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A new Keynesian theory of inflation and growth in the long run

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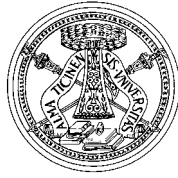
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Quaderni di Dipartimento

A New Keynesian Theory of Inflation and Growth in the Long Run

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August 6, 2007

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A New Keynesian Theory of Inflation and Growth in the Long Run

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Abstract

This paper explores the influence of inflation on economic growth. In order to match the empirical stylized fact of a threshold level of inflation, beyond which inflation ceases to have a positive impact on growth and begins to harm it, we propose to merge an endogenous growth model of learning by doing with a New Keynesian one with sticky wages. In this way, we mimic the stylized fact of a hump shaped relationship between inflation and economic growth.

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1 Introduction

This paper shows that changes in the inflation rate can produce permanent changes in the growth rate of output - even though the model here proposed contains no money illusion, no permanent nominal rigidities, and no departure from rational expectations. We argue that when the money supply grows in the presence of temporary nominal frictions (in the form of staggered nominal contracts), nominal adjustments never have a chance to work themselves out fully. Thus, in an endogenous growth context, changes in money growth affect the marginal product of capital and thereby the rate of economic growth.

The theoretical literature on the relationship between inflation and growth had problems in mimicking the stylized fact of a threshold level of inflation below which inflation has a positive impact on growth and above which inflation harms growth (see the literature review below). On the contrary the New Keynesian literature on the relationship between inflation and the output level found a hump-shaped relationship between the long-run level of output and inflation (Ascari, 1998 and Graham and Snower, 2003).

As a consequence, we suggest to merge an endogenous growth model of learning-by-doing with a New Keynesian one with sticky wages allowing to explore how the effect of inflation on output growth is connected not only to the direct effects of inflation on capital accumulation, but also to indirect ones passing through the labour market. This strategy allows to

mimic the empirical results of Vaona and Schiavo (2007), where, relying on both nonparametric and semiparametric estimators, it is found that inflation has a weak positive impact on output growth at low inflation levels and a strong negative one at high inflation levels.

The basic intuition for the present contribution can be described as follows.

- When nominal wage contracts are staggered, current contract wages depend on the current and future prices that prevail over the contract period. On account of time discounting, the current contract wage is affected more strongly by current prices than by expected future prices.
- The greater is the rate of money growth, the faster do prices rise, and the more the contract wage lags behind the current price level. Thus the lower is the average real wage over the contract period. Consequently, the more labor is demanded by firms and the more output they produce.
- In an endogenous growth context, an increase in the labour input rises the marginal product of capital, leading to faster economic growth.

In short, the above influences imply a positive long-run relation between inflation and output growth. There is however an important countervailing effect:

- As money growth - and consequently inflation - increases, relative prices become more volatile, i.e. the real wage varies more over the contract

period, since the nominal wage is constant over the contract period whereas the aggregate price level rises gradually through time.

- This real wage volatility induces employment fluctuations, i.e. "employment cycling". Since there are diminishing returns to labor, this is inefficient leading to a reduction in the marginal product of capital and therefore in output growth.¹

These influences imply a negative long-run relation between inflation and output growth.

The discounting effect is dominant at low money growth rates whereas the employment cycling effect is dominant at high money growth rates. By implication, the long-run relation between inflation and output growth is backward bending: growth rises with inflation at low inflation rates, but falls with inflation at high inflation rates.

The paper is organized as follows. Section 2 relates our contribution to the existing literature. Section 3 presents the underlying model. Section 4 sketches the model solution, which is illustrated in detail in Appendices A and B. Section 5 contains our results regarding the relations between inflation and growth and between inflation and welfare. Section 6 concludes.

¹Note that employment cycling takes place over a time period short enough (the contract period) for diminishing returns to be relevant.

2 Relation to the Literature

2.1 Empirical Literature

Many empirical contributions found a non-linear relationship between inflation and real growth. Gylfason and Herbertsson (2001) refer to various studies pointing out that an increase of inflation from 5% to 50% decreases the real growth rate, however they found that this effect is non-linear and that inflation rates below 10% are positively correlated with growth, but the opposite holds for inflation rates above 10%.

Similar results were found by Chari *et al.* (1996) and Barro (2001). From the former study would result that an increase in inflation from 10 to 20% would decrease growth by an amount ranging from 0.2 to 0.7%. Kahn and Senhadji (2001) found the threshold to be around 1% for industrialized countries and 11% for developing ones. The results of Ghosh and Phillips (1998) would point it to be at 2.5%, whereas Judson and Orphanides (1999) at 10%.

A threshold effect was found also by Thirlwall and Barton (1971) at an annual inflation rate ranging from 8% to 10%, a similar value was suggested also by Sarel (1996). Gylfason (1991) found that economies with inflation above 20% grew less rapidly than economies with inflation below 5% a year. Bruno and Easterly (1998) report that inflation rates above 40 percent a year for at least two years in a row are generally harmful to growth. Also Fischer (1993) noted the existence of a positive relationship at low inflation rates and a negative one as inflation rises. Finally, Vaona and Schiavo (2007) used

both a nonparametric and a semiparametric instrumental variable estimators and they showed that the threshold inflation level is about 4% for developed countries. For developing countries they found that too high variability does not allow to reach clear-cut results. The effect of inflation on growth does not appear to be large.

2.2 Theoretical Literature

With the exception of a few recent studies, the theoretical literature had major difficulties in explaining the threshold effect highlighted by the empirical literature.

One of the first theoretical studies concerning inflation and output is Tobin (1965), according to which inflation is beneficial to the output level because it lowers the interest rate and therefore the opportunity cost to invest. This increases the capital-labour ratio and therefore output. Stockman (1981) pointed to the possible existence of an inverse Tobin effect, whereby an increase in the inflation rate causes the capital stock to decrease, once supposing a cash in advance constraint for capital accumulation and given that inflation raises the cost of money holding.

More recently, the literature has shifted from the level of output to the output growth rate and from Solow models to endogenous growth ones. Gillman and Kejak (2005a) proposed to distinguish between physical capital models, labelled as Ak , human capital ones, labelled as Ah , and combined models, with both human and physical capital. The results have been mixed.

Some contributions have produced insignificant long-run inflation-growth effects, such as the Ak models of Ireland (1994) and Dotsey and Sartre (2000) and the combined model of Chari *et al.* (1996). Other contributions have produced a negative and significant inflation-growth effect, such as the Ak models of Haslag (1998) and Gillman and Kejak (2004), the Ah model of Gillman *et al.* (1999), the combined models of Gomme (1993) and that of Gillman and Kejak (2002, 2005b). Gillman and Kejak (2005a) propose a model that nests most of the models proposed before. Their result is that in the Ak model inflation works as a tax on physical capital, implying a negative Tobin (1965) effect, whereas in the Ah model inflation works as a tax on human capital, implying a positive Tobin (1965) effect. Finally, in the combined model, inflation works more like a tax on human capital than on physical capital implying a positive Tobin (1965) effect.

