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## Congestion and Residential Moving Behaviour

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# Congestion and Residential Moving Behaviour

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# Congestion and Residential Moving Behaviour

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## Abstract

In this paper we study how congestion and residential moving behaviour are interrelated using a two-region job search model. Workers choose optimally between interregional commuting and residential moving to live closer to the place of work. This choice affects the external costs of commuting due to congestion. The welfare maximizing road tax is derived. We demonstrate that road pricing may not only reduce congestion but also increase total residential moving costs in the economy. One of the main consequences is that the road tax does not necessarily increase welfare.

**JEL codes: R23, R41.**

**Keywords: Congestion, Residential Moving and Job Search.**

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# 1 Introduction

In the large literature on road pricing and congestion, it has been recognised that road pricing will not only affect commuting behaviour directly but may also affect the optimal location of households and firms. In particular, road pricing may induce workers to locate closer to firms (see for example Anas and Xu (1999)). One of the consequences is that housing rents, wages and spatial structure will change.

In the analysis of optimal location of households it is generally presumed that households may move residence at no costs (Arnott (1998); Anas and Xu (1999); Boyce and Mattson (1999); Eliasson and Mattson (2001)). This assumption has many advantages in the context of commuting, because it simplifies the analysis to a large extent. It ignores however that residential moving costs are relevant, in particular because information on available vacancies at different locations is incomplete. Workers are therefore not able to find the job which is closest to their residence, but search for jobs given incomplete information and will therefore accept jobs which do not minimise commuting costs<sup>1</sup>. The commuting model developed in this paper allows for incomplete information in the labour market combined with residential moving behaviour and positive residential moving costs.<sup>2</sup>

In the current paper, the commuting model is essentially a two-region job search model where unemployed job seekers seek for job offers which arrive randomly over time from both regions (so information is spatially incomplete). Job seekers accept jobs in both regions. The basic decision job seekers have to make is whether to commute between regions or to move residence to another

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<sup>1</sup>This theoretical result is in line with the empirical literature on 'excess commuting' (see for example Hamilton (1982), White (1988), or Small and Song (1992)). One of the conclusions from this literature is that workers commute further than might be thought if residential moving costs would be absent and information on the labour market would be complete, although studies dispute by how much.

<sup>2</sup>Hence, we study road pricing, in the context of commuting employing a labour market model which allows for search imperfections. For other labour market studies of environmental externalities, see Bovenberg and de Mooij (1994b) and Bovenberg and van der Ploeg (1994).

region, which is costly, taking into account future labour and residential mobility. One of the main characteristics of the model is 'excess commuting': some workers commute to the other region although they are not compensated for the excess commuting costs. This characteristic is the consequence of the combination of imperfect labour market information and positive moving costs. Congestion is introduced in the model and it is assumed that congestion depends positively on the number of workers who commute between regions.

One of the aims of the paper is to derive the optimal road tax given the presence of the congestion externality taking into account imperfect labour market information combined with the presence of residential moving costs. In order to do so, we characterise different equilibriums, which are defined by the value of the incurred commuting costs (inclusive the road tax) relative to the discounted residential moving costs.<sup>3</sup>

We examine three types of equilibriums outcomes. In the first equilibrium, interregional job offers induce interregional commuting (but no moving). In the second equilibrium, these job offers induce interregional moving (but no commuting). In a third equilibrium, interregional commuting and residential moving both occur. Road pricing causes then a welfare gain due to the reduction in the congestion externality, but only when the type of equilibrium does not change. It reduces the real commuting costs of all interregional commuters by an amount equal to the tax. Further it reduces the number of interregional commuters since it increases residential mobility. The reduction in commuting costs due to the decrease of interregional commuters is equal to the increased expenses on the moving costs. Hence, the welfare gain of the road tax is equal to the road tax multiplied by the number of interregional commuters (after the introduction of the tax). Consequently, a tax revenue-maximising road tax maximises welfare. This result makes sense, because the opportunity of costly moving induces the demand for commuting to become perfect price elastic. One of the consequences is that a private monopolist company that levies the

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<sup>3</sup>In this paper, we distinguish between the commuting costs which may include a road tax, and the real commuting costs which are the incurred commuting costs exclusive road tax.

road tax would set the road tax optimally from a welfare perspective. But a positive welfare effect of road pricing is not always guaranteed. Under some specific circumstances, a road tax may induce a welfare loss<sup>4</sup>. For example, when commuting between regions is more cost effective than moving residence to the other region where the job is located, and a road tax induces workers to move residence to the other region, then a negative welfare contribution may result.

The outline of the paper is as follows. In section 2, the model is introduced. In section 3 the equilibriums are characterised. Section 4 discusses the welfare implications of a road tax. Section 5 concludes.

## 2 The Model

In this section we define a model which consists of a labour market model including commuting and allows for moving in the housing market. Our starting point is a model with two regions and a given number of (ex-ante) identical workers. We presume the presence of (endogenously determined) involuntary unemployment due to job search imperfections. Unemployed workers search for jobs in both regions. Within a region all jobs are identical. Employed workers do not search, but are laid off each period with a fixed probability. The probability of receiving a job offer in a period does not depend on the region of residence. All job offers are accepted<sup>5</sup>. We consider the case where an unemployed worker

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<sup>4</sup>In general there could be a number of reasons why an environmental tax such as a road tax may induce a welfare loss due to market imperfections (see, for example, Bovenberg and de Mooij (1994a), Bovenberg and de Mooij (1994b), Bovenberg and van der Ploeg (1994). Parry and Bento (2001) emphasizes that a road tax reduces overall quantity of labour supply and therefore it is important how the road tax revenue is recycled. Nonetheless the optimal congestion tax is still the Pigouvian tax in Parry and Bento (2001). The latter is not necessarily the case in this paper. Note that in this paper, we will see that the quantity of labour supply is not affected.

<sup>5</sup>This assumption implies that the lifetime utility of being unemployed is less than the lifetime utility of being employed in both regions. We show at the end of section 3 under which assumptions of job search behaviour the assumption that all jobs are accepted is valid.

chooses between two strategies: a *commuting strategy* (CS) or a *residential moving strategy* (MS). The CS implies that a worker who finds a job in the other region will commute and not move residence. The MS implies that the worker will move residence to the other region. After accepting a job in the other region, a worker with MS pays residential moving costs, whereas a worker with CS pays the costs for commuting to the other region.

