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Optimal Pricing of Flights and Passengers at Congested Airports: The Efficiency of Atomistic Charges

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Optimal pricing of flights and passengers at congested airports and the efficiency of atomistic charges

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Abstract

This paper investigates and compares airport pricing policies under various types of competition, considering both per-passerenger and per-flight charges at congested airports. We show that an airport requires both pricing instruments to achieve the first-best outcome, and we distinguish their role by showing that congestion externalities need to be addressed through per-flight tolls whereas the inefficiency caused by airlines’ market power exertion must be corrected with per-passerenger subsidies. We also show that Bertrand competition with differentiated products, a type of behavior recently pointed out by the empirical literature as pertinent, has policy implications that diverge from analyses that assume Cournot competition. The welfare gains and congestion reductions of congestion pricing would be higher than what has been advanced before; the degree of self-financing of airport infrastructure under optimal pricing would be increased and may approach exact self-financing; and the implied differentiation of charges between (asymmetric) airlines would be significantly smaller, presumably enhancing the political feasibility of welfare maximizing congestion pricing, as the potential distributional concerns would be decreased. Finally, we numerically analyze second-best policies, and find that atomistic pricing may offer a relatively attractive alternative to first-best congestion pricing.

Keywords: Airport pricing, Congestion internalization, Airline conduct

1. Introduction

Delays at airports have been consistently increasing over the past years, becoming a major problem worldwide (see, for example, Rupp (2009) and Santos and Robin (2010)). Besides capacity enlargements, the price mechanism has been widely discussed and proposed to manage congestion. This approach involves an airport authority setting user charges, with the possibility to charge passengers, airlines or both.1 The

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1Another approach to manage airport congestion is to use slot sales and slot trading; see Brueckner (2009), Basso and Zhang (2010) and Verhoef (2010).
basic economic motivation for such charges is the congestion externality, as already identified by Pigou (1920) in the context of road traffic. What makes the airport literature different is its focus on congestion pricing when there is market power. This, obviously, introduces a second distortion into the analysis, namely non-competitive pricing. Many papers have studied optimal airport pricing: for example, Brueckner (2002) first showed that in oligopoly, airlines competing in a Cournot fashion internalize congestion imposed on themselves; therefore, the optimal charge should account only for the fraction of congestion that is imposed on competitors. We will refer to this as the “Cournot toll”, as opposed to the “atomistic toll” that considers marginal congestion costs imposed on all flights and passengers, regardless of the operator. One important implication of a Cournot toll is that a dominant airline should pay a lower congestion charge per flight than small airlines (Brueckner, 2005), which is likely to decrease its political feasibility due to distributional issues. Furthermore it would imply that self-financing of airport capacity from the revenues from optimal congestion charges, would become less realistic than in the benchmark case considered originally by Mohring and Harwitz (1962), who showed that with atomistic congestion charges and neutral scale economies in capacity supply, as well as some other technical assumptions, exact self-financing is obtained. Pels and Verhoef (2004) extend the analysis by explicitly considering market power distortions. They show that a welfare maximizing airport has to deal with two inefficiencies: airlines’ market power, that has to be corrected by subsidizing them, and congestion externalities, that requires charging of the Cournot toll. Further extensions have this congestion pricing rule intact, a consequence of these theoretical works assuming that Cournot competition is representative for airline markets.²

However, the Cournot assumption has recently been questioned from the empirical side. Fischer and Kamerschen (2003) estimate airlines conduct parameters with U.S. data, finding substantial deviations from Cournot behavior. Fageda (2006) rejects the suitability of the Cournot assumption as representative of the Spanish airline market. Perloff et al. (2007) study airlines’ conduct in a duopoly market using the dataset of Brander and Zhang (1990) and Oum et al. (1993), that has traditionally been used to support Cournot behavior. They, allowing airlines to provide differentiated services on a route, show that in some routes the outcomes implied by Bertrand behavior are virtually the same as the observed outcomes, while Cournot predictions lie in a less competitive region, not consistent with the data.³ Finally, Nazarenus (2011) revisit Brander and Zhang’s (1990) study, using data from 2007, concluding that the industry has experienced a regime change from Cournot towards more competitive behavior.⁴ This, at least, indicates that other behavioral assumptions, such as Bertrand competition with differentiated products, may be as relevant for

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² Zhang and Zhang (2006) explicitly incorporate airport’s costs and capacity decisions, and Basso (2008) includes airline differentiation, origin and destination airports and schedule delay costs. Basso and Zhang (2007) analyze rivalry between two congestible facilities, which again could be airports.

³ Recall that with differentiated products, the Bertrand outcome no longer entails marginal cost pricing.

⁴ Empirical evidence for leadership behavior also exists. See for example Morrison and Winston (1995).
aviation markets as the traditional Cournot view.

The purpose of this paper is to contribute to an ongoing aviation policy debate which focuses on the extent to which airlines indeed internalize congestion effects imposed upon their own flights. Notably, Daniel (1995) and Daniel and Harback (2008) have questioned this feature in Cournot models, while Brueckner and Van Dender (2008) showed how airline behavior approaches atomistic-like non-internalization when there is a Stackelberg leader. Our paper contributes on a number of respects. First, we derive the optimal pricing policy at congested airports when consumers perceive airlines as imperfect substitutes, and airline behavior follows the Bertrand assumption. Although, as argued above, this case has been shown to be empirically relevant, it has received almost no attention in the theoretical literature. Our aim is to fill this void, by providing the pertinent policy analysis. Second, we distinguish between the role of per-flight and per-passengers tolls for welfare maximization, in order to understand the policy implications of using the one versus the other. To do this, we model the long-run choice of seat capacity as made by airlines, and thus endogenously differentiate between charges per flight and charges per passenger. Third, we assess the relative efficiency of second-best policies, such as atomistic charges, with numerical examples.

We show that, in a duopoly setting where outputs are imperfect substitutes, Bertrand behavior implies that airlines internalize less than the self-imposed congestion, because they take into account the fact that an extra flight imposes congestion on its competitor’s passengers, affecting positively its own demand and profit. This yields an optimal congestion toll that lies between the marginal congestion cost imposed on the competitors’ passengers (the Cournot toll) and the atomistic toll. We also find that the size of the deviation from the self-imposed internalization result of Cournot competition depends on the degree of product substitutability. The entire range of tolls can be optimal: when the substitutability is low, the optimal congestion charge is close to the Cournot toll. Conversely, if the substitutability is high, airlines should be charged a toll very close to the atomistic one, even when they fully recognize the impact of their flight scheduling on airport congestion and even when one of them is a dominant airline with a high market share. We also reproduce the result of Brueckner and Van Dender (2008) for a Stackelberg leader with a Cournot follower, and extend it to the case of a Stackelberg leader with a Bertrand follower, finding optimal tolls that lie between the Cournot toll and the atomistic toll for both players. In this last setting, the optimal toll again approaches the atomistic toll more closely as the substitutability is higher.

Various policy conclusions follow from the analysis. We show that a welfare maximizing airport can only reach the first-best outcome by using two tax instruments, namely per-flight and per-passenger tolls. Moreover, congestion and market power effects are separate: the market power exertion can only be corrected by means of a per-passenger subsidy, while the optimal congestion charge should only be charged with a per-flight toll. As a consequence, the welfare maximizing per-passenger toll is below the airport’s marginal cost per passenger (due to the subsidy) and the welfare maximizing toll per flight is above airport’s marginal cost per flight (due to the congestion charge). This finding, common to all studied behavioral assumptions,
provides important insights on the role of each of the two mentioned instruments and directions on how they should be set. In addition, this result conflicts with a growing tendency of replacing per-movement charges by per-passenger charges, and also with the International Air Transport Association (IATA) position of recovering costs through passenger based charges instead of other aeronautical based charges (IATA, 2010).

We further find that the optimal pricing strategy, in the cases where the Cournot assumption is not representative of the market, includes a congestion charge that is above the marginal congestion cost imposed on the competitors’ passengers, and is likely to be close to the atomistic toll. This has significant policy implications. First, optimal congestion pricing would bring more significant welfare gains and congestion reductions than what has been advanced before on the basis of Cournot assumptions, hence increasing its relevance and efficiency. Second, the degree of self-financing of congested airports would be higher and, in absence of subsidies, it may be close to exact self-financing. Third, the political feasibility of welfare maximizing congestion pricing would be enhanced as the implied differentiation of charges is considerably smaller. For instance, under the Cournot assumption, a firm with 75% market share should pay a congestion toll equal to 25% of the total marginal congestion cost, whereas we find that, considering the behavior and parameters implied by the empirical study of Perloff et al. (2007) for Chicago-based markets, a firm with 75% of market share should pay between 55% and 77% of the total marginal congestion cost. This decreases significantly the potential distributional concerns of optimal congestion pricing.

Finally, we present numerical examples to assess the relative efficiency of second-best policies, and, for example, find that only using a per-flight congestion charge and levying atomistic tolls yield substantial and similar benefits when airlines do not behave in a Cournot fashion, and when the degree of product substitutability is not too low. This complements the findings of Morrison and Winston (2007), who argue in favor of levying atomistic tolls at congested airports, because they find a small net benefit loss when an airport charges the atomistic toll instead of the Cournot toll.

