

Bianchi, Carluccio; Menegatti, Mario

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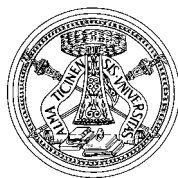
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## **Quaderni di Dipartimento**

### **Rules versus Discretion in Fiscal Policy**

Carluccio Bianchi  
(Università di Pavia)

Mario Menegatti  
(Università di Parma)

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Dipartimento di economia politica  
e metodi quantitativi  
Università degli studi di Pavia  
Via San Felice, 5  
I-27100 Pavia

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# Rules versus Discretion in Fiscal Policy

*Carluccio Bianchi\**

and

*Mario Menegatti\*\**

**Abstract.** This paper purports to apply the Kydland-Prescott framework of dynamic inconsistency to the case of fiscal policy, by considering the trade-off between output and debt stabilization. The Government budget constraint provides the link between debt dynamics and the level of activity, influenced by fiscal policy. Contrary to what happens in the monetary policy framework, however, a commitment is not always superior to discretion, even in the absence of uncertainty, but only when the public debt-GDP ratio is sufficiently large. The introduction of uncertainty, as usual, implies a reduction in the net benefit generated by the adoption of a fixed rule.

**Key words:** rules, discretion, time inconsistency, fiscal policy.

**JEL Classification:** E61, E62, H62, H63, H68.

*\* Dipartimento di Economia Politica e Metodi Quantitativi - Università di Pavia*

*Via S. Felice, 7 – 27100 Pavia*

*Tel. 0382-986212 – Fax 0382-304226*

*Email: [cbianchi@eco.unipv.it](mailto:cbianchi@eco.unipv.it)*

*\*\* Dipartimento di Economia - Università di Parma*

*Via J. F. Kennedy, 6 - 43100 Parma*

*Tel. 0521-032450 - Fax 0521-032402*

*Email: [mario.menegatti@unipr.it](mailto:mario.menegatti@unipr.it)*

## 1. Introduction

The use of fiscal policy as a stabilizing tool implies a typical trade-off between output growth and a deterioration of public finances, possibly leading to an undesirable increase in the public debt-GDP ratio. The negative consequences of this trade-off have been recently experienced by many economies. In Europe, for instance, a few countries violated the budget requirements established by the Growth and Stability Pact in order to stimulate output growth and promote a recovery after a long period of slump. This, however, led to an increase in public debts, relative to GDP, which is particularly worrisome in the case of countries, like Italy, where the current ratio is very high and where a ten-year historical tendency of reduction has been interrupted. A similar experience of budget deterioration and public debt increase has been experienced by the United States, where a strong countercyclical fiscal policy has been adopted since the beginning of the new century, and by Japan, where the long fight against the stagnation begun in the 1990's has led the public debt-GDP ratio to reach an astonishingly high value.

The recent episodes of rising public debts have given a new stimulus to the economic debate about debt sustainability and the opportunity to adopt rules rather than discretion in the conduct of fiscal policy.<sup>1</sup>

A large public finance literature highlights many reasons, of differing nature, behind the suggestion to impose constraints on fiscal policy. First of all, the theory of fiscal constitutionalism, attributable to the Public Choice School (Buchanan and Wagner, 1977), suggests that the Government budget should be balanced because of the distortions and costs implicit in deficit financing. The crowding out argument (Carlson and Spencer, 1975) suggests that the creation of a budget deficit ultimately leads to an increase in the interest rate that displaces productive private investment, while leaving output unchanged. The principle of Ricardian equivalence (Barro, 1974) arrives at the same final suggestion of the desirability of a balanced budget on the theoretical ground that Government deficits are wholly ineffective. The golden rule of public finance, instead, admits the possibility of a public deficit, of an amount equal to the value of net public investment.<sup>2</sup> Finally, the vast

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<sup>1</sup> This debate has been particularly heated in Europe, where, as we said, the prescriptions of the Growth and Stability Pact have been violated and then institutionally relaxed. For a recent position on this debate see also Wyplosz (2005).

<sup>2</sup> It has been suggested that the golden rule of public finance might be one of the rational arguments behind the Maastricht Treaty prescription of a 3% upper limit to the public deficit-GDP ratio, where the actual figure corresponds to the average incidence of public investment on output at the time when the Treaty was

literature on public debt sustainability, which can even be dated back to an old work by Domar (1944), and has returned to public attention with the seminal contribution by Sargent and Wallace (1976), suggests that Government budgets should be targeted to the goal of stabilizing the debt-GDP ratio, in order to avoid the harmful long run and short run consequences of the potential crises connected to an ever-increasing debt-GDP ratio.<sup>3</sup>

The previous discussion outlines the existence of several different arguments behind the suggestion to impose rules on fiscal policy; also, from a practical point of view, specific criteria have been defined in order to identify the desirable features of “good rules” (Kopits and Symansky, 1998). To our knowledge, however, no attempt has been made until now to derive the prescription for imposing constraints on fiscal policy from the “rules versus discretion” framework, as it has been successfully applied to the case of monetary policy (Kydland and Prescott, 1977; Barro and Gordon, 1983).<sup>4</sup> As well known, according to this approach monetary policy should follow fixed rules, rather than discretion, in order to avoid the inflationary bias generated by the monetary authorities incentive to behave opportunistically by trying to stabilize output above the natural level rather than commit themselves to maintaining a low and stable rate of inflation. In a world of rational expectations, this incentive is understood by optimizing agents who fix their labour contracts by anticipating the behaviour of monetary authorities and thus the correct rate of future inflation. The result of this game between rational agents and monetary authorities is the achievement of a suboptimal (third best) equilibrium where output is at its natural level and the rate of inflation is higher than under commitment. Thus monetary

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signed (Buiter, Corsetti and Roubini, 1993). For a recent discussion about the possibility and consequences of the application of the golden rule in EMU cf. Balassone and Franco (2000).

<sup>3</sup> The original argument by Sargent and Wallace warned against the possibility that under fiscal dominance a given sequence of Government deficits would ultimately lead to debt monetization, which would inevitably generate inflation (that under rational expectations would occur immediately). The subsequent debate also emphasized the possibility of Government insolvency and thus the possibility that public debt might be ultimately consolidated or repudiated. Again, in a world of forward-looking agents, this possible long-run outcome would be anticipated and might give rise to short-run financial crises (for a recent reassessment of the debt sustainability issue, cf. Blanchard et al., 1990). It is worth noticing again that one of the possible rationalizations of the Maastricht Treaty requirement on public debt is that, in order to avoid financial crises and bailing-out, every member country of EMU should at least stabilize the debt-GDP ratio: given the existing relationship between debt dynamics and deficit, this would justify the twin prescriptions in terms of public debts and deficits (respectively 60% and 3% of GDP), under the hypothesis that inflation is around 2% and output growth 3% (cf. Buiter, Corsetti and Roubini, 1993). Finally it should be emphasized that limits to fiscal policy have been recently devised not only in EMU, but also in other countries, such as the US, the UK, New Zealand, Chile and Brazil.

