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Internet Retailing as a Marketing Strategy

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Internet Retailing as a Marketing Strategy

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Abstract

We analyze the incentives for incumbent bricks-and-mortar firms and new entrants to start an online retail channel in a differentiated goods market. To this end we set up a two-stage model where firms first decide whether or not to build the infrastructure necessary to start an online retail channel and then compete in prices using the channels they have opened up. Consumers trade-off the convenience of online shopping and the ease to compare prices, with online uncertainties. Without a threat of entry by a third pure online player we find that for most parameter constellations firms’ dominant strategy is not to open an online retail channel as this cannibalizes too much on their conventional sales. As the cannibalization effect is not present for a pure Internet player, we show that these firms will start online retail channels under a much wider range of parameter constellations. The threat of entry may force incumbent bricks-and-mortar firms to deter entry by starting up an Internet retail channel themselves. We also show that a low cost of building up an online retail channel or online shopping conveniences may not be to the benefit of online shopping as the strategic interaction between firms may be such that no online retail channel is built when the circumstances seem to be more favourable.

Keywords: E-Commerce, Internet, multichannel competition, online uncertainty, online shopping convenience.

JEL Codes: D43, L13, M30.

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1 Introduction

The importance of the Internet as a marketplace has substantially grown over the past decade, even though expectations have been dramatically tempered since early 2000. A distinctive feature of doing business electronically is that transactions no longer require the physical coordination of buyers and sellers: market participants find each other at their screens. There are many aspects of market interaction which are affected by this online nature of trade. On the supply side, we may think of all kinds of cost reductions, resulting from new ways of organizing production and sales processes. On the demand side, the major impact of the Internet is on consumers’ ability to acquire information about firms and their prices.

While electronic commerce may have considerable potential, it is still of minor importance compared to other retail channels even for books and CDs. To understand the role of the Internet as a retail channel, we have to explain how its features influence market behaviour. This paper provides a theoretical framework that analyzes firms’ incentives to sell online and the extent to which consumers will substitute the Internet for the conventional retail channel. The interaction between incentives on both market sides sheds light on the different factors that determine the channel structure of the industry and the market share of electronic sales. Moreover, the analysis sheds a new light on the empirical evidence concerning prices on the electronic channel relative to prices in the conventional retail channel.

For consumers, online shopping makes it easy to find and ‘visit’ firms and compare their prices, but the online nature of the transaction leads to uncertainties. For example, in some product categories such as clothing and furniture, it is more difficult to assess how well a particular product fits a consumer’s needs. Other uncertainties and inconveniences are related to the payment method, poor product declarations, slow delivery of goods, and unclear redemption policies. A consumer weighs the conveniences of online shopping with the inconveniences and uncertainties. We explicitly take this trade-off into account in the consumer’s choice of retail channel.

For firms, the Internet may be used as an alternative retail channel to gain market

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1 According to a study by the Boston Consulting Group (2000) in the United States and Europe, e-commerce accounts for less than 5 percent of total sales in the books and CDs categories.

2 A survey by Taylor Nelson Sofres (2001) indicates that the most important reason for not purchasing online is related to payment security and the second most important reason is given by ‘You don’t know what you get.’ and ‘Easier/more fun to buy goods/services in a store.’ In Taylor Nelson Sofres (2000) it is added that many “E-commerce web sites ... are not good at making the shopping experience easy and trustworthy for the online users.”
share. Moreover, if the online shopping convenience is large enough, firms may try to exploit the fact that consumers are willing to pay for this convenience by charging higher prices online. These positive aspects are offset by the fact that online competition is stronger as it is easier for consumers to compare prices. Moreover, the online infrastructure and the reorganization of sales processes requires that firms make an investment upfront. The paper models these different market forces in a consistent way and attempts to answer which factor is dominant and under what circumstances.

We present a two-stage game in which firms first decide whether to build an online retail channel before engaging in price competition, using the retail channels at their disposal. This game is analyzed in a variety of settings where initially there are always two incumbent bricks-and-mortar firms that sell horizontally differentiated products. Differentiation between firms is modelled a la Hotelling (1929) where firms are located at opposite sides of a line segment. That is, incumbent firms have built a reputation that appeals more to some consumers than to others. Traditionally, differentiation between firms in Hotelling type of analyses is either with respect to physical location and transportation cost or with respect to heterogeneous products and tastes. In our model, differentiation between the two bricks-and-mortar firms is comprised of both factors. On the Internet, physical location is not important so that only the product differentiation dimension remains in that retail channel. Accordingly, there is less differentiation online than in the conventional stores.

We analyse two settings of strategic interaction. The first setting is one where there is no threat of entry and consumers may or may not experience any additional convenience from buying online. The second setting we analyze is one where we allow for a pure online player to threaten the incumbent firms to enter the market and we investigate the possibilities for entry deterrence. In each setting equilibrium channel structures and prices are characterized.

Consumers know which of the two incumbent bricks-and-mortar firms they prefer the most. If these firms start an online retail channel, they bring the reputation that is built up in the conventional stores with them. A pure online player does not have any reputation among consumers so that consumers have to form expectations about their position on the product line.

We arrive at the following insights. First, in the absence of a threat of entry and no additional online shopping convenience, bricks-and-mortar firms do not become clicks-and-mortar firms. The reason is that a firm with a lower online price (than on its conventional channel) attracts two types of consumers: those who would otherwise buy from the rival and those who would otherwise buy form the conventional store.

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3When we model the consumer’s utility of buying from the entrant, it turns out that it is more convenient to interpret the line as a circle a la Salop (1979). The difference between the circle and the line segment is of no further importance, however, and therefore, we prefer to speak about differentiation along the line.
of the same firm. The first effect can be called the business-stealing effect, while the second effect may be termed the cannibalisation effect. We find that the cannibalization effect dominates the business-stealing effect, and hence, firms find it optimal not to build an online presence as alternative retail channel.

Second, if we allow for online shopping conveniences, some consumers will buy online even if the online prices are not lower. The cannibalization effect is in this case weaker as even at identical prices across retail channels, some consumers prefer to buy online. Moreover, we find that the business-stealing effect is strengthened in case the other incumbent firm did not open an alternative retail channel. This is because consumers who are fairly indifferent about the two firms, are more inclined to buy online than those who have a strong preference for a certain firm. Consequently, if the convenience of online shopping is high enough, individual firms have an incentive to start-up an online retail channel. However, if both firms become clicks-and-mortar, profits are lower than if both had stayed bricks-and-mortar firms. The reason is that even though some consumers are willing to pay more for a product online than in the conventional store, online competition for costumers is stronger as there is less differentiation online. Therefore, online prices will be lower. Depending on the cost of setting up an online retail channel, different equilibrium configurations are possible. If the cost is relatively low, firms face a Prisoner’s Dilemma: individually they have an incentive to start an alternative retail channel, but they are both worse off if they both do so. For intermediate values of this cost, there is scope for only one click-and-mortar firm and there is a first-mover advantage of starting an online retail channel. When the cost is relatively high, none of the two firms will start an online channel. Interestingly, the only case where online equilibrium prices are higher than in the conventional store is when there is only one firm selling online and when the inconvenience associated with shopping online is not too large.

We next look into the possibilities for a pure online player to enter the market. As an entrant does not eat up its installed costumer base, the incentives for an entrant to go online are stronger than those of the bricks-and-mortar firms. Indeed, we find that both in the case where the incumbent firms do not have an online retail channel and in case they do, an entrant may make positive operating profits. Again, online prices can only be higher than in the conventional retail channel if firms enjoy some "monopoly power" online. We then analyze the incentives of both incumbents and entrants to make a strategic decision to be the first to start an online retail channel. For intermediate values of the cost of starting up an online retail channel, we show that an incumbent firm may set up an online infrastructure to deter entry and that if these costs are even lower, the incumbent firms face a situation of strategic uncertainty in the sense that they want to take identical strategies: either they want to jointly deter entry (as this is the way to deter entry) or they both want to remain a bricks-and-mortar firm.
In the case where the entrant considers making a strategic decision to be the first online player, many possible equilibrium reactions by incumbent firms may follow. This by itself shows the difficult situation online entrants face. Details of the interaction matter a lot; details that may be difficult to foresee. One important difficulty is that when online shopping convenience becomes larger or when the cost of building an online presence becomes smaller, it may be more profitable for incumbents to follow an entrant’s decision to build an online retail channel thereby destroying the profitability of the entrant’s decision. Another difficulty that may arise is that incumbents may face a coordination problem such that the entrant’s decision to enter is only profitable if the incumbents solve the problem by not going online themselves. However, it may well be that the incumbents work out the other solution to the coordination problem in which case the entrant would have been better off not to enter in the first place. We see these theoretical possibilities as ways to explain the difficulties online players have been confronted with in the real world and as possible reasons why online shopping has not been as successful as it promised to be at the end of the previous millennium.

The empirical literature on the implications of the Internet for the competitiveness of markets has compared average prices in electronic marketplaces against those in conventional markets. The empirical evidence is mixed. Some studies find that prices in electronic markets are lower than in conventional markets (see, e.g., Brynjolfsson and Smith (1999)), while others report that prices in electronic markets are approximately equal to (Clay et al. (2000)) or even higher than in conventional markets (see, e.g., Lee (1997) and Bailey (1998)). Finally, Friberg et al. (2001) find that pure online retailers charge lower prices than the online channel of clicks-and-mortar firms. Our theoretical model provides an explanation for the latter result in terms of the fact that the online player does not compete with its own conventional store and therefore has less to lose from charging lower prices online. Our paper attributes the other mixed empirical evidence concerning conventional and online prices to different degrees of online market power and differences in online shopping convenience.