We also believe that our results may help explaining why the empirical and simulation studies provide a wide range of estimations regarding internalization of congestion at airports. In contrast to the road case, where users behave atomistically, the relevant question in aviation markets is what share of congestion airlines actually do internalize when making scheduling decisions of flights. If they internalize a high proportion of congestion costs, charges that optimally account for this will be relatively low and thus should have a small impact on flight patterns and social welfare. The internalization hypothesis, based on Brueckner’s analysis, is supported by empirical evidence by Mayer and Sinai (2003) with U.S. data and by Santos and Robin (2010) with European data, who show that delays are lower at highly concentrated airports. On the other hand, Daniel (1995), who first identified the potential for internalization of congestion, argues—with a simulation model—that atomistic behavior may in fact be more pertinent from an empirical point of view; i.e., that airlines do not take into account self-imposed congestion when making scheduling decisions. As a consequence, the optimal toll should be the so-called atomistic toll that ignores any internalization, and
that is equal to total marginal congestion costs. The atomistic behavior of airlines is further supported by empirical evidence by Daniel and Harback (2008) and by Rupp (2009). While the outcome of Cournot competition is in conflict with this evidence, the outcome of the Bertrand setting predicts both a negative relationship between delays and concentration, as well as congestion levels that can be significantly close to the atomistic level.\footnote{This divergence between views on the extent to which atomistic versus Cournot tolls are more desirable has been addressed before. For example, Brueckner and Van Dender (2008) present a theoretical analysis showing that when one airline acts as a Stackelberg leader and interacts with a large number of fringe carriers, the leader behaves atomistically as long as the products are perfect substitutes. Czerny and Zhang (2011) present a different argument and state that, when travelers with different values of time are considered, and in absence of price discrimination by carriers, it might be useful to increase the airport charge towards the atomistic toll. This is to protect passengers with a relatively high value of time from congestion caused by passengers with a relatively low value of time. Obviously, these analyses cannot explain differences in findings between models that consider homogeneous values of time or Nash competition.}

The paper is organized as follows. First, in Section 2, we introduce the model that includes aircraft size, fare and frequency decisions in an oligopolistic airline market, and that formally takes into account market power exertion and (potential) congestion internalization. In Section 3 we derive analytical solutions for the airports’ problem, specifically first-best tolls and optimal capacity investment. Section 4 presents numerical exercises to quantify the analytical results, to assess the efficiency of second-best policies, and to study the performance of levying atomistic tolls. Finally, Section 5 concludes.

\section{Airlines’ duopoly model}

For the analysis, we consider a vertical setting on a single market, i.e., a single origin-destination pair. In the first stage an airport chooses capacity, toll per flight and toll per passenger charged to the carriers that use the facility. In the second stage, a duopoly of airlines compete with aircraft size and frequency as decision variables, in addition to the fare or number of passengers. We choose to analyze analytically a duopoly of carriers in order to keep the simplest and most transparent possible focus on congestion internalization results, the effects of endogenous aircraft size, and the comparison between airlines’ behavioral assumptions; leaving the extension of more than two airlines for the numerical analyses of Section 4. Following Zhang and Zhang (2006), we model only one airport for analytical simplicity, but the conclusions remain the same if the other airport is included, as long as the airports share the objective function (this is, in our case, they perform joint welfare maximization).\footnote{If airports are not regulated by the same authority or if airports independently maximize profits both airports have to be formally modeled. For a discussion on the implications of two local welfare maximizing airports see Pels and Verhoef (2004) and for a discussion on independent profit maximization see Basso (2008).}

For the airlines’ market, we consider the differentiated duopoly proposed by Dixit (1979), assuming that
demands arise from the following quadratic utility function:

$$U(q_i, q_j) = A \cdot (q_i + q_j) - (B \cdot q_i^2 + 2 \cdot E \cdot q_i \cdot q_j + B \cdot q_j^2) / 2,$$

where $q_i$ is the amount of good $i$ (hereafter, when subscript $j$ appears in the same expression with $i$, it refers to the rival airline), $A$, $B$ and $E$ are positive parameters with $B \geq E \geq 0$ so that goods are imperfect substitutes typically, with the special cases of perfect substitutes occurring when $B = E$, and independent goods when $E = 0$. We consider airlines as imperfect substitutes to account for the fact that not all passengers choose the airline with the most attractive fare-delay combination, therefore allowing airlines with different generalized prices having passengers in equilibrium. This is motivated by the fact that there are other factors such as loyalty (e.g. due to frequent flyer programs), service levels (e.g. meals and drinks), and consumer preferences for other particular aspects of airlines (e.g. language) that may differ across carriers and make passengers perceive airlines as imperfect substitutes.\(^7\)

This utility function gives rise to a linear demand structure, with equivalent inverse and direct demands:

$$\theta_i = A - B \cdot q_i - E \cdot q_j,$$  \hspace{1cm} (2)

$$q_i = a - b \cdot \theta_i + c \cdot \theta_j,$$  \hspace{1cm} (3)

where $\theta_i$ is the full price of good $i$ and parameters $a$, $b$ and $c$ satisfy $a = A / (B + E)$, $b = B (B^2 - E^2)$ and $c = E / (B^2 - E^2)$. Note that the ratio $e/b$ directly measures the substitutability between airlines, as it ranges from 0 when products are completely independent, to 1 when products are perfect substitutes.

The full price of traveling with airline $i$ is assumed to be:

$$\theta_i = p_i + D + g_i.$$  \hspace{1cm} (4)

The first term, $p_i$, is the fare. $D$ is the passengers’ cost of congestion delays experienced at airports and depends on airport capacity ($K$) and on the total number of take offs and landings at the congested airport ($F = f_i + f_j$). Finally, $g_i$ is the schedule delay cost faced by a passenger that travels with airline $i$, which depends only on the flight frequency of the airline ($f_i$). The fact that schedule delay does not depend on rival’s frequency, as congestion does, reflects our assumption that in the differentiated duopoly, frequency is perceived as an airline-specific attribute.

We make the plausible assumptions that $D$ is differentiable in $F$, that $g_i$ is differentiable in $f_i$ and that:

$$\frac{\partial D}{\partial f_i} > 0, \quad \frac{\partial^2 D}{\partial f_i^2} \geq 0, \quad \frac{\partial^2 D}{\partial f_i \partial f_j} \geq 0, \quad \frac{\partial D}{\partial K} < 0,$$

$$\frac{\partial^2 D}{\partial K \partial f_i} < 0, \quad \frac{\partial g_i}{\partial f_i} < 0, \quad \frac{\partial^2 g_i}{\partial f_i^2} > 0, \quad \forall i.$$  \hspace{1cm} (5)

\(^7\)The fact that an airline can have demand despite having a higher generalized price than the competitor has been also modeled with a brand-loyalty variable that gives the additional gain from traveling with a specific airline relative to travel with the other airline (Brueckner and Flores-Fillol, 2007; Flores-Fillol, 2010). In our model it is also possible to model a preference for a specific carrier by letting the demand parameters ($A$, $B$ and $E$) vary across carriers.
Congestion thus increases with the number of flights and the marginal effect is stronger when congestion is more severe; congestion decreases with airport’s capacity; schedule delay cost decreases with airline-specific frequency, and that effect is smaller when frequency is higher.\footnote{This set of assumptions is common in the literature: the linear delay function used by Pels and Verhoef (2004) and the convex function used by Zhang and Zhang (2006) satisfy the assumptions regarding $D$. The schedule delay function that is inversely proportional to the airline frequency satisfies the conditions for $g_i$ (see Brueckner (2004) and Basso (2008) for a discussion).}

Following Brueckner (2004), we model airlines’ cost ($C_i$) as a function of aircraft size and frequency in the following way:

$$ C_i = f_i \cdot \left( \gamma_f^i + \gamma_s^i \cdot s_i \right), \quad (6) $$

where $\gamma_f^i$ and $\gamma_s^i$ are positive cost parameters and $s_i$ is the number of seats per flight. The underlying assumption is that cost per flight is a linear function of the number of seats, a relation that has been also found in a cost-engineering study for airlines by Swan and Adler (2006).\footnote{In this work, a cost function per trip is calibrated using distance and aircraft size as explanatory variables; then, holding distance fixed, the function is linear in number of seats.}

Congestion costs for airlines are not considered in the analysis because we focus on passengers’ congestion, but including them would not change the results in any essential way.\footnote{Congestion imposed on airlines works out in a way similar to congestion imposed on passengers. The intuition is that passenger congestion costs reduce fare that can be charged at given output levels on a dollar by dollar basis. Therefore, the firm weighs “own” passenger congestion costs as heavily as it would weigh “own” congestions costs, and the two types of congestion costs would enter the optimization problem in identical ways. If congestion costs are included, airlines would not internalize congestion costs imposed on the competitors’ flights and this should be corrected in first-best tolls, as found by Basso (2008), Brueckner (2009) and Verhoef (2010).}

With the cost function defined, we can now write the profit of airline $i$ as:

$$ \pi_i = q_i \cdot p_i - f_i \cdot \left( \gamma_f^i + \gamma_s^i \cdot s_i \right) - f_i \cdot \tau_f^i - q_i \cdot \tau_q^i, \quad (7) $$

where $\tau_f^i$ is the per-flight toll charged by the airport and $\tau_q^i$ the toll per passenger.