When unemployed workers choose the optimal strategy, they are assumed to maximize the expected present value of future utilities, the so-called *lifetime utility*. The lifetime utility can then be written as a function of the utility enjoyed during the current period, the so-called *flow utility*, and the expected lifetime utility enjoyed in the future periods. We will present a discrete version of the Bellmann equation, McKenna (1985). We use a number of subscripts. The first subscript refers to the region of residence ( $i$  or  $j$ ), the second to the labour market status (0 when unemployed,  $i$  when employed in region  $i$ ,  $j$  when employed in region  $j$ ), and the third subscript refers to the time period.

If an unemployed worker living in region  $i$  in period  $t$  chooses MS, lifetime utility,  $V_{i,0,t}^M$ , can be written as follows:

$$V_{i,0,t}^M = \frac{1}{1+\delta} \left( v_{i,0,t} + \theta_i V_{i,i,t+1} + \theta_j (V_{j,j,t+1} - m) + (1 - \theta_i - \theta_j) V_{i,0,t+1}^M \right) \quad (1)$$

$\delta$  denotes the discount rate.  $\theta_i$  is the probability of becoming employed in region  $i$ . Note that with a positive probability equal to  $1 - \theta_i - \theta_j$  the unemployed worker will remain unemployed. Residential moving costs,  $m$ , incurred if the worker moves to another region, are paid at the beginning of the period in which the move takes place.  $V_{i,i,t+1}$  denotes the lifetime utility of an employed worker living and working in region  $i$  in period  $t + 1$ . Lifetime utility of an unemployed who chooses MS can thus be written as the sum of flow utility,  $v_{i,0,t}$ , and the expected utility of finding employment in region  $i$ . The flow utility should be noted that the unemployed would only search in a region if the probability of acceptance is positive. Given identical jobs in a region this implies the probability of acceptance is one.



utility of being unemployed,  $v_{i,0,t}$ , is exogenously given for the individual and may include unemployment benefit, but may depend on regional characteristics such as the regional housing rent<sup>6</sup>.

The lifetime utility of the employed living in  $i$  and working in  $i$  at time  $t$ ,  $V_{i,i,t}$ , is determined by the employed's flow utility,  $v_{i,i,t}$ , the probability of being laid off,  $\lambda_i$ , and the discount rate and can be written as:

$$V_{i,i,t} = \frac{1}{1+\delta} (v_{i,i,t} + \lambda_i \max \{V_{i,0,t+1}^M, V_{i,0,t+1}^C\} + (1 - \lambda_i) V_{i,i,t+1}) \quad (2)$$

The probability of being laid off determines the probability of staying employed or becoming unemployed in the next period,  $t + 1$ . If the worker is laid off, he will choose the strategy ( $MS$  or  $CS$ ) that maximizes his lifetime utility. At the end of period  $t$  the employed worker receives the flow utility  $v_{i,i,t}$ . This flow utility is thought to consist of labour income (wage), commuting costs, but may also depend on housing rent.

From now, we will consider only the steady state and ignore the subscript,  $t$ , so,  $V_{i,0,t}^M = V_{i,0,t+1}^M = V_{i,0}^M$ . Given the choice of moving residence, lifetime utility,  $V_{i,0}^M$ , can be expressed in terms of flow utilities and exogenous parameters (see Appendix A):

$$V_{i,0}^M = \frac{1}{\delta} (\mu_1 (v_{i,0} - \theta_j m) + \mu_2 (v_{j,0} - \theta_i m) + \mu_3 v_{i,i} + \mu_4 v_{j,j}) \quad (3)$$

where

$$\begin{aligned} \mu_1 &= \frac{1 - \frac{\lambda_j \frac{\theta_j}{\delta + \lambda_j}}{\delta + \theta_i + \theta_j}}{1 + \frac{\theta_i}{\delta + \lambda_i} + \frac{\theta_j}{\delta + \lambda_j}}, \mu_2 = \frac{\frac{\lambda_j \frac{\theta_j}{\delta + \lambda_j}}{\delta + \theta_i + \theta_j}}{1 + \frac{\theta_i}{\delta + \lambda_i} + \frac{\theta_j}{\delta + \lambda_j}} \\ \mu_3 &= \frac{\frac{\theta_i}{\delta + \lambda_i}}{1 + \frac{\theta_i}{\delta + \lambda_i} + \frac{\theta_j}{\delta + \lambda_j}}, \mu_4 = \frac{\frac{\theta_j}{\delta + \lambda_j}}{1 + \frac{\theta_i}{\delta + \lambda_i} + \frac{\theta_j}{\delta + \lambda_j}} \end{aligned}$$

Let us interpret equation (3). The term " $\frac{1}{\delta}$ " applies since the future is discounted at rate  $\delta$ . It can be easily seen that  $\mu_1 + \mu_2 + \mu_3 + \mu_4 = 1$  and

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<sup>6</sup>In the next section we will specify the flow utilities.

the  $\mu$ 's can therefore be interpreted as the weight attached of being in a certain combined labour/ housing market state. We distinguish between four weights associated with labour/ housing market states: being unemployed in region  $i$  ( $\mu_1$ ), being unemployed in region  $j$  ( $\mu_2$ ), being employed region  $i$  ( $\mu_3$ ), or being employed in region  $j$  ( $\mu_4$ ). These weights depend on the exogenous parameters  $\delta$ ,  $\theta_i$ ,  $\theta_j$ ,  $\lambda_i$ , and  $\lambda_j$ . The discounted lifetime utility of being unemployed,  $V_{i,0}^M$ , can be written as the weighted average of flow utilities taking the expected moving costs ( $\theta_i m$  and  $\theta_j m$ ) into account.

In a similar way as above, the steady state lifetime utility  $V_{i,0}^C$  of an unemployed worker choosing CS can be written as:

$$V_{i,0}^C = \frac{1}{\delta} ((\mu_1 + \mu_2) v_{i,0} + \mu_3 v_{i,i} + \mu_4 v_{i,j}) \quad (4)$$

Hence, lifetime utility  $V_{i,0}^C$  can be written as the weighted sum of flow utilities, where the same weights are used as in equation (3).

