Ascari, Guido; Ropele, Tiziano

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The Optimal Stability-Oriented Monetary Policy: Optimal Monetary Policy Under Low Trend Inflation

Guido Ascari
(Università di Pavia)

Tiziano Ropele
(Università di Milano-Bicocca)

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Dipartimento di economia politica
e metodi quantitativi
Università degli studi di Pavia
Via San Felice, 5
I-27100 Pavia

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Guido Ascari
University of Pavia

Tiziano Ropele*
University of Milano-Bicocca

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Abstract

In this paper we generalise the standard optimal monetary policy literature as in Galí (2003) to the case of positive trend inflation. We present a simple framework that provides straightforward analytical results directly comparable with the standard case. Optimal monetary policy is strongly influenced by trend inflation and becomes less effective in controlling inflation as trend inflation increases. Moreover: (i) under discretion, optimal monetary policy may not be implementable (i.e., indeterminacy arises) and the efficient policy frontier worsens; (ii) under commitment, the degree of interest rate smoothing increases with trend inflation and the gains from commitment are highly sensitive to the level of underlying inflation. An ECB-like stability oriented monetary policy (i.e., 2% target inflation rate in the medium term) determines a substantial percentage loss in welfare with respect to a zero inflation target policy.

JEL classification: E24, E32.

Keywords: Optimal monetary policy, trend inflation

Address: Department of Economics, University of Milano-Bicocca, Piazza dell'Ateneo Nuovo, 1 - 20126 Milano, Italy. *Tel: +39 02 6448——; Fax: +39 02 6448——; e-mail:* tiziano.ropele@unimib.it.

"The Council also clarifies that, in the pursuit of price stability, it aims to maintain inflation rates below, but close to, 2% over the medium term" (ECB, 2003, p.79)

1 Introduction

In May 2003 the ECB stated exactly what it meant for price stability: an inflation rate of 2% over the medium term. It is unfortunate that most of the theoretical model employed by the fastly growing field of the optimal monetary literature cannot say much about how the choice of a positive target level of inflation would affect the optimal short-run stabilisation policy of the ECB. Indeed, with few notable exception (e.g., Khan, King, and Wolman (2003) and Schmitt-Grohe and Uribe (2004)), the literature always used a version of the model log-linearised around a zero inflation steady state.

This paper aims to fill this gap and solve this important inconsistency by looking at how optimal monetary policy is affected by positive trend inflation.

There are two important reasons why the literature was concerned with zero inflation steady state. The first one is analytical convenience. The second one is that zero inflation is argued to be optimal in a so-called cashless economy where relative price distortion is the only one caused by inflation (see Goodfriend and King (2001) and Woodford (2003)).

We think that there are even more compelling reasons to look at the case of positive trend inflation.

First, the zero inflation case is plainly unrealistic, as the post-war economic history of the industrialised countries shows. Schmitt-Grohe and Uribe (2004) use the average US GDP deflator growth in the period 1960-1998 to calibrate the steady state inflation rate to 4.2 percent. The average inflation rate for European countries in post-war years ranges from approximately 3% in Germany to almost 10% in Spain. Hence we should model how an optimal stabilisation policy would act in that environment. This is most compelling if these models are to be used to empirically assess the behavior of central banks in post war data.

Second, the practice of central banks suggest that zero inflation steady state is not an actual target. In other words, zero inflation does not coincide with the central bankers' "price stability", as the ECB case illustrates. The ECB provides one fundamental

argument that suggests that a moderate positive rate of inflation would be desirable: the risk of deflation.[1] In a deflationary situation monetary policy could be constrained by the zero lower bound for the nominal interest rate and, besides, the real burden of debt increases augmenting the risk of financial instability. The ECB then distinguishes between the long-run target level of inflation and stabilisation policy, that would also allow monetary policy to be concerned with the level of output.[2]

Third, what we just said above suggests that there are important elements (e.g., debt burden, downward nominal rigidity) which are not considered in the standard Neo-keynesian model. That is, the prescription of an optimal zero inflation in the long run is based on a model that is necessarily a limited depiction of reality. We think this model captures the main aspects of the short-run behavior of the economy and it therefore is the right tool to analyse short-run stabilisation policy. It may however miss some important features that standard economics and central banks' practice suggests as important for the determination of the optimal inflation target in the long-run.

It is therefore important to look at how the optimal monetary policy as defined in the standard model is influenced by trend inflation levels in the long-run moderately different from zero inflation. This paper takes trend inflation rate as exogenous to the model, while we assume that the model is well suited to describe the short-run response of the economy after a shock. We then ask the model what is the optimal monetary policy from a short-run stabilisation perspective for a given long-run target and investigate how different long-run inflation targets affects the optimal monetary policy in the short-run.

This paper owes a lot to the seminal works by Clarida, Gali, and Gertler (1999) and Gali (2003) and it can actually be interpreted as a generalisation of those contributions to positive trend inflation. We are able to define a simple and neat framework that nests the standard one in Clarida, Gali, and Gertler (1999) and Gali (2003) and allows it to deal with a general steady state inflation. The main change to the standard framework is

[1] The ECB actually provides three main arguments that suggests that a moderate positive rate of inflation would be desirable: the risks of deflation, the possibility of upward measurement bias in inflation and the presence of downward nominal rigidities. However, the ECB considers the last two of "minor importance" (see ECB (2003)).

[2] "*...the medium-term orientation also allow monetary policy to take into account concerns about output fluctuations, without prejudice to attaining the primary objective*" (ECB (2003), p. 80).

just the adding of one equation that derives from a generalisation of the New Keynesian Phillips Curve. This allows us to provide intuitive analytical results for the case of discretionary monetary policy and to develop a straightforward comparison with the standard case. In order to do so, we had to postulate the monetary authority loss function as in Clarida, Gali, and Gertler (1999).

Our main finding is that optimal monetary policy is highly sensitive to the underlying trend inflation level. In particular, monetary policy progressively looses efficacy in stabilising inflation after a cost push shock, as trend inflation increases. This happens because the New Keynesian Phillips Curve becomes flatter as trend inflation increases, so that, the current output gap level decreases its influence on current inflation.

Moreover, for the case of discretionary monetary policy, we find that: (i) optimal monetary policy is not always implementable, in the language of Schmitt-Grohe and Uribe (2004), since it can lead to indeterminacy; (ii) the efficient policy frontier worsens substantially with trend inflation. Under commitment, we find that: (i) the impulse responses and the gains from commitment are highly sensitive to the level of trend inflation; (ii) the degree of interest rate smoothing increases with trend inflation. Finally, our model is able to match a very robust feature of the data, since it implies a positive correlation between the level of average inflation and the variance of inflation. Note that the theoretical motivation behind this empirical implication differs from other ones proposed in the literature, since it is due to the optimal response of monetary policy both under discretion and commitment.

The paper is linked to two paper recently appeared in the literature. Khan, King, and Wolman (2003) studies the optimal monetary policy in a somewhat richer model both from a long-run and short-run perspective. They show that the optimal inflation rate in the long-run in their model is actually negative but close to zero, since it is basically a compromise between the Friedman rule and the relative price distortion, with the latter being larger than the former. While this is a sensible theoretical results, it contrasts with the view of many central banks (see again ECB (2003)). In any case, when Khan, King, and Wolman (2003) look at the optimal policy in the short-run in response to shocks they eliminate the money demand distortions and log-linearise the model around a zero inflation steady state. Schmitt-Grohe and Uribe (2004), instead, look at the optimal

monetary and fiscal policy just from a short-run stabilisation perspective. Hence, exactly as in this paper, they take the level of trend inflation as exogenous and calibrate it to US post war data.

Our paper is complementary to these two papers, since it differs from them in two important respects. First, providing a tractable framework, our paper largely hinges on analytics, while the two papers mentioned above rely mainly on numerical results. Second, neither of the two papers investigate how the optimal monetary policy is affected by changes in trend inflation.

Finally, our results are obviously sensitive to two standard features of the standard model: no indexation and fix contract length. A relaxation of these two standard assumptions would weaken our results. With respect to the former, Ascari (2003) showed that any kind of indexation reduces the effect that trend inflation has on the NKPC and that these effects actually vanishes with full indexation (either trend inflation indexation, as in Yun (1996), or a backward-looking indexation, as in Christiano, Eichenbaum, and Evans (2001)). It must be said however that we are concerned with *low levels* of trend inflation, as post world war II data in developed economies show positive, but low levels of average inflation. In such an environment the sticky price assumption is usually believed to be valid. Moreover (i) in reality we do not observe indexed prices, (ii) we know at least since Gray (1976) that full indexation is not optimal, (iii) the theoretical microfoundations of such price indexation scheme is rather questionable and indeed the Christiano, Eichenbaum, and Evans (2001) justification for using it is mainly empirical. Moreover, given that our concern is with moderate level of trend inflation we also keep the expected duration of prices exogenously fixed.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 presents the problem facing the monetary authority. Section 4 provides analytical results and basic intuition of the effects trend inflation has on the optimal monetary policy under discretion. Section 5 present the optimal monetary policy under commitment. Section 6 briefly analyses the case of strategic complementarity. Section 7 concludes.

2 The Model

In this section we describe a simple New Keynesian stochastic general equilibrium model, similar to Clarida, Gali, and Gertler (1999), Gali (2003) and Woodford (2003), generalized to allow for positive trend inflation.

2.1 Households

The economy is populated by infinitively lived households whose instantaneous utility function is increasing in the consumption of the final good (C_t) and real money balances (M_t/P_t) and decreasing in labor (N_t) according to

$$U\left(C, \frac{M}{P}, N\right) = \frac{C_t^{1-\sigma_c} - 1}{1-\sigma_c} + \chi_m \frac{(M_t/P_t)^{1-\sigma_m} - 1}{1-\sigma_m} - \chi_n \frac{N_t^{1+\sigma_n}}{1+\sigma_n} \tag{1}$$

where the positive parameters σ_c, σ_m and σ_n represents the inverse of the intertemporal elasticity of substitution in consumption, real money balances and labor supply, respectively, while χ_m and χ_n are positive constants.