<sup>4</sup> A different approach to time inconsistency in fiscal policy is proposed by Alesina and Tabellini (1990) and Persson and Svensson (1989) within a political economy framework of partisan models where political parties have different preferences. We shall say something more on this point in the last section of this paper.

policy is dynamically inconsistent, and this provides a strong argument in favour of a rule constraining the Central Bank to pursue a low and stable inflation rate. This conclusion is only partially mitigated by the consideration of uncertainty: in the presence of supply shocks there is indeed a trade-off between rigour and flexibility, i.e. between the benefits of a fixed rule in reducing the “inflation bias” and the income losses due to the inability of monetary authorities to stabilize output under a more discretionary regime (Rogoff, 1985).

The purpose of this paper is to apply the Kydland-Prescott framework of dynamic inconsistency to the case of fiscal policy, by considering the trade-off between output and debt stabilization under the constraint given by the equation describing debt dynamics, under the hypothesis of the existence of a risk premium on the yield of public bonds tied to the possibility of a default. The model will show, in particular, that, when the Government wants to increase the level of output beyond its equilibrium level, a public announcement to pursue debt stabilization proves to be a second-best solution and involves a problem of time inconsistency. Similarly to what happens in the Barro-Gordon (1983) model, then, the opportunistic incentive to forsake debt stabilization and increase output will be completely anticipated by rational agents, thus generating a “deficit bias”, aggravated by the additional burden to the Government deficit, due to the increase in the interest rate premium required to finance a growing debt-output ratio.

Unlike in the case of monetary policy, however, this outcome does not necessarily imply the univocal conclusion that the adoption of a fixed policy rule is always superior to discretion even in the absence of uncertainty. Indeed it will be shown that, in the case of fiscal policy and in a deterministic framework, fiscal time inconsistency suggests the opportunity to limit Government choices only when the public debt is sufficiently large; on the contrary, discretion is a better solution in the opposite case of a small debt. This result is coherent with the nature of the problem actually met in the implementation of expansionary fiscal policies, whereby the negative effects of such policies on the Government budget and debt are stronger when public finances are already structurally disordered.

Finally, it will be shown that, similarly to what occurs in the dynamic inconsistency framework concerning monetary policy, the introduction of uncertainty implies a reduction in the net benefit generated by the adoption of a fixed rule because of the loss triggered by the commitment to forsake an output-stabilizing discretionary policy in the presence of random shocks.

The rest of the paper proceeds as follows. Section two outlines the main assumptions of the model and examines the elements characterizing Government behaviour. Section three compares the results obtained in the case of a discretionary policy against a commitment to a fixed rule. Section four studies the implications of random shocks. Section five concludes.

## 2. The model

In this section we examine the basic features of the economy and the elements describing Government behaviour. We start by considering the equation describing public debt dynamics, given by the usual Government budget constraint, under the hypothesis that no monetization is possible. Thus:<sup>5</sup>

$$B_t - B_{t-1} = D_t + r_t B_{t-1} \quad [1]$$

where  $B_t$  is the stock of outstanding public debt at time  $t$ ,  $D_t$  is the primary deficit (i.e. the deficit net of interest payments) and  $r_t$  is real interest rate. Relating both sides of eq. (1) to the level of output  $Y_t$ , we get the well-known equation describing the dynamics of the debt-output ratio, given by:<sup>6</sup>

$$\Delta b_t = b_t - b_{t-1} = d_t + (r_t - g_t)b_{t-1} \quad [2]$$

where lower case letters indicate the derived ratios, so that  $b_t = B_t/Y_t$  and  $d_t = D_t/Y_t$ , while  $g_t$  is output growth.

We assume that the real interest rate paid on Government bonds incorporates a risk premium. Under this hypothesis, the bonds yield is given by:

$$r_t = \bar{r}_t + \gamma(\Delta b_t^e) \text{ with } \gamma > 0 \quad [3]$$

where  $\bar{r}_t$  is the rate of interest that would prevail in the absence of a default risk and the function  $\gamma(\Delta b_t^e)$  indicates the risk premium attributable to expected debt growth  $\Delta b_t^e$  (it might be plausible to assume that  $\gamma$  is also an increasing function of  $\Delta b_t^e$ ; this further assumption, however, is not relevant for our results). If agents expect public debt to be

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<sup>5</sup> It is worth noticing that all variables are expressed in real terms; in our framework we do not deal with the determination of the absolute price level which is not relevant for our purposes.

<sup>6</sup> Eq. (2) is obtained under the usual approximation that  $r_t - g_t = (r_t - g_t)/(1 + g_t)$ .

constant or decreasing over time (i.e. if  $\Delta b_t^e \leq 0$ ) then they will exclude the chance of a default and  $\gamma = 0$ . If agents instead expect public debt to increase (i.e. if  $\Delta b_t^e > 0$ ) then there will be a default risk and  $\gamma > 0$ . Agents are assumed to pursue an optimal allocation of wealth among different existing financial assets, which implies, ceteris paribus, a lower demand for public bonds when they turn out to be a risky asset.<sup>7</sup>

We assume that only one good is produced in the economy. We also assume for simplicity that an increase in the Government deficit is determined by a corresponding reduction in taxes. A lower level of taxation generates effects both on the demand and on the supply-side of the economy that can be analysed through the appropriate market-clearing condition:

$$C(T_t, \bar{r}_t + \gamma(\Delta b_t^e)) + I(T_t, \bar{r}_t) + G_t = Y(T_t) \quad [4]$$

where  $C_t$  is consumption,  $I_t$  investment,  $T_t$  taxes,  $G_t$  Government expenditure,  $Y_t$  output, and where  $\frac{dC_t}{dT_t} < 0$ ,  $\frac{dC_t}{d\bar{r}_t} < 0$ ,  $\frac{dI_t}{dT_t} < 0$ ,  $\frac{dI_t}{d\bar{r}_t} < 0$ ,<sup>8</sup> and  $\frac{dY_t}{dT_t} < 0$ .

From the supply-side perspective (right-hand side of eq. (4)), assuming that taxes are at least partially distortionary, a reduction in  $T_t$  will determine an increase in the output level, triggered by a higher labour supply. From the demand-side perspective (left-hand side of eq. (4)) a reduction in taxes will increase both consumption (if Ricardian equivalence does not fully hold, for instance because at least some consumers are myopic (Campbell and Mankiw, 1989) or liquidity constrained) and investment (if entrepreneurial decisions on capital accumulation depend on the internal rate of return net of taxes).<sup>9</sup> Market clearing between demand and supply is ensured by the adjustment of  $\bar{r}_t$ , which will affect both consumption and investment decisions.

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<sup>7</sup> It might be argued that the risk premium on public bonds depends not only upon the expected debt growth but also upon the debt size. This latter effect would be justified by the fact that, given debt dynamics, the probability of default is higher when the public debt is larger. If this additional influence were considered, equation [3] would become  $r_t = \bar{r}_t + \gamma(\Delta b_t^e, b_{t-1})$ ; the new formulation, however, would not change the basic results of the paper, as it can be easily checked from the analysis presented in the next section.