### 3 Spatial Equilibrium

In this section we will characterize the spatial equilibrium in the labour/ housing market. The equilibrium is defined such that no unemployed worker would gain from choosing another strategy in each period. Hence, the unemployed who find a job in the other region and decide to commute to the job will not gain from moving residence to the other region. Similarly, the unemployed who find a job in the other region and decide to move will not gain from not move and commute instead. First, we suppose that the flow utility  $v_{i,j}$  can be decomposed into a region  $j$  work-related utility flow, a region  $i$  related residence-related utility flow, and a region  $i$  and  $j$  commuting-related utility flow. So the flow utility  $v_{i,j}$  is an additive function of the work-related utility flow  $w_j$ , the residence-related utility flow  $a_i$ , and the costs of commuting  $c_{i,j}$ , between regions  $i$  and  $j$ . So:

$$v_{i,j} = w_j - a_i - c_{i,j} \quad (5)$$

For example,  $w_j$  and  $a_i$  may be interpreted as the wage earned in region  $j$

and the housing rent paid in region  $i$  respectively. For notational convenience, we standardise  $c_{i,i} = c_{j,j} = 0$ , so intraregional commuting costs are zero<sup>7</sup>.

In equilibrium interregional commuting and moving to the other region may, or may not occur. Let us consider the case where *both* interregional commuting and moving between regions occur. This implies that the lifetime utility of an unemployed worker who selects MS must equal the lifetime utility of an unemployed worker who choose CS, so the unemployed worker is indifferent between two strategies,  $V_{i,0}^M = V_{i,0}^C$ . Hence using equation (3) and (4) in equilibrium the following condition must hold:

$$\mu_2(v_{j,0} - v_{i,0}) + \mu_4(v_{j,j} - v_{i,j}) = (\mu_1\theta_j + \mu_2\theta_i)m \quad (6)$$

This equilibrium condition shows that the sum of the weighted regional differences in the flow utilities is equal to the expected moving costs. The regional difference in the flow utilities are weighted with the probability of being employed or unemployed in region  $j$  ( $\mu_2$  and  $\mu_4$ ), so both size of the regional difference in the flow utility as well as how often the worker expects to be employed or unemployed matter. For example, it could be the case that the flow utility of being unemployed is not region specific, so  $v_{j,0} = v_{i,0}$ . In this case, it appears a necessary condition for equilibrium is that  $v_{j,j} > v_{i,j}$ . When the flow utilities defined by equation (5) are substituted into equation (6) then the equilibrium condition can be reformulated as:

$$(\mu_2 + \mu_4)(a_i - a_j) + \mu_4c_{i,j} = (\mu_1\theta_j + \mu_2\theta_i)m \quad (7)$$

This shows that the sum of the regional weighted difference in housing rent and the interregional commuting costs is equal to the expected moving costs. Equilibrium condition (7) does not depend on difference in the regional wages ( $w_i - w_j$ ). This occurs because the location of residence does not influence where you expect to find a job in the future<sup>8</sup>. If the housing rent is lower in

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<sup>7</sup>Presuming positive intraregional commuting costs does not change any result.

<sup>8</sup>If the model was extended by the job arrival rate to depend on place of residence, differences in regional wages would be relevant.

region  $i$  than in  $j$  (so  $a_i < a_j$ ), then  $\frac{c_{i,j}}{m} > \frac{(\mu_1\theta_j + \mu_2\theta_i)}{\mu_4}$  is needed for equilibrium. Equation (7) is derived for  $V_{i,0}^M = V_{i,0}^C$ , but for  $V_{j,0}^M = V_{j,0}^C$  we could derive a similar condition.

From now on, we suppose that regions are identical<sup>9</sup>. So,  $V_{i,0}^M = V_{j,0}^M = V_0^M$  and  $V_{i,0}^C = V_{j,0}^C = V_0^C$ . We consider *three* equilibriums which are characterized by the difference between  $V_0^M$  and  $V_0^C$ . It must be the case that either (i)  $V_0^M > V_0^C$ , (ii)  $V_0^M < V_0^C$  or (iii)  $V_0^M = V_0^C$ . No other equilibriums exist because we have assumed that unemployed workers are identical. In the first equilibrium, the lifetime utility of moving exceeds the lifetime utility of commuting. Hence commuting between regions does not occur and all interregional job offers induce a residential move to the other region. In the second, the opposite is the case. Moving to the other region does not occur and all interregional job offers induce commuting to the other region. In the remainder of this section, we focus on the third equilibrium:  $V_0^M = V_0^C$ , and we suppose that this equilibrium exists before and after the introduction of a road tax.

We denote the number of unemployed workers as  $n_0$ , the number of intraregional commuters who have received and accepted while unemployed an intraregional offer as  $n_s$  (*short* distance), the number of intraregional commuters who have received and accepted while unemployed an interregional job offer but who move residence as  $n_m$ , and the number of interregional commuters as  $n_l$  (*long* distance). The size of the labour force is normalized to 1. It follows that  $n_0 + n_s + n_m + n_l = 1$  so  $n_0$ ,  $n_s$ ,  $n_m$ , and  $n_l$  can be interpreted as probabilities of being in a certain labour/ housing market state. Further, we distinguish between unemployed individuals with a moving or commuting strategy, and employ therefore  $n_0^M$  and  $n_0^C$ , where  $n_0^M + n_0^C = n_0$ .

Following the literature on congestion, let us presume now that the interregional commuting costs are endogenously determined, because these costs depend positively on the number of interregional commuters due to road

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<sup>9</sup>Given identical regions,  $a_i = a_j$ , and we will see later on that the value of the housing rents do not play any role in the model, so our results are consistent with endogenous and exogenous housing rents.

congestion. So,  $c = c[n_l] > 0$ , where  $c[\cdot]$  is a continuous increasing function of its argument.

The assumption of identical regions implies that equation (7) can be written as:

$$c[n_l] = m(\delta + \lambda) \quad (8)$$

Hence, in this equilibrium, the interregional commuting costs are equal to the discounted residential moving costs, where the discounting occurs based on the sum of the discount and separation rate. Discounting occurs because the residential moving costs are paid upfront whereas the commuting costs are paid each period during the whole job spell. So workers take into account the risk of becoming unemployed, because the increase in the flow utility due to moving is lost when the workers become unemployed.

