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# Working Paper A note on the Cournot-Bertrand profit differential: A reversal result in a differentiated model with wage bargaining

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#### Quaderni di Dipartimento

#### A Note on The Cournot-Betrand Profit Differential: A Reversal Result in a Differentiated Model with Wage Bargaining

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# A Note on The Cournot-Betrand Profit Differential: A Reversal Result in a Differentiated Model with Wage Bargaining

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May 2005

#### Abstract

We consider an oligopolistic model with product differentiation in which firm's costs are not given exogenously but are the result of a wage bargaining process between firms and local unions. Using a generalised version of the model of Lopez and Naylor (2004) we compare Cournot and Bertrand equilibria. We show that, contrary to standard results that Cournot equilibrium profits always exceed those under Bertrand competition, Bertrand profits can be higher than Cournot profits for some particular values of the parameters of the wage bargaining. This holds even if there are more than two firms in the economy. However, there is a critical level in the number of firms above which, independently on the values of the parameters of the model, the standard result conitnues to hold.

# 1 Introduction

Lopez and Naylor (2004) discuss the nature of competition in Bertrand and Cournot markets using the differentiated duopoly framework developed by Singh and Vives (1984). They extended the analysis of Singh and Vives (1984) introducing endogneous costs, by assuming that the wages paid by each firm is the outcome of a strategic bargain with its labor union. They show that the standard result that Cournot equilibrium profits exceed those under Bertrand competition-when the differentiated duopoly game is played in imperfect substitutes- is reversible. Whether equilibrium

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profits are higher under Cournot or Bertrand competition is shown to depend upon the nature of the labor unions' preferences and the distribution of bargaining power over the wage paid by each firm.

Häckner (2000) extends th3 analysis of Singh and Vives (1984) allowing for  $n \ge 2$ in the economy. He shows that the result concerning the dominance of Cournot over Bertrand profits is sensitive to the duopoly assumption. In our paper, we use a similar framework as Häckner (2000), but while Häckner (2000) extends the standard model horizontally through increasing the number of firms within the product market, our paper extends the analysis vertically by examining the consequences of introducing upstream suppliers to the downstream firms, as in Lopez and Naylor (2004). Thus, our paper can be thought as a generalization of Lopez and Naylor (2004), since we allow for  $n \ge 2$  firms, but also a generalization of Häckner (2000), since we endogenise the costs faced by firms through a wage bargaining process.

Our aim is to check whether the standard result on the ranking of Cournot and Bertrand equilibrium outcomes under a differentiated oligopoly are robust to the inclusion of a decentralised wage-bargaining game played by each firm and a firm specific labour union. Our model is a two-stage game, with wage bargaining played in the first stage, while in the second stage, we consider both Cournot and Bertrand solutions to the non-cooperative product market game. In our analysis we assume that there is symmetry across all the union-firm wage bargains as in Lopez and Naylor (2004). The paper is structured as follows. In Section 2 we consider Cournot competition, in Section 3 we consider the Bertrand competition case, in Section 3 we compare the two outcomes, finally, Section 4 concludes.

# 2 Cournot Equilibrium under Unionised Market

We consider a differentiated oligopoly market as in Häckner (2000), that is a generalisation of Singh and Vives (1984) and Qiu (1987). We analyse a two-stage noncooperative game in which firms produce imperfect substitutes goods. As in Lopez and Naylor (2004), in the first stage, each firms independently bargains over its wage with a local labour union, while in the second stage, each firm sets its output, give the wage outcome of stage one, to maximise profits. Our equilibrium notion is the standard sub-game perfect and the model is solved through backward induction. The utility function of the representative consumer is given by:<sup>1</sup>

$$U(\mathbf{q}) = \alpha \sum_{i=1}^{n} q_i - \frac{1}{2} \left( \sum_{i=1}^{n} q_i^2 + 2\gamma \sum_{k \neq i}^{n} q_i q_k \right) + I$$

The parameter  $\gamma \in [0, 1]$  represents the degree of substitutability between the products,  $\alpha > 0$ . If  $\gamma = 0$ , each firm has monopolistic market power, while if  $\gamma =$ 

<sup>&</sup>lt;sup>1</sup>See Häckner (2000).