On the theoretical side, there are papers by Bakos (1997) and Janssen, Moraga and Wildenbeest (2005), among others, studying the implications of a reduction in search cost due to the emergence of online shopping on the competitiveness of markets. Bakos presents a model of circular product differentiation where consumers search for prices and product features. In his model, consumers can get to know all product characteristics if they engage in costly search. In our model, in contrast, consumers cannot get around some of the uncertainties associated with buying online. Commenting on Bakos’ paper, Harrington, Jr. (2001) questions the validity of some of Bakos’ results.

There are a few papers addressing the issue of channel substitution. Zettelmeyer (2000) focuses on the incentives firms have to increase the ease with which consumers
can evaluate their products. In his paper, consumer search cost is a strategic variable of firms. He shows that firms may provide selected groups of consumers with different information (level of search cost), thereby providing monopoly power to the firms and reducing market competition. The focus of our model is different. The source of market power in our model lies in the fact that firms sell differentiated products. More recently, Liu et al. (2003) analyze an issue that is more closely related to the present paper, namely to what extent can a bricks-and-mortar retailer’s online expansion affect a pure online player’s entry decision? They find that for some parameter values, a bricks-and-mortar’s decision to go online may trick an entrant to go online as well, making it more profitable for a brick-and-mortar firm not to start an online retail channel in the first place. To get this result, Liu et al. (2003) assume that firms are restricted to set identical prices across different retail channels, that there is perfect competition online between homogeneous goods producing firms and that the pricing game is sequential. As we study a setting with heterogeneous goods and allow firms to set different prices across channels, this result cannot occur in our setting.

Lal and Sarvary (1999) pose a different question: When is the Internet likely to decrease price competition? In their model, the specific inconvenience of online shopping is that products cannot be evaluated physically to learn about non-digital attributes. Firms are present on two retail channels. Since digital attributes can be communicated over the web, the Internet changes the effective search cost structure: consumers do not have to visit the store to buy their familiar brand, and thus, the cost of trying another brand is higher than the cost of buying the familiar brand. In this way, the Internet may increase the effective cost of search, which decreases price competition.

Finally, Mazón and Pereira (2000) also analyze whether firms have incentives to open electronic retail channels and the different price equilibria these incentives generate. However, they focus on different issues such as retail cost reductions and their results depend crucially on the assumption that some (“old”) consumers do not have Internet access.

The paper is organized as follows. Section 2 presents the model. Section 3 analyzes equilibrium properties in the absence of a threat of entry. Section 4 discusses the case of a pure online player threatening to enter. The issue of which market structures can obtain in equilibrium is discussed in Section 5. Conclusions and managerial implications are contained in Section 6 and proofs and calculations can be found in the Appendix.

2 The Model

There are two types of firms. Two incumbent bricks-and-mortar firms have established a certain brand reputation among consumers, while the possible pure online
entrant does not have such a reputation. In the first stage of the game, firms decide whether or not to invest in setting up an online retail channel at fixed cost $f$. For the pure online player this decision is equivalent to the decision whether or not to enter the market. This first stage can be analyzed as a simultaneous move or as a sequential move game where one or two firms may try to pre-empt the other(s).

In the second stage, there are two (or three) vertically integrated firms on the supply side of the market. Firms produce the good at constant returns to scale and production cost is normalized w.l.o.g. to zero. Firms are horizontally differentiated, as in Hotelling (1929)'s linear city model. On the demand side of the market, there is a mass of consumers, normalised to 1. Incumbent firms 1 and 2 are located at $x = 0$ and $x = 1$, respectively.

Every consumer has a location $x$ on the line segment and the preference for a firm is represented by the disutility of travelling the distance between the consumer’s location and the firm. This travel cost consists of two components: the physical travel cost related to travelling to a firm’s physical shop and the "utility cost" of buying a good that differs from her most preferred taste. For each unit travelled, consumers incur a linear cost $t > 0$. The utility a consumer $x$ gets from buying the product from firm 1 in its physical shop is given by

$$v - tx - p_{1c},$$

where $v$ is the consumer’s maximal willingness-to-pay$^4$ and $p_{1c}$ is the price firm 1 sets for its conventional retail channel (i.e., its physical shop). Similarly, the utility consumer $x$ gets from buying the product at firm 2’s physical shop is given by

$$v - t(1 - x) - p_{2c}.$$

Firm $i$ charges price $p_{ic}$ in its store and $p_{iE}$ on its electronic retail channel (if it has one). Consumers purchase at most one unit and to do so they can either go online, or they can visit one store.$^5$ The advantages and disadvantages of the different retail channels are modelled as follows. A first advantage of buying online is that consumers can easily compare the firms’ prices. Moreover, the conveniences and inconveniences/uncertainties of buying on the internet such as poor product declarations and slow delivery of goods are modelled in two steps. First, when buying in a firm’s online shop, the willingness-to-pay is multiplied by a factor $\lambda$. If $\lambda < 1$ ($\lambda > 1$), then the consumer’s willing-to-pay is lower (larger) for an online purchase compared

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$^4$It is assumed throughout that $v$ is large enough so that the market is covered.

$^5$Here we recognize that, although often suggested otherwise, searching on the Internet does take time. The equilibria we characterize are such that given the price and product offerings available, no consumer regrets the choice it has made and, therefore, allowing consumers to visit the Internet and the conventional channel would not alter the results.
to a purchase in a conventional store. Second, a consumer does not have to travel to a firm’s conventional store in order to buy, i.e., he only “pays” the mismatch between a firm’s product and his most preferred commodity. This is modelled by multiplying the cost of “travelling” to a firm by a factor $\beta$, where $0 \leq \beta < 1$. The parameter $\beta$ measures how much of the disutility of buying from firm $i$ is attributed to product heterogeneity: for example $\beta = 0$ implies that products are homogeneous and differentiation between incumbent firms is entirely due to differences in their locations, whereas $\beta = 1$ implies that differentiation is entirely due to the heterogeneity of products. Hence, consumer $x$ derives the following utility from buying from one of the incumbent firms’ online retail channels:

$$\max[\lambda v - \beta tx - p_{1E}, \lambda v - \beta t(1 - x) - p_{2E}]$$

It turns out to be useful to define a parameter $\alpha$ as $\alpha = (1 - \lambda)v/(1 - \beta)t$. The interpretation of $\alpha$ is straightforward whenever it is positive ($\lambda < 1$). If firm 1, resp. 2, sets identical prices in its conventional channel and its online channel, consumer $\alpha$, resp. $(1 - \alpha)$, is indifferent between buying in the conventional store and buying online. Another interpretation of $\alpha$ is that it measures the size of the online shopping inconvenience relative to the inconvenience of having to travel to the conventional shop. We will assume that $-1/2 < \alpha < 1/2$.

Since the potential third firm in the market is a newcomer and did not build up any reputation in its conventional channel, we assume that consumers do not know the $x$ location of this firm. Each consumer therefore has to “travel” an expected distance of $1/2$. The expected utility of buying from firm 3, a pure Internet player, is then given by

$$\lambda v - \beta t/2 - p_{3E}.$$ 

An equilibrium of the second stage of the game is a set of prices, one for each retail channel on which a firm is active, such that each individual firm $i$ maximizes its profits given the prices set by the other firms. Consumers buy at the firm where utility is maximized. We will focus on symmetric subgame perfect equilibria of the two-stage game.

## 3 No Threat of Entry

In this section we study incentives to open online channels and the subsequent price competition between two incumbent bricks-and-mortar firms. We study a two stage

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6Strictly speaking, this is only true along a circle. Along the line, a consumer’s expected travel to the Internet player depends on his location and is smaller the closer a consumer is located towards the middle of the line segment. Working with the proper line segment interpretation only complicates the analysis without bringing additional insights.
game in which firms first decide whether or not to sell online, and then compete in prices. The two incumbent players have both the possibility to open an additional retail channel, yielding four possible market structures in the second stage. The case in which neither firm opens an online presence is standard as it confirms the logic of Hotelling’s (1929) linear city model. It is easy to see that in this case both firms set their prices equal to $t$ and the two firms share the market equally so that each firm’s profit is equal to $t/2$. The case in which only firm 2 sells online is equivalent to the case in which only firm 1 sells online. This leaves us with two cases to investigate further.

We first concentrate on the case where each duopolist is a multi-channel retailer.

3.1 Two Clicks-and-Mortar Players

Whenever $\lambda < 1$ and the prices $(p_{1c}, p_{1E}, p_{2c}, p_{2E})$ are in the relevant range, the picture of channel substitution is as in Figure 1, where $x_{1Ec}$ and $x_{2Ec}$ represent the consumer that is indifferent between buying from a respective firm’s conventional and online channel and $x_{1E2E}$ represent the consumer that is indifferent between buying from the two online channels. One can easily calculate that

$$x_{1Ec} = \frac{(1 - \lambda)v - (p_{1E} - p_{1E})}{(1 - \beta)t}; x_{2Ec} = 1 - \frac{(1 - \lambda)v - (p_{2E} - p_{2E})}{(1 - \beta)t}; x_{1E2E} = \frac{\beta t - (p_{1E} - p_{2E})}{2 \beta t}.$$

In this picture, the area to the left of $x_{1Ec}$ forms the demand for firm 1’s conventional channel, while the area between $x_{1Ec}$ and $x_{1E2E}$ constitutes the consumers who buy via firm 1’s online channel. Mirror areas represent Firm 2’s demand on its conventional and online channel, respectively. From Figure 1 it is clear that the profit function for firm 1 is given by:

$$\pi_1 = p_{1c} x_{1Ec} + p_{1E}(x_{EE} - x_{1Ec}) = \frac{(1 - \lambda)v - (p_{1E} - p_{1E})}{(1 - \beta)t}(p_{1E} - p_{1E}) + p_{1E} \frac{\beta t - (p_{1E} - p_{2E})}{2 \beta t}.$$

A similar equation holds for firm 2. The equation has a simple interpretation. By means of its online channel, a firm competes with the other firm to get a larger market share. The division of a firm’s market share between the conventional and

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7 The main reason why we focus in the text mostly on this case is that the expressions that hold true when $\lambda > 1$ are less complicated and easily follow once the case $\lambda < 1$ is clearly understood. Also, one may argue that this is the most natural case to look at as the main advantage of shopping online, the fact that one does not incur geographical travelling cost, is already captured by the parameter $\beta$.

