

Suzuki, Soushi; Nijkamp, Peter

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## A Generalized Goals-achievement Model in Data Envelopment Analysis

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# A Generalized Goals-achievement Model in Data Envelopment Analysis

*Soushi Suzuki<sup>a</sup>*

*Peter Nijkamp<sup>b</sup>*

<sup>a</sup> *Hokkai-Gakuen University, Sapporo, Japan;*

<sup>b</sup> *VU University Amsterdam, and Tinbergen Institute.*

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**Tinbergen Institute Amsterdam**

Roetersstraat 31  
1018 WB Amsterdam  
The Netherlands  
Tel.: +31(0)20 551 3500  
Fax: +31(0)20 551 3555

**Tinbergen Institute Rotterdam**

Burg. Oudlaan 50  
3062 PA Rotterdam  
The Netherlands  
Tel.: +31(0)10 408 8900  
Fax: +31(0)10 408 9031

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# **A generalized goals-achievement model in data envelopment analysis: An application to efficiency improvement in local government finance in Japan**

Soushi Suzuki<sup>a</sup>

Peter Nijkamp<sup>b</sup>

<sup>a</sup>*Hokkai-Gakuen University, Department of Civil and Environmental Engineering, South 26-West 11, 1-1, Chuo-ku, 064-0926 Sapporo, Japan*

<sup>b</sup>*VU University Amsterdam, Department of Spatial Economics, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands*

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## **Abstract**

Data Envelopment Analysis (DEA) has become an established tool in comparative analyses of efficiency strategies in both the public and the private sector. The aim of this paper is to present and apply a newly developed, adjusted DEA model – emerging from a blend of a Distance Friction Minimization (DFM) and a Goals Achievement (GA) approach on the basis of the Charnes-Cooper-Rhodes (CCR) method – in order to generate a more satisfactory efficiency-improving projection model in conventional DEA.

Our DFM model is based on a generalized Euclidean distance minimization and serves to assist a Decision Making Unit (DMU) in improving its performance by the most appropriate movement towards the efficiency frontier surface. Standard DEA models use a uniform proportional input reduction or a uniform proportional output increase in the improvement projections, but our DFM approach aims to generate a new contribution to efficiency enhancement strategies by deploying a weighted projection function. In addition, at the same time, it may address both input reduction and output increase as a strategy of a DMU. A suitable form of multidimensional projection functions that serves to improve efficiency is given by a Multiple Objective Quadratic Programming (MOQP) model using a Euclidean distance.

Another novelty of our approach is the introduction of prior goals set by a DMU by using a GA approach. The GA model specifies a goal value for efficiency improvement in a DFM model. The GA model can compute the input reduction value or the output increase value in order to achieve a pre-specified goal value for the efficiency improvement in an optimal way. Next, using the integrated DFM-GA model, we are able to develop an operational efficiency-improving projection that provides a clear, quantitative orientation for the actions of a DMU.

The above-mentioned DFM-GA model is illustrated empirically by using a data set of efficiency indicators for cities in Hokkaido prefecture in Japan, where the aim is to increase the efficiency of local government finance mechanisms in these cities, based on various input and output performance characteristics. In summary, this paper presents a practical policy instrument that may have great added value for the decision making and planning of both public and private actors.

**Keywords:** Distance Friction Minimization, Goals Achievement, Data Envelopment Analysis (DEA), Efficiency-improving Projection, Local Government Finance

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\*Corresponding author: Soushi Suzuki (e-mail address: soushi-s@cvl.hokkai-s-u.ac.jp)

## 1. Introduction

In recent years, the public sector has been under increasing pressure to increase its efficiency, through innovative strategies (see, e.g., Windrum and Koch, 2008). To this end, it is necessary to use reliable and operational methods that can be used for benchmark and performance analysis. Data Envelopment Analysis (DEA) has become an established approach in the analysis of efficiency problems in both the public and the private sector. A large number of studies show that efficiency analysis is an important but difficult topic. DEA was developed to analyse the relative efficiency of 'Decision Making Units' (DMUs) by constructing a piecewise linear production frontier, and projecting each agent (DMU) onto the frontier. A DMU that is located on the frontier is efficient, while a DMU that is not on the frontier is inefficient. An inefficient DMU can become efficient by reducing its inputs (or increasing its outputs). In the standard DEA approach, this is achieved by a uniform reduction in all inputs (or uniform increase in all outputs). But in principle, there are an infinite number of improvements to reach the efficient frontier, and hence there are many solutions for a DMU to enhance efficiency.

The existence of an infinite number of solutions to reach the efficient frontier has led to a stream of literature on the integration of DEA and Multiple Objective Linear Programming (MOLP), which was initiated by Golany (1988). In short, this literature proposes trajectories to efficiency by taking into account the preferences of the decision maker (DMU). Thus, the challenge is now to develop a methodology for projecting DMUs on the efficient frontier that does not include subjective valuations.

Suzuki et al. (2007a, 2007b, 2007c) proposed a Distance Friction Minimization (DFM) model in a DEA model that is based on a generalized distance friction function and serves to assist a DMU in improving its performance by an appropriate movement towards the efficiency frontier surface. This DFM approach aims to generate a new contribution to efficiency enhancement strategies by deploying a weighted projection function, and at the same time it may address both input reduction and output increase as a strategy of a DMU. A suitable form of multidimensional projection functions that serves to improve efficiency is given by a Multiple Objective Quadratic Programming (MOQP) model in conformity with a Euclidean distance.

A general efficiency-improving projection model in combination with our DFM model is able to calculate either an input reduction value or an output increase value to reach an efficient score 1.000, although in reality this may be hard to achieve.

The aim of this paper is to present and apply a newly developed, adjusted DEA model – emerging from a blend of a Distance Friction Minimization (DFM) and a Goals Achievement (GA) approach on the basis of the Charnes-Cooper-Rhodes (CCR) method – in order to generate a more appropriate efficiency-improving projection model in conventional DEA. The GA model specifies a Goal Improvement Rate (GIR) of the total efficiency gap in the framework of a DFM model. The GA model can compute an input reduction value or an output increase value in order to achieve a prior goal value for the efficiency improvement in an optimal way.

The above-mentioned CCR-DFM-GA model will be empirically illustrated by using a data set of cities in Hokkaido prefecture in Japan, where the aim is to increase the efficiency of local government finance, based on various input and output performance characteristics of these cities. The relevance of our approach can be illustrated by referring to recent public financial deficits in Yubari city in Hokkaido prefecture, which was close to financial bankruptcy in March 2007. In particular, the White Paper on local public finance (Ministry of Internal Affairs and Communications 2007) illustrated clearly that the issue of the public financial deficits of cities and prefectures is an urgent concern in Japan. This paper thus proposes a policy instrument that may have great added value for the decision making and planning of public finance actors.

The paper is organized as follows. Section 2 discusses DEA and efficiency-improvement projection methods. Next, Section 3 introduces our DFM methodology, while Section 4 proposes the new model which is a GA model in the framework of a DFM model. Section 5 then presents an application of the methodology to a comparative study of local government finance efficiency analysis in Japan. Finally, Section 6 draws some conclusions.

## 2. Efficiency Improvement Projection in DEA

The original formulation for DEA was given by Farrell (1957), who aimed to develop a measure for production efficiency. This work was elaborated by Charnes et al. (1978), who presented a quantitative measure for assessing the relative efficiency of DMUs in the case of a frontier method that aims to determine the maximum volume of outputs, given a set of inputs. In this framework, it is possible to assess ex post the (in)efficiency of a production system using the distance to the production frontier (without any explicit assumptions on the production technology concerned). This is usually a deterministic analysis, which has a close resemblance to non-parametric linear programming. Over the years, DEA has become an operational tool for analysing efficiency problems in both the private and the public sector, where (in)efficiency is interpreted as the relative distance from an actual situation to the optimal production frontier function.

DEA has been fully developed by Charnes et al. (1978) and later on by Banker et al. (1984) to analyse the efficient operation of DMUs, as well as to determine improvements of inefficiency by means of an appropriate projection choice of a DMU, based on the ratio of the weighted sum of outputs to the weighted sum of inputs, given the requirement that these ratios are less than (or equal to) 1 for each DMU under consideration. The main goal is to determine in numerical terms the weights associated with each DMU in such a way that it may maximize the improvement of its efficiency. The Charnes et al. (1978) model (abbreviated hereafter as the CCR-input model) for a given DMU<sub>*j*</sub> ( $j = 1, \dots, J$ ) to be evaluated in any trial generally designated as DMU<sub>*o*</sub> (where *o* ranges over 1, 2 ..., *J*) may then be represented as the following fractional programming (*FP<sub>o</sub>*) problem:

$$\begin{aligned}
(FP_o) \quad & \max_{v,u} \quad \theta = \frac{\sum_s u_s y_{so}}{\sum_m v_m x_{mo}} \\
\text{s.t.} \quad & \frac{\sum_s u_s y_{sj}}{\sum_m v_m x_{mj}} \leq 1 \quad (j = 1, \dots, J) \\
& v_m \geq 0, \quad u_s \geq 0,
\end{aligned} \tag{2.1}$$

where  $\theta$  is an objective variable (efficiency score);  $x_{mj}$  is the volume of input  $m$  ( $m=1, \dots, M$ ) for DMU $_j$  ( $j=1, \dots, J$ );  $y_{sj}$  is the output  $s$  ( $s=1, \dots, S$ ) of DMU $_j$ ; and  $v_m$  and  $u_s$  are the weights given to input  $m$  and output  $s$ , respectively.