1, the products are perfect substitutes.<sup>2</sup> The utility function is quadratic in the consumption of q-goods while is linear in the consumption of the other goods I. The budget constraint is simply:  $\sum_{i=1}^{n} q_i p_i + I \leq M$ , where M is the income level. The first-order conditions determining the optimal consumption of good i is

$$\frac{\partial U}{\partial q_i} = \alpha - q_i - \gamma \sum_{k \neq i}^n q_k - p_i = 0 \tag{1}$$

The firm i's inverse demand function can be solved for directly from Eq. (1):

$$p_i(q_i, \mathbf{q}_{-i}) = \alpha - q_i - \gamma \sum_{k \neq i}^n q_k.$$
 (2)

with  $p_i$  as the price of the product and  $\mathbf{q}_{-i} = \sum_{k \neq i}^n q_k$ . Production technology is very simple. Each firm produces one unit of the output by the use of one unit of labour. So the profit function of a typical firm in the industry is

$$\pi_i \left( \mathbf{q} \right) = \left( \alpha - q_i - \gamma \sum_{k \neq i}^n q_k - w_i \right) q_i \tag{3}$$

with  $w_i$  as the wage paid by firm i and  $\mathbf{q} = \sum_{i=1}^{n} q_i$ .

Firms set quantities to maximise profits,  $\pi_i$ , taking the other firms' quantities as given. Firm *i*'s reaction function is

$$q_i\left(\mathbf{q}_{-i}\right) = \frac{\alpha - w_i - \gamma \sum_{k \neq i}^n q_k}{2}.$$
(4)

As  $\gamma > 0$ , by assumption, the best-reply functions are downward sloping: Under the Cournot assumption, the product market game is played in strategic substitutes. Summing over all firms equations (4), and using the fact that  $\sum_{i=1}^{n} q_i = q_i + \sum_{k \neq i}^{n} q_k$ , we can solve for labour demand in equilibrium of firm *i*, given  $w_i$  and  $\mathbf{w}_{-i}$ :

$$q_{i}(\mathbf{w}_{i}) = \frac{(2-\gamma)\alpha - (\gamma(n-2)+2)w_{d,i} + \gamma \mathbf{w}_{d,-i}}{(2-\gamma)(\gamma(n-1)+2)}$$
(5)

with  $\mathbf{w}_{-i} = \sum_{k=1}^{n} w_k - w_i$ . The equilibrium output of a firm (and hence, equilibrium level of employment) is decreasing with its own wage, while it is increasing with the other firm's wage. The profit function under Cournot competition is then given by:

 $<sup>^{2}</sup>$ When goods are substitutes, the degree of substitutability could be interpreted in terms of horizontal product differentiation.

$$\pi_{i} = \left(\frac{(2-\gamma)\alpha - (\gamma(n-2)+2)w_{d,i} + \gamma \mathbf{w}_{d,-i}}{(2-\gamma)(\gamma(n-1)+2)}\right)^{2}$$
(6)

A firm's profit rises with the wage of the competitors in that industry, because the competition situation improves. Accompanied with this improvement is an increase in the equilibrium output. Furthermore, as we expect, firm's profit falls with firm's specific wage. Notice that when n = 2 we obtain the same profit function as in Lopez and Naylor (2004).