Recall that we have presumed that  $V_0^C = V_0^M$ . Based on (8), it can easily be seen given which parameter values this type of equilibrium exists. If  $c[0] > m(\delta + \lambda)$ , then  $V_0^M > V_0^C$ ; if  $c[1 - n_0] < m(\delta + \lambda)$  then  $V_0^C > V_0^M$ . Hence,  $V_0^M = V_0^C$  if  $c[0] < m(\delta + \lambda) < c[1 - n_0]$ .

According to equation (8), the number of commuters,  $n_l$ , is endogenously determined and depend on the moving costs, since the interregional commuting costs depend on the number of commuters, and the interregional commuting costs are equal to the discounted moving costs. In equilibrium, the number of interregional commuters  $n_l$  are an increasing function of residential moving costs. Because the residential moving costs are exogenous, equation (8) implies that the commuting costs are given and hence the demand for commuting is perfect price elastic.

Now suppose that the government introduces a road tax on the congested roads to deal with the external costs of commuting. The equilibrium condition in equation (8) yields then that the commuting costs including the road tax are equal to the discounted residential moving costs. Hence,  $c[n_l] = g[n_l] + \tau$ , where  $g[n_l]$  denotes the real commuting costs ( $g' > 0$ ) and  $\tau$  denotes the tax. Thus,

$\frac{\partial c[n_l]}{\partial \tau} = 0$ ,  $\frac{\partial n_l}{\partial \tau} < 0$ ,  $\frac{\partial n_s}{\partial \tau} > 0$ ,  $\frac{\partial n_0^M}{\partial \tau} > 0$ ,  $\frac{\partial n_0^C}{\partial \tau} < 0$ . The commuting costs, inclusive road tax, do not depend on the tax, because the demand for commuting is perfect price elastic. The tax induces more unemployed workers to choose MS, which results in less interregional commuters and less congestion but at the same time induces more regional moves that are costly. The welfare implications of the road tax are further analysed in the following section.

## 4 Welfare Effects of Road Pricing

Road pricing makes it more expensive to commute between regions and we have seen that it induces more workers to choose MS. Hence, road pricing induces more costly moves but less congested interregional connections. This raises the question what is the optimal road tax? Further, and importantly, to what extent does the optimal tax differ from the standard Pigouvian tax policy, which claims that the optimal tax is such that the tax is equal to the marginal external costs, but which is usually applied to a static model. These questions are answered using a social welfare function. We assume that the revenue from the tax is redistributed as a lump sum transfer to each individual in the labour force.

In the current paper, the focus is how congestion and residential moving behaviour are interrelated based on a labour market search model. Congested roads are not only used by commuters, but also by other road users. To simplify the analysis, we impose the assumption that other users are perfect price inelastic. This assumption can be interpreted as a simplification of the assumption that other users are less price elastic than commuters. This assumption seems valid in the light of (8) which shows that commuters are perfect price elastic<sup>10</sup>.

We distinguish again between the three types of equilibriums that may occur before and after the introduction of the road tax: i)  $V_{i,0}^M > V_{i,0}^C$ , ii)  $V_{i,0}^M < V_{i,0}^C$  and iii)  $V_{i,0}^M = V_{i,0}^C$ . We will call a combination of an equilibrium before the road tax with an equilibrium after the road tax an 'outcome'.

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<sup>10</sup>This result applies only in the equilibrium defined by  $V_0^M = V_0^C$ .

To investigate the welfare implications for the different outcomes we define an additive social welfare function (SWF) which is equal to the sum of lifetime utilities enjoyed by all types of workers. Hence:

$$SWF = n_0 V_0 + n_s V_s + n_l V_l + n_m (V_s - m) + T \quad (9)$$

where  $T$  is the total discounted revenue from the road tax in all future periods which is equal to  $\frac{1}{\delta} n_l \tau$ . This SWF is based on the lifetime utilities of workers when the economy is in steady state equilibrium<sup>11</sup>. In the steady state, the SWF measures the weighted lifetime utility a worker expects to enjoy where the weights are determined by the probability of being in a certain labour/ housing market state. Note that workers who move residence pay moving costs to enjoy life time utility  $V_s$ , so  $n_m$  workers pay  $m$ .

To evaluate the effect of a road tax, the  $SWF$ 's in two specific equilibriums are compared ( $SWF^1 - SWF^0$ ). From now on, the superscript 0 refers to the baseline economy without a road tax and the superscript 1 refers to an economy with a road tax.

The equilibrium condition,  $m(\delta + \lambda) = c[n_l] = g[n_l] + \tau$ , which has been derived in the previous section, can be applied. Note that  $m(\delta + \lambda)$  is not affected by the road tax, so this condition implies that  $g[n_l^0] = g[n_l^1] + \tau$ , i.e. the real commuting costs before the road tax,  $g[n_l^0]$ , are equal to the real commuting costs inclusive road pricing,  $g[n_l^1] + \tau$ .

It can be easily seen that the road tax has no effect on the unemployment rate and also no effect on the number of workers who find a job in their place of residence,  $n_s$ , because neither the job offer probabilities nor the job acceptance probability are affected by the road tax (for a formal proof, see Appendix C)<sup>12</sup>.

Table 1 presents all six relevant outcomes which may occur<sup>13</sup>. We emphasize

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<sup>11</sup>No intermediate dynamics are measured or valued.

<sup>12</sup>Note that in Parry and Bento (2001) and other labour market studies, the quantity of labour supply is reduced. This is *not* the case in the current paper.

<sup>13</sup>Three of the nine outcomes mentioned in Table 1 do not occur, because these outcomes imply an increase in interregional commuting due to road pricing, which is inconsistent with the model (and intuition).

here that the tax may not be optimally set e.g. due to absence of information by the government, so we focus on an arbitrarily set road tax.