8 Here, it is implicitly assumed that equilibrium prices are such that all channels by all firms are visited by at least some consumers. Below, we will specify the parameters for which this is the case.
Figure 1: Two clicks-and-mortar players

the electronic channel is entirely determined by a firm’s internal pricing policy: a firm has some monopoly power over the consumers who buy from them and can set the price difference \( p_{1c} - p_{1E} \) so as to maximize profits. It is easy to see then that whenever all channels are visited by some consumers, equilibrium prices are given by:

\[
\begin{align*}
    p_{1E} &= p_{2E} = \beta t \\
    p_{1c} &= p_{2c} = \beta t + \frac{(1 - \lambda)v}{2}
\end{align*}
\]

and the equilibrium indifferent consumers are given by

\[
\begin{align*}
    x_{1Ec}^* &= \frac{(1 - \lambda)v}{2(1 - \beta)t}; \\
    x_{2Ec}^* &= 1 - \frac{(1 - \lambda)v}{2(1 - \beta)t}; \\
    x_{EE}^* &= 1/2.
\end{align*}
\]

As it has to be the case that \( 0 \leq x_{1Ec} \leq 1/2 \), it follows that this equilibrium holds whenever \( 0 < (1 - \lambda)v \leq (1 - \beta)t \), or given the restrictions we have imposed on \( \alpha \), whenever \( \alpha \) is positive.\(^9\) The Proposition below summarizes the above and also considers the case where \( \alpha < 0 \).

**Proposition 1** When both firms have built the infrastructure to sell online, there are

\(^9\)One may observe that many relevant expressions in this paper, like the ones here for the indifferent consumers (in equilibrium), include a term like \( (1 - \lambda)v/(1 - \beta)t \). One of the advantages of defining \( \alpha \) in the way we did is that one does not have to impose each time additional and different restrictions on \( \beta \) (to have it bounded away from 1). One should note, however, that given the values of the other parameters, \( \alpha \) negatively depends on \( \beta \) and in particular that \( \alpha \) is close to 0 whenever \( \beta \) is close to 1.
three cases to consider: (a) if $\alpha \leq 0$, all consumers will buy online, $p_{1E} = p_{2E} = \beta t$, and equilibrium profits will be $\beta t/2$; (b) if $\alpha > 0$, all retail channels will be used and $p_{1E} = p_{2E} = \beta t, p_{1c} = p_{2c} = \beta t + \frac{(1-\lambda)v}{2}$ and equilibrium profits will be $\beta t/2 + \frac{[(1-\lambda)v]^2}{4(1-\beta)t}$.

Proposition 1 is easily understood by taking the Hotelling result as a reference point and therefore, a more formal proof is omitted. If Internet purchases are considered to be relatively inconvenient overall ($\alpha > 0$), firms use both channels. Competition between the firms mostly takes the form of competition online as the consumer that is indifferent between buying from the two firms is a consumer that buys online. The online prices that result are exactly equal to the equilibrium prices in a Hotelling model where transportation costs are equal to $\beta t$ (the online "transportation costs"). When both firms are clicks-and-mortar firms, online prices are always lower than prices in the conventional channel. More precisely, conventional prices are set as a monopoly mark-up on the online prices: there is no effective competition for these inframarginal consumers and firms will use their monopoly power over these consumers. This also helps to explain the result in case online purchases are considered to be more convenient ($\alpha < 0$). In this case, firms have to set lower conventional prices than online prices to motivate consumers to buy in their conventional stores. From the discussion provided above for the case $\alpha > 0$ it follows that this cannot be profit maximizing. Therefore, we get a situation where the online retail channel dominates all sales. As online "transportation costs" are lower, competition is more severe and prices are lower than when firms compete with their conventional stores.

As competition online is more severe, we can therefore arrive at a preliminary conclusion that it is not in the interest of incumbent bricks-and-mortar firms to make the online shopping experience very convenient.

### 3.2 Clicks-and-Mortar vs. Bricks-and-Mortar

When one firm (say firm 1) has opened an electronic retail channel and the other has decided not to do so, the picture of channel substitution for the relevant range of prices and parameter values is as in Figure 2, where

\[
x_{1Ec} = \frac{(1 - \lambda)v - (p_{1c} - p_{1E})}{(1 - \beta)t}; \quad x_{1E2c} = \frac{t - (1 - \lambda)v + (p_{2c} - p_{1E})}{(1 + \beta)t}.
\]

In this picture, the segment to the left of $x_{1Ec}$ forms the demand for firm 1’s conventional channel, while the segment between $x_{1Ec}$ and $x_{1E2c}$ constitutes the consumers who buy via firm 1’s Internet channel.

The remaining segment represents firm 2’s demand on its conventional channel.
Using Figure 2 the profit function for firms 1 and 2 are given by:

\[
\begin{align*}
\pi_1 &= p_{1c} x_{1Ec} + p_{1E} (x_{1E2c} - x_{1Ec}) \\
&= \frac{(1 - \lambda)v - (p_{1c} - p_{1E})(p_{1c} - p_{1E}) + p_{1E} \left(t - (1 - \lambda)v + (p_{2c} - p_{1E})\right)}{(1 + \beta)t}; \\
\pi_2 &= p_{2c} x_{1E2c} + p_{2E} \left(\beta t + (1 - \lambda)v - (p_{2c} - p_{1E})\right)/(1 + \beta)t.
\end{align*}
\]

The first-order conditions that yield the subgame perfect equilibrium prices are given in the Appendix. Solving these equations and substituting them back into the profit function given above gives the Proposition below.

**Proposition 2** Suppose one firm has decided to build the infrastructure to sell online, say firm 1, and firm 2 has not. Then, if \(\lambda < 1\), the unique symmetric subgame equilibrium prices are given by

\[
\begin{align*}
p_{1c} &= \frac{(4 + 2\beta)t + (1 - \lambda)v}{6}; \\
p_{2c} &= \frac{(1 + 2\beta)t + (1 - \lambda)v}{3}; \\
p_{1E} &= \frac{(2 + \beta)t - (1 - \lambda)v}{3}.
\end{align*}
\]

In case \(\lambda > 1\), firm 1 will not use its conventional channel, the remaining two equilibrium prices will not be affected and equilibrium profits are simply given by

\[
\begin{align*}
\pi_1 &= \frac{(2 + \beta)t + (1 - \lambda)v}{9(1 + \beta)t}^2 \\
\pi_2 &= \frac{(1 + 2\beta)t + (1 - \lambda)v}{9(1 + \beta)t}^2.
\end{align*}
\]
and equilibrium profits equal

\[
\pi_1 = \frac{[(1 - \lambda)v]^2}{4(1 - \beta)t} + \frac{[(2 + \beta)t - (1 - \lambda)v]^2}{9(1 + \beta)t};
\]

\[
\pi_2 = \frac{[(1 + 2\beta)t + (1 - \lambda)v]^2}{9(1 + \beta)t}.
\]

There are a few interesting observations to make. First, if only one firm opens an electronic retail channel, it will set higher prices online than the other firm with its conventional and it may even set higher prices online than the average conventional price. The first point follows as \((1 - \beta)t > 2(1 - \lambda)v\) whenever \(\alpha < 1/2\) and is explained by the fact that the clicks-and-mortar firm exploits its monopoly position online. The second point holds true whenever \(\alpha < 1/5\) and is explained by the fact that in this case the online inconveniences are considered to be fairly small (or non-existent in case \(\alpha < 0\)) so that the clicks-and-mortar firm can further exploit its online monopoly position. A second observation is that the operating profits of the clicks-and-mortar firm are always higher than those of the bricks-and-mortar firm. Finally, both the online and the conventional prices of the clicks-and-mortar firm are lower than in case both firms had stayed out of online retailing altogether. The main reason for this is that the bricks-and-mortar firm considers the online channel of its competitor more aggressive than the competitor’s conventional retail channel. In reaction, it will price lower, which forces the clicks-and-mortar firm also to lower its price.

### 3.3 Clicks or just Bricks?

In the previous two subsections we have characterized the price equilibria of the second stage of the game. We now go one step back and analyze the first stage decision whether or not to build an online retail infrastructure. The above analysis can be summarized in the payoff matrix below (for the case where \(\lambda < 1\)),\(^{12}\) where because of symmetry only the pay-offs of firm 1 are mentioned.