One of the goals of this paper is to assess the impact of different kinds of strategic interaction on optimal pricing policy at congested airports. In modeling the airlines’ competition, we study the traditional setting for the airlines market, namely Cournot competition, where airlines simultaneously choose aircraft size, frequency and number of passengers taking the rivals decision as given. We thereafter look at game with airlines as Bertrand oligopolists, where fare (besides aircraft size and frequency) is the strategic variable instead of quantity. Finally, we study two Stackelberg settings, where the leader chooses all the relevant variables prior to a follower who takes the rival’s number of passengers (output) as given, or the rival’s fare as given.
2.1. Cournot behavior

In this game setting, we assume that airlines are Cournot oligopolists in that they choose aircraft size, frequency and number of passengers. Because having idle seat capacity only decreases profit in our model, it is straightforward that an airline will set the number of seats such that the aircrafts are filled \( s_i = q_i / f_i \); this allows us to express profit in terms of number of passengers and frequency. Rewriting equation (7), using (2) and (4) we get:

\[
\pi_i = q_i \cdot \left( A - B \cdot q_i - E \cdot q_j - D - g_i \right) - f_i \cdot \left( \gamma_i^f + \tau_i^f \right) - q_i \cdot \left( \gamma_i^s + \tau_i^q \right),
\]

Then, first-order conditions with respect to number of passengers and frequency yield:

\[
\frac{\partial \pi_i}{\partial q_i} = 0 \Rightarrow p_i = \gamma_i^s + \tau_i^q + q_i \cdot B,
\]

\[
\frac{\partial \pi_i}{\partial f_i} = 0 \Rightarrow \gamma_i^f + \tau_i^f = -q_i \cdot \left( \frac{\partial D}{\partial f_i} + \frac{\partial g_i}{\partial f_i} \right),
\]

Equation (9) states that the fare charged by an airline has three terms: (i) the marginal cost per capacity unit \( \gamma_i^s \); (ii) the airport charge per passenger \( \tau_i^q \); and (iii) a conventional monopolistic markup reflecting carrier’s market power, which is related to the sensitivity of demand and own number of passengers \( q_i \cdot B \). Equation (10) states that airline’s marginal cost per flight equals marginal benefits for own passengers (marginal congestion savings plus marginal schedule delay benefits); therefore, airlines internalize own-passenger congestion. These rules basically describe that airlines internalize congestion on their own passengers and charge a markup which equals \( q_i \cdot \partial \theta_i / \partial q_i \), a result analogous to the rules obtained previously in Cournot competition (e.g. Pels and Verhoef, 2004; Zhang and Zhang, 2006; Basso, 2008).\(^{11}\) From now on, to simplify notation, we refer to this game as Cournot competition and we refer to Cournot internalization to the result obtained in this game setting, i.e. perfect internalization of congestion imposed on own-passengers.

It is worth noting at this point that this reduced form of Cournot competition can also be interpreted as a two-stage game where airlines first simultaneously choose aircraft size and frequency and, in the second stage, they compete on fares. As an airline can transport at most \( f_i \cdot s_i \) passengers, in the first stage they are also making a capacity decision, thus the well-known result that a two-stage capacity-constrained competition leads to Cournot outcomes holds.\(^{12}\)

\(^{11}\)The second term in the right-hand side of equation (10) is present in previous studies including schedule delay cost. It is in Brueckner (2004) for a monopoly and it is in Basso (2008), but does not appear in the pricing rule of airports because it is set optimally by a private airline from a social welfare perspective.

\(^{12}\)For this result to hold, we only need to assume that when a price-setting firm is capacity constrained, it adjust prices so that the demanded quantity equals its capacity. The intuition comes from the seminal paper of Kreps and Scheinkman (1983), but does not apply directly with imperfect product differentiation. A formal proof and textbook treatment in absence of externalities can be found in Martin (2002), and the formal proof accounting for congestion is available from the authors upon request.
It is interesting to note some aspects about airlines’ behavior that arise from this model. Airlines do not charge passengers directly for congestion because they set frequency and number of passengers separately; for any given demand, they internalize own-passengers congestion by setting frequency according to (10) and adjusting aircraft size to accommodate the passengers. In a fixed-proportions model, an additional passenger necessarily increases delays and the only way to internalize this is by charging self-imposed marginal congestion costs to passengers. But, when aircraft size is a strategic variable, this is no longer desirable, because they can accommodate a new passenger, without raising delays, by increasing aircraft size by \(1/f_i\) at a cost of \(\gamma_s^i\) (which they do charge to passengers, see (9)). This also explains why the per-flight toll \((\tau_f^i)\) is absent in the airlines’ fare: it affects frequency and aircraft size setting, while keeping the cost per passenger constant at \(\gamma_s^i + \tau_f^i\).

2.2. Bertrand behavior

In this game, the problem faced by an airline is to maximize profit (equation 7) with strategic variables being frequency, aircraft size and fare. Again, having idle seat capacity only decreases profit, so that \(s_i = q_i/f_i\) holds and we can rewrite profit in terms of fare and frequency. Rewriting equation (7) and explicitly including the functions’ arguments (without including rival’s variables since they are taken as given) we get:

\[
\pi_i(p_i, f_i) = q_i(p_i, f_i) \cdot (p_i - \gamma_s^i - \tau_f^i) - f_i \cdot (\gamma_f^i + \tau_f^i).
\]  

Then, first-order conditions with respect to fare and frequency yield:

\[
\frac{\partial \pi_i}{\partial p_i} = 0 \Rightarrow p_i = \gamma_s^i + \tau_f^i + \frac{q_i}{b}, \tag{12}
\]

\[
\frac{\partial \pi_i}{\partial f_i} = 0 \Rightarrow \gamma_f^i + \tau_f^i = (p_i - \gamma_s^i - \tau_f^i) \cdot \frac{\partial q_i}{\partial f_i}. \tag{13}
\]

Again, the fare charged by an airline includes the marginal cost per capacity unit (\(\gamma_s^i\)), the airport charge per passenger, and a market-power markup. The difference with the Cournot game is that the market power effect is now weaker \((1/b < b/(b^2 - e^2) = B)\). The intuition of this comes directly from the type of game: when airlines take rival’s price as given, the outcome is more competitive than when they take rival’s quantity as given (see Singh and Vives (1984) for a discussion in a general context).

From the frequency first-order condition (13), we get that frequency is set optimally by equating marginal cost per flight to the revenue gain from an extra flight. For further interpretation, note that using equation (3) we can rewrite \(\partial q_i/\partial f_i\) as:

\[
\frac{\partial q_i}{\partial f_i} = -b \cdot \frac{\partial \theta_i}{\partial f_i} + e \cdot \frac{\partial \theta_j}{\partial f_i} = -b \cdot \left(\frac{\partial D}{\partial f_i} + \frac{\partial g_i}{\partial f_i}\right) + e \cdot \frac{\partial D}{\partial f_i}. \tag{14}
\]

This expression shows that the effect of an increase in the airline’s number of flights, has two effects on its demand: it changes both schedule delay cost and congestion for own passengers, but it also increases
the congestion experienced by competitor’s passengers when its frequency is fixed (or taken as given). The second effect has a positive impact for the airline, since increasing competitor’s congestion raises own demand due to the fact that airlines offer (imperfect) substitute outputs.

Using \( p_i - \gamma_s^i - \tau_q^i = q_i/b \) from equation (12) and equation (14), we can rewrite (13) as:

\[
\gamma_f^i + \tau_f^i = -q_i \cdot \left( \frac{\partial D}{\partial f_i} \cdot \left(1 - \frac{e}{b}\right) + \frac{\partial g_i}{\partial f_i} \right).
\] (15)

This equation defines how an airline sets frequency. It differs from equation (10) for the Cournot case in the term multiplying marginal congestion costs. In this game, an airline internalizes congestion imposed on its own passengers but also takes into account the congestion imposed on its competitor, as explained above. This is represented by the degree of substitutability \( e/b \) that appears in equation (15).

This term causes a difference with the common internalization finding, because now airlines are not taking the competitor’s output as given. In Cournot competition, airlines believe that they are not able to influence the competitor’s number of passengers by raising congestion, simply because they do not “see” the effect by assumption. On the other hand, when taking the competitor’s fare together with frequency as given, output is the result of setting the generalized price through the two variables. Therefore, airlines realize that they can influence competitor’s output, or increase own demand, by raising the rival’s congestion.

The size of the deviation from the traditional result of internalization depends directly on the degree of substitutability \( e/b \). Recall that this ratio ranges from 0 when products are completely independent to 1 when products are perfect substitutes. The fraction of runway congestion internalized by an airline, when setting frequency, is given by the ratio of congestion terms from equation (15) and total marginal congestion costs:

\[
\frac{q_i \cdot D' \cdot (1 - e/b)}{(q_i + q_j) \cdot D'} = \frac{q_i}{q_i + q_j} \cdot \left(1 - \frac{e}{b}\right).
\] (16)

As \( e < b \), carriers act as if they internalize less congestion than what is imposed on their own passengers, as in the Cournot model. Only when products are close to be independent, the effective internalization approaches the market share. When they are close to be homogeneous, airline behavior approaches atomistic behavior. For example, if the output is symmetric and the ratio of substitutability is 0.5, airlines internalize only 25 percent of congestion costs, instead of one half.

As in the previous case, the Bertrand reduced form used in this section to represent the airlines market has alternative interpretations. This game setting is equivalent to a two-stage game where airlines first choose aircraft size and, in a second stage, they compete on frequency and fares, as long as they do not directly care about rival’s aircraft size. It is also equivalent to the two-stage game where airlines first choose aircraft size and frequency, and in the second stage they compete on fares, as long as they cannot observe the rival’s actions. In other words, the open-loop equilibrium of the latter two-stage game corresponds to the Bertrand setting analyzed here, and the closed-loop equilibrium to the Cournot setting of Section 2.1.
From now on, to simplify notation we refer to this game as Bertrand competition and the result regarding internalization in this setting as Bertrand internalization.

Which one of the two closed form settings is more appropriate to describe the airline market depends, obviously, on market-specific conditions. Estimations by Brander and Zhang (1990) and Oum et al. (1993)—which have been used to support Cournot behavior—are well summarized by the latter’s conclusion that “the overall results indicate that the duopolists’ conduct may be described as somewhere between Bertrand and Cournot behavior, but much closer to Cournot, in the majority of the sample observations” (p. 189). As their estimations assume perfect substitutability, the Bertrand outcome would correspond to perfectly competitive conditions. Therefore, their conclusion is that fares generally exceed marginal costs, and by less than the “Cournot markup”. This statement is consistent with the Bertrand outcome with differentiated products described above.