**Table 1. Effects of road tax on welfare**

		Equilibrium after road tax		
		$V_0^M > V_0^C$	$V_0^M = V_0^C$	$V_0^M < V_0^C$
Equilibrium before road tax	$V_0^M > V_0^C$	<i>I</i> : No		
	$V_0^M = V_0^C$	<i>II</i> : No	<i>IV</i> : Gain	
	$V_0^M < V_0^C$	<i>III</i> : Loss	<i>V</i> : Ambiguous	<i>VI</i> : No

Let us first concentrate on three outcomes that result in a road tax regime for which holds that  $V_0^M > V_0^C$  so interregional commuting does not occur when the road tax is introduced. These outcomes are labelled in the table as outcomes *I*, *II*, and *III*. For outcome *I* there is obviously no welfare effect since interregional commuting does not occur before the road tax. Outcome *II* occurs when before the road tax was introduced, interregional commuting was equally alternative as residential moving, but due to the road tax commuting is not an acceptable option any more. So after the introduction of the road tax the condition  $c[0] > m(\delta + s)$  holds. Interestingly, forcing all commuters off the interregional roads has no negative (or positive) welfare implications. Although we will see that the tax has been set too high, in the sense that the tax is not welfare maximizing, it does not reduce welfare. Clearing the roads from interregional commuters does not result in a welfare loss, because the interregional commuters are not worse off by switching to residential moving. The latter is true, since moving was an equally alternative before the road tax. We find that this result is relevant, because it is often unknown how commuters will react to an introduction of a road tax. This result provides some room for a learning process for the tax collectors, because there is no welfare loss connected with overtaxing.

In outcome *III* residential moving does not occur before the road tax is introduced, but commuting does not occur after the road tax. It illustrates



the effect of a road tax scheme with sufficiently high taxes to eliminate all interregional commuting. All commuters are 'forced' to choose to move, which none of them preferred in the baseline situation, and this results in a welfare loss. Because interregional commuting is absent after the road tax scheme has been implemented, there is no tax revenue collected, which may have compensated the welfare loss. This result is also important, because it shows explicitly when a road tax will reduce welfare. The message is here that if all commuters switch to moving residence, the tax has been set too high.

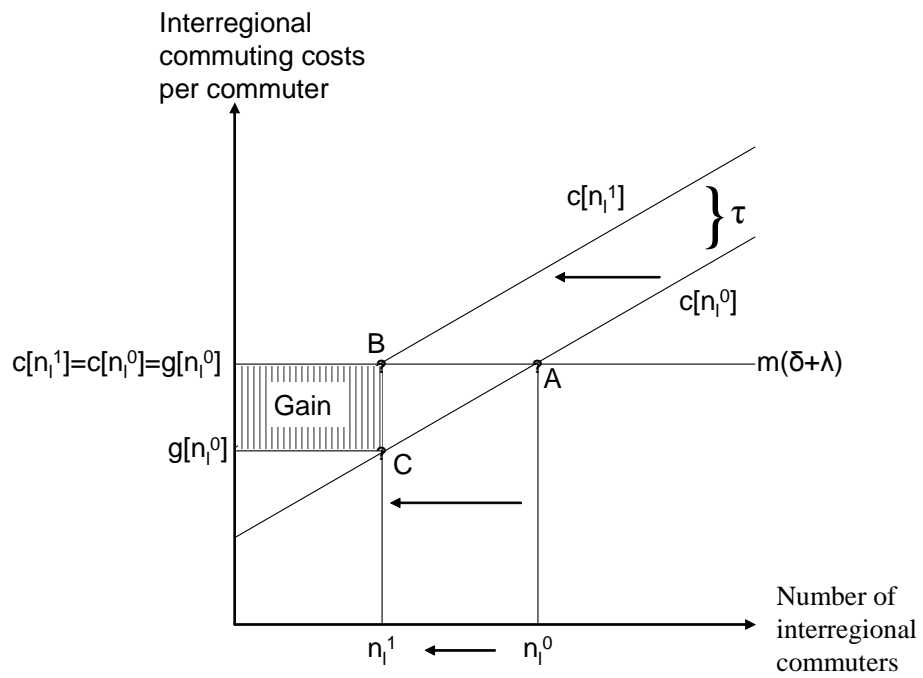
Let us now consider outcome *IV*, when one observes both interregional commuting and residential moves before and after the introduction of road tax, so  $V_0^M = V_0^C$  both before and after the road tax. Outcome *IV* implies that the number of interregional commuters and the number of residential movers change as a result of the road tax. It can be shown that lifetime utilities,  $V_0$ ,  $V_s$ ,  $V_l$ , and  $(V_s - m)$  do not change as a result of the road tax (see Appendix B for a formal proof). The lifetime utility of the interregional commuters does not change because the commuting costs (inclusive the tax) do not change. Further it follows that  $V_l = V_s - m$ . As a consequence, any change in  $n_l$  and  $n_m$  does not have any impact on the *SWF* (see equation (9)). The impact of road pricing on the *SWF* is therefore equal to the discounted tax revenue paid in all future periods (see appendix B):

$$SWF^1 - SWF^0 = \frac{1}{\delta} n_l^1 \tau = T > 0 \quad (10)$$

The welfare gain arises because of the reduction in congestion externality. The gain is equal to the standard first-best Pigouvian tax. Equation (10) implies that there is a positive welfare gain because  $n_l^1 > 0$ . Figure 1 illustrates the welfare effect of a road tax.

As discussed in section 2, the opportunity of moving residence induces the demand for commuting to become perfect price elastic (see Figure 1). In Figure 1, the horizontal line is the inverse demand function for commuting, which is equal to the interregional commuting costs and the discounted residential

Figure 1: Welfare effects of a road tax - outcome *IV*



**Note:**  $c[n_i]$  denotes the commuting costs including the road tax.  $g[n_i]$  denotes the commuting costs excluding the road tax.

