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The Fisher hypothesis and nominal interest rate decomposition: A structural VAR approach

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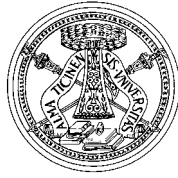
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Quaderni di Dipartimento

**The Fisher hypothesis and nominal interest rate decomposition:
A Structural VAR approach**

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The Fisher hypothesis and nominal interest rate decomposition: A Structural VAR approach

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April 14, 2007

Abstract

Using Treasury Bill nominal interest rates data for the sample period 1957Q1-2005Q1, we apply the structural VAR methodology developed by Blanchard and Quah (1989) to achieve twofold objective. First of all, we are interested to estimate the time-path of the unobservable variables that, according to the Fisherian theory of interest, determine fluctuations of the nominal interest rates, i.e. the expected inflation and the *ex-ante* real interest rate components. Secondly, we try to solve the puzzling evidence regarding the apparently nonstationarity of the real interest rates. After accounting for potential infrequent mean shifts in the *ex-post* real interest rate series, we impose valid long-run neutrality restriction to obtain Structural (identified) VAR and test the magnitude of the Fisher effect over the entire sample of analysis.

1 Introduction

According to the Fisher's theory, nominal interest rates incorporate information on economic agents' expectations about future inflation and real economic activity. Extracting both the *ex-ante* real interest rate and inflation expectations components from Treasury bond rates makes possible to interpret the central bank's monetary policy features regarding two main targets: output stabilization and price stability. The *ex-ante* real interest rate represents a key variable by which policymakers induce shocks to real economic activities influencing saving and investment decisions. At the same time, monitoring market's expectations of future inflation provides an indicator of central bank's policy credibility to low inflation. Despite their informative relevance for policy analysis purposes, estimation of both the inflation expectations and *ex-ante* real rate has proven to be a difficult exercise. In empirical literature, a variety of estimation methodologies exist and they all have advantages and shortcomings. A most popular way to acquire information about inflation expectations involves conducting regular and frequent surveys of many "informed" market agents,

such as professional forecasters for financial institutions, and then constructing an average inflation forecast from the survey responses¹.

However, using survey-based forecasts is sometime problematic for a series of reasons. First, survey responses taken from consumers, businesses or professionals are quite different to each other. Moreover, responses usually are not consistent with the movements in the market prices. Secondly, inflation forecasts are not always available because no survey was taken for that period. Finally, forecasts are limited to predetermined fixed horizons. A second approach, used in empirical literature, infers expected inflation (for a given forecast horizon) comparing nominal interest rates on equal-maturity nominal indexed bond rates. However, this estimation method is not widely available because of few countries have active markets for indexed bonds. Among the industrialized countries, only the United Kingdom provides series of treasury inflation-indexed securities prices that span more than a few years. For the United States, real bonds, i.e. treasury inflation protected securities (T.I.P.S.) have traded only post 1997.

In this paper, we try to overcome the above discussed difficulties in estimating expectations of future inflation and propose to estimate a Structural Vector Autoregressive (SVAR) model with the objective to decompose nominal interest rate series into its embedded components: the *ex-ante* real interest rate and inflationary expectations. After discussing conditions ensuring the validity of the Fisher's hypothesis test (see Rose (1988), Mishkin (1992) and Evans and Lewis (1995)), we follow the Blanchard-Quah (1989) methodology and identify the two sources of fluctuations in the nominal interest rate imposing the long-run restriction that *ex-ante* real interest rate shocks have but a temporary impact on the nominal interest rate.

In our analysis, we focus our attention on the short-side of the term structure. Using quarterly data for the U.S. Treasury Bill market, we decompose three-month T-bill nominal interest rates into one-quarter ahead expected inflation and the corresponding *ex-ante* real interest rate. However, our estimation procedure can be easily extended to nominal interest rates with different maturity. This may result particularly useful when inflation forecasts are not provided by surveys over longer horizon. The remainder of the paper is organized as follows. Section 2 summarizes the Fisher's theory of interest and the assumptions behind the decomposition of the nominal interest rate into its unobservable components. Moreover, we introduce some reasons for the puzzling evidence when testing for the presence of the Fisher effect. In section 3, the Blanchard-Quah (1989) methodology is introduced and applied for our purposes. Section 4 reports empirical findings including a comparison between our one-quarter ahead estimates of expected inflation and one-quarter ahead inflation forecasts provided by Survey of Professional Forecasters (SPF). Finally, section 5 concludes.

¹In United States, for example, survey of the University of Michigan and survey of Professional Forecasters (now produced by the Federal Reserve Bank of Philadelphia) provide information about expected inflation over different horizon.