At a given period t, the representative household faces the following nominal flow budget constraint

$$P_t C_t + M_t + B_t \leq W_t N_t + M_{t-1} + (1 + i_{t-1}) B_{t-1} + \Pi_t + TR_t \tag{2}$$

where P_t is the price of the final good, M_t represents holding of nominal money, B_t represents holding of bonds offering a one-period nominal return i_t, W_t is the nominal wage and Π_t are firms profits rebated to the households. In addition, each period the government makes lump-sum nominal transfers to households equal to TR_t. The household's problem is to maximize the lifetime expected utility subject to budget constraints (2), that is

$$\max_{\left\{C_t, \frac{M_t}{P_t}, N_t, B_t\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma_c} - 1}{1-\sigma_c} + \chi_m \frac{(M_t/P_t)^{1-\sigma_m} - 1}{1-\sigma_m} - \chi_n \frac{N_t^{1+\sigma_n}}{1+\sigma_n} \right) \tag{3}$$

$$s.t. \qquad C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} \leq \frac{W_t}{P_t} N_t + \frac{M_{t-1}}{P_t} + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + \frac{TR_t}{P_t}$$

where $\beta \in (0,1)$ is the subjective rate of time preference and E_0 denotes the expectation operator conditional on the time $t = 0$ information set. The resulting first order

conditions yield:

$$\textit{labor supply} \quad : \qquad \chi_n \frac{N_t^{\sigma_n}}{C_t^{-\sigma_c}} = \frac{W_t}{P_t} \tag{4}$$

$$\textit{money demand} \quad : \qquad \chi_m \frac{(M_t/P_t)^{-\sigma_m}}{C_t^{-\sigma_c}} = \frac{i_t}{1+i_t} \tag{5}$$

$$\textit{consumption Euler eq.} \quad : \qquad 1 = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma_c}}{C_t^{-\sigma_c}} (1+i_t) \frac{P_t}{P_{t+1}} \right\}. \tag{6}$$

(4), (5), (6) have the usual straightforward economic interpretation.[3]

2.2 Final Good Producers

In each period t, a final good Y_t is produced by perfectly competitive firms, combining a continuum of intermediate inputs $Y_t(i)$, according to the following standard CES production function:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \qquad \theta > 1. \tag{7}$$

Taking prices as given the final good producer chooses the quantities of intermediate goods $Y_t(i)$ that maximize its profits, i.e., $\max_{Y_t(i)} \left\{ P_t \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} - \int_0^1 P_t(i) Y_t(i) di \right\}$, resulting in the following demand function for each intermediate good i:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\theta} Y_t. \tag{8}$$

The zero profit condition in the final good sector brings about the following expression for the aggregate price index

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \tag{9}$$

2.3 Intermediate Good Producers

The intermediate inputs $Y_t(i)$ are produced by a continuum of firms indexed by $i \in [0,1]$, with the following simple production technology with constant returns to scale to labor:

$$Y_t(i) = N_t(i). \tag{10}$$

[3] Note that the momentary utiltiy function is additively separable in all the three arguments, consumption, real money balances and labor, so that it follows that real money balances will not enter in any of the other structural equations of the model. That is, the money demand equation becomes completely recursive to the rest of the system equations.

The intermediate goods sector is characterized by the fact that prices are *sticky*. In particular, intermediate good producers act as monopolistic competitors and set prices according to a standard discrete version of the mechanism put forward by Calvo (1983). In each period, there exists a fixed probability $(1-\alpha)$ according to which a firm can re-optimize its nominal price. On the contrary, with probability α the firm cannot set a new price and must keep the price unchanged. The problem of a price-resetting firm can thus be formulated as

$$\max_{p_t^*(i)} E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j} \left[\frac{p_t^*(i)}{P_{t+j}} Y_{t+j}(i) - TC_{t+j}^r \left(Y_{t+j}(i) \right) \right] \tag{11}$$

$$s.t. \qquad Y_{t+j}(i) = \left[\frac{p_t^*(i)}{P_{t+j}} \right]^{-\theta} Y_{t+j}$$

where $p_t^*(i)$ denotes the new optimal price of producer i , $TC_{t+j}^r \left(Y_{t+j}(i) \right)$ the real total cost function and $\Delta_{t,t+j}$ is the stochastic discount factor. The solution to this problem yields the familiar formula for the optimal resetted price in a Calvo's setup:

$$p_t^*(i) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j} \left[P_{t+j}^{\theta} Y_{t+j} MC_{t+j}^r(i) \right]}{E_t \sum_{j=0}^{\infty} \alpha^j \Delta_{t,t+j} \left[P_{t+j}^{\theta-1} Y_{t+j} \right]} \tag{12}$$

where $MC_t^r(i)$ denotes the real marginal costs function, which, given the production function (10), is simply $MC_t^r \equiv \frac{\partial TC(i)}{\partial Y(i)} = \frac{W_t}{P_t}$, and hence equal across producers i. Real marginal costs function thus depends only upon aggregate quantities, namely the real wage.

2.4 Government

The government injects money into the economy through nominal transfers, such that:

$$TR_t = M_t^s - M_{t-1}^s \tag{13}$$

where M^s is aggregate nominal money supply. Most importantly, we assume that in steady state money supply evolves according to the following fixed rule

$$M_t^s = \gamma M_{t-1}^s \tag{14}$$

where γ is the (gross) rate of nominal money supply growth. As explained in the introduction, we are assuming thus that monetary policy has a specific inflation target

in the long-run which is exogenous to the model. While generally the optimal monetary policy is also intended to refer to the steady state or long-run behavior of the economy, we argued in the introduction about the importance to disentangle long-run objectives from short-run stabilization policy. In particular, in the long-run monetary policy is modelled as to control the (gross) rate of nominal money to implement a given long-run inflation target. In the short run, instead, optimal monetary policy aims at stabilising inflation and output gap around the long-run targets in response to exogenous shocks buffeting the economy, and is implemented through the control of the short-term nominal interest rate.

2.5 Market clearing conditions

The market clearing conditions in the goods markets, in the money market and in the labour market are simply:

$$\begin{aligned} Y_t &= C_t; \qquad Y_t^s(i) = Y_t^D(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} Y_t \qquad \forall i \\ M_t &= M_t^s; \qquad \text{and} \qquad N_t = \int_0^1 N_t(i)di. \end{aligned} \tag{15}$$

2.6 The steady state

In a deterministic steady state equation (12) becomes

$$\frac{p^*(i)}{P} = \frac{\theta}{\theta-1}\frac{\sum_{j=0}^{\infty}\left(\alpha\beta\gamma^{\theta}\right)^j \chi_n Y^{\sigma_n+\sigma_c}}{\sum_{j=0}^{\infty}\left(\alpha\beta\gamma^{\theta-1}\right)^j} \tag{16}$$

which converges to

$$\frac{p^*(i)}{P} = \frac{\theta}{\theta-1}\frac{1-\alpha\beta\gamma^{\theta-1}}{1-\alpha\beta\gamma^{\theta}}\chi_n Y^{\sigma_n+\sigma_c} \tag{17}$$

if and only if $\alpha\beta\gamma^{\theta} < 1$.[4] In the steady state the aggregate price index evolves according to

$$\frac{p^*(i)}{P} = \left(\frac{1-\alpha\gamma^{1-\theta}}{1-\alpha}\right)^{\frac{1}{1-\theta}} \tag{18}$$

which then can be used to obtain and expression for the steady state output level

$$Y = \left[\left(\frac{1-\alpha\gamma^{1-\theta}}{1-\alpha}\right)^{\frac{1}{1-\theta}}\left(\frac{\theta-1}{\theta}\right)\left(\frac{1-\alpha\beta\gamma^{\theta}}{1-\alpha\beta\gamma^{\theta-1}}\right)\frac{1}{\chi_n}\right]^{\frac{1}{\sigma_n+\sigma_c}}. \tag{19}$$

[4] We will then assume that this condition holds in what follows.

It is immediately evident that money is not superneutral in this model, and it can be shown that the rate of growth of money has substantial effects on steady state output.[5]

2.7 A generalized New Keynesian Phillips Curve

Log-linearizing (4) and (6) we obtain

$$\sigma_n \hat{N}_t + \sigma_c \hat{C}_t = \hat{W}_t - \hat{P}_t \tag{20}$$

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \sigma_c^{-1} \left[\hat{\imath}_t - E_t \hat{\pi}_{t+1} \right] \tag{21}$$

where hatted variables denote percentage deviations from deterministic steady state and $\hat{\imath}_t \equiv \log\left(\frac{1+i_t}{1+\bar{\imath}}\right)$. Moreover, we used the market clearing condition $\hat{Y}_t = \hat{C}_t$ to obtain the standard forward-looking IS curve (21).

Regarding the equation for optimal price (12), the log-linearization under trend inflation is definitely more cumbersome. As shown in the appendix, it is possible to describe a generalized New Keynesian Phillips curve (NKPC) with trend inflation as the following system of two first-order expectational difference equations

$$\begin{cases} \hat{\pi}_t = \left[\frac{(1-\gamma)(1-\sigma_c)}{\left(1-\alpha\beta\gamma^{\theta}\right)} + (\sigma_n + \sigma_c) \right] \bar{\lambda}(\gamma) \hat{Y}_t + \beta\gamma E_t \hat{\pi}_{t+1} + \frac{\bar{\lambda}(\gamma)(\gamma-1)}{\left(1-\alpha\beta\gamma^{\theta}\right)} \hat{\phi}_t \\ \hat{\phi}_t = \left(1-\alpha\beta\gamma^{\theta-1}\right)(1-\sigma_c) \hat{Y}_t + \alpha\beta\gamma^{\theta-1} \left[(\theta-1) E_t \hat{\pi}_{t+1} + E_t \hat{\phi}_{t+1} \right] \end{cases} \tag{22}$$

where $\bar{\lambda}(\gamma) \equiv \frac{\left(1-\alpha\gamma^{\theta-1}\right)\left(1-\alpha\beta\gamma^{\theta}\right)}{\alpha\gamma^{\theta-1}}$ and $\hat{\phi}_t$ is just an auxiliary variable with no obvious interpretation. Our generalized version encompasses the standard NKPC used in the literature, since if $\gamma = 1$ then we are back to the standard expression for the New-Keynesian Phillips curve as in Gali (2003)

$$\hat{\pi}_t = (\sigma_n + \sigma_c) \lambda \hat{Y}_t + \beta E_t \hat{\pi}_{t+1} \tag{23}$$

where $\lambda = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}$. Indeed, when $\gamma = 1$ the second equation becomes irrelevant for the dynamics of inflation and there is no need for the auxiliary variable $\hat{\phi}_t$.

Two main points are worth stressing. First, the system of three equations composed by (21) and (22) is a neat and compact generalisation of the standard IS-AS Neo-keynesian two equations model popularised by the work of Clarida, Gali, and Gertler

[5] See Ascari (2003) for a thorough discussion of this point, of the properties of the steady state and the dynamics of this model.

(1999), Gali (2003) and Woodford (2003). Through the definition of the auxiliary variable $\hat{\phi}_t$, we are able to express the NKPC under trend inflation just by adding one equation to the model. This allows us to derive some simple analytical results and provide a straightforward and intuitive comparison with the standard model. On the other hand, one extra dynamic equation enlarges the dynamics of the model, forcing us to resort to numerical results more often than in the standard case. Second, trend inflation dramatically alters the dynamics of inflation with respect to the usual Calvo's model with $\gamma = 1$. The first equation in (22) can equivalently be written as

$$\begin{aligned} \hat{\pi}_t &= \left[(\gamma-1)(\sigma_c-1)(1-\alpha\gamma^{\theta-1}) + (\sigma_n+\sigma_c)\bar{\lambda}(\gamma)\right]\hat{Y}_t + \\ &+[\beta\gamma + (\gamma-1)(\theta-1)(1-\alpha\gamma^{\theta-1})]E_t\hat{\pi}_{t+1} + \frac{\bar{\lambda}(\gamma)}{(1-\alpha\beta\gamma^{\theta})}E_t\hat{\phi}_{t+1} \end{aligned} \tag{24}$$

which shows that the sensitivity of rate of inflation to contemporaneous output gap is, as usual, an increasing function of σ_n and σ_c, but it is now also dependent on γ. For standard calibration values, we can say that the higher the growth rate of money supply the smaller the coefficients on the current variables $(\hat{Y}_t, \hat{\phi}_t)$ and the higher the coefficient on future expected inflation, $E_t\hat{\pi}_{t+1}$ and $E_t\hat{\phi}_{t+1}$. In other words, trend inflation makes inflation dynamics more forward-looking.