#### 2.1 The Wage Bargaining

We now solve for stage 1 of our game. Assume that, in stage 1, firm i bargains over the wage  $w_i$ , with a local union, union i, whose utility function is given by

$$u_i = (w_i - \overline{w})^{\theta} q_i^{1-\theta}, \tag{7}$$

where  $\theta$  denotes the relative strength of the union's preference for wages over employment and  $0 \le \theta \le 1$  and  $\overline{w}$  is the disagreement payoff available to the union (for example, the level of unemployment subsidies). This functional form is quite general and encompasses common assumptions such as rent-maximization, arising when  $\theta = \frac{1}{2}$  and total wage bill maximization when  $\theta = \frac{1}{2}$  and  $\overline{w} = 0$ . The general asymmetric Nash bargain over wages between union-firm pair *i* solves:

$$\Phi_{i} = \operatorname*{arg\,max}_{w_{i}} \left( u_{i} - \overline{u} \right)^{\beta} \left( \pi_{i} - \overline{\pi} \right)^{1-\beta}$$
(8)

where  $\beta$  is the union's Nash bargaining parameter and  $\beta \in [0, 1]$ . In the two-stage sequential game, the union and firm bargain over wages only: the firm is assumed to have the right-to-manage autonomy over employment. We rule out the special case in which  $\beta = \theta = 1$ .

Substituting (5), (6) and (7) into (8) yields

$$\Phi_{i} = \operatorname*{arg\,max}_{w_{i}} \left(w_{i} - \overline{w}\right)^{\beta\theta} \left(q_{i}\right)^{2-\beta(1+\theta)}$$
(9)

From (5) and (9), the first-order condition yields

$$w_i^C = \frac{\beta\theta\left[(2-\gamma)\alpha + \gamma \mathbf{w}_{-i}\right] + \left(\gamma\left(n-2\right) + 2\right)\left(2-\beta\left(1+\theta\right)\right)\overline{w}}{\left(\gamma\left(n-2\right) + 2\right)\left(2-\beta\right)}$$
(10)

which defines the sub-game perfect best-reply function in wages of union-firm pair i under the assumption of a non-cooperative Cournot-Nash equilibrium in the product market. From (10), the slope of union-firm pair i's best-reply function is given by

$$\frac{\partial w_i^C}{\partial \mathbf{w}_{-i}} = \frac{\beta \theta \gamma}{\left(\gamma \left(n-2\right)+2\right) \left(2-\beta\right)} \tag{11}$$

The slope of the best-reply wage function is positive for  $\gamma > 0, \theta > 0, \beta > 0$ , confirming that the labour market game is played in strategic complements. In a symmetric sub-game perfect equilibrium we have that  $w_i = \mathbf{w}_{-i}$  and hence, from (10), equilibrium wages are given by

$$w_{i}^{C} = \mathbf{w}_{-i}^{C} = \frac{\beta\theta \left(2 - \gamma\right)\alpha + (\gamma \left(n - 2\right) + 2)\left(2 - \beta \left(1 + \theta\right)\right)\overline{w}}{\left[(2 - \beta)\left(\gamma \left(n - 2\right) + 2\right)\right] - \beta\theta\gamma \left(n - 1\right)}$$
(12)

Note that  $w^C = \overline{w}$  if either  $\theta = 0$  or  $\beta = 0$ . From substitution of (12) in (6), we conclude that sub-game perfect equilibrium profits under Cournot competition are given by

$$\pi^{C} = \left(\frac{\left[\gamma \left(n-2\right)+2 \left(2-\beta \left(1+\theta\right)\right)\right] \left(\alpha-\overline{w}\right)}{\left(\gamma \left(n-1\right)+2\right) \left[\left(2-\beta\right) \left(\gamma \left(n-2\right)+2\right)-\beta \theta \gamma \left(n-1\right)\right]}\right)^{2}$$
(13)

### **3** Betrand Equilibrium under Unionised Market

In this section of the paper, we suppose that the product market game in stage 2 is characterized by price-setting behaviour by firms. Summing over all firms, Eq. (1) can be written as

$$n\alpha - \sum_{i=1}^{n} q_i - \gamma \left(n-1\right) \sum_{i=1}^{n} q_i - \sum_{i=1}^{n} p_i = 0$$
(14)

