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# Endogenous growth and changing sectoral composition in advanced economies

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## Endogenous growth and changing sectoral composition in advanced economies

## ABSTRACT

Despite the striking evidence of the changing sectoral composition in employment and output shares characterizing the growth process, structural change is usually disregarded in growth modeling. In contrast, we focus on how structural change can affect aggregate growth by presenting a two-sector model with a "progressive" industry ("manufacturing"), which exhibits endogenous technological progress and produce both for consumption and for investment, and a technologically "stagnant" industry ("services"), which produces only for consumption. Within this framework, we show under what conditions on preferences perpetual growth can be generated. In particular, the paper demonstrates that positive long-term growth is possible even if what households spend on services tends to increase more than proportionally than their total consumption expenditure, namely when preferences are non-homothetic. This is at odds with previous literature arguing that Baumol's "asymptotic stagnancy" applies when the stagnant industries supply final products. Moreover, the paper does not limit its attention to the balanced growth path: numerical examples illustrate how the transition path displays the regularities which appear to characterize the structural dynamics in advanced economies.

KEY WORDS: Balanced growth, structural change, non-homothetic preferences.

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#### 1 INTRODUCTION

There is a striking evidence that dramatic changes in the sectoral output and employment shares occur during any development and growth episode. In particular, a sharp increase in service-sector employment share to the detriment of manufacturing has taken place in industrialized economies during the last fifty years. This notwithstanding, growth theoreticians usually treats the economy as if its sectoral composition were constant for very long periods. In general, this literature does not provide an adequate framework for explaining structural change and its implication for aggregate growth. In contrast, this paper aims at modeling the changing sectoral composition that characterizes the economic dynamics of the advanced countries by developing a two-sector endogenous-growth framework.

This model has two main features that are crucial for explaining the structural change which is peculiar to the growth process in the advanced economies. On the supply side, we assume that there is a "progressive" industry ("manufacturing"), which exhibits endogenous technological progress and produce both for consumption and for investment, and a technologically "stagnant" industry ("services"), which produces only for consumption.<sup>1</sup> The stagnant industry uses an input (physical capital) that is produced by the progressive industry, thus benefiting indirectly by the possible improvements in total factor productivity (TFP) achieved in the latter. On the demand side, we consider both homothetic and non-homothetic consumers' preferences, so as to analyze the consequences for the growth process of different hypotheses on the evolution of final demand. This formal set-up is especially suited to study how aggregate growth is affected by the interaction between technological progress, which is generated endogenously and has a stronger positive impact on the manufacturing sector, and the demand for services, which tends to increase— other things being equal—more than proportionally than total expenditure in consumption. To our knowledge, indeed, no other growth model—even among those recent theoretical contributions dealing with sectoral changes (see Echevarria, 1997; Laitner, 2000; Kongsamut et al. 2001)—captures the joint effect of

<sup>&</sup>lt;sup>1</sup> The "progressive" sector can be identified with manufacturing sector, with the possible inclusion of some service branches (Transport, Communications, Financial services)., which have experienced radical changes in their production processes because of the massive introduction of information and communication technologies (ICT). One can include in the "stagnant" sector the remaining branches of services. A distinction along similar lines was proposed (but at the early stage of the ICT revolution) by Baumol (1967) and Baumol et al. (1985).

non-homothetic preferences and endogenous technological progress having an uneven impact on different industries' TFP.

An important result of our paper is that positive long-term growth is possible even if what households spend on services tends to increase more than proportionally than their total consumption expenditure, namely when their preferences are non-homothetic. Even in this case, indeed, the model shows that asymptotic stagnancy can occur only if an excessively large portion of what households spend on consumption is devoted to the service. This is at odds with previous literature arguing that Baumol's "asymptotic stagnancy" applies when the stagnant industries supply final goods or services.<sup>2</sup>

However, one may claim that the study of the asymptotic properties of such economic system can provide useful insights on the direction towards which it will proceed as long as preferences and technologies are not subject to major changes, but that for any practical purpose what really matters is its behavior along the transition path. In this spirit, we present two numerical examples where we show that starting from an initial employment share of the manufacturing sector in overall employment greater than its long-run equilibrium share, the gradual shift of employment shares towards the service sector is accompanied by rates of growth of output and capital stock that are higher in the service sector than in manufacturing. Moreover, along this transition path, the relative price of the service is growing and the economy's GDP tend to grow at a higher rate than along the balanced growth path of the economy: the gradual shift of labor towards the service sector is accompanied by a decline in the aggregate rate of growth. In other words, the pattern resulting from these numerical examples seems to be consistent with the stylized facts both in the case where preferences are assumed to be homothetic and in the case with non-homothetic preferences, although the latter case appears to be more relevant in the light of empirical estimates showing an income elasticity of demand greater than one for the services and lower than one for the manufactured goods.

This paper is organized as it fallows. *Section 2* presents the main stylized facts about structural change and briefly reviews some theoretical and empirical contributions. *Section 3* presents the model. *Section 4* 

 $<sup>^{2}</sup>$  Oulton (2001) shows that Baumol's stagnationist conclusion does not apply when the stagnant industries supply intermediate products.

characterizes the equilibrium path of the economy. The case with homothetic preferences is analyzed in *section 5*, while the case with non-homothetic preferences is treated in *section 6*. *Section 7* concludes.

#### 2 MOTIVATIONS

#### Stylized facts

We present some stylized facts that may help understanding the changes in sectoral composition that have occurred in the advanced countries, together with their implications for aggregate growth.

- 1. It is typically observed in industrialized economies a first phase of increase in manufacturing and services shares to the detriment of agriculture, followed by a second phase characterized by the sharp increase in the services share in overall employment to the detriment of manufacturing. Looking at Table 1 in the Appendix, we see that starting, at the beginning of last century, from an employment share of 27.1% in France, 16% in Italy, 26.2% in Germany, 43.1% in the UK and 31.4% in the US, services have reached in 1990 respectively a share of 64.6% in France, 59,7% in Italy, 58.7% in Germany, and about 70% in the US and in the UK. Among services, the initially weightiest and then ever growing activities have been the "wholesale and retail trade, restaurants and hotels" and the "community, social and personal services", with a share—respectively--of 20% and 30% in overall employment. The activities showing the sharpest increase, starting from very low levels, are the "finance, insurance, real estate and business services", in contrast with the quasi-constant levels of "transport, storage and communications".<sup>3</sup>
- 2. As pointed out by some recent papers (Easterly, 1999; O' Mahony and Van Ark, 2003), the aggregate income and productivity growth rates are not constant. The growth rates of GDP and productivity have decreased since the second half of the 1970s in industrialized economies, compared with their values in the previous decades. As Table 4 in the Appendix shows, GDP growth has decreased, in most of the industrialized economies, from yearly rates well above 4% in the

<sup>&</sup>lt;sup>3</sup> See Table 7 and 8 in Borzaga and Villa (1999). As for some recent evidence about structural dynamics of employment, see also Castells and Aoyama (1994), Wieczorek (1995), Oecd (1994, 2000), Martinelli and Gadrey (2000), Schettkat and ten Raa (2001). In 1998, the services share in overall employment reached 70.7% in France, 64.1% in Italy, 62.1% in Germany, 71% in the UK and 73.8% in the US (see Table 3.2 in Oecd, 2000).

decades before 1970, to rates about 2% in the post-1970 period. The trend of aggregate productivity appears to be similar.<sup>4</sup> In the US both the rates have risen since the mid 90's, while there is no evidence of a similar recovery in the EU countries (see for productivity first line of Table 5 in the Appendix).<sup>5</sup>

- 3. The services share in total expenditure remains constant or rises slightly as income grows, when expressed in real terms (constant prices), while it is sharply increasing when measured in nominal terms (current prices). Table 2 in the Appendix shows the evolution of the service share in total GDP over the period 1957-1978, and the evolution of the services share in total expenditure<sup>6</sup> (in real and nominal terms), for the US, the UK and France: in real terms, the shares in total GDP and in total expenditure remained constant, except for the US where they both increase, whereas in nominal terms all the three countries exhibit a massive increase.<sup>7</sup> In more recent analysis focused on the US (see Appelbaum and Schettkat, 1999; Mattey, 2001), it is presented evidence of increasing relative prices of services, together with an increasing share of services in nominal product, while the share of services in real output is shown to be more or less constant until the mid-1970s and to be increasing since then.<sup>8</sup>
- 4. The relative price of services increases with income. As mentioned before, the services share is growing more in nominal terms than in real ones. This is explained by the positive correlation

<sup>6</sup> The difference is given by the final expenditures of government, included in the service share in total GDP, excluded in the share of services in total expenditure.

<sup>7</sup> In 1998, the services share in nominal value added for the three countries is about 70% (Oecd, 2000, in particular Table 3.8). The same evidence is provided by Summers (1985) in his cross-sectional analysis carried out over 39 countries in 1975 on the relationship between real (inflation adjusted) per capita GDP and share of services in total real GDP expenditure and in total nominal GDP expenditure. The results obtained by Summers corroborate what Fuchs (1968), Baumol (1967, 2001) and Kuznets (1971) argued: the share of services in nominal output increases more than the share of services in real output (see also Echevarria, 1998, and—for specific countries—Inman, 1985).

<sup>8</sup> For contrasting results, see Falvey and Gemmel (1996).

<sup>&</sup>lt;sup>4</sup> For a general discussion on these trends, see OECD (1994a), OECD (1994b), OECD (1995, in particular Table 2.3 and 2.4).