In addition, as we discuss in Section 1, there are empirical studies that support the internalization hypothesis as well as studies that reject it. We believe that our model helps in explaining such a wide range of findings. As we discuss above, the degree of internalization depends on demand-structure parameters (the ratio of substitutability $e/b$) and, therefore, it is possible that in some markets an airline behaves almost atomistically regarding frequency setting, even if it has a large market share, while in others it internalizes a big share of congestion. Importantly, this still predicts a negative relationship between airport delays and concentration.

Naturally, the suitability of the settings above still remains an empirical question and it can perfectly vary across markets. As suggested by Perloff et al. (2007) with 1980s data, the appropriate setting for the Chicago-Wichita and Chicago-Providence markets is Bertrand competition, analyzed in this Section. In addition, Nazarenus’s (2011) analysis of 37 Chicago-based routes with 2007 data rejects the Cournot hypothesis as representative on average (in contrast to Brander and Zhang (1990)), suggesting that Bertrand behavior with imperfect substitution is more appropriate.13

2.3. Stackelberg behavior with a Cournot follower

In the next setting we consider a Stackelberg model, where we suppose that airline $i$ is a leader and airline $j$ the follower that chooses output ($q_j$), frequency ($f_j$) and aircraft size ($s_j$) viewing the leader’s strategic variables ($q_i$, $f_i$ and $s_i$) as parametric. We refer to this game as Stackelberg-Cournot as a convenient shorthand. As a result of the assumptions, the follower’s behavior is characterized by first-order conditions (9)-(10). The leader maximizes profit knowing the response of the follower to its own decisions. As in

13 They reject Cournot behavior as representative in most of the cases, and sometimes the Bertrand behavior as well, because, as they do not consider product differentiation, it is equivalent to reject the perfectly competitive outcome. They conclude that the change of regime is from Cournot towards a more competitive one without reaching perfect competition, which is again consistent with our Bertrand model.
previous settings, it is not optimal to have idle capacity, so the profit function of the leader is:

\[
\pi_i(q_i, f_i) = q_i \cdot [p_i(q_i, f_i, q_j, f_j) - \gamma_i^s - \tau_i^q] - f_i \cdot (\gamma_i^f + \tau_i^f),
\]

where both \(q_j\) and \(f_j\) depend on the leader’s choice variables.

First-order conditions with respect to output and frequency yield:

\[
\frac{\partial \pi_i}{\partial q_i} = 0 \Rightarrow p_i = \gamma_i^s + \tau_i^q + q_i \cdot \left( B + E \cdot \frac{\partial q_i}{\partial q_i} + \frac{\partial D}{\partial f_i} \cdot \frac{\partial f_j}{\partial q_i} \right), \tag{18}
\]

\[
\frac{\partial \pi_i}{\partial f_i} = 0 \Rightarrow \gamma_i^f + \tau_i^f = -q_i \cdot \left[ \frac{\partial D}{\partial f_i} \cdot \left( 1 + \frac{\partial f_j}{\partial f_i} \right) + \frac{\partial g_i}{\partial f_i} + E \cdot \frac{\partial q_i}{\partial f_i} \right]. \tag{19}
\]

Since the follower’s responses are downward-sloping (the proof is in Appendix A), the leader sets a higher quantity than an airline that takes rival’s output as given (like the follower does). As a consequence, the leader’s market power effect is weaker (for a given frequency, a higher number of passengers implies a lower fare).

As the leader anticipates the way the follower reacts, the incentives to reduce frequency (first-order condition 19) are different from those in the Cournot case, because of two effects. First, the leader predicts that any frequency reduction is partially offset by an increase in the number of flights by the follower \((\partial f_j/\partial f_i < 0)\). As can be seen in (19), the term involving marginal congestion is reduced by this expression. This situates the leader’s internalization in between the Cournot case of self-imposed congestion and atomistic behavior, just as pointed out by Brueckner and Van Dender (2008). The second effect—not directly related to marginal congestion—is the last term multiplying \(q_i\) on the right hand-side of equation (19), which further reduces internalization. The leader realizes that any frequency increase induces a reduction on follower’s output, therefore the frequency reduction incentive is diminished. The overall effective internalization is in between congestion imposed on own passengers and atomistic behavior. The exact degree, however, depends, among other things, on the degree of substitutability. In Section 4, we expand more on this.

2.4. Stackelberg behavior with a Bertrand follower

Finally we solve a Stackelberg game with airline \(i\) as a leader and airline \(j\) as a follower that chooses fare \((p_j)\), frequency \((f_j)\) and aircraft size \((s_j)\). The only difference with the previous setting is that the follower takes the leader’s fare as given, instead of output. We refer to this setting as the Stackelberg-Bertrand game.

The problem for the follower is the same as in the Bertrand game, i.e. maximize profit (equation 11) with respect to fare and frequency, yielding the same first-order conditions (equations 12 and 13). For the leader, the profit function now is:

\[
\pi_i(p_i, f_i) = q_i(p_i, f_i, p_j, f_j) \cdot (p_i - \gamma_i^s - \tau_i^q) - f_i \cdot (\gamma_i^f + \tau_i^f). \tag{20}
\]
First-order conditions with respect to fare and frequency yield:

\[
\frac{\partial \pi}{\partial p_i} = 0 \Rightarrow p_i = \gamma_i^* + \tau_i^f + \frac{q_i}{b},
\]

(21)

\[
\frac{\partial \pi}{\partial f_i} = 0 \Rightarrow \gamma_i^f + \tau_i^f = -q_i \cdot \left[ \frac{\partial D}{\partial f_i} \left( 1 - \frac{e}{b} \right) \left( 1 + \frac{\partial f_j}{\partial f_i} \right) + \frac{\partial g_i}{\partial f_i} - \frac{e}{b} \left( \frac{\partial p_j}{\partial f_i} + \frac{\partial g_j}{\partial f_j} \frac{\partial f_j}{\partial p_i} \right) \right] \cdot \tilde{b},
\]

(22)

where \( \tilde{b} \equiv b - e \cdot \partial p_j / \partial p_i - \partial q_i / \partial f_j \cdot \partial f_j / \partial p_i \).

In this game, the follower’s responses to an increase of frequency by the leader (\( \partial p_j / \partial f_i, \partial f_j / \partial f_i \)) are downward-sloping while the responses to fare increases (\( \partial p_j / \partial p_i, \partial f_j / \partial p_i \)) are upward-sloping (see Appendix A). This feature prevents us from quantifying analytically whether the market power effect and the congestion internalization are higher or lower in comparison with the Bertrand game of Section 2.2. This is because, with general functional forms, the sign of \( \tilde{b} - b \) and the sign of the last term in brackets on the right-hand side of equation 22 cannot be determined analytically. In Section 4 we study the magnitude of these effects with numerical examples, suggesting that the leader’s internalization can be less than in the Bertrand game, but still in between what is found for Cournot and atomistic behavior.

With the airlines’ market characterized, we now analyze airport pricing and capacity investments.

3. Airport pricing and capacity investment

We consider the first-best case of a (unweighted) welfare maximizing airport, with capacity and per-flight as well as per-passenger tolls as instruments. We solve the airport maximization problem analytically in this section, and numerically in Section 4. The derivation of first-order conditions presented in this section is in Appendix B.

Social welfare is defined as the sum of net benefits for all agents: consumer surplus, airlines’ profits and airport’s profit. The first of this, with quantities and full-prices being \( q_i, q_j, \theta_i \), and \( \theta_j \), is just \( U(q_i, q_j) − \theta_i \cdot q_i − \theta_j \cdot q_j \). Using (1) and (2), straightforward calculations yield the following expression:

\[
CS = \frac{B}{2} \cdot (q_1^2 + q_2^2) + E \cdot q_1 \cdot q_2.
\]

(23)

We assume that airport’s costs are separable and proportional to number of passengers, frequency and capacity, shaping airport profit in the following way:

\[
\Pi = \sum_i \left( q_i \cdot (\tau_i^q - c_q) + f_i \cdot (\tau_i^f - c_f) \right) - K \cdot r,
\]

(24)

where \( c_q \) is the (constant) operating cost per-passenger, \( c_f \) the (constant) operating cost per-flight, \( r \) the cost of capital and \( K \) the capacity of the airport.

Adding airlines profit (equation 7) we can express social welfare as a function of traffic, frequencies, aircraft sizes, and fares:

\[
SW = \left[ \frac{B}{2} (q_1^2 + q_2^2) + E \cdot q_1 \cdot q_2 \right] + \left[ \sum_i q_i \cdot (p_i - c_q) \right] - \left[ \sum_i f_i \cdot (\gamma_i^f + s_i \cdot \gamma_i^s + c_f) \right] - [K \cdot r],
\]

(25)
where the first bracketed term is consumer surplus, the second bracketed term is airlines’ and airport’s per-
passenger revenues minus costs (airport’s revenues from tolls cancel out against airlines’ costs from tolls),
the third is per-flight revenues minus costs (again airport’s tolls cancel out) and the last bracketed term
is capacity costs. Both numbers of passengers are a function of fares and frequencies, but we omit the
arguments here.