Eq. (1) and (14) then yield firm *i*'s demand function,

$$q_{i}(p_{i}, \mathbf{p}_{-i}) = \frac{(1-\gamma)\alpha - (\gamma(n-2)+1)p_{i} + \gamma \mathbf{p}_{-i}}{(1-\gamma)(\gamma(n-1)+1)}$$
(15)

where  $\mathbf{p}_{-i} = \sum_{k \neq i}^{n} p_k$ . Profits of firm *i* is are then given by

$$\pi_i \left( p_i, \mathbf{p}_{-i} \right) = \left( \frac{\left( 1 - \gamma \right) \alpha - \left( \gamma \left( n - 2 \right) + 1 \right) p_i + \gamma \mathbf{p}_{-i}}{\left( 1 - \gamma \right) \left( \gamma \left( n - 1 \right) + 1 \right)} \right) \left( p_i - w_i \right)$$
(16)

From (16), the first-order condition for profit-maximisation gives

$$p_{i}(\mathbf{p}_{-i}) = \frac{(1-\gamma)\alpha + (\gamma(n-2)+1)w_{i} + \gamma \mathbf{p}_{-i}}{2(\gamma(n-2)+1)}$$
(17)

Summing equations (17) over all firms, and using the fact that  $\sum_{i=1}^{n} p_i = p_i + \sum_{k \neq i}^{n} p_k$ , we obtain the price setting rule for each firm.

$$p_{i}(\mathbf{w}) = \frac{\left[ (1-\gamma) \left( \gamma \left( 2n-3 \right) +2 \right) \right] \alpha + \gamma \left( \gamma \left( n-2 \right) +1 \right) \mathbf{w}_{-i} + \left[ \left( \gamma \left( n-2 \right) +1 \right) \left( \gamma \left( n-2 \right) +2 \right) \right] w_{i}}{\left( \gamma \left( 2n-3 \right) +2 \right) \left( \gamma \left( n-3 \right) +2 \right)}$$
(18)

and hence, for  $\gamma > 0$ , the Betrand product market game is played in strategic complements. Hence, substituting (18) in (15) yields the equilibrium quantities and profits, given wages, under Bertrand competition:

$$q_{i} = \frac{\left(\gamma \left(n-2\right)+1\right) \left[\begin{array}{c} \left(1-\gamma\right) \left(\gamma \left(2n-3\right)+2\right) \alpha + \gamma \left(\gamma \left(n-2\right)+1\right) \mathbf{w}_{-i} \right) \\ -\left[\left(\gamma \left(n-2\right)+1\right) \left(\gamma \left(n-2\right)+2\right)-\gamma^{2} \left(n-1\right)\right] w_{i} \end{array}\right]}{\left(1-\gamma\right) \left(\gamma \left(n-1\right)+1\right) \left(\gamma \left(2n-3\right)+2\right) \left(\gamma \left(n-3\right)+2\right)}$$
(19)

$$\pi_{i} = \frac{(1-\gamma)\left(\gamma\left(n-1\right)+1\right)q_{i}^{2}}{(\gamma\left(n-2\right)+1)}$$
(20)

#### 3.1 The Wage Bargaining

As for the case of Cournot competition, the general asymmetric Nash bargain over wages between union-firm pair i solves:

$$\Phi_{i} = \operatorname*{arg\,max}_{w_{i}} \left( u_{i} - \overline{u} \right)^{\beta} \left( \pi_{i} - \overline{\pi} \right)^{1-\beta}$$
(21)

and as before,  $\beta$  is the union's Nash bargaining parameter and  $\beta \in [0, 1]$ . In the two-stage sequential game, the union and firm bargain over wages only: the firm is assumed to have the right-to-manage autonomy over employment. Again, we rule out the special case in which  $\beta = \theta = 1$ .