2 Theoretical framework: the Fisher effect

The Fisher's equation states that movements in nominal bond interest rates originate in two sources: changes in real interest rates and changes in expected rate of inflation:

$$i_{t,k} = rr_{t,k}^e + \pi_{t,k}^e \quad (1)$$

where, $i_{t,k}$ is the nominal interest rate at time t on a bond with k -periods till maturity, $rr_{t,k}^e$ is the k -periods real rate of interest expected for the bond maturing at time $t+k$. $\pi_{t,k}^e$ denotes inflation expectations by the agents for the time from t to $t+k$. By assuming rational expectations hypothesis, expected inflation term in equation (1) can be defined as the sum of the realized inflation rate $\pi_{t,k}$ and a stationary inflation forecast error term $\varepsilon_{t,k} \sim I(0)$. Therefore (1) can be expressed in regression form as follows:

$$i_{t,k} = rr_{t,k}^e + \pi_{t,k} + \varepsilon_{t,k} \quad (2)$$

where $i_{t,k}$ and $\pi_{t,k}$ are implicitly assumed to be cointegrated with vector $b = [1, -1]'$ and $rr_{t,k}^e \sim I(0)$. In general, assuming that the real interest rate process has got a mean equal to α , we can test for the Fisher effect in the context of the following regression:

$$i_{t,k} = \alpha + \beta\pi_{t,k} + v_{t,k} \quad (3)$$

Full Fisher effect holds if i) $i_{t,k}$ and $\pi_{t,k}$ are cointegrated series, i.e. $v_{t,k} \sim I(0)$ and ii) $\beta = 1$, i.e. nominal interest rate, $i_{t,k}$, moves *one-to-one* with expected inflation rate, $\pi_{t,k}^e$. The first of these conditions is supported by the bulk of empirical evidence in the literature. On the other hand, when dealing with the second condition, estimates of β appear to be significantly different from unity, leading to the so-called Fisher effect 'puzzle'. Before discussing this puzzle, we remark that combining equation (3) and (1), the *ex-post* real interest rate can be expressed as:

$$rr_{t,k} = (i_{t,k} - \pi_{t,k}) = \alpha - (1 - \beta)\pi_{t,k} + v_{t,k} \quad (4)$$

Therefore, if a full Fisher effect holds ($\beta = 1$), the *ex-post* real interest rate is a stationary process, such that $rr_{t,k} \sim VWN(\alpha, \sigma_v^2)$, otherwise $rr_{t,k}$ shares the same permanent disturbances as the expected inflation. In particular, the case $\beta < 1$ indicates the existence of a negative correlation between real interest rate and inflation.

2.1 The Fisher effect 'puzzle'

Many studies for the U.S. country suggest a positive relationship between nominal interest rates and inflation but they do not establish a clearly *one-to-one* relationship as postulated by the Fisherian theory. Many authors suggest a resolution of the Fisher effect 'puzzle' in terms of estimation method's choice.

The general lack of consensus in empirical literature about the true size of the Fisher effect should originate from differences in methodological approaches used to estimate it. During the 1970s, the lack of robust statistical methods certainly contributed to make results about the validity of the fisherian theory quite controversial. Since the early 1980s, the innovations in the analysis of non stationary time-series and the growing interest in the innovative concept of Granger's cointegration motivated researchers to focus their attention on the stochastic properties of the time series and the problems of spurious regressions described by Granger and Newbold (1974). Mishkin (1992) pointed out that the Fisher's relation should be treated within the context of a cointegrated system, as in Engle and Granger (1987). He used the Engle-Granger OLS procedure to estimate the Fisher effect but did not derive any strong conclusion due to the large standard errors of the estimated parameters. Many authors argued that, even when applying the more appropriate cointegration analysis, severe problems may arise associated with the performance of the various estimators in small samples. Crowder and Hoffman (1996), for example, suggest that the different conclusions reached in the literature are attributable to the differences in the small sample properties of the Ordinary Least Squares (OLS), the Dynamic Least Squares (DOLS) and the Johansen's (JOH) maximum likelihood estimators².

Other strand of literature consider that occasional exogenous events may contribute to complicate the Fisher effect puzzle. For example, Bekdache (1999), Malliaropulos (2000), and Johnson and Garcia (2000) show that events, such as oil price shocks or shifts in monetary policy regimes can be reflected in the form of infrequent changes in the mean or the drift rate of both the interest rate series and inflation rate series. These events potentially involve difficulties in assessing stochastic properties of the series using standard unit-root testing procedures, such as ADF-type tests. In fact, these latter tests are biased toward nonstationarity since they misinterpret structural breaks as permanent stochastic disturbances. It follows that unit-root testing procedures can potentially be responsible for the apparent puzzling evidence. Starting from these basic premises, Evans and Lewis (1995) proposed a reexamination of the Fisher effect assuming that the process followed by inflation shifts infrequently. After characterizing the shifts in inflation series by a Markov switching model, they demonstrate that not accounting for these shifts induce significant small sample biases in estimates of the long-run Fisher relationship. These small sample biases may create the appearance of permanent shocks to the real interest rates even when none are truly present. Contrary, incorporating infrequent shifts in inflation process, the authors are unable to reject the hypothesis that in the long-run nominal interest rates reflect expected inflation *one-to-one*. Analogously, Garcia and Perron (1996), using a three-regimes Markov-Switching model in a similar fashion of Hamilton (1989), analyzed the unusual time-path behavior of U.S. 90-day treasury bill real rate over the period 1961-1986 and concluded that

²Crowder and Hoffman (1996) used Johansen's (1988,1991) maximum likelihood estimator (MLE) to estimate cointegration relation between interest rate and inflation rate series. They found, for the period from 1951Q1 to 1991Q4, an evidence of full Fisher effect.

real interest rate series can be essentially described as a stationary process with mean subject to occasional shifts.