Finally, Gylfason and Herbertsson (2001) proposed to insert real money balances into the production function, as a proxy of the effect of financial depth on production (King and Levine, 1993; Levine, 1997; Gylfason and Zoega, 2001), and they found a negative effect of inflation on growth through three channels: it lowers the real interest rate, and therefore savings, it reduces efficiency by driving a wedge between the returns to real and financial capital and, finally, it reduces financial depth harming output.

In Wang and Yip (1992) inflation is negatively related to growth, because a reduction in real balances arising from an increase in the rate of monetary growth raises transaction time and therefore transaction costs. On the con-

trary Mino and Shibata (1995), in an overlapping generation framework, show that inflation may have a redistributive impact from one generation to the other and foster capital accumulation. Bonatti (2002a, b) argue that, when multiple balanced growth paths exist in a non-monetary economy, inflation targeting cannot resolve the resulting indeterminacy, whereas a fixed monetary growth rule can do it and it also determines the growth path of the economy. Furthermore, a restrictive monetary policy may select a lower growth path than a more expansive one.

In Paal and Smith (2000), as in the empirical literature, the relationship between money growth and real growth is showed to be characterized by a threshold. At low money growth rates, banks perceive a small opportunity cost in detaining reserves instead of lending funds for investments. As money growth rises, the nominal interest rate rises too increasing the opportunity cost of holding reserves and spurring lending and therefore investment and growth. When the nominal interest rate grows beyond a certain threshold level credit rationing badly affects lending, reducing capital accumulation and growth.

On the other hand, Funk and Kromen (2005, 2006) investigated the connection between inflation and growth in a Schumpeterian framework with short-run price rigidity. They also found an hump-shaped inflation-growth locus due to the distortionary effects produced by inflation on the incentive to innovate.

It is possible to conclude that the theoretical literature focused on the

effect of inflation on growth passing through the accumulation of either human or physical capital, through the credit market or through the product market. This study, instead, deals with the effect of inflation on real growth passing through the labour market in presence of wage staggering. In addition, the theoretical literature, with the exceptions of Paal and Smith (2000) and Funk and Kromen (2005, 2006), did not manage to produce a threshold effect as implied by the empirical literature, while here a model is offered able to do it.

With difference to the model proposed by Paal and Smith (2000), we offer calibration results and our model produces a continuous, not a discontinuous relationship between inflation and economic growth, consistently with the empirical evidence presented in Vaona and Schiavo (2007).

This study gives different insights into the inflation-growth nexus than Funk and Kromen (2005, 2006) as we use a learning by doing model and not a Schumpeterian framework, wage-staggering and not price-staggering and, more importantly, labour supply is not exogenously given, but determined by the optimizing behavior of economic agents.

For a broad review of both the theoretical and the empirical literature see Temple (2000).

3 A Model of Nominal Rigidities and Endogenous Growth

In our analysis, the nominal rigidity takes the form of staggered Taylor wage contracts². Our model economy contains a continuum of households, supplying differentiated labor, and a large number of identical firms, producing output by means of all labor types and capital. The labor types are imperfect substitutes in the production function, exhibiting diminishing returns to labor and constant returns to scale. Thus each household faces a downward-sloping labour demand curve in the short run. The government prints money and it returns its seigniorage proceeds to households in the form of lump sum tax rebates.

3.1 The household's problem

Suppose there are N cohorts of households. Each cohort sets its nominal wage contract $(W_{j,t})$ for N periods. Each household maximizes the present value of its utility (U) with respect to its consumption (C) , wage (W) , real money balances (M/P) and capital (K) , subject to its budget constraint, its labor demand function and the capital law of motion.

²As noted by Graham and Snower (2004), for "sufficiently high levels of money growth, Calvo contracts are not appropriate. The reason is straightforward. With Calvo contracts, some households keep their nominal wage unchanged for a very long period of time, which means that, in the presence of inflation, the real value of this wage approaches zero. This implies that the firm will wish to hire as much of the labor of these households as possible, and as little of the other households. This is very inefficient so output approaches zero."

Specifically, the problem of household h is

$$\max_{\left\{C_{t+j}(h), \frac{M_{t+j}(h)}{P_{t+j}}, W_{j,t}(h), K_{t+j+1}(h), B_{t+j}(h)\right\}} \sum_{j=0}^{N-1} \beta^{t+j} U \left(C_{t+j}(h), \frac{M_{t+j}(h)}{P_{t+j}}, n_{j,t+j}(h) \right) \quad (1)$$

$$s.t. \sum_{j=0}^{N-1} \beta^{t+j} \left[P_{t+j} C_{t+j}(h) + M_{t+j}(h) - M_{t+j-1}(h) + \frac{1}{1 + E(r_{t+j+1}^b)} B_{t+j}(h) - \right. \quad (2)$$

$$\left. -B_{t+j-1}(h) + P_{t+j} I_{t+j}(h) - T_{t+j}(h) \right] = \sum_{j=0}^{N-1} \beta^{t+j} [W_{j,t}(h) n_{j,t}(h) + \quad (3)$$

$$+ P_{t+j} \Pi_{t+j}(h) + P_{t+j} r_{t+j} K_{t+j}(h)] \quad (4)$$

$$n_{j,t+j}(h) = \left(\frac{W_{j,t}(h)}{W_{t+j}} \right)^{-\theta_w} \hat{n}_{t+j} \quad (5)$$

$$K_{t+j+1}(h) = I_{t+j}(h) + (1 - \delta) K_{t+j}(h) \quad (6)$$

where β is the discount factor, \hat{n} is the aggregate labour input, n_j is the labour input of cohort j , I is investment in physical capital, B are bond holdings, T are lump sum transfers, E is the expectation operator, Π is profit income, r^b is the interest rate paid to bond holders and r is the user's cost of capital. After detrending nominal variables for nominal growth (μ) and real variables for real growth (γ), it is possible to restate problem (1) - (6) in the following form:

$$\left\{ c_{t+j}(h), \frac{m_{t+j}(h)}{p_{t+j}}, w_{j,t}(h), k_{t+j+1}(h), b_{t+j}(h) \right\} \max \sum_{j=0}^{N-1} \beta^{t+j} U \left[c_{t+j}(h), \frac{m_{t+j}(h)}{p_{t+j}}, n_{j,t+j}(h) \right] \quad (7)$$

$$s.t. \sum_{j=0}^{N-1} \beta^{t+j} \left[p_{t+j} c_{t+j}(h) + m_{t+j}(h) - m_{t+j-1}(h) \frac{1}{\mu\gamma} + \frac{1}{1 + E(r_{t+j+1}^b)} b_{t+j}(h) - b_{t+j-1}(h) \frac{1}{\mu\gamma} + \right. \quad (8)$$

$$\left. + p_{t+j} i_{t+j}(h) - t_{t+j}(h) \right] = \sum_{j=0}^{N-1} \beta^{t+j} \left[\frac{w_{j,t}(h)}{\mu^j} n_{j,t+j}(h) + p_{t+j} \pi_{t+j}(h) + p_{t+j} r_{t+j} k_{t+j}(h) \right] \quad (9)$$

$$n_{j,t+j}(h) = \left[\frac{w_{j,t}(h)}{w_{t+j}} \frac{1}{\mu^j} \right]^{-\theta_w} \hat{n}_{t+j} \quad (10)$$

$$\gamma k_{t+j+1}(h) = i_{t+j}(h) + (1 - \delta) k_{t+j}(h) \quad (11)$$

The first constraint is the household's budget constraint: the sum of the household's expenditures on consumption, money balance accumulation, bond accumulation and investment are equal to disposable income, namely the sum of the household's wage, profit and capital incomes. The second constraint is the labor demand for the cohort's services (derived below), and the final constraint is the law of motion of capital.