<table>
<thead>
<tr>
<th></th>
<th>(y/n)</th>
<th>(n)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>((1 - \lambda)v)^2 + (\frac{(2 + \beta)t - (1 - \lambda)v)^2}{9(1 + \beta)t}) - (f)</td>
<td>(\frac{[1 - \lambda]v}{4(1 - \beta)t} + \frac{(2 + \beta)t - (1 - \lambda)v)^2}{9(1 + \beta)t} - \frac{\beta t}{2} + \frac{[1 - \lambda]v^2}{4(1 - \beta)^2} - \frac{f}{\beta})</td>
<td></td>
</tr>
<tr>
<td>(n)</td>
<td>(\frac{1}{2})</td>
<td>(\frac{(1 - \lambda)v}{4(1 - \beta)t} + \frac{(2 + \beta)t - (1 - \lambda)v)^2}{9(1 + \beta)t})</td>
<td></td>
</tr>
</tbody>
</table>

\(^{12}\)It follows from the analysis above that the pay-off matrix in case \(\lambda > 1\) differs only to the extent that the term \(\frac{(1 - \lambda)v^2}{4(1 - \beta)^2}\) is missing in both cells in the bottom row of the matrix.
A firm contemplating to attract additional consumers via its online retail channel has to trade-off two effects. First, it will steal some consumers away from its competitor, but at the expense of lower overall prices. Moreover, the online channel also steals consumers from its own conventional store. The calculations in the Appendix and Figure 3 below show that unless $\alpha$ is a relatively large negative number the first, business-stealing, effect is weaker than the second, cannibalization, effect. In fact, the Appendix and Figure 3 show that for many parameter constellations the firms have a dominant strategy not to open an online channel. This implies that without a threat of entry, incumbent firms will not find it profitable to start an online retail channel for most parameter values. It is interesting to observe that this result holds true even if $f$, the cost of starting such a retail channel, is very small.

If $\alpha$ is a relatively large negative number and $f$ is small enough, a firm may be better off having an online channel and the situation where both firms did not build an online retail channel cannot arise in equilibrium. Two cases may result. First, for a range of parameters where $\alpha$ is not close to $-1/2$ when $\beta$ takes on middle range values (see the picture below for an example), there exist asymmetric equilibria where only one firm opens an online channel. In this case one firm effectively uses only its online retail channel, while the other firm uses the only channel it has, the conventional one. Online prices in this case will be higher than offline prices. If $\alpha$ is quite close to $-1/2$ and $\beta$ is not close 0 or to 1, firms face a Prisoner’s Dilemma: both firms have a dominant strategy to build on online channel and to use it as their only effective retailing channel, but as competition online is more severe than off-line competition, both firms would be better off if they had not opened an online channel!

The possible equilibrium configurations are summarized in Figure 3 below for the case when $f/t = 0.01$. The Figure shows for which parameter constellations, which equilibrium structure arises. The Figure shows that the Prisoner’s Dilemma situation described above cannot arise when $\beta$ is close to 0. In this case, locational differences are the main source of product differentiation, hence online competition is very severe. In this case, firms want to avoid both having an online presence. When $\beta$ is close to 1, differences between firms are due to differences in the products they sell and in this case firms cannot benefit from an online presence, no matter how large the online shopping conveniences are! In this case without a threat of entry, geographical differences between shops and the consumer cost of travelling associated with it are thus important features explaining whether or not firms will start an online retail channel. Note also that on average firms are worse off, or not better off, if online shopping conveniences increase ($\alpha$ decreases). This is easily seen in the two regions where a symmetric equilibrium exists. In the asymmetric equilibrium case, one can

\[13\] Of course, when $f$ is relatively large, it is never optimal to build an online channel.
Figure 3: Equilibrium structures when no threat of entry, \( f/t = 0.01 \).

easily show that the average pay-off, relevant if firms do not know which one of them will be the one with an online presence, is declining in \( \alpha \).

4 Threat of Entry by a pure Internet player

We now turn to the role the Internet can play as an alternative marketing and distribution channel in case a pure online retailer threatens to enter the market. We are interested in the incentives to enter and in the question whether the two conventional firms have an incentive to start operating an Internet channel in view of this threat of entry. Assuming the entrant enters, there are three relevant situations to consider depending on how many incumbent firms open an online channel: no incumbent firm sells online, one incumbent sells online or both incumbents sell online. Each of these three cases is analyzed in turn in the next Subsections. The case where the entrant, firm 3, decides not to enter is analyzed in the previous section.

4.1 Bricks-and-Mortar vs. Pure Internet Retailer

We first consider the subgame where the two incumbent firms have not built the infrastructure to sell online, while the entrant has done so and uses the online channel as the sole retail channel. In this case, it is easy to see that for the set of relevant
prices, the indifferent consumers are given by the following expressions:

\[ x_{1cE} = \frac{(1 - \lambda)v - (p_{1c} - p_{1E}) + \frac{\beta t}{2}}{t} \]
\[ x_{2cE} = 1 - \frac{(1 - \lambda)v - (p_{2c} - p_{2E}) + \frac{\beta t}{2}}{t} \]

where for example \( x_{1cE} \) denotes the consumer who is indifferent between buying from firm 1’s conventional store and the entrant’s (firm 3’s) online store. Consumers located to the left of \( x_{1cE} \) buy from firm 1’s conventional store, those located to the right of \( x_{2cE} \) buy from firm 2’s conventional store and those located between \( x_{1cE} \) and \( x_{2cE} \) buy from the entrant’s online store. Given these indifferent consumers, one can easily derive the firms’ profit functions.

**Proposition 3** When the two incumbent firms have committed not to use the Internet channel and a pure internet retailer has entered, the subgame equilibrium prices are given by\(^\text{14}\)

\[ p_{1c} = p_{2c} = \frac{(1 + \beta)t + 2(1 - \lambda)v}{6} \]
\[ p_{3E} = \frac{(1 - \frac{1}{2}\beta)t - (1 - \lambda)v}{3} \]

Operating profits are given by

\[ \pi_1 = \pi_2 = \frac{[(1 - \lambda)v + \left(\frac{1}{2} + \frac{1}{2}\beta\right)t]^2}{9t} \]
\[ \pi_3 = \frac{2[(1 - \frac{1}{2}\beta)t - (1 - \lambda)v]^2}{9t} \]

The Proposition shows that entry by a pure online retailer forces incumbent bricks-and-mortar firms to lower their prices considerably: straightforward calculations show that the subgame perfect equilibrium prices are smaller than \( \frac{1}{2} \) implying that they are more than two times smaller than without the presence of the pure online retailer. In this case, the entrant has a monopoly position online and because of this may even enter with higher prices than the incumbent firms who have an established reputation. Online prices are larger whenever \( (2 - \beta)t - 2(1 - \lambda)v > (1 + \beta)t + 2(1 - \lambda)v \). This can be rewritten as: \( \alpha < \frac{1 - 2\beta}{4(1 - \beta)} \). This inequality is satisfied whenever \( \alpha \) and \( \beta \) are both relatively small, i.e., in sectors where the online inconvenience is perceived to be fairly small (or online shopping is considered to be more convenient) and products are close substitutes. This has a clear economic significance. In case \( \beta \) is small, consumers do not have a clear preference for one of the incumbents’ brands; they mainly dislike travelling to the conventional shop. The online retailer benefits in case the online shopping convenience is not too bad. Note that it is in the interest of the entrant to increase online shopping convenience if possible.

\(^{14}\) Note that equilibrium prices are nonnegative for \(-1/2 \leq \alpha \leq 1/2\).
4.2 Clicks-and-Mortar vs. Pure Internet Retailer

Let us now consider the case where all firms opened the possibility of selling through the online channel. In case \( \lambda < 1 \), Figure 4 applies for relevant values of the parameters and prices, where (in addition to the notation used earlier) \( x_{1E3E} \) denotes the consumer who is indifferent between buying from the online channel of firm 1 and the online channel of firm 3. Consumers located relatively close to 1/2 buy from firm 3.

As firms 1 and 2 are symmetric with respect to each other, we only concentrate on firm 1. Using this notation and the figure one can derive that the indifferent consumers are given by

\[
x_{1Ec} = \frac{(1 - \lambda)v - (p_{1c} - p_{1E})}{(1 - \beta)t} \quad \text{and} \quad x_{1E3E} = \frac{1}{2} - \frac{p_{1E} - p_{3E}}{\beta t}.
\]

Using these equations and the derivations in the Appendix, we can state the following Proposition.

**Proposition 4** If \( \lambda < 1 \),\(^{15}\) the unique symmetric subgame equilibrium prices are as follows in case all three firms have opened an online retail channel:

\[
\begin{align*}
p_{1E} &= p_{2E} = \frac{\beta t}{3} \quad \text{(1)} \\
p_{1c} &= p_{2c} = \frac{\beta t}{3} + \frac{(1 - \lambda)v}{2}, \quad \text{(2)}
\end{align*}
\]

\(^{15}\)In case \( \lambda > 1 \), it easily follows that the conventional channel will not be used by the incumbent firms and the equilibrium profits of these firms will be simply \( \beta t/9 \). All other expressions remain the same.
and

\[ p_{3E} = \frac{\beta t}{6}. \]  

(3)

Some consumers buy via the incumbents Internet channel. Operating profits are given by

\[
\begin{align*}
\pi_1 &= \pi_2 = \frac{\beta t}{9} + \frac{[(1 - \lambda)v]^2}{4(1 - \beta)t} \\
\pi_3 &= \frac{\beta t}{18}.
\end{align*}
\]

With three retailers with online presence, online prices are lower than prices in the conventional retail channel. Moreover, the fact that pure online retailers do not have an established reputation, forces them to gain market share by setting quite low prices: their equilibrium prices are just half of the online prices of incumbent firms and these are again lower than their conventional prices. Also, when \( \beta \) is close to 0, online prices are close to 0 as well: in this case, products are almost homogeneous and due to a lack of locational difference, online competition is very severe.\(^{16}\)

### 4.3 Asymmetric Incumbents

The last case to consider in the presence of entry is when one incumbent has set up an online retail channel, say firm 1, and the other has not. The profit functions in this case are given by the following equations:

\[
\begin{align*}
\pi_1 &= \frac{(1 - \lambda)v - (p_{1c} - p_{1E})(p_{1c} - p_{1E})}{(1 - \beta)t} + p_{1E}\left(\frac{1}{2} - \frac{p_{1E} - p_{3E}}{\beta t}\right);
\\
\pi_2 &= \frac{(1 - \lambda)v - (p_{2c} - p_{3E})}{t} + \frac{\beta t}{2} p_{2c};
\\
\pi_3 &= \left[\frac{1}{2} - \frac{(1 - \lambda)v - (p_{2c} - p_{3E}) + \frac{\beta t}{2}}{t} + \frac{p_{1E} - p_{3E}}{\beta t}\right] p_{3E};
\end{align*}
\]

Using the derivations in the Appendix, we can state the following Proposition.