Substituting (7), (19) and (20) into (21) yields

$$\Phi_{i} = \arg\max_{w_{i}} \left(w_{i} - \overline{w}\right)^{\beta\theta} \left(q_{i}\right)^{2-\beta(1+\theta)} \left[\frac{(1-\gamma)\left(\gamma\left(n-1\right)+1\right)}{(\gamma\left(n-2\right)+1)}\right]^{1-\beta}$$
(22)

where  $\overline{w}$  is again the disagreement payoff. From (22), the first-order condition yields

$$w_{i}^{B} = \frac{\beta\theta \left[ (1-\gamma) \left( \gamma \left( 2n-3 \right) + 2 \right) \alpha + \gamma \left( \gamma \left( n-2 \right) + 1 \right) \mathbf{w}_{-i} \right] + \left[ (\gamma \left( n-2 \right) + 1 \right) \left( \gamma \left( n-2 \right) + 2 \right) - \gamma^{2} \left( n-1 \right) \right] \left( 2-\beta \left( 1+\theta \right) \right) \overline{w}}{\left[ (\gamma \left( n-2 \right) + 1 \right) \left( \gamma \left( n-2 \right) + 2 \right) - \gamma^{2} \left( n-1 \right) \right] \left( 2-\beta \right)}$$
(23)

which defines the sub-game perfect best-reply function in wages of union-firm pair i under the assumption of a non-cooperative Bertrand-Nash equilibrium in the product market. From (23), the slope of union-firm pair i's best-reply function is given by

$$\frac{\partial w_i^C}{\partial \mathbf{w}_{-i}} = \frac{\beta \theta \gamma \left(\gamma \left(n-2\right)+1\right)}{\left[\left(\gamma \left(n-2\right)+1\right) \left(\gamma \left(n-2\right)+2\right)-\gamma^2 \left(n-1\right)\right] \left(2-\beta\right)}$$
(24)

The slope of the best-reply wage function is positive for  $\gamma > 0, \theta > 0, \beta > 0$ , confirming that the labour market game is played in strategic complements. In symmetric sub-game perfect equilibrium,  $w_i = \mathbf{w}_{-i}$  and hence, from (23), equilibrium wages are given by

$$w_{i}^{B} = \mathbf{w}_{-i}^{B} = \frac{\left[\begin{array}{c} \beta\theta \left(1-\gamma\right) \left(\gamma \left(2n-3\right)+2\right) \alpha + \\ \left[\left(\gamma \left(n-2\right)+1\right) \left(\gamma \left(n-2\right)+2\right)-\gamma^{2} \left(n-1\right)\right] \left(2-\beta \left(1+\theta\right)\right) \overline{w}\end{array}\right]}{\left[\left(\gamma \left(n-2\right)+1\right) \left(\gamma \left(n-2\right)+2\right)-\gamma^{2} \left(n-1\right)\right] \left(2-\beta\right) \\ -\beta\theta\gamma \left(n-1\right) \left(\gamma \left(n-2\right)+1\right)}\right]$$
(25)

Note that  $w^B = \overline{w}$  if either  $\theta = 0$  or  $\beta = 0$ . From substitution of (25) in (20), we conclude that sub-game perfect equilibrium profit under Bertrand competition ais given by

$$\pi^{B} = \frac{\left[ \begin{array}{c} (\gamma (n-2)+1) \\ \left\{ \begin{array}{c} (1-\gamma) (\gamma (2n-3)+2) \alpha - \\ [(\gamma (n-2)+1) (\gamma (n-2)+2) - \gamma (n-1) (\gamma (n-1)+1)] \overline{w} \end{array} \right\}^{2} \right]}{\left[ \begin{array}{c} (1-\gamma) (\gamma (n-1)+1) \\ \left\{ \left[ \begin{array}{c} \beta \theta \left[ (\gamma (n-2)+1) (\gamma (n-2)+2) - \gamma (n-1) (\gamma (n-1)+1) \right] \\ [(\gamma (2n-3)+2) (\gamma (n-3)+2)] \end{array} \right] \right\}^{2} \right]}$$
(26)