Preferences are assumed to be as follows:

$$U = \sum_{j=0}^{N-1} \beta^{t+j} \left\{ \log c_{t+j}(h) - \varsigma n_{j,t+j}(h) + V \left[\frac{m_{t+j}(h)}{p_{t+j}} \right] \right\} \quad (12)$$

The first order conditions for consumption, capital and each cohort's wage are of particular importance for the solution of the model. Given that there exist complete asset markets and that the only heterogeneity among households is due to wage staggering, they can be written as follows:

$$\frac{1}{c_{t+j}} = \lambda_{t+j} \quad (13)$$

$$\gamma \lambda_{t+j} = \beta \lambda_{t+j+1} (1 - \delta + r_{t+j}) \quad (14)$$

$$E_t \sum_{j=0}^{N-1} n_{j,t+j} \beta^{t+j} = \frac{\theta_w - 1}{\theta_w \varsigma} E_t \sum_{j=0}^{N-1} \frac{\beta^{t+j} w_{j,t} n_{j,t+j}}{p_{t+j} \mu^j c_{t+j}} \quad (15)$$

3.2 The supply side of the economy

As far as the supply side of the economy is concerned, we assume the existence of two good sectors. In the final good sector a set of perfectly competitive firms transforms a continuum of horizontally differentiated inputs into an homogenous good. In the intermediate good sector a continuum of monopolistically competitive firms produces different varieties of a horizontally differentiated good using both labour and capital inputs. The continuum of monopolistically competitive firm is normalized on the $[0, 1]$ interval.

We further suppose that, while there is a perfectly competitive capital market, there are two labour markets: an intermediate and a final one. In the intermediate labour market, households belonging to different cohorts set their wages in a monopolistically competitive environment and sell their

labour force to a set of perfectly competitive intermediaries that produce an undifferentiated labour factor to be sold to the continuum of monopolistically competitive firms of the intermediate good sector.

3.2.1 The final good sector

In the final good sector, due to the existence of a set of differentiated inputs with a constant elasticity of substitution, the production function assumes the form of a CES aggregator $y_t = \left(\int_0^1 y_{ft}^{\frac{\theta_p-1}{\theta_p}} df \right)^{\frac{\theta_p}{\theta_p-1}}$, namely in the final good sector firms just use intermediate inputs, y_{ft} , to produce their output, y_t . Firms maximize profits subject to their production function:

$$\begin{aligned} \max_{\{y_{ft}\}} \quad & p_t y_t - \int_0^1 p_{ft} y_{ft} df \\ \text{s.t. } y_t = \quad & \left(\int_0^1 y_{ft}^{\frac{\theta_p-1}{\theta_p}} df \right)^{\frac{\theta_p}{\theta_p-1}} \end{aligned}$$

By solving this maximization problem it is possible to get the demand function for each good variety:

$$y_{ft} = \left(\frac{p_{ft}}{p_t} \right)^{-\theta_p} y_t \tag{16}$$

Furthermore, by imposing the zero profit condition and substituting (16) into the profit equation it is possible to obtain the price index:

$$p_t = \left(\int_0^1 p_{ft}^{1-\theta_p} df \right)^{\frac{1}{1-\theta_p}} \quad (17)$$

3.2.2 The intermediate good sector

In the intermediate good sector each firm f buys capital and labour to produce one variety of good. In so doing, it minimizes costs subject to the constraint of its production function and it maximizes the spread between the price it charges and its marginal cost. So the first problem is:

$$\begin{aligned} \min_{\{\hat{n}_{ft}, k_{ft}\}} & \frac{w_t}{p_t} \hat{n}_{ft} + r_t k_{ft} \\ \text{s.t. } & y_{ft} = (\hat{n}_{ft} k_t)^\alpha (k_{ft})^{1-\alpha} \end{aligned} \quad (18)$$

where w_t is the nominal aggregate wage rate, r_t is the remuneration for capital services, \hat{n}_{ft} is the labour input in efficiency units used by firm f , k_{ft} is the amount of capital used by firm f and k_t is the aggregate capital stock. (18) is a typical production function with knowledge externalities, whereby an increase in a firm's capital stock leads to an increase in its stock of knowledge. Assuming that each firm's knowledge is a public good, the aggregate increase in knowledge is proportional to the aggregate capital stock (Barro and Xala-i-Martin, 1995).

The first order conditions for labour and capital are:

$$\frac{w_t}{p_t} = mc_t \alpha (\hat{n}_{ft} k_t)^{\alpha-1} (k_{ft})^{1-\alpha} k_t \quad (19)$$

$$r_t = mc_t (1 - \alpha) (\hat{n}_{ft} k_t)^\alpha (k_{ft})^{-\alpha} \quad (20)$$

Having chosen the amount of labour and capital that minimize costs, firms of the intermediate output sector maximize profits by maximizing the spread between the price they charge and their marginal cost under the constraint of the demand for the specific good variety they produce (16):

$$\max_{\{p_{ft}\}} \frac{p_{ft} - mc_t^n}{p_t} y_{ft} \quad (21)$$

$$s.t. \ y_{ft} = \left(\frac{p_{ft}}{p_t} \right)^{-\theta_p} y_t \quad (22)$$

where p_{ft} and y_{ft} are respectively the price and the output quantity of variety f and mc_t^n is the nominal marginal cost.

By solving the problem (21)-(22), it is possible to show that the price charged by each firm is just a mark-up over the real marginal cost:

$$\frac{p_{ft}}{p_t} = \frac{\theta_p}{\theta_p - 1} mc_t \quad (23)$$

3.2.3 The final labour market

Let us now move from the good markets to the factor ones. In the final labour market a set of perfectly competitive intermediaries buy labour services from N cohorts of workers, transforming them in an homogenous labour input to be sold to the firms of the intermediate good sector. Again we assume that labour services of different kinds have a constant elasticity of substitution (θ_w). Labour intermediaries maximize their profit subject to their production function:

$$\max_{\{n_{j,t}\}} w_t \hat{n}_t - \sum_{j=0}^{N-1} \frac{w_{j,t}}{\mu^j} n_{j,t} \quad (24)$$

$$s.t. \hat{n}_t = \left(\sum_{j=0}^{N-1} n_{j,t}^{\frac{\theta_w-1}{\theta_w}} \right)^{\frac{\theta_w}{\theta_w-1}} \quad (25)$$

\hat{n}_t is the labour input in efficiency units not to be confused with n_t which is aggregate employment, defined as just the sum of all the quantities of labour supplied by the different cohorts: $n_t = \sum_{j=1}^N n_{j,t}$. The term $\frac{w_{j,t}}{\mu^j}$ is the wage of cohort j that has been detrended because money is growing in steady state but the wage of each cohort is staggered.