Let us assume log-utility, that is, $\sigma_c \to 1$. The system above simplifies to

$$\begin{cases} \hat{\pi}_t = (1+\sigma_n)\bar{\lambda}(\gamma)\hat{Y}_t + \beta\gamma\hat{\pi}_{t+1} + \frac{\bar{\lambda}(\gamma)(\gamma-1)}{(1-\alpha\beta\gamma^{\theta})}\hat{\phi}_t \\ \hat{\phi}_t = \alpha\beta\gamma^{\theta-1}\left[(\theta-1)\hat{\pi}_{t+1} + \hat{\phi}_{t+1}\right]. \end{cases} \tag{25}$$

In this case the second equation for $\hat{\phi}_t$ depends uniquely on variables dated at $t+1$. Moreover, it can be easily shown that $\bar{\lambda}(\gamma)$ is decreasing in γ in the relevant parameter space (i.e., $\alpha\beta\gamma^{\theta} < 1$). As we shall see, the combination of these two facts is very important for the optimal policy under discretion. Indeed if the monetary authority is unable to control or influence expectations of future variables (i.e., under discretionary monetary policy) then we can ignore the second equation. Furthermore, the higher trend inflation, the flatter the short-run Phillips curve, that is, for given future expectations, the less current inflation would react to a given change in current output (or the more current output has to vary to cause a given change in current inflation).

3 Optimal Monetary Policy

Following Gali (2003) we add to the first equation in (25) a cost-push shock u_t, whose law of motion is

$$u_t = \rho u_{t-1} + \varepsilon_t \tag{26}$$

where $0 \leq \rho < 1$ and ε_t is a i.i.d. random variable with zero mean and variance σ_ε^2. The model economy is therefore described by the system of three equations composed by (21) and (25). As usual, we assume that the monetary authority controls the nominal interest rate so to minimize a welfare-loss criterion. We define this latter to be the discounted sum of expected instantaneous loss functions defined over inflation and output gap according to

$$\mathcal{W} = \frac{1}{2} E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left(\hat{\pi}_{t+j}^2 + \chi \hat{Y}_{t+j}^2 \right) \right\} \tag{27}$$

where $0 < \beta < 1$ is the discount factor and χ measures the relative weight placed on output gap stabilization with respect to inflation. Rotemberg and Woodford (1997) and Rotemberg and Woodford (1999) show that the specific form assumed here can be derived, under certain assumptions, as a second-order Taylor approximation to the theoretically correct welfare measure, i.e., the household's utility function, around the zero inflation steady state. Despite the limitation of assuming an ad hoc loss function for the monetary authority, we choose to use (27). Indeed, using (27) will allow us to derive simple analytical results easily comparable with the standard case in the literature with $\gamma = 1$, because (27) is the one commonly employed in the literature. Moreover, we can then provide straightforward intuition about the effects trend inflation has on an otherwise standard Neo-Keynesian model.[6] We therefore assume (27) where χ is taken as exogenous, but we will investigate the results of the model for different values of the parameter χ, as a robustness analysis. Moreover, the value of χ influences the result in an obvious way.

The policy problem is then to choose the optimal time path for the nominal interest

[6] With general trend inflation, instead, it would not be possible to derive a neat and tractable expression from a second order approximation of the utility function of the representative agent. The only comparison then could be numerical. In this paper, we therefore restrain ourselves to do that and leave it to future research.

rate, $\hat{\imath}_t$, to engineer time paths of the target variables $\hat{\pi}_t$ and $\hat{Y}_t$ that minimizes (27) subject to the IS curve (21) and the NKPC (25). We proceed to solve this problem in the two cases of discretion and commitment.

4 Optimal Policy Under Discretion

Under discretion, the monetary authority cannot make credible announcements about future policy and therefore it must reoptimize each period taking as given future expectations. This simplifies the problem which becomes a one period problem at each point in time, that is

$$\begin{aligned} & \min_{\hat{\pi}_t, \hat{Y}_t} \quad \frac{1}{2}\left(\hat{\pi}_t^2 + \chi \hat{Y}_t^2 + F_t\right) \\ \text{s. t.} \quad & \hat{\pi}_t = \kappa \hat{Y}_t + f_t + u_t \end{aligned} \tag{28}$$

where $\kappa = (1+\sigma_n)\,\bar{\lambda}(\gamma)$, and F_t and f_t contains all the future expected variables which are taken as given by the monetary authority. The solution to this simple problem is

$$\hat{Y}_t = -\frac{\kappa}{\chi}\hat{\pi}_t. \tag{29}$$

Condition (29) simply states the general prescription for a discretionary policy is to "lean against the wind". The solution is very similar to the standard one obtained for the zero inflation steady state case, as in Clarida, Gali, and Gertler (1999) and Gali (2003). There is however an important difference: the degree of "aggressiveness" with which output gap ought to respond to inflation along the optimal path now depends on trend inflation. The higher trend inflation, the lower κ, the flatter the Phillips curve, and the less aggressive the central bank will fight inflation and the more it will stabilize output. The intuition is straightforward. Everything rests on the slope of the Phillips curve: an higher trend inflation worsen the trade-off for the monetary authority, in the sense that it lowers the gain in reduced inflation per unit of output loss. The increase in the cost of reducing inflation then induces a less aggressive policy response to inflation. In sum, the higher trend inflation, the steeper the optimal policy rule, and the more the shock will be passed on to inflation and less to output. Hence the higher is the relative variability between inflation and output ($\sigma_{\hat{\pi}}/\sigma_{\hat{Y}}$).

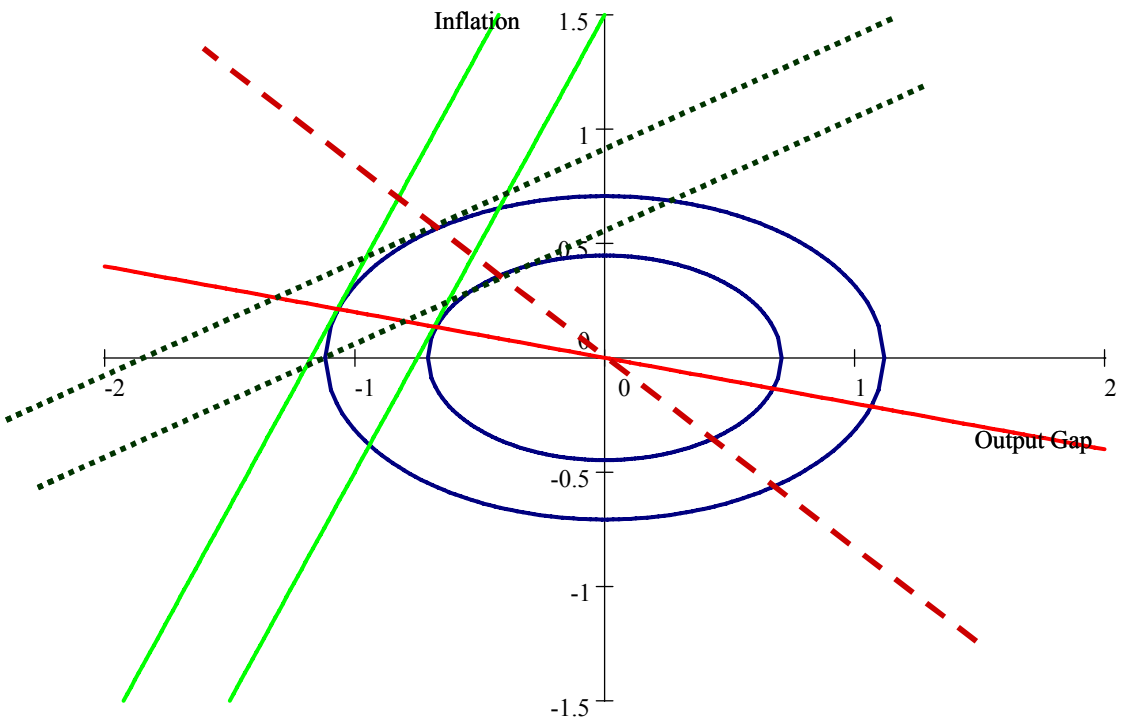

Figure 1. The effects of trend inflation on the NKPC and on the optimal discretionary policy.

Figure 1 depicts the main result implied by (29). The ellipses represent the map of the indifference curves of the monetary authority's loss function in (28), while the positively sloped lines are the map of NKPC in (28) for different values of the shock. The negatively sloped line is hence the optimal reaction function of the monetary authority under discretion, that is, the locus of the tangency points between the indifference curve and the NKPC. As trend inflation increases, the NKPC gets flatter and thus the reaction function turns clockwise (see the dashed lines): the variance of inflation is increased relative to the one of output.

Note that this result has an appealing empirical implication, since it delivers a positive correlation between the average inflation level and the variance of inflation, which is a very robust feature of the data both across time periods and countries. As we will see, the same implication holds for the optimal monetary policy under commitment. It is worth noting that this positive correlation between the level and the variance of

inflation results from the optimal response of monetary policy as trend inflation varies.

4.1 An indeterminacy problem

Substituting the optimal condition (29) into (25), we obtain a linear system of two first order difference equations from which we compute the optimal time path of $\hat{\pi}_t$ as function of the only state variable present in the model, that is u_t. Then, we can recover $\hat{Y}_t$ exploiting condition (29) and finally the time path for $\hat{\imath}_t$ using the IS curve. Therefore, equations (25) can be compactly written in matrix notation as:

$$\varpi_t = \mathbf{N}_1 E_t \varpi_{t+1} + \mathbf{N}_2 u_t \tag{30}$$

where $\varpi_t = \left[\hat{\pi}_t; \hat{\phi}_t\right]^T$ and the transition matrix $\mathbf{N}_1$ is given by[7]

$$\mathbf{N}_1 = \begin{bmatrix} \frac{\beta\left[\gamma+(\theta-1)(\gamma-1)\left(1-\alpha\gamma^{\theta-1}\right)\right]}{1+\frac{1}{\chi}(1+\sigma_n)^2\bar{\lambda}^2} & \frac{\beta(\gamma-1)\left(1-\alpha\gamma^{\theta-1}\right)}{1+\frac{1}{\chi}(1+\sigma_n)^2\bar{\lambda}^2} \\ \\ \alpha\beta\gamma^{\theta-1}(\theta-1) & \alpha\beta\gamma^{\theta-1} \end{bmatrix}$$

and $N_2^T = [\frac{\chi}{\chi+(1+\sigma_n)^2\bar{\lambda}^2}\ ; 0]$.

Since both $\hat{\pi}_t$ and $\hat{\phi}_t$ are forward-looking variables, a necessary and sufficient condition for determinacy requires both eigenvalues of $\mathbf{N}_1$ to lie inside the unit circle. If this condition is not satisfied then the optimal rule would deliver indeterminacy and hence it is not implementable (in the language of Schmitt-Grohe and Uribe (2004)). As shown in the Appendix, if $\gamma > 1$, the necessary and sufficient condition for a rational expectations equilibrium to be unique requires that

$$tr\mathbf{N}_1 - \det \mathbf{N}_1 < 1 \tag{31}$$

Assuming $\beta = 1$ allows us to state the following proposition.