According to Garcia and Perron (1996), we consider the possibility that potential nonstationarity of the *ex-post* real interest rate process may derive by infrequent but significant breaks which produce jumps in the time-varying dynamics of the series. Therefore, we investigate the timing for structural breaks using Bai and Perron (2003a, 2003b) sequential unit-root testing procedure. This latter extends the traditional ADF-type test by accounting for possible multiple mean shifts in the data process with no *a priori* knowledge of the break dates. We expect that, if the *ex-post* real interest rates series can be regarded as a stationary process around the estimated levels of mean, a strong cointegrating relation (β near the unity) between nominal interest rate and its embedded expected inflation component exists. We can obtain confirmation of this latter consideration by estimating our bivariate SVAR model that allows us to decompose nominal interest rate series into its embedded components.

3 Decomposing nominal interest rate: the SVAR approach

In order to identify the two sources of fluctuations in the nominal interest rate, we proceed as follows. We estimate a VAR(p) model:

$$\begin{aligned} Z_t &= A_1 Z_{t-1} + A_2 Z_{t-2} + \dots + A_p Z_{t-p} + e_t & (5) \\ (I_2 - A_1 L - A_2 L^2 - \dots - A_p L^p) Z_t &= e_t \\ A(L) Z_t &= e_t, \quad e_t \sim N(0, \Omega) \\ E(e_t e_{t+j}) &= 0 \quad j \neq 0; \\ E(e_t e'_{t+j}) &= \Omega \quad j = 0; \end{aligned}$$

where $Z_t = [\Delta i_t, rr_t]$ is a (2×1) vector of stationary variables. Δi_t denotes changes in nominal interest rates and rr_t is the *ex-post* real interest rates series. $A(L)$ is the matrix polynomial in the lag operator L and e_t is a (2×1) normally distributed vector white noise. By the Wold decomposition theorem, the above reduced-form VAR(p) can be written in the following moving average representation (VMA) form³:

$$\begin{aligned} Z_t &= e_t + C_1 e_{t-1} + \dots = \sum_{j=0}^{\infty} C_j e_{t-j} & (6) \\ Z_t &= C(L) e_t \end{aligned}$$

³Wold representation requires that $A(L)$ polynomial matrix can be inverted. This implies that the determinantal polynomial, $|A(L)|$, has all its roots greater than one.

where $C(L)$ is a matrix polynomial which can be of infinite order and for which we assume that the multivariate invertibility conditions hold, i.e. $\det[C(L)] = 0$ has all the roots outside the unit circle, so that $C(L) = A(L)^{-1}$. $C_0 = I_2$ and $C(1) = (I_2 - A_1 - A_2 \dots - A_p)^{-1}$ is the estimated accumulated responses to the reduced-form (observed) shocks, e_t . From equations (2) and (4), we assume that the behavior of Z_t is governed by two independent structural shocks $\varepsilon_t = [\varepsilon^\pi, \varepsilon^{rr}]$. The first element of ε_t is defined as the expected inflation shocks while the second element (ε^{rr}) represents the *ex-ante* real interest rate shocks. Let suppose that the VMA representation with the structural shocks takes the following form:

$$Z_t = D_0 \varepsilon_t + D_1 \varepsilon_{t-1} \dots = \sum_{j=0}^{\infty} D_j \varepsilon_{t-j} \quad (7)$$

equivalent to:

$$Z_t = \begin{bmatrix} \Delta i_t \\ rr_t \end{bmatrix} = D(L) \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^{rr} \end{bmatrix} \quad \varepsilon_t \sim VWN(0, I_2) \quad (8)$$

where the variance of the structural shocks is normalized so that ε_t are non-correlated and orthogonal to each other. D_j denote the structural impulse responses functions. From equations (6) and (7) we note that residuals of the reduced-form model are related to the structural residuals in the following way:

$$e_t = D_0 \varepsilon_t \quad (9)$$

This particular structural relation implies that each variable in the system (5) is affected by a set of orthonormal disturbances whose impact effect is explicitly modelled via the D_0 matrix, assumed to be an (2×2) invertible matrix. Therefore, identification of the D_0 matrix makes possible to recover structural innovations from the reduced-form residuals. Identification of D_0 necessitates four restrictions. If we calculate the variance-covariance matrix of $D_0 \varepsilon_t$ we have:

$$\begin{aligned} E(e_t e_t') &= D_0 E(\varepsilon_t \varepsilon_t') D_0' \\ \Omega &= D_0 D_0' \end{aligned} \quad (10)$$