Model (2.1) is often called an input-oriented CCR model, while its reciprocal (i.e. an interchange of the numerator and denominator in objective function (2.1), with a specification as a minimization problem under an appropriate adjustment of the constraints) is usually known as an output-oriented CCR model. Model (2.1) is obviously a fractional programming model, which may be solved stepwise by first assigning an arbitrary value to the denominator in (2.1), and then maximizing the numerator. But it is preferable to transform (2.1) into a linear programming model, as shown below.

The CCR model (2.1) can be shown to have the following equivalent linear programming ( $LP_o$ ) specification for any DMU $_j$ :

$$\begin{aligned}
(LP_o) \quad & \max_{v,u} \quad \theta = \sum_s u_s y_{so} \\
\text{s.t.} \quad & \sum_m v_m x_{mo} = 1 \\
& - \sum_m v_m x_{mj} + \sum_s u_s y_{sj} \leq 0 \\
& v_m \geq 0, \quad u_s \geq 0.
\end{aligned} \tag{2.2}$$

The dual problem of (2.2),  $DLP_o$ , can be expressed by means of a real variable  $\theta$ , using the following vector notation:

$$\begin{aligned}
(DLP_o) \quad & \min_{\theta, \lambda} \quad \theta \\
\text{s.t.} \quad & \theta x_o - X\lambda \geq 0 \\
& Y\lambda \geq y_o \\
& \lambda \geq 0,
\end{aligned} \tag{2.3}$$

where the transposed (T) presentation  $\lambda = (\lambda_1, \dots, \lambda_J)^T$  is a non-negative vector (corresponding to the presence of slacks for each DMU),  $X$  an  $(M \times J)$  input matrix, and  $Y$  an  $(S \times J)$  input matrix.

We can now define the input excesses  $s^- \in R^m$  and the output shortfalls  $s^+ \in R^s$ , and identify them as ‘slack’ vectors as follows:

$$s^- = \theta x_o - X\lambda; \quad (2.4)$$

$$s^+ = Y\lambda - y_o. \quad (2.5)$$

We can then solve the following two-stage LP problem in a straightforward way:

1. Solve DLP. Let the optimal objective value be  $\theta^*$ .
2. Given the value of  $\theta^*$ , solve the following LP model using  $(\lambda, s^-, s^+)$  as slack variables:

$$\max_{\lambda, s^-, s^+} \omega = es^- + es^+ \quad (2.6)$$

$$\text{s.t.} \quad s^- = \theta^* x_o - X\lambda \quad (2.7)$$

$$s^+ = Y\lambda - y_o \quad (2.8)$$

$$\lambda \geq 0, \quad s^- \geq 0, \quad s^+ \geq 0, \quad (2.9)$$

where  $\omega$  is an objective variable, and  $e$  a unit vector. For any inefficient DMU<sub>o</sub>, we can now define the reference set  $E_o$ , based on the max-slack solution as obtained in Steps 1 and 2, as follows:

$$E_o = \{j | \lambda_j^* > 0\} \quad (j \in \{1, \dots, J\}), \quad (2.10)$$

where  $E_o$  is a reference set for any inefficient DMU<sub>o</sub>. An optimal solution can then be expressed as follows:

$$\theta^* x_o = \sum_{j \in E_o} x_j \lambda_j^* + s^{-*}; \quad (2.11)$$

$$y_o = \sum_{j \in E_o} y_j \lambda_j^* - s^{+*}. \quad (2.12)$$

The improvement projection  $(\hat{x}_o, \hat{y}_o)$  is now defined in (2.13) and (2.14) as:

$$\hat{x}_o = \theta^* x_o - s^{-*}; \quad (2.13)$$

$$\hat{y}_o = y_o + s^{+*}. \quad (2.14)$$



These equations suggest that the efficiency of  $(x_o, y_o)$  for  $DMU_o$  can be improved if the input values are reduced radially by the ratio  $\theta^*$ , and the input excesses  $s^{-*}$  are eliminated (see Figure 1). Similarly, the efficiency can be improved, if the output values are increased by the output shortfall  $s^{+*}$ .

The original DEA models presented in the literature have thus far only focused on a uniform input reduction or a uniform output increase in the efficiency-improvement projections, as shown in Figure 1 ( $\theta^* = OC'/OC$ ). But, in principle, there are an infinite number of efficiency-improvement projections on the efficient frontier line. The efficiency-improvement projection of the original DEA models is only one solution, based on a projection related to a uniform input reduction or a uniform output increase. If we adopt a different perspective, this will, of course, lead to another projection.

In the past decade several attempts have been made to integrate the DEA and the MOLP models (see, e.g., Belton 1992, Belton and Vickers 1993, and Doyle and Green 1993). Most of the research was inspired by the pioneering research of Golany (1988) who tried to find efficient solutions in order to map out the efficiency frontier in an interactive way. Later on, Kombluth (1991) was able to show the similarity between DEA problems and fractional MOLP problems. This similarity holds for both input-oriented and output-oriented models.

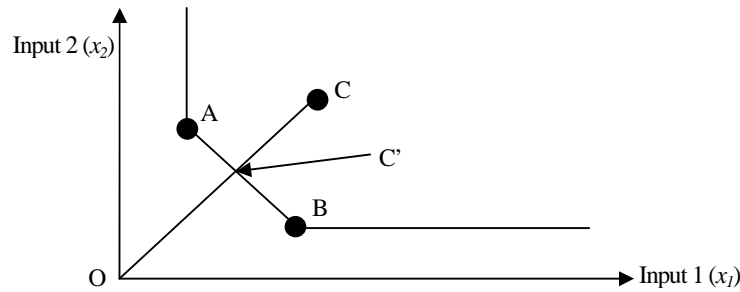


Figure 1 Illustration of original DEA projection in input space

Most contributions on the integration of the DEA and the MOLP models find their origin in the standard CCR model or in the Banker et al. (1984) (abbreviated as BCC) model, which provide the foundations of DEA. All such models aim to find an appropriate projection for an efficiency improvement for each inefficient DMU, based on a radial projection in which the input volumes are reduced (or the output values are increased) by a uniform ratio.

It is noteworthy that the existence of an infinite number of efficiency-improvement solutions has in recent years prompted a rich literature on the methodological integration of the MOLP and the DEA models. As mentioned, the first contribution was offered by Golany (1988), who proposed an interactive MOLP procedure which aimed at generating a set of efficient points for a DMU. This model allows a decisionmaker to select the preferred set of output levels, given the input levels, and it was used as a support tool for the selection of effective and efficient points for a decision-making agency. Thanassoulis and Dyson (1992) then developed adjusted models which can be used to estimate alternative

input and output levels in order to render relatively inefficient DMUs more efficient. These models are able to incorporate preferences for a potential improvement of individual input and output levels. The resulting target levels reflect the user's relative preference over alternative paths to efficiency. Joro et al. (1998) demonstrated the analytical similarity between a DEA model and a Reference Point Model in a MOLP formulation from a mathematical standpoint. Additionally, the Reference Point Model provides suggestions which make it possible to freely search on the efficiency frontier for good solutions or for the most preferred solution based on the decisionmaker's preference structure. More recently, Halme et al. (1999) developed a Value Efficiency Analysis (VEA), which included the decisionmaker's preference information in a DEA model. The foundation of VEA originates from the Reference Point Model in a MOLP context. Here the decisionmaker identifies the Most Preferred Solution (MPS), so that each DMU can be evaluated by means of the assumed value function based on the MPS approach. A further development of this approach was made by Korhonen and Siljamäki (2002) who addressed several practical aspects related to the use of VEA. In addition, Korhonen et al. (2003) developed a multiple objective approach which allows for changes in the time frame. And, finally, Lins et al. (2004) proposed two multi-objective approaches that determine the basis for an a posteriori preference incorporation. The first model is known as MORO (Multiple Objective Ratio Optimization), which optimizes the ratios between the observed and the target inputs (or outputs) of a DMU. The second model is known as MOTO (Multiple Objective Target Optimization), which directly optimizes the target values.

These approaches dealt with the challenge to identify a target or a direction to render relatively inefficient DMUs more efficient, based on the decisionmaker's preference information. The various approaches have suggested that the solution of an efficient improvement problem is not only a search for just one point. In particular, the Reference Point Model (see Joro et al. 1998) has many possibilities to generate a great variety of solutions to render inefficient DMUs more efficient. Clearly, one remark is in order here: these approaches have to incorporate the decisionmaker's preference information. In this regard, Angulo-Meza and Lins (2002) make the following observation:

*"There are disadvantages in the methods that incorporate a priori information, concerning subjectivity:*

- The value judgments, or a priori information can be wrong or biased, or the ideas may not be consistent with reality.*
- There may be a lack of consensus among the experts or decision-makers, and this can slow down or adversely affect the study.*

*Indeed, one may want to preserve the DEA spirit in the sense of not including a priori information."* (p. 232).