Proposition 1 *Let $\sigma_c = \beta = 1$. Then, the dynamic system defined by the optimal monetary policy under discretion admits a unique rational expectation equilibrium if*

[7] Under the standard assumption of zero inflation steady state, i.e., $\gamma = 1$, the transition matrix $\mathbf{N}_1$ becomes lower triangular indicating that the solution for $\hat{\pi}_t$ is independent from $\hat{\phi}_t$. Therefore, to describe $\hat{\pi}_t$ one just needs a *single* equation which, if solved forward, yields the same expression reported in Clarida, Gali, and Gertler (1999).

and only if:

$$\frac{1+\theta(\gamma-1)}{1+\frac{\left[(1+\sigma_n)\bar{\lambda}(\gamma)\right]^2}{\chi}}<1. \tag{32}$$

Corollary 1. Comparative statics. Ceteris paribus, indeterminacy arises:

the higher the level of trend inflation, γ

the higher the elasticity of substitution among goods, θ

the higher the weight on output in the monetary authority loss function, χ

the lower the elasticity of substitution in labour supply, σ_n

the higher the probability of non-adjusting prices, α.

Corollary 2. If $\gamma = 1$, then the equilibrium is always unique in the admissible parameters space.

In particular, it seems important to note that higher trend inflation brings about the possibility of indeterminacy of the rational expectations equilibrium. While under the standard assumption of no trend inflation, i.e., $\gamma = 1$, indeterminacy never arises (this is true also for values of σ_c and β different from 1), we show that, instead, the optimal policy under discretion could lead to indeterminacy in the general case, even for very low levels of trend inflation. However, the more conservative the central banker, i.e., the lower χ, the less likely indeterminacy arises.

Corollary 3. The higher the value of trend inflation, the more "conservative" a central bank needs to be to guarantee the uniqueness of the equilibrium under optimal discretionary policy. Moreover, whatever the value of trend inflation (among the admissible ones), there always exist a sufficiently low value of χ such that the equilibrium is unique.[8]

This corollary suggests that the higher the target inflation rate in the long-run, the lower χ should the central bank have to avoid the risk of non implementable policies. The higher the trend inflation level, the steeper the optimal policy reaction function in Figure1. If this latter becomes too steep, however, then indeterminacy can arise. For an

[8] $\bar{\lambda}(\gamma)$ tends to zero as γ tends to its upper bound (defined by the condition $\alpha\beta\gamma^{\theta}<1$), so that it has a finite value within the range of admissible value of γ.

implementable optimal policy to exist, it is required a sufficiently low value of χ, which flattens the optimal policy response in Figure 1.

More generally, it is not possible to derive any meaningful analytical expression of (31), because of the obscure convolution of parameters. It is however interesting to investigate how much the central bank needs to be conservative as trend inflation increases for the optimal policy to be implementable. For the sake of comparison, we hence choose the benchmark calibration values in Gali (2003), as in Table 1.[9]

Parameter	β	σ_c	σ_n	α	θ
Value	0.99	1	1	0.75	11

Table 1: Benchmark Calibration (quarterly)

Figure 2 shows the feasible combinations of (γ, χ) that ensure uniqueness of the rational equilibrium in this case. The values of γ in Figure 2 go from 1 to 1.02, corresponding to 8% annual inflation. We choose to vary the values of χ from 0 to 1, that is from a "pure inflation target" central bank to a central bank that gives equal weight to the inflation and output targets in the loss function.[10] The graph evidently depicts the analytical results of Proposition 1 and its corollaries.

[9] Changes in the values of θ, σ_n and σ_c have the usual effects as in a standard model (see equation (24)). The results in the paper regarding the effects of trend inflation, however, qualitatively do not change. Therefore, in the paper we present numerical results only for these benchmark calibration values.

[10] If we had been able to microfound such a loss function, the value of χ would have been endogenous and dependent on γ. On the one hand, for that reason we choose a wide range for χ such that is sensible to assume that it falls within it, given the existing calibration of χ in the literature (see Rotemberg and Woodford (1997), Rotemberg and Woodford (1999) and Gali (2003)).

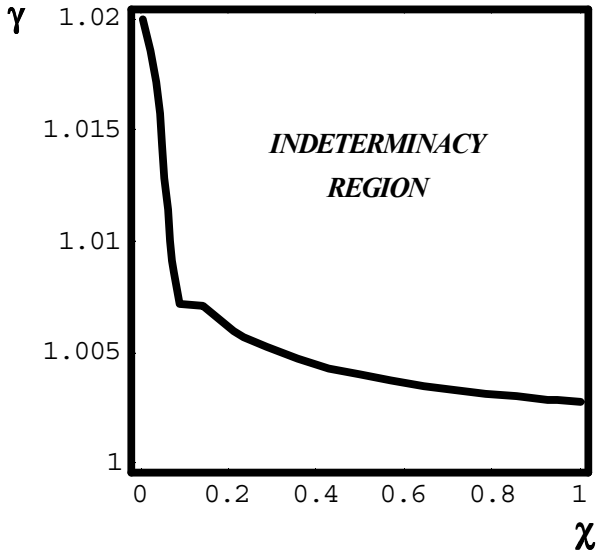

Figure 2. The indeterminacy region as γ and χ changes.

It is important to stress the following points. First, the ECB stability-oriented monetary policy target inflation rate of 2%, corresponding to a value of $\gamma = 1.005$, would require a value of χ lower than 0.32 to make the optimal monetary policy under discretion implementable. In other words, if it has to be true that "the medium-term orientation also allows monetary policy to take into account concerns about output fluctuations, without prejudice to attaining the primary objective" (p. 80, ECB (2003)), then the ECB should put a weight on inflation fluctuations at least more than three times as high as the one on output fluctuactions.[11]

Second, from a theoretical point of view, Gali (2003) calibrates the microfunded value of χ equal to 0.0156. In this case levels of trend inflation lower than 5.6% annually (i.e., $\gamma = 1.0137$) can support optimal discretionary monetary policy.

Third, Schmitt-Grohe and Uribe (2004), instead, calibrate trend inflation to be 4.2%, equal to the U.S. average GDP deflator growth rate in the period 1960-1989. This means that the FED's weight on inflation should have been twenty times higher than the one on output fluctuactions to maintain its discretionary monetary policy implementable.

Finally, it is clear that very low levels of χ are necessary for determinacy as soon

[11] Unless one is willing to make the Euro Area monetary policy to rest on the risky assumption of commitment, see next section.

as trend inflation reaches values which are in line with the historical experience of many developed countries after the second world war. This casts some shadows on the monetary policy of many developed countries in the 70's and 80's, when the level of trend inflation would have required basically a pure inflation target central banker. If we are willing to reasonably assume that many central banks had no commitment power and were not purely concerned with inflation in that historical period (see, e.g., Clarida, Gali, and Gertler (1999)), then we must conclude that their monetary policy was simply not implementable. And indeed in many countries inflation got out of hand, following the oil shocks in the 70's.

4.2 The efficient frontier

Given (26), the solution to the system (31) is simply given by

$$\varpi_t = (I - \rho N_1)^{-1} N_2 u_t \tag{33}$$

which in the case of no persistent shock (i.e., $\rho = 0$) greatly simplifies to deliver the following analytical solution

$$\hat{Y}_t = -\frac{\kappa}{\kappa^2 + \chi} u_t \qquad \text{and} \qquad \hat{\pi}_t = \frac{\chi}{\kappa^2 + \chi} u_t. \tag{34}$$

Given (21), the response of the interest rate is exactly the opposite of $\hat{Y}_t$. As usual, following a cost push shock, the policymaker induces an increase in the real interest rate and, as predicted by the IS curve, the output gap reduces, causing the desired reduction on inflation.

Note that (34) exactly parallels the solutions in Clarida, Gali, and Gertler (1999) (i.e., equations (3.4) and (3.5) at p. 1672) and in Gali (2003) (i.e., equations (39) and (40)), showing another time that our framework is able to generalises those results in a very simple and intuitive way. It is evident that trend inflation modifies the response of output gap and inflation under the optimal policy. The incentive to split the effects of the shock between inflation and output now depends on the loss function (i.e., χ) but also on trend inflation since $\kappa(\gamma) = \left[(1 + \sigma_n)\,\bar{\lambda}\,(\gamma)\right]$ is a decreasing function of γ. Trend inflation therefore modifies the trade-off the optimal policy is facing.

Proposition 2 *As trend inflation increases, the reaction of inflation to the cost push shock is higher and the one of output is instead ambiguous, depending on the relative importance of the gain, measured by* χ*, and the cost, measured by* κ*.*

The intuition is as in Figure 1 above. The effectiveness of a fall in the output gap to curb inflation decreases quite strongly with trend inflation. Hence, $\frac{\partial \widehat{i}_t}{\partial \gamma} < 0$ if $\chi > 2\kappa(\gamma)$. Assume that for zero trend inflation: $\chi < 2\kappa(0)$, as for the benchmark calibration. Then as trend inflation increases the response of the monetary policy is firstly more aggressive, but then it will it starts decreasing. Numerically this would happen for something less than 1% annual inflation in the benchmark calibration. There is a simple reason to do that: monetary policy becomes less effective on inflation as trend inflation increases, so the optimal response is to be increasingly cautious and passive. Indeed, the current output cost necessary to correct inflation is increasing with trend inflation. This has an immediate implication for the usual trade-off between inflation and output variability. Low values of inflation variability can be obtained only at the expenses of a great output variability.[12]

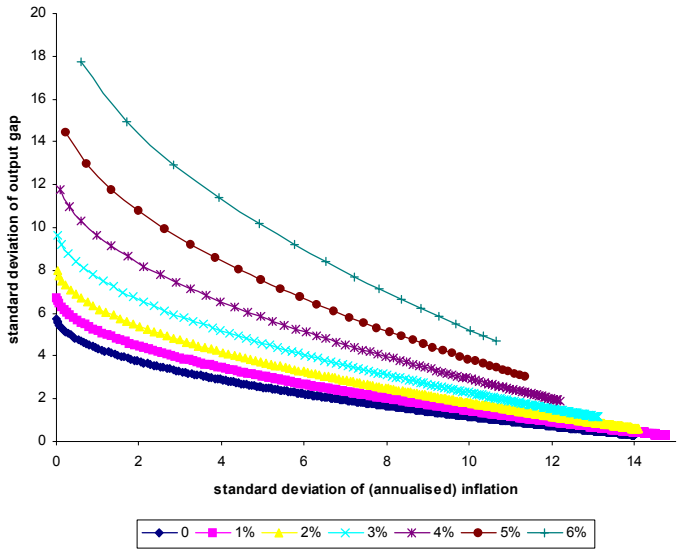

Figure 3. Efficient frontier as trend inflation varies ($\rho = 0.5$)

[12] For example, in the extreme cases when $\chi = 0$ (pure inflation target which implies $\sigma_\pi = 0$) and $\gamma = 1.02$ (8% annual trend inflation), then $\sigma_y = 45.6$, that is, 45 times higher than the standard deviation of the cost push shock.