<sup>8</sup> It would be plausible to assume that investment depends upon the specific risk premium paid on private bonds. The size of this premium, however, is not relevant for our analysis.

<sup>9</sup> Similar effects on demand and supply would occur if the assumed increase in the public deficit were due to a rise in Government expenditure, under the hypothesis that Government purchases have a positive influence on input productivity.



If Government spending is supposed to be constant, then eq. (4) implies that<sup>10</sup>:

$$Y_t = f(D_t) \quad \text{with } f' > 0 \quad [5]$$

According to eq. (5), given our previous definitions, we have  $d_t = D_t / f(D_t)$  and  $g_t = [f(D_t) - Y_{t-1}] / Y_{t-1}$ . By substituting these results and eq. (3) in eq. (2) we get then:

$$\Delta b_t = h(D_t) + \gamma (\Delta b_t^e) b_{t-1} \quad [6]$$

where  $h(D_t) = D_t / f(D_t) + b_{t-1} [\bar{r}(D_t) - [f(D_t) - Y_{t-1}] / Y_{t-1}]$ . The equilibrium rate  $\bar{r}$  will be a function of  $D_t$  since, as it follows from the previous arguments, a change in the Government deficit will affect the market clearing condition (4)<sup>11</sup>.

Let us assume now that the Government pursues two different goals: output and debt stabilization<sup>12</sup>. The introduction of the latter objective in the Government loss function may appear unusual; however the assumption can be easily justified if we suppose, as in the debt sustainability literature, that an economy with a growing public debt exhibits a positive risk of default. Since the chance of a default generates disutility to the society as a whole, then debt stabilization may be considered, as it actually is, a fundamental goal for the Government.

Let us define, as usual,  $Y^*$  as the desired level of output and  $\Delta b^*$  as the optimal value of debt dynamics. We assume, as in the Kydland-Prescott framework, that  $Y^*$  is higher than  $Y_t$  because of the existence of distortions in the tax system and in the labour market. In line with previous arguments, we finally assume that  $\Delta b^* = 0$ , so that the Government wants to stabilize the debt-output ratio as a sufficient condition for debt sustainability. Under these assumptions the Government loss function can be written as:

$$L = a [\Delta b_t]^2 + [Y_t - Y^*]^2 \quad [7]$$

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<sup>10</sup> In our model, since the effects of tax cuts on output ultimately depend upon the supply-side of the economy (as eq. (4) clearly shows), the so-called non-Keynesian effects of fiscal policy are irrelevant.

<sup>11</sup> In our model while a tax cut unambiguously raises income, it has no definite effect on the market clearing rate of interest. If however the effect of taxes on demand is higher than that on supply, then the rate of interest will definitely rise. This result is consistent with the empirical findings of Kneller, Bleaney and Gemmel (1999) and Ohaian, Raffo and Rogerson (2006) from the supply-side, and of Perotti (2006) and especially Favero and Giavazzi (2007), from the demand-side.

<sup>12</sup> Actually one might also assume that, especially in the case of countries with a high debt-output ratio, the second goal might be represented by debt reduction; this hypothesis, however, would not alter the conclusions reached under the assumption that the Government pursues debt stabilization.

Substituting eqs. (5) and (6) in (7), we can then express  $L$  as a function of  $D_t$  in the following way:

$$L(D_t) = a \left[ h(D_t) + \gamma (\Delta b_t^e) b_{t-1} \right]^2 + \left[ f(D_t) - Y^* \right]^2 \quad [8]$$

The properties of eq. (8) can be derived from those of function  $h(D_t)$ , whose form depends upon that of function  $f(D_t)$  in eq. (5). In general, one cannot say whether  $h(D_t)$  is an increasing, decreasing or even non-monotone function of  $D_t$  since this will depend upon the effects of an increase in the Government deficit on  $B_t$  and  $Y_t$  respectively.

From now on, however, we shall make an explicit hypothesis about the monotonicity of  $h(D_t)$  by assuming  $h' > 0$  (thus implying that an expansion in the primary deficit will determine an increase in the debt-output ratio). This assumption may be justified on many different grounds. First of all, if it were  $h' < 0$ , then a rise in the primary deficit would cause a reduction in the debt-output ratio; this case does not seem interesting or relevant, since it would imply that the Government, by expanding the public deficit, could at the same time raise growth and reduce the debt-output ratio. Secondly, the assumption that  $h' > 0$  seems to be confirmed both by the actual experience of a few industrial countries and by the results of some recent empirical studies concerning the effects of Government taxation on output.<sup>13</sup>

If  $h' > 0$  is assumed, then minimization of the loss function  $L$  implies:<sup>14</sup>

$$ah'(D_t) \left[ h(D_t) + \gamma (\Delta b_t^e) b_{t-1} \right] = -f'(D_t) \left[ f(D_t) - Y^* \right] \quad [9]$$

Let us label  $D^{opt}$  as the optimal level of  $D_t$  satisfying eq. (9).

Let us define now  $D^{st}$  as the level of  $D_t$  that ensures public debt stabilization, i.e.  $\Delta b_t = 0$ . This level is given by the condition:

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<sup>13</sup> The possible occurrence of Laffer effects connected to the adoption of tax cuts is clearly rejected by the actual experience of the United States, under the Reagan and Bush administrations, and of Japan, both in the 1990's and in the most recent years of the new decade. In these cases tax reductions led to a substantial worsening of the Government deficit and a strong upsurge of the debt-output ratio.

From an empirical standpoint, a series of recent papers purporting to estimate the effects on output of tax reductions (but also of expenditure increases) have come to the conclusion that these effects are small and that they also tend to reduce in size if the most recent years are considered (such as the period after 1980) and if the dynamics of the debt-output ratio is explicitly introduced into the analysis (cf. Blanchard and Perotti (2002), Perotti (2004) and Favero and Giavazzi (2007)).

<sup>14</sup> It is worth noticing that minimization is possible if functions  $h(\cdot)$  and  $f(\cdot)$  ensure that the necessary second order conditions are satisfied. For this reason we also introduce the technical assumptions that  $h'' > 0$  and  $f'' > 0$ .

$$h(D^{st}) = -\gamma(\Delta b_t^e) b_{t-1} \quad [10]$$

It is easy to see that if  $D_t = D^{st}$  then the left-hand side of eq. (9) is zero; hence condition (9) will be satisfied only if its right-hand side is also zero, i.e. if  $Y^* = f(D^{st})$ . In this case, then, the optimal level of Government deficit is also the one that ensures debt stabilisation ( $D^{opt} = D^{st}$ ).

Let us assume now that  $Y^* > f(D^{st})$ . This assumption introduces a trade-off between the goal of reaching the desired output level and that of stabilizing public debt. Indeed, when  $Y^* > f(D^{st})$  the right-hand side of eq. (9) is positive, implying that  $h(D^{opt}) > -\gamma(\Delta b_t^e) b_{t-1}$ . Since  $h(D_t)$  is by definition an increasing function, this implies that  $D^{opt} > D^{st}$ , meaning that the Government chooses to increase the deficit in order to push output closer to its desired level. This behaviour, which stems from the assumed trade-off between the two conflicting objectives of output and debt stabilization, will however generate a case of dynamic inconsistency, as described in the next section.