Given these considerations, we propose in our study a new efficiency-improvement projection model, known as the Distance Friction Minimization (DFM) approach, which does not need to incorporate a value judgment of a decision-maker. In this approach a generalized distance friction function will be presented to assist a DMU in improving its efficiency by the most appropriate movement towards the efficiency frontier surface. The direction of this efficiency improvement depends on the input/output data characteristics of the DMU. Each of these characteristics may have a different weight for the DMU. To achieve the required rise in efficiency, it is thus necessary to take into account the various most appropriate input/output weights of these characteristics. It is then possible to define the projection

functions for the minimization of the distance friction, using a Euclidean distance in weighted spaces. Here we will use a MOQP model.

### 3. The Distance Friction Minimization (DFM) Approach

As mentioned, the efficiency improvement solution in the original CCR-input model requires that the input values are reduced radially by a uniform ratio  $\theta^*$  ( $\theta^* = OD'/OD$  in Figure 2). That is to say, the improvement solution for any arbitrarily inefficient  $DMU_D$  is  $D'$  in Figure 2 (in cases where the input space is a non-weighted (i.e. normal)  $x$ -space). The general specification of a CCR model was frequently based on a normal  $x$ - or  $y$ -space (non-weighted space) (see Figure 1), in contrast to Figures 2 and 3, which are based on weighted  $x$ - or  $y$ -spaces. Weighted spaces can be investigated regarding the distance frictions in efficiency-improvement projections for input and output variables in the following way (see Cooper et al. 2006).

The  $(v^*, u^*)$  values obtained as an optimal solution for formula (2.2) result in a set of optimal weights for  $DMU_o$ . Then the efficiency score can be evaluated by:

$$\theta^* = \frac{\sum_s u_s^* y_{so}}{\sum_m v_m^* x_{mo}} . \quad (3.1)$$

The denominator may arbitrarily be set equal to 1, and hence:

$$\theta^* = \sum_s u_s^* y_{so} . \quad (3.2)$$

As mentioned earlier,  $(v^*, u^*)$  is the set of most favourable weights for  $DMU_o$ , in the sense of maximizing the ratio scale.  $v_m^*$  is the optimal weight for the input item  $m$ , and its magnitude expresses how much in relative terms the item is contributing to efficiency. Similarly,  $u_s^*$  does the same for the output item  $s$ . Furthermore, if we examine each item  $v_m^* x_{mo}$  in the total input:

$$\sum_m v_m^* x_{mo} (= 1), \quad (3.3)$$

we can derive the relative importance of each item with reference to the value of each  $v_m^* x_{mo}$ . The same holds for  $u_s^* y_{so}$ , where  $u_s^*$  provides a measure of the relative contribution of  $y_{so}$  to the overall value of  $\theta^*$ . These values show not only which items contribute to the performance of  $DMU_o$ , but also to what extent they do so. In other words, it is possible to express the distance frictions (or alternatively, the potential increases) in improvement projections.

In this study, we use the optimal weights  $u_s^*$  and  $v_m^*$  from (3.1), and then develop next our new efficiency improvement projection model. A visual presentation of this new approach is given in Figures 2 and 3.

In this approach a generalized distance friction is deployed to assist a DMU in improving its efficiency by a

movement towards the efficiency frontier surface. The direction of efficiency improvement depends on the input/output data characteristics of the DMU. It is then appropriate to define the projection functions for the minimization of distance friction by using a Euclidean distance in weighted spaces. As mentioned, a suitable form of multidimensional projection functions that serves to improve efficiency is given by a MOQP model which aims to minimize the aggregated input reduction frictions, as well as the aggregated output increase frictions. Thus, the DFM approach can generate a new contribution to efficiency enhancement problems in decision analysis, by deploying a weighted Euclidean projection function, and at the same time it may address both input reduction and output increase.

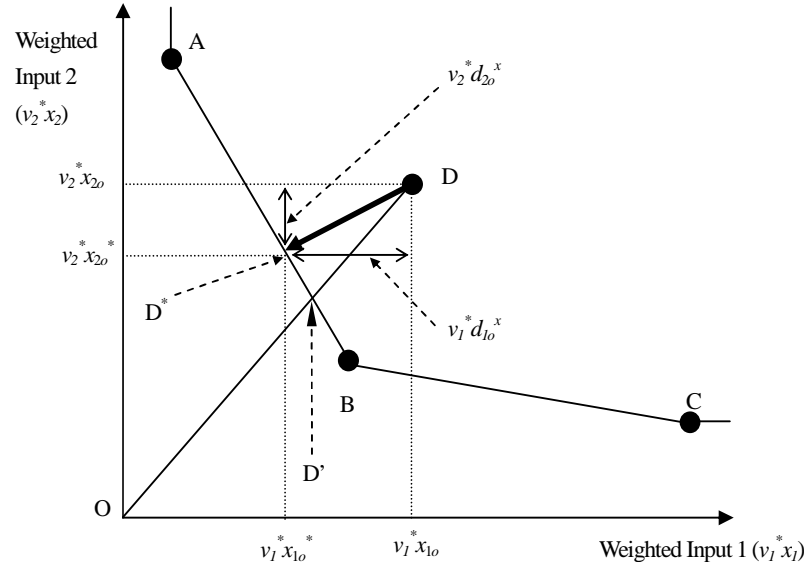


Figure 2 Illustration of the DFM approach (Input-  $v_i^* x_i$  space)

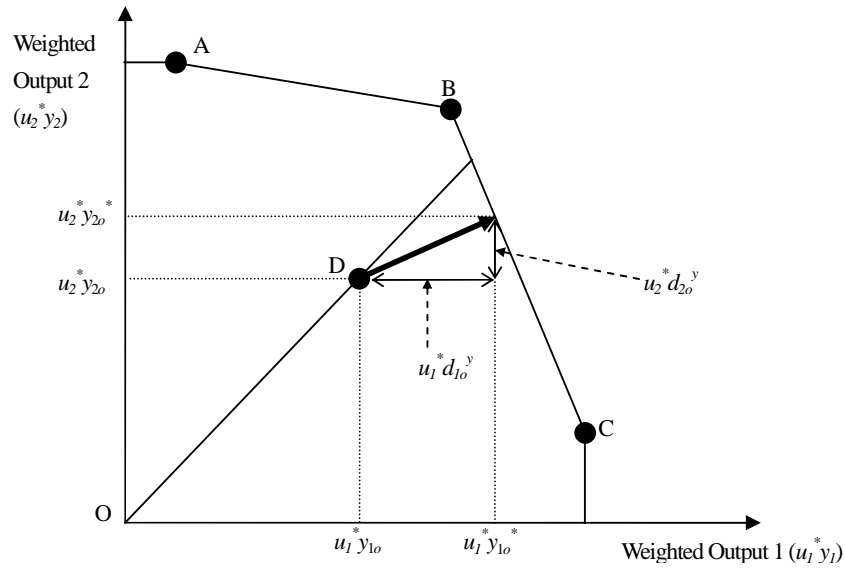


Figure 3 Illustration of the DFM approach (Output -  $u_r^* y_r$  space)

Our DFM approach contains 5 stages which will now briefly be presented.

1. Solve  $DLP_0$  in (2.3). Let the optimal objective value be  $\theta^*$ , and the obtained optimal weights  $u_s^*$  and  $v_m^*$ .
2. Using  $\theta^*$ , solve (2.6)-(2.9), so that we obtain  $s^{-*}, s^{+*}$ . Each DMU can then be categorized by  $\theta^*$ ,  $s^{-*}$  and  $s^{+*}$  as follows:

- (a) if  $\theta^*=1$ ,  $s^{-*}=s^{+*}=0$ : a situation of an efficient DMU.
- (b) if  $\theta^*=1$ ,  $s^{-*} \neq 0$  or  $s^{+*} \neq 0$ : improvement solutions are generated by formulas (2.13) and (2.14).
- (c) if  $\theta^* \neq 1$ ,  $s^{-*} \neq 0$  or  $s^{+*} \neq 0$ : improvement solutions are generated by subsequent steps 3,4 and 5.

3. Introduce the distance friction function  $Fr^x$  and  $Fr^y$  by means of (3.4) and (3.5), which are defined by the Euclidean distance shown in Figures 2 and 3. And solve the following MOQP using  $d_{mo}^x$  (a reduction distance for  $x_{io}$ ) and  $d_{so}^y$  (an increase distance for  $y_{so}$ ) as variables:

$$\min Fr^x = \sqrt{\sum_m (v_m^* x_{mo} - v_m^* d_{mo}^x)^2} \quad (3.4)$$

$$\min Fr^y = \sqrt{\sum_s (u_s^* y_{so} - u_s^* d_{so}^y)^2} \quad (3.5)$$

$$\text{s.t.} \quad \sum_m v_m^* (x_{mo} - d_{mo}^x) = \frac{2\theta^*}{1 + \theta^*} \quad (3.6)$$

$$\sum_s u_s^* (y_{so} + d_{so}^y) = \frac{2\theta^*}{1 + \theta^*} \quad (3.7)$$

$$x_{mo} - d_{mo}^x \geq 0 \quad (3.8)$$

$$d_{mo}^x \geq 0 \quad (3.9)$$

$$d_{so}^y \geq 0, \quad (3.10)$$

where  $x_{mo}$  is the amount of input item  $m$  for an arbitrarily inefficient DMU<sub>o</sub>, and  $y_{so}$  is the amount of output item  $s$  for arbitrarily inefficient DMU<sub>o</sub>.