As in Clarida, Gali, and Gertler (1999), it is very revealing to calculate the efficient policy frontier to fully understand the effect of the low level of trend inflation on the trade-off facing the optimal monetary policy under discretion. The efficient policy frontier links the standard deviations of output and inflation for different values of χ in (27). Figure 3 shows how the efficient frontier changes with trend inflation for the benchmark calibration values in the case of $\rho = 0.5$. Figure 3 illustrates an important result which is stated in the following proposition.

Proposition 3 *The efficient policy frontier gets worse with trend inflation.*

Figure 3 shows that as trend inflation rises, the efficient policy frontier turns clockwise. Hence, the attainable points with zero trend inflation in the space (σ_y, σ_π) are not anymore so as γ rises: either an higher value of σ_y is necessary for the same σ_π or viceversa. Moreover, the graph plots the points as χ varies from 0.001 to 1, with a step increase of 0.001. One can see that most of the points are concentrated on the right-lower corner, that is, only very low values of χ can deliver low values of the standard deviation of inflation. Furthermore, the number of points that composes the frontier decreases with γ, since the model enters the indeterminacy region in Figure 2. Indeed, we know already that the higher γ, the smaller the interval of admissible values for χ. In the case $\gamma = 1.02$, for example, if χ is slightly above $\chi = 0.016$, then the optimal policy would lead to indeterminacy. In other words, as trend inflation increases, the frontier tilts upwards and becomes steeper and shorter: the trade-off tends to vanish, because monetary policy is loosing its efficacy.

5 Optimal Policy Under Commitment

In the presence of a credible commitment mechanism, the monetary authority does not take private future expectations as exogenously given but recognises instead that its policy actions effectively influence such beliefs. In this case, we can not provide analytical results, but we are forced to rely on numerical simulations. A first important result is the following.

Proposition 4 *Let χ be bounded between $(0,1)$ and γ be bounded between $(1,1.03)$. Then, rational expectations equilibrium is always determinate under commitment in the admissible parameter space.*[13]

In other words, contrary to the discretionary case, if there exists a credible commitment mechanism, the optimal path for the nominal interest rate is implementable, in the sense that it ensures uniqueness of the rational expectations equilibrium, regardless to the underlying parameterisation of the model economy.

Moreover, optimal monetary policy shows a certain degree of inertia in the commitment case (see Woodford (1999)). Interestingly enough, our numerical results clearly illustrate that as trend inflation increases, optimal monetary policy inertia increases. Formally, it can be shown that the magnitude of the stable eigenvalues is significantly increasing in the value of trend inflation. We can therefore states the following proposition.

Proposition 5 *In response to a cost-push shock, the monetary authority, optimally acting under commitment, chooses a trajectory for the short term nominal interest rate whose degree of persistence is positively correlated to the level of trend inflation.*

Hence, with positive trend inflation, the nominal interest rate, as well as inflation and output gap, will take several quarters before they settle back to steady state, even if the shock is purely transitory. Our model is therefore able to generate inertia in the interest rate optimal response without resorting to changes in the loss function.

5.1 Impulse responses to a cost push shock

We now turn to the analysis of the impulse responses of the three endogenous variables, namely $\hat{Y}_t$, $\hat{\pi}_t$ and $\hat{\imath}_t$, to a unit cost-push shock under commitment.[14] In each simulation exercise we stick to the benchmark parameterization in Table 1 and we put $\chi = 0.0156$, following Gali (2003).

[13] For robustness, we consider the sensitivity of the results to different parametrization of the intertemporal elasticity of substitution (i.e., $\sigma_c = 5$) and the degree of nominal stickiness (i.e., $\alpha = 0.5$).

[14] In computing the impulse response function we employed the algorithm proposed by Soderlind (1999).

Figure 4 displays impulse responses of inflation, output, nominal and real interest rate to a purely transitory cost-push shock ($\rho = 0$) under commitment. For each of the panels, we report the impulses for six different levels of trend inflation: $\gamma = 1.0$, $\gamma = 1.005$, $\gamma = 1.01$, $\gamma = 1.015$, $\gamma = 1.02$ and $\gamma = 1.025$, corresponding respectively to 0, 2%, 4%, 6%, 8% and 10% trend inflation rates.

Consider first the standard and well-known case of zero inflation steady state. In the event of a cost-push shock, the monetary authority responds by engineering an aggressive deflation and a persistent adjustment pattern in output gap. Within one period, inflation goes from 0.27 to -0.2 while output gap, on impact, jumps down to -3.05 and remains under steady state for about four quarters. Such a pattern for $\hat{Y}$ and $\hat{\pi}$ is achieved by raising the nominal interest rate and keeping it above steady state for few quarters. As known, with respect to discretion, the responses of the interest rate and of the output gap are smaller, but more prolonged. The intuition is straightforward and builds on the forward-looking nature of price setters. The forward looking price setters in fact anticipate the protracted period of tight monetary policy and therefore they do not increase prices to much in first place.

As trend inflation comes about the overall picture changes quite substantially both qualitatively and quantitatively. Recall that trend inflation produces two effects on the model economy. First, it makes the price setters, and hence, the dynamics of inflation more forward-looking, in the sense that the relative weight of future expectations with respect to current output gap in the NKPC is increasing in trend inflation (see 2.7). As such, we would expect the ability to commit, and thus to influence future expectations, to become even more important with trend inflation, and the features of optimal monetary policy under commitment to be strenghtened: lower impact effect and more persistent policy. Second, it makes monetary policy less effective in controlling inflation, because it diminishes the effect that the intermediate target ($\hat{Y}$) has on $\hat{\pi}$ (see 2). From this, instead, we would expect that monetary policy will react less and less as trend inflation increases, as we already saw in the discretionary case. Intuition would therefore predict that both the effects tend to decrease the impact multipliers on $\widehat{i}$ and $\hat{Y}$, while the first one tends to increase the degree of inertia.

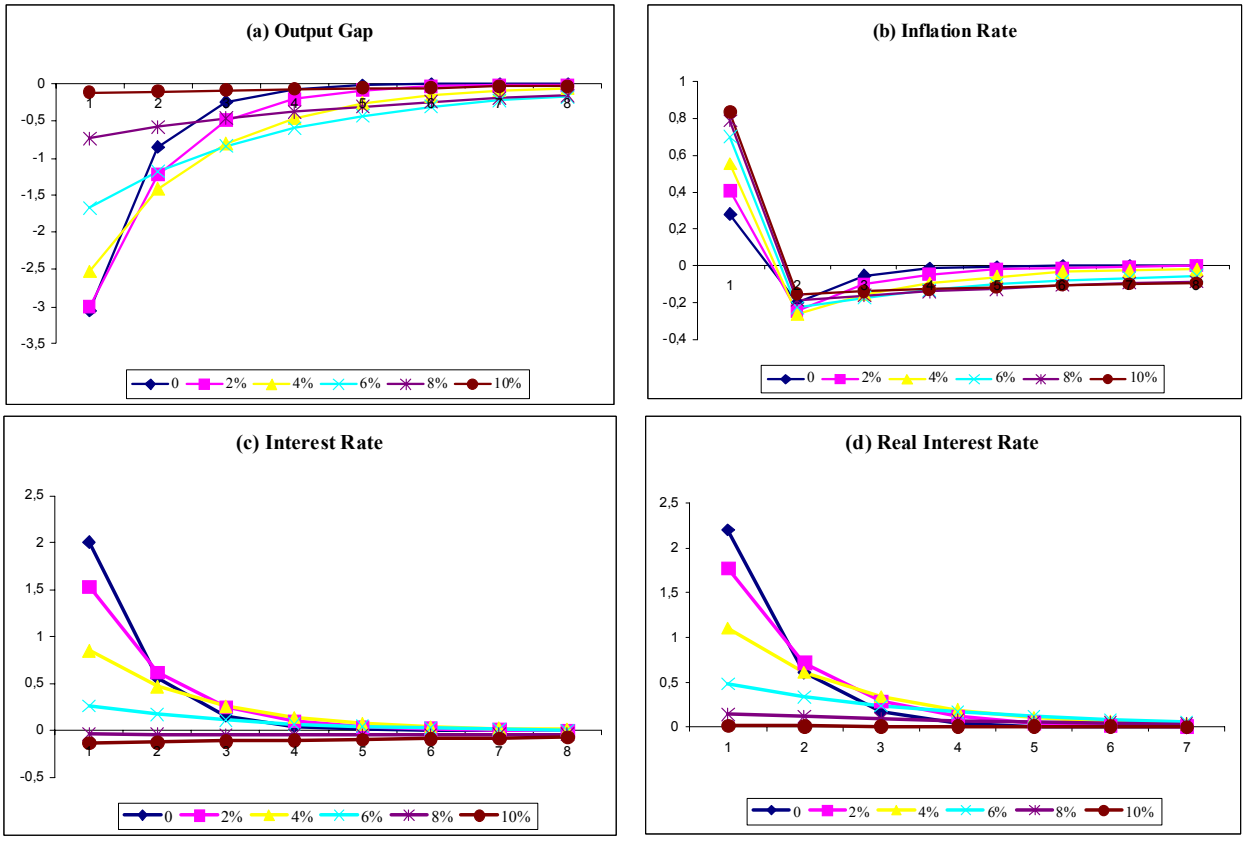

Figure 4. Optimal impulse responses under commitment ($\rho = 0$)

Panels (a), (b) and (c) clearly illustrates that this is indeed the case. Increasing levels of trend inflation generates more smoothed responses of the endogenous variables and dampens the impact effects on $\widehat{i}$ and $\widehat{Y}$. For levels of trend inflation between 0% and 6% the patterns of the three endogenous variables are quite intuitive and plausible. As trend inflation increases, the interest rate responds less aggressively, implementing a milder output contraction and hence the response of inflation is higher. Moreover, the paths of the endogenous variables exhibit an higher persistence. In correspondence to a level of trend inflation of 8% and 10%, however, the dynamic properties of the system are quite striking. For these levels of trend inflation the gain in reducing inflation per unit of output loss is very low. Therefore, the policymaker will find optimal to keep the output gap almost constant, setting the interest rate slightly above expected future inflation, as evident from panel (d). This policy obviously produces a more volatile time

pattern for inflation.