### 3. Rules versus discretion

Within the framework illustrated in the previous section, we can compare the outcomes of the two alternative cases of a discretionary choice made by the Government and of a rule compelling fiscal policy to stabilize public debt. In the case of discretion we assume that the Government declares that it will pursue public debt stabilization and hence choose the subsequent level of deficit given its preferences described by the loss function  $L(D_t)$ . As in the usual time inconsistency literature, we shall consider both the case in which the Government announcement is trusted (corresponding to the so-called case of “fooling”) and the case where it is not. We shall thus compare three situations:

- 1) Discretion and fooling (i.e.  $D_t = D^{opt} = D^f$  and  $\Delta b_t^e = 0$ )
- 2) Discretion and rational expectations on public debt dynamics (i.e.  $D_t = D^{opt} = D^d$  and  $\Delta b_t^e = \Delta b_t = \Delta b^d$ )
- 3) Commitment (i.e.  $D_t = D^{st} = D^c$  and  $\Delta b_t^e = \Delta b_t = 0$ ).

In the first case the Government announcement of debt stabilization is assumed to be believed. The Government then will choose the optimal level of  $D_t$  under the condition

$\Delta b_t^e = 0$ . From eq. (9) it is easy to see that the optimal choice of  $D_t$  (labelled in this case  $D^f$ ) is given by

$$ah'(D^f)h(D^f) = -f'(D^f)[f(D^f) - Y^*] \quad [11]$$

As previously explained, since  $Y^* > f(D^{st})$ , we have that  $D^f > D^{st}$ . This means that agents expectations about debt stabilization ( $\Delta b_t^e = 0$ ) are wrong, since the Government choice implies that  $\Delta b_t > 0$ . If agents are supposed to have rational expectations, then,  $D^f$  cannot be an equilibrium level of  $D_t$ .

If agents have rational expectations, however, they will not trust the Government announcement and will instead anticipate its actual choice and behaviour. In this case we shall have  $\Delta b_t^e = \Delta b_t$ ,<sup>15</sup> implying that the optimal level of  $D_t$  (labelled in this case  $D^d$ ) is given by the condition:

$$ah'(D^d)[h(D^d) + \gamma(\Delta b_t)b_{t-1}] = -f'(D^d)[f(D^d) - Y^*] \quad [12]$$

Comparing the loss suffered by the Government when  $D_t$  is equal to  $D^f$  or  $D^d$ , it is easy to see that  $L(D^d) > L(D^f)$ . Indeed, since  $\gamma(\Delta b_t)b_{t-1} > 0$  and since  $D^f$  is the optimal choice in the case where  $\Delta b_t^e = 0$ , we have that:

$$\begin{aligned} L(D^d) &= a[h(D^d) + \gamma(\Delta b_t)b_{t-1}]^2 + [f(D^d) - Y^*]^2 > \\ &a[h(D^d)]^2 + [f(D^d) - Y^*]^2 > \\ &a[h(D^f)]^2 + [f(D^f) - Y^*]^2 = L(D^f) \end{aligned} \quad [13]$$

The meaning of this result is straightforward. In equilibrium, agents will anticipate the growth of public debt and will ask for a risk premium that will increase the interest rate paid on public bonds. This additional interest payment will raise the Government loss with respect to the case of fooling.

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<sup>15</sup> In the Kydland-Prescott framework private agents optimal strategy is to do their best to forecast inflation in order to avoid the income losses due to an underestimation of the future change in prices that will influence their real wage in the process of wage bargaining. In our model private agents optimal strategy is to do their best to forecast the future dynamics of public debt in order to avoid the capital loss due to an underestimation of the risk premium embodied in the interest rate that will influence their real return on bonds in the process of deciding the optimal composition of their portfolios.

Finally we can study the case in which the Government has a binding commitment to stabilize the debt-output ratio. In this case the Government will follow the rule  $D_t = D^{st}$  (labelled in this case  $D^c$ ), implying  $\Delta b_t = 0$  and, under rational expectations,  $\Delta b_t^e = 0$ . It is easy to see that the value of the loss function in this case will be:

$$L(D^c) = [f(D^c) - Y^*]^2 \quad [14]$$

We can now compare the value of loss function in the case of discretion  $L(D^d)$ , corresponding to the first row of inequality (13), with that in eq. (14) in order to see whether commitment may reduce the Government loss. The difference between  $L(D^d)$  and  $L(D^c)$  is given by:

$$L(D^d) - L(D^c) = a[h(D^d) + \gamma(\Delta b_t)b_{t-1}]^2 + \left\{ [f(D^d) - Y^*]^2 - [f(D^c) - Y^*]^2 \right\} \quad [15]$$

It is clear that, since  $\Delta b_t > 0$  when  $D_t = D^d$ , then the first term in the right-hand side of eq. (15) is positive while, since  $Y^* > f(D^d) > f(D^c)$ , the second term is negative. This result implies that, in general, we cannot say whether commitment is better or worse than discretion. This occurs because the two choices involve differentiated advantages: discretion entails larger interest payments due to the risk premium but a level of output closer to the desired value, while commitment implies lower interest payments, due to the absence of the risk premium, but also a lower level of output.

It is interesting to notice that this conclusion differs from that obtained in the application of the dynamic inconsistency approach to monetary policy. As pointed out by Barro and Gordon (1983), in the case of monetary policy, discretion generates an inflation bias with an unchanged output; thus, in that framework, commitment is always preferable (the second-best solution). In the case of fiscal policy, the difference in the conclusion is due to the fact that in our model the equilibrium level of output is higher under discretion because of the positive effects of fiscal policy on production.

Although in general commitment can either reduce or increase the loss function, a clear conclusion as to its superiority can be drawn on the basis of the value of the inherited debt-output ratio  $b_{t-1}$ . In fact, eq. (15) implies the following:

*Proposition 1.*  $\exists \hat{b}$ : if  $b_{t-1} < \hat{b}$  then  $L(D^d) > L(D^c)$ , if  $b_{t-1} = \hat{b}$  then  $L(D^d) = L(D^c)$  and if  $b_{t-1} > \hat{b}$  then  $L(D^d) < L(D^c)$  (see Appendix A for the proof).

This result indicates that there is a threshold level of the debt-output ratio  $\hat{b}$  which, when exceeded, makes commitment more desirable. We can therefore conclude that if the debt-output ratio in the economy is small, then discretion is preferable, while, on the contrary, commitment is a better alternative for highly indebted economies.

The interpretation of this result is straightforward. The size of the inherited debt-output ratio ( $b_{t-1}$ ) influences the value of the additional interest payments incurred in the case of discretion. If  $b_{t-1}$  is small, then the increase in interests payments will also be small and the advantage related to a larger level of output under discretion will prevail. On the contrary, if  $b_{t-1}$  is high enough, then the increase in interest payments will also be large and the disadvantage connected to the risk premium paid under discretion will overcome the advantage deriving from a higher level of output.

