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Working Paper

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Tinbergen Institute Discussion Paper, No. 07-097/1

Provided in Cooperation with:
Tinbergen Institute, Amsterdam and Rotterdam

TI 2007-097/1
Tinbergen Institute Discussion Paper

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Dynamic efficiency of product market competition: Cournot versus Bertrand

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December 17, 2007

Abstract

We consider the efficiency of Cournot and Bertrand equilibria in a duopoly with substitutable goods where firms invest in process R&D. Under Cournot competition firms always invest more in R&D than under Bertrand competition. More importantly, Cournot competition yields *lower* prices than Bertrand competition when the R&D production process is efficient, when spillovers are substantial, and when goods are not too differentiated. The range of cases for which total surplus under Cournot competition exceeds that under Bertrand competition is even larger as competition over quantities always yields the largest producers' surplus.

Key words: Bertrand competition; Cournot competition; process R&D; efficiency.

JEL Classification: L13.

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[‡]Thanks are due to seminar participants at the Katholieke Universiteit Leuven and the University of Amsterdam. The usual disclaimer applies.

1 Introduction

Competition over price (Bertrand competition) is known to yield lower prices than competition over quantities (Cournot competition). This result was first established by Sing and Vives (1984) for a symmetric duopoly supplying demand substitutes (see also Cheng, 1985). It is robust to various generalizations, including the extension to an oligopoly (Vives, 1985), to differences in costs under Cournot and Bertrand competition (Qiu, 1997; Lopez and Naylor, 2004; Zanchettin, 2006), to differences in product quality under the two types of competition (Linn and Saggi, 2002; Symeonidis, 2003), and to differences in market structure associated with price and quantity competition (Cellini *et al.*, 2004; Mukherjee, 2005).[1]

In this paper we qualify the celebrated result of Sing and Vives (1984) by showing that Cournot competition can yield *lower* prices than Bertrand competition in a duopoly with endogenous production costs that supplies demand substitutes. This occurs when products are relatively homogenous, when technological spillovers are strong, and when the R&D production process is sufficiently efficient. It is precisely under these circumstances that the incentives to conduct R&D are much larger under Cournot competition than under Bertrand competition as in this case much more of the benefits of any cost reduction are given to consumers when competition is over price. As a result, post innovation costs are much lower under Cournot competition which translates into a lower equilibrium price. The range of cases for which total surplus under Cournot competition exceeds that under Bertrand competition is even larger as profits under Bertrand competition are always below those under Cournot competition.

Our analysis is related to that of Qiu (1997). The main difference is that we consider technological spillovers to occur during the R&D process while Qiu (1997) assumes that final R&D results spill over. That is, we consider input spillovers rather than output spillovers. There are at least three important reasons for doing so. First, empirical studies indicate that spillovers indeed occur during the R&D process (Kaiser, 2002). This finding corresponds to the three channels that Geroski (1995) identifies through which a technological spillover can occur: (i) the exchange of ideas through publications, casual encounters and at seminars, (ii) the flow of knowledge when a knowledge worker changes employer, and (iii) the deduction of the line of reasoning of rivals by observing their behavior.

Second, Qiu (1997) assumes the R&D results of one firm to be *perfectly additive* to its rival's R&D results. There are at least three reasons to question this assumption. Note that the two firms operate in the same product market

[1] For an oligopoly supplying demand complements with quality differences an exception exists. Häckner (2000) shows that in this case the switch from Cournot competition to Bertrand competition induces the high-quality firm to charge a lower price. The resulting upward pressure on the demand for the low-quality complement then allows for a price increase of this complement.

while initially using the same production technology. It is then most likely that there will be some overlap in their independently obtained research results that are aimed at reducing the costs of production. Also, the parts that do not overlap are expected not to be a perfect match to rivals' research results. Finally, differences in corporate culture, research strategies, and internal organization hamper any firm's ability to appropriate fully rival's research results. In sum, high levels of technological output spillovers are not likely to be observed (Gerschbach and Schmutzler (2003) take an extreme position here by assuming that *all* of any firm's R&D results are perfectly additive to any of its rivals' R&D results).

Third, Qiu (1997) assumes diminishing returns to scale in R&D. In combination with additive output spillovers this has a counter-intuitive implication. If one firm has spent more on R&D than its rival, it could be in the interest of the former to donate its next R&D investment dollar to its rival and to appropriate the R&D results through the technological spillover. If these spillovers are substantial this could be a more effective additional cost reduction than spending this last R&D dollar on own R&D (Amir, 2000).

For these reasons we re-examine the dynamic efficiency of Cournot and Bertrand competition assuming input spillovers. In passing we reveal a technical error in Qiu (1997) related to the stability of equilibria when R&D is a strategic substitute.