The aim of function  $Fr^x$  (3.4) is to find a solution that minimizes the sum of input reduction distances which is incorporated in the improvement friction. The aim of function  $Fr^y$  (3.5) is to find a solution that minimizes the sum of

output increase distances which is incorporated in the improvement friction.

Constraint functions (3.6) and (3.7) refer to the target values of input reduction and output increase. An illustration of a target value and a 'fair' allocation between input efforts and output efforts is shown in Figure 4.

The balance in the distribution of contributions from the input and output side to achieve efficiency is established as follows. The total efficiency gap to be covered by inputs and outputs is  $(1-\theta^*)$ . The input and output side contribute according to their initial levels 1 and  $\theta^*$ , implying shares  $\theta^*/(1+\theta^*)$  and  $1/(1+\theta^*)$  in the efficiency-improvement contribution. Thus the contributions from both sides equal  $(1-\theta^*)[\theta^*/(1+\theta^*)]$ , and  $(1-\theta^*)[1/(1+\theta^*)]$ .

Hence we find for the input reduction target and the output increase targets:

$$\text{Input reduction target: } \sum_m v_m^* (x_{mo} - d_{mo}^x) = 1 - (1 - \theta^*) \times \frac{1}{(1 + \theta^*)} = \frac{2\theta^*}{1 + \theta^*} ; \quad (3.11)$$

$$\text{Output increase target: } \sum_s u_s^* (y_{so} + d_{so}^y) = \theta^* + (1 - \theta^*) \times \frac{\theta^*}{(1 + \theta^*)} = \frac{2\theta^*}{1 + \theta^*} . \quad (3.12)$$

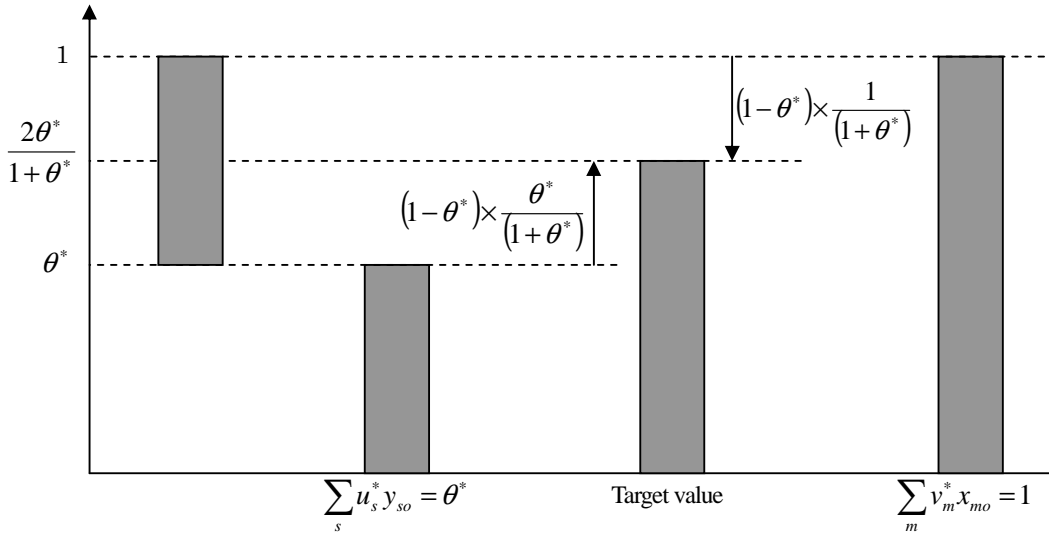


Figure 4 Presentation of balanced allocation for the total efficiency gap  $(1-\theta^*)$

Constraint function (3.8) refers to a limitation of input reduction, while constraint functions (3.9) and (3.10) express simultaneously the pressure of input reduction and output increase. It is now possible to determine each optimal distance  $d_{mo}^{x*}$  and  $d_{so}^{y*}$  by using MOQP (3.4)-(3.10).

4. The friction minimization solution for an inefficient DMU<sub>o</sub> can now be expressed by means of formulas (3.13) and (3.14):

$$x_{mo}^* = x_{mo} - d_{mo}^{x*} \quad (3.13)$$

$$y_{so}^* = y_{so} + d_{so}^{y*} \quad (3.14)$$

5. In order to ascertain the presence of slacks for input and output variables, we have to solve formulas (2.3) and (2.6)-(2.9). By using  $x_{mo}^*, y_{so}^*$ , we can obtain  $\theta^{**}, s^{-**}, s^{+**}$ . In this case, we are sure that  $\theta^{**}$  is calculated as 1. An optimal solution for an inefficient DMU<sub>o</sub> can be now expressed by means of formulas (3.15) and (3.16):

$$x_{mo}^{**} = x_{mo}^* - s^{-**} ; \quad (3.15)$$

$$y_{so}^{**} = y_{so}^* + s^{+**} \quad (3.16)$$

By means of the DFM model, it is possible to present a new efficiency-improvement solution based on the standard CCR projection. This means an increase in options for efficiency-improvement solutions in DEA. The main advantage of the DFM model is that it yields an outcome on the efficient frontier that is as close as possible to the DMU's input and output profile (see Figure 5).

In addition, the DFM model retains the property of the standard DEA approach that the measurement units of the different inputs and outputs need not be identical, while the efficiency-improvement projection in a DFM model does not need to incorporate a priori information.

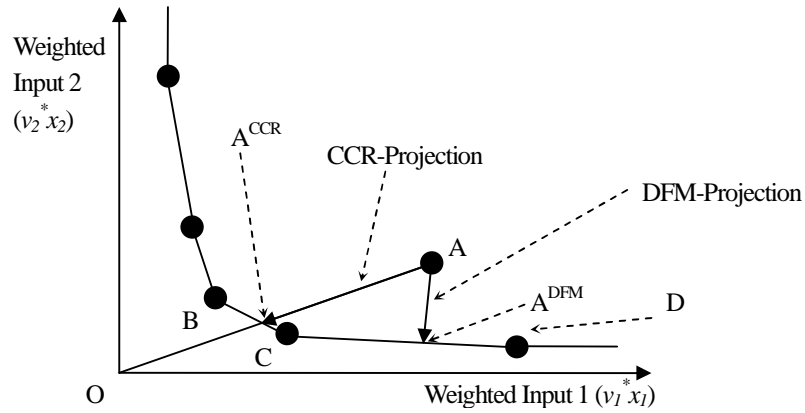


Figure 5 Degree of improvement of a DFM-projection and a CCR-projection in weighted input space

#### 4. A Goals Achievement Model in a DFM Approach

In our study we aim to integrate a GA model in the framework of the CCR-DFM model. The GA model specifies a Goal Improvement Rate (GIR) of the total efficiency gap  $(1 - \theta^*)$  in the DFM model. The value of the GIR ranges from

0 to 1. For example, if GIR is specified to be 0.1, then the GA model can compute an input reduction value and an output increase value in order to achieve an efficiency-improvement that is equivalent to 10 percent of the total efficiency gap  $(1-\theta^*)$ . This model will use the constraint functions (4.1) and (4.2) instead of constraint functions (3.6) and (3.7) in the DFM model. Thus, we have the following model specification for the Goals-Achievement Values (GAVs):

$$GAV^x = \sum_m v_m^* (x_{mo} - d_{mo}^x) = 1 - \frac{(1-\theta^*)}{(1+\theta^*)} + \frac{(1-\theta^*)(1-GIR)}{(1+\theta^*)} = \frac{2\theta^* + (1-\theta^*)(1-GIR)}{1+\theta^*}; \quad (4.1)$$

$$GAV^y = \sum_s u_s^* (y_{so} + d_{so}^y) = \theta^* + \frac{(1-\theta^*)\theta^*}{(1+\theta^*)} - \frac{(1-\theta^*)(1-GIR)\theta^*}{(1+\theta^*)} = \frac{2\theta^* - (1-\theta^*)(1-GIR)\theta^*}{1+\theta^*}. \quad (4.2)$$

A visual presentation of constraint functions (4.1) and (4.2) is given in Figure 6, which will now be clarified concisely.