<sup>&</sup>lt;sup>5</sup> See also Table 2 and Table 4 in Mc Guckin and Van Ark (2003); Table I.1 and Table I.3 in Oecd (2002).

between the price of services and GDP, as come out from the cross-sectional and longitudinal analysis presented by Kravis et al. (1983) and from the cross-sectional evidence presented by Summers (1985). Moreover, it is also confirmed in the more recent analysis cited in the point above.<sup>9</sup>

- 5. Services are more labor intensive than manufacturing. The capital intensity (capital per hour worked) in 2000 is lower in most of the service sectors, with the exceptions of "transport and communication" and of "financial services" in some countries (the US, and slightly, France and Netherlands), as O'Mahony and Van Ark (2003) have pointed out.<sup>10</sup> This is true despite the fact that the pace of capital accumulation appears to be faster in services than in manufacturing. Looking at Table 3 in the Appendix, one can see that before 1973 trends in capital accumulation were similar in industry and services. After 1973 they diverged sharply in a number of countries, with capital accumulation slackening in manufacturing and being maintained in services (see Glyn, 1997, 2001).
- 6. The income elasticity of demand is estimated to be above unity for most of the service branches and for services as an aggregate. The same elasticity is sharply below unity for manufacturing branches and for the whole sector (see Curtis and Murthy, 1998; Rowthorn and Ramaswamy, 1999; Inman in Oecd, 2000; Möller, 2001).<sup>11</sup>
- 7. The recent empirical evidence reaches a general consensus in pointing out the negative productivity differential of most of the service branches compared with manufacturing ones (see Kravis et al., 1983; Summers, 1985; Sakuray, 1995; Rowthorn and Ramaswamy, 1999; Inman in Oecd, 2000).<sup>12</sup> We can see from Table 5 in the Appendix that the growth rates of productivity in services are lower

<sup>&</sup>lt;sup>9</sup> For the correlation between prices and productivity, see Rowthorn and Ramaswami (1999). According to them, the relative price of services increases because their productivity grows more slowly.

<sup>&</sup>lt;sup>10</sup> See in particular Table II.6. A similar evidence is reported also by Mohnen and ten Raa (2001) for Canada.

<sup>&</sup>lt;sup>11</sup> Kraving et al. (1983) and Summers (1985) find income elasticity of demand for services slightly different from unity for the sector as a whole (in contrast they are far above unity for some branches). Falvey and Gemmel (1996), extending Summers (1985), reach similar conclusions. In the same papers, services as a whole appear to be highly price inelastic. Price rigidities are also found by Curtis and Murthy (1998), Inman in Oecd (2000) and Möller (2001), although the evidence on the existence of price rigidities appears to be less univocal than on income elasticity.

<sup>&</sup>lt;sup>12</sup> For specific countries see also Inman (1985), Mohnen and ten Raa (2001); while for comparisons among industrialized countries of sectoral productivity in the long run (1913-1987), see Maddison (1991, Table 5.13).

than in manufacturing with the exception of branches like "Transport and Communications" and "Finance".<sup>13</sup>

## Theoretical literature

As we already pointed out, despite the stylized facts presented above, growth modeling has not generally focused on structural change.<sup>14</sup> The long-run dynamics is generally analysed along the balanced growth path, where all the relevant variables grow at constant rates and the system is not supposed to change in its sectoral composition. The omission of structural change and the priority given to balanced growth analysis probably depends on the acceptance of the so-called "Kaldor facts"<sup>15</sup> as a good description of the behavior of aggregate variables in the long run by most of the growth literature (including endogenous growth theory).

Some recent papers (Meckl, 2000; Foellmi and Zweimüller, 2002) seek to reconcile the Kaldor facts (in particular, steady aggregate growth) with the existence of structural change. They derive a dynamic equilibrium characterized by continuous structural change: in both these models, the driving force behind structural change is the difference in the income elasticity of demand across sectors, while technological progress is uniform across the sectors producing the final products. In other words, the changing sectoral composition of the economy originates only from non-homothetic preferences. This approach makes the structural change neutral with respect to aggregate growth, but--at the same time--it ignores another fundamental force driving structural dynamics, namely the fact that sectors differ in their permeability to technological progress. Under this respect, our model differ from these papers because of our attempt of accounting for both forces underlying structural change.

<sup>13</sup> Although the existence of a productivity bias in favor of manufacturing is widely accepted, it is not evident whether this differential will be preserved in the future, when ICT will increasingly affect the services sector. In this respect, a consequence of the application of ICT is the possibility of separating production and consumption for many service activities, increasing their "stockability" and "transferability". For evidence and discussions on the impact of ICT on sectoral productivities, see Petit and Soete (1997), Mattey (2001), Triplett (2003), O' Mahony and van Ark (2003).

<sup>14</sup> Among the noteable exceptions, one can cite Pasinetti (1984), Reati (1998), Metcalfe (2000), Montobbio (2001), Aoki and Yoshikawa (2001).

<sup>15</sup> That is, *per capita* output grows at a rate that is roughly constant, the capital-output ratio is roughly constant, the real rate of return to capital is roughly constant, the share of labour and capital in national income are roughly constant.

In contrast with this approach, some other recent papers (Echevarria, 1998; Kongsamut et al., 2001) argue that the long-run economic dynamics has to be analyzed out of the balanced growth path. In particular, Kongsamut et al. find a knife-edge condition on parameters which must be satisfied for a generalized balanced growth path (GBGP) to exist, characterizing it as a path which features a constant real interest rate but a time-varying allocation of inputs across sectors. It is worth noting that along this GBGP the relative prices and the growth rates of GDP, productivity and final expenditure are time varying. Also in this model—as in those previously discussed--structural change is driven by non-homothetic preferences, but the case of sector-specific (exogenous) technological progress is taken into account. Moreover, structural change vanishes asymptotically as the economy approaches its balanced growth path. Therefore, the possibility of analyzing the changing sectoral composition of the economy relies on the existence of the GBGP, which in its turn depends on very particular combinations of values of the parameters entering both the utility and the production. In contrast, our approach introduces endogenous technological progress and does not hinges on special assumptions on the parameters values in order to deal with the changing sectoral composition of the economy.

Finally, one should consider Oulton (2001), where it is shown that Baumol's stagnationist argument does not hold when services are used as intermediate products. This approach is consistent with the literature identifying the cause of services expansion in the increasing demand for services as intermediate products (see Stanback, 1979; Gershuny and Miles, 1983; Momigliano and Siniscalco, 1986; Petit and Soete, 1997; Klodt, 1997). However, consensus has not yet been reached about the relative weight of the use of services as intermediate products in the overall growth of the services sector (see Russo and Schettkat, 2001),<sup>16</sup> while it is widely recognized the importance of physical capital as an input in most service industries, which is a feature that is captured by our model.

#### 3 THE MODEL

We consider an economy in discrete time with an infinite time horizon. This economy is assumed to have two sectors of market activity (manufacturing and services). The manufactured good, which is the numéraire of the system (its price is set to be one), can be both consumed and used for investment purposes.

<sup>&</sup>lt;sup>16</sup> For a recent review of the literature of the shift to services, see Schettkat and Yocarini (2003).

The service can be only consumed. Moreover, consistently with the Baumol's distinction between "progressive" and "stagnant" sectors, we assume that there is (endogenous) technological progress only in the manufacturing sector. Finally, all markets are assumed to be perfectly competitive.

### Households

For simplicity and without loss of generality, it is assumed that the population is constant and that each household contains one adult working member of the current generation. Thus, there is a fixed and large number (normalized to be one) of identical adults who take account of the welfare and resources of their actual and perspective descendants. Indeed, following Barro and Sala-i-Martin (1995), this intergenerational interaction is modeled by imaging that the current generation maximizes utility and incorporates a budget constraint over an infinite future. That is, although individuals have finite lives, the model considers immortal extended families ("dynasties").<sup>17</sup> The current adults expect the size of their extended family to remain constant, since expectations are rational (in the sense that they are consistent with the true processes followed by the relevant variables). In this framework in which there is no source of random disturbances, this implies perfect foresight.

Again for simplicity and without loss of generality, it is assumed that all households--being the firms' owners--are entitled to receive an equal share of the firms' net profits and that bequests are accidental.<sup>18</sup>

Households decide in each t what fraction of their labor income and gross returns on wealth to spend on consumption rather than on buying corporate bonds. Simultaneously, they decide how to allocate their consumption expenditure over the manufactured good and the service. Hence, the representative household's problem amounts to deciding a contingency plan for  $C_{Mt}$ ,  $C_{St}$  and  $B_{t+1}$  in order to maximize:

<sup>&</sup>lt;sup>17</sup> As Barro and Sala-i-Martin (1995, p. 60) point out, "this setting is appropriate if altruistic parents provide transfers to their children, who give in turn to their children, and so on. The immortal family corresponds to finite-lived indiiduals who are connected via a pattern of operative intergenerational transfers that are based on altruism".

<sup>&</sup>lt;sup>18</sup> In other words, it is ruled out the existence of actuarially fair annuities paid to the living investors by a financial institution collecting their wealth as they die: the wealth of someone who dies is inherited by some newly born individual.

$$\sum_{s=t}^{\infty} \theta^{s-t} C_{Ms}^{\eta} (C_{Ss} + \varepsilon)^{\gamma}, \ 0 \le \theta \le 1, 0 \le \eta \le 1, 0 \le \gamma \le 1, \varepsilon \ge 0,$$
(1)

subject to

$$B_{t+1} + C_{Mt} + P_t C_{St} \le W_t + (1+r_t)B_t + \pi_{Mt} + \pi_{St}, B_0 \text{ given},$$
(2)

where  $C_{Mt}$  and  $C_{St}$  are, respectively, the manufactured good and the service consumed by the representative household in period t,  $B_t$  are corporate bonds with maturity in period t and issued in t-1,  $\theta$  is a timepreference parameter,  $\varepsilon$  can be interpreted as the amount of service that is produced at home,  $P_t$  is the price of the service (the units of manufactured good that are necessary to buy one unit of service),  $W_t$  is the wage rate (the quantity of labor supplied by each household is assumed to be fixed and set to be one),  $r_t$  is the oneperiod market rate of interest, and  $\pi_{Mt}$  and  $\pi_{St}$  are the net profits generated in period t, respectively, by the manufacturing firms and the service-producing firms. It is worth to note that in the special case where  $\varepsilon=0$ the period-utility function is Cobb-Douglas, while for  $\varepsilon>0$  preferences are not homothetic: in the latter case, the elasticity of the demand for the service with respect to the household's consumption expenditure is more than unitary, while the elasticity of the demand for the manufactured good with respect to the household's consumption expenditure is less than unitary.