Straightforward calculations lead to the following conditions for optimal fares:

$$p_i - \gamma_i^s - c_q = 0, \quad \forall i.$$  \hspace{1cm} (26)

This result states that optimal fare must equal airlines’ marginal cost per capacity unit plus airport marginal
operating cost per passenger. The welfare maximizing fare that should be charged to passengers does not
include any congestion term because—as explained in the previous section—airlines take congestion into
account only in their frequency setting. From first-order conditions for frequency we obtain the following:

$$-(q_i + q_j) \cdot \frac{\partial D}{\partial f_i} - q_i \cdot \frac{\partial g_i}{\partial f_i} = c_f + \gamma_i^f, \quad \forall i.$$  \hspace{1cm} (27)

This means that the optimal frequency must be such that the marginal cost per flight (right-hand side) equals
marginal net benefits of all passengers from congestion, plus schedule delay savings (left-hand side). We
define “total marginal congestion costs” as the congestion cost that an extra flight imposes on all passengers
($\partial D/\partial f_i \cdot (q_i + q_j)$). If airlines do not internalize any congestion at all, they should be charged this amount
plus the airport’s marginal operating cost per flight ($c_f$), the so-called atomistic toll.

Finally, the optimal investment rule for the airport is:

$$-(q_i + q_j) \cdot \frac{\partial D}{\partial K} = r.$$  \hspace{1cm} (28)

This shows that airport capacity should be increased until marginal cost equals marginal benefits from
congestion reductions. Having established the first-order conditions for social optimal fares and frequencies,
we can now derive the optimal tolls per passenger ($\tau^q_i, \tau^q_j$) and per flight ($\tau^f_i, \tau^f_j$), by using airlines’ first-
order conditions for each game (e.g., for Bertrand competition, we use equations (12), (15), (26) and (27)).
Results now follow, ordered by game type.

- Cournot behavior

For the simultaneous game of Cournot behavior, it can be expected that optimal tolls are consistent with
the earlier airport pricing literature. Indeed, the per-passenger toll in (29) is marginal operating cost plus
a subsidy equal to market power markup, and the per-flight toll in (30) equals marginal operating cost plus
congestion imposed on the rival airline passengers.

$$\tau^q_i = c_q - q_i \cdot B,$$  \hspace{1cm} (29)
\[ \tau_i^f = c_f + q_j \cdot D'. \] (30)

However, the subsidy is separate from the congestion toll, and the per-passenger first-best toll is negative when the market power effect is bigger than airport per-passenger marginal operating cost. Hereafter, we use the term “Cournot toll” for the per-flight charge that accounts only for the congestion costs imposed on the competitor’s passengers (equation (30)).

- Bertrand behavior

For the simultaneous game of Bertrand behavior, we find:

\[ \tau_i^q = c_q - q_i \cdot \frac{1}{b}, \] (31)

\[ \tau_i^f = c_f + \left( q_j + \frac{e}{b} \cdot q_i \right) \cdot D'. \] (32)

The per-passenger toll in this game is marginal operating cost plus a subsidy equal to the market power markup. For the per-flight toll, we obtain that it is marginal operating cost per flight plus a congestion toll, which is, however, different to the traditional Cournot toll. A first-best airport charges the congestion costs imposed on rival’s passengers \((q_j \cdot D')\) plus an additional term. This new term is the own-passengers marginal congestion cost \((q_i \cdot D')\) times the degree of substitutability \((e/b)\). When this ratio is zero, the goods are independent and the optimal per-flight toll is the congestion imposed on the rival’s passengers, as it is in the Cournot game. When it is one, goods are perfect substitutes, and the first-best toll is the so-called atomistic toll. Any other feasible value of \(e/b\) (between zero and one) yields a charge that is somewhere in between the atomistic toll and the Cournot toll. The toll reflects that airlines are internalizing less than self-imposed congestion. Hence, first-best tolls are closer to the atomistic toll than in Cournot competition; and, the higher the degree of substitutability, the higher the first-best toll should be.

The intuition of this result, which to the best of our knowledge is new in the airport pricing literature, is that when goods are independent, there are two monopolies using the same facility in order to serve two independent markets. Therefore, a global welfare maximizing airport charges to each carrier the congestion imposed on the competitor, which was entirely ignored by the operators because of the independence. In the other extreme, where the ratio equals one, goods are perfect substitutes and—since airlines take the rival’s fare as given—the Nash equilibrium is the perfectly competitive outcome (as in Bertrand competition with homogeneous goods). In this extreme case, an airline will expect that any reduction in its own flight volume will be offset by an equally big increase in the competitor’s flight volume. As a consequence, the total number of flights, thus airport congestion, will remain unchanged, and internalization of own congestion makes no sense.

This new result on optimal congestion pricing suggests that welfare maximizing per-flight tolls may be higher than what is suggested by the Cournot model, hence have a more substantial impact in airlines’
decisions, and therefore may yield more sizable welfare gains. Even if the market is highly concentrated, the dominant airline has to be charged for a high proportion of total marginal congestion costs if the substitutability between the airlines is not too low.

- Stackelberg leader and Cournot follower game

Since the first-order conditions for the follower are the same as in the simultaneous setting with Cournot behavior, the first-best tolling rules also coincide. The per-passenger toll is marginal operating cost plus the market power subsidy (see equation 29) and the per-flight toll is marginal operating cost plus congestion imposed on the rival (equation 30). For the leader \((i)\), the first-best tolls are:

\[
\tau^q_i = c_q - q_i \cdot \tilde{B},
\]

\[
\tau^f_i = c_f + \left( q_i + q_i \cdot \frac{\partial f_i}{\partial f_j} \right) \cdot D' + q_i \cdot E \cdot \frac{\partial q_j}{\partial f_i},
\]

where \(\tilde{B} = B + E \cdot \partial q_j / \partial q_i + D' \cdot \partial f_j / \partial q_i\) and derivatives in absolute value are negative (see Appendix B).

The interpretation of these tolls is the same as in previous settings. The first-best per-passenger toll is the marginal operating cost plus a subsidy equal to the market power markup, and the per-flight toll corrects for uninternalized congestion. Because the leader—when considering the effect of its own decisions on the followers—alters internalization, the optimal congestion toll charged is in between the Cournot and the atomistic toll. This optimal congestion toll conceptually reproduces the result of Brueckner and Van Dender (2008) in their Stackelberg behavior with a Cournot follower game (note that the last term in (34) is not present in their analysis due to the assumptions of perfectly elastic demand and perfect substitution).\(^{14}\)

- Stackelberg leader and Bertrand follower game

The optimal tolling rules for the follower are identical to the tolling rules with Bertrand behavior (equations 31 and 32). On the other hand, the optimal tolls charged to the leader \((i)\) are:

\[
\tau^q_i = c_q - q_i \cdot \frac{1}{b},
\]

\[
\tau^f_i = c_f + \left( q_j + q_i \cdot \frac{c}{b} \right) \cdot D' + q_i \left[ \frac{\tilde{b} - b}{b} \left( D' + g_i \right) + \frac{c - b}{b} D' \frac{\partial f_i}{\partial f_j} \right] + \frac{c}{b} \left( \frac{\partial p_i}{\partial f_i} + g_j \frac{\partial f_j}{\partial f_i} \right),
\]

where \(\tilde{b} = b - c \cdot \partial p_j / \partial p_i - \partial q_i / \partial f_j \cdot \partial f_j / \partial p_i\).

The optimal toll per passenger is airport’s marginal operating cost per passenger plus the subsidy that corrects market power. Since \(\tilde{b}\) is the derivative of own traffic with respect to own fare (taking into account

\(^{14}\)They relax both assumptions but only in the case of a firm acting as a Stackelberg leader and interacting with a competitive fringe.
the effect on the follower) the interpretation is the same as usual for pricing with market power. The per-flight toll corrects the frequency setting so that the leader sets the welfare maximizing frequency. As the sign of \( \hat{b} - b \) cannot be determined a priori, the per-flight toll for the leader has to be studied numerically. In Section 4 we do this, finding that in the numerical examples the optimal toll is in between the Cournot and the atomistic toll, and that it can be above or below the Bertrand toll depending on the degree of substitutability.

For each of the four game types we considered, we found that a welfare maximizing airport needs to use two taxes, namely per-passenger and per-flight tolls, to reach the first-best outcome. It corrects the market power effect with a per-passenger toll and the frequency inefficiency with a per-flight charge. The former (\( \tau^q_i \)) is below airport marginal operating cost per passenger, because it counteracts the airline market power exertion by means of a subsidy. As a consequence, this toll is negative when airlines’ markups exceed airport’s marginal operating costs. Conversely, the first-best per-flight toll is always above airport marginal operating cost, because airlines do not fully internalize congestion. If only one tax can be applied, the airport is facing a second-best problem, and which instrument is better to apply depends on market specific conditions. The stronger the airlines’ market power effect compared to the congestion effect, the more likely is that using only per-passenger subsidies is more efficient than charging only per-flight. In the extreme case of monopoly operation, the per-flight toll is unnecessary, and the first-best is attained with per-passenger subsidies only.

The results also imply that the first-best outcome cannot be reached by only charging passengers, because also charges per movement are necessary. If the authority or the facility wants to charge airlines per flight and passengers per trip, a per-passenger tax above the operating costs per passenger is not consistent with welfare maximization. We further expand on this in the numerical analysis below.

Two recent analyses have put a question mark on the desirability of the traditional Cournot congestion toll, i.e. total marginal congestion costs times the market share of each airline (in our model the Cournot per-flight toll in equation 30). Morrison and Winston (2007) find a small difference between the net benefits of charging the Cournot toll versus the atomistic toll that ignores any internalization. Then, Brueckner and Van Dender (2008) shows that Stackelberg behavior with a Cournot follower yields optimal airport tolls that lie in between of both policies.