i.e.,

$$E \begin{bmatrix} (e_{1t})^2 & e_{1t} e_{2t} \\ e_{1t} e_{2t} & (e_{2t})^2 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}' \quad (11)$$

where d_{ij} represents the generic element of D_0 matrix. Both the symmetry of the matrix Ω and normalization conditions (which define the variance-covariance matrix of the structural shocks, I_2) impose, implicitly, three restrictions on the D_0 matrix, leaving one free element. Therefore, we need only one more restriction to identify D_0 . Combining equations (6), (7) and (9), we can express

structural impulse-response functions, D_j , in term of the reduced-form impulse responses, C_j , via D_0 matrix:

$$\begin{aligned} C_j D_0 &= D_j \\ C(L) D_0 &= D(L) \end{aligned} \quad (12)$$

In the long-run, we have:

$$D(1) = C(1)D_0 \quad (13)$$

where $D(1)$ and $C(1)$ are the long-run impulse-response functions of structural and reduced-form VAR, respectively. We can rewrite expression (13) as:

$$\begin{bmatrix} D_{11}(1) & D_{12}(1) \\ D_{21}(1) & D_{22}(1) \end{bmatrix} = \begin{bmatrix} C_{11}(1) & C_{12}(1) \\ C_{21}(1) & C_{22}(1) \end{bmatrix} \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \quad (14)$$

where $D_{ij}(1)$ and $C_{ij}(1)$ indicate the long-run response of the i^{th} -variable to the j^{th} -shock in the structural and reduced-form specification, respectively. Following Blanchard and Quah (1989), identification of D_0 matrix is obtained imposing one homogeneous restriction (a zero restriction) on the long-run structural impulse response function $D(1)$. In our analysis, if we consider the *ex-post* real interest rate, $rr_{t,k}$, a stationary variable, then we can assume that the *ex-ante* real rate shock, ε^{rr} , has no long run effects on nominal interest rate $i_{t,k}$. Since nominal interest rate enters the VAR model in differenced form, the corresponding long-run neutrality assumption is equivalent to the restriction that the cumulative effect of the *ex-ante* real interest rate disturbances on changes of nominal interest rate, $\Delta i_{t,k}$, is zero. Formally, considering the order of variables entering the VAR model (see expression 8), this restriction can be written as:

$$D_{12}(1) = C_{11}(1) d_{12} + C_{12}(1) d_{22} = 0 \quad (15)$$

After imposing the above additional restriction (15), the SVAR model results *exactly* identified. Once we have computed the structural shocks, ε_t , and the sequence of structural impulse-response functions (D 's matrices), we are able to express our set of stationary endogenous variables Z_t as the sum of the two structural shocks:

$$\begin{bmatrix} \Delta i_{t,k} \\ rr_{t,k} \end{bmatrix} = \sum_{j=0}^{\infty} \begin{bmatrix} D_{11,j} & D_{12,j} \\ D_{21,j} & D_{22,j} \end{bmatrix} \begin{bmatrix} \varepsilon_{t-j}^{\pi} \\ \varepsilon_{t-j}^{rr} \end{bmatrix} \quad (16)$$

The above VMA structural representation allows to decompose changes in interest rate series, $\Delta i_{t,k}$, into a part attributable to the expected inflation shocks and a part due to the *ex-ante* real interest rate shocks. Formally, we have:

$$\Delta i_{t,k} = \underbrace{\sum_{j=0}^{\infty} (D_{11,j} \varepsilon_{t-j}^{\pi})}_{\Delta \pi_{t,k}^e} + \underbrace{\sum_{j=0}^{\infty} (D_{12,j} \varepsilon_{t-j}^{rr})}_{\Delta rr_{t,k}^e} \quad (17)$$

Since we are interested in investigating the decomposition of the nominal interest rate series, $i_{t,k}$, it will be sufficient to cumulate separately the effects on $\Delta i_{t,k}$ induced by the two shocks, ε^{π} and ε^{rr} , to obtain an estimate of the expected inflation and *ex-ante* real rate components.

4 Empirical analysis

4.1 Data

The data consist of quarterly U.S. three-month Treasury Bill interest rates and Consumer Prices Index (CPI) series. Data are taken from International Financial Statistics (IFS) for a time span beginning in 1957Q1 and ending in 2005Q1. In order to calculate the *ex-post* real interest rate series, we compute the annualized one-quarter inflation rate, $\pi_{t,3}$, as the annualized rate of change of the CPI over the previous quarter. Deflating the nominal interest rate series, $i_{t,3}$, from one-quarter inflation rate, we obtain the *ex-post* real rate series, $rr_{t,3}$. All the series are depicted in figure [1].