2 The model

We consider a two-stage game. In the first stage firms invest in cost-reducing R&D. In the second stage they compete either over price or quantity. Market demand in indirect form is given by:[2]

$$p_i = a - (q_i + \theta q_j)\,, \tag{1}$$

$i, j = 1, 2$, $i \neq j$, where p_i and q_i are the respective price and quantity of product i, and where θ captures the extent to which products are differentiated; in case $\theta = 1$ products are homogeneous while $\theta = 0$ corresponds to completely differentiated products (i.e. both firms have a local monopoly). These polar cases are further ignored, that is, $\theta \in]0, 1[$. Unless stated otherwise, $i, j = 1, 2$, $i \neq j$ holds throughout the rest of the paper. Market demand in direct form is then given by:

$$q_i = \frac{1}{1-\theta^2}\left[(1-\theta)a - (p_i - \theta p_j)\right]. \tag{2}$$

The industry consists of two firms each producing one version of the differentiated product. *Ex ante* marginal costs of production, c, are fixed. We assume that

[2] This follows from a standard quadratic utility function, see Singh and Vives (1984).

both firms are active, that is, $c < a$. The fixed production costs can be reduced by investing in process-innovating R&D. Note that if one firm conducts R&D, the rival firm can absorb part of this effort without having to pay for it.[3] Accordingly, if firm i invests x_i in R&D, its *effective* R&D investments X_i are given by:

$$X_i = x_i + \beta x_j. \tag{3}$$

In (3) $\beta \in [0, 1]$ represents the technological spillover. The reduction in marginal cost brought about by these R&D investments is determined by an R&D production function f. This function is a mapping from effective R&D inputs to cost reductions. Following Kamien *et al.* (1992) we assume diminishing returns to scale in R&D: $f' > 0$, $f'' < 0$ and $f(0) = 0$. In particular we set:

$$f(X_i) = \sqrt{\frac{X_i}{\gamma}}, \tag{4}$$

whereby $\gamma > 0$ determines the efficiency of the R&D phase. A higher value of γ corresponds to a less efficient production of R&D results. Note that in this setting the technological spillover is an input of the R&D process. Firm i's profits then equal

$$\pi_i = p_i q_i - (c - y_i)\, q_i - x_i, \tag{5}$$

with $y_i = \sqrt{(x_i + \beta x_j)/\gamma}$.

3 Market equilibria

3.1 Second-stage Bertrand competition

Maximizing (5) over price yields equilibrium prices conditional on effective R&D efforts:[4]

$$\widehat{p}_i(X_i, X_j) - c = \frac{(a-c)(2+\theta)(1-\theta) - 2y_i - \theta y_j}{4 - \theta^2}. \tag{6}$$

[3] It is understood that firms have to conduct at least some R&D themselves to share in rival's R&D activities (for an early recognition of this point see Cohen and Levinthal, 1989). We abstain from modelling this absorptive capacity as it would make the analysis intractable (cfr. Kamien and Zang, 2000).

[4] A hat refers to a conditional equilibrium outcome.

Inserting (6) into (5) and maximizing the resulting profits over R&D investments result in the following cost reduction:[5,6]

$$\widetilde{y}^B = \frac{(a-c)\left(2-\theta^2-\theta\beta\right)}{\gamma(1+\theta)(2-\theta)(4-\theta^2)-(2-\theta^2-\theta\beta)}, \tag{7}$$

and concomitant total output:

$$\widetilde{Q}^B = \frac{2\gamma(a-c)(4-\theta^2)}{\gamma(1+\theta)(2-\theta)(4-\theta^2)-(2-\theta^2-\theta\beta)}. \tag{8}$$

Single-firm equilibrium profits then equal:

$$\widetilde{\pi}^B = \frac{\gamma(1+\beta)(1-\theta^2)(4-\theta^2)^2-(2-\theta^2-\theta\beta)^2}{\gamma(1+\beta)(4-\theta^2)^2}\left(\widetilde{q}^B\right)^2, \tag{9}$$

where $\widetilde{Q}^B = 2\widetilde{q}^B$. Consumers' surplus and total surplus are then respectively given by:

$$\widetilde{CS}^B = (1+\theta)\left(\widetilde{q}^B\right)^2, \tag{10}$$

and

$$\widetilde{TS}^B = \frac{\gamma(1+\beta)(1+\theta)(4-\theta^2)^2(3-2\theta)-2(2-\theta^2-\theta\beta)^2}{\gamma(1+\beta)(4-\theta^2)^2}\left(\widetilde{q}^B\right)^2. \tag{11}$$

3.2 Second-stage Cournot competition

Maximizing (5) over quantities gives us:

$$\widehat{q}_i(X_i,X_j) = \frac{(a-c)(2-\theta)+2y_i-\theta y_j}{4-\theta^2}. \tag{12}$$

Maximizing firm profits over R&D investments after inserting (12) into (5) yields as cost reduction and concomitant output level:[7]

$$\widetilde{y}^C = \frac{(a-c)(2-\theta\beta)}{\gamma(2+\theta)(4-\theta^2)-(2-\theta\beta)}, \tag{13}$$

and:

[5] A tilde refers to an unconditional equilibrium expression; superscript B stands for second-stage Bertrand competition.