Solving the problem above it is possible to find the demand function for each cohort's labour services (10).

By substituting (10) in the zero profit condition it is possible to obtain the aggregate wage index:

$$w_t = \left[\sum_{j=0}^{N-1} \left(\frac{w_{j,t}}{\mu^j} \right)^{1-\theta_w} \right]^{\frac{1}{\theta_w-1}} \quad (26)$$

3.2.4 The intermediate labour market

Finally, on the intermediate labour market, the demand and the supply for the different kinds of labour meet. The equality between labour demand and labour supply is assumed to find (15).

3.3 The Government

For simplicity, the government is assumed to distribute its seigniorage in the form of lump-sum transfers to households:

$$m_{t+j} - m_{t+j-1} \frac{1}{\mu^\gamma} = t_{t+j} \quad (27)$$

4 The Model Solution

The macroeconomic model is constituted by the equations: (10), (13), (14), (15), (16), (17), (18), (19), (20), (23), (26) and (27). The procedure to solve the model is outlined in more detail in Appendix A.

It is possible to obtain the solution for the steady state growth rate of the economy, by using (13), (23), (20), (14) and (18):

$$\gamma = \beta \left[1 - \delta + \frac{\theta_p - 1}{\theta_p} (1 - \alpha) (\hat{n})^\alpha \right] \quad (28)$$

It is therefore necessary to find the steady state level of labour in efficiency units in order to solve the model. In the appendix we show that labour in efficiency units is given by the roots of the following polynomial

$$A^3 \hat{n}^3 - 3A^2 B \hat{n}^2 + (3AB^2 - C^3) \hat{n} - B^3 = 0 \quad (29)$$

where A , B and C are defined as follows:

$$A : = \left[1 - \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \right] \quad (30)$$

$$B : = \left(\frac{1 - \mu^{(\theta_w - 1)}}{1 - \mu^{N(\theta_w - 1)}} \frac{\theta_p - 1}{\theta_p} \alpha \frac{1 - \beta^N \mu^{N(\theta_w - 1)}}{1 - \beta \mu^{(\theta_w - 1)}} \right) \frac{1 - \beta \mu^{\theta_w}}{1 - (\beta \mu^{\theta_w})^N} \frac{\theta_w - 1}{\theta_w \varsigma} \left(\frac{1 - \mu^{N(\theta_w - 1)}}{1 - \mu^{\theta_w - 1}} \right)^{\frac{\theta_w}{\theta_w - 1}} \quad (31)$$

$$C : = (1 - \delta) (\beta - 1) \quad (32)$$

The possibility to have multiple steady states within a learning-by-doing model is well known in the literature (Benhabib and Farmer, 1994). We calibrate the model using standard parameter values. ($\beta = 0.96^{\frac{1}{N}}$, $N = 4$, $\delta = 1 - 0.92^{\frac{1}{N}}$, $\theta_p = 10$, $\alpha = 0.67$, $\theta_w = 2$) like in Ascari (2003), Graham and Snower (2003) and Huang and Liu (2002). ς has been adjusted for the model to produce realistic growth rates. Under this calibration two of the three roots of (29) turn out to be complex numbers which can be ruled out being

without economic meaning. This leaves us with a unique solution for the model.

Figures 1 and 2 show how the long-run growth rate of the economy changes as a function of θ_p and θ_w . An increase in θ_p reduces monopolistic rents in the intermediate product market, implying a positive income effect and enhancing economic growth. This effect is captured by terms depending on θ_p in (28), (30) and (31). The same does not happen for θ_w as its impact on long run growth is the result of a number of different effects. First of all, an increase in θ_w reduces monopolistic rents on the intermediate labour market, entailing a greater labour supply. However, due to wage staggering, this is not the complete story. Consider the ratio between the labour demanded to cohort 0 and cohort j :

$$\frac{n_0}{n_j} = \mu^{-\theta_w j} \tag{33}$$

An increase in θ_w entails more labour cycling (as different labour kinds can be more easily substituted among one another), which produces more inefficiencies. This negative effect more than offset the beneficial reduction of rents leading to a net fall in labour in efficiency units and therefore in economic growth.

5 The Effects of Monetary Policies

5.1 The Real Effects of Monetary policy

Figures 3 and 4 show that the relationship between money and real growth is clearly non-linear. As highlighted by the empirical literature there is a threshold: increasing the money growth rate has a positive impact on real growth until about 2% and a negative one above this value. Underlying this non-linearities there are the discounting and the employment cycling effect. At low inflation rates, the time discounting effect prevails leading to a greater labour supply and therefore to faster capital accumulation and growth. On the contrary, at high inflation rates the employment cycling effect is stronger leading to less labour demand and therefore to slower growth. The labour cycling effect is due to the fact that firms substitute between different kinds of labour because agents belonging to different cohorts have different wages, being some of them locked in past contracts. Similarly to the empirical results obtained by Vaona and Schiavo (2007), the effects of money growth on real growth are sizeable but not large.

The analysis above, regarding how the elasticity of substitution between different goods and different labour kinds affect the real growth rate, already gives some insights on how they affect the long-run relationship between money growth and real growth. Figures 5 and 6 show that the money growth-real growth locus moves upward as θ_p increases and downward as θ_w decreases. Economic growth appears to be much more sensitive to the

structural parameters of the model than to money growth.

5.2 Optimal Monetary Policy

To individuate the optimal rate of money growth we use a specification of the welfare function similar to Woodford (1998), Benigno (2004) and Aoki (2001) where the weight of money holdings is assumed to be very small:

$$W = \sum_{j=0}^{N-1} \beta^{t+j} [\log C_{t+j} - \varsigma n_{jt+j}]$$

C_{t+j} is growing at the rate γ , therefore $C_{t+j} = C_0 \gamma^{t+j}$ and the C_0 term can be dropped without loss of generality.