**Proposition 5** In the asymmetric case where only incumbent firm 1 and the entrant have an online retail channel and firm 2 not, the unique symmetric subgame

\(^{16}\)It is easy to see that \( x_{1Ec} \leq 1/3 \) whenever \( \alpha \leq 2/3 \), which is always satisfied given the assumption on \( \alpha \) we imposed.
equilibrium prices are, in case $\lambda < 1$,\footnote{When $\lambda > 1$, there are two subcases to consider. In both cases, as before, the incumbent firm with an online channel decides not to use its conventional retail channel implying equation (5) stops being relevant. The first subcase arises when the expression for $p_{2c}$ remains positive. This is the case as long as $\alpha (3 + 2\beta)(1 - \beta) + \beta(3 + \beta) > 0$. Apart from the fact that we have to delete the expression $\frac{(1 - \lambda)v^2}{4(1 - \beta)t}$ in the equilibrium profits of the incumbent, all other expressions remain unchanged. When $\alpha (3 + 2\beta)(1 - \beta) + \beta(3 + \beta) < 0$, firm 2 cannot make positive profits and drops out of the market altogether. One can easily check that in this case, the equilibrium prices are $p_{1E} = p_{3E} = \beta/2$ and equilibrium profits are given by $\pi_1 = \pi_3 = \beta t/4$. One may easily verify that given the condition on $\alpha$ and $\beta$, even the consumer located at 1 prefers buying from the entrant at this price (yielding a pay-off of $\lambda v - \beta t$) to getting the good for free from firm 2 (yielding a pay-off of $v$).} given by:

\begin{align*}
  p_{1E} &= \frac{\beta}{1+\beta} \left[ \frac{(3+\beta)t}{6} - \frac{(1-\lambda)v}{6} \right]; \\
  p_{1c} &= p_{2c} = \frac{\beta}{1+\beta} \left[ \frac{(3+\beta)t}{6} + \frac{3+2\beta(1-\lambda)v}{1+\beta} \right]; \\
  p_{3E} &= \frac{\beta}{1+\beta} \left[ \frac{(3-\beta)t}{6} - \frac{(1-\lambda)v}{3} \right].
\end{align*}

Operating profits in this case are given by

\begin{align*}
  \pi_1 &= \frac{[(1-\lambda)v]^2}{4(1-\beta)t} + \frac{\beta}{36(1+\beta)^2t}[(3+\beta)t - (1-\lambda)v]^2; \\
  \pi_2 &= \frac{1}{36(1+\beta)^2t}[(3\beta + \beta^2)t + (3 + 2\beta)(1-\lambda)v]^2; \\
  \pi_3 &= \frac{\beta}{36(1+\beta)t}[(3-\beta)t - 2(1-\lambda)v]^2.
\end{align*}

With asymmetric incumbent firms there are again some interesting results that deserve some further elaboration. First, in case online shopping is considered to be less convenient ($\alpha > 0$) both incumbents charge the same equilibrium offline price. This price is simply equal to the online price plus the mark-up $\frac{(1-\lambda)v}{2}$ we have seen in previous cases. Second, the entrant’s online price can never be larger than the incumbent’s online price. From equations (4) and (6) one can conclude that the entrant’s online price is larger in case $\alpha (3 + 2\beta)(1 - \beta) + \beta(3 + \beta) < 0$. However, in this case the condition that $p_{2c} \geq 0$ is violated, implying that the equilibrium online prices of both firms equal $\beta t/2$ (see footnote 17). We can therefore conclude already that the entrant’s (online) prices can only be larger than the incumbents’ prices in case the entrant has a monopoly position online, the case examined in subsection 4.1.
5 Equilibrium Market Structures: Only Bricks or also Clicks?

We now analyze the first stage of the game and ask the question what are the incentives of firms to develop an infrastructure to sell online at a fixed cost $f$. In other words, under what conditions on the exogeneous parameters can which of the above market structures arise? We consider two different ways the first stage may be played. First, we analyze the case where the incumbents decide whether or not to start an online channel before the entrant does. Here, we ask the question whether starting an online channel may be a way to deter a pure online player to enter. Second, we analyze the case where the entrant decides whether or not to start an online channel before the incumbents do.

Before we delve into the implications of these two different decision sequences, we provide some analysis that is used in both cases. We first summarize the pay-offs in the different cases where the entrant always enters in the following two matrices, where due to the symmetry between firm 1 and 2, we have only one matrix for both of them.\(^{18}\)

\[
\begin{array}{cccc}
\text{y/n} & \text{n} & \text{y} \\
\hline
\text{n} & \dfrac{[(1-\lambda)v^2 + (\frac{\lambda}{4} + \frac{\beta}{3})t]^2}{96} & \dfrac{[(3\beta + \beta^2)t + (3 + 2\beta)(1-\lambda)v]^2}{36(1+\beta)^2t} \\
\text{y} & \dfrac{[(1-\lambda)v^2 + 2[(4+\beta)t - (1-\lambda)v]^2 - f}{36(1+\beta)^2t} & \dfrac{\beta t}{18} + \dfrac{[(1-\lambda)v]^2}{4(1-\beta)t} - f
\end{array}
\]

Table 2. Pay-offs for firm 1 in case $\lambda < 1$ depending on whether or not itself and its rival incumbent start an online channel and given that a pure online retailer enters.\(^{19}\)

\[
\begin{array}{cccc}
\text{y/n} & \text{n} & \text{y} \\
\hline
\text{n} & \dfrac{2[(1-\lambda)v^2 - (1-\lambda)v]^2}{96} - f & \dfrac{\beta [(3-\beta)t - 2(1-\lambda)v]^2}{36(1+\beta)^2t} - f \\
\text{y} & \dfrac{2[(3-\beta)t - 2(1-\lambda)v]^2}{36(1+\beta)^2t} - f & \dfrac{\beta t}{18} - f
\end{array}
\]

Table 3. Pay-offs for firm 3 in case $\lambda < 1$ depending on which of the incumbent firms starts an online channel and given that itself enters.\(^{20}\)

\(^{18}\)Note that the cells are defined in terms of the actions taken by firms 1 and 2. Hence, this is not a proper pay-off matrix.

\(^{19}\)In case $\lambda > 1$ the term $\dfrac{[(1-\lambda)v]^2}{(1-\beta)t}$ has to be deleted from the two cells in the bottom row. In case $\lambda > 1$ and, in addition, $\alpha(3 + 2\beta)(1-\beta) + \beta(3 + \beta) < 0$, the bottom left cell will have to be replaced by $\beta t/4 - f$ (see footnote 17) and the upper right cell becomes zero.

\(^{20}\)In case $\lambda > 1$ and $\alpha(3 + 2\beta)(1-\beta) + \beta(3 + \beta) > 0$, the expressions remain identical. In case $\lambda > 1$ and $\alpha(3 + 2\beta)(1-\beta) + \beta(3 + \beta) < 0$, the bottom left cell will have to be replaced by $\beta t/4 - f$. 

---

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On the basis of Table 3, we can easily argue that firm 3’s incentive to enter are monotone in the number of firms that have an online channel: the more incumbents have an online retail channel, the less operating profits a pure online retailer can get.\textsuperscript{21}

Looking at the matrices, one can see that the parameters $\lambda$ and $v$ only enter together in the combination $(1 - \lambda)v$. Using the definition of $\alpha$ we can therefore substitute this expression without loss of generality for $\alpha(1 - \beta)t$, where $\alpha$ is relatively small (large) when the shopping inconvenience or the willingness-to-pay is relatively small. Once the substitution is made, one can also see that all operating profits are linear in $t$ so that in fact we are left with three relevant parameters: $\alpha$, $\beta$ and $f/t$. Figure 5 shows for $\alpha = 0.4$, the monotonicity of the incentives of firm 3 to enter: for any $\beta$, if $f/t$ is relatively large firm 3 will never enter even, not even if no incumbent has built the facilities to sell online. If $f/t$ becomes smaller we first enter an area of parameter values in which firm 3 can only profitably enter when no incumbent firm has an online retail channel. If $f/t$ becomes even smaller we next enter an area of parameter values in which firm 3 can only profitably enter if at most one incumbent firm has an online retail channel. Finally, if $f/t$ is very small firm 3 will always want to enter.

(see footnote 17).

\textsuperscript{21}See the Appendix for the calculations.
5.1 Online Retailing as an Entry Deterrence Strategy

We first consider whether or not incumbent firms have an incentive to make a strategic move by opening an online retail channel first. We have seen that in the absence of an entry threat the incentives to open an online channel are limited to the case where the online shopping conveniences are relatively large. By looking at the case where incumbents decide first whether or not to start an online retail channel before a pure online player decides to enter the market, we therefore inquire whether the possibility to deter entry gives incumbent firms more incentives to open online retail channels.