[6] The concomitant second-order and stability conditions are dealt with below.

[7] Superscript C stands for second-stage Cournot competition.

$$\widetilde{Q}^C = \frac{2\gamma(a-c)(4-\theta^2)}{\gamma(2+\theta)(4-\theta^2)-(2-\theta\beta)}. \tag{14}$$

Single-firm profits are given by:

$$\widetilde{\pi}^C = \frac{\gamma(1+\beta)(4-\theta^2)^2-(2-\theta\beta)^2}{\gamma(1+\beta)(4-\theta^2)^2}\left(\widetilde{q}^C\right)^2, \tag{15}$$

with $\widetilde{Q}^C = 2\widetilde{q}^C$. Consumers' surplus and total welfare under second-stage Cournot competition then equal:

$$\widetilde{CS}^C = (1+\theta)\left(\widetilde{q}^C\right)^2, \tag{16}$$

and

$$\widetilde{TS}^C = \frac{\gamma(1+\beta)(3+\theta)(4-\theta^2)^2-2(2-\theta\beta)^2}{\gamma(1+\beta)(4-\theta^2)^2}\left(\widetilde{q}^C\right)^2. \tag{17}$$

3.3 Regularity conditions

The R&D stage gives rise to eight regularity conditions. In addition to the two second-order conditions, post-innovation costs have to be positive and the equilibrium has to be stable. The second-order conditions under Bertrand and Cournot competition require, respectively:

$$\gamma \geq \frac{\left(2-\theta^2-\theta\beta\right)^3}{(1-\theta^2)(4-\theta^2)^2(2-\theta^2-\theta\beta^2)}, \tag{R1}$$

and

$$\gamma \geq \frac{(2-\theta\beta)^3}{(2-\theta\beta^2)(4-\theta^2)^2}. \tag{R2}$$

Under Bertrand and Cournot competition positive post-innovation costs respectively imply:

$$\gamma > \frac{a(2-\theta^2-\theta\beta)}{c(2-\theta)(1+\theta)(4-\theta^2)}, \tag{R3}$$

and

$$\gamma > \frac{a(2-\theta\beta)}{c(2+\theta)(4-\theta^2)}. \tag{R4}$$

Finally, the Routh-Hurwitz stability condition is that:

$$\frac{\partial^2\widehat{\pi}_i(x_i,x_j)}{\partial x_i^2}\frac{\partial^2\widehat{\pi}_j(x_i,x_j)}{\partial x_j^2} - \frac{\partial^2\widehat{\pi}_i(x_i,x_j)}{\partial x_j\partial x_i}\frac{\partial^2\widehat{\pi}_j(x_i,x_j)}{\partial x_i\partial x_j} > 0. \tag{18}$$

This condition depends on the strategic nature of the R&D process. Following Bulow *et al.* (1985), label decision variable x a strategic substitute in case $\partial^2\widehat{\pi}_i(x_i,x_j)/\partial x_i\partial x_j < 0$, and a strategic complement if $\partial^2\widehat{\pi}_i(x_i,x_j)/\partial x_i\partial x_j > 0$. Accordingly, in a symmetric equilibrium condition (18) boils down to:

$$\frac{\partial^2\pi_i(x_i,x_j)}{\partial x_i^2} < \frac{\partial^2\pi_i(x_i,x_j)}{\partial x_i\partial x_j}, \tag{19}$$

for strategic substitutes. For strategic complements it reads as:

$$\frac{\partial^2\pi_i(x_i,x_j)}{\partial x_i^2} < -\frac{\partial^2\pi_i(x_i,x_j)}{\partial x_i\partial x_j}. \tag{20}$$

Under Bertrand competition these two stability conditions respectively translate into:

$$\gamma > \frac{(2-\theta^2-\theta\beta)^2}{(4-\theta^2)(2+\theta)(1-\theta)(2-\theta^2+\theta\beta)}, \tag{R5}$$

and

$$\gamma > \frac{(2-\theta^2-\theta\beta)}{(4-\theta^2)(2-\theta)(1+\theta)}. \tag{R6}$$

In case of Cournot competition the two stability conditions are:

$$\gamma > \frac{(2-\theta\beta)^2}{(4-\theta^2)(2-\theta)(2+\theta\beta)}, \tag{R7}$$

and

$$\gamma > \frac{(2-\theta\beta)}{(4-\theta^2)(2+\theta)}. \tag{R8}$$

Five of these regularity conditions are redundant as the following lemma shows.