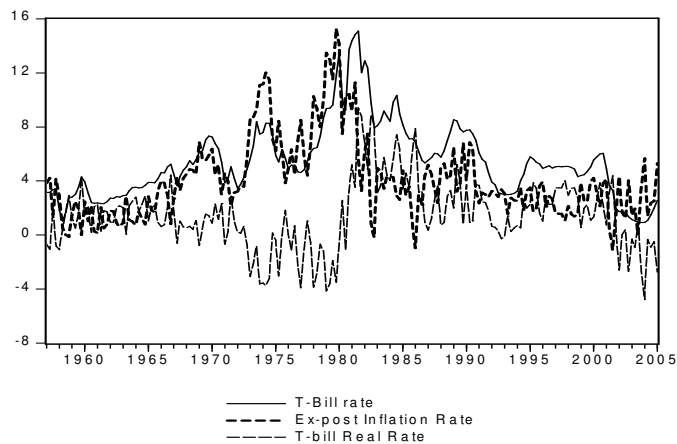


Figure 1: Time series for the sample period 1957Q1 to 2005Q1

Table 1 reports results of the unit-root test obtained using the regular Augmented Dickey -Fuller test (ADF).

Variable	ADF Test		
	Levels	First differences	Order of Integration
$i_{t,3}$	-2.50	-6.21***	I(1)
$\pi_{t,3}$	-2.35	-17.2***	I(1)
$rr_{t,3}$	-2.59	-17.7***	I(1)

Table 1: Unit-root test. The number of lag lengths considered are selected according to Schwartz information criterion. *, **, *** denote statistical significance at the 10%, 5% and 1% levels, respectively. Critical values are tabulated in Dickey and Fuller (1979) and MacKinnon (1991).

Consistent with the bulk of empirical evidence in the literature, table 1 indicates that both the nominal interest rate, $i_{t,3}$, and actual three-month inflation rate, $\pi_{t,3}$, are nonstationary series. Moreover, it's interesting to note that we cannot reject (at 5% level of significance) the null hypothesis of unit root for the *ex-post* real interest rate series, $rr_{t,3}$. This latter result would suggest that a full Fisher effect doesn't hold. However, in the previous section, we have discussed that the apparently puzzling evidence suggesting that $rr_{t,3}$ are affected by permanent shocks might be explained by the infrequent deterministic structural breaks in the series. Not accounting for these infrequent structural breaks, ADF-type tests are biased towards nonstationarity since they misinterpret structural breaks as permanent stochastic disturbances. After all, a simple visual inspection of the *ex-post* real rate path, depicted in figure [1], seems to suggest that its unusual behavior might be attributable to occasional mean shifts. Formally, we proceed to determine the appropriate numbers of possible mean shifts, m , present in the $rr_{t,3}$ series, employing the Bai and Perron (2003a, 2003b) sequential unit-root testing procedure. A brief description of this latter procedure is reported in appendix A. Results are reported in table 2.

In panel A, we report the $SupF_T(k)$ statistics testing for no structural break ($m = 0$) versus the alternative hypothesis that there is an arbitrary fixed number of breaks ($m = k$). Considering $k = 5$, we find that at least one breakpoint exists. Panel B reports the values of the $SupF_T(l + 1 | l)$ statistics. This latter makes possible to decide the optimal number of break points in the series testing the null hypothesis of l breaks against the alternative that an additional break ($l + 1$) exists. A sequential examination of the $SupF_T(l + 1 | l)$ statistics suggests to consider 3 breaks. In panel C, we report the estimated optimal break dates and their intervals at 95% level of confidence. Our results suggest that the *ex-post* real interest rate series, $rr_{t,3}$, can be best characterized by a four-regime model. The break dates are estimated at 1972Q3, 1980Q3 and 1987Q4. However, while the first two dates are precisely estimated (their 95% confidence intervals cover only a few quarters before and after), the third date has a rather large confidence interval. Finally panel D reports the parameter estimates of the *pure* structural change model⁴ used to consistently determine the appropriate numbers of breaks present in the time series. In figure [2],

⁴Our estimated model is defined as:

Tests				
Panel A				
Sup F_T (1)	Sup F_T (2)	Sup F_T (3)	Sup F_T (4)	Sup F_T (5)
7.68*	12.41***	18.06***	15.06***	12.28***
Panel B				
Sup F_T (2 1)	Sup F_T (3 2)	Sup F_T (4 3)		
31.02***	31.02***	2.51		
Selected break dates				
Panel C				
T_1	T_2	T_3		
1972Q3	1980Q3	1987Q4		
(1969Q4–1973Q2)	(1979Q1–1981Q1)	(1983Q2–1992Q3)		
Parameters estimates of the pure structural change model				
Panel D				
$\hat{\delta}_1$	$\hat{\delta}_2$	$\hat{\delta}_3$	$\hat{\delta}_4$	
1.26	-1.57	4.83	1.50	
(0.23)	(0.32)	(0.33)	(0.22)	

Table 2: Bai and Perron test results. Standard errors (robust to serial correlation) and 95% confidence intervals for break dates in parenthesis. *, **, *** denote statistical significance at the 10%, 5% and 1% levels, respectively. Critical values are provided by Bai and Perron(2003b).

we plot the three-month *ex-post* real interest rate, $rr_{t,3}$, and the estimated structural mean shifts (dotted line).