The results of this model give new insights into the debate concerning the desirability of the traditional Cournot congestion toll: first, the optimal toll might well be close to the atomistic toll even without assuming leadership behavior and without abstracting from airlines’ market power exertion. This is the case with Bertrand behavior, simultaneous competition with aircraft size, fare and frequency as strategic variables, and the related two-stage setting. From equation (32), it is straightforward that the closeness of the optimal toll to the atomistic toll depends on market-specific characteristics (ratio of substitutability \( e/b \)). Therefore, within the same setting but with conditions varying over a network, the entire range of tolls can be optimal.
We also confirm the internalization result of Brueckner and Van Dender (2008) for a Stackelberg leader with a Cournot follower, and extend it to the case of a Stackelberg leader with a Bertrand follower, finding even higher optimal tolls. These findings may lead to optimism on the relative efficiency of atomistic congestion pricing in aviation markets. As we cannot compare welfare and equilibrium values analytically, in Section 4 we solve numerically the equilibrium for an airport charging the atomistic toll, and make the comparisons with the first-best to assess the relative efficiency.

4. Numerical analysis

In this section we present a numerical analysis that allows making comparisons that are not possible analytically. We also analyze the performance of second-best policies. Despite the simplified structure of the model, we use parameters that are as much as possible calibrated so as to reflect realistic values. We use the following functional forms for the schedule delay cost \( g_i \) and passengers’ congestion cost \( D \):

\[
g_i = \gamma \cdot \frac{1}{f_i}, \tag{37}
\]

\[
D = \alpha \cdot \left( \frac{f_i + f_j}{K} \right)^\beta, \tag{38}
\]

where the schedule delay cost in (37) is inversely proportional to the airline frequency, and \( \gamma \) is a constant representing the monetary value of a unit of schedule delay time. The functional form in (37) would be consistent with uniformly distributed desired departure times, and equally spaced flights, and is often used in the literature (e.g. Brueckner, 2004; Basso, 2008). The congestion delay at the airport in (38) is a function of the volume capacity ratio, with \( \alpha \) being proportional to the passengers’ value of travel time, \( K \) the capacity, and \( \beta \) the power of the function.

Our reference scenario for calibration has symmetric airlines and assumes a marginal-operating-cost pricing airport, i.e. a toll per passenger of \( c_q \) and a toll per flight of \( c_f \). The parameter calibration considered equilibria in the Cournot and Bertrand settings, and the following tables summarize parameters and equilibrium outputs of the calibration case.

<table>
<thead>
<tr>
<th>Demand parameters</th>
<th>Cost parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airlines</td>
<td>Airport</td>
</tr>
<tr>
<td>( A )</td>
<td>1250</td>
</tr>
<tr>
<td>( B )</td>
<td>1.04</td>
</tr>
<tr>
<td>( E )</td>
<td>0.63</td>
</tr>
</tbody>
</table>
Table 2: Equilibrium outputs, marginal operating cost pricing airport.

<table>
<thead>
<tr>
<th>Game</th>
<th>Total traffic</th>
<th>Fare Frequency</th>
<th>Total Aircraft size</th>
<th>K</th>
<th>Airlines Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bertrand</td>
<td>745</td>
<td>596</td>
<td>2.5</td>
<td>295</td>
<td>3.6</td>
</tr>
<tr>
<td>Cournot</td>
<td>639</td>
<td>680</td>
<td>2.1</td>
<td>311</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Table 3: Demand elasticities, generalized price and share of generalized price due to fare and delays.

<table>
<thead>
<tr>
<th>Game</th>
<th>Gen. Fare Congestion Schedule Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bertrand</td>
<td>629 0.95 0.014 0.036 -1.01</td>
</tr>
<tr>
<td>Cournot</td>
<td>718 0.95 0.011 0.039 -1.35</td>
</tr>
</tbody>
</table>

Airlines’ operating cost parameters are from Swan and Adler (2006), using a reference distance of 1.500 km and an adjustment in the per-passenger cost as suggested by the authors. Values of time are from Morrison and Winston (1989), while congestion, delay and demand parameters are set such that equilibrium elasticity with respect to generalized price in both settings is close to -1.146, the mean value of 204 observations reported by Brons et al. (2002). Values are set to the same monetary units (U.S. dollars) throughout the calibration process.

We provide numerical examples below for the duopoly setting considered analytically in the previous sections, and also extend the analysis to the case with several airlines. Throughout the analysis, we illustrate possible policy implications of our results by considering the substitutability ratios estimated by Perloff et al. (2007) for the Orlando-Wichita and the Orlando-Providence markets. According to their data, United Airlines and American Airlines had a combined share of passengers above 98% in both markets, and their estimations indicate that the markets are well represented by a Bertrand differentiated duopoly with substitutability ratios between 0.4 and 0.7. Nazarenus (2011), using data from 2007, finds that 37 Orlando based markets fit a duopoly criteria and conclude that the outcome is more competitive than Cournot behavior implies and less competitive than perfect competition, which is exactly the outcome of differentiated Bertrand competition.¹⁵

¹⁵The duopoly criteria they use is the one proposed by Brander and Zhang (1990), which considers a market as duopolistic when United Airlines and American Airlines together had a market share exceeding 75%, and each carrier had at least 100
4.1. Internalization

We first solve the problem with a welfare maximizing airport and a duopoly of symmetric airlines, to assess the equilibrium share of congestion that is internalized by each player. In this case, each carrier has an equilibrium share of 50 percent of passengers. Figure 1 shows the percentage of congestion that is internalized by each agent for a feasible range of the ratio of substitutability, $e/b$. Recall that a value of 0 means independent goods, whereas 1 represents pure substitutes. According to the first-order conditions derived in Section 2, in the Cournot case each airline internalizes half of the total marginal congestion cost (see equation 10). For the Bertrand case, internalization is in between one half and zero, and decreases linearly in the ratio of substitutability; see (16). For the game with a Stackelberg leader and a Cournot follower, the leader internalizes less than half of total marginal congestion and the follower acts as in the simultaneous Cournot game, internalizing exactly half of total marginal congestion costs. Finally, Bertrand follower’s internalization is the same as in the simultaneous Bertrand game, while the leader behavior cannot be quantified a priori.

The self-imposed internalization of Cournot behavior and the linear decrease of the internalization for airlines in the Bertrand setting are shown in Figure 1 in black lines. To illustrate the possible implications of our results, we may look at parameters estimated by Perloff et al. (2007) for a linear demand structure (as in equation (3)) in a duopolistic competition. They find values for the ratio $e/b$ between 0.4 and 0.7, which implies that airlines would internalize between 12 and 30 percent of congestion costs, and therefore should be charged for a remaining 70 to 88 percent of the total marginal congestion costs.

For both Stackelberg games, we find that the leader internalization is less than the self-imposed congestion, and decreases towards atomistic behavior as the substitutability as measured by the ratio $e/b$ increases. In the Stackelberg-Cournot case (the two solid lines), the leader always internalizes less than the follower, whose internalization is always the congestion imposed on own passengers. For the Stackelberg-Bertrand setting (the two dashed lines), we find that the leader internalizes roughly the same congestion as the follower, both being less than the self-imposed and approaching to zero as the ratio of substitutability grows. Brueckner and Van Dender (2008) also analyze Stackelberg behavior with a Cournot follower, but suppressing market-power by assuming perfectly elastic demand, and assuming that the outputs of the carriers are perfect substitutes. We extend this to the case of price-sensitive demand and imperfect substitution, finding similar results regarding internalization of congestion: the leader internalizes less than self-imposed congestion, but does not fully reach atomistic behavior. This is represented in Figure 1 by the solid gray line.

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passengers in the 10% sample available. Nazarenus (2011) finds that the combined market share of the two airlines is higher than 91% in 29 markets in the third quarter of 2007. Unfortunately the literature on conduct parameters is scarce, and we are not aware of other estimations for the degree of substitutability in the airline industry (following Brander and Zhang (1990), Nazarenus (2011) does not allow for imperfect substitutability).
From this analysis it follows that, for Bertrand behavior both in simultaneous competition as well as in Stackelberg competition, optimal congestion charges are close to the atomistic charge when the substitutability is not too low (near the right-hand end of Figure 1). For this reason, optimal per-flight congestion tolls might have a more significant impact on airlines’ scheduling decisions and, therefore, in alleviating congestion, than what was suggested by earlier Cournot models.

4.2. Alternative policies

We next study the following alternative policies: (i) the second-best cases where an airport can use only one tax instrument, and (ii) the relative performance of atomistic pricing. The motivation of studying one instrument is to gain insight on the impact of each tolling instrument separately. The purpose of assessing the performance of atomistic pricing is to better understand to what extent this policy is attractive from an efficiency point of view.

4.2.1. Using one tax instrument

As shown in Section 3, a welfare maximizing airport needs two pricing instruments to reach the first-best outcome. It corrects the market power effect with a per-passenger subsidy and the frequency inefficiency with a per-flight charge. We now look at what happens when it can use only one instrument. For this purpose, we define the relative efficiency $\Omega^p$ as the welfare gain due to a policy ($p$), relative to the first-best gain:

$$\Omega^p = \frac{SW^p - SW^{mc}}{SW^{fb} - SW^{mc}},$$

where superscript $fb$ refers to the first-best case, and $mc$ to an airport charging marginal operating costs (as in the reference scenario for calibration).
Figure 2: Relative efficiency by game type.

Figure 2 shows the relative efficiency of both second-best policies for the base calibration. The results show that, in our calibration, the market power effect dominates the congestion effect, yielding a high relative efficiency for a per-passenger subsidy and a low one for the per-flight toll. The results also show a significant effect of game type on the performance of a policy. As we discuss in Section 2, the market power exertion and the amount of internalized congestion are always higher in a Cournot setting than in an Bertrand setting. This leads to a higher relative efficiency of the per-passenger subsidy, and a lower efficiency of the per-flight charge, in the Cournot competition than in the Bertrand competition. For the Stackelberg games, the relative efficiency lies in between the Cournot and Bertrand cases, but the ranking is sensitive to the parameters (as is the degree of internalization). The results furthermore show that, for the chosen parametrization, the social gains for the two instruments are nearly additive. That is, the gains from introducing the one instrument are almost insensitive to the other instrument being in place already. This underlines the lack of substitutability between the two instruments.