Lemma 1 *The parameter space is bounded by regularity conditions R4, R5 and R7.*

Proof. It is immediate that R4 dominates R3, that R5 dominates R6, and that R7 dominates R8. Also, R5 dominates R1 and R7 dominates R2. ■

Note that Qiu (1997) considers the stability conditions only in case of R&D being a strategic complement. In his model the stability conditions for R&D as a strategic substitute under Cournot and Bertrand competition are respectively given by (using the notation in Qiu, 1997):

$$v > \frac{2(2-\theta\gamma)(1-\theta)}{(2-\gamma)(4-\gamma^2)}, \tag{21}$$

and

$$v > \frac{2(1-\theta)(2-\theta\gamma-\gamma^2)}{(1-\gamma)(2+\gamma)(4-\gamma^2)}, \tag{22}$$

where $\theta \in [0,1]$ is the output spillover, where v is the measure of the efficiency of the R&D process, and where $\gamma \in [0,1]$ indicates the extent of product differentiation. The analysis of Qiu (1997) applies only to R&D that is a strategic complement as it is straightforward to show that conditions (21) and (22) are more binding than the stability conditions when R&D is a strategic complement.

4 Cournot versus Bertrand

4.1 R&D investments

Comparing the effective R&D efforts of the different competition modes leads to the following proposition:

Proposition 1 *For any given* $\theta \in]0,1[$ *and* $\beta \in [0,1]$, $\widetilde{y}^C > \widetilde{y}^B$ *under R4, R5 and R7.*

Proof. $\widetilde{y}^C > \widetilde{y}^B \Leftrightarrow (1+\theta)(2-\theta)(2-\theta\beta) > (2+\theta)(2-\theta^2-\theta\beta)$, or $\beta > -1$. ■

According to Proposition 1, R&D activity is higher under Cournot competition than under Bertrand competition. This result replicates Qiu (1997) who points out that there is a strategic effect at work when firms decide upon their R&D investments. In Cournot markets this strategic effect is positive. The firm with the lower production costs is the tougher competitor that has the largest market share. In Bertrand markets this strategic effect is negative. Any reduction in production costs induces rivals to cut price which is not in the interest of either firm. The switch from output spillovers to input spillovers does not affect this reasoning. The ranking in Proposition 1 is also found by Breton *et al.* (2004) who replicate the analysis of Qiu (1997) within an infinite horizon setting.

The actual difference in R&D activity that leads to the ranking in Proposition 1 is closely related to the efficiency of the R&D process. That is:

Lemma 2 *Under R4, R5 and R7, the difference in R&D activity under Cournot and Bertrand competition is larger the more efficient is the R&D process.*

Proof. Note that

$$\widetilde{y}^C - \widetilde{y}^B = \frac{\gamma\theta^3(4-\theta^2)(1+\beta)(a-c)}{[\gamma(2+\theta)(4-\theta^2)-(2-\theta\beta)][\gamma(1+\theta)(2-\theta)(4-\theta^2)-(2-\theta^2-\theta\beta)]}.$$

Then observe that:

$$\frac{\partial\left(\widetilde{y}^C-\widetilde{y}^B\right)}{\partial\gamma}<0 \;\Leftrightarrow\; \gamma^2>\frac{(2-\theta\beta)(2-\theta^2-\theta\beta)}{(1+\theta)(4-\theta^2)^3}.$$

This last conditions is less binding than condition R7 if, and only if $(2-\theta\beta)^3(1+\theta)(4-\theta^2)-(2-\theta^2-\theta\beta)(2-\theta)^2(2+\theta\beta)^2>0$. Considering the left-hand side (LHS) of this last inequality, the result then follows as $\min_{\{\theta,\beta\}}LHS=\lim_{\theta\to 0}\,LHS|_{\beta=1}=0$. ■

The larger is the reduction in production costs for any level of R&D investment, the more prominent is the strategic effect that affects any firms' incentive to conduct R&D. Hence, the more efficient is the R&D process, the larger is the difference in R&D investments under Cournot competition vis-à-vis Bertrand competition.

4.2 Profits

Under Cournot competition firms invest more in R&D than under Bertrand competition (Proposition 1). And larger R&D investments reduce profits, all else equal. The following proposition shows however that these higher R&D costs under Cournot competition are more than offset by the concomitant reduction in production cost:

Proposition 2 *For any given $\theta\in]0,1[$ and $\beta\in[0,1]$, $\widetilde{\pi}^C>\widetilde{\pi}^B$ under R4, R5 and R7.*