The analysis of this section is facilitated by the fact presented above that the entrant’s incentives to enter are monotonic in the number of incumbent firms having an online retail channel at their disposal. It turns out that in line with Figure 5, there are four possible cases for the entrant’s reaction to the incumbents’ online presence. To discuss these cases in an easy way, we denote by $\pi_i(a, b, c)$ the total payoff (including the cost of opening an online channel) of player $i$ when player 1, 2 and 3 take decisions $a, b$ and $c$ respectively, where $a, b$ and $c$ are either equal to $n$ (not to start an online retail channel) or $y$ (start an online channel).

First, if the cost of setting up an electronic retail channel are relatively high, $\pi_3(n, n, y) < 0$, the entrant will never enter. In the appendix we show that if the costs are really that high that an entrant will never enter, then incumbents will also not start an online channel. In other words, the incentives to start an online retail channel are larger for a pure online entrant than for an incumbent. This is easily explained by the fact that an incumbent partially competes with its own conventional store when it sets up its own online channel.

Second, if the cost of setting up an electronic retail channel is somewhat smaller, $\pi_3(n, n, y) > 0 > \pi_3(y, n, y)$, the entrant will enter if, and only if no incumbent firm has started an electronic channel. In the appendix it is shown that in this case the only subgame perfect equilibrium has one incumbent incurring the cost of starting an online retail channel. Depending on the parameters, the incumbent firms then may face a free-rider problem whenever both of them prefer the other to incur the cost of deterring entry. Free riding may occur for example when $\beta$ is close to 1 and $\alpha$ is close to 0 so that product differentiation is mostly due to product heterogeneity and if there is an online shopping inconvenience, it is fairly small. As we have seen in Section 3, online prices may be higher in this market structure (when $\alpha$ is small) than average conventional prices due to the fact that one of the incumbents has some online monopoly power. Unlike in Section 3, this market structure may now also arise when $\alpha > 0$ as incumbent firms have an incentive to deter entry.

Third, if the cost of setting up an electronic retail channel is even smaller, $\pi_3(y, n, y) > 0 > \pi_3(y, y, y)$, the entrant will enter if and only if, at most one incumbent firm has started an electronic channel. The appendix shows that in this case two things
may happen. The more interesting possibility occurs when \( \alpha > 0 \) and it may also arise when \( \alpha \) is not too small. In this case, the incumbent firms face a situation of severe strategic uncertainty in the sense that they only would like to start an online retail channel if the other firm will do likewise so that they together deter entry. This strategic uncertainty is reflected by the fact that the incumbent firms face a coordination game and there are two subgame perfect equilibria: either they both start an online retail channel and together deter entry, or none of them builds an online channel and entry occurs. When entry occurs, as explained in subsection 4.1., online prices may be higher than the prices in the conventional stores due to the "monopoly power" of the entrant. When there are large online shopping conveniences, only one online shop owned by the entrant is a very big threat to the incumbents and it becomes a dominant strategy for the incumbent firms to start an online channel irrespective of the behavior of the other incumbent. The net effect in this case is that entry is deterred.

Fourth, if the cost of setting up an electronic retail channel is very small, \( f < \frac{\beta}{\pi} \), the entrant will always enter. If the cost of setting up the online retail channel is really very low, it may be (as a subcase) that both incumbent firms nevertheless decide to go online, sometimes even when \( \alpha > 0 \). Hence, even if entry does occur it may facilitate incumbents also to start an online retail channel as without entry they would not do so.

An overview of the different cases presented in this subsection is given in \((\alpha, \beta)\) space in Figure 6 below. The vertical line in the Figure represents the boundary between a region where the entrant always enters (to the right) and a region where the entrant’s decision is conditional on what incumbents do. In the left-hand part of the Figure we see the different possibilities to deter entry (either by one firm, or jointly as part of a coordination game or due to dominant strategies) as discussed above. The right-hand part shows that when \( \alpha \) decreases it becomes more attractive for incumbents to start an online retail channel as well even though this will not affect the entrant’s choice.

### 5.2 Following the entrant’s lead

Next, we analyze the case where the entrant is considering whether or not to make a strategic move and invest already early on (i.e., before the incumbents consider doing so) in building an online retail channel. More formally, the entrant now decides first whether or not to enter before the incumbents decide whether or not they follow and start their own retail channel. Of the cases considered in the previous subsection, the first and the last case \((\pi_3(n,n,y) < 0 \text{ and } \pi_3(y,y,y) > 0)\) are very similar to the previous analysis where the incumbents decide first and are therefore omitted. So, we concentrate on the intermediate cases where \(\pi_3(n,n,y) > 0 > \pi_3(y,y,y)\).
Figure 6: Equilibrium structures, $f/t = 0.02$.

Unlike in the previous subsection, now many possibilities arise and we don’t attempt to give a complete characterization. The most obvious cases arise when the incumbents have a dominant strategy either to start or not to start an online channel after the entrant has decided to enter. For quite a few parameter values when $\alpha > 0$, the incumbents have a dominant strategy not to follow the entrant and the market outcome is one where the entrant is the only one having online presence. On the other hand, when $\alpha$ is relatively small, there are many parameters values (especially when $\beta$ is not too close to 0) for which the incumbents have a dominant strategy also to start an online retail channel whenever the entrant has done so. In this case, the entrant is better off not to start an online channel when $\pi_3(y, y, y) < 0$. Whether or not the incumbents will then set up an online channel depends on the specific parameter values and this case is already analyzed in section 3.3.

There are, however, also quite a few other more interesting situations that may occur. First, it may be that the incumbents face a situation of strategic uncertainty, formally represented by a coordination game, following the entrant’s decision to go online. This is for example the case in an area around the following parameter constellation: $\alpha = 0, \beta = 0.1$ and $f/t = 0.007$. The incumbent firms would either like to enter both or not at all as $\pi_1(y, y, y) > \pi_1(n, y, y)$ and $\pi_1(n, n, y) > \pi_1(y, n, y)$. Turning back to the decision of the entrant makes clear that when the incumbents face a coordination problem, the entrant also faces a very risky decision: if incumbents will not follow, it can make a profit starting an online channel as $\pi_3(n, n, y) > 0$. 

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However, if incumbents follow suit, the entrant cannot recover its start-up costs and goes bankrupt as $\pi_3(y, y, y) < 0$. As the entrant cannot logically predict what the incumbents will do, as both possibilities may occur as equilibrium responses, the entrant cannot really figure out what to do. We may interpret this theoretical possibility as one of the possible rational explanations for the shake-out we have seen in recent years in the online business world: Entrants may have expected that incumbent brick-and-mortar firms will not follow and they had good reasons to expect so as not going online is an equilibrium response by incumbent firms. However, there exists another equilibrium path where incumbents do go online and where entrants make losses. The model tells that this possibility arises when both $\alpha$ and $\beta$ are close to 0, i.e., when the online shopping inconvenience is fairly small and goods are relatively homogeneous. Interestingly, these are exactly the sectors where one may expect online retailing to be quite successful!

A second interesting observation is that the number of firms deciding to start an online retail channel may very well be non-monotonic in the cost of building the infrastructure to sell online. The following parameter constellation serves as an example. For $\alpha = 0, \beta = 0.3$ and $f/t = 0.10$, it is optimal for the entrant to enter and start a pure online shop as $\pi_3(n, n, y) > 0$, knowing that the incumbents will not follow suit as $\pi_1(y, n, y) < \pi_1(n, n, y)$ and $\pi_1(y, y, y) < \pi_1(n, y, y)$ . However, when $f/t$ drops to $f/t = 0.05$, ceteris paribus, one incumbent will find it optimal also to start an online retail channel following the entrant’s decision to do so as $\pi_1(y, n, y) > \pi_1(n, n, y)$. This, however, makes it unprofitable for the entrant to start an online retail channel in the first place as $\pi_3(y, n, y) < 0$. When the entrant does in fact abstain from entering (as it is able to accurately predict the incumbent’s response), the incumbents will find it optimal not to start an online channel themselves. Thus, when $f/t = 0.05$, no online retail channel is built: the threat to enter in this case is, in a sense, incredible in case the incumbents can respond by going online themselves!

When the cost of building an online retail channel falls further to, for example, $f/t = 0.03$ the entrant will find it optimal to enter again ($\pi_3(n, n, y) > 0$) even if it knows that an incumbent will do likewise.\footnote{The implicit point in the above discussion is that for some parameter configurations the incumbent’s decision to go online is positively affected by the entrant’s presence online. This can be explained by the fact that by doing so an incumbent will want to recover some of the business it has lost to the entrant.} Thus, this numerical example shows that it is not generally true that by subsidizing the building of online retail channels, more online retail channels will be built.

A similar situation may arise with respect to the online shopping convenience perceived by the consumers. For $\alpha = 0, \beta = 0.4$ and $f/t = 0.07$, it is optimal for the entrant to enter and start a pure online shop, knowing that the incumbents will not follow suit. However, when the online shopping inconvenience drops to $\alpha = -0.3$,
keeping the other parameter values equal, one incumbent will find it optimal to go online as well, making the entrant’s decision to do so unprofitable. As above, when the entrant does not enter in the first place, the incumbents will not go online at all. This makes the point that it is not generally true that increasing the online shopping convenience as perceived by consumers, will increase the possibility that firms build an online presence. Hence, increasing online shopping convenience may actually not be in the interest of consumers!

6 Conclusions and Managerial Implications

In this paper we have explored the implications of the possibility for firms of building a retail channel online. The setting we analyze is one where two incumbent firms compete for customers in a market with horizontally differentiated products. Firms are differentiated because they sell differentiated products and because their physical locations are different. In the online market, only the first type of differentiation plays a role. Consumers know which of the two incumbent firms sell products closest to their tastes. They can decide to go either to the conventional store of a firm or they can go online. Consumers find one aspect of online shopping certainly better than conventional shopping, which is the fact that they do not have to travel a physical distance. Apart from that, consumers may find online shopping more or less convenient from going to a conventional shop depending on whether or not they feel confident about things like payment method, delivery time and the trustworthiness of redemption policies.