## 4 The Cournot-Bertrand Profits Comparison

In this section we compare the Cournot profit function, given by equation (13), and the Bertrand profit function given by (26). We shall show which profit function is higher given some values of the parameters  $(\beta, \gamma, \theta, n)$ . In order to analyse the Cournot-Bertrand profit differential, we define the following ratio under a symmetric equilibrium:

$$\frac{(\gamma(n-2)+2)^2 (\gamma(n-1)+1)(2+\gamma(n-3))^2}{\pi^B} = \frac{\left[ ((\gamma(n-2)+1)(\gamma(n-2)+2)-\gamma^2(n-1))^2 (2-\beta)-\theta\beta\gamma(\gamma(n-2)+1) \right]}{(\gamma(n-1)+2)^2 ((2-\beta)(\gamma(n-2)+2)-\beta\theta\gamma(n-1))^2} (\gamma(n-2)+1)(1-\gamma) \left[ (\gamma(n-2)+1)(\gamma(n-2)+2)-\gamma^2(n-1) \right]^2} (27)$$

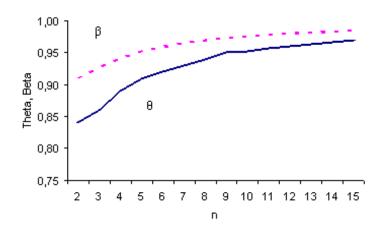


Figure 1:  $\beta$  and  $\theta$  as a function of n.

We want to show under which values of the parameters of the model, that ration is greater or less than 1. Notice that equation (27) does not depend on the disagreement payoff  $\overline{w}$  and on the intercept of the demand function  $\alpha$ .Lopez and Naylor (2004) showed that, with 2 firms, the ratio in (27) is less than 1 for high values of  $\theta$  and  $\beta$ . That means that in a differentiated duopoly with wage bargaining, in the sub-game perfect equilibrium, the Bertrand profits exceed Cournot when unions are both relatively powerful in the wage bargaining process and attach relatively high importance to the wages in their objective function.<sup>3</sup> Here we shall show if their result is still valid when there are more than two firms in the market. Following Lopez and Naylor (2004) we evaluate ratio (27) for  $\gamma = 0.5$ . We first define the critical values of  $\beta$  and  $\theta$  that can make the ratio (27) equal to 1. Consider first the case with  $\gamma = 0.5$ and  $\beta = 1$ , we find the values of  $\theta$  as a function of n, that can make  $\frac{\pi^C}{\pi^B} = 1$ . Then consider the case  $\gamma = 0.5$  and  $\theta = 1$ , we find the values of  $\beta$  as a function of n, that can make  $\frac{\pi^C}{\pi^B} = 1$ . Those relationships between  $\theta$ ,  $\beta$  and n are plotted in the following figure:

We can notice that the initial values of  $\beta = 0.91$  and  $\theta = 0.84$  for n = 2 are the ones considered by Lopez and Naylor (2004). The values of  $\beta$  and  $\theta$  described in the Figure above may be thought as the critical values (for a given n and  $\gamma = 0.5$ ) above which the ratio in 27) may fall under one. As we can see those critical values are increasing in the number of firms. In the following proposition we state the main

result about the Cournot-Bertrand profit differential in our model:

**Proposition 1** In the sub-game perfect equilibrium, for high values of  $\beta$  and  $\theta$ , the

<sup>&</sup>lt;sup>3</sup>Lopez and Naylor (2004) use numerical evaluation to find their result. In particular, they use  $\gamma = 0.5, \theta > 0.84$  and  $\beta > 0.91$ .