An inspection of eq. (15) shows that the threshold value of  $\hat{b}$  depends upon many features of the economy and upon Government preferences. Two elements seem however to be more relevant. This leads us to the following result:

*Proposition 2.* *Ceteris paribus the value of  $\hat{b}$  is decreasing in  $\gamma(\Delta b_t)$  and in  $a$ .*

A larger risk premium connected to a non-stabilized debt entails a larger interest cost for each unit of outstanding debt; this implies that a lower threshold debt is sufficient to make discretion disadvantageous. On the other hand, a Government more interested in debt stabilization (i.e. having a higher  $a$ ) makes the relative cost of commitment lower; this implies that discretion is disadvantageous only if the total cost due to the risk premium is also lower, i.e. if, ceteris paribus,  $\hat{b}$  is reduced.

The conclusions just reached about the relative desirability of discretion and commitment can be illustrated with the help of a graphical analysis (figures 1 and 2).

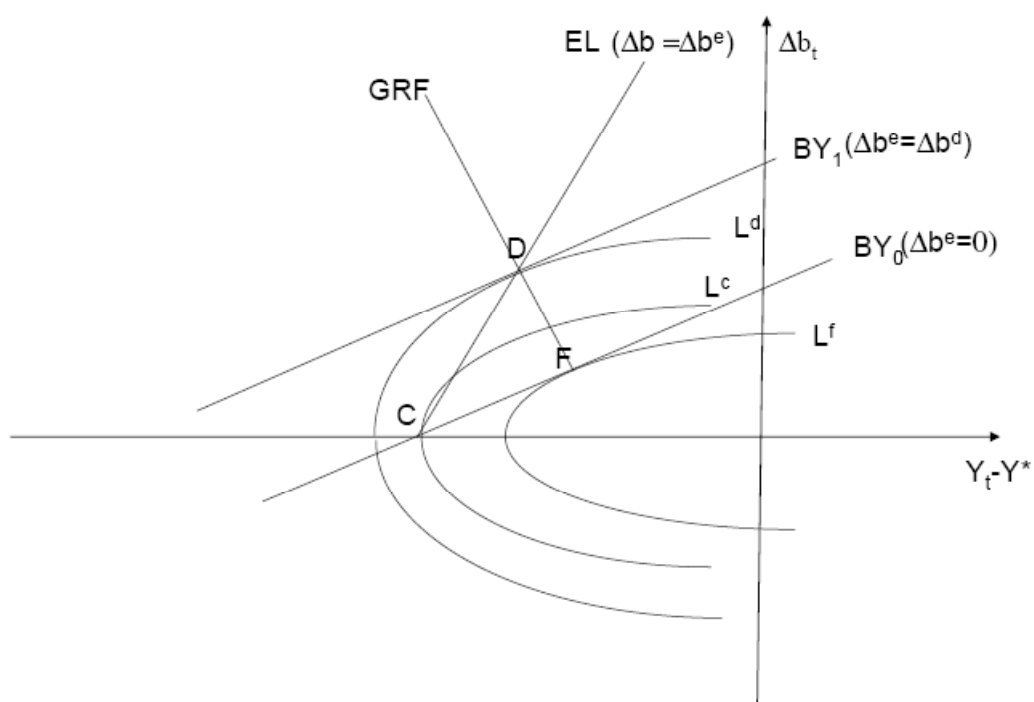


Figure 1. Possible outcomes when commitment is preferable.

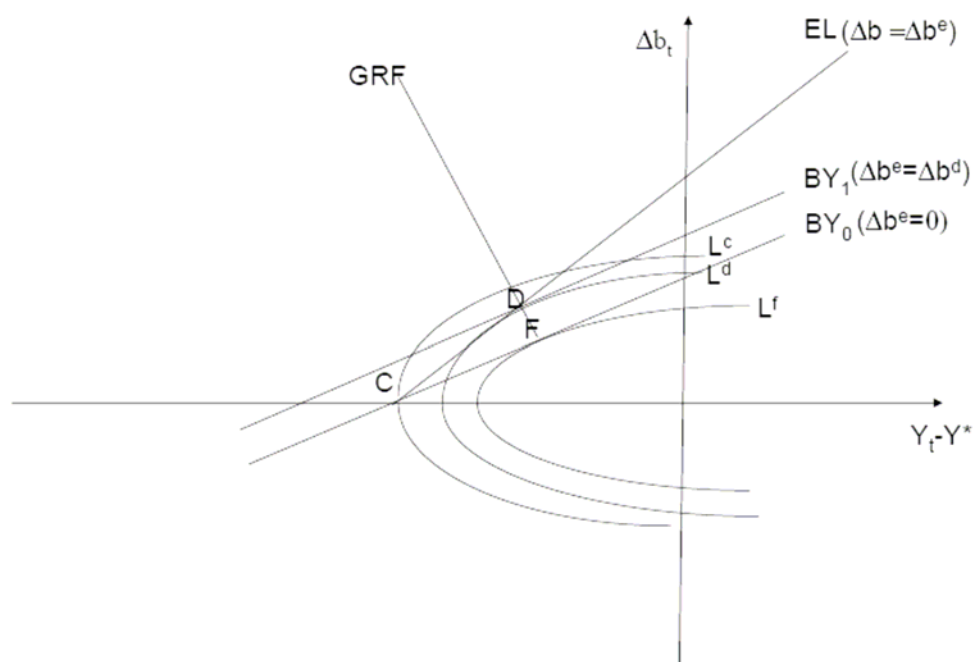


Figure 2. Possible outcomes when discretion is preferable.

The BY schedules represent the trade-off between public debt stabilization and output implied by equations (5) and (6).<sup>16</sup> The two schedules drawn refer respectively to the case where the expected debt growth is null ( $\Delta b_t^e = 0$ :  $BY_0$ ) and to the case where the expected debt growth corresponds to the discretionary outcome ( $\Delta b_t^e = \Delta b^d$ :  $BY_1$ ); thus the distance between any two curves is given by the additional interest payments paid on Government bonds, due to a varying risk premium.<sup>17</sup> Along each BY schedule agents expectations about debt dynamics are given and may be wrong; the EL schedule, instead, represents the locus of all possible equilibria under rational expectations, i.e. the locus of all combinations between  $\Delta b$  and  $Y$  for which  $\Delta b_t^e = \Delta b_t$ .<sup>18</sup> As a consequence of the assumptions made, the slope of the EL schedule is steeper than that of the BY schedules, and will, ceteris paribus, be increasing in the size of the risk premium and the value of Government debt.<sup>19</sup> In the two figures the L curves represent the Government indifference (or iso-loss) curves: along each curve Government loss is unchanged, while a curve closer to the axes origin involves a lower loss.