Firms play a two-stage game. In the first stage they decide whether or not to build the infrastructure needed to sell online at a fixed cost. The interesting cases arise when this fixed cost is not prohibitively high so as to make it unprofitable to start an online retail channel. The main question at this stage is what are the incentives for a bricks-and-mortar firm to go online and what are the incentives to enter for a pure online player. In the second stage, firms decide which prices to charge on different retail channels.

The more interesting managerial implications are as follows. First, bricks-and-mortar firms are worse off if they open an online retail channel as the cannibalization effect (stealing consumers from its own conventional store at lower prices) is likely to be more important than the business stealing effect (attracting demand from rival firms). When online shopping is considered to carry some conveniences, incumbent firms also will not have an incentive to start an online retail channel. When there are large online shopping conveniences, firms may individually have an incentive to go online, but total profits decline as online competition is more severe due to the absence of differentiation in location. In some cases, firms may in fact face a Prisoner’s Dilemma.
A second managerial implication relates to the question who benefits from online shopping conveniences. By increasing the online shopping conveniences (or reducing the inconveniences), firms are able to direct more of their costumers through the online channel. If there are no important cost advantages related to this shift and there is no (threat of) entry, incumbent firms will be worse off as more bricks-and-mortar firms will open an online retail channel and competitive pressure online is larger. When there is a threat of entry, however, increasing online shopping convenience may be a way for incumbent firms to credibly commit to start an online channel if pure only players do so. In such situations, online players may abstain from entry as entry will be followed by an incumbent going online, making the entry unprofitable. These are situations where incumbent firms may benefit from online shopping conveniences at the expense of entrants and consumers.

A third implication relates to the viability of pure online retailers. Our analysis shows that in most circumstances these retailers have to charge fairly low prices in order to get a fair market share. The operating profits of online retailers are therefore relatively low as they do not benefit from a bricks-and-mortar reputation. Moreover, they may find it difficult to predict the response of incumbent firms to their entering the market. There are situations when incumbent firms face a coordination problem after entry. This makes the entry decision a risky decision as the consequences in terms of profitability are difficult to predict. Moreover, it may be that incumbent firms will face lower cost of building up an online retail channel as they may learn from the entrant’s experience. We have seen how sensitive the entrant’s decision problem is to the non-monotonicity in the cost of building an online retail channel. All these factors question the viability of pure online retailers, especially in markets where product differentiation is largely due to the geographical location of bricks-and-mortar firms.

A last managerial implication relates to a firm’s pricing policy. Online prices depend on the type of firm: a clicks-and-mortar firm charges higher online prices than a pure online player. Also, online prices depend on the market structure and on the size of the online shopping convenience. Online prices will only be higher than prices in the conventional stores if firms have some market power online, which is partly founded by the size of the online shopping convenience. In future research it would be interesting to see whether this theoretical prediction can be empirically verified.

The results may explain the fact that online shopping has not become as important a retail channel as many believed at the end of the previous millennium. The online shop may cannibalize a firm’s own customer base and when it does not, it may increase competition between firms if everyone builds an online retail channel. Managers should be aware of these facts and should weigh the importance of them against the possibility that new customers are attracted to the market by using an alternative...
retail channel. In future theoretical work, it would be interesting to see to what extent the conclusion drawn here for a more or less saturated market continue to hold when the online retail channel is able to attract new costumers.

7 Appendix

First-order conditions Section 3.2.

There are three first-order conditions, one for firm 2 and two for firm 1:

\[ p_{1c} = p_{1E} + \frac{(1 - \lambda)v}{2}; \]
\[ 0 = t - (1 - \lambda)v + p_{2c} - 2p_{1E}; \]
\[ 0 = \beta t - (1 - \lambda)v - 2p_{2c} + p_{1E}; \]

Solving these three equations for the three unknown prices, yield the subgame equilibrium prices given in the main text.

Calculations establishing firms have a dominant strategy in Section 3.3 when \( \lambda < 1 \)

When firm 2 does not start an online channel, it is better for firm 1 to do likewise iff

\[ \frac{1}{2} \geq \frac{\alpha^2(1 - \beta)}{4} + \frac{[2 - \alpha + (1 + \alpha)\beta]^2}{9(1 + \beta)} - \frac{f}{t}. \]

This inequality certainly holds if it holds for \( f = 0 \). In this case, the inequality can be rewritten as

\[ 18(1 + \beta) \geq 9\alpha^2(1 - \beta^2) + 4[2 - \alpha + (1 + \alpha)\beta]^2. \]

It is easy to verify that this inequality holds true for both \( \beta = 1 \) (with equality) and \( \beta = 0 \). As the coefficient in front of the \( \beta^2 \) term of the RHS is positive for \( \alpha > 0 \), the inequality has therefore to hold for all \( 0 \leq \beta \leq 1 \).

When firm 2 starts an online channel, it is better for firm 1 not to do likewise iff

\[ \frac{[1 + \alpha + (2 - \alpha)\beta]^2}{9(1 + \beta)} \geq \frac{\beta}{2} + \frac{\alpha^2(1 - \beta)}{4} - \frac{f}{t}. \]

For \( f = 0 \), this reduces to

\[ 4[1 + \alpha + (2 - \alpha)\beta]^2 \geq 18\beta(1 + \beta) + 9\alpha^2(1 - \beta^2). \]

It is easy to verify that this inequality holds true for both \( \beta = 1 \) (with equality) and \( \beta = 0 \). If we bring all terms to the LHS the coefficient in front of \( \beta^2 \) is negative, implying that the inequality has to hold for all \( 0 \leq \beta \leq 1 \).
Calculations establishing the equilibrium structure in Section 3.3 when \( \lambda > 1 \)

When firm 2 does not start an online channel, it is better for firm 1 to do likewise iff

\[
\frac{1}{2} \geq \frac{[2 - \alpha + (1 + \alpha)\beta]^2}{9(1 + \beta)} - \frac{f}{t}.
\]

In this case, the inequality can be rewritten as

\[
\left(1 + 2\frac{f}{t}\right) 9(1 + \beta) \geq 2[2 - \alpha + (1 + \alpha)\beta]^2.
\]

The upper curve in Figure 2 depicts this relation in case we replace the inequality by an equality sign and \( f/t = 0.01 \).

When firm 2 starts an online channel, it is better for firm 1 not to do likewise iff

\[
\frac{[1 + \alpha + (2 - \alpha)\beta]^2}{9(1 + \beta)} \geq \frac{\beta}{2} - \frac{f}{t}.
\]

The lower curve in Figure 2 depicts this relation when \( f/t = 0.01 \) and when replacing the inequality by an equality sign.

\textit{First-order conditions Section 4.1.}

There are three first-order conditions, one for each firm. As firms 1 and 2 are in a symmetric position and we are looking for symmetric equilibria, we only give the first-order conditions for firm 1 and 3:

\[
(1 - \lambda)v - 2p_{1c} + p_{3E} + \frac{\beta t}{2} = 0;
\]
\[
t - 2(1 - \lambda)v + p_{1c} + p_{2c} - 4p_{3E} - \beta t = 0;
\]

Imposing symmetry \( (p_{1c} = p_{2c}) \) we can solve these two equations and obtain the subgame equilibrium prices given in Proposition 3.

\textit{First-order conditions Section 4.2.}

In case \( \lambda < 1 \), there are five first-order conditions, two for firms 1 and 2 and one for firm 3. As firms 1 and 2 are in a symmetric position and we are looking for symmetric equilibria, we only give the first-order conditions for firm 1 and 3, taking \( p_{1E} \) and \( p_{1c} - p_{1E} \) as choice variables:

\[
p_{1c} - p_{1E} = \frac{(1 - \lambda)v}{2};
\]
\[
\frac{1}{2} - \frac{2p_{1E} - p_{3E}}{\beta t} = 0;
\]
\[
p_{1E} + p_{2E} - 4p_{3E} = 0;
\]
Imposing symmetry \((p_{1c} = p_{2c} \text{ and } p_{1E} = p_{2E})\) we can solve these three equations and obtain the subgame equilibrium prices given in the main text.

**First-order conditions Section 4.3.**

In case \(\lambda < 1\), there are four first-order conditions, two for firm 1, one for firm 2 and one for firm 3:

\[
p_{1c} - p_{1E} - \frac{(1 - \lambda)v}{2} = 0; \\
\frac{1}{2} - \frac{2p_{1E} - p_{3E}}{\beta t} = 0; \\
(1 - \lambda)v - 2p_{2c} + p_{3E} + \frac{\beta t}{2} = 0; \\
\frac{1}{2} - \frac{(1 - \lambda)v - p_{2c} + 2p_{3E} + \frac{\beta t}{2} + p_{1E} - 2p_{3E}}{\beta t} = 0;
\]

Conditions 2 and 3 together imply that \(p_{2c} = p_{1E} + \frac{(1 - \lambda)v}{2}\). It is important to note that this is independent of whether or not the first-order condition holds, i.e., in case \(\lambda > 1\) and the first condition does not hold the second firm’s offline price is still the usual mark-up over the online price. Substituting this equation for \(p_{2c}\) into the fourth first-order condition and using the second yields

\[
\frac{1}{2} - \frac{(1 - \lambda)v - p_{2c} + 2p_{3E} + \frac{\beta t}{2} + p_{1E} - 2p_{3E}}{\beta t} = 0;
\]

This can be solved for the subgame perfect equilibrium value of \(p_{3E}\) given in the Proposition. Substituting this value into the second and third condition yields the subgame perfect equilibrium values of \(p_{1E}\) and \(p_{2c}\). The expressions for equilibrium profits, can be obtained by substituting the equilibrium prices in the expression for profits given in the text.