Proof. First note that $\widetilde{\pi}^C-\widetilde{\pi}^B=\gamma(a-c)^2(A-B)/(1+\beta)$, where

$$A=\frac{\gamma(1+\beta)(4-\theta^2)^2-(2-\theta\beta)^2}{\left[\gamma(2+\theta)(4-\theta^2)-(2-\theta\beta)\right]^2},$$

and

$$B=\frac{\gamma(1+\beta)(1-\theta^2)(4-\theta^2)^2-(2-\theta^2-\theta\beta)^2}{\left[\gamma(1+\theta)(2-\theta)(4-\theta^2)-(2-\theta^2-\theta\beta)\right]^2}.$$

Then observe that:

$$\widetilde{\pi}^C-\widetilde{\pi}^B>0\Leftrightarrow\gamma>\frac{2(4-3\theta^2)-\theta(1-\beta)(\theta^2-2\theta-4)}{2(1+\theta)(4-\theta^2)^2}.$$

This last condition is less binding than condition R7 if, and only if, $(1-\beta)\left[32+16\theta-12\theta^2-16\theta\beta-2\theta^3(1+\beta)+8\theta^3\beta\right]+\theta^2\beta\left[8\beta-\theta^2(1+\beta)\right]>0$. Considering the LHS of this last inequality the result then follows as $\min_{\{\theta,\beta\}}LHS=\lim_{\theta\to 0}\,LHS|_{\beta=1}=0$. ■

Proposition 2 states that producers' surplus under Cournot competition is always larger than under Bertrand competition. Because post-innovation production costs are lower under Cournot competition, this larger producers' surplus can exceed the lower consumers' surplus in Cournot markets compared to Bertrand markets. But before we analyze total surplus we first consider consumers' surplus.

4.3 Price

For comparing prices under Cournot and Bertrand competition we introduce the following assumption:

$$\gamma < \frac{1}{4-\theta^2} \tag{A1}$$

If assumption A1 holds the R&D process is labelled 'efficient'. According to Lemma 2 this corresponds to situations where post-innovation cost under Cournot competition are particularly low compared to post-innovation costs under Bertrand competition. As will be shown below, this allows the equilibrium price under Cournot competition to be lower than under Bertrand competition. First note that assumption A1 does not rule out the existence of equilibria:

Lemma 3 *The set where regularity conditions R4, R5, R7 and assumption A1 hold is not empty.*

Proof. For A1 and R4 to hold jointly it must be that $1 < a/c < (2+\theta)/(2-\theta\beta)$, or $2(a-c) < \theta(a\beta+c)$. Indeed, a and c can always be chosen such that this inequality holds. For A1 and R5 to hold jointly it must be that $1 > (2-\theta^2-\theta\beta)^2/(2+\theta)(1-\theta)(2-\theta^2+\theta\beta)$, or $\beta > \left(6-3\theta^2-\theta-\sqrt{(1-\theta)(36+16\theta-19\theta^2-9\theta^3)}\right)\Big/ 2\theta = f(\theta)$. Note that $f(\theta)$ is continuous and strictly increasing in $\theta \in]0,1[$, that $\lim_{\theta\to 0} f(\theta) = \frac{1}{3}$, and that $\lim_{\theta\to 1} f(\theta) = 1$. For A1 and R7 to hold jointly it must be that $1 > (2-\theta\beta)^2/(2-\theta)(2+\theta\beta)$, or $\beta > \left(6-\theta-\sqrt{(18-\theta)(2-\theta)}\right)\Big/ 2\theta = g(\theta)$. Note that $g(\theta)$ is continuous and strictly increasing in $\theta \in]0,1[$, that $\lim_{\theta\to 0} g(\theta) = \frac{1}{3}$, and that $\lim_{\theta\to 1} g(\theta) = (5-\sqrt{17})/2 \approx 0.438$. ■

Figure 1 displays the admissible parameter space and assumption A1 for particular values of a, c, and γ. Note that from the proof of Lemma 3 follows that $f(\theta) - g(\theta) > 0 \ \forall\theta \in]0,1[$. Hence, under assumption A1 the admissible parameter space is confined by conditions R4 and R5.

We can now state the main result of our analysis:

Proposition 3 *For any given $\theta \in]0,1[$ and $\beta \in [0,1]$, $\widetilde{p}^C < \widetilde{p}^B$ under R4, R5, R7, and A1.*