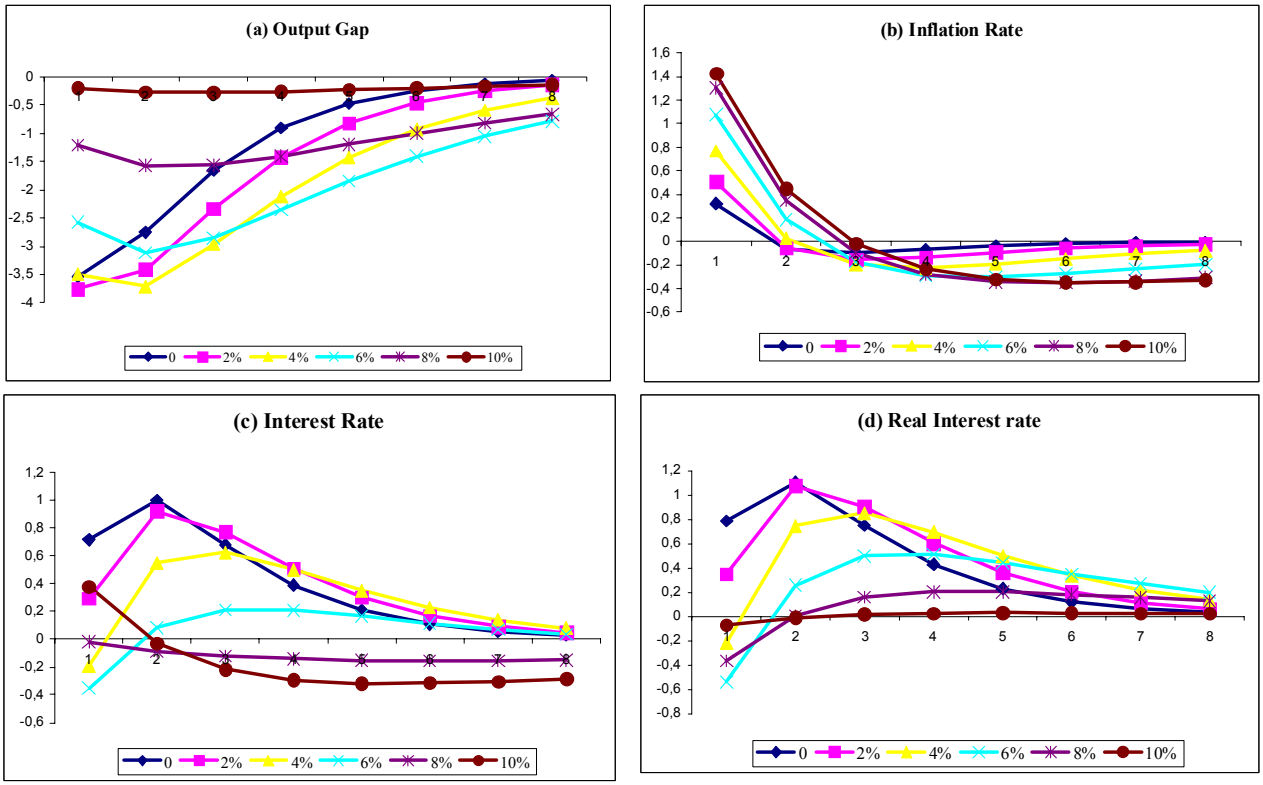

Figure 5. Optimal impulse responses under commitment ($\rho = 0.5$)

Figure 5 displays the same graphs for the impulse response functions under commitment in the case the cost-push shock follows an autoregressive process with persistence parameter $\rho = 0.5$. The persistence in the shock makes more evident the same features described above. In particular, as trend inflation increases, the monetary authority faces a worsening of the trade-off and therefore it is bound to prolong the contractionary policy, implementing a long-lasting recession. It is noteworthy how the path of the nominal interest rate becomes more persistent with trend inflation, despite there is no term in the monetary authority's loss function to induce interest rate smoothing. Finally for very high trend inflation rates (i.e., 10%) the monetary authority again simply quits from controlling inflation.

5.2 Gains from Commitment relative to Discretion

In this section we assess the welfare implications of positive levels of trend inflation under discretion and under commitment. We do so by calculating the value of the loss function and the percentage gain from commitment. The former is computed as:

$$E\left(W\right) = Var(\hat{\pi}_t) + \chi Var(\hat{Y}_t) \tag{35}$$

The percentage gain from commitment, instead, is computed as:

$$\%\ gain = 100 * \left[1 - \frac{L_c}{L_d}\right] \tag{36}$$

where L_c and L_d denote the loss under commitment and under discretion respectively.

In Table 2 we report the unconditional variances $\hat{\pi}$ and $\hat{Y}$, the value of the expected loss and the percentage gain from commitment, for six values of trend inflation (i.e., 0%, 2%, 4%, 6%, 8% and 10%) and two values of the autoregressive coefficient of the cost push shock (i.e., $\rho = 0$ and $\rho = 0.5$).

Several interesting features are worth stressing. First, as obvious, numerical results indicate that discretionary policy always delivers an expected welfare loss greater than the one obtained under commitment. The economic intuition for this finding is well known and impinges on the capacity to influence future expectations that characterizes commitment. In fact, because current inflation crucially depends on future output gaps, the monetary policy can better stabilize inflation by credibly announcing a long-lived sequence of future output contractions, also when the shock has died out. Such a promise to spread the shock over multiple periods is thus welfare enhancing.

Second, and most importantly for the matter of this paper, the welfare loss is always increasing in the level of trend inflation both in the case of discretion and commitment and regardless of the persistence of the shock. With respect to a policy that targets zero inflation, an ECB-like stability oriented monetary policy (i.e, 2% target inflation rate in the medium term) determines a substantial percentage loss in welfare. The size of this loss is about 50% if $\rho = 0$[15], and 100% in the case of discretion and 70% in the case of commitment, if $\rho = 0.5$. Hence, we can conclude that even very low levels of

[15] More precisely, 55% in the case of discretion and 45% in the case of commitment.

trend inflation quite substantially deteriorate the performance of the optimal monetary policy.

Third, in the last column of Table 2 we report a measure of the percentage gain from commitment relative to discretion. We saw above that the higher trend inflation, the more forward looking the dynamics of inflation. Hence, one would expect that the gains from commitment should be increasing with trend inflation, because of the ability to influence future expectations under commitment. It turns out that this is true only for moderate levels of trend inflation. For instance, in the case $\rho = 0$ the percentage gain from commitment is increasing for levels of trend inflation up to 4% annual. After such level the gain is, of course, positive but starts to decline. In the case $\rho = 0.5$ this threshold shifts to 6% annual. The economic intuition for this apparently at odds finding rests on the fact that there is another effect induced by positive trend inflation, as we saw above. Trend inflation indeed reduces the effectiveness of monetary policy. After a certain threshold level of trend inflation, this effect dominates and monetary policy starts disregarding the behavior of inflation, because it is too costly. This is true both under discretion and under commitment: the two policies get close one another, hence the gain is reduced. This result is well captured by the behavior of the unconditional variances of $\hat{\pi}$ and $\hat{Y}$.

Fourth, Table 2 also reveals that trend inflation tends to reinforce the well known fact that the percentage gain from commitment is more relevant when the shock exhibits some degree of persistence. If $\rho = 0$ the gain from commitment increases of about 12 and 34 percentage points, moving from 0% to 2% trend inflation, and from 0% to 4%, respectively. If $\rho = 0.5$, the percentage increase in the gain is much higher: 44% and 81% respectively. Once again, the economic intuition for this finding impinges on the effect of trend inflation in making the dynamics of inflation more forward looking. In this sense, if the monetary authority can credibly commit to future policy actions then firms discount future actions and in first instance they do not increase prices to much. In turn this reduces the overall variability of the endogenous variables enhancing the overall welfare.

Parameter values		Discretion			Commitment			%Gain
		$\text{Var}(\hat{\pi}_t)$	$\text{Var}(\hat{Y}_t)$	Loss	$\text{Var}(\hat{\pi}_t)$	$\text{Var}(\hat{Y}_t)$	Loss	
$\gamma = 1$	$\rho = 0.0$	0.119	14.508	0.346	0.120	10.098	0.278	19.67
$\gamma = 1.005$	$\rho = 0.0$	0.289	15.933	0.537	0.237	10.797	0.405	24.61
$\gamma = 1.01$	$\rho = 0.0$	0.570	11.843	0.755	0.410	9.343	0.556	26.34
$\gamma = 1.015$	$\rho = 0.0$	0.842	4.826	0.918	0.613	5.658	0.701	23.61
$\gamma = 1.02$	$\rho = 0.0$	0.973	1.475	0.986	0.771	1.475	0.794	19.45
$\gamma = 1.025$	$\rho = 0.0$	0.999	0.018	0.999	0.838	0.044	0.839	16.04
$\gamma = 1$	$\rho = 0.5$	0.232	28.170	0.672	0.125	23.920	0.498	25.82
$\gamma = 1.005$	$\rho = 0.5$	0.728	40.106	1.353	0.316	34.24	0.85	37.17
$\gamma = 1.01$	$\rho = 0.5$	2.035	42.236	2.694	0.763	42.877	1.432	46.83
$\gamma = 1.015$	$\rho = 0.5$	4.103	23.502	4.470	1.651	37.955	2.243	49.80
$\gamma = 1.02$	$\rho = 0.5$	5.488	4.680	5.561	2.760	12.631	2.957	46.81
$\gamma = 1.025$	$\rho = 0.5$	5.703	0.107	5.705	3.361	0.423	3.368	40.95

Table 2. Welfare Analysis

5.3 Pure Inflation Targeting

Finally, it is instructive to analyse what would be the effects of appointing a central bank who pursue a strict inflation targeting policy. The strict definition of the practically unique ECB objective in terms of price stability could well depict the ECB as a central bank that is exclusively concerned with inflation stabilisation, that is, $\chi \to 0$. Recall that in the case $\chi \to 0$ the solution under discretion is always determinate. Actually there is basically no difference between the optimal policy under discretion and commitment. Both policies achieve a zero variability of inflation, that is, they completely stabilize inflation, fulfilling the objective.[16] In contrast, output variability is then much higher and actually increasing with trend inflation. This is not surprising given the discussion above: monetary policy becomes less effective with trend inflation, and hence it needs to engineer much higher variations in the output gap to neutralise the effects of a given shock on inflation. What is surprising, however, just by looking at Table 3, are the

[16] The loss function would then be zero in this case.

numbers: $\text{Var}(\hat{Y}_t)$ increases esponentially with trend inflation. Under pure inflation targeting, changing the target from 0 to 2% inflation would imply to more than double the variability of output, while passing from 0 to 4% inflation target would actually mean to cause the variability of output to be amplified by a factor of almost 6. This obviously happens because the central bank does not care about $\text{Var}(\hat{Y}_t)$ (since $\chi \to 0$), but it just aims at stabilising inflation at any output cost. This cost however is again extremely sensitive to trend inflation levels.

Annual Trend Inflation	$\text{Var}(\hat{Y}_t)$
0	33.93
2%	74.60
4%	198.00

Table 3. Pure Inflation Targeting

6 Strategic Complementarity

In the recent optimal monetary policy literature it has been emphasised the importance of including in the model economy a source of real rigidities or strategic complementarity which makes a firm's optimal relative price less sensitive to the quantity it supplies. In particular, Woodford (2003) argues that such an adding is relevant in order to better describe output and inflation dynamics in response to monetary shocks.

Woodford (2003) shows that one simple way to get strategic complementarity (SC henceforth) is assuming specific-factor markets. Our model economy, instead, implies strategic substitutability (SS henceforth) because of the common-factor labour market set-up. In this section, we therefore briefly reports how our results changes if we assume imperfect substitution in labour skills, as in Blanchard and Kiyotaki (1987).

It is worth stressing from the very outset that focusing the paper on a model with SS in the previous sections, we considered the worst case scenario for our results. Indeed, SC greatly strengthen all our results.