Differences in the estimated means over each segment are significant and point to a decrease of 0.31% in 1972Q3, an increase of 6.4 % in 1980Q3 and a moderate decrease of 3.3% at the end of 1987. The abrupt mean shifts in the real interest rate process seem to be due to the dramatic reversal in inflation psychology during the time. Thus, upward level shift in the inflation process during the 1970s coincides with downward level shift in $rr_{t,3}$. Contrary, downward reversal in the inflation rate since the 1980s (the disinflation era) coincides with upward shift in real interest rate. However, we remark that the uncertainty

$$y_t = z_t' \delta_j + u_t \quad (t = T_{j-1} + 1, \dots, T_j)$$

for $j = 1, \dots, m+1$, where m denotes the number of breaks. We use the convention that $T_0 = 0$ and $T_{m+1} = T$. y_t is the observed dependent variable ($rr_{t,3}$) at time t ; z_t is the assumed constant regressor and δ_j ($j = 1, \dots, m+1$) is the corresponding vector of coefficients; u_t is the disturbance at time t . The indices (T_1, \dots, T_m) represent the break dates (treated as unknown). A description of the general model is reported in appendix A.

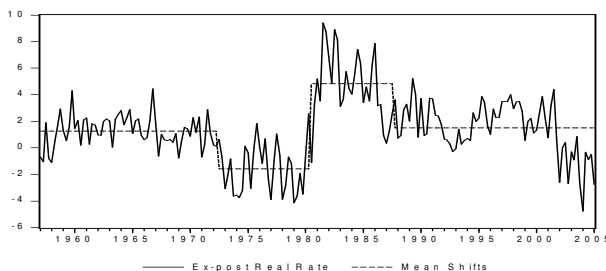


Figure 2: Three-month real interest rate, $rr_{t,3}$ and estimated mean shifts.

surrounding the third estimated break date (1987Q4) can be potentially interpreted as a signal of some degree of persistence in the $rr_{t,3}$ series. Apart from considerations about potential mean shifts, this evidence should imply that, since the mid-1980s, the Fisher's relation may appear weaker than in the past. We can obtain confirmation of this latter hypothesis after estimating our SVAR model.

In section 3, we have pointed out that all variables entering the VAR(p) model must be stationary. Despite stochastic properties of the real interest rate series appear quite controversial after the break date 1987Q4, we can interpret $rr_{t,3}$, over the entire period of analysis, as a stationary variable with mean subject to occasional shifts. Defining $\tilde{r}_{t,3}$ series as the difference between $rr_{t,3}$ and its means over each segment obtained by the estimated pure structural model (see panel D of table 2), we have that $\tilde{r}_{t,3}$ is $I(0)$ process by definition. Therefore, given that we are interested to remove the biases in estimating the Fisher relationship due to the presence of infrequent shifts in real interest rate process, we consider $\tilde{r}_{t,3}$ as the candidate endogenous variable entering our VAR model. Moreover, given that $\tilde{r}_{t,3}$ is a stationary process, we can obtain an *exact* identification of the SVAR model, imposing the reasonable condition (15) that real interest rate shocks have only transitory effects on nominal interest rate $i_{t,3}$. After estimating the SVAR model, we proceed to compute the sequence of structural impulse-response functions (SIRFs) of $\Delta i_{t,3}$. It will be sufficient to cumulate these latter functions to obtain the SIRFs relative to the $i_{t,3}$ variable. In figure [3] are depicted the responses of $i_{t,3}$ to a one standard deviation inflation expectations shock, ε^π , and *ex-ante* real rate shock, ε^{rr} , respectively. The horizontal axis measures the number of quarters. Dotted lines represent 90% significance bands calculated by bootstrap simulations.

According to the imposed long-run identifying restriction, the $i_{t,3}$ response to the *ex-ante* real interest rate structural shock approaches to zero (neutrality condition).

In order to decompose the three-month nominal interest rate, $i_{t,3}$, into its two (unobservable) source of fluctuations, $\pi_{t,3}^e$ and $rr_{t,3}^e$, we use *ad hoc* formulation

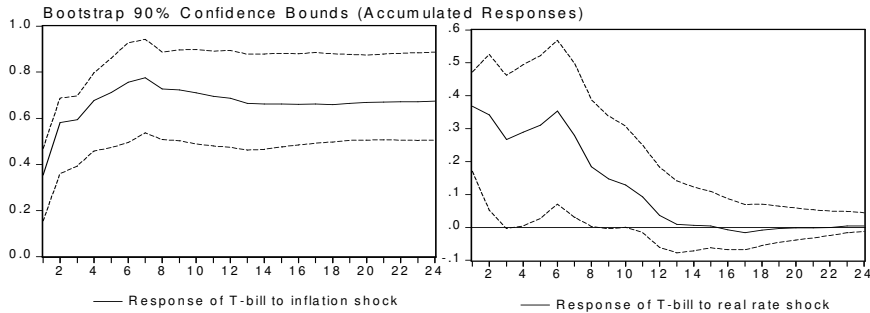


Figure 3: $i_{t,3}$ structural impulse-response functions

of equation (17) accounting for the estimated infrequent mean shifts in the real interest rate process:

$$\Delta i_{t,3} = \underbrace{\sum_{j=0}^N (D_{11,j} \varepsilon_{t-j}^{\pi})}_{\Delta \pi_{t,3}^e} + \underbrace{\sum_{j=0}^N (D_{12,j} \varepsilon_{t-j}^{rr})}_{\Delta \hat{r}_{t,3}} + \Delta \mu_t \quad (18)$$

where N indicates the number of periods chosen such that SIRFs approach to zero (i.e. $D_{11,N} \approx 0$ and $D_{12,N} \approx 0$) and $\Delta \mu_t$ term indicates changes in the estimated means of the *ex-post* real interest rate series, $rr_{t,3}$. After imposing expected inflation structural shocks to zero ($\varepsilon_{t-j}^{\pi} = 0$ for $j = 1, \dots, N$), estimates of the *ex-ante* real interest rates, $rr_{t,3}^e$, are obtained as follows:

$$\hat{r}_{t+h,3}^e = \mu_t + \sum_{k=0}^h \Delta \hat{r}_{t+k,3} \quad (\forall h > 0) \quad (19)$$

Finally, subtracting the estimated *ex-ante* real rate, $\hat{r}_{t,3}^e$, from the nominal T-bill rate, $i_{t,3}$, yields the corresponding expected inflation series, $\hat{\pi}_{t,3}^e$. Figure [4] depicts T-bill nominal interest rate series, $i_{t,3}$, the one-quarter ahead expected inflation, $\hat{\pi}_{t,3}^e$, and the *ex-ante* real interest rate series, $\hat{r}_{t,3}^e$.

A simple visual inspection of figure [4] suggests that only before the early-1980s movements of expected inflation rates are fully reflected in changes of T-bill rate. We find that over the period 1963Q4 to 1983Q4 the correlation between nominal interest rate and expected inflation component is near to one ($\hat{\beta} = 0.9$). Since the mid-1980s, the acquisition of credibility to low inflation caused the market's inflationary expectations to be more stable than in the past. The effect is that the volatility in the *ex-ante* real interest rates accounts for a relevant part of the volatility in the nominal interest rate and the Fisher effect appears to weaken. However, we expected this latter result. In fact, we remark that, given the uncertainty surrounding the estimation of the third break date,

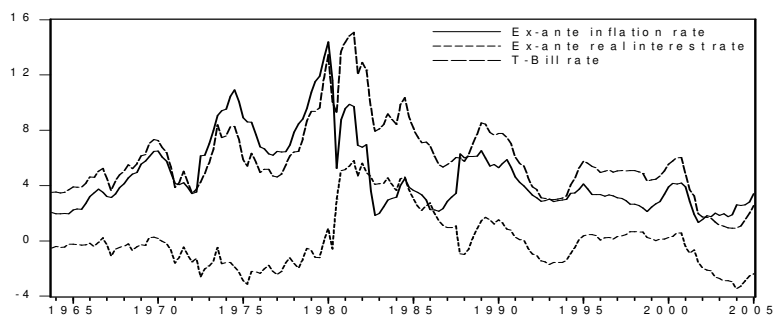


Figure 4: Decomposition of the T-bill nominal interest rate.

we have feared it was a signal of some degree of persistence in the real interest rate series. Therefore, SVAR estimates confirmed our suspicion. We conclude this section comparing in figure [5] our one-quarter ahead estimates of expected inflation to the corresponding one-quarter ahead inflation forecasts provided by the Survey of Professional Forecasters (SPF) for the common sample 1981Q3 to 2005Q1.

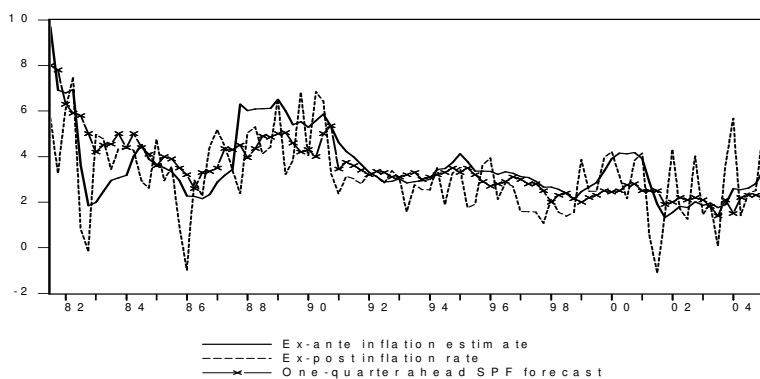


Figure 5: SPF series and our SVAR-based estimates of expected inflation. Dotted line denotes current inflation.