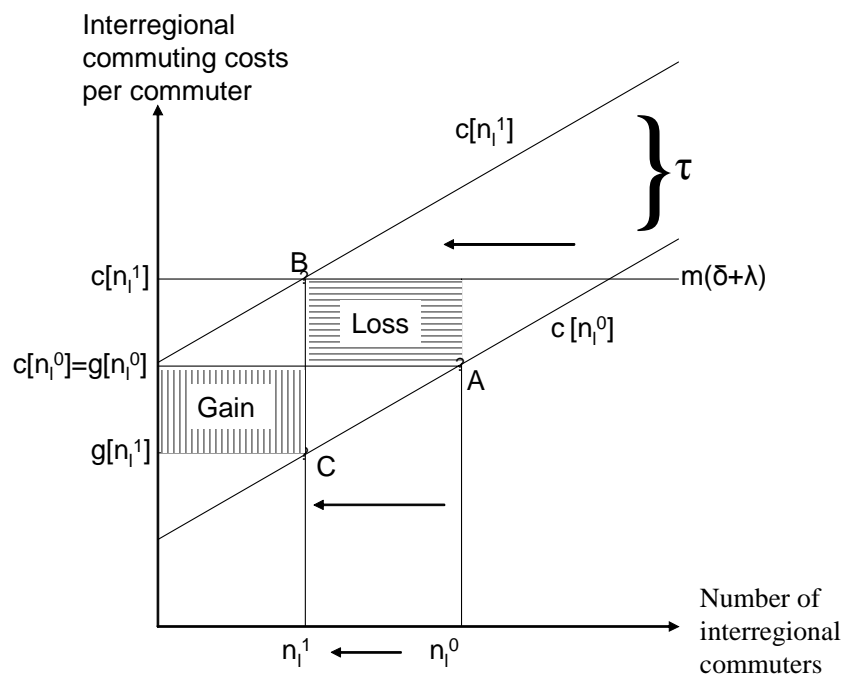
moving costs  $m(\delta + \lambda)$ . The congestion cost function  $c[n_l^0]$  indicates that the commuting costs per commuter increase in the total number of commuters. The intersection of the inverse demand function for commuting and the congestion cost function is point A, which indicates an equilibrium with  $n_l^0$  commuters. Due to the road tax, the congestion cost curve moves upwards because of the extra costs the interregional commuters have to pay, so the equilibrium shifts to point B. The number of interregional commuters drops from  $n_l^0$  to  $n_l^1$ . The area between  $n_l^0$  and  $n_l^1$  and the horizontal commuting inverse demand function is equal to the reduction in the commuting costs in the economy,  $(c[n_l^1] - c[n_l^0])(n_l^0 - n_l^1)$ . The reduction in commuting costs is equal to the increase in residential moving costs,  $(\delta + \lambda)(m(n_l^1 - n_l^0))$ . Clearly,  $c[n_l^1] = c[n_l^0] = g[n_l^0] = g[n_l^1] + \tau$ , so the cost of commuting do not change due to the road tax, but the costs  $g[n_l^1]$  are less than  $g[n_l^0]$ . The commuting costs excluding road tax,  $g[n_l^0]$ , are determined by point C. The tax revenue in the figure is the shaded rectangle. The rectangle should be maximized to maximize welfare (see Verhoef (2004)). Because the demand for commuting is perfect price elastic the welfare gain of road pricing is equal to the tax revenue.

Now consider outcome V. The baseline situation is that workers do not move between regions, but given the road tax, interregional moving and commuting occur both. For this outcome, the commuting costs increase due to the road tax, because  $m(\delta + \lambda) = g[n_l^1] + \tau > c[n_l^0]$ . The welfare effects can be written as (see Appendix B):

$$SWF^1 - SWF^0 = n_l^0 \left( \frac{g[n_l^0] - g[n_l^1] - \tau}{(\delta + \lambda)} \right) + \frac{1}{\delta} n_l^1 \tau \quad (11)$$

The first term on the right hand side measures the welfare loss due to the increase in the costs of finding a job in another region. It is negative because  $n_l^0 < n_l^1$  and  $g'[n_l] > 0$ . The second term measures the welfare gain because the congestion externality is internalized. It is not the case that one of the opposite effects dominates the other. Therefore the welfare effects are ambiguous in outcome V. Figure 2 illustrates the welfare effect of a road tax for outcome V.

Figure 2: Welfare effects of a road tax - outcome  $V$



**Note:**  $c[n_i]$  denotes the commuting costs including the road tax.  $g[n_i]$  denotes the commuting costs excluding the road tax.

The initial equilibrium is point A, so workers do not move residence between regions, ( $n_m^0 = 0$ ), because the discounted moving costs exceed the commuting costs. This can be seen in Figure 2 because the horizontal commuting inverse demand function is above point A. As in Figure 1, the congestion cost curve moves upwards because of the road tax, so the new equilibrium is in point B. In the new equilibrium some workers choose MS, so the commuting costs in point B are equal to  $m(\delta + \lambda)$ . The road tax results in lower real commuting  $g[n_t^1]$  as indicated by point C. The welfare gain due to less congestion is equal to the rectangle with the shaded vertical lines. The gain is internalized via the road tax. The welfare gain is not equal to the tax revenue, because the commuting costs increase. The ambiguity of outcome  $V$  arises because of another effect which results in a welfare loss that is illustrated by the rectangle with the shaded horizontal lines. This welfare loss is due to the additional cost of moving for  $n^0 - n^1$  workers, who experience higher costs compared to the initial equilibrium without road tax. Whether or not the overall welfare effect is positive or negative in outcome  $V$  depends on the specific parameters of the model. For example, if  $g[n_t^1]$  is large, then the welfare effects are more likely to be negative. The standard Pigouvian tax policy does *not* hold here, because *in equilibrium* an alternative option (moving residence) is not acceptable.

When workers never move residence (before and after the introduction of the road tax), then the demand for interregional commuting is price inelastic (outcome  $VI$ ), so the road tax does not affect welfare.

Let us now discuss the main assumptions which drive our results. These are the assumptions regarding the presence of residential moving costs and imperfect information in the labour market. Unemployed workers search for jobs given incomplete information about the location of the job openings and one of the implications is excess commuting: workers commute to other regions for which they are not compensated by means of higher wages or lower housing rents. The main results are that under specific circumstances the welfare maximizing road tax maximizes road tax revenue, but that a road tax may have negative welfare implications under some circumstances.