#### Manufacturing firms

The manufactured good is denoted by  $Y_{Mt}$  and is produced by a large number (normalized to be one) of identical firms according to the technology

$$Y_{Mt} = A_t L_{Mt}^{\alpha} K_{Mt}^{1-\alpha}, \ 0 < \alpha < 1,$$
(3)

where  $A_t$  is a variable measuring the state of technology,  $K_{Mt}$  is the capital installed in the manufacturing sector (capital can be interpreted in a broad sense, inclusive of all reproducible assets) and  $L_{Mt}$  is labor employed in the manufacturing sector. It is assumed that  $A_t$  is a positive function of the stock of capital existing in the manufacturing sector:  $A_t = K_{Mt}^{\alpha}$ .<sup>19</sup> Furthermore, consistently with Frankel (1962), it is supposed that although  $A_t$  is endogenous to the economy, each firm takes it as given, since a single firm

<sup>&</sup>lt;sup>19</sup> Consistently with this formal set-up, one can interpret technological progress as labor augmenting.

would only internalize a negligible amount of the effect that its own investment decisions have on the aggregate stock of capital.

The period net profits  $\pi_{Mt}$  of a manufacturing firm are given by:

$$\pi_{Mt} = Y_{Mt} - W_t L_{Mt} - (1 + r_t) B_{Mt} , \qquad (4)$$

where  $B_{Mt}$  are the bonds with maturity in period t and issued by a manufacturing firm in t-1 to finance its investment expenditure in that period.

#### Service-producing firms

The output of a service-producing firm is denoted by  $Y_{St}$  and is produced by a large number (normalized to be one) of identical firms according to the technology

$$Y_{St} = L_{St}^{\beta} K_{St}^{1-\beta}, \ 0 < \beta < 1,$$
(5)

where K<sub>St</sub> is the capital installed in the service sector and L<sub>St</sub> is labor employed in the service sector.

The period net profits  $\pi_{St}$  of a service-producing firm are given by:

$$\pi_{\rm St} = P_{\rm t} Y_{\rm St} - W_{\rm t} L_{\rm St} - (1 + r_{\rm t}) B_{\rm St} , \qquad (6)$$

where  $B_{St}$  are the bonds with maturity in period t and issued by a service-producing firm in t-1 to finance its investment expenditure in that period.

## Investment

The process of installing new capital and adapting the existing production facilities to the new machinery and equipment reduces the manufactured good available for consumption purposes and for adding to the stock of capital. One may think of this adjustment cost indifferently as if the producers of the manufactured good must divert resources from production in order to assist the capital users in installing the new capital, or as if some manufactured good is used up in the process of installing the new capital. Since firms finance their investment costs  $c(I_{it},K_{it})$  by issuing debt, one has:

$$c(I_{it},K_{it})=I_{it}+\frac{I_{it}^2}{K_{it}}=B_{it+1}, i=M,S,$$
 (7)

where investment costs are assumed to be the sum of gross investment  $I_{it}$  and adjustment costs, that are a quadratic function of  $I_{it}$  and a decreasing function of  $K_{it}$ .

The capital stock installed in each sector evolves according to

$$K_{it+1} = I_{it} + (1-\delta)K_{it}, 0 \le \delta \le 1, K_{i0} \text{ given, } i=M,S,$$
 (8)

where  $\delta$  is a capital depreciation parameter.

## Firms' objective

In t, a firm chooses  $\{L_{is}\}_{s=t}^{\infty}$  and  $\{I_{is}\}_{s=t}^{\infty}$  in order to maximize its discounted sequence of net profits:

$$\sum_{s=t}^{\infty} \frac{\pi_{is}}{\prod_{v=t+1}^{s} (1+r_v)}, \quad i=M,S,$$
(9)

subject to (5), (6) and (7), where 
$$\prod_{v=t+1}^{t} (1+r_v) = 1$$
.

Markets equilibrium

Equilibrium in the product markets requires, respectively,

$$Y_{Mt} = C_{Mt} + I_{Mt} + \frac{I_{Mt}^2}{K_{Mt}} + I_{St} + \frac{I_{St}^2}{K_{St}}$$
(10)

and

$$Y_{St} = C_{St}.$$
 (11)

Equilibrium in the labor market requires

$$1 = L_{Mt} + L_{St}.$$
 (12)

Equilibrium in the asset market requires

$$B_{Mt} + B_{St} = B_t.$$
(13)

## 4 CHARACTERIZATION OF THE GENERAL EQUILIBRIUM PATH

## Households' optimal behavior

One can solve the intertemporal problem of the representative household by maximizing

$$\sum_{s=t}^{\infty} \theta^{s-t} \left\{ C_{Ms}^{\eta} (C_{Ss} + \varepsilon)^{\gamma} + \chi_s [W_s + (1+r_s)B_s + \pi_{Ms} + \pi_{Ss} - B_{s+1} - C_{Ms} - P_s C_{Ss}] \right\} \text{ with respect to } C_{Mt}, C_{St}, B_{t+1} = C_{Ms} - C_{Ms$$

and  $\chi_t,$  and then by eliminating the multiplier  $\chi_t,$  thus obtaining:

$$C_{Mt} = \frac{\eta P_t}{\gamma} (C_{St} + \varepsilon)$$
(14)

and

$$\theta^{-1} \left(\frac{P_{t+1}}{P_t}\right)^{1-\eta} \left(\frac{C_{St+1} + \varepsilon}{C_{St} + \varepsilon}\right)^{1-\eta-\gamma} = 1 + r_{t+1}.$$
(15)

Therefore, along an optimal path a household must satisfy (14), (15) and the transversality condition

$$\lim_{t \to \infty} \theta^{t} B_{t} \eta \left( \frac{\eta P_{t}}{\gamma} \right)^{\eta - 1} (C_{St} + \varepsilon)^{\gamma + \eta - 1} = 0.$$
 (16)

## Manufacturing firms' optimal behavior

The manufacturing firms' optimality condition with respect to the choice of the labor input is

$$\alpha A_t K_{Mt}^{1-\alpha} L_{Mt}^{\alpha-1} = W_t, \qquad (17)$$

thus implying that employment in the manufacturing sector is:

$$L_{Mt} = K_{Mt} \left(\frac{\alpha A_t}{W_t}\right)^{1/(1-\alpha)}.$$
(18)

By using (3), (7) and (18), one can rewrite (4) as

$$\pi_{\rm Mt} = (1 - \alpha) K_{\rm Mt} \left[ A_{\rm t} \left( \frac{\alpha}{W_{\rm t}} \right)^{\alpha} \right]^{\frac{1}{(1 - \alpha)}} - (1 + r_{\rm t}) \left( I_{\rm Mt-1} + \frac{I_{\rm Mt-1}^2}{K_{\rm Mt-1}} \right).$$
 Hence, one can solve the intertemporal problem of

the representative firm by maximizing

$$\sum_{s=t}^{\infty} \frac{(1-\alpha)K_{Ms} \left[A_s \left(\frac{\alpha}{Ws}\right)^{\alpha}\right]^{\frac{1}{(1-\alpha)}} - (1+r_s) \left(I_{Ms-t} + \frac{I_{Ms-1}^2}{K_{Ms-1}}\right) + \lambda_{Ms} [I_{Ms} + (1-\delta)K_{Ms} - K_{Ms+1}]}{\prod_{v=t+1}^{s} (1+r_v)}$$
 with respect to  $I_{Mt}$ ,

 $K_{Mt+1}$  and  $\lambda_{Mt},$  and then by eliminating the multiplier  $\lambda_{Mt},$  thus obtaining (8) and

$$\frac{(1-\alpha)\left[A_{t+1}\left(\frac{\alpha}{W_{t+1}}\right)^{\alpha}\right]^{\frac{1}{(1-\alpha)}} + X_{Mt+1}^{2} + (1-\delta)(1+2X_{Mt+1})}{1+2X_{Mt}} = 1 + r_{t+1}, \quad X_{Mt} \equiv \frac{I_{Mt}}{K_{Mt}}.$$
(19)

Therefore, along an optimal path a manufacturing firm must satisfy (8), (19) and the transversality condition

$$\lim_{s \to \infty} \frac{(1+2X_{Ms})K_{Ms}}{\prod_{v=t+1}^{s} (1+r_v)} = 0.$$
 (20)

## Service-producing firms' optimal behavior

The service-producing firms' optimality condition with respect to the choice of the labor input is

$$\beta \mathsf{P}_{\mathsf{t}} \mathsf{K}_{\mathsf{St}}^{\mathsf{1}-\beta} \mathsf{L}_{\mathsf{St}}^{\beta-\mathsf{1}} = \mathsf{W}_{\mathsf{t}} \,, \tag{21}$$

thus implying that employment in the manufacturing sector is:

$$L_{St} = K_{St} \left(\frac{\beta P_t}{W_t}\right)^{1/(1-\beta)}.$$
(22)