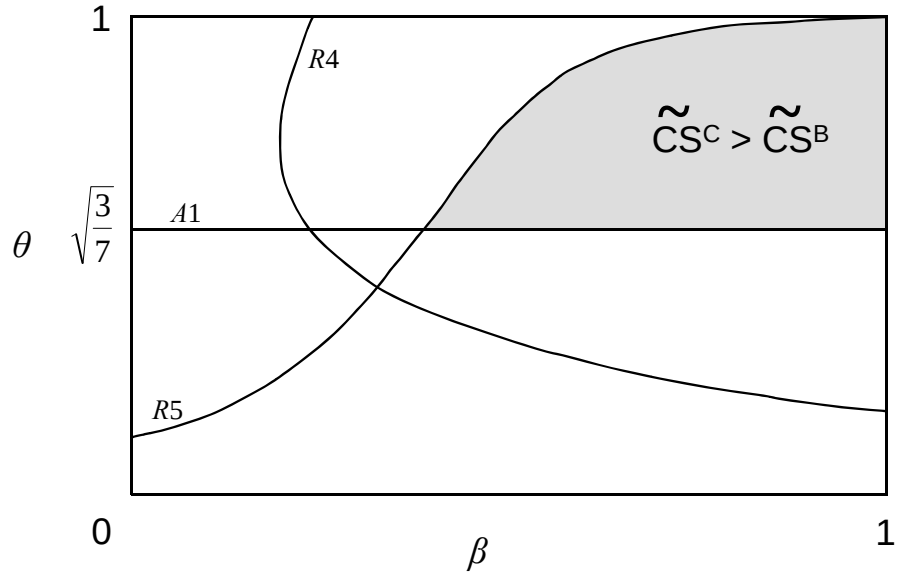

Figure 1: Comparing consumers' surplus with Cournot and Bertrand competition under assumption A1 and regularity conditions R4 and R5 ($a = 100$, $c = 70$, $\gamma = \frac{7}{25}$).

Proof. Lower prices obtain under Cournot competition than under Bertrand competition if, and only if, $\widetilde{Q}^C > \widetilde{Q}^B$, or $\gamma < 1/(4 - \theta^2)$. ■

Proposition 3 conveys our new message. In a duopoly with substitutable products, prices can be *lower* under Cournot competition than under Bertrand competition. This happens when post-innovation costs under Cournot competition are sufficiently below post-innovation costs under Bertrand competition. Considering the admissible parameter space in Lemma 3, this occurs when the R&D process is efficient, when spillovers are substantial, and when products are not too differentiated. It is precisely under these circumstances that the benefits of any cost reduction are transferred much more to consumers under Bertrand competition than under Cournot competition. Hence, production costs under Cournot competition are much lower than under Bertrand competition which allows the equilibrium price to be lower as well.

4.4 Welfare

As producers' surplus is always higher under Cournot competition than under Bertrand competition (Proposition 2), the result in Proposition 3 carries over to total surplus:

Proposition 4 *For any given* $\theta \in]0,1[$ *and* $\beta \in [0,1]$, $\widetilde{TS}^C > \widetilde{TS}^B$ *under R4, R5, R7, and A1.*

For a less efficient R&D production process it is still possible that total surplus under Cournot competition exceeds total surplus under Bertrand competition. In that case consumers' surplus is lower when firms compete over quantities (Proposition 3). But this lower consumers' surplus is then more than compensated for by the higher producers' surplus under Cournot competition. To establish this result it is convenient to distinguish two cases: (i) no input spillovers, and (ii) positive input spillovers.

Proposition 5 *For any given* $\theta \in]0,1[$ *and* $\beta = 0$, $\widetilde{TS}^C < \widetilde{TS}^B$ *under R4, R5, R7, and* $\neg A1$.

Proof. See Appendix. ■

Absent input spillovers the traditional welfare comparison emerges provided that the R&D production process is not too efficient. For positive input spillovers the difference in R&D investment incentives under Cournot and Bertrand competition becomes more pronounced. Indeed, a threshold value of the input spillover exists beyond which total surplus is larger if firms compete over quantity rather than over price:

Proposition 6 *Suppose that* $\beta \in]0,1]$, *and that R4, R5, R7 and* $\neg A1$ *hold. Then, given* $\theta \in]0,1[$, $\exists\ \overline{\gamma}(\theta)$ *such that*

(i) *if* $\gamma > \overline{\gamma}(\theta)$, *then* $\widetilde{TS}^B - \widetilde{TS}^C > 0\ \forall \beta \in]0,1]$; *and*

(ii) *if* $\gamma < \overline{\gamma}(\theta)$, *then* $\exists\ \overline{\beta}(\theta) \in]0,1]$ *such that*

$$\widetilde{TS}^B - \widetilde{TS}^C \begin{cases} > 0 & \forall \beta < \overline{\beta}(\theta) \\ = 0 & \text{if } \beta = \overline{\beta}(\theta) \\ < 0 & \forall \beta > \overline{\beta}(\theta). \end{cases}$$

Proof. See Appendix. ■

Technological spillovers carry a positive externality that raises total surplus. The combination of large R&D investments and strong technological spillovers contributes in particular to total surplus. Hence, as under Cournot competition R&D investments exceed those under Bertrand competition, total surplus can be larger under quantity competition when the input spillover is strong enough.