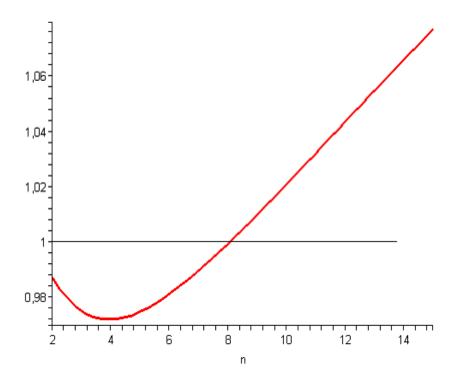


Figure 2: Relationship between ratio (27) and n.

profits under Bertrand is higher than the profits under Cournot competition even if n > 2. However, there is a critical value of  $n^* > 2$ , such that, for any value of the parameters  $(\beta, \theta \text{ and } \gamma)$  the profits under Cournot are always higher than profits under Bertrand competition.

The results in Proposition 1) are displayed in the following figure, that is obtained with the following parametrisation:  $\gamma = 0.5$ ,  $\beta = 0.98\theta$  and a = 0.98. In that figure is plotted the relationship between the ratio (27) and the number of firms. As we can notice, for *n* not too high but greater than 2, the ratio (27) is less than one. We can notice that in the figure the critical value of  $n^*$  is equal to 8, however, that critical value depends on the values of  $\beta$ ,  $\theta$  and  $\gamma$ .

Another important feature of the relationship between the ratio  $\frac{\pi^C}{\pi^B}$  and the number of firms is the role of product differentiation  $\gamma$ . We can see that higher is  $\gamma$  and higher is the possibility that the ratio  $\frac{\pi^C}{\pi^B}$  is less than the one, for n not too high. In the next figure, we show the relationship between  $\frac{\pi^C}{\pi^B}$  and  $\gamma$  for two different values of n. The figure is obtained using  $\beta = 0.98$  and  $\theta = 0.97$ , that are two values that can assure that ratio (27) can be less than 1 for some values of n.

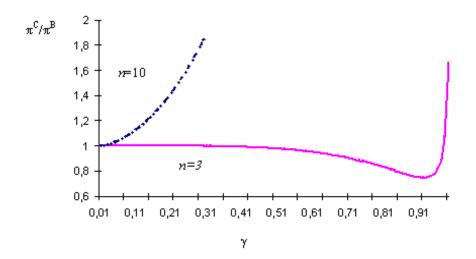


Figure 3: Relationship between ratio (27) and  $\gamma$ , for different values of n.

When n = 3, higher is  $\gamma$  and higher are profits under Bertrand respect to the ones under Cournot. When n = 10, there are no values of the parameters that can make the Bertrand's profits higher that Cournot's ones. Thus, there are two different elements that affects the Cournot-Bertrand profit differential in our model. On one hand, there is the effect of the wage bargaining parameters ( $\beta$  and  $\theta$ ) and product differentiation ( $\gamma$ ) on the level of competition. On the other hand there is the effect of the number of firms. When n is small enough, the effect of higher values of  $\beta$ ,  $\theta$ and  $\gamma$  is dominant, and Bertrand competition can lead to higher profits than Cournot competition. As n increases, the effect of the number of firms on the competition level becomes more relevant and thus, Bertrand competition becomes fiercer than Cournot competition, and so the profits under price setting behaviour become lower.

# 5 Conclusions

In this paper, we have considered an oligopolistic model with product differentiation in which firm's costs are not given exogenously but are the result of a wage bargaining process between firms and local unions. Our aim was to test the robustness of the standard result arising in duopoly model with product differentiation and exogenous costs that profits under Cournot competition is always higher than profits under Bertrand competition. Using a generalised version of the model of Lopez and Naylor (2004), we showed that if unions are sufficiently powerful and care about wage more than employment, then Bertrand profits exceed Cournot profits in the sub-game perfect equilibrium when goods are imperfect substitutes and firms bargain over costs. That result holds even if firms in the economy are more than two. Furthermore, higher is the level of product differentiation and higher are Betrand profits relative to Cournot profits if the number of firms is small enough. However, there is a critical level in the number of firms above which, independently on the values of the parameters of the model, Cournot profits are always higher than Bertrand profits.

There are obviously various directions for further work. For example, it may be interesting to consider the case of efficient bargaining where firms and unions bargain over wages and employment levels simultaneously.

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