By using (5), (7) and (22), one can rewrite (6) as

$$\pi_{\text{St}} = (1 - \beta) K_{\text{St}} \left[ P_{\text{t}} \left( \frac{\beta}{W_{\text{t}}} \right)^{\beta} \right]^{\frac{1}{(1 - \beta)}} - (1 + r_{\text{t}}) \left( I_{\text{St-1}} + \frac{I_{\text{St-1}}^2}{K_{\text{St-1}}} \right). \text{ Hence, one can solve the intertemporal problem of}$$

the representative firm by maximizing

$$\sum_{s=t}^{\infty} \frac{(1-\beta)K_{Ss} \left[ P_s \left(\frac{\beta}{Ws}\right)^{\beta} \right]^{\frac{1}{(1-\beta)}} - (1+r_s) \left( I_{Ss-t} + \frac{I_{Ss-1}^2}{K_{Ss-1}} \right) + \lambda_{Ss} [I_{Ss} + (1-\delta)K_{Ss} - K_{Ss+1}]}{\prod_{v=t+1}^{s} (1+r_v)}$$
 with respect to  $I_{St}$ ,  $K_{St+1}$ 

and  $\lambda_{St}$ , and then by eliminating the multiplier  $\lambda_{St}$ , thus obtaining (8) and

$$\frac{(1-\beta)\left[P_{t+1}\left(\frac{\beta}{W_{t+1}}\right)^{\beta}\right]^{\frac{1}{(1-\beta)}} + X_{St+1}^{2} + (1-\delta)(1+2X_{St+1})}{1+2X_{St}} = 1+r_{t+1}, \quad X_{St} \equiv \frac{I_{St}}{K_{St}} \quad .$$
(23)

Therefore, along an optimal path a service-producing firm must satisfy (8), (23) and the transversality condition

$$\lim_{s \to \infty} \frac{(1+2X_{Ss})K_{Ss}}{\prod_{v=t+1}^{s} (1+r_v)} = 0.$$
 (24)

## General equilibrium path

Considering (12), (17), (22) and the fact that  $A_t = K_{Mt}^{\alpha}$ , one can obtain

$$P_{t} = \frac{\alpha K_{Mt} (1 - L_{Mt})^{1 - \beta}}{\beta K_{St}^{1 - \beta} L_{Mt}^{1 - \alpha}}.$$
(25)

Considering (5), (11) and (12), one can also obtain

$$C_{St} = K_{St}^{1-\beta} (1 - L_{Mt})^{\beta}$$
(26)

One can use (15), (17), (25), (26) and the fact that  $A_t = K_{Mt}^{\alpha}$  to write (19) as

$$\frac{(1-\alpha)L_{Mt+1}^{\alpha} + X_{Mt+1}^{2} + (1-\delta)(1+2X_{Mt+1})}{1+2X_{Mt}} = \theta^{-1} \left[ \frac{K_{Mt+1}(1-L_{Mt+1})^{1-\beta}K_{St}^{1-\beta}L_{Mt}^{1-\beta}}{K_{Mt}(1-L_{Mt})^{1-\beta}K_{St+1}^{1-\beta}L_{Mt+1}^{1-\alpha}} \right]^{1-\eta} \left[ \frac{K_{St+1}^{1-\beta}(1-L_{Mt+1})^{\beta} + \varepsilon}{K_{St}^{1-\beta}(1-L_{Mt})^{\beta} + \varepsilon} \right]^{1-\eta-\gamma}$$

$$(27)$$

Similarly, one can use (12), (15), (21), (25) and (26) to write (23) as

$$\frac{\frac{\alpha(1-\beta)K_{Mt+1}(1-L_{Mt+1})}{\beta K_{St+1}L_{Mt+1}^{1-\alpha}} + X_{St+1}^{2} + (1-\delta)(1+2X_{St+1})}{1+2X_{St}} = \theta^{-1} \left[ \frac{K_{Mt+1}(1-L_{Mt+1})^{1-\beta}K_{St}^{1-\beta}L_{Mt}^{1-\alpha}}{K_{Mt}(1-L_{Mt})^{1-\beta}K_{St+1}^{1-\beta}L_{Mt+1}^{1-\alpha}} \right]^{1-\eta} \left[ \frac{K_{St+1}^{1-\beta}(1-L_{Mt+1})^{\beta} + \varepsilon}{K_{St}^{1-\beta}(1-L_{Mt})^{\beta} + \varepsilon}} \right]^{1-\eta-\gamma}$$

$$(28)$$

Considering (8), one can obtain:

$$\frac{K_{\text{St+1}}}{K_{\text{St}}} = X_{\text{St}} + 1 - \delta, \qquad (29)$$

$$\frac{K_{Mt+1}}{K_{Mt}} = X_{Mt} + 1 - \delta .$$
(30)

Finally, one can use (3), (14), (25), (26) and the fact that  $A_t = K_{Mt}^{\alpha}$  to write (10) as

$$K_{Mt}L_{Mt}^{\alpha} = \frac{\alpha\eta K_{Mt}(1-L_{Mt})}{\beta\eta L_{Mt}^{1-\alpha}} \left[ 1 + \frac{\varepsilon(1-L_{Mt})^{-\beta}}{K_{St}^{1-\beta}} \right] + K_{Mt}(X_{Mt} + X_{Mt}^{2}) + K_{St}(X_{St} + X_{St}^{2}).$$
(31)

The system (26)-(31) governs the general equilibrium path of the economy. Moreover, equation (31) can be used to obtain:

$$\frac{K_{St}}{K_{Mt}} = n(L_{Mt}, X_{Mt}, X_{St}, Q_t) = \frac{L_{Mt}^{\alpha} - \frac{\alpha\eta}{\beta \gamma L_{Mt}^{1-\alpha}} \left(1 - L_{Mt} + \varepsilon Q_t^{1-\beta}\right) - (X_{Mt} + X_{Mt}^2)}{(X_{St} + X_{St}^2)}, \quad Q_t \equiv \frac{(1 - L_{Mt})}{K_{St}}.$$
 (32)

## **5 HOMOTHETIC PREFERENCES**

In the Cobb-Douglas case ( $\epsilon$ =0), equation (32) is such that  $\frac{K_{St}}{K_{Mt}} = n(L_{Mt}, X_{Mt}, X_{St})$ . Using (30)

and (32), one can rewrite (27)-(29) as a system of three difference equations in  $L_{Mt}$ ,  $X_{Mt}$  and  $X_{St}$  governing the general equilibrium path of the economy:

$$\Psi(L_{Mt+1}, X_{Mt+1}, X_{St+1}, L_{Mt}, X_{Mt}, X_{St}) = \frac{(1-\alpha)L_{Mt+1}^{\alpha} + X_{Mt+1}^{2} + (1-\delta)(1+2X_{Mt+1})}{1+2X_{Mt}} - \frac{-\theta^{-1}\left[\frac{(X_{Mt}+1-\delta)(1-L_{Mt+1})L_{Mt}^{1-\alpha}}{(1-L_{Mt})L_{Mt+1}^{1-\alpha}}\right]^{1-\eta-\gamma}\left[\frac{(X_{Mt}+1-\delta)\beta L_{Mt}^{1-\alpha}[n(L_{Mt}, X_{Mt}, X_{St})(1-L_{Mt+1})]^{1-\beta}}{L_{Mt+1}^{1-\alpha}[n(L_{Mt+1}, X_{Mt+1}, X_{St+1})(1-L_{Mt})]^{1-\beta}}\right]^{\gamma} = 0, \quad (33)$$

$$\Phi(L_{Mt+1}, X_{Mt+1}, X_{St+1}, L_{Mt}, X_{Mt}, X_{St}) = \frac{\alpha(1-\beta)(1-L_{Mt+1})}{\beta n(L_{Mt+1}, X_{Mt+1}, X_{St+1})L_{Mt+1}^{1-\alpha}} + X_{St+1}^{2} + (1-\delta)(1+2X_{St+1})}{1+2X_{St}} - \frac{-\theta^{-1}\left[\frac{(X_{Mt}+1-\delta)(1-L_{Mt+1})L_{Mt}^{1-\alpha}}{(1-L_{Mt})L_{Mt+1}^{1-\alpha}}\right]^{1-\eta-\gamma}\left[\frac{(X_{Mt}+1-\delta)\beta L_{Mt}^{1-\alpha}[n(L_{Mt}, X_{Mt}, X_{St})(1-L_{Mt+1})]^{1-\beta}}{1+2X_{St}}\right]^{\gamma} = 0, \quad (34)$$

$$\Lambda(L_{Mt+1}, X_{Mt+1}, X_{St+1}, L_{Mt}, X_{Mt}, X_{St}) = \frac{(X_{Mt}+1-\delta)n(L_{Mt+1}, X_{Mt+1}, X_{St+1})}{n(L_{Mt}, X_{Mt}, X_{St})} - X_{St} - 1+\delta = 0. \quad (35)$$

#### Balanced growth path

Along a balanced growth path (BGP), one must have  $L_{Mt+1}=L_{Mt}=L_M$ ,  $X_{Mt+1}=X_{Mt}=X_M$  and  $X_{St+1}=X_{St}=X_S$  in equations (33)-(35). If a BGP exists, it is characterized by  $X_S^{\circ} = X_M^{\circ}$ ,  $L_M^{\circ} = f(X_M^{\circ})$ ,  $X_M^{\circ} = g(X_M^{\circ})$  (" $^{\circ}$ " denotes the BGP value of a variable when  $\varepsilon = 0$ ), where

$$f(X_{\rm M}) = \left\{ \left[ \frac{(X_{\rm M} + 1 - \delta)^{1 - \eta - \gamma(1 - \beta)}}{\theta} - 1 + \delta \right] \frac{(1 + 2X_{\rm M})}{(1 - \alpha)} - \frac{(X_{\rm M})^2}{(1 - \alpha)} \right\}^{1/\alpha}$$
(36)

and

$$g(\mathbf{X}_{\mathrm{M}}) = -\frac{1}{2} + \left\{ \frac{1}{4} + \frac{(1-\alpha)[(\beta\gamma + \alpha\eta)(f(\mathbf{X}_{\mathrm{M}}))^{1+\alpha} - \alpha\eta(f(\mathbf{X}_{\mathrm{M}}))^{\alpha}]}{\gamma[\alpha(1-\beta) + (\beta-\alpha)f(\mathbf{X}_{\mathrm{M}})]} \right\}^{\frac{1}{2}}.$$
 (37)