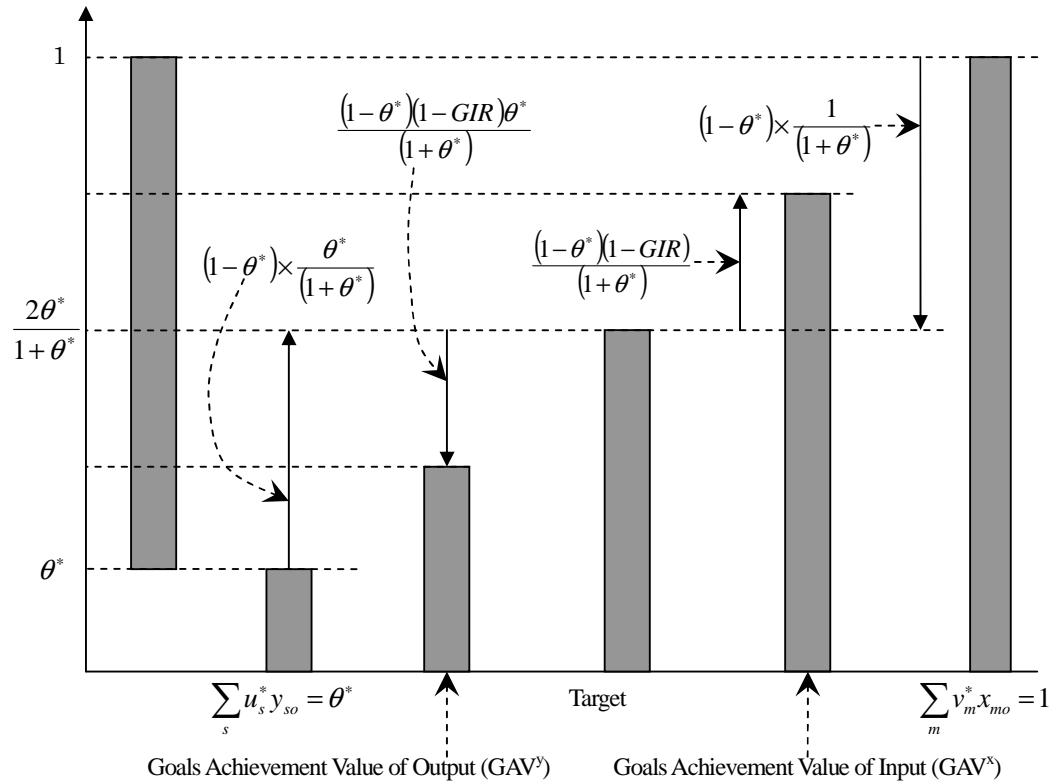


Figure 6 Presentation of a GA model

Firstl, the GA model has arbitrarily specified a GIR of the total efficiency gap equal to  $(1-\theta^*)$ . Next, the  $GAV^x$  and



the  $GAV^y$ , which are fairly allocated between input efforts and output efforts, are computed in Figure 6 using constraint functions (4.1) and (4.2). Finally, we can compute an input reduction value and an output increase value in order to achieve a  $GAV^x$  and a  $GAV^y$  using our CCR-DFM model. If the  $GIR = 1.0$ , then constraint functions (4.1) and (4.2) completely accord with constraint functions (3.6) and (3.7). In other words, the case of  $GIR = 1.0$  represents a full improvement in the total efficiency gap  $(1 - \theta^*)$ . Alternatively, a case of  $GIR = 0.0$  indicates a negligible improvement in the total efficiency gap  $(1 - \theta^*)$ .

## 5. Application to Local Government Finance Efficiency by Means of the CCR-DFM-GA Model

### 5.1 Analysis framework and database of local government finance efficiency in Hokkaido, Japan

In our empirical work, we use input and output data for a set of 34 cities (the capital Sapporo City - population 1,880,863 – was eliminated from our list of DMUs in order to avoid the extreme biased effects caused by scale differences) in Hokkaido prefecture in Japan. The cities (DMUs) used in our analysis are listed in Table 1. These cities were categorized, on the basis of their population size, into two groups: those with populations of more than 50,000, and those with populations of less than 50,000, in order to avoid biased effects caused by scale differences in government finance.

For our DEA, we use the following inputs and outputs:

- Input:
  - (a) Number of municipal employees (in 2005);
  - (b) Expenditures by local government (in million yen) (with elimination of employment costs) (in 2005);
  - (c) Amount of outstanding city bonds (in million yen) (in 2005).
- Output:
  - (d) Tax revenues by local government (in million yen) (in 2005);
  - (e) Public service level (in 2005).

Data on ‘(a) Number of municipal employees’ were obtained from ‘The local authority regular data base 2005, Ministry of Internal Affairs and Communications, Japan’. Data on ‘(b) Expenditures by local government’, and ‘(c) Amount of outstanding city bonds’, and ‘(d) Tax revenues by local government’, were obtained from ‘The Municipality Accounting Card 2005, Ministry of Internal Affairs and Communications, Japan’. Data on ‘(e) Public service level’ were calculated by a standardized score method using 6 types of data, viz. ‘Number of elementary and junior high schools’, ‘Number of community centres and libraries’, ‘Road extensions (municipality road)’, ‘Number of urban parks’, ‘Number of care facilities for the elderly’, and ‘Number of day-care centres for children’, which were obtained from ‘Statistical observations of SHI, KU, MACHE, MURA 2005, Ministry of Internal Affairs and Communications, Japan’.

Table 1 DMUs (Hokkaido prefecture's cities)

<i>Group 1 (More than 50000 population)</i>			<i>Group 2 (Less than 50000 population)</i>		
<i>No.</i>	<i>DMU</i>	<i>Population</i>	<i>No.</i>	<i>DMU</i>	<i>Population</i>
1	Asahikawa	355,004	1	Hokuto	48,056
2	Hakodate	294,264	2	Takikawa	45,562
3	Kushiro	190,478	3	Abashiri	42,045
4	Tomakomai	172,758	4	Wakkanai	41,592
5	Obihiro	170,580	5	Date	37,066
6	Otaru	142,161	6	Nayoro	31,628
7	Kitami	129,365	7	Nemuro	31,202
8	Ebetsu	125,601	8	Bibai	29,083
9	Muroran	98,372	9	Rumoi	26,826
10	Iwamizawa	93,677	10	Monbetsu	26,632
11	Chitose	91,437	11	Fukagawa	25,838
12	Eniwa	67,614	12	Furano	25,076
13	Kitahiroshima	60,677	13	Shibetsu	23,411
14	Ishikari	60,104	14	Sunagawa	20,068
15	Noboribetsu	53,135	15	Ashibetsu	18,899
	Sapporo	1,880,863	16	Akabira	14,401
			17	Yubari	13,001
			18	Mikasa	11,927
			19	Utashinai	5,221

In our application, we first applied the standard CCR model, while next the results of this analysis were used to determine the CCR-DFM and CCR-DFM-GA projections. The steps followed in our analysis are shown in Figure 7.

In Subsection 5.2, we present the efficiency evaluation results based on the CCR model. Next, in Subsection 5.3, we present the efficiency-improvement projection results based on the CCR-DFM model, and compare these with the CCR projections and outcomes. Finally, in Subsection 5.4, we present the efficiency-improvement projection results based on the CCR-DFM-GA model.

## 5.2 Efficiency evaluation based on the CCR model

The efficiency evaluation results for the 15 larger cities (more than 50,000 population) and the smaller 19 cities (less than 50,000 population) based on the CCR model are given in Figures 8 and 9.

From Figure 8, it can be seen that Tomakomai city, Obihiro city, Chitose city, Kitahiroshima city, and Ishikari city are efficiently-operating cities. It should be noted that Tomakomai city and Ishikari city have a large-scale industrial area and a harbour, while Chitose city has the New Chitose International Airport. Obihiro city produces a high agricultural output, and well-known confectionary companies are also based in the city. And finally, Kitahiroshima city has many industrial complexes and printing factories.

On the other hand, Iwamizawa city has a low efficiency (i.e. an efficiency score around 50 percent) in terms of

government finance. It is also clear that this city has in the past flourished on the basis of its coal production and its railway links, but most coal mines in Hokkaido were closed down after 1970s.