In the case of discretion the Government declares that it will stabilize public debt and choose the level of  $D_t$  (and thus the levels of  $\Delta b_t$  and  $Y_t$ ) so as to minimize its loss. If the Government is trusted, and agents may therefore be fooled, the equilibrium that will be reached is represented by point F in figure 1 and figure 2. In point F, however, the deficit is too large for debt stabilization and public debt grows; this implies that agents

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<sup>16</sup> Thus the BY schedules are defined by the condition  $\Delta b_t = h \left[ f^{-1}(Y_t) \right] + \gamma \left( \Delta b_t^e \right) b_{t-1}$ . In the graphs, for the sake of simplicity, the schedules are assumed to be linear, as if they were of the form  $\Delta b_t = \alpha + \beta Y_t + \gamma b_{t-1} \Delta b_t^e$ .

<sup>17</sup> Actually there are infinite BY schedules, one for every value of agents expectations about public debt dynamics, and the distance between any two BY schedules is given by  $\gamma \left( \Delta b_t^e \right) b_{t-1}$ . In our framework, therefore, the BY schedules are the equivalent of the expectations-augmented Phillips curves used in the traditional time-inconsistency approach to monetary policy.

<sup>18</sup> Thus the EL schedule is given by the condition  $\Delta b_t = h \left[ f^{-1}(Y_t) \right] + \gamma \left( \Delta b_t \right) b_{t-1}$ , which, in the linear case, would take the form  $\Delta b_t = \frac{\alpha}{1-\gamma b_{t-1}} + \frac{\beta}{1-\gamma b_{t-1}} Y_t$ . In our framework, therefore, the EL schedule is the equivalent of the long-run vertical Phillips curve.

<sup>19</sup> In the general case the slope of the EL schedule would be given by  $\frac{h' f^{-1}}{1-\gamma' b_{t-1}}$ , which, in the linear case, would simplify to the expression  $\frac{\beta}{1-\gamma b_{t-1}}$ . Assuming  $\gamma' b_{t-1}$  (or  $\gamma b_{t-1}$  in the linear case) to be less than one, the slope of the EL schedule is necessarily steeper than that of the BY schedules.



expectations ( $\Delta b_t^e = 0$ ) are wrong. Since, however, rational agents will anticipate that the Government will renege on its announcement, the statement will not be trusted and agents will expect public debt to grow ( $\Delta b_t^e = \Delta b_t = \Delta b^d$ ). This rational expectation implies that equilibrium under discretion will be in point D instead of point F. The loss suffered in point D is always larger than that in point F, as shown in both figures 1 and 2, since point F represents the Government first-best outcome.

In the case of commitment the Government is compelled to stabilize public debt by choosing  $D=D^c$ . Because of the commitment, the Government is trusted and the equilibrium will be in point C along the  $BY_0$  line. The comparison between points C and D confirms that in general we cannot say whether commitment is better or worse than discretion. However, as illustrated in the previous discussion and as stated in Proposition 1, the comparison between the loss suffered in points C and D will, *ceteris paribus*, depend upon the size of  $b_{t-1}$ . If  $b_{t-1}$  is large enough, then the slope of the EL schedule will be steeper and the distance between the two BY schedules will be wider, so that the loss in point D will be higher than in point C, making commitment preferable (figure 1). The opposite situation will occur, *ceteris paribus*, when  $b_{t-1}$  is small, so that the slope of the EL schedule will be flatter and the distance between the two BY schedules narrower: in that case the loss in point D will be smaller than in point C, making discretion preferable (figure 2).<sup>20</sup>

#### 4. A possible extension of the basic model: the effects of exogenous shocks

The model studied in section three assumes that there are no random shocks in the economy. The time inconsistency literature applied to monetary policy, however, shows that the presence of uncertainty is important in influencing the conclusions about the relative desirability of rules versus discretion (cf. Rogoff, 1985). In that framework, in particular, the existence of shocks provides an argument in favour of discretion since a countercyclical monetary policy can reduce the output loss triggered by an adverse supply shock. In this section we show that a partially similar conclusion is true also with

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<sup>20</sup> It is worth noticing that, as in the traditional time inconsistency framework applied to monetary policy, the discretionary outcome may be derived, in a game-theory approach, as the intersection between the private agents reaction function, given by the EL schedule, and the Government reaction function, given by the GRF schedule, which is defined as the locus of all tangency points between the iso-loss curves and the BY schedules.

reference to fiscal policy, since the threshold level of debt  $\hat{b}$ , under which discretion is superior to commitment, turns out to be higher under uncertainty.

In order to draw this conclusion we re-examine the model of the previous section and introduce the possibility of exogenous random shocks. Therefore we assume that eq. (5) is substituted by:

$$Y_t = f(D_t) + \tilde{\varepsilon}_t \quad [16]$$

where  $\tilde{\varepsilon}$  is a random variable with a null expected value and a known variance  $\sigma^2$ , which can represent either a demand or a supply shock.

The Government expected loss function in this case becomes:

$$E[L(D_t)] = E \left\{ a \left[ h(D_t) + \gamma(\Delta b_t^e) b_{t-1} \right]^2 + \left[ f(D_t) - Y^* + \tilde{\varepsilon}_t \right]^2 \right\} \quad [17]$$

Let us suppose now, as is normally done in the time inconsistency literature, that agents form their expectations before the realization of  $\tilde{\varepsilon}_t$ , while the Government chooses  $D_t$  after this realization. Let us label  $D^u$  the value of  $D_t$  minimizing the loss function. Its value is determined by the condition:

$$ah'(D^u) \left[ h(D^u) + \gamma(\Delta b_t^e) b_{t-1} \right] = -f'(D^u) \left[ f(D^u) - Y^* + \varepsilon_t \right] \quad [18]$$

where  $\varepsilon_t$  is the realization of the random variable  $\tilde{\varepsilon}_t$ .

It is worth noticing that the term  $\gamma(\Delta b_t^e) b_{t-1}$  in eq. (18) is the same as in the case of discretion without uncertainty (eq. (12)), since agents do not know the realization of  $\varepsilon_t$  and form their expectations by setting  $E\tilde{\varepsilon}_t = 0$ .

A comparison between eqs. (12) and (18) shows that:

$$D^u = D^d + \varphi(\varepsilon_t) \quad [19]$$

where  $\varphi(0) = 0$ ,  $\varphi(\varepsilon_t) < 0$  when  $\varepsilon_t > 0$ ,  $\varphi(\varepsilon_t) > 0$  when  $\varepsilon_t < 0$ , and  $\varphi'(\varepsilon_t) < 0$ . Obviously, when there are no shocks, the optimal Government choice is the same as that reached in the absence of uncertainty. When there is a positive shock, instead, the output level will be larger (and closer to the desired level), so that the Government will reduce the deficit in order to decrease the growth of public debt. When there is a negative shock, the opposite conclusion holds.

