;\mathrm{h}\left(x_{t-1},\,\ell_{t-1}^c,\,n\,\ell_{t-1}^w,\,k_{n,t-1},\,\pi_{t-1}\right) \qquad \text{(A.14)}\end{aligned}$$

Similar to case of perfect foresight, the solution under learning requires an initial conjectured sequence of values for the nonlinear function $\mathrm{h}(\cdot)$ from $t=0$ to $t=1500$. As before, we construct the initial conjectured sequence using a π_t-weighted average of the two sets of decision rules from the log-linearized learning model. At each time t, the conjectured value for $\mathrm{h}(t+1)$ and the realized lagged value $\mathrm{h}(t-1)$ are both substituted into the right side of equation (A.14), thus allowing equations (A.1), (A.2) and (A.14) to solved simultaneously for x_t, ℓ_t^c, and $n\,\ell_t^w$. This procedure is repeated each time period, yielding a new conjectured sequence for $\mathrm{h}(\cdot)$. The learning solution is obtained when the conjectured sequence for $\mathrm{h}(\cdot)$ does not change from one simulation to the next.

B Appendix: Welfare Computation

An individual worker's lifetime utility can be written as

$$\begin{aligned} V^w \quad &= \quad \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^w - \frac{D^w}{\gamma} H_t\,(\ell_t^w)^\gamma\right]^{1-\alpha} - 1}{1-\alpha} \\ &= \quad \sum_{t=0}^{\infty} \beta^t \frac{\left[\frac{\gamma-1}{\gamma}\,c_t^w + \frac{1}{\gamma}\,T_t/n\right]^{1-\alpha} - 1}{1-\alpha}, \qquad \text{(B.1)}\end{aligned}$$

where we have substituted in $D^w H_t\,(\ell_t^w)^\gamma = w_t^w \ell_t^w$ from the labor supply equation (6) and $w_t^w \ell_t^w = c_t^w + T_t/n$ from the budget equation (3). Similarly, an individual capital owner's lifetime utility can be written as

$$V^c \quad = \quad \sum_{t=0}^{\infty} \beta^t \frac{\left[\frac{\gamma-1}{\gamma}\,c_t^c + \frac{1}{\gamma}\,(d_t - T_t)\right]^{1-\alpha} - 1}{1-\alpha}, \qquad \text{(B.2)}$$

where the terms in square brackets in (B.1) and (B.2) are the agents' composite consumption baskets. Both (B.1) and (B.2) show the direct influence of transfers T_t on lifetime utility.

The welfare effect of the technology change is calculated as the constant percentage amount by which the agent's composite consumption basket in the no-change economy (which holds income shares constant at their initial levels) must be adjusted upward or downward each period to make lifetime utility equal to that in the transition economy. Specifically, we find ϕ^w and ϕ^c that solve the following two equations

$$\sum_{t=0}^{\infty} \beta^t \frac{[C_t^w]^{1-\alpha} - 1}{1-\alpha} = \sum_{t=0}^{\infty} \beta^t \frac{\left[\overline{C}_t^w (1+\phi^w)\right]^{1-\alpha} - 1}{1-\alpha}, \tag{B.3}$$

$$\sum_{t=0}^{\infty} \beta^t \frac{[C_t^c]^{1-\alpha} - 1}{1-\alpha} = \sum_{t=0}^{\infty} \beta^t \frac{\left[\overline{C}_t^c (1+\phi^c)\right]^{1-\alpha} - 1}{1-\alpha}, \tag{B.4}$$

where C_t^w and C_t^c are the composite consumption baskets in the transition economy and $\overline{C}_t^w$ and $\overline{C}_t^c$ are the composite consumption baskets obtained along the no-change trend. The infinite sums in (B.3) and (B.4) are approximated by sums over a 1500 period simulation, after which the results are not changed. The initial conditions at $t = 0$ correspond to the steady state with $\pi_t = 0$.

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