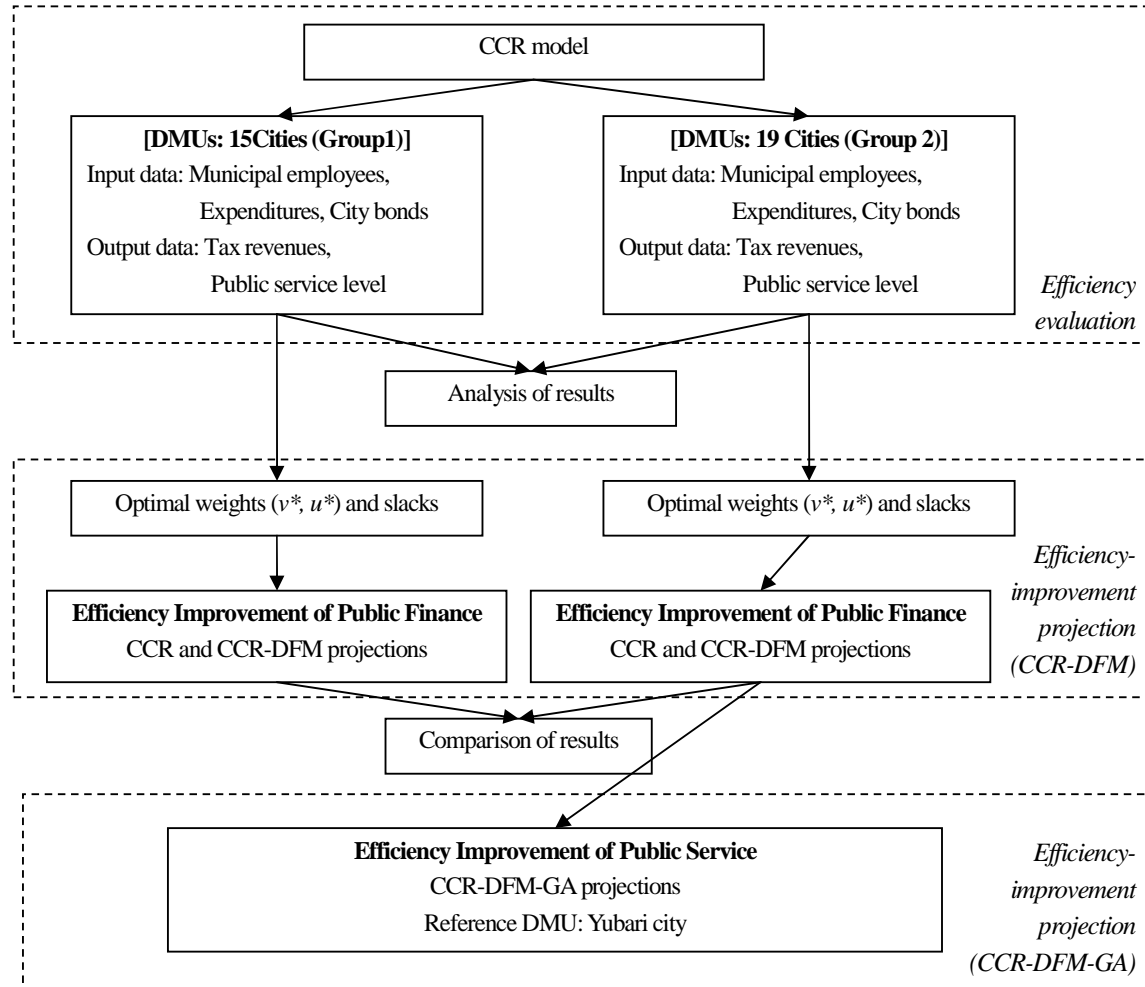


Figure 7 Analysis framework

From Figure 9, it can be seen that Hokuto city, Furano city, and Utashinai city are efficient. It is noteworthy that Hokuto city has promoted mergers of cities, towns and villages, in order to improve the efficiency of the city administration. Furthermore, this city has a large-scale factory which is a subsidiary of a cement company in Japan. On the other hand, Yubari city and Bibai city are low-efficiency cities in terms of government finance. It is also noteworthy that these cities have flourished as former coal mining areas, but now they have been deprived from their main industry.

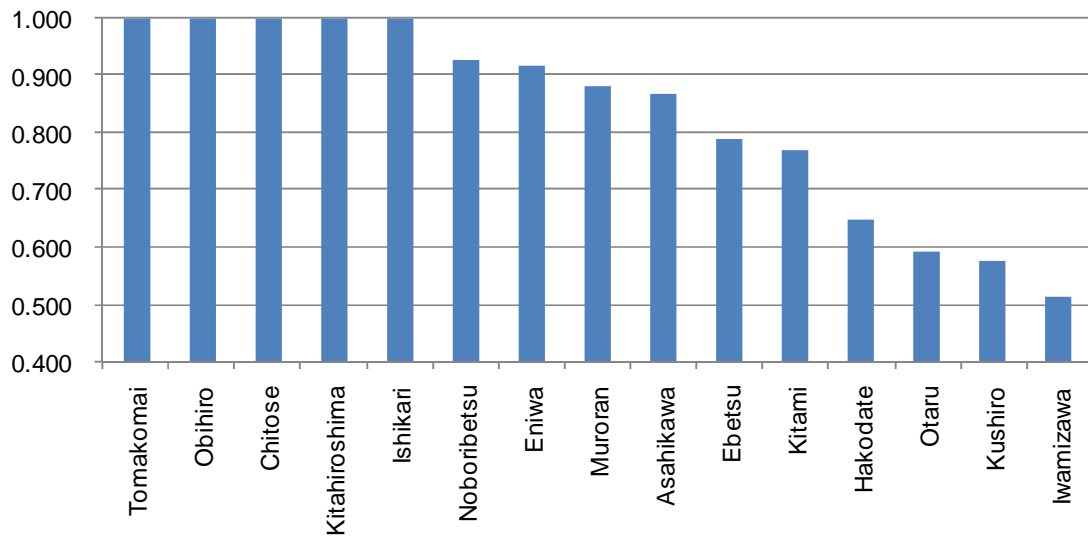


Figure 8 Efficiency score based on the CCR model (15 larger cities: more than 50,000 population)

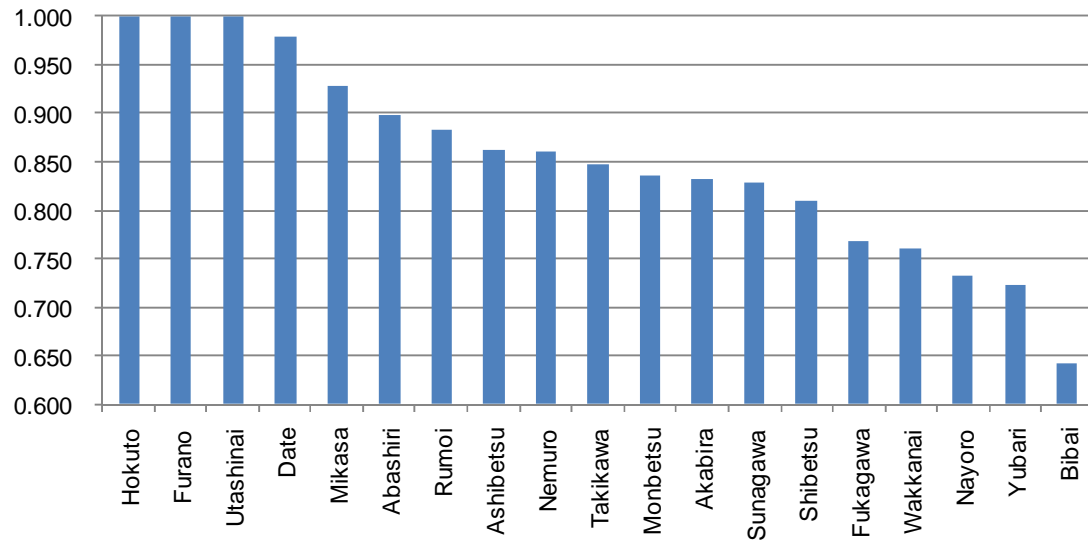


Figure 9 Efficiency score based on CCR model (19 smaller cities: less than 50,000 population)

### 5.3 Efficiency improvement projection based on the CCR and CCR-DFM models

The efficiency improvement projection results based on the CCR and CCR-DFM model for inefficient cities are presented below (see Tables 2 and 3).

In Tables 2 and 3, it appears that the ratios of change in the CCR-DFM projection are smaller than those in the CCR projection, as was expected. In Table 2, this particularly applies to Kushiro, Kitami, Iwamizawa and Eniwa. (the larger Group 1 cities in Table 1), which are non-slack type cities (i.e.  $s^{**}$  and  $s^{+**}$  are zero). The CCR-DFM projection

involves both input reduction and output increase, and, clearly, the CCR-DFM projection does not involve a uniform ratio because this model looks for the optimal input reduction (i.e., the shortest distance to the frontier, or distance friction minimization). For instance, the CCR projection shows that Eniwa should reduce the urban Employees and City bonds by 8.1 percent and its Expenditures by 25.7 percent in order to become efficient. On the other hand, CCR-DFM results show that a reduction in City bonds of 7.5 percent and an increase in the Tax revenues of 4.9 percent are required to become efficient. Apart from the practicality of such a solution, the models show clearly that a different, and a perhaps more efficient solution is available than the standard CCR projection to reach the efficiency frontier.