Considering (29) and (30), note that  $X_{S}^{\circ} = X_{M}^{\circ}$  entails  $\mu_{M}^{\circ} = \mu_{S}^{\circ}$ , where  $\mu_{it} = \frac{K_{it+1} - K_{it}}{K_{it}}$ , i=M,S

(along a BGP, the capital stock grows at the same rate in the two sectors). Note also that  $\mu_{M}^{\circ} = \mu_{S}^{\circ} \begin{cases} > \\ = \\ < \end{cases} 0$ 

whenever  $X_{M}^{\circ} = X_{S}^{\circ} \begin{cases} > \\ < \end{cases} \delta$  (the steady-state rate of growth of capital is positive if and only if the steady-state

ratio between gross investment and capital stock is larger than the capital-depreciation parameter). In its turn,

one can check by manipulating (36) and (37) that  $X_{M}^{\circ} = X_{S}^{\circ} \begin{cases} > \\ = \\ < \end{cases} \delta$  whenever  $Z \begin{cases} < \\ = \\ > \end{cases} \delta + \delta^{2}$ , where

$$Z = \frac{(1-\alpha)\left\{ (\beta\gamma + \alpha\eta) \left[ f(X_{\rm M}) \Big|_{X_{\rm M}} = \delta \right]^{1+\alpha} - \alpha\eta \left[ f(X_{\rm M}) \Big|_{X_{\rm M}} = \delta \right]^{\alpha} \right\}}{\gamma \left[ \alpha(1-\beta) + (\beta-\alpha) f(X_{\rm M}) \Big|_{X_{\rm M}} = \delta \right]},$$
(38)

$$f(\mathbf{X}_{\mathbf{M}})\Big|_{\mathbf{X}_{\mathbf{M}}=\boldsymbol{\delta}} = \left[\frac{1+2\boldsymbol{\delta}-\boldsymbol{\theta}-\boldsymbol{\theta}\boldsymbol{\delta}(1-\boldsymbol{\delta})}{\boldsymbol{\theta}(1-\boldsymbol{\alpha})}\right]^{1/\boldsymbol{\alpha}}.$$
(39)

One can also check that Z increases with  $\gamma$  and decreases with  $\eta$ : given the other parameter values, a larger  $\gamma$  shifts consumer demand towards the service, thus making less likely that the steady-state rate of growth of the capital stock is positive, while a larger  $\eta$  tends to have the opposite effect.

Considering (3) and the fact that  $A_t = K_{Mt}^{\alpha}$ , one has  $\rho_M^{\circ} = \mu_M^{\circ}$ ; while--considering (5)--one has

$$\rho_{\rm S}^{\circ} = (1 + \mu_{\rm S}^{\circ})^{1-\beta} - 1$$
, where  $\rho_{\rm it} = \frac{Y_{\rm it+1} - Y_{\rm it}}{Y_{\rm it}}$ , i=M,S. Together with  $\mu_{\rm M}^{\circ} = \mu_{\rm S}^{\circ}$ , this entails  $\rho_{\rm M}^{\circ} \begin{vmatrix} z \\ z \end{vmatrix} = \rho_{\rm S}^{\circ}$  whenever

$$X_{M}^{\circ} = X_{S}^{\circ} \begin{cases} > \\ < \\ < \end{cases} \delta$$
 (along a BGP, the output of the manufacturing sector grows at a higher rate than the output of

the service sector if and only if the steady-state ratio between gross investment and capital stock is larger than the capital-depreciation parameter). Note also that the GDP of this economy grows along a BGP at the same rate as the capital stock:  $\rho^{\circ} = \mu_{M}^{\circ} = \mu_{S}^{\circ}$ , where  $\rho_{t} \equiv \frac{\text{GDP}_{t+1} - \text{GDP}_{t}}{\text{GDP}_{t}}$  and

 $GDP_t$ 

$$GDP_{t} = Y_{Mt} + P_{t}Y_{St} = K_{Mt}L_{Mt}^{\alpha} + \frac{\alpha K_{Mt}(1 - L_{Mt})}{\beta L_{Mt}^{1 - \alpha}}.$$
(40)

This implies the following proposition:

**Proposition 1**. With homothetic preferences ( $\varepsilon$ =0), the economy displays perpetual growth ( $\rho^{\circ}>0$ ) whenever the parameter values are such that  $X_{M}^{\circ} > \delta$ . In particular, a smaller share of service in total consumption

expenditure (smaller  $\gamma$ ) and a larger share of manufacturing in total consumption expenditure (larger  $\eta$ ) can contribute to generate a positive steady-state rate of growth.

Finally, considering (25) and (32), one has  $\omega^{\circ} = (1 + \mu_{\rm M}^{\circ})^{\beta} - 1$ , where  $\omega_{\rm t} \equiv \frac{P_{\rm t+1} - P_{\rm t}}{P_{\rm t}}$ . Note that

$$\omega^{\circ} \begin{cases} > \\ < \end{cases} 0$$
 whenever  $X_{M}^{\circ} = X_{S}^{\circ} \begin{cases} > \\ < \end{cases} \delta$  (the steady-state rate of growth of the relative price of the service is

positive if and only if the steady-state ratio between gross investment and capital stock is larger than the capital-depreciation parameter).

## The transition path: a numerical example

As a numerical example, let  $\alpha$ =0.6,  $\beta$ = $\gamma$ =0.7,  $\delta$ =0.008,  $\varepsilon$ =0,  $\eta$ =0.3 and  $\theta$ =0.851797. Given these parameter values<sup>20</sup>, one can show that there exists a unique BGP<sup>21</sup> characterized by  $L_{M}^{\circ} \approx 0.28$  and  $X_{M}^{\circ} = X_{S}^{\circ} \approx 0.0096$ , thus entailing  $\mu_{M}^{\circ} = \mu_{S}^{\circ} = \rho_{M}^{\circ} \approx 0.0016$ ,  $\rho_{S}^{\circ} \approx 0.00049$  and  $\omega^{\circ} \approx 0.0011$ . Furthermore, by linearizing (33)-(35) around  $(L_{M}^{\circ}, X_{M}^{\circ}, X_{S}^{\circ})$ , one can show that the linearized system is saddle-path stable, since the characteristic roots are:  $\sigma_{1}\approx 0.8923$ ,  $\sigma_{2}\approx 1.3062+0.1421i$  and  $\sigma_{3}\approx 1.3062-0.1421i$ . The unique path converging to  $(L_{M}^{\circ}, X_{M}^{\circ}, X_{S}^{\circ})$  is governed by

$$L_{Mt} - L_M^{\circ} = Ze_1 \sigma_1^t, \qquad (41)$$

$$X_{Mt} - X_{M}^{\circ} = Ze_2 \sigma_1^t, \qquad (42)$$

<sup>&</sup>lt;sup>20</sup> The values of the parameters  $\eta$  and  $\gamma$  entering the utility function have been chosen looking at the expenditure shares for the two sector as reported by Mattey (1997), Oecd (2000), Business Statistic of the US (2002). These expenditure shares include government expenditure. The parameter  $\alpha$ —entering the production function of the progressive sector-is consistent with the evidence reported in the Survey of Current Business (2003) for US. A larger value is assigned to the corresponding parameter entering the production function of the stagnant sector ( $\beta$ ), so as to account for the evidence showing that this sector is more labor intensive (see O'Mahony and Van Ark, 2003; particularly Table II.6). These parameters values are in line with those chosen by Kongsamut et al. (2003) in their examples.

<sup>&</sup>lt;sup>21</sup> The existence and the uniqueness of the BGP are guaranteed by the following facts: i) both  $f(X_M)$  and  $g(X_M)$  are continuous and monotonically increasing in  $X_M$  for  $0 \le X_M \le \overline{X}$ , where  $\overline{X}$  is that value of  $X_M$  such that  $f(X_M)=1$ ; ii)  $g(X_M)-X_M<0$  at  $X_M=0$  and  $g(X_M)-X_M>0$  at  $X_M=\overline{X}$ , and iii) g'>1 for  $\underline{X} \le X_M \le \overline{X}$ , where  $\underline{X} > 0$  is that value of  $X_M$  such that  $g(X_M)=0$ .

$$X_{St} - X_S^{\circ} = Ze_3\sigma_1^t, \qquad (43)$$

where  $\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \approx \begin{bmatrix} 0.3965 \\ 0.1386 \\ 0.4725 \end{bmatrix}$  are the characteristic vectors associated with the stable root  $\sigma_1$ , Z is a constant to be

determined, 
$$X_{S0} = -\frac{1}{2} + \left\{ \frac{1}{4} + \frac{K_{M0}}{K_{S0}} \left[ L_{M0}^{\alpha} \left( 1 + \frac{\alpha \eta}{\beta \gamma} \right) - \frac{\alpha \eta}{\beta \gamma} L_{M0}^{\alpha - 1} - X_{M0} - X_{M0}^{2} \right] \right\}^{1/2}$$
 is obtained from (31) and  $\frac{K_{M0}}{K_{S0}}$  is given.

Recalling that  $\mu_{Mt} = X_{Mt} - \delta$  and  $\mu_{St} = X_{St} - \delta$ , equations (41)-(43) tell us that--whenever  $L_{M0} > L_{M}^{\circ}$ -both  $\mu_{Mt}$  and  $\mu_{St}$  are larger along the transition path than along the BGP. Moreover--along the transition path-- $\mu_{St}$  tends to be larger than  $\mu_{Mt}$ : along this path, the capital stock tends to grow at a faster rate in the service sector than in the manufacturing sector when the share of the manufacturing sector on total employment tends to decline. Finally, the combined effect of a declining share of the manufacturing sector on total employment and of  $\mu_{St} > \mu_{Mt}$  may imply that for some t  $\rho_{St} \ge \rho_{Mt}$ .