We have presumed that all workers are identical, but this is generally not the case. It may be useful to allow for heterogeneity of moving costs (moving costs may be higher for some workers than for others, for example, due to stronger local personal networks). Heterogeneous moving costs affect the main finding for outcome *IV*, which states that the welfare maximizing road tax maximizes road tax revenue. This result relies on the perfect price elastic demand for commuting, which is derived from homogenous moving costs. Given heterogeneity of moving costs the demand for commuting is not perfect price elastic anymore. Heterogeneous moving costs do *not* affect the main finding of outcome *V*, which is that a road tax may have negative welfare implications.

## 5 Conclusion

We have studied how congestion and residential moving behaviour are related to each other employing a job search model allowing for search imperfections. Depending on the amount of commuting and residential moving between regions, we demonstrate that a congestion tax may lead to both welfare losses and gains. Under the following circumstances the model predicts when to expect welfare losses or gains:

i) When workers move residence and commute interregionally at the same time before and after the introduction of a road tax, a road tax induces a positive welfare gain, because of the reduction in the congestion externality. In this situation the road tax that maximizes the road tax revenue will maximize overall welfare. Even if the tax collectors set the road tax price too high and clear the roads from commuting traffic it does not induce a welfare loss.

ii) When interregional residential moves do not occur before the introduction of the road tax, then the welfare effect of a road tax may be positive or negative. When interregional commuting does not occur after the road tax has been introduced the welfare effect is negative.

The model can easily be extended in many ways. For example, we have focussed on workers who belong to one-earner households, but the case of

two-earner households deserves attention, since for these households, the residential moving decision is less straightforward. Further we would like to consider endogenous wages, non-identical regions, and different kind of price formation in housing markets. These are to be examined in future work.

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## A Appendix - Derivation of lifetime utility

In this appendix we first define the steady state lifetime utilities of workers who choose MS. Then we write the lifetime utility of an unemployed who chooses MS,  $V_{i,0,t}^M$ , as a function of flow utilities.

Note that given equation (2), the lifetime utility of an unemployed worker who knows what he will choose MS when becoming unemployed,  $V_{i,i,t}^M$ , is defined as:

$$V_{i,i,t}^M = \frac{1}{1+\delta} (v_{i,i,t} + \lambda_i V_{i,0,t+1}^M + (1-\lambda_i) V_{i,i,t+1}^M) \quad (12)$$

In steady state the above equation can be written as:

$$V_{i,i}^M = \frac{v_{i,i} + \lambda_i V_{i,0}^M}{\delta + \lambda_i} \quad (13)$$

The steady state lifetime utility of an unemployed in region  $i$ ,  $V_{i,0}^M$ , and in region  $j$ ,  $V_{j,0}^M$ , as defined in equation (1) can be written as:

$$V_{i,0}^M = \frac{v_{i,0} + \theta_i V_{i,i}^M + \theta_j (V_{j,j}^M - m)}{\delta + \theta_i + \theta_j} \quad (14)$$

and:

$$V_{j,0}^M = \frac{v_{j,0} + \theta_j V_{j,j}^M + \theta_i (V_{i,i}^M - m)}{\delta + \theta_i + \theta_j} \quad (15)$$

Substituting  $V_{j,0}^M$  into  $V_{j,j}^M$  (as defined by equation (13)) we obtain:

$$V_{j,j}^M = \frac{(\delta + \theta_i + \theta_j) v_{j,j} + \lambda_j v_{j,0} + \theta_i \lambda_j (V_{i,i}^M - m)}{(\delta + \lambda_j) (\delta + \theta_i + \theta_j) - \theta_j \lambda_j} \quad (16)$$

Substituting equation (16) and (13) into equation (14), we obtain:

$$V_{i,0}^M = \frac{1}{\delta} (\mu_1 (v_{i,0} - \theta_j m) + \mu_2 (v_{j,0} - \theta_i m) + \mu_3 v_{i,i} + \mu_4 v_{j,j}) \quad (17)$$

where the  $\mu$ 's are defined in section 2 after equation (3). Table A.1 shows how the  $\mu$ 's depend on the exogenous parameters  $(\delta, \lambda, \theta)$ .

**Table A.1 Weights**

	$\delta$	$\lambda$	$\theta$
$\mu_1$	+	+	-
$\mu_2$	-	+	-/+
$\mu_3$	-	-	+
$\mu_4$	-	-	+

## B Appendix - Welfare analysis

In this appendix, we will define the social welfare function ( $SWF$ ). The additive  $SWF$  is defined using lifetime utilities:

$$SWF = n_0 V_0 + n_s V_s + n_l V_l + n_m (V_s - m) + T \quad (18)$$

where  $T = \frac{1}{\delta} n_l \tau$  is the discounted revenue from the road tax in all future periods<sup>14</sup>.

To evaluate the welfare implications of a road tax we compare the  $SWF$  in two steady states ( $SWF^1 - SWF^0$ ), where the superscript 0 defines the equilibrium before the introduction of a road tax and the superscript 1 defines an equilibrium after:

$$\begin{aligned} SWF^1 - SWF^0 &= n_0^1 V_0^1 + n_s^1 V_s^1 + n_l^1 V_l^1 + n_m^1 (V_s^1 - m) + T \\ &- (n_0^0 V_0^0 + n_s^0 V_s^0 + n_l^0 V_l^0 + n_m^0 (V_s^0 - m)) \end{aligned} \quad (19)$$

It can easily be shown (see Appendix C) that the number of unemployed,  $n_0$ , and intraregional commuters,  $n_s$ , do not depend on  $\tau$ , so  $n_0^1 = n_0^0 = n_0$  and  $n_s^1 = n_s^0 = n_s$ . Hence (19) can be rewritten as:

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<sup>14</sup>We have assumed that the revenue is redistributed as a lump sum transfer to each individual in the labour force, which does not affect the labour market search strategies. This is identical to assuming that the tax collector keeps the tax revenue for lump sum transfers and let the tax collector enter the  $SWF$ . Consequently, the lump sum transfers are not included in the flow utilities.

$$SWF^1 - SWF^0 = n_0 V_0^1 + n_s V_s^1 + n_l V_l^1 + n_m (V_s^1 - m) + T \quad (20)$$

$$- (n_0 V_0^0 + n_s V_s^0 + n_l V_l^0 + n_m (V_s^0 - m))$$

Equation (20) will be the basis of the welfare analysis. We will explicitly use it here to derive the welfare changes for outcomes *IV* (equation (26)) and outcome *V* (equation (29)). The welfare changes for the other outcomes can be analysed similarly.