In general, both the two approaches provide similar information about the expected future inflation. However, despite the same trend, differences arise from the basic consideration that we have assumed that market's expectations are rational. Contrary, survey-based forecasts are not necessary consistent with the prices movements. As result, sharply changes in CPI index, and then in current inflation, determine our market-based estimates appear more volatile than survey-based forecasts.

5 Conclusions

In this paper, we have proposed a methodology which makes possible to achieve twofold objective: i) to extract both inflation expectations and *ex-ante* real interest rate components directly from nominal interest rates and ii) to test, implicitly, the magnitude of the so-called Fisher effect. We account for the puzzling evidence, discussed in literature, suggesting the apparently nonstationarity of real interest rates. Employing the Bai and Perron (2003a, 2003b) procedure, we test for the presence of infrequent deterministic structural breaks in the *ex-post* real interest rate series and find that this latter can be characterized by a four-regime model. Then, we use this latter result to impose a long-run identification condition in the SVAR model. After estimating the SVAR model, we obtain that only before the mid-1980s nominal interest rates and expected inflation rate are strongly cointegrated. The strong anti-inflationary policies, pursued by monetary authorities since 1980s, caused a downward adjustment of market's expectations of future inflation toward a more stable level with the consequence to reduce the degree of correlation between short-term interest rates and inflationary expectations.

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A Appendix: Bai & Perron testing procedure

Bai and Perron (2003a, 2003b) propose a sequential general to specific modeling strategy which allows to consistently determine the appropriate numbers of breaks present in time series. Under the general framework, they consider the following multiple linear regression with m breaks (which implies $m+1$ regimes):

$$y_t = x_t' \beta + z_t' \delta_j + u_t \quad (t = T_{j-1} + 1, \dots, T_j) \quad (20)$$

for $j = 1, \dots, m+1$ and where we use the convention that $T_0 = 0$ and $T_{m+1} = T$. In this model, y_t is the observed dependent variable ($rr_{t,3}$, in our case) at time t ; x_t ($p \times 1$) and z_t ($q \times 1$) are vectors of covariates and β and δ_j ($j = 1, \dots, m+1$) are the corresponding vectors of coefficients; u_t is the disturbance at time t . The indices (T_1, \dots, T_m) represent the break dates and are treated as unknown. The purpose is to estimate the unknown regression coefficients together with the breakpoints (T_1, \dots, T_m) when T observations on (y_t, x_t, z_t) are available. The core of testing method rests upon the principle of global minimizers of the sum of squared residuals of (20) and consistently determines the number of break points over all possible partitions (T_1, \dots, T_m) . More specifically, for each possible partition, $\{T_j\}$, the associated least-squares estimates of β and δ_j are obtained by minimizing the sum of squared residuals:

$$\sum_{i=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} \left[y_t - x_t' \beta - z_t' \delta_j \right]^2 \quad (21)$$

Substituting the resulting estimates, $\hat{\beta}(\{T_j\})$ and $\hat{\delta}(\{T_j\})$, into the objective function and denoting the resulting sum of squared residuals as $S_T(T_1, \dots, T_m)$, the estimated break points, $(\hat{T}_1, \dots, \hat{T}_m)$ are such that:

$$(\hat{T}_1, \dots, \hat{T}_m) = \arg \min_{T_1, \dots, T_m} S_T(T_1, \dots, T_m) \quad (22)$$

where the minimization is taken over all partition (T_1, \dots, T_m) such that $T_i - T_{i-1} \geq q$. Thus, $(\hat{T}_1, \dots, \hat{T}_m)$ are global minimizers of the objective function and the associated parameter estimates are $\hat{\beta} = \hat{\beta}(\{T_j\})$ and $\hat{\delta} = \hat{\delta}(\{T_j\})$.

In order to make inference on the number of breaks, Bai and Perron suggest to perform a set of tests different for power characteristics. Given the limiting distribution of these tests and critical values depend on the nature of regressors and the presence or absence of serial correlation in the residuals, we refer to Bai and Perron (1998, 2003a, 2003b) for further details on these aspects and we limit to summarize the testing procedure recommended by the authors for empirical applications.

In the first step, it is suggested to perform a $\text{Sup}F_T(k)$ test of no structural break ($m = 0$) versus the alternative hypothesis that there is an arbitrary fixed number of breaks ($m = k$). If these statistics are significative, this is a sign of the presence of at least one breakpoints.

In this case, Bai and Perron (2003a) suggest to decide the optimal number of break points performing a sequential examination of the $\sup F_T(l+1 | l)$ statistics. This latter tests the null hypothesis of l breaks against the alternative that an additional break exists. Ideally, this test is based on the difference between the sum of squared residuals obtained with l breaks and that obtained with $(l+1)$ breaks. We conclude for a rejection in favor of a model with $(l+1)$ breaks if the minimal value of the sum of squared residuals (over all segments where an additional break is included) is sufficiently smaller than the sum of squared residuals from the l break model. The asymptotic critical values are provided by Bai and Perron(2003b).