Obviously, the value that  $L_{M0}$  must assume along the path converging to the BGP depends on the initial condition  $\frac{K_{M0}}{K_{S0}}$ . In particular, it is apparent that  $L_{M0} = L_M^{\circ}$  and  $X_{M0} = X_M^{\circ} = X_{S0} = X_S^{\circ}$  whenever

 $\frac{K_{M0}}{K_{S0}} = \left(\frac{K_M}{K_S}\right)^{\circ} \approx 0.6$  (the system is at its steady state starting from period 0 if the initial value of the ratio between the capital stock installed in the manufacturing sector and the capital stock installed in the service

sector is equal to its steady-state value, which is approximately equal to 0.6). One can also check that

$$\frac{\frac{\partial L_{M0}}{\partial \frac{K_{M0}}{K_{S0}}} \left| \frac{K_{M0}}{K_{S0}} = \left( \frac{K_{M}}{K_{S}} \right)^{\circ} > 0$$
(44)

(as the initial value of the ratio between the capital stock installed in the manufacturing sector and the capital stock installed in the service sector tends to be larger than its steady-state value, also the initial value of the share of the manufacturing sector on total employment tends to be larger than its steady-state value). Finally, one can easily check that

$$\frac{\frac{\partial (X_{S0} - X_{M0})}{\partial \frac{K_{M0}}{K_{S0}}} \bigg|_{\frac{K_{M0}}{K_{S0}} = \left(\frac{K_{M}}{K_{S}}\right)^{\circ} > 0$$
(45)

(as the initial value of the ratio between the capital stock installed in the manufacturing sector and the capital stock installed in the service sector tends to be larger than its steady-state value, the initial value of the gross investment-installed capital ratio tends to be larger in the service sector than in the manufacturing sector).

For instance, take  $\frac{K_{M0}}{K_{S0}} \approx 0.6592 > \left(\frac{K_M}{K_S}\right)^\circ$ . Given this initial condition, one has  $L_{M0} \approx 0.3 > L_M^\circ$  and

 $X_{S0} \approx 0.03347 > X_{M0} \approx 0.016626 > X_{M}^{\circ} = X_{S}^{\circ}$ . Furthermore--in a neighborhood of the BGP--one has:

$$\omega_{0} = \omega^{\circ} + (X_{M0} - X_{M}^{\circ})(1 + X_{S}^{\circ} - \delta)^{\beta - 1} - (1 - \beta)(X_{S0} - X_{S}^{\circ})(1 + X_{S}^{\circ} - \delta)^{\beta - 2}(1 + X_{M}^{\circ} - \delta) - \left[\frac{(1 - \beta)}{(1 - L_{M}^{\circ})} + \frac{(1 - \alpha)}{L_{M}^{\circ}}\right](1 + X_{S}^{\circ} - \delta)^{\beta - 1}(1 + X_{M}^{\circ} - \delta)(L_{M1} - L_{M0}) \approx 0.00492, \quad (46)$$

$$\rho_{M0} = \rho_{M}^{\circ} + X_{M0} - X_{M}^{\circ} + \frac{\alpha(1 + X_{M}^{\circ} - \delta)(L_{M1} - L_{M0})}{L_{M}^{\circ}} \approx 0.0039356,$$
(47)

$$\rho_{\rm S0} = \rho_{\rm S}^{\circ} + (1 - \beta)(1 + X_{\rm S}^{\circ} - \delta)^{-\beta}(X_{\rm S0} - X_{\rm S}^{\circ}) - \frac{\beta(1 + X_{\rm S}^{\circ} - \delta)^{1 - \beta}(L_{\rm M1} - L_{\rm M0})}{(1 - L_{\rm M}^{\circ})} \approx 0.009727, \tag{48}$$

$$\rho_{0} - \rho^{\circ} = X_{M0} - X_{M}^{\circ} - \alpha \left[ \frac{1 - \alpha - L_{M}^{\circ}(\beta - \alpha)}{\alpha L_{M}^{\circ} + (L_{M}^{\circ})^{2}(\beta - \alpha)} \right] (1 + X_{M}^{\circ} - \delta)(L_{M1} - L_{M0}) \approx 0.0097298.$$
(49)

We have from equation (46) that the relative price of the service tends to grow along a transition path characterized by a declining employment level in the manufacturing sector. In addition, one can see by comparing (47) and (48) that along such a path the output of the service sector may grow at a higher rate than the output of the manufacturing sector. Finally, equation (49) shows that along this transition path the economy's GDP may increase at a higher rate than along the BGP: the economy's rate of growth tends to decline over time as the share of the two factors of production used in the manufacturing sector shrinks.

#### **6 NON-HOMOTHETIC PREFERENCES**

As  $\epsilon > 0$ , one can use (32) to rewrite (27)-(30) as a system of four difference equations in  $L_{Mt}$ ,  $X_{Mt}$ ,  $X_{St}$  and  $Q_t$  governing the general equilibrium path of the economy:

$$\Omega(L_{Mt+1}, X_{Mt+1}, X_{St+1}, Q_{t+1}, L_{Mt}, X_{Mt}, X_{St}, Q_t) = \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + X_{Mt+1}^2 + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + X_{Mt+1}^2 + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + X_{Mt+1}^2 + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + X_{Mt+1}^2 + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + X_{Mt+1}^2 + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt+1}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt+1}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt+1}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt+1}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt+1}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt+1}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt+1}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt+1}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt+1}} - \frac{(1 - \alpha)L_{Mt+1}^{\alpha} + (1 - \delta)(1 + 2X_{Mt+1})}{1 + 2X_{Mt+1}}$$

$$-\theta^{-1}\left\{\left[\frac{(X_{Mt}+1-\delta)L_{Mt}^{1-\alpha}}{L_{Mt+1}^{1-\alpha}}\right]\left[\frac{(1-L_{Mt+1})+\varepsilon Q_{t+1}^{1-\beta}}{(1-L_{Mt})+\varepsilon Q_{t}^{1-\beta}}\right]\right\}^{1-\eta-\gamma}\left[\frac{(X_{Mt}+1-\delta)^{\beta}L_{Mt}^{1-\alpha}[n(L_{Mt},X_{Mt},X_{St},Q_{t})(1-L_{Mt+1})]^{1-\beta}}{L_{Mt+1}^{1-\alpha}[n(L_{Mt+1},X_{Mt+1},X_{St+1},Q_{t+1})(1-L_{Mt})]^{1-\beta}}\right]^{\gamma}=0, (50)$$

$$\Gamma(L_{Mt+1}, X_{Mt+1}, X_{St+1}, Q_{t+1}, L_{Mt}, X_{Mt}, X_{St}, Q_{t}) = \frac{\frac{\alpha(1-\beta)(1-L_{Mt+1})}{\beta n(L_{Mt+1}, X_{Mt+1}, X_{St+1}, Q_{t+1})L_{Mt+1}^{1-\alpha}} + X_{St+1}^{2} + (1-\delta)(1+2X_{St+1})}{1+2X_{St}} - \frac{\alpha(1-\beta)(1-L_{Mt+1})}{1+2X_{St}} + (1-\delta)(1+2X_{St+1})}{1+2X_{St}} - \frac{\alpha(1-\beta)(1-L_{Mt+1})}{1+2X_{St}} + (1-\delta)(1+2X_{St+1})}{1+2X_{St}} - \frac{\alpha(1-\beta)(1-L_{Mt+1})}{1+2X_{St}} + (1-\delta)(1+2X_{St+1})}{1+2X_{St}} - \frac{\alpha(1-\beta)(1-L_{Mt+1})}{1+2X_{St}} + \frac{\alpha(1-\beta)(1-L_{Mt+1})}{1+$$

$$-\theta^{-1}\left\{\left[\frac{(X_{Mt}+1-\delta)L_{Mt}^{1-\alpha}}{L_{Mt+1}^{1-\alpha}}\right]\left[\frac{(1-L_{Mt+1})+\varepsilon Q_{t+1}^{1-\beta}}{(1-L_{Mt})+\varepsilon Q_{t}^{1-\beta}}\right]\right\}^{1-\eta-\gamma}\left[\frac{(X_{Mt}+1-\delta)^{\beta}L_{Mt}^{1-\alpha}[n(L_{Mt},X_{Mt},X_{St},Q_{t})(1-L_{Mt+1})]^{1-\beta}}{L_{Mt+1}^{1-\alpha}[n(L_{Mt+1},X_{Mt+1},X_{St+1},Q_{t+1})(1-L_{Mt})]^{1-\beta}}\right]^{\gamma}=0, (51)$$

$$\Theta(L_{Mt+1}, X_{Mt+1}, X_{St+1}, Q_{t+1}, L_{Mt}, X_{Mt}, X_{St}, Q_t) = (X_{Mt} + 1 - \delta) - \frac{(X_{St} + 1 - \delta)n(L_{Mt}, X_{Mt}, X_{St}, Q_t)}{n(L_{Mt+1}, X_{Mt+1}, X_{St+1}, Q_{t+1})} = 0, \quad (52)$$

$$\Sigma(L_{Mt+1}, Q_{t+1}, L_{Mt}, X_{St}, Q_t) = (1 - L_{Mt+1})Q_t - (X_{St} + 1 - \delta)(1 - L_{Mt})Q_{t+1} = 0.$$
(53)

Balanced growth paths

Along a BGP, one must have  $L_{Mt+1}=L_{Mt}=L_M$ ,  $X_{Mt+1}=X_{Mt}=X_M$ ,  $X_{St+1}=X_{St}=X_S$  and  $Q_{t+1}=Q_t=Q$  in equations (50)-(53). If a BGP exists, it is characterized by  $X_S^* = X_M^* > \delta$ ,  $L_M^* = f(X_M^*)$ ,  $X_M^* = g(X_M^*)$ ,

Q\*=0 whenever Z<
$$\delta+\delta^2$$
, and by  $X_M^* = X_S^* = \delta$ ,  $L_M^* = f(X_M)|_{X_M = \delta}$ ,  $Q^* = \frac{(1-L_M^*)}{K_S^*}$ ,

$$K_{M}^{*} = \frac{\beta K_{S}^{*} L_{M}^{*} (1 - \alpha)}{\alpha (1 - L_{M}^{*})(1 - \beta)}, \quad K_{S}^{*} = \left\{ \left[ (L_{M}^{*})^{\alpha} - \delta - \delta^{2} \right] \frac{\beta \gamma (1 - L_{M}^{*})^{\beta - 1}}{\alpha \eta \varepsilon (L_{M}^{*})^{\alpha - 1}} - \frac{(1 - L_{M}^{*})^{\beta}}{\varepsilon} - \frac{\gamma (\delta + \delta^{2})(1 - \beta)(L_{M}^{*})^{-\alpha}}{\varepsilon \eta (1 - \alpha)(1 - L_{M}^{*})^{-\beta}} \right\}^{\frac{1}{(\beta - 1)}}$$

whenever  $Z \ge \delta + \delta^2$  ("\*" denotes the BGP value of a variable when  $\varepsilon > 0$ ), where  $f(X_M)$ ,  $g(X_M)$ , Z and  $f(X_M)|_{X_M = \delta}$  are given, respectively, by (36), (37), (38) and (39)<sup>22</sup>.