In order to see the effects of uncertainty upon the relative desirability of rules versus discretion we have to compare the difference  $L(D^d) - L(D^c)$  with  $E[L(D^u)] - E[L(D^c)]$ , where  $E[L(D^u)]$  is the expected loss when  $D_t = D^u$ , and  $E[L(D^c)]$  is the expected loss under commitment. This comparison leads to the following result:

*Proposition 3. Ceteris paribus, the threshold level  $\hat{b}$ , defined in Proposition 1, is higher under uncertainty (see Appendix B for the proof).*

The explanation of proposition 3 is quite simple. The presence of (an adverse) shock affects the level of output by generating a cost that is considered in the loss function. Under discretion the Government can reduce this cost by choosing an optimal policy that takes into account the effect of the shock; under commitment, the whole cost of uncertainty is instead borne by the economy. For this reason, random shocks introduce a new element in favour of discretion, additional to the advantage related to the output level described in Section three. Hence the overall advantage associated to discretion is larger under uncertainty, implying that commitment is preferable only if the loss associated with discretion is larger too, which happens if, ceteris paribus, the risk premium embodied in the interest rate is higher; this situation occurs only if the initial debt level is sufficiently high, and anyway higher than without uncertainty.

The additional advantage associated to discretion in the presence of uncertainty can be illustrated with the help of figure 3. The occurrence of exogenous random shocks, impacting on output, will determine a corresponding shift in the position of the BY schedules, which will range from  $BY_{i-\varepsilon}$  and  $BY_{i+\varepsilon}$  (with  $i=0,1$ ). A Government committed to stabilize public debt will not take any action to offset the consequences of the shock, so that output will fluctuate between  $C'$  and  $C''$ . On the other hand, a Government acting under discretion can take measures to partially stabilise output by choosing the optimal position situated along the GRF line, between  $D_1$  and  $D_2$ . Hence under discretion output variability will be limited in the range  $D' - D''$ , which is much smaller than the analogous range  $C' - C''$ , occurring under commitment.

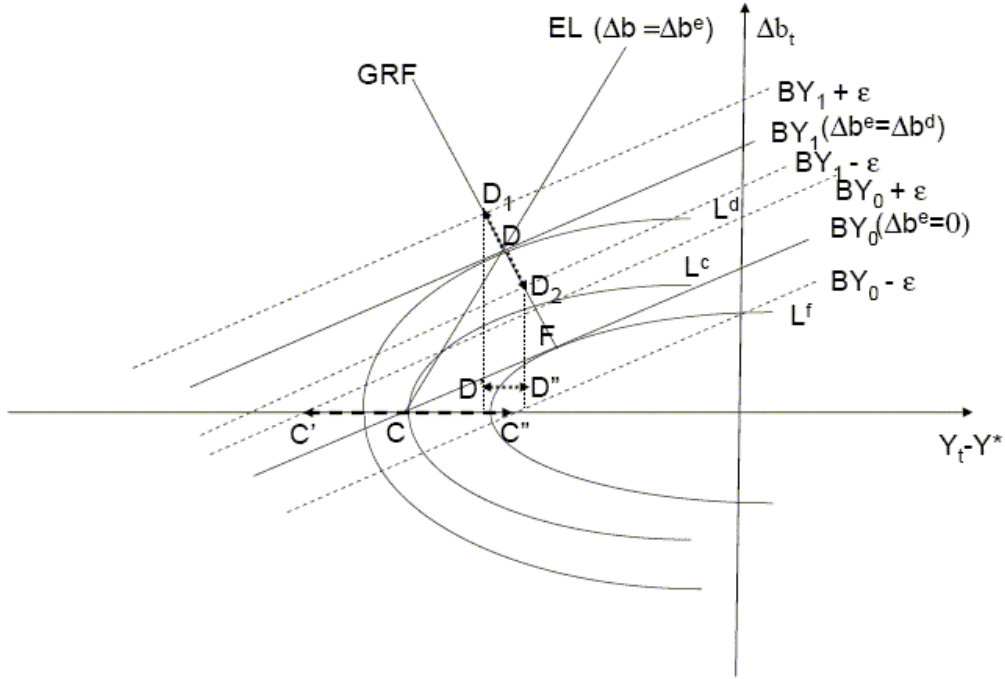


Figure 3. The trade-off between commitment and flexibility under uncertainty.

## 5. Conclusions

This paper studied the relative advantages and costs of rules versus discretion in the conduct of fiscal policy within a framework in which agents' expectations about Government choices affect the risk premium paid on public bonds. The achieved results are relevant under many perspectives.

In the first place, in the proposed model, the Government will choose a level of the public deficit larger than announced because of its desire to increase output beyond the current equilibrium level. Since this time-inconsistent behaviour is correctly anticipated by rational agents, it will generate a “deficit bias”, due to the increase in the risk premium paid on public bonds, as a consequence of the debt growth entailed by the increased deficit. This “deficit bias” is analogous to the “inflation bias” generated by monetary policy in the traditional time inconsistency literature and provides a new argument in the analysis of the reasons behind the emergence of public deficits. In fact, Alesina and Tabellini (1990) show that a “deficit bias” can arise when there are differences in political parties' preferences about the composition of public expenditure, such that an incumbent

Government will not fully internalize the cost of bequeathing debt to the new entrant. Persson and Svensson (1989) show that a conservative Government will run a larger deficit if it knows that it will be replaced by a liberal one. This paper shows another potential source of a deficit bias, due to the ability of financial markets to incorporate the effects of time inconsistent Government behaviour into the risk premium paid on public bonds.

In second place, the deficit bias generated by fiscal dynamic inconsistency does not imply that the equilibrium under discretion is always inferior to that under commitment, because output is different in the two cases. Since the value of the deficit depends on the level of the initial public debt, the relative desirability of either commitment or discretion depends upon that level as well. This result is different from that obtained in the case of monetary policy where, without uncertainty, commitment is always preferable.

The introduction of uncertainty in the model does not change the conclusions just reached. However, the threshold level of public debt that makes commitment preferable is, in the case of uncertainty, higher than in the absence of exogenous random shocks.

Furthermore, from a normative point of view, our results highlight the elements necessary to identify the economies that will benefit from a commitment to fiscal policy. In this perspective, we point out a new reason why economies with a large amount of outstanding public debt should fix binding rules to their fiscal policy, while, on the contrary, discretion would be a better alternative for economies endowed with a small stock of public debt.<sup>21</sup>

Finally, our analysis also shows that the advantages of commitment are lower if the risk premium connected to a non-stabilized debt is small or if the financial markets response to a change in expected debt dynamics is not very large. The benefit of a fiscal policy rule also depends on Government (or society) preferences and is greater if the relative importance assigned to the debt stabilization goal is higher.

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<sup>21</sup> It is perhaps worth noticing that, with reference to the recent experience of European countries, our conclusions suggest that the softening of the Growth and Stability Pact will be disadvantageous for high-debt economies and favourable for the others.