The second analysis we perform has the purpose of assessing the relative efficiency of both second-best policies when the market power and the congestion effect have a different comparative importance. We do this by increasing the number of (symmetric) airlines that participate in the market, for a given demand structure, because it captures in one parameter the relative importance of both effects: increasing the number of firms makes the market power effect weaker because of the increased competition and, for the same reason, the congestion externality becomes more severe. As Figure 3 shows, the number of firms participating in the market affects the policies in a different manner: the relative efficiency of the per-flight toll increases with the number of airlines, while the opposite occurs for the per-passenger subsidy. The intuition is straightforward: when the number of airlines increases, each airline’s market share of passengers decreases, which leads to less internalization as well as to less ability for exerting market power. Both effects explain the performance improvement of the per-flight toll and the reduction of gains from counteracting the airlines’ markup with a per-passenger subsidy. The negative relative efficiency of the per-passenger toll,
in Bertrand competition for 5 or more airlines, is because congestion inefficiencies are significantly more important than market power exertion in those cases. Therefore, the positive per-flight toll ($\tau_f = c_f$) of the reference scenario, which is removed in the per-passenger toll scenario, gives higher social welfare gains than the second-best per-passenger toll, with a zero per-flight toll.

Which second-best option is better clearly depends on the balance between the inefficiencies and the market structure. The relative efficiency of per-passenger subsidies is higher than the per-flight toll efficiency for Cournot competition up to 8 airlines; on the other hand, in Bertrand competition, the per-flight toll outperforms the per-passenger subsidy already with 4 airlines, and exceeds 70 percent of the first-best with 6 airlines.

We illustrated these points by means of increasing the number of firms, but the increasing performance of the per-flight toll in the Bertrand case can also be found with an increase of the ratio of substitutability $e/b$. As the substitutability increases, a smaller share of congestion is internalized spontaneously, and the performance of the per-flight toll rises.\(^{16}\)

This exercise provides some useful insights into the performance of second-best policies. When the market power effect is stronger than the congestion effect, it is better for social welfare to give a per-passenger subsidy instead of charging a per-flight toll, and vice versa. If negative tolls are not feasible, it is attractive to charge only a per-flight congestion toll, which will perform better if the internalization is low, because of small market shares, or because substitutability is not too low when airlines behave as in the Bertrand setting. Finally, a positive per-passenger toll cannot be supported from an efficiency perspective, unless the per-flight toll is not feasible and the airlines’ market power markup is small compared to the

\(^{16}\)We assess this numerically, finding that for a ratio $e/b$ of 0.9, $\Omega^f$ reaches 67 percent for a Bertrand duopoly.
Figure 4: Atomistic pricing performance relative to second-best case.

airport’s marginal operating cost per passenger.

4.2.2. Atomistic pricing

Finally, we assess the efficiency of levying atomistic congestion tolls to airlines. For this purpose, we look at the welfare gain due to atomistic tolls relative to the second-best case of only having per-flight tolls ($SW^f$), and having a marginal-operating-cost pricing airport as a reference ($SW^{mc}$). The aim of measuring the efficiency relative to this second-best policy, is to isolate the welfare gain that comes from the per-passenger subsidy.\footnote{When the comparison is carried out with respect to the first-best, the welfare gains of charging atomistic tolls are also almost as high as the gains of using the (optimal) per-flight toll. On the other hand, the relative efficiency of atomistic tolls together with (second-best) per-passenger (negative) tolls varies between 99 and 100 percent. This is because market power effect is dominating.} The performance measure, for this case, is defined in the following way (using superscript $atom$ for atomistic tolls):

$$\Omega^{atom} = \frac{SW^{atom} - SW^{mc}}{SW^f - SW^{mc}}.$$  \hspace{1cm} (40)

Figure 4 shows that $\Omega^{atom}$ has an intuitive relationship with the amount of congestion internalized by carriers. This policy achieves the lowest benefits when airlines behave as Cournot oligopolists, where the first-best toll is half of total marginal congestion costs. Moreover, the performance of atomistic pricing in Cournot competition is only moderately sensitive to the substitutability ratio, as it varies between 77 and 79 percent.
When the airlines market is characterized by Bertrand behavior, the performance of atomistic pricing rapidly improves as the substitutability is higher \((e/b)\) approaches 1). This is because the amount of congestion that is internalized diminishes (see Figure 1) and, therefore, the first-best toll approaches the atomistic toll as we showed analytically in Section 3. Figure 4 also shows that the efficiency measure used \((\bar{\Omega}^{\text{atom}})\) is close to 1 when the degree of substitutability is not too low. Using again the ratios \(e/b\) obtained by Perloff et al. (2007), atomistic pricing would yield roughly between 85 and 95 percent of the maximum social benefit that can be obtained with per-flight tolls.

The benefits that atomistic pricing generates in Stackelberg games follow the internalization patterns; for the Stackelberg-Bertrand game, \(\bar{\Omega}^{\text{atom}}\) is almost the same as in Bertrand competition, because carriers are internalizing approximately the same amount of congestion in both games. For the Stackelberg-Cournot game, we showed, both analytically as well as numerically, that the follower internalizes the same amount of congestion as in the static game with Cournot behavior, and that the leader’s internalization is similar to the one observed in the Bertrand static game. As a consequence, the performance of atomistic pricing, in the Stackelberg-Cournot setting, is lower than in the Bertrand setting, and higher than in the Cournot setting.

In Figure 5, we show the relative efficiency of atomistic pricing compared to the first-best (i.e. using \(\Omega^{\text{atom}}\) from (39) as the performance measure), when the number of symmetric firms increases in the Bertrand and Cournot cases. The performance of atomistic pricing, in this case, is also very similar to the performance of the second-best policy of charging only per-flight tolls, as can be seen by comparing Figure 3 and Figure 5. As the number of airlines increases, the performance of atomistic pricing is better, reaching high values when
 airlines behavior is well represented by the Bertrand assumption. This implies that atomistic congestion pricing can perform, in terms of social welfare, in a way comparable to the optimal per-flight toll.

These results also give new insights to the airport pricing literature: if congestion is a major issue and the industry is more adequately described as in the Bertrand case, atomistic pricing may offer a more attractive instrument. When per-passenger subsidies are given or, as a second-best policy, per-passenger tolls are set to zero, an airport’s financial deficit can then be reduced without significant welfare losses. If Cournot behavior is more adequate, then naïve atomistic pricing for a duopoly can be less attractive. For the Stackelberg games, where first- and second-best tolls differ among carriers, the uniform atomistic toll still produces a small welfare loss with respect to the maximum benefit that can be obtained with a per-flight charge.

5. Conclusion

The present analysis shows that the amount of congestion that airlines internalize may be smaller than the simple market shares formulae from Cournot models, and more so if firms are closer substitutes. As a consequence, the welfare gains and congestion reductions from optimal congestion pricing may be higher. We also show that the airport revenue may be increased and, as a result, a congested airport would be closer to exact self-financing under optimal congestion pricing. Furthermore, the optimal congestion charges may be less differentiated than what has been advanced before, hence diminishing the perception of charges being unequitable and enhancing its implementation feasibility. To what extent these results apply depends on the prevailing type of strategic interaction in a particular market and on the degree of substitutability between airlines. In addition, the paper differentiates between per-passenger and per-flight charges, showing their unique and non-interchangeable roles in welfare maximization.

We also provide numerical examples to assess the performance of levying atomistic tolls and the relative efficiency of second-best policies. The analysis confirms that when the airlines’ market power effect is larger than the congestion effect, it is wiser to give a per-passenger subsidy instead of a per-flight charge, and vice versa. Numerical examples also suggest that only using a per-flight charge and levying atomistic tolls can yield similar and substantial benefits when the degree of substitutability is not too low, and airlines do not behave in a Cournot fashion. This is, in our framework, when they behave as Bertrand oligopolists, either competing simultaneously (Nash) or in a Stackelberg-Bertrand fashion. The good performance of atomistic pricing, although to a lesser extent, is also found numerically in the Stackelberg-Cournot setting.

From the analysis, a number of issues emerge for future research. As airline behavior determines the optimality of congestion charges, and in some settings the degree of substitutability also has a major influence on the size of optimal tolls, estimation of airline conduct parameters stands as an important topic for future research. This is of particular relevance as airlines’ conduct is likely to vary across markets and, in order to implement the correct policy, it is necessary to know which behavior is representative of the market in study.
Our approach abstracts from studying entry barriers, which might yield different incentives and outcomes for existent firms, and this framework can be used for studying such potential incentives from a dual tax perspective. Another qualification of our model is that it relies on symmetric product differentiation and, although this does not critically affects our main conclusions, a more realistic demand structure should be considered especially for studying interactions between asymmetric airlines. For example, studying the interactions between legacy and low-cost carriers may require a more elaborate specification of the demand structure.

We see regulation of private airports and the role of commercial (concession) operations in airport pricing as another natural extension of the present analysis. Finally, we perform the analysis in a single market, with one airport for analytical simplicity and to focus on the main insights, but network effects and airports having different objective functions are also seen as a logical extension for future research.