<sup>22</sup> Given the definition of  $Q_t \left( Q_t \equiv \frac{(1 - L_{Mt})}{K_{St}} \right)$ , one can easily see that there cannot exist a BGP characterized by  $X_S^* = X_M^*$ ,  $L_M^* = f(X_M^*)$ ,  $X_M^* = g(X_M^*)$ ,  $Q^*=0$  whenever  $Z \ge \delta + \delta^2$ . In this case, indeed, one would have  $X_S^* = X_M^* \le \delta$ , entailing  $\mu_S^* = \mu_M^* \le 0$ , which is inconsistent with the fact that both  $\lim_{t \to \infty} L_{Mt} = L_M^* = f(X_M^*)$  and  $\lim_{t \to \infty} Q_t = Q^* = 0$  must hold. Moreover, to see that there cannot exist a BGP characterized by fixed levels of  $K_{Mt}$  and  $K_{St}$  whenever  $Z < \delta + \delta^2$ , consider that

$$(K_{\rm S}^*)^{\beta-1} = \left[ (L_{\rm M}^*)^{\alpha} - \delta - \delta^2 \right] \frac{\beta \gamma (1 - L_{\rm M}^*)^{\beta-1}}{\alpha \eta \varepsilon (L_{\rm M}^*)^{\alpha-1}} - \frac{(1 - L_{\rm M}^*)^{\beta}}{\varepsilon} - \frac{\gamma (\delta + \delta^2) (1 - \beta) (L_{\rm M}^*)^{-\alpha}}{\varepsilon \eta (1 - \alpha) (1 - L_{\rm M}^*)^{-\beta}} < 0 \qquad \text{if} \qquad Z < \delta + \delta^2, \qquad \text{where}$$

By comparing the case with  $\varepsilon=0$  to the case with  $\varepsilon>0$ , one can see that whenever  $Z<\delta+\delta^2$  (which tends to be satisfied when  $\gamma$  is small and/or  $\eta$  is large) the economy with homothetic preferences and the economy with non-homothetic preferences share the same BGP, that is characterized by perpetual growth  $(\rho^* = \rho_M^* = \mu_M^* = \mu_S^* > 0, \omega^* > 0, \rho_S^* > 0)$ . In contrast, whenever  $Z>\delta+\delta^2$ , the economy with  $\varepsilon=0$ -displays a negative steady-state rate of growth  $(\rho^* = \rho_M^* = \mu_M^* = \mu_S^* < 0, \omega^* < 0, \rho_S^* < 0)$ , while the economy with  $\varepsilon>0$  has a BGP along which the levels of capital and output are fixed in both sectors  $(\rho^* = \rho_M^* = \rho_S^* = \mu_M^* = \mu_S^* = \omega^* = 0)$ . Hence, one can conclude that, even in the case where the elasticity of demand for the service with respect to the household's consumption expenditure is greater than one ( $\varepsilon>0$ ), the economy displays perpetual growth if the share of total consumption expenditure devoted to the manufactured good is not too small. In other words, even with non-homothetic preferences, asymptotic stagnancy can occur only if an excessively large portion of what households spend on consumption is devoted to the service.

Intuitively, one can think that when final demand is not too much unbalanced towards the product of the stagnant industry, a virtuous circle can be ignited, whereby growing market production of both the manufactured good and the service makes progressively less relevant the fixed amount of service that is produced at home. Thanks to this virtuous circle, the elasticity of demand for each of the two goods approaches asymptotically one, thus avoiding that aggregate growth could vanish in the long run because of the structural burden of increasing labor and capital shares getting used in the stagnant sector. Finally, note that a negative steady-state rate of growth cannot be a BGP, since--in a shrinking market economy— the elasticity of the two goods with respect to the household's consumption expenditure would increasingly diverge, thus progressively unbalancing the composition of final demand.

This discussion can be summarized by the following proposition:

**Proposition 2.** With non-homothetic preferences ( $\varepsilon > 0$ ), the economy displays perpetual growth ( $\rho *>0$ ) whenever the parameter values are such that  $Z < \delta + \delta^2$  and asymptotic stagnancy ( $\rho *=0$ ) whenever the parameter values are such that  $Z \ge \delta + \delta^2$  In particular, a smaller share of service in total consumption

 $<sup>\</sup>mathcal{L}_{\mathbf{M}}^{*}=f(\mathbf{X}_{\mathbf{M}})\Big|_{\mathbf{X}_{\mathbf{M}}=\boldsymbol{\delta}}\ .$ 

expenditure (smaller  $\gamma$ ) and a larger share of manufacturing in total consumption expenditure (larger  $\eta$ ) can contribute to avoid asymptotic stagnancy.

Note that--when the economy is asymptotically stagnant-- $L_M^*$  depends neither on the parameters of the households' period-utility function nor on the parameter of the service-producing firms' production function. Indeed, the steady-state share of the manufacturing sector on total employment increases with the steady-state rate of interest (1/ $\theta$ -1): other things being equal, the marginal profitability of capital must be boosted in the manufacturing sector by employing more workers in order to accommodate a higher rate of interest. Similarly,  $L_M^*$  increases with the capital-depreciation parameter ( $\delta$ ): the gross rate of return on capital investment in the manufacturing sector must be higher in order to accommodate a faster capital depreciation. Note also that--in the case where  $Z \ge \delta + \delta^2 - L_M^*$  increases with the share of labor on the income generated in the manufacturing sector ( $\alpha$ ) and that the steady-state ratio between the capital stock installed in the manufacturing sector and the capital stock installed in the service sector increases with  $L_M^*$ . In contrast, both  $K_M^*$  and  $K_8^*$  are sensitive to the parameters of the households' period-utility function. In particular still in the case where  $Z \ge \delta + \delta^2$ --everything that (other things being equal) induces the households to devote a larger fraction of their consumption expenditure to the manufactured good (higher  $\eta$  or  $\varepsilon$ , lower  $\gamma$ ) leads to larger  $K_M^*$  and  $K_8^*$ , thus boosting  $Y_M^*$  and  $Y_8^*$ .

#### The transition path: a numerical example

As a numerical example, let  $\alpha=2/3$ ,  $\beta=\gamma=0.8$ ,  $\delta=0.05$ ,  $\varepsilon=0.1$ ,  $\eta=0.2$  and  $\theta=0.93$ . Given these parameter values, one has  $Z \ge \delta + \delta^2$ , and -the unique BGP is characterized by  $L_M^* \approx 0.25859$ ,  $X_M^* = X_S^* = 0.05$ ,  $K_M^* \approx 0.33453$  and  $K_S^* \approx 0.47958$ , thus entailing  $\frac{K_M^*}{K_S^*} \approx 0.6976$ . Furthermore, by linearizing (26)-(29) around  $(L_M^*, X_M^*, K_M^*, K_S^*)$ , one can show that the linearized system is saddle-path stable, since the characteristic roots are:  $\xi_1 \approx 0.99167$ ,  $\xi_2 \approx 0.8723$ ,  $\xi_3 \approx 1.22847 + 0.2078i$  and  $\xi_4 \approx 1.22847 - 0.2078i$ . The unique

path converging to  $(L_M^*, X_M^*, K_M^*, K_S^*)$  is governed by

$$L_{Mt} - L_{M}^{*} = H_{1}d_{11}\xi_{1}^{t} + H_{2}d_{12}\xi_{2}^{t},$$
(54)

$$X_{Mt} - X_{M}^{*} = H_{1}d_{21}\xi_{1}^{t} + H_{2}d_{22}\xi_{2}^{t},$$
(55)

$$K_{Mt} - K_M^* = H_1 d_{31} \xi_1^t + H_2 d_{32} \xi_2^t,$$
(56)

$$K_{St} - K_{S}^{*} = H_{1}d_{41}\xi_{1}^{t} + H_{2}d_{42}\xi_{2}^{t},$$
(57)

where  $\begin{bmatrix} d_{11} \\ d_{21} \\ d_{31} \\ d_{41} \end{bmatrix} \approx \begin{bmatrix} -0.0766 \\ -0.0264 \\ 1.06150 \\ 1.17214 \end{bmatrix}$  and  $\begin{bmatrix} d_{12} \\ d_{22} \\ d_{32} \\ d_{42} \end{bmatrix} \approx \begin{bmatrix} -0.5455 \\ -0.1427 \\ 0.37593 \\ 2.57116 \end{bmatrix}$  are the characteristic vectors associated—respectively--with  $\xi_1$  and

with  $\xi_2,\,H_1$  and  $H_2$  are two constants to be determined,  $K_{M0}$  and  $K_{S0}$  are given.