5 Conclusions

We have shown that in a duopoly with substitutable goods where firms invest in process R&D, price can be lower under Cournot competition than under Bertrand competition. This occurs when the R&D process is efficient, when spillovers are substantial, and when products are not too differentiated. Under these circumstances much more of the benefits of any cost reduction are given to consumers under Bertrand competition than under Cournot competition. As a result the post-innovation costs are much lower under Cournot competition than under Bertrand competition leading to lower prices when firms compete over quantities.

The robustness of our result should be checked along several dimensions. An obvious scenario would be to consider cooperative R&D prior to the production stage. Allowing firms to cooperate in R&D is an important policy tool to enhance incentives towards investments in R&D. As this policy is driven foremost by the concomitant internalization of the technological spillover, it needs to be examined whether it affects the conclusion that price can be lower under Cournot competition than under Bertrand competition.

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6 Appendix[8]

6.1 Proof of Proposition 5

First note that:

$$\widetilde{TS}^B - \widetilde{TS}^C = \frac{\gamma(a-c)^2}{\Delta_B^2 \Delta_C^2} F(\gamma;\theta),$$

where $\Delta_B = \gamma(1+\theta)(2-\theta)(4-\theta^2)-(2-\theta^2)$, $\Delta_C = \gamma(2+\theta)(4-\theta^2)-2$, and $F(\gamma,\theta) = \left[\gamma(4-\theta^2)^2(1+\theta)(3-2\theta) - 2(2-\theta^2)^2\right]\Delta_C^2 - \left[\gamma(4-\theta^2)^2(3+\theta)-8\right]\Delta_B^2$. Define $G(\gamma;\theta) = F(\gamma;\theta)/\left(\gamma\theta^2(4-\theta^2)\right)$. Obviously, $sign\left(\widetilde{TS}^B - \widetilde{TS}^C\right) = sign(G(\gamma;\theta))$. Note that $G(\gamma;\theta) = \gamma^2 g_1 + \gamma g_2 + g_3$, where $g_1 = (4-\theta^2)^3(1+\theta)(4-2\theta-\theta^2)$, $g_2 = -2(4-\theta^2)^2(1+\theta)(4-\theta-\theta^2) + 2\theta(4-\theta^2)(8+4\theta-4\theta^2-\theta^3)$, and $g_3 = (4-\theta^2)(4+4\theta-3\theta^2-\theta^3) - 8\theta(2-\theta^2)$. It follows that $G(\gamma;\theta)$ is strictly convex in γ as $\partial^2 G(\gamma;\theta)/\partial\gamma^2 = 2g_1 > 0$ (indeed: $\min_{\{\theta\}} g_1 = \lim_{\theta\to 1} g_1 = 54$). Moreover, $g_2^2 - 4g_1g_3 > 0 \;\forall\; \theta \in]0,1[$. Hence, given any $\theta \in]0,1[$, there are two real solutions to $G(\gamma;\theta) = 0$, in particular:

$$\overline{\gamma}_1(\theta) = \frac{-g_2 - \sqrt{g_2^2 - 4g_1g_3}}{2g_1}, \text{ and } \overline{\gamma}_2(\theta) = \frac{-g_2 + \sqrt{g_2^2 - 4g_1g_3}}{2g_1}.$$

When $\beta = 0$, regularity condition R5 is most binding. Label the resulting threshold value on the efficieny parameter γ^*. The result then follows as $min_\theta\{\gamma^* - \overline{\gamma}_2(\theta)\} = \lim_{\theta\to 0}\{\gamma^* - \overline{\gamma}_2(\theta)\} = 0$. (see also Figure 2).

6.2 Proof of Proposition 6

This proofs is a general version of that in Section 6.1. Observe that:

$$\widetilde{TS}^B - \widetilde{TS}^C = \frac{\gamma(a-c)^2}{(1+\beta)\Delta_B^2 \Delta_C^2} F(\gamma;\beta,\theta),$$

where $\Delta_B = \gamma(1+\theta)(2-\theta)(4-\theta^2) - (2-\theta^2-\theta\beta)$, $\Delta_C = \gamma(2+\theta)(4-\theta^2) - (2-\theta\beta)$, and $F(\gamma,\beta,\theta) = \left[\gamma(1+\beta)(4-\theta^2)^2(1+\theta)(3-2\theta) - 2(2-\theta^2-\theta\beta)^2\right]\Delta_C^2 - \left[\gamma(1+\beta)(4-\theta^2)^2(3+\theta) - 2(2-\theta\beta)^2\right]\Delta_B^2$. Again we consider the related function $G(\gamma;\beta,\theta) = F(\gamma;\beta,\theta)/\left(\gamma\theta^2(4-\theta^2)\right)$. It follows that $sign\left(\widetilde{TS}^B - \widetilde{TS}^C\right) =$

[8] The proofs in this appendix are inspired by Qiu (1997, pp. 225 – 228).