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Appendix A. Reaction functions

- Stackelberg-Cournot

First-order conditions for the follower $j$ can be written as:

$$A - 2 \cdot B \cdot q_j - E \cdot q_i - D - g_j - \gamma_j^s - \tau_j^q = 0,$$

(A.1)

$$-q_j \cdot \left( \frac{\partial D}{\partial f_j} + \frac{\partial g_j}{\partial f_j} \right) - \gamma_j^f - \tau_j^f = 0.$$

(A.2)

To derive how the follower outputs vary when the leader changes quantity and frequency, we differentiate the system, write the result in matrix notation, and apply Cramer's rule. After some straightforward calculations, we get:

$$\frac{\partial q_j}{\partial q_i} = \frac{-E \cdot q_j \cdot (D'' + g''_j)}{R} \leq 0,$$

(A.3)

$$\frac{\partial f_j}{\partial q_i} = \frac{E \cdot (D' + g'_j)}{R} \leq 0,$$

(A.4)

$$\frac{\partial q_j}{\partial f_i} = \frac{-q_j \cdot (D'' \cdot g''_j - D'' \cdot g''_j)}{R} \leq 0.$$

(A.5)
\[
\frac{\partial f_j}{\partial f_i} = \frac{-2 \cdot B \cdot q_j \cdot D'' + D' \cdot (D' + g_j')}{R} \leq 0, 
\]
where \( R = 2 \cdot B \cdot q_j \cdot (D'' + g_j') - (D' + g_j')^2 \) is, by definition, the determinant of the Hessian matrix of airlines' profit. Since we assume existence of a maximum, \( R > 0 \). Because of assumptions (5), and the fact that an equilibrium with positive traffic implies \( D' + g_j' < 0 \), it is clear that all the reaction functions in this case are non-positive.

- **Stackelberg-Bertrand**

We proceed in the same way as above to calculate the reaction functions. First-order conditions for the follower \( j \) are:

\[
p_j - \gamma_j^* - \tau_j = 0, 
\]

\[
\gamma^f_j + \tau_j'^j - (p_j - \gamma_j^* - \tau_j^j) \cdot \frac{\partial q_j}{\partial f_j} = 0. 
\]

And denoting \( \hat{p}_j = p_j - \gamma_j^* - \tau_j^j \), we obtain:

\[
\frac{\partial p_j}{\partial p_i} = \frac{-e \cdot \frac{\partial^2 q_j}{\partial f_i^2} \cdot \hat{p}_j}{R} \geq 0, 
\]

\[
\frac{\partial p_j}{\partial f_i} = \frac{-1 \cdot \frac{\partial q_j}{\partial f_i} \cdot \frac{\partial^2 q_j}{\partial f_i^2} \cdot \hat{p}_j + 1 \cdot \frac{\partial q_j}{\partial f_j} \cdot \frac{\partial^2 q_j}{\partial f_i \partial f_j} \cdot \hat{p}_j}{R} \leq 0, 
\]

\[
\frac{\partial f_j}{\partial f_i} = \frac{2 \cdot \frac{\partial^2 q_j}{\partial f_i \partial f_j} \cdot \hat{p}_j + 1 \cdot \frac{\partial q_j}{\partial f_j} \cdot \frac{\partial q_j}{\partial f_i}}{R} \leq 0, 
\]

where \( R = -2 \cdot \frac{\partial^2 q_j}{\partial f_i^2} \cdot \hat{p}_j - \frac{1}{b} \cdot (\frac{\partial q_j}{\partial f_i})^2 > 0 \), as we assume existence of a maximum. Because of assumptions (5), \( \partial q_j / \partial f_i < 0 \), \( \partial^2 q_j / \partial f_i^2 < 0 \) and \( \partial^2 q_j / \partial f_i \partial f_j < 0 \). An equilibrium with positive traffic implies \( \partial q_j / \partial f_j > 0 \), therefore we can show that reaction functions have the sign that is presented above.

**Appendix B. Welfare maximizing airport first-order conditions**

Since airlines choose aircraft size such that \( q_i = s_i \cdot f_i \), we use this relation before maximizing welfare. Then, first-order condition with respect to fare is:

\[
\frac{\partial SW}{\partial p_i} = Bq_i \frac{\partial q_i}{\partial p_i} + Bq_j \frac{\partial q_j}{\partial p_i} + Eq_i \frac{\partial q_i}{\partial p_i} + Eq_j \frac{\partial q_j}{\partial p_i} + (p_i - \gamma_i^* - e_q) \cdot \frac{\partial q_i}{\partial p_i} + q_i + (p_j - \gamma_j^* - e_q) \cdot \frac{\partial q_j}{\partial p_i} 
\]
\[
q_i \left( B \frac{\partial q_i}{\partial f_i} + E \frac{\partial q_j}{\partial f_i} + 1 \right) + q_j \left( B \frac{\partial q_j}{\partial f_i} + E \frac{\partial q_i}{\partial f_i} \right) + (p_i - \gamma_i^s - c_q) \cdot \frac{\partial q_i}{\partial p_i} + (p_j - \gamma_j^s - c_q) \cdot \frac{\partial q_j}{\partial p_i} = 0. \tag{B.1}
\]

And noting that
\[
B \frac{\partial q_i}{\partial p_i} + E \frac{\partial q_j}{\partial p_i} = B \cdot -b + E \cdot c = \frac{-b^2}{b^2 - e^2} + \frac{c^2}{b^2 - e^2} = -1, \tag{B.2}
\]

\[
B \frac{\partial q_j}{\partial p_i} + E \frac{\partial q_i}{\partial p_i} = B \cdot e + E \cdot -b = \frac{b \cdot e}{b^2 - e^2} + \frac{-b \cdot e}{b^2 - e^2} = 0, \tag{B.3}
\]

we obtain, using the analogous calculations for \( \partial SW / \partial p_j \), first-order conditions for fares:
\[
\frac{\partial SW}{\partial p_i} = (p_i - \gamma_i^s - c_q) \cdot \frac{\partial q_i}{\partial p_i} + (p_j - \gamma_j^s - c_q) \cdot \frac{\partial q_j}{\partial p_i} = 0, \tag{B.4}
\]

\[
\frac{\partial SW}{\partial p_j} = (p_i - \gamma_i^s - c_q) \cdot \frac{\partial q_i}{\partial p_j} + (p_j - \gamma_j^s - c_q) \cdot \frac{\partial q_j}{\partial p_j} = 0. \tag{B.5}
\]

Both conditions can only be satisfied if fares fulfill:
\[
(p_i - \gamma_i^s - c_q) = 0, \quad \forall i. \tag{B.6}
\]

For the frequency first-order conditions, let \( p_i = \gamma_i^s - c_q \) and \( p_j = \gamma_j^s - c_q \). Then, the derivative of \( SW \) with respect to \( f_i \) is:
\[
\frac{\partial SW}{\partial f_i} = B q_i \frac{\partial q_i}{\partial f_i} + B q_j \frac{\partial q_j}{\partial f_i} + E q_i \frac{\partial q_i}{\partial f_i} + E q_j \frac{\partial q_j}{\partial f_i} + \tilde{p}_i \cdot \frac{\partial q_i}{\partial f_i} + \tilde{p}_j \cdot \frac{\partial q_j}{\partial f_i} - (\gamma_i^f + c_f). \tag{B.7}
\]

Using \( \tilde{p}_i = \tilde{p}_j = 0 \) from equation (26), we can write,
\[
\frac{\partial SW}{\partial f_i} = q_i \left( B \frac{\partial q_i}{\partial f_i} + E \frac{\partial q_j}{\partial f_i} \right) + q_j \left( B \frac{\partial q_j}{\partial f_i} + E \frac{\partial q_i}{\partial f_i} \right) - (\gamma_i^f + c_f). \tag{B.8}
\]

Note that,
\[
\frac{\partial q_i}{\partial f_i} = -b \frac{\partial D}{\partial f_i} - b \frac{\partial g_i}{\partial f_i} + e \frac{\partial D}{\partial f_i}, \tag{B.9}
\]

\[
\frac{\partial q_j}{\partial f_i} = -b \frac{\partial D}{\partial f_i} + e \frac{\partial D}{\partial f_i} + e \frac{\partial g_i}{\partial f_i}. \tag{B.10}
\]

And using these equations, first-order condition can be written as,
\[
\frac{\partial SW}{\partial f_i} = -(\gamma_i^f + c_f) + q_i \left( \frac{\partial D}{\partial f_i} (-bB + eE) + \frac{\partial D}{\partial f_i} (eB - bE) + \frac{\partial g_i}{\partial f_i} (-bB + eE) \right) \\
+ q_j \left( \frac{\partial D}{\partial f_i} (-bB + eE) + \frac{\partial D}{\partial f_i} (eB - bE) + \frac{\partial g_i}{\partial f_i} (eB - bE) \right). \tag{B.11}
\]

Finally, because \(-b \cdot B + e \cdot E = -1 \) and \( e \cdot B - b \cdot E = 0 \), the first-order condition is:
\[-(q_i + q_j) \frac{\partial D}{\partial f_i} - q_i \cdot \frac{\partial q_i}{\partial f_i} = c_f + \gamma_i. \tag{B.12}\]

The derivative of social welfare with respect to capacity is:

\[
\frac{\partial SW}{\partial K} = Bq_i \frac{\partial q_i}{\partial K} + Bq_j \frac{\partial K}{\partial f_i} + Eq_i \frac{\partial q_i}{\partial K} + Eq_j \frac{\partial q_i}{\partial K} + p_i \cdot \frac{\partial q_i}{\partial K} + p_j \cdot \frac{\partial q_j}{\partial K} - r. \tag{B.13}\]

Noting that,

\[
\frac{\partial q_i}{\partial K} = -b \frac{\partial D}{\partial K} + e \frac{\partial D}{\partial K}, \forall i, \tag{B.14}\]

and using \(\overline{p} = \overline{p} = 0\) from equation (26), \(-b \cdot B + e \cdot E = -1\) and \(e \cdot B - b \cdot E = 0\), we can write first-order condition for capacity as:

\[-(q_i + q_j) \cdot \frac{\partial D}{\partial K} = r. \tag{B.15}\]

References


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