First, when SC is accounted for the upper bound that needs to be placed on trend inflation in order to have a well-defined problem becomes very restrictive. In particular, using the benchmark calibration in Table I, it falls quite dramatically from 10.9% under

SS to 5.4% under SC (in annualized terms).[17]

Second, and most importantly, with regards to the slope of the short-run Phillips curve it can be easily shown that for the benchmark calibration the coefficient on output gap under strategic substitutability (κ_{ss}) is always greater than the one under strategic complementarity (κ_{sc}), that is:

$$\underbrace{\frac{\left(1-\alpha\gamma^{\theta-1}\right)\left(1-\alpha\beta\gamma^{\theta}\right)}{\alpha\gamma^{\theta-1}}\left(1+\sigma_n\right)}_{\kappa_{ss}} \quad > \quad \underbrace{\frac{\left(1-\alpha\gamma^{\theta-1}\right)\left(1-\alpha\beta\gamma^{\theta(\sigma_n+1)}\right)}{\alpha\gamma^{\theta-1}}\frac{\left(1+\sigma_n\right)}{\left(1+\theta\sigma_n\right)}}_{\kappa_{sc}}$$

In light of the discussions in previous sections, the fact that the short-run Phillips curve becomes flatter reduces to a large extent, for a given level of trend inflation, the effectiveness of the monetary policy in controlling inflation. Moreover, the elasticity of κ_{sc} with respect to γ is much higher than the one of κ_{ss}, meaning that the model with κ_{sc} is more sensitive to variation in γ.

Third, the parallel in the SC case of the condition in 1 for the equilibrium under discretion to be determinate is:

$$\frac{1+\theta\left[\gamma^{\theta\sigma_n+1}\left(1+\sigma_n\right)-1\right]}{\left(1+\frac{\kappa_{sc}^2}{\chi}\right)\left(1+\theta\sigma_n\right)}<1 \tag{37}$$

which can be easily showed to be more restrictive that (32) for any admissible value of the parameters. Using the benchmark calibration, we can plot Figure 6 that is the analog of Figure 2 for the SC case. The determinacy region in the space (χ,γ) not only remains always below the corresponding one under SS, but it shrunk towards the axis. In turn, this implies that for a given level of γ the central bank acting under discretion must be to a greater extent more concerned with inflation rather than output stabilisation so to guarantee determinacy. In particular, the ECB stability-oriented monetary policy target inflation rate of 2%, corresponding to a value of $\gamma = 1.005$, would require a value of χ lower than 0.001 to make the optimal monetary policy under discretion implementable.[18] In other words, for an optimal stability-oriented monetary policy to be implementable

[17] See also Bakhshi, Burriel-Llombart, Khan, and Rudolf (2002). We compute the upper bounds on trend inflation assuming a constant return to scale production function in the intermediate sector.

[18] Recall from section 4.1 that for $\gamma = 1.005$, χ had to be lower than 0.32 for the optimal monetary policy under discretion to be implementable in the SS case.

under discretion, the ECB has to be basically a pure inflation target central bank, which in turn implies the results shown in the previous section.

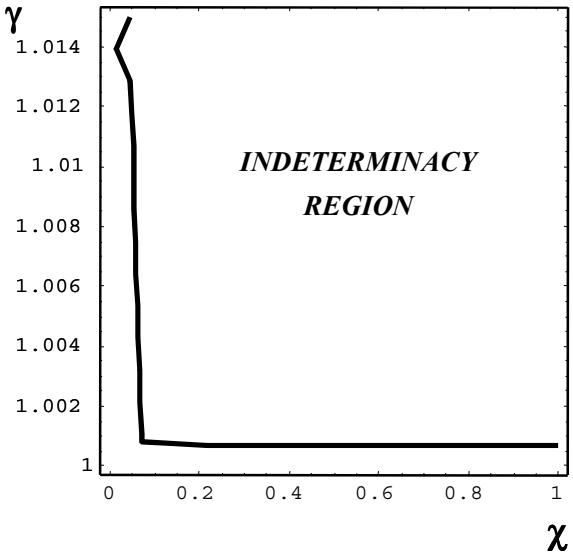

Figure 6. The indeterminacy region as γ and χ changes in the strategic complementarity case

Finally, simulations suggests that it is still true that under commitment the optimal solution leads always to a determinate equilibrium. Moreover, also the results presented in section 5 concerning the commitment solution are strengthen by SC.

7 Conclusion

This paper generalises the seminal works by Clarida, Gali, and Gertler (1999) and Gali (2003). We were able to provide a simple and neat framework that nests the standard one in Clarida, Gali, and Gertler (1999) and Gali (2003) and allows it to deal with a generic steady state inflation rate. The main change to the standard framework is just the adding of one equation that derives from a generalisation of the New Keynesian Phillips Curve. This allows us to provide intuitive analytical results for the case of

discretionary monetary policy and to develop a straightforward comparison with the standard case.

Our main finding is that optimal monetary policy is highly sensitive to the underlying trend inflation level. In particular, monetary policy looses efficacy with trend inflation in stabilising the economy after a cost push shock. This happens because the New Keynesian Phillips Curve become flatter as trend inflation increases, so that, the current output gap level decreases its influence on current inflation.

Moreover, for the case of discretionary monetary policy, we find that: (i) optimal monetary policy is not always implementable, in the language of Schmitt-Grohe and Uribe (2004), since it can lead to indeterminacy; (ii) the efficient policy frontier worsens substantially with trend inflation. Under commitment, we find that: (i) the impulse responses and the gains from commitment are highly sensitive to the level of trend inflation; (ii) the degree of interest rate smoothing increases with trend inflation.

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A APPENDIX

A.1 Derivation of the Phillips Curve Under Trend Inflation

In this appendix we provide details of the log-linearization of the supply side of our model economy which leads to the New Keynesian Phillips curve (22) in the main text. We begin by re-writing numerator and denominator of (12) as:

$$\frac{p_t^*(i)}{P_t} = \frac{\theta}{\theta - 1}\left(\frac{\psi_t}{\phi_t}\right)$$

where

$$\begin{aligned}
\psi_t &= E_t \sum_{j=0}^{\infty} (\alpha\beta)^j u_c(t+j) \left[\left(\frac{P_{t+j}}{P_t}\right)^{\theta} Y_{t+j} MC_{t+j}(i)\right] \\
\phi_t &= E_t \sum_{j=0}^{\infty} (\alpha\beta)^j u_c(t+j) \left[\left(\frac{P_{t+j}}{P_t}\right)^{\theta-1} Y_{t+j}\right]
\end{aligned}$$

The denominator can also be written as:

$$\phi_t = u_c(t) Y_t + E_t \sum_{j=0}^{\infty} (\alpha\beta)^{j+1} \left[\left(\frac{P_{t+j+1}}{P_t}\right)^{\theta-1} u_c(t+j+1) Y_{t+j+1}\right]$$

Next, considering the definition for ϕ_{t+1} and collecting the term $\left(\frac{P_{t+1}}{P_t}\right)^{\theta-1}$ yields the following expression for ϕ_t:

$$\phi_t = u_c(t) Y_t + \alpha\beta E_t\left(\Pi_{t+1}^{\theta-1}\phi_{t+1}\right) \tag{38}$$

where $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$. Doing exactly the same steps for the numerator gives rise to the following expression for ψ_t:

$$\psi_t = u_c(t) Y_t MC_t(i) + \alpha\beta E_t\left(\Pi_{t+1}^{\theta}\psi_{t+1}\right) \tag{39}$$

Now we take a log-linear approximation of (38) and (39). ϕ is linearized around $\frac{u_c Y}{\left(1-\alpha\beta\gamma^{\theta-1}\right)}$, ψ around $\frac{u_c Y MC(i)}{1-\alpha\beta\gamma^{\theta}}$, Y_t around Y, Π aroundγ and $u_c(t)$ around $Y^{-\sigma_c}$:

$$\hat{\phi}_t \simeq \left(1-\alpha\beta\gamma^{\theta-1}\right)\left[\hat{u}_c(t) + \hat{Y}_t\right] + \alpha\beta\gamma^{\theta-1}\left[(\theta-1)\hat{\pi}_{t+1} + \hat{\phi}_{t+1}\right] \tag{40}$$

$$\hat{\psi}_t \simeq \left(1-\alpha\beta\gamma^{\theta}\right)\left[\hat{u}_c(t) + \hat{Y}_t + \hat{MC}_t\right] + \alpha\beta\gamma^{\theta}\left[\theta\hat{\pi}_{t+1} + \hat{\psi}_{t+1}\right] \tag{41}$$

With this results at hand we can compactly rewrite the log-linearized optimal price (12) as:

$$\hat{p}_t^*(i) - \hat{P}_t = \hat{\psi}_t - \hat{\phi}_t$$

In order to find the New Keynesian Phillips curve we have to combine the last equation with the log-linear expression of the aggregate price, which is given by $\hat{p}_{it}^{*} - \hat{P}_t = \left(\alpha\gamma^{\theta-1}\right)\left(1-\alpha\gamma^{\theta-1}\right)^{-1}\hat{\pi}_t$. Thus, the New Keynesian Phillips curve under positive trend inflation is fully described by the three equations:

$$\hat{\pi}_t = \left[\frac{1-\alpha\gamma^{\theta-1}}{\alpha\gamma^{\theta-1}}\right]\left(\hat{\psi}_t - \hat{\phi}_t\right) \tag{42}$$

$$\hat{\phi}_t = \left(1-\alpha\beta\gamma^{\theta-1}\right)\left[\hat{u}_c(t) + \hat{Y}_t\right] + \alpha\beta\gamma^{\theta-1}\left[(\theta-1)\hat{\pi}_{t+1} + \hat{\phi}_{t+1}\right] \tag{43}$$

$$\hat{\psi}_t = \left(1-\alpha\beta\gamma^{\theta}\right)\left[\hat{u}_c(t) + \hat{Y}_t + \hat{MC}_t\right] + \alpha\beta\gamma^{\theta}\left[\theta\hat{\pi}_{t+1} + \hat{\psi}_{t+1}\right] \tag{44}$$

Interestingly enough the above system can be reduced to only two equations. First write the difference between $\hat{\psi}_t$ and $\hat{\phi}_t$ as:

$$\begin{aligned}\left(\hat{\psi}_t - \hat{\phi}_t\right) &= \alpha\beta\gamma^{\theta-1}(1-\gamma)(1-\sigma_c)\hat{Y}_t + \left(1-\alpha\beta\gamma^{\theta}\right)\hat{MC}_t + \\ &+\theta\alpha\beta\gamma^{\theta-1}(\gamma-1)\hat{\pi}_{t+1} + \alpha\beta\gamma^{\theta-1}\hat{\pi}_{t+1} + \alpha\beta\gamma^{\theta-1}\left(\gamma\hat{\psi}_{t+1} - \hat{\phi}_{t+1}\right)\end{aligned}$$

where we also used $\hat{u}_c(t) = -\sigma_c\hat{Y}_t$. Next add and subtract $\alpha\beta\gamma^{\theta-1}\gamma\hat{\phi}_{t+1}$ so to have:

$$\begin{aligned}\left(\hat{\psi}_t - \hat{\phi}_t\right) &= \alpha\beta\gamma^{\theta-1}(1-\gamma)(1-\sigma_c)\hat{Y}_t + \left(1-\alpha\beta\gamma^{\theta}\right)\hat{MC}_t + \\ &+\theta\alpha\beta\gamma^{\theta-1}(\gamma-1)\hat{\pi}_{t+1} + \alpha\beta\gamma^{\theta-1}\hat{\pi}_{t+1} + \alpha\beta\gamma^{\theta-1}\gamma\left(\hat{\psi}_{t+1} - \hat{\phi}_{t+1}\right) \\ &+\alpha\beta\gamma^{\theta-1}(\gamma-1)\hat{\phi}_{t+1}\end{aligned}$$