Given (54)-(57), one can ascertain that initial conditions such that  $K_{M0} < K_M^*$  and  $\frac{K_{M0}}{K_{S0}} > \frac{K_M^*}{K_S^*}$  are

consistent with a saddle path displaying a declining level of employment in the manufacturing sector, a positive (but declining) rate of growth of the capital stock in both sectors, and with the tendency of  $K_{St}$  to grow at a higher rate than  $K_{M0}$ . For instance, take  $K_{M0}\approx 0.27876$  and  $\frac{K_{M0}}{K_{S0}}\approx 1$ . Given these initial conditions,

one has  $L_{M0} \approx 0.29609 > L_M^*$  and  $X_{S0} \approx 0.11073 > X_{M0} \approx 0.06 > X_M^* = X_S^*$ . Furthermore--in a neighborhood of the BGP--one has:

$$\omega_{0} = \omega^{*} + (X_{M0} - X_{M}^{*})(1 + X_{S}^{*} - \delta)^{\beta - 1} - (1 - \beta)(X_{S0} - X_{S}^{*})(1 + X_{S}^{*} - \delta)^{\beta - 2}(1 + X_{M}^{*} - \delta) - \left[\frac{(1 - \beta)}{(1 - L_{M}^{*})} + \frac{(1 - \alpha)}{L_{M}^{*}}\right](1 + X_{S}^{*} - \delta)^{\beta - 1}(1 + X_{M}^{*} - \delta)(L_{M1} - L_{M0}) \approx 0.0048575, \quad (58)$$

$$\rho_{\rm M0} = \rho_{\rm M}^* + X_{\rm M0} - X_{\rm M}^* + \frac{\alpha (1 + X_{\rm M}^* - \delta)(L_{\rm M1} - L_{\rm M0})}{L_{\rm M}^*} \approx -0.00158266,$$
(59)

$$\rho_{\rm S0} = \rho_{\rm S}^* + (1 - \beta)(1 + X_{\rm S}^* - \delta)^{-\beta}(X_{\rm S0} - X_{\rm S}^*) - \frac{\beta(1 + X_{\rm S}^* - \delta)^{1-\beta}(L_{\rm M1} - L_{\rm M0})}{(1 - L_{\rm M}^*)} \approx 0.016993, (60)$$

$$\rho_0 - \rho^* = X_{M0} - X_M^* - \alpha \left[ \frac{1 - \alpha - L_M^* (\beta - \alpha)}{\alpha L_M^* + (L_M^*)^2 (\beta - \alpha)} \right] (1 + X_M^* - \delta) (L_{M1} - L_{M0}) \approx 0.0149369.$$
(61)

As in the numerical example analyzed in the previous section, one can see from (58) that the relative price of the service tends to grow along a transition path characterized by a declining employment level in the manufacturing sector. Furthermore, by comparing (59) and (60), one can verify that along such a path the

output of the service sector may grow at a higher rate than the output of the manufacturing sector. Again, equation (61) shows that along this transition path the economy's GDP may increase at a higher rate than along the BGP: as along the transition path considered in the Cobb-Douglas case, the economy's rate of growth tends to decline over time as the share of the two factors of production used in the manufacturing sector shrinks.

## 7 CONCLUSIONS

The massive reallocation of resources among sectors and, in particular, the reallocation from manufacturing to services in the industrialized economies which have characterized the latest decades, has induced us to develop a model that can account for these impressive evidence. This formal set-up has permitted to study how aggregate growth is affected by the interaction between technological progress, which is generated endogenously and has a stronger positive impact on the manufacturing sector, and the demand for services, which tends to increase—other things being equal—more than proportionally than total expenditure in consumption.

Indeed, we have presented two numerical examples where it is shown that starting from an initial employment share of the manufacturing sector in overall employment greater than its long-run equilibrium share, the gradual shift of employment shares towards the service sector is accompanied by rates of growth of output and capital stock that are higher in the service sector than in manufacturing. Moreover, along this transition path, the relative price of the service is growing and the economy's GDP tend to grow at a higher rate than along the balanced growth path of the economy: the gradual shift of labor towards the service sector is accompanied by a decline in the aggregate rate of growth. In other words, the pattern resulting from these numerical examples seems to be consistent with the stylized facts.

In addition, we have shown within this analytical framework that positive long-term growth is possible even if what households spend on services tends to increase more than proportionally than their total consumption expenditure, namely when their preferences are non-homothetic: perpetual growth cannot take place only if an excessively large portion of what households spend on consumption is devoted to the service. This implies that tastes and attitudes of households may have relevant consequences for the long-term growth performances of an economy by affecting the composition of consumers' demand. More in

general, one may conclude that every factor affecting the composition of final demand can influence longterm growth. Such important conclusion suggests an interesting extension of this paper: introducing public demand for final products in order to model its effects on growth via its impact on the composition of final demand.

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## APPENDIX

Table 1. Sectoral employment shares.							
	Agriculture	Manuf.	Services		Agriculture	Manuf.	Services
France				Germany			
1901	41.4	31.5	27.1	1907	33.9	39.9	26.2
1949	29.6	33.1	37.3	1950	22.1	44.7	33.2
1960	22.0	36.9	41.1	1960	13.8	47.7	38.5
1970	13.3	38.7	47.9	1970	8.5	48.4	43.1
1980	8.6	35.4	56.0	1980	5.2	42.8	52.0
1990	6.1	29.2	64.6	1990	3.5	39.1	57.4
Italy				Japan			
1901	61.7	22.3	16.0	1906	61.8	16.2	22.0
1951	43.9	29.5	26.7	1950	48.3	22.6	29.0
1960	32.2	36.2	31.6	1960	32.6	29.7	37.6
1970	19.6	38.4	42.0	1970	17.4	35.7	46.9
1980	13.3	36.9	49.2	1980	10.4	35.3	54.2
1990	8.7	31.6	59.7	1990	7.2	34.1	58.7
UK				US			
1901	13.0	43.9	43.1	1900	40.4	28.2	31.4
1951	5.0	47.4	47.6	1950	12.8	31.5	55.7
1961	3.7	48.4	47.9	1960	8.6	30.6	60.8
1970	3.2	44.1	52.7	1970	4.4	33.0	62.6
1980	2.6	37.2	60.3	1980	3.5	29.9	66.6
1990	2.1	28.7	69.2	1990	2.8	25.7	71.5

Sources: OECD, Job Study, 1994.

 Table 2. Changes in Services Shares over time. France, UK., US.

Shares of					
	Services in privat	e consumption.	Services in GDP		
	At current prices At constant prices		At current prices	At constant prices	
France					
1959-60	28.9	36.3	31.3	38.0	
1977-78	37.5	37.7	37.9	36.5	
UK					
1957-58	22.8	35.2	39.5	51.6	
1967-68	29.4	33.6	43.8	50.5	
1977-78	31.6	33.2	47.6	49.7	
US					
1947-48	31.4	39.6	33.2	42.4	
1957-58	38.4	41.5	44.9	49.6	
1967-68	42.2	43.6	48.8	50.8	
1977-78	45.6	45.5	49.7	49.0	
Source Kravin-Heston-Summers (1983)					

		1960-64	1963-68	1968-73	1973-78	1979-83	1983-88	1988-93
France	Man	6.1	5.6	6.4	3.4	2.0	2.0	2.2
	Ser			7.6	6.2	4.3	3.6	4.8
Ger	Man	7.9	6.0	5.5	2.0	1.4	1.2	2.1
	Ser			6.7	5.9	6.2	4.6	4.5
Italy	Man	8.4	4.7	4.9	5.1	3.8	2.5	2.7
Japan	Man	16.5	12.8	13.7	5.8	5.2	5.9	7.7
	Ser			14.9	9.4	7.6	11.9	9.9
UK	Man	3.9	3.8	3.3	2.3	1.7	1.4	0.9
	Ser			4.4	3.0	2.6	3.8	4.5
US	Man	2.7	5.1	4.0	4.1	3.5	2.0	2.5
	Ser			4.7	3.7	4.2	4.7	3.3
Source	Source Glyn (1997)							

Table 3. Manufacturing and services capital stock growth (average annual % changes)

## Table 4. GDP growth.

	GDP growth			
	1960-73	1974-95		
France	5.4	2.2		
Ger	4.4	2.6		
Italy	5.3	2.4		
Japan	9.4	3.2		
UK	3.2	1.8		
EC 12	4.8	2.2		
US	4.3	2.5		
Source Table 2.3, ILO (1996)				

Table 5. Annual labour productivity growth by sector.

	EU-15			US		
	1979-90	1990-95	1995-01	1979-90	1990-95	1995-01
Total Economy	2.2	2.3	1.7	1.4	1.1	2.3
Agr., Forestry, Fish.	5.2	4.8	3.3	6.4	1.7	9.1
Mining, quarrying	2.9	13.1	3.5	4.4	5.1	-0.2
Manufacturing	3.4	3.5	2.3	3.4	3.6	3.8
Elect., gas, water	2.7	3.6	5.7	1.1	1.8	0.1
Construction	1.6	0.8	0.7	-0.8	0.4	-0.3
Distributive trade	1.3	1.9	1.0	1.8	1.5	5.1
Transport	2.8	3.8	2.3	3.9	2.2	2.6
Communications	5.2	6.2	8.9	1.4	2.4	6.9
Financial services	2.2	1.0	2.8	-0.7	1.7	5.2
Business services	0.7	0.7	0.3	0.1	0.0	0.0
Community, social, personal serv.	-0.3	0.4	0.3	1.2	0.9	-0.4
Public Ad., Education, Health	0.6	1.1	0.8	-0.4	-0.8	-0.6
Source: Table 1.4b in O'Mahony-Van Ark (2003)						

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