Table 2 Efficiency-improvement projection results of the CCR and the CCR-DFM model  
(more than 50,000 population cities)

		CCR Projection		CCR-DFM Projection	
DMU	Score( $\theta^*$ )	Score( $\theta^{**}$ )		Score( $\theta^{**}$ )	
I/O	Data	Difference	%	Difference	%
				$d_{mo}^{x^*+s^{***}}$	$d_{so}^{y^*+s^{***}}$
Asahikawa	0.869	1.000		1.000	
(I)Employees	3229	-424.5	-13.2%	-238.8	-7.4%
(I)Expenditures	126886.6	-21040.3	-16.6%	-9232.0	-7.3%
(I)City bonds	194947.5	-25629.9	-13.2%	0.0	0.0%
(O)Tax revenues	38607.7	0.0	0.0%	2716.5	7.0%
(O)Public service	73.3	17.0	23.1%	26.7	36.4%
Hakodate	0.649	1.000		1.000	
(I)Employees	4054	-1485.0	-36.6%	-937.8	-23.1%
(I)Expenditures	103328.9	-36288.0	-35.1%	-22008.6	-21.3%
(I)City bonds	157613.8	-67455.3	-42.8%	-48252.0	-30.6%
(O)Tax revenues	31918.6	0.0	0.0%	6798.5	21.3%
(O)Public service	63.8	50.5	79.2%	74.9	117.4%
Kushiro	0.578	1.000		1.000	
(I)Employees	2847	-1202.2	-42.2%	-793.4	-27.9%
(I)Expenditures	90247.1	-38841.7	-43.0%	0.0	0.0%
(I)City bonds	119382.3	-50411.3	-42.2%	0.0	0.0%
(O)Tax revenues	22248.4	0.0	0.0%	6060.9	27.2%
(O)Public service	52.9	0.0	0.0%	0.0	0.0%
Otaru	0.594	1.000		1.000	
(I)Employees	2012	-858.6	-42.7%	-611.3	-30.4%
(I)Expenditures	54218.1	-22965.4	-42.4%	-13936.4	-25.7%
(I)City bonds	69480.5	-28184.3	-40.6%	-17677.5	-25.4%
(O)Tax revenues	14628.9	0.0	0.0%	3739.5	25.6%
(O)Public service	46.4	0.0	0.0%	0.0	0.0%
Kitami	0.772	1.000		1.000	
(I)Employees	1300	-296.8	-22.8%	-168.7	-13.0%
(I)Expenditures	52923.4	-12084.7	-22.8%	0.0	0.0%
(I)City bonds	85613.4	-22256.8	-26.0%	0.0	0.0%
(O)Tax revenues	13612.5	0.0	0.0%	1846.3	13.6%
(O)Public service	54.9	0.0	0.0%	0.0	0.0%
Ebetsu	0.789	1.000		1.000	
(I)Employees	1169	-256.7	-22.0%	-134.6	-11.5%
(I)Expenditures	30744.6	-6487.0	-21.1%	-3626.1	-11.8%
(I)City bonds	44192.8	-10356.7	-23.4%	-7389.5	-16.7%
(O)Tax revenues	11483.6	0.0	0.0%	1409.7	12.3%
(O)Public service	48.1	0.0	0.0%	0.0	0.0%
Muroran	0.883	1.000		1.000	
(I)Employees	1432	-304.9	-21.3%	-245.2	-17.1%
(I)Expenditures	35125.4	-4175.8	-11.9%	-1997.0	-5.7%
(I)City bonds	46054.1	-5407.9	-11.7%	-2872.6	-6.2%
(O)Tax revenues	14401.7	0.0	0.0%	902.3	6.3%
(O)Public service	43.6	0.0	0.0%	0.0	0.0%
Iwamizawa	0.515	1.000		1.000	
(I)Employees	1228	-595.5	-48.5%	-282.8	-23.0%
(I)Expenditures	38822.4	-20818.4	-53.6%	0.0	0.0%
(I)City bonds	53304.3	-25849.1	-48.5%	-20847.6	-39.1%
(O)Tax revenues	8337.1	0.0	0.0%	3097.8	37.2%
(O)Public service	48.1	0.0	0.0%	0.0	0.0%
Eniwa	0.919	1.000		1.000	
(I)Employees	609	-49.1	-8.1%	0.0	0.0%
(I)Expenditures	21537.9	-5531.0	-25.7%	0.0	0.0%
(I)City bonds	26370.4	-2125.9	-8.1%	-1985.9	-7.5%
(O)Tax revenues	7395.8	0.0	0.0%	359.3	4.9%
(O)Public service	41.6	0.0	0.0%	0.0	0.0%
Noboribetsu	0.927	1.000		1.000	
(I)Employees	514	-37.3	-7.3%	-19.4	-3.8%
(I)Expenditures	18035	-4506.5	-25.0%	-3997.1	-22.2%
(I)City bonds	30716.5	-9595.5	-31.2%	-8800.1	-28.6%
(O)Tax revenues	5066.6	1216.2	24.0%	1452.8	28.7%
(O)Public service	39.3	0.0	0.0%	1.5	3.8%

Table 3 Efficiency improvement projection results of the CCR and the CCR-DFM model  
(less than 50000 population cities)

DMU	Score( $\theta^*$ )	CCR Projection		CCR-DFM Projection	
		Score( $\theta^{**}$ )		Score( $\theta^{**}$ )	
		Difference	%	Difference	%
I/O	Data			$d_{mo}^{x^*} + s^{+**}$ $d_{so}^{y^*} + s^{+**}$	
Takikawa	0.846	1.000		1.000	
(I)Employees	805	-513.6	-63.8%	-489.4	-60.8%
(I)Expenditures	17260.4	-2734.5	-15.8%	-1526.6	-8.8%
(I)City bonds	24001.6	-3685.3	-15.4%	-1995.9	-8.3%
(O)Tax revenues	4227.3	0.0	0.0%	351.5	8.3%
(O)Public services	55.4	4.7	8.4%	9.7	17.5%
Abashiri	0.899	1.000		1.000	
(I)Employees	396	-67.0	-16.9%	-49.5	-12.5%
(I)Expenditures	18248.7	-1849.7	-10.1%	-974.3	-5.3%
(I)City bonds	49073.7	-26137.6	-53.3%	-24913.1	-50.8%
(O)Tax revenues	4772.4	0.0	0.0%	254.8	5.3%
(O)Public services	52.1	15.7	30.2%	19.3	37.1%
Wakkanai	0.760	1.000		1.000	
(I)Employees	783	-484.8	-61.9%	-444.2	-56.7%
(I)Expenditures	19555	-4689.3	-24.0%	-2664.1	-13.6%
(I)City bonds	29764.8	-8973.1	-30.2%	-6140.6	-20.6%
(O)Tax revenues	4326.2	0.0	0.0%	589.4	13.6%
(O)Public services	52.9	8.6	16.2%	17.0	32.0%
Date	0.979	1.000		1.000	
(I)Employees	448	-135.0	-30.1%	-136.3	-30.4%
(I)Expenditures	13112.7	-277.7	-2.1%	-140.3	-1.1%
(I)City bonds	22046.1	-4652.9	-21.1%	-4419.7	-20.0%
(O)Tax revenues	3442.1	0.0	0.0%	61.5	1.8%
(O)Public services	58.1	0.0	0.0%	0.2	0.3%
Nayoro	0.732	1.000		1.000	
(I)Employees	885	-552.0	-62.4%	-460.5	-52.0%
(I)Expenditures	15769.7	-4229.3	-26.8%	-2442.2	-15.5%
(I)City bonds	23971.5	-8850.9	-36.9%	-6911.0	-28.8%
(O)Tax revenues	2822.8	0.0	0.0%	226.1	8.0%
(O)Public services	56.9	0.0	0.0%	12.4	21.8%
Nemuro	0.860	1.000		1.000	
(I)Employees	663	-370.9	-55.9%	-328.0	-49.5%
(I)Expenditures	12662	-1771.6	-14.0%	-952.4	-7.5%
(I)City bonds	23825.3	-9334.5	-39.2%	-8454.2	-35.5%
(O)Tax revenues	2780.3	0.0	0.0%	99.0	3.6%
(O)Public services	51.7	0.0	0.0%	5.8	11.2%
Bibai	0.641	1.000		1.000	
(I)Employees	544	-256.7	-47.2%	0.0	0.0%
(I)Expenditures	14462.6	-5192.4	-35.9%	-3164.2	-21.9%
(I)City bonds	25707.6	-13760.9	-53.5%	0.0	0.0%
(O)Tax revenues	2162.8	0.0	0.0%	234.7	10.9%
(O)Public services	47.5	0.0	0.0%	14.5	30.5%
Rumoi	0.882	1.000		1.000	
(I)Employees	608	-358.5	-59.0%	-344.2	-56.6%
(I)Expenditures	11490.7	-1352.6	-11.8%	-718.6	-6.3%
(I)City bonds	26076	-12360.6	-47.4%	-11489.4	-44.1%
(O)Tax revenues	2706.7	0.0	0.0%	176.4	6.5%
(O)Public services	46.1	0.0	0.0%	2.8	6.0%
Monbetsu	0.837	1.000		1.000	
(I)Employees	327	-70.3	-21.5%	-47.5	-14.5%
(I)Expenditures	12399.5	-2024.8	-16.3%	-1102.4	-8.9%
(I)City bonds	28617.2	-14595.6	-51.0%	-13348.1	-46.6%
(O)Tax revenues	2762.6	0.0	0.0%	246.1	8.9%
(O)Public services	47.3	0.0	0.0%	4.2	8.9%
Fukagawa	0.768	1.000		1.000	
(I)Employees	605	-268.0	-44.3%	-205.2	-33.9%
(I)Expenditures	13233.4	-3076.0	-23.2%	-1740.3	-13.2%
(I)City bonds	27701.6	-14729.5	-53.2%	-11898.7	-43.0%
(O)Tax revenues	2236.8	0.0	0.0%	0.0	0.0%
(O)Public services	54.3	0.0	0.0%	11.9	21.9%
Shibetsu	0.809	1.000		1.000	
(I)Employees	682	-329.8	-48.4%	-277.7	-40.7%
(I)Expenditures	12951.9	-2477.4	-19.1%	-1369.7	-10.6%
(I)City bonds	23623.4	-9960.3	-42.2%	-7612.7	-32.2%
(O)Tax revenues	2231.8	0.0	0.0%	0.0	0.0%
(O)Public services	57.2	0.0	0.0%	9.9	17.2%
Sunagawa	0.828	1.000		1.000	
(I)Employees	746	-492.5	-66.0%	-434.1	-58.2%
(I)Expenditures	10496.5	-1803.3	-17.2%	-986.4	-9.4%
(I)City bonds	17954.2	-6590.4	-36.7%	-5871.0	-32.7%
(O)Tax revenues	2112.5	0.0	0.0%	15.5	0.7%
(O)Public services	43.1	0.0	0.0%	7.2	16.7%
Ashibetsu	0.863	1.000		1.000	
(I)Employees	527	-264.6	-50.2%	-237.6	-45.1%
(I)Expenditures	9045.9	-1238.6	-13.7%	-664.8	-7.3%
(I)City bonds	13433.7	-3258.0	-24.3%	-2042.0	-15.2%
(O)Tax revenues	1665.7	0.0	0.0%	0.0	0.0%
(O)Public services	42.6	0.0	0.0%	5.1	12.0%
Akabira	0.832	1.000		1.000	
(I)Employees	460	-205.6	-44.7%	-176.8	-38.4%
(I)Expenditures	8029.2	-1350.5	-16.8%	-737.3	-9.2%
(I)City bonds	12777.5	-2254.4	-17.6%	-954.7	-7.5%
(O)Tax revenues	949.4	0.0	0.0%	0.0	0.0%
(O)Public services	44.1	0.0	0.0%	5.5	12.4%
Yubari	0.722	1.000		1.000	
(I)Employees	406	-146.9	-36.2%	-108.4	-26.7%
(I)Expenditures	10183.9	-3408.9	-33.5%	-2590.6	-25.4%
(I)City bonds	14873.9	-4136.3	-27.8%	-2402.2	-16.2%
(O)Tax revenues	946.7	0.0	0.0%	0.0	0.0%
(O)Public services	45	0.0	0.0%	7.3	16.2%
Mikasa	0.929	1.000		1.000	
(I)Employees	388	-137.7	-35.5%	-129.3	-33.3%
(I)Expenditures	8245.4	-1551.7	-18.8%	-1372.3	-16.6%
(I)City bonds	11051.9	-789.8	-7.2%	-409.6	-3.7%
(O)Tax revenues	1026	0.0	0.0%	0.0	0.0%
(O)Public services	43	0.0	0.0%	1.6	3.7%