Plugging into the last expression $\left[\frac{\alpha\gamma^{\theta-1}}{1-\alpha\gamma^{\theta-1}}\right]\hat{\pi}_t = \left(\hat{\psi}_t - \hat{\phi}_t\right)$ yields to:

$$\begin{aligned}\hat{\pi}_t &= \left(1-\alpha\gamma^{\theta-1}\right)\beta(1-\gamma)(1-\sigma_c)\hat{Y}_t + \frac{\left(1-\alpha\gamma^{\theta-1}\right)\left(1-\alpha\beta\gamma^{\theta}\right)}{\alpha\gamma^{\theta-1}}\hat{MC}_t + \\ &+\left(1-\alpha\gamma^{\theta-1}\right)\theta\beta(\gamma-1)\hat{\pi}_{t+1} + \left(1-\alpha\gamma^{\theta-1}\right)\beta\hat{\pi}_{t+1} + \\ &+\alpha\beta\gamma^{\theta}\hat{\pi}_{t+1} + \left(1-\alpha\gamma^{\theta-1}\right)\beta(\gamma-1)\hat{\phi}_{t+1}\end{aligned}$$

Now using the definition $\hat{\phi}_t$ we can substitute for $\frac{\hat{\phi}_t}{\alpha\beta\gamma^{\theta-1}} - \frac{\left(1-\alpha\beta\gamma^{\theta-1}\right)(1-\sigma_c)}{\alpha\beta\gamma^{\theta-1}}\hat{Y}_t - (\theta-1)\hat{\pi}_{t+1} = \hat{\phi}_{t+1}$ and therefore

$$\begin{aligned}\hat{\pi}_t &= \left[\frac{\left(1-\alpha\gamma^{\theta-1}\right)(1-\gamma)(1-\sigma_c)}{\alpha\gamma^{\theta-1}}\right]\hat{Y}_t + \frac{\left(1-\alpha\gamma^{\theta-1}\right)\left(1-\alpha\beta\gamma^{\theta}\right)}{\alpha\gamma^{\theta-1}}\hat{MC}_t + \\ &+\beta\gamma\hat{\pi}_{t+1} + \frac{\left(1-\alpha\gamma^{\theta-1}\right)(\gamma-1)}{\alpha\gamma^{\theta-1}}\hat{\phi}_t\end{aligned}$$

Finally, plugging into the last equation $\hat{MC}_t(i) = (\sigma_n + \sigma_c)\hat{Y}_t$ and defining $\bar{\lambda}(\gamma) \equiv \frac{\left(1-\alpha\gamma^{\theta-1}\right)\left(1-\alpha\beta\gamma^{\theta}\right)}{\alpha\gamma^{\theta-1}}$ leads to (22).

A.2 Determinacy Condition under Discretion

We start by stating the following proposition:

Proposition A1 *Under plausible values for structural parameters and provided that* $\gamma \in [1, \infty)$*, the matrix* N_1 *possesses two real, distinct and positive eigenvalues.*

Proof. For a 2×2 matrix the characteristic equation can be written as:

$$\lambda^2 - tr\lambda + \det = 0 \tag{45}$$

where tr and det respectively denote trace and determinant of the matrix. (45) real roots if and only if the discriminant Δ is greater than zero, that is:

$$\Delta = (n_{11} - n_{22})^2 + 4n_{12}n_{21} \geq 0$$

where n_{ij} denotes a generic element of the matrix. If Δ is strictly greater than zero then the roots are real and distinct. As long as the extra diagonal elements n_{12} and n_{21} have the same algebraic sign the discriminant will be always greater (or equal) than zero, $\Delta \geq 0$. Therefore, having extra diagonal elements with the same sign represents a *sufficient condition* for real roots. As all the elements of N_1 are non-negative provided that $\gamma \geq 1$ and $\alpha\beta\gamma^{\theta} < 1$, then the eigenvalues are *real*. Given also that $n_{11} \neq n_{22}$, for plausible parametrization, it follows that the roots are *distinct*. As the the determinant of N_1 is bounded between zero and one it also follows that the two eigenvalues are *positive*. **QED.**

Now using the property whereby the determinant is equal to the product of eigenvalues ($\det \mathbf{N}_1 = \lambda_1\lambda_2$), we can state the following necessary condition:

Necessary Condition *Necessary condition to ensure determinacy of the rational expectations equilibrium requires the determinant of the matrix* N_1 *to be bounded between zero and one:*

$$0 < \det \mathbf{N}_1 < 1 \tag{46}$$

In our case this condition is always satisfied. Additionally, in light of the fact that the eigenvalues are also real and distinct, it must be true that $0 < \lambda_1 < \lambda_2$. Hence, to

ensure determinacy it suffices to require the dominant eigenvalue (λ_2) being less than one. This entails solving the following inequality:

$$\left[(tr\mathbf{N}_1)^2 - 4\det\mathbf{N}_1\right]^{\frac{1}{2}} < 2 - tr\mathbf{N}_1 \tag{47}$$

Given (46), the right-hand side of (47) is always greater than zero. Finally, squaring both sides yields the necessary and sufficient condition that is:

$$tr\mathbf{N}_1 - \det\mathbf{N}_1 < 1$$

A.3 Commitment

To find the optimal contingency plan for the triple $\left\{\hat{Y}_t, \hat{\pi}_t, \hat{\imath}_t\right\}_{t\geq 0}$ the policymaker minimizes the welfare-loss criterion (27) subject to the New Keynesian Phillips curve (22). Setting to zero the partial derivatives of the Lagrangian function yields the following first order conditions:

$$\hat{\pi}_t \quad : \quad 0 = \hat{\pi}_t + \psi_{2t} - \gamma\psi_{2t-1} - \psi_{3t-1}\alpha\gamma^{\theta-1}(\theta - 1) \tag{48}$$

$$\hat{Y}_t \quad : \quad 0 = \chi\hat{Y}_t - \alpha_0\psi_{2t} - \alpha_1\psi_{3t} \tag{49}$$

$$\hat{\phi}_t \quad : \quad 0 = -\frac{\bar{\lambda}(\gamma - 1)}{(1 - \alpha\beta\gamma^{\theta})}\psi_{2t} + \psi_{3t} - \alpha\gamma^{\theta-1}\psi_{3t-1} \tag{50}$$

where ψ_{2t} and ψ_{3t} denote the Lagrangian multipliers of the first and second equation of (22) respectively; $\alpha_0 \equiv \left[\frac{(1-\gamma)(1-\sigma_c)}{1-\alpha\beta\gamma^{\theta}} + \sigma_n + \sigma_c\right]\bar{\lambda}$ and $\alpha_1 \equiv \left(1 - \alpha\beta\gamma^{\theta-1}\right)(1 - \sigma_c)$. Each of conditions (48)-(50) must hold $\forall t \geq 1$, whereas at time $t = 0$ the same conditions also hold with the additional stipulation that

$$\psi_{2-1} = \psi_{3-1} = 0$$

By inspecting the above efficiency conditions, several features warrant some words of comment. Firstly, the presence of lagged Lagrangian multipliers clearly indicates that the optimal plan under commitment is not *time consistent*. Note, in fact, that at the date optimization takes place, say $t = 0$, the policymaker sets $\psi_{2-1} = \psi_{3-1} = 0$ while in subsequent periods the time paths for ψ_{2t} and ψ_{3t} not necessarily will be zero. Now, a policymaker that solves the same problem at date $T > 0$ will choose instead initial conditions such that

$$\psi_{2T-1} = \psi_{3T-1} = 0$$

But these last conditions will be in general different from those implied by the optimal plan chosen at date zero.

Another, and perhaps more important, consequence due to the presence of ψ_{2t-1} and ψ_{3t-1} is that endogenous variables at date t, and in particular the policy instrument, should not depend solely upon current and forecasted future cost-push shocks but also on the two predetermined Lagrangian multipliers. These predetermined shadow prices represent thus a source of inertia in the optimal monetary policy which is independent from any inertia that might be present in the exogenous process of u_t.

Appropriately rearranging equations (48)-(50) together with (22) yields the following linear system of difference equations:

$$\mathbf{A}_1 \begin{bmatrix} \mathbf{z}_t \\ \boldsymbol{\psi}_{t-1} \end{bmatrix} = \mathbf{A}_0 \begin{bmatrix} E_t \mathbf{z}_{t+1} \\ \boldsymbol{\psi}_t \end{bmatrix} + \mathbf{A}_2 u_t \tag{51}$$

where $\mathbf{z}_t^T \equiv \begin{bmatrix} \hat{\pi}_t & \hat{\phi}_t \end{bmatrix}$, $\boldsymbol{\psi}_t^T \equiv \begin{bmatrix} \psi_{2t} & \psi_{3t} \end{bmatrix}$ and $\mathbf{A}_1$ and $\mathbf{A}_0$ contains the following elements

$$\begin{aligned}
a_0(1,1) &= \beta\gamma & a_0(4,4) &= 1 \\
a_0(1,3) &= \tfrac{\alpha_0^2}{\chi} & a_1(1,1) &= 1 \\
a_0(1,4) &= \tfrac{\alpha_0\alpha_1}{\chi} & a_1(1,2) &= -\tfrac{\bar{\lambda}(\gamma-1)}{1-\alpha\beta\gamma^{\theta}} \\
a_0(2,1) &= \alpha\beta\gamma^{\theta-1}(\theta-1) & a_1(2,2) &= 1 \\
a_0(2,2) &= \alpha\beta\gamma^{\theta-1} & a_1(3,1) &= -1 \\
a_0(2,3) &= \tfrac{\alpha_0\alpha_1}{\chi} & a_1(3,3) &= \gamma \\
a_0(2,4) &= \tfrac{\alpha_1^2}{\chi} & a_1(3,4) &= \alpha\gamma^{\theta-1}(\theta-1) \\
a_0(3,3) &= 1 & a_1(4,4) &= \alpha\gamma^{\theta-1} \\
a_0(4,3) &= -\tfrac{\bar{\lambda}(\gamma-1)}{\left(1-\alpha\beta\gamma^{\theta}\right)}
\end{aligned}$$

Similarly to what we have done under discretion, from (51) we compute the optimal path of $\hat{\pi}_t$, as function of the cost-push shock and the two predetermined variables, i.e. ψ_{2t-1} and ψ_{3t-1}, then recover $\hat{Y}_t$ exploiting (29) and finally the optimal path of $\hat{\imath}_t$ from the IS curve.

Dipartimento di economia politica e metodi quantitativi
Università degli studi di Pavia

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