### B.1 Outcome *IV*

We first derive equation (10). Outcome *IV* implies the presence of both residential moves and interregional commuting before and after the introduction of the road tax. For outcome *IV*, we defined that  $V_0^M = V_0^C = V_0$  (see section 3). To derive the welfare changes we will show that 1) lifetime utilities do not change due to the road tax, so  $V_0^{M0} = V_0^{M1}$ ,  $V_s^0 = V_s^1$ ,  $V_l^0 = V_l^1$  and 2) the number of interregional commuters and residential movers do not change due to the tax. Thus, the welfare gain will be equal to the tax revenue.

1) The steady state lifetime utility of an unemployed who chooses MS can be written as (see appendix A, equation (17) and impose identical regions and use equation (5)):

$$V_0^{M1} = \frac{1}{\delta \left(1 + \frac{2\theta}{\partial + \lambda}\right)} \left( (-a - \theta m) + \frac{2\theta}{\partial + \lambda} (w - a) \right) = V_0^{M0} \quad (21)$$

None of the variables in equation (21) depend on the road tax, hence  $V_0^1 = V_0^0 = V_0$ .

Similarly, the lifetime utilities of the employed can be written as:

$$V_s^1 = \frac{v_s + \lambda V_0}{(\delta + \lambda)} = \frac{w - a + \lambda V_0}{(\delta + \lambda)} = V_s^0 \quad (22)$$

$$V_l^1 = \frac{v_l^1 + \lambda V_0}{(\delta + \lambda)} = \frac{w - a - c^1 - \tau + \lambda V_0}{(\delta + \lambda)} = \frac{w - a - c^o + \lambda V_0}{(\delta + \lambda)} = V_l^0 \quad (23)$$

Hence  $V_s^1 = V_s^0 = V_s$  and  $V_l^1 = V_l^0 = V_l$ .  $V_s$  does not depend on  $\tau$ , because  $V_0$  does not depend on  $\tau$ . Furthermore,  $V_l$  does not depend on  $\tau$  because, in equilibrium:  $m(\delta + \lambda) = g[n_l] + \tau$ . The equilibrium condition also implies that  $g[n_l^0] = g[n_l^1] + \tau$ , i.e. the total commuting costs does not depend on  $\tau$ .

Further in equilibrium:

$$\begin{aligned}
V_l &= \frac{w - a - g[n_l^1] - \tau + \lambda V_0}{(\delta + \lambda)} \\
&= \frac{w - a - m(\delta + \lambda) + \lambda V_0}{(\delta + \lambda)} \\
&= \frac{w - a + \lambda V_0}{(\delta + \lambda)} - m \\
&= V_s - m
\end{aligned} \tag{24}$$

Hence, the lifetime utility of interregional commuters is equal to the lifetime utility of workers who move residence. Further, the increase in the number of workers who has changed place of residence ( $n_m^1 - n_m^0$ ) must be equal to the decrease in number of interregional commuters:

$$n_m^1 - n_m^0 = n_l^1 - n_l^0 \tag{25}$$

Using equations (21) to (25) implies that (19) can be written as:

$$SWF^1 - SWF^0 = T^1 = \frac{1}{\delta} n_l^1 \tau \tag{26}$$

## B.2 Outcome $V$

To derive equation (11), we analyse outcome  $V$ . For this outcome,  $m(\delta + \lambda) = g[n_l^1] + \tau > c[n_l^0]$ , because  $V_0^{M0} < V_0^{C0}$  and  $V_0^{M1} = V_0^{C1}$ . The welfare effect can then be written as:

$$SWF^1 - SWF^0 = n_l^1 V_l^1 + n_m^1 (V_s^1 - m) + T - n_l^0 V_l^0 \tag{27}$$

The number of unemployed workers and the number of employed workers is exogenous as shown in Appendix C. The condition:  $m(\delta + \lambda) = c[n_l^1] + \tau$

implies that  $V_l^1 = (V_s^1 - m)$ . The welfare effect is:

$$\begin{aligned} SWF^1 - SWF^0 &= (n_l^1 + n_m^1) \left( \frac{w - a - c^1 - \tau + \lambda V_0}{(\delta + \lambda)} \right) + T \\ &\quad - n_l^0 \left( \frac{w - a - c^0 + \lambda V_0}{(\delta + \lambda)} \right) \end{aligned} \quad (28)$$

The number of workers who find a job in another region does not depend on  $\tau$ , so  $(n_l^1 + n_m^1) = n_l^0$ , because  $V_0^{M0} < V_0^{C0}$  implies that  $n_m^0 = 0$ . Hence equation (28) can be written as:

$$\begin{aligned} SWF^1 - SWF^0 &= n_l^0 \left( \frac{w - a - g[n_l^1] - \tau + \lambda V_0}{(\delta + \lambda)} \right) + T \\ &\quad - n_l^0 \left( \frac{w - a - g[n_l^0] + \lambda V_0}{(\delta + \lambda)} \right) \\ &= n_l^0 \left( \frac{g[n_l^0] - g[n_l^1] - \tau}{(\delta + \lambda)} \right) + \frac{1}{\delta} n_l^1 \tau \end{aligned} \quad (29)$$

## C Appendix - Number of unemployed and employed workers in steady state

In the steady state, the number of employed who become unemployed must equal the number of unemployed who become employed. Note that  $2\theta$  times the number of unemployed workers,  $n_0$ , will find a job during a period. Furthermore, the exogenous separation rate times the number of employed,  $1 - n_0$ , is equal to the number of workers who become unemployed each period. In the steady state:

$$2\theta n_0 - \lambda(1 - n_0) = 0 \quad (30)$$

So,

$$n_0 = \frac{\lambda}{2\theta + \lambda} \quad (31)$$

Using a similar approach, it can be shown that the number of workers  $n_s, n_l$ , and  $n_m$  are defined by:

$$n_s = \frac{\theta}{2\theta + \lambda} = n_l + n_m \quad (32)$$

Because  $\theta$  and  $\lambda$  are exogenously given, it follows that  $n_0$  and  $n_s$  do not depend on the road tax  $\tau$ .