As showed in Appendix B one can further write

$$W = \left[\beta \frac{1 - \beta^N}{(1 - \beta)^2} - \frac{\beta^N N}{1 - \beta} \right] \log \gamma - \varsigma n_0 \frac{1 - \beta^N \mu^{\theta_w N}}{1 - \beta \mu^{\theta_w}} \quad (34)$$

Figure 7 shows that the optimal money growth rate is about 8.5%. This value is much larger than the one that maximizes growth. This happens because money growth affects (34) through three channels: the real growth, the labour input of cohort zero and the weight of the labour input of cohort zero, that includes the term $\frac{1 - \beta^N \mu^{\theta_w N}}{1 - \beta \mu^{\theta_w}}$. The relation between money growth and real growth is shown in Figure 3. On the other hand, the greater is money growth the smaller are both n_0 and $\frac{1 - \beta^N \mu^{\theta_w N}}{1 - \beta \mu^{\theta_w}}$. Indeed, the faster is money growth, the larger will be the real wage of cohort zero and firms will demand less and less n_0 . n_j increases more for $j = 1, \dots, N - 1$ the faster

is money growth. However, the consequences on welfare are reduced due to time discounting. In the end, welfare increases up to 8.5%, when the real growth effect becomes so depressive to reduce it.

6 Conclusions

This paper extends the results of the New-Keynesian literature with wage-staggering from the relation between inflation and the level of output to the inflation-growth nexus. In this way, it is possible to explore how inflation affects growth passing through the labour market instead of capital or credit markets. For standard values of the calibrated parameters, real growth benefits from increasing inflation up to 2%. Beyond this threshold inflation harms growth. The overall effect of inflation on growth does not appear to be large, consistently with the relevant empirical literature. Regarding welfare, the threshold level of inflation is about 8.5%.

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7 Appendices

A The Solution of The Model

Recall that the macroeconomic model consists of the equations: (10), (13), (14), (15), (16), (17), (18), (19), (20), (23), (26), and (27).

Let us first consider the price index (17) in steady state. Given the absence of price staggering and due to symmetry all the firms will charge the same price p_{ft} . So (17) will become:

$$\frac{pf}{p} = 1$$

By substituting the steady state price index into (23) it is possible to obtain that in steady state the marginal cost is the same across all the firms and that in its turn it can be substituted, together with (22), into the first order conditions for capital and labour in efficiency units to obtain

$$\frac{w}{p} = \frac{\theta_p - 1}{\theta_p} \alpha \frac{y}{\hat{n}} \tag{35}$$

$$r = \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \frac{y}{k} \tag{36}$$

Having in mind the equation for capital remuneration, it is possible to obtain the solution for the steady state growth rate of the economy, by using (13) and (14):

$$\gamma = \beta \left[1 - \delta + \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \frac{y}{k} \right] \quad (37)$$

Consider the production function in steady state, the output-capital ratio is given by:

$$\frac{y}{k} = (\hat{n})^\alpha \quad (38)$$

where the firm subscript was dropped because all the firms are symmetric.

Substituting (38) into (37), it is possible to find that in steady state the real growth rate of the economy depends on labour in efficiency units:

$$\gamma = \beta \left[1 - \delta + \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \hat{n}^\alpha \right] \quad (39)$$

Therefore it is necessary to find the steady state level of labour in efficiency units in order to solve the model. As a first step let us focus on (15). As showed in Graham and Snower (2003), the labour supply of each cohort can be reduced to the labour supply of cohort zero times a term accounting for money growth:

$$\frac{n_0}{n_j} = \mu^{-\theta_w j}$$

Therefore the left hand side of (15), becomes in steady state

$$E_t \sum_{j=0}^{N-1} n_{j,t+j} \beta^{t+j} = n_0 \frac{1 - \beta^N \mu^{N\theta_w}}{1 - \beta \mu^{\theta_w}} \quad (40)$$

As far as the right hand side of (15) is concerned, divide both the numerator and the denominator by y_{t+j} :

$$\frac{\theta_w - 1}{\theta_w \varsigma} E_t \sum_{j=0}^{N-1} \frac{\beta^{t+j} w_{j,t} n_{j,t+j}}{p_{t+j} \mu^j c_{t+j}} = \frac{\theta_w - 1}{\theta_w \varsigma} E_t \sum_{j=0}^{N-1} \frac{\frac{\beta^{t+j} w_{j,t} n_{j,t+j}}{p_{t+j} \mu^j} \frac{n_{j,t+j}}{y_{t+j}}}{\frac{c_{t+j}}{y_{t+j}}} \quad (41)$$

Focusing on the numerator, in steady state, one has:

$$\sum_{j=1}^{N-1} \beta^{t+j} \left(\frac{w_{j,t} n_{j,t+j}}{p_{t+j} \mu^j y_{t+j}} \right) = \frac{w^* n_0}{p y} \sum_{j=1}^{N-1} \beta^{t+j} \mu^{(\theta_w - 1)j} \quad (42)$$

where w^* is the steady state nominal wage rate of cohort zero.

Consider that, due to the zero profit condition of the final labour market, one has:

$$\frac{w \hat{n}}{p y} = \frac{\theta_p - 1}{\theta_p} \alpha = \frac{\sum_{j=0}^{N-1} \frac{w_{j,t} \hat{n}_{j,t}}{\mu^j}}{p y} = \frac{w^* n_0}{p y} \sum_{j=0}^{N-1} \mu^{(\theta_w - 1)j} = \frac{w^* n_0}{p y} \frac{1 - \mu^{N(\theta_w - 1)}}{1 - \mu^{(\theta_w - 1)}}$$

hence

$$\frac{w^* n_0}{p y} = \frac{1 - \mu^{(\theta_w - 1)}}{1 - \mu^{N(\theta_w - 1)}} \frac{\theta_p - 1}{\theta_p} \alpha$$

So it is possible to rewrite (42) as follows

$$\sum_{j=1}^{N-1} \beta^{t+j} \left(\frac{w_{j,t} n_{j,t+j}}{p_{t+j} \mu^j y} \right) = \frac{1 - \mu^{(\theta_w - 1)}}{1 - \mu^{N(\theta_w - 1)}} \frac{\theta_p - 1}{\theta_p} \alpha \frac{1 - \beta^N \mu^{N(\theta_w - 1)}}{1 - \beta \mu^{(\theta_w - 1)}} \quad (43)$$

Considering the denominator of the right hand side of (41), both consumption and output are growing at the same rate so their ratio is a constant in steady state and one can extract it from the sum. Furthermore, given the aggregate budget constraint

$$\frac{c}{y} = 1 - \frac{i}{y} = 1 - \frac{i}{y} \quad (44)$$

Consider that

$$\frac{i}{y} = \frac{i}{k} \frac{k}{y}$$

The capital output ratio can be recovered by the first order condition for cost minimization ($r = \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \frac{y}{k}$), whereas the investment capital ratio can be recovered from the law of motion of capital:

$$\frac{i}{k} = \gamma - 1 + \delta$$

So by substituting the capital output ratio and the investment capital ratio in the previous equation and taking into consideration (37), it is possible to get:

$$\frac{i}{y} = \frac{i}{k} \frac{k}{y} = [(1 - \delta) \beta + \beta r + \delta - 1] \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \frac{1}{r}$$

Simplifying and substituting for r by taking into consideration (36), it is

possible to perform two further steps

$$\begin{aligned}
\frac{i}{y} &= (1 - \delta) (\beta - 1) \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \frac{1}{r} + \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \\
&= (1 - \delta) (\beta - 1) (\hat{n})^{-\alpha} + \frac{\theta_p - 1}{\theta_p} (1 - \alpha)
\end{aligned} \tag{45}$$

Therefore the investment-output ratio depends on labor in efficiency units.