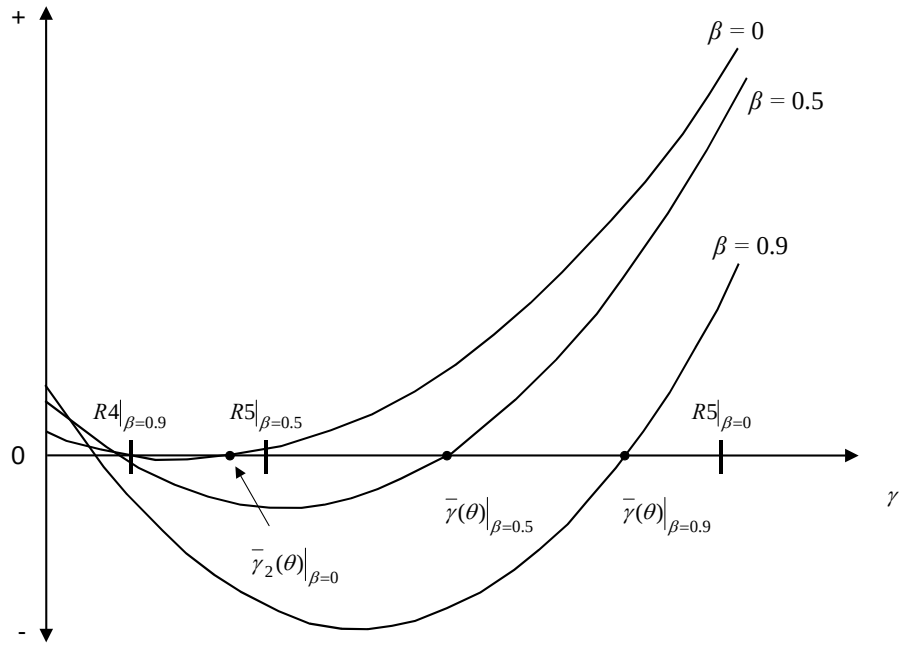

Figure 2: $G(\gamma;\beta,\theta)$ for different levels of R&D input spillovers; $a = 100$, $c = 70$, $\theta = 0.9$.

$sign(G(\gamma;\beta,\theta))$. Note that $G(\gamma;\beta,\theta) = \gamma^2 g_1 + \gamma g_2 + g_3$, where $g_1 = (1+\beta)(4-\theta^2)^3(1+\theta)(4-2\theta-\theta^2)$, $g_2 = -2(1+\beta)(4-\theta^2)^2(1+\theta)(4-\theta^2-\theta(1-\beta)) - 2(4-\theta^2)\left[(4+2\theta-\theta^2)(2-\theta\beta)^2 - (2+\theta)^2(4-2\theta\beta-\theta^2)\right]$, and $g_3 = (1+\beta)(4-\theta^2)\left[2(2-\theta\beta)(1+\theta+\theta\beta) - (3+\theta)\theta^2\right] - 4\theta(1+\beta)(2-\theta\beta)(2-\theta^2-\theta\beta)$. Then note that $G(\gamma;\beta,\theta)$ is strictly convex in γ as $\partial^2 G(\gamma;\beta,\theta)/\partial\gamma^2 = 2g_1 > 0$ (indeed: $\min_{\{\theta,\beta\}} g_1 = \lim_{\theta\to 1,\beta\to 0} g_1 = 54$). Moreover, $g_2^2 - 4g_1g_3 > 0 \;\forall\; \theta \in]0,1[$. Hence, given any $\theta \in]0,1[$, there are two real solutions to $G(\gamma;\beta,\theta) = 0$, in particular:

$$\overline{\gamma}_1(\theta) = \frac{-g_2 - \sqrt{g_2^2 - 4g_1g_3}}{2g_1}, \text{ and } \overline{\gamma}_2(\theta) = \frac{-g_2 + \sqrt{g_2^2 - 4g_1g_3}}{2g_1}.$$

Only the larger root needs to be considered as $min_{\theta,\beta}\{\gamma^* - \overline{\gamma}_1(\theta)\} = \lim_{\theta\to 0}\{\gamma^* - \overline{\gamma}_2(\theta)\}\big|_{\beta=1} = 0$, where γ^* is the threshold value induced by R7. Label the larger root $\overline{\gamma}(\theta)$. Then observe that $min_{\theta,\beta}\{\partial\overline{\gamma}(\theta)/\partial\beta\} = \lim_{\theta\to 0}\partial\overline{\gamma}(\theta)/\partial\beta\big|_{\beta=0.5} = 0$. This gives rise to the different lines as drawn in Figure 2 for different values of β. Obviously, for any $\gamma > \overline{\gamma}(\theta)$ we are in situation (i) while situation (ii) emerges for any $\gamma > \overline{\gamma}(\theta)$. The rest of the proof then follows.