## 5.4 Efficiency improvement projection of the CCR-DFM-GA models

We will now provide a comprehensive picture of the results of our integrated CCR-DFM-GA model, and use Yubari city as a reference ('target') city. It should be noted that Yubari city was in a state of financial crisis in March 2007. Now, however, this city has a local government that is responsible for a financial reconstruction, and hence it has put local public finance on the road to recovery. But the city does not have resources to achieve a full efficiency improvement, as shown in Table 3.

In this subsection, we will use as an inefficient reference city (DMU) Yubari city, and present an efficiency improvement projection result based on the CCR-DFM-GA model. We assume that the GIR uses steps from 0.0 to 1.0 at intervals of 0.1. Next, the efficiency scores and the input reduction values and the output increase values based on the CCR-DFM-GA model are calculated in Table 4 and Figure 10.

Table 4 Efficiency improvement projection results based on the CCR-DFM-GA model (Yubari city)

GIR	Score	$d_x+s^{**}$ (Employees)	$d_x+s^{**}$ (Expenditures)	$d_x+s^{**}$ (City bonds)	$d_y+s^{**}$ (Revenues)	$d_y+s^{**}$ (Public services)	Employees (%)	Expenditures (%)	City bonds(%)	Revenues (%)	Public services(%)
0.0	0.722	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0%	0.0	0.0
0.1	0.746	0	0.0	-240.2	0.0	0.7	0.0	0.0	-1.6	0.0	1.6
0.2	0.770	0	0.0	-480.4	0.0	1.5	0.0	0.0	-3.2	0.0	3.2
0.3	0.795	0	0.0	-720.7	0.0	2.2	0.0	0.0	-4.8	0.0	4.8
0.4	0.822	0	0.0	-960.9	0.0	2.9	0.0	0.0	-6.5	0.0	6.5
0.5	0.849	0	0.0	-1201.1	0.0	3.6	0.0	0.0	-8.1	0.0	8.1
0.6	0.877	0	0.0	-1441.3	0.0	4.4	0.0	0.0	-9.7	0.0	9.7
0.7	0.906	0	0.0	-1681.5	0.0	5.1	0.0	0.0	-11.3	0.0	11.3
0.8	0.936	0	0.0	-1921.7	0.0	5.8	0.0	0.0	-12.9	0.0	12.9
0.9	0.967	0	0.0	-2162.0	0.0	6.5	0.0	0.0	-14.5	0.0	14.5
1.0	1.000	-108.4	-2590.6	-2402.2	0.0	7.3	-26.7	-25.4	-16.2	0.0	16.2

These results show that, if the city implements an efficiency improvement plan with a GIR amounting to 0.5 (i.e. 50 percent of the total efficiency gap), only a reduction in the City bonds of 8.1 percent and an increase in Public services of 8.1 percent are required, and then the efficiency score improves from 0.722 to 0.849. Furthermore, the results of a plan with a GIR of 1.0 (i.e. 100 percent of the total efficiency gap) accord with the result of our CCR-DFM model in Table 3. Yubari city is an Input-slack type of city (i.e.  $s^{**}$  is not zero). If a new plan with a GIR of 1.0 is implemented in this case, it would have to incorporate both a slack of Employees (-108.4) and a slack of Expenditures (-2590.6).

These results may offer a meaningful contribution for the decision making and planning for the efficiency improvement of local government finance. And this new model may thus become a policy instrument that may have great added value for the decision making and planning of both public and private actors.

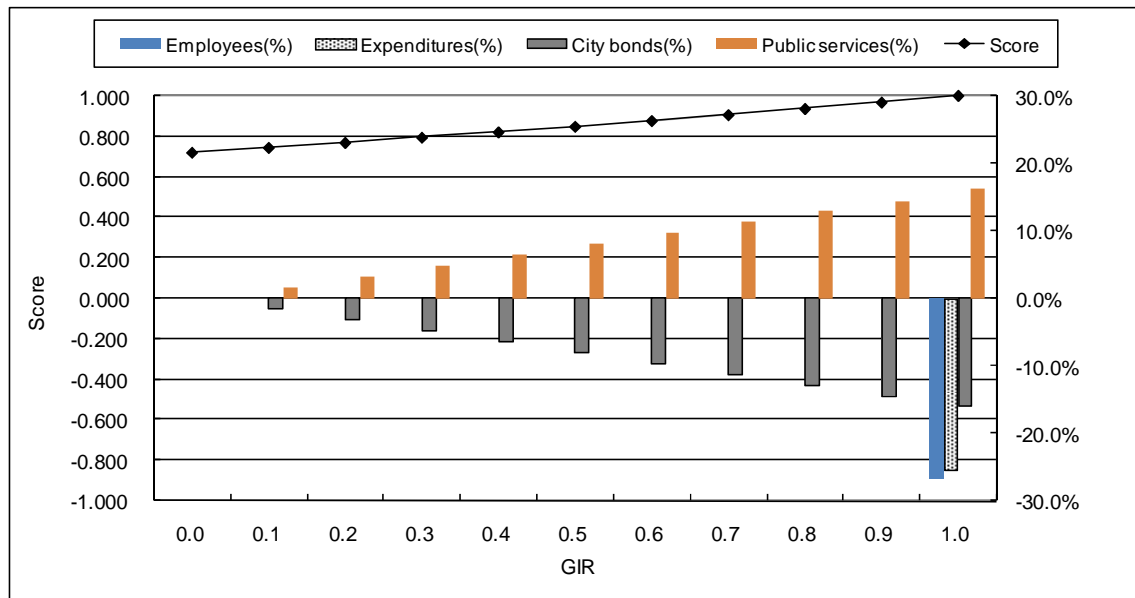


Figure 10 Efficiency-improvement projection results based on the CCR-DFM-GA model (Yubari city)

## 6. Conclusion

In this paper we have presented a new methodology for an inefficient city to reach the efficiency frontier and to achieve the prior goal set by a DMU. This methodology does not require a uniform reduction of all inputs, as in the standard model. Instead, the new method minimizes the distance friction for each input and output separately. As a result, the reductions in inputs and increases in outputs necessary to reach the efficiency frontier are smaller than in the standard model. Furthermore, our CCR-DFM-GA model can present a more realistic efficiency-improvement plan, and may thus provide a meaningful contribution to the decision making and planning for the efficiency improvement of relevant agents. The results for our Hokkaido case study are illustrative: they are able to identify the weak municipalities in the region and to identify the factors that are responsible for a non-optimal performance of these actors.

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