It is time to take stock. By considering (40),(42),(44) and (45), it is possible to rewrite (15) as:

$$n_0 = \frac{1 - \beta \mu^{\theta_w}}{1 - \beta^N \mu^{N\theta_w}} \frac{\left(\frac{1 - \mu^{(\theta_w - 1)}}{1 - \mu^{N(\theta_w - 1)}} \frac{\theta_p - 1}{\theta_p} \alpha \frac{1 - \beta^N \mu^{N(\theta_w - 1)}}{1 - \beta \mu^{(\theta_w - 1)}} \right)}{\left[1 - (1 - \delta) (\beta - 1) (\hat{n})^{-\alpha} - \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \right]} \frac{\theta_w - 1}{\theta_w \varsigma} \tag{46}$$

It is worth noting that the labour supply of cohort zero depends on the aggregate labour input in efficiency units via the investment share of output.

In steady state, the demand for cohort zero is equal to:

$$n_0 = \left(\frac{w^*}{w} \right)^{-\theta_w} \hat{n} \tag{47}$$

where

$$\frac{w^*}{w} = \left(\frac{1 - \mu^{N(\theta_w - 1)}}{1 - \mu^{\theta_w - 1}} \right)^{\frac{1}{\theta_w - 1}}$$

By substituting (46) into (47), it is possible to obtain:

$$\hat{n} = \frac{\left(\frac{1 - \mu^{(\theta_w - 1)}}{1 - \mu^{N(\theta_w - 1)}} \frac{\theta_p - 1}{\theta_p} \alpha \frac{1 - \beta^N \mu^{N(\theta_w - 1)}}{1 - \beta \mu^{(\theta_w - 1)}} \right)}{\left[1 - (1 - \delta)(\beta - 1)(\hat{n})^{-\alpha} - \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \right]} \frac{1 - \beta \mu^{\theta_w}}{1 - (\beta \mu^{\theta_w})^N} \frac{\theta_w - 1}{\theta_w \varsigma} \left(\frac{1 - \mu^{N(\theta_w - 1)}}{1 - \mu^{\theta_w - 1}} \right)^{\frac{\theta_w}{\theta_w - 1}} \quad (48)$$

Simplifying

$$\hat{n} \left[1 - \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \right] - (1 - \delta)(\beta - 1) \hat{n}^{1 - \alpha} - \quad (49)$$

$$\left(\frac{1 - \mu^{(\theta_w - 1)}}{1 - \mu^{N(\theta_w - 1)}} \frac{\theta_p - 1}{\theta_p} \alpha \frac{1 - \beta^N \mu^{N(\theta_w - 1)}}{1 - \beta \mu^{(\theta_w - 1)}} \right) \frac{1 - \beta \mu^{\theta_w}}{1 - (\beta \mu^{\theta_w})^N} \frac{\theta_w - 1}{\theta_w \varsigma} \left(\frac{1 - \mu^{N(\theta_w - 1)}}{1 - \mu^{\theta_w - 1}} \right)^{\frac{\theta_w}{\theta_w - 1}} = 0 \quad (50)$$

Suppose, as customary, that $\alpha = \frac{2}{3}$:

$$\hat{n} \left[1 - \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \right] - (1 - \delta)(\beta - 1) \hat{n}^{\frac{1}{3}} - \quad (51)$$

$$\left(\frac{1 - \mu^{(\theta_w - 1)}}{1 - \mu^{N(\theta_w - 1)}} \frac{\theta_p - 1}{\theta_p} \alpha \frac{1 - \beta^N \mu^{N(\theta_w - 1)}}{1 - \beta \mu^{(\theta_w - 1)}} \right) \frac{1 - \beta \mu^{\theta_w}}{1 - (\beta \mu^{\theta_w})^N} \frac{\theta_w - 1}{\theta_w \varsigma} \left(\frac{1 - \mu^{N(\theta_w - 1)}}{1 - \mu^{\theta_w - 1}} \right)^{\frac{\theta_w}{\theta_w - 1}} = 0 \quad (52)$$

Define:

$$\begin{aligned}
A & : = \left[1 - \frac{\theta_p - 1}{\theta_p} (1 - \alpha) \right] \\
C & : = (1 - \delta) (\beta - 1) \\
B & : = \left(\frac{1 - \mu^{(\theta_w - 1)}}{1 - \mu^{N(\theta_w - 1)}} \frac{\theta_p - 1}{\theta_p} \alpha \frac{1 - \beta^N \mu^{N(\theta_w - 1)}}{1 - \beta \mu^{(\theta_w - 1)}} \right) \frac{1 - \beta \mu^{\theta_w}}{1 - (\beta \mu^{\theta_w})^N} \frac{\theta_w - 1}{\theta_w \varsigma} \left(\frac{1 - \mu^{N(\theta_w - 1)}}{1 - \mu^{\theta_w - 1}} \right)^{\frac{\theta_w}{\theta_w - 1}}
\end{aligned}$$

As a consequence the solution for labour in efficiency units is given by the roots of the following polynomial

$$A^3 \hat{n}^3 - 3A^2 B \hat{n}^2 + (3AB^2 - C^3) \hat{n} - B^3 = 0$$

With \hat{n} in mind, one can easily compute γ from (39).

B Optimal Monetary Policy

As stated above welfare is given by:

$$U = \sum_{j=0}^{N-1} \beta^{t+j} (\log C_{t+j} - \varsigma n_{j,t+j})$$

Keeping in mind (33) it is possible to write:

$$U = \sum_{j=0}^{N-1} \beta^{t+j} [\log \gamma^{t+j} - \varsigma n_0 \mu^{\theta_w(t+j)} + \text{const.}]$$

Setting $t = 0$, one obtains without loss of generality:

$$U = \log \gamma \sum_{j=0}^{N-1} \beta^j j - \varsigma n_0 \frac{1 - \beta^N \mu^{\theta_w N}}{1 - \beta \mu^{\theta_w}}$$

Consider that

$$(1 - \beta) \sum_{j=0}^{N-1} \beta^j j = \beta + \beta^2 + \dots + \beta^{N-1} + \beta^N - \beta^N N$$

and consequently

$$\sum_{j=0}^{N-1} \beta^j j = \beta \frac{1 - \beta^N}{(1 - \beta)^2} - \frac{\beta^N N}{1 - \beta}$$

Recalling (46), it is possible to write:

$$U' = \left[\beta \frac{1 - \beta^N}{(1 - \beta)^2} - \frac{\beta^N N}{1 - \beta} \right] \log \gamma - \varsigma n_0 \frac{1 - \beta^N \mu^{\theta_w N}}{1 - \beta \mu^{\theta_w}}$$