**Section 5: The entrant’s incentive to enter are monotone in the number of incumbents with an online channel**

On the basis of Table 3 we first show for the case when \(\lambda < 1\) or the case where \(\lambda > 1\) and \(\alpha(3 + 2\beta)(1- \beta) + \beta(3 + \beta) > 0\) that the operating profits of the entrant are larger when no incumbent has an online channel than when one of them has one. This is the case when\(^{23}\)

\[
8(1 + \beta)[1 - \alpha + (\alpha - \frac{1}{2})\beta]^2 > \beta[3 - 2\alpha + (2\alpha - 1)\beta]^2.
\]

\(^{23}\)In case \(\lambda > 1\) and \(\alpha(3 + 2\beta)(1- \beta) + \beta(3 + \beta) < 0\), the inequality takes the form \(8(1 + \beta)[1 - \alpha + (\alpha - \frac{1}{2})\beta]^2 > 9\beta\). Straightforward calculations show that whenever this inequality is relevant, it holds.
As $\beta/(1 + \beta) < 1/2$, it follows that this is certainly the case when

$$1 - \alpha + (\alpha - \frac{1}{2})\beta > \frac{3 - 2\alpha + (2\alpha - 1)\beta}{4}.$$ 

One can easily verify that this is the case if $\alpha < 1/2$.

Next, we show that the operating profits of the entrant are larger when one incumbent has an online channel than when both of them have one.\(^{24}\) This is the case when

$$[3 - 2\alpha + (2\alpha - 1)\beta]^2 > 2(1 + \beta).$$

As the LHS is decreasing in $\alpha$, this inequality certainly holds if it holds for $\alpha = 1/2$. Substituting this into the inequality yields $4 > 2(1 + \beta)$, which holds for all $\beta < 1$.

**Section 5.1:** If $f$ is too large for the entrant to enter in case the incumbents have no online retail channel, then incumbent will not start an online retail channel either.

We have to show that $\pi_3(n, n, y) < 0$ implies $\pi_1(y, n, n) > \pi_1(n, n, n)$. This is the case when \(\frac{2(1 - \frac{1}{2}\beta)(t - (1 - \beta))}{9} < f/t\) implies \(\frac{(2 + \beta)(t - (1 - \beta))}{9(1 + \beta)} - f/t < \frac{1}{2}\). This implication holds true if

$$\begin{align*}
(1 + \beta)\{2 - 2\beta + \frac{1}{2}\beta^2 - 4\alpha(1 - \beta)(1 - \frac{1}{2}\beta) + \alpha^2(1 - \beta)^2\} + \frac{9}{2}(1 + \beta) & > 4 + 4\beta + \beta^2 - 4\alpha(1 - \beta)(1 + \frac{1}{2}\beta) + \alpha^2(1 - \beta)^2.
\end{align*}$$

This inequality can be rewritten as \(2\frac{1}{2} + \frac{1}{2}\beta - 2\frac{1}{2}\beta^2 + \frac{1}{2}\beta^3 + 2\alpha\beta^2 + \alpha^2(1 - \beta)^2 > 0\). The terms involving $\alpha$ reach a minimum value of $-\beta^3$ at $\alpha = -\beta/(1 - \beta)$. Given this, it is easily seen that the inequality always holds true.

**Section 5.1:** Entry deterrence when the entrant enters if, and only if, no incumbent sells online

We have to show that $\pi_3(n, n, y) > 0$ implies $\pi_1(y, n, n) > \pi_1(n, n, y) > 0$. In case $\lambda < 1$ (and when $\lambda > 1$) this implication certainly holds true if \(\frac{2(1 - \frac{1}{2}\beta)(t - (1 - \lambda))}{9} > f\) implies \(\frac{(2 + \beta)(t - (1 - \lambda))v}{9(1 + \beta)t} - f > \frac{(1 - \lambda)v + (\frac{1}{2} + \frac{1}{2}\beta)t}{9(1 + \beta)}\). This can be rewritten as

$$\frac{\left[2 - \alpha + (1 + \alpha)\beta\right]^2}{9(1 + \beta)} > \frac{\left[\frac{1}{2} + \alpha + (\frac{1}{2} - \alpha)\beta\right]^2}{9} + \frac{2[1 - \alpha + (\alpha - \frac{1}{2})\beta]^2}{9}.$$ 

The LHS of this inequality is certainly larger than $(2 - \alpha)^2/9$, while the RHS is certainly smaller than $(1 + 2(1 - \alpha)^2)/9$. It is easily seen that $(2 - \alpha)^2 > 1 + 2(1 - \alpha)^2$ for all values of $\alpha < 1/2$.

**Coordination issue between incumbents when the entrant enters if, and only if, at**

\(^{24}\)In this case the relevant expressions for the entrant’s pay-off are independent of whether or not $\lambda < 1$. 

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most one incumbent firm has started an electronic channel (Section 5.1).

We have to show two things: (i) $\pi_3(y,y,y) < 0$ and $\alpha > 0$ implies $\pi_1(n,n,y) > \pi_1(y,n,y)$ and (ii) $\pi_3(y,n,y) > 0$ implies $\pi_1(n,y,y) < \pi_1(y,y,n)$.

(i) The first implication says $f > \frac{\beta}{18}$ implies $\frac{(1-\lambda)e + \frac{1}{2}f + \frac{1}{2}t)^2}{9f} > \frac{(1-\lambda)e^2 + \frac{1}{2}f(3+\beta)(1-\lambda)e^2}{36(1+\beta)^2} - f$. We think of the LHS and the RHS of this inequality as function of $\alpha$. We prove three facts: (a) When $\alpha = 0$, the implication holds true if the following inequality holds: $\frac{(1+\beta)^2}{36} > \frac{\beta(3+\beta)^2}{36(1+\beta)^2} - \frac{\beta}{18}$, which can be rewritten as $1 - 3\beta + 4\beta^2 + 5\beta^3 + \beta^4 > 0$, which holds for all $0 \leq \beta \leq 1$; (b) When $\alpha = 1/2$, the implication holds true if the following inequality holds: $\frac{1}{9} > \frac{\beta((5+3\beta)^2 + 1 - \beta - \beta^3}{144(1+\beta)^2}$, which can be rewritten as $16(1 + \beta)^2 > \beta(5 + 3\beta)^2 + 9(1 - \beta) - 8\beta$, or, $-\beta^3 - 14\beta^2 + 24\beta + 7 > 0$, which also holds for all $0 \leq \beta \leq 1$; (c) Further, it is easy to evaluate the derivative of both LHS and RHS with respect to $\alpha$. The derivative of the LHS is positive on the relevant domain of $\alpha$, whereas the derivative of the RHS is first negative and then positive. These three facts together imply that the implication always holds.

When $\lambda > 1$ and $\alpha(3 + 2\beta)(1 - \beta) + \beta(3 + \beta) > 0$, the implication does not need to hold. To see this note that the implication holds if, and only if, $\frac{|\alpha(\beta(3-\beta)(1-\lambda)e^2)|}{36(1+\beta)^2} > \frac{\beta(3+\beta)(1-\lambda)e^2}{36(1+\beta)^2} - \frac{\beta}{18}$. One may check that for example when $\alpha = -1/2$ and $\beta = 0.4$ the inequality does not hold.

(ii) In case $\lambda < 1$ the second implication certainly holds true if $\frac{\beta(3-\beta)(1-\lambda)e^2}{36(1+\beta)^2} > f$ implies $\frac{\beta(3-\beta)(1-\lambda)e^2}{36(1+\beta)^2} > \frac{\beta}{2} + \frac{(1-\lambda)e^2}{4(1-\beta)^2} - f$. This implication holds true if

$$\beta + \frac{\alpha^2(1-\beta)}{4} > \frac{\beta(3+\beta)^2(1-\lambda)e^2}{36(1+\beta)^2} + \frac{\beta(3-2\alpha + 2(\alpha - 1)\beta)^2}{36(1+\beta)^2}$$

As the last term of the RHS is decreasing in $\beta$, this term is smaller than $\beta/4$ when $\alpha > 0$, and thus, this inequality holds if

$$\frac{\beta}{4} + \frac{\alpha(1-\beta)}{4} \left( \frac{\alpha(3+2\beta)^2(1-\beta)}{9(1+\beta)^2} - \frac{2\beta(3+2\beta)(3+\beta)}{9(1+\beta)^2} \right) > \frac{\beta^2(3+\beta)^2}{36(1+\beta)^2}.$$  

As the term on the LHS in brackets is increasing in $\alpha$, this is certainly the case when

$$\frac{\beta}{4} - \frac{\alpha(1-\beta)}{4} \frac{2\beta(3+2\beta)(3+\beta)}{9(1+\beta)^2} > \frac{\beta^2(3+\beta)^2}{36(1+\beta)^2}.$$  

As $\alpha < 1/2$, it is a straightforward exercise to show this is the case.

Similar calculations hold when $\lambda > 1$ and $\alpha(3 + 2\beta)(1 - \beta) + \beta(3 + \beta) > 0$.\footnote{When $\lambda > 1$ and $\alpha(3 + 2\beta)(1 - \beta) + \beta(3 + \beta) < 0$ the implication reads as $\beta t/4 - f > 0$ implies $\beta t/2 - f > 0$, which obviously holds true.}
this case the implication