## References

- Alesina, A. and Tabellini G. (1990), A Positive Theory of Fiscal Deficits and Government Debt, *Review of Economic Studies*, v. 57 (3), pp. 403-14.
- Balassone, D. and Franco D. (2000), Public Investment, the Stability Pact and the Golden Rule”, *Fiscal Studies*, v. 21 (2), pp. 207-229.
- Barro, R. J. and Gordon D. B. (1983), A Positive Theory of Monetary Policy in a Natural Rate Model, *Journal of Political Economy*, v. 91 (4), pp. 589-610.
- Blanchard, O. and Perotti R. (2002), An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output, *The Quarterly Journal of Economics*, v. 1178 (4), pp. 1329-1368.
- Blanchard, O. et al. (1990), The Sustainability of Fiscal Policies: New Answers to an Old Question, *OECD Economic Studies*, v. 15, pp. 7-34.
- Buchanan, J. and Wagner R. (1977), *Democracy in Deficit*, Academic Press, New York
- Buiter, W., Corsetti G. and Roubini N. (1993), Excessive Deficits: Sense and Nonsense in the Treaty of Maastricht, *Economic Policy*, v. 8 (16), pp. 57-100.
- Campbell, J. Y. and Mankiw G. N. (1989), Consumption, Income and Interest Rates: Reinterpreting the Time Series Evidence, *NBER Macroeconomics Annual*, v. 4, pp. 185-216.
- Carlson, K. M. and Spencer R.W. (1975), Crowding-out and its Critics, *Federal Reserve of St. Louis Review*, v. 57 (2), pp. 2-17.
- Davis, S. J. and Henrekson M. (2004), Tax Effects on Work Activity, Industry Mix and Shadow Economy Size: Evidence from Rich-country Comparisons, *NBER Working Paper Series*, n. 10509.
- Domar, E.D. (1944), The Burden of the Debt and the National Income, *American Economic Review*, v. 34 (4), pp. 798-827.
- Favero, C. and Giavazzi F. (2007), Debt and the Effects of Fiscal Policy, *NBER Working Paper Series*, n. 12822.
- Kneller R., Bleaney M. F. and Gemmel N. (1999), Fiscal Policy and Growth: Evidence from OECD Countries, *Journal of Public Finance*, v. 74, pp. 171-190.
- Kopits, G. and Symansky S. (1998), Fiscal Policy Rules, *IMF Occasional Paper*, n.162.
- Kydland, F. E. and Prescott E. C. (1977), Rules Rather Than Discretion: The Inconsistency of Optimal Plans, *Journal of Political Economy*, v. 85 (3), pp. 473-91.

- Ohanian L., Raffo A. and Rogerson R. (2006), Long-term Changes in Labor Supply and Taxes: Evidence from OECD Countries, 1956-2004, *NBER Working Paper Series*, n. 12786.
- Perotti, R. (2004), Estimating the Effects of Fiscal Policy in OECD Countries, *IGIER Working Papers*, Bocconi University.
- Persson, T. and Svensson, L. E. O. (1989), Why a Stubborn Conservative Would Run a Deficit: Policy with Time-Inconsistent Preferences, *Quarterly Journal of Economics*, v. 104 (2), pp. 325-45.
- Rogoff, K. (1985), The Optimal Degree of Commitment to an Intermediate Monetary Target, *Quarterly Journal of Economics*, v. 100 (4), pp. 1169-1190.
- Sargent, T. J. and Wallace N. (1985), Some Unpleasant Monetarist Arithmetic, *Federal Reserve Bank of Minneapolis Quarterly Review*, v. 9 (1), pp. 15-31.
- Wyplosz, C. (2005), Fiscal Policy: Institutions versus Rules, *National Institute Economic Review*, v. 191, pp. 64-78.

## Appendix A. Proof of Proposition 1.

A simple inspection of eq. (15) shows that the difference  $L(D^d) - L(D^c)$  is positive for  $b_{t-1} \rightarrow \infty$  and is an increasing function of  $b_{t-1}$ .

The further conclusion that the difference  $L(D^d) - L(D^c)$  is negative for  $b_{t-1} = 0$  can be derived from the following observations. First, for  $b_{t-1} = 0$  we have no effect of the risk premium on debt growth (the term  $\gamma(\Delta b_t)b_{t-1}$  is zero); this implies that commitment does not reduce the interest payments with respect to discretion. Given this result, and since  $D^d$  is, by assumption, the optimal choice of the Government given the loss function, we have that  $L(D^d) \leq L(D^c)$ . Finally, under the hypothesis that  $Y^* > L(D^c)$ ,  $D^c$  cannot be the optimal level of  $D_t$ . This excludes the possibility that  $L(D^d) = L(D^c)$ .

The three results we have just shown, according to which the difference  $L(D^d) - L(D^c)$  is negative for  $b_{t-1} = 0$ , positive for  $b_{t-1} \rightarrow \infty$  and increasing in  $b_{t-1}$ , prove Proposition 1.



### Appendix B. Proof of Proposition 3.

We first prove that ceteris paribus, for any given value of  $b_{t-1}$ ,  $E[L(D^u)] - E[L(D^c)]$  is lower than  $L(D^d) - L(D^c)$ . Given the expected loss function (17) we have:

$$E[L(D^u)] = E\left[a\left[h(D^u) + \gamma(\Delta b_t^c)b_{t-1}\right]^2 + \left[f(D^u) - Y^* + \tilde{\varepsilon}\right]^2\right] \quad [20]$$

where  $D^u$  satisfies condition (18) in section four. The expected loss under commitment is instead:

$$E[L(D^c)] = E\left[\left[f(D^c) - Y^* + \tilde{\varepsilon}\right]^2\right] = L(D^c) + \sigma^2 \quad [21]$$

where the last equality holds since  $E\tilde{\varepsilon} = 0$ .

In order to determine  $E[L(D^u)] - E[L(D^c)]$  we first compute the expected loss obtained when the Government chooses  $D^d$  (determined by condition (12)) under uncertainty. Since  $E\tilde{\varepsilon} = 0$  we get:

$$E[L(D^d)] = E\left[a\left[h(D^d) + \gamma(\Delta b_t^c)b_{t-1}\right]^2 + \left[f(D^d) - Y^* + \tilde{\varepsilon}\right]^2\right] = L(D^d) + \sigma^2 \quad [22]$$

Eqs. (21) and (22) imply that  $E[L(D^d)] - E[L(D^c)] = L(D^d) - L(D^c)$ . However, since  $D^u$  is different from  $D^d$  (cf. eq. (19)) and is the optimal choice under uncertainty we have that  $E[L(D^d)] > E[L(D^u)]$  implying that  $L(D^d) - L(D^c) > E[L(D^u)] - E[L(D^c)]$  for any given value of  $b_{t-1}$ .

This conclusion has relevant implications for the comparison between the value of  $\hat{b}$  under certainty ( $\hat{b}^c$ ) or uncertainty ( $\hat{b}^u$ ). Indeed, for  $b_{t-1} = \hat{b}^c$ , we have  $L(D^d) = L(D^c)$  and  $E[L(D^c)] > E[L(D^u)]$ . Since  $E[L(D^u)]$  is increasing in  $b_{t-1}$  this implies that  $\hat{b}^u > \hat{b}^c$ .