Figure 1 – The effect of θ_p on growth

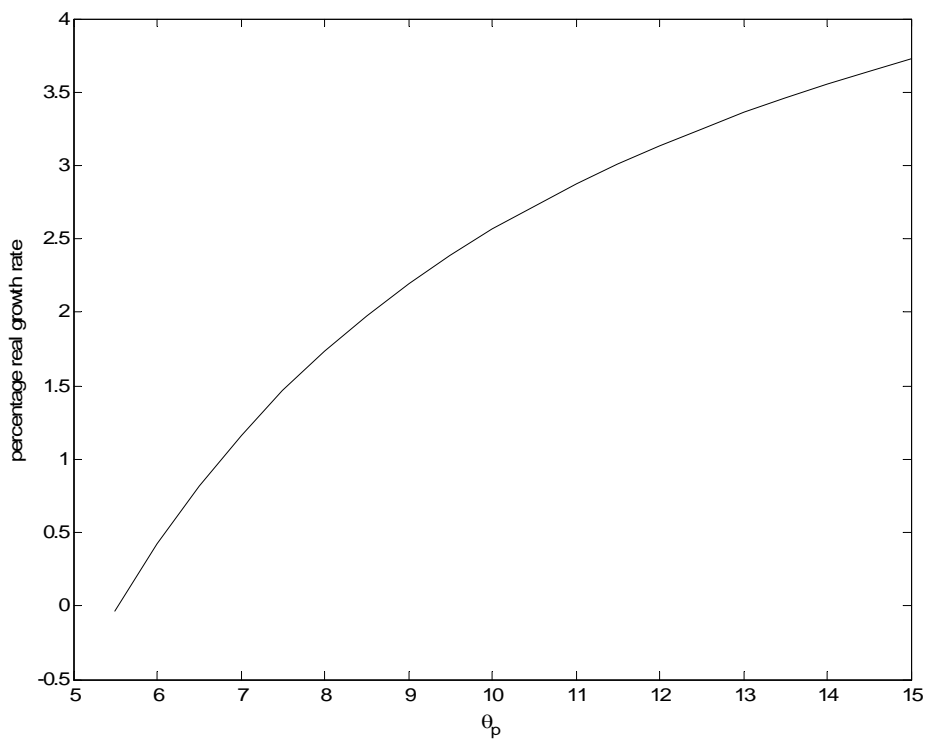


Figure 2 – The effect of θ_w on growth

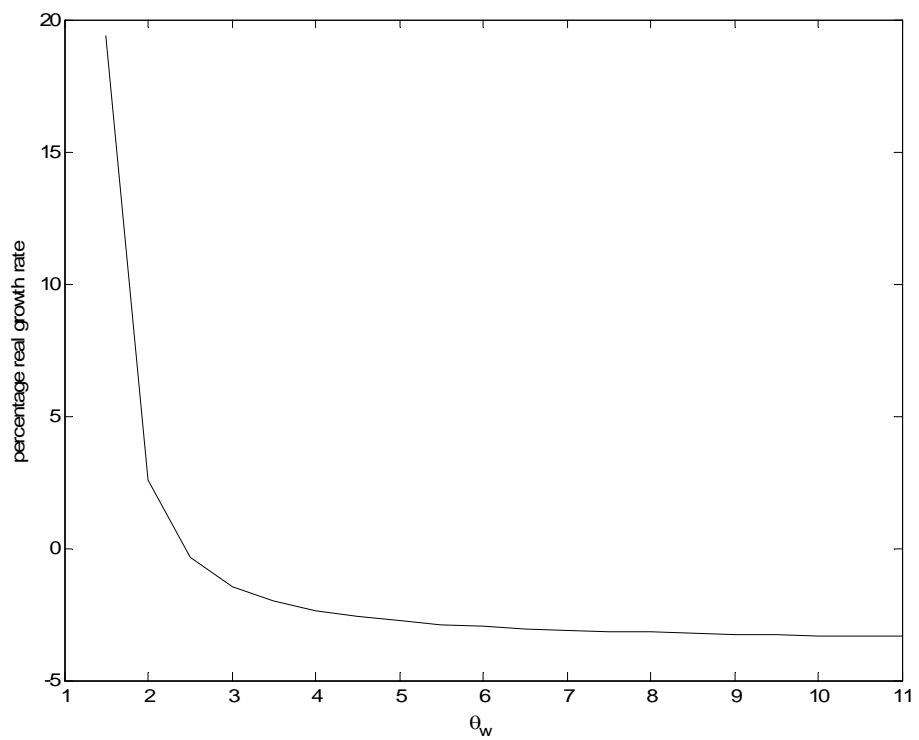


Figure 3 – The effect of money growth on real growth

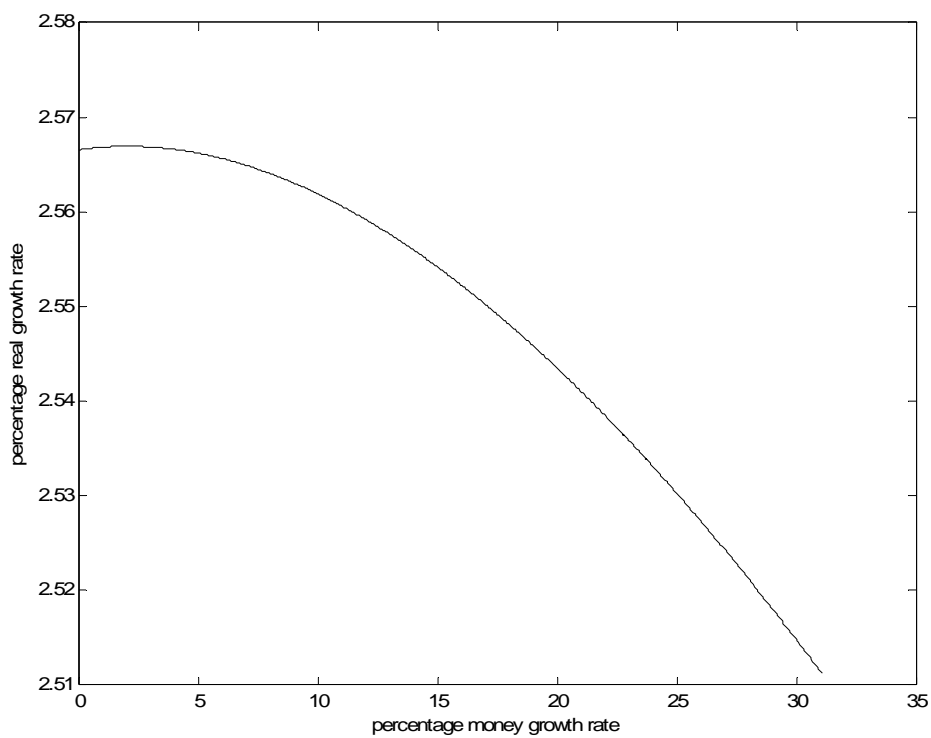


Figure 4 – The effect of money growth on real growth – detail.

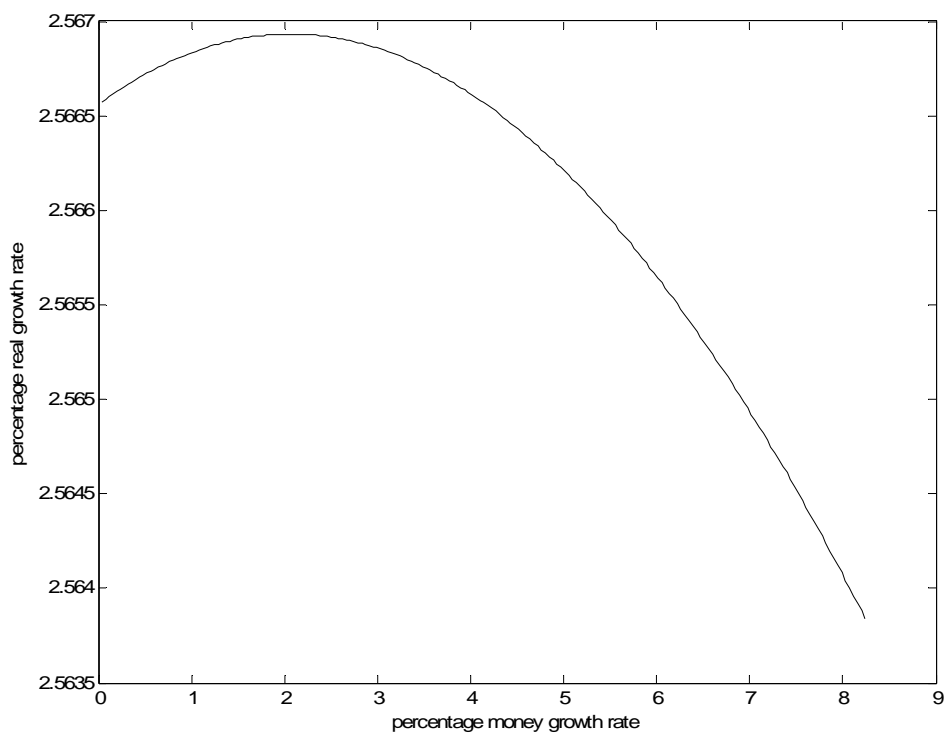


Figure 5 – The effect of money growth on real growth for different values of θ_p

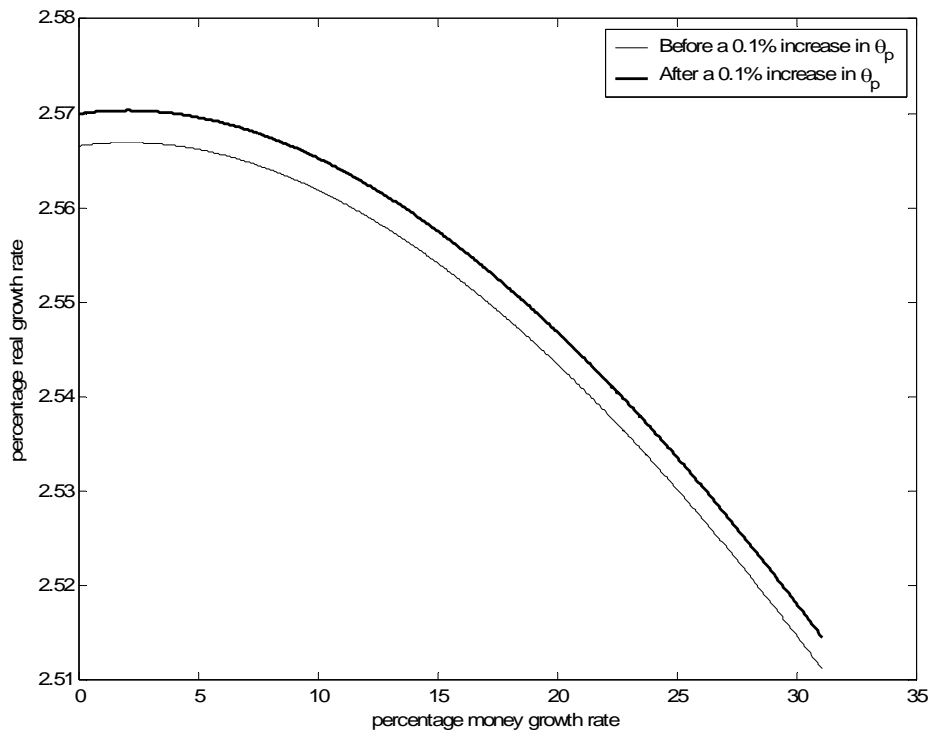


Figure 6 – The effect of money growth on real growth for different values of θ_w

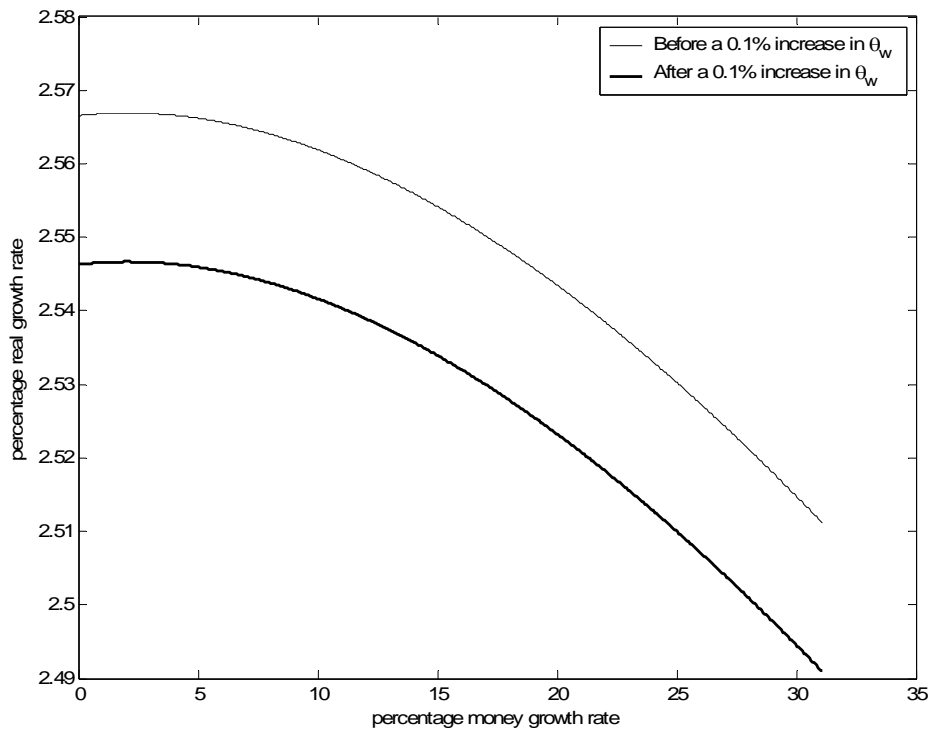


Figure 7 – The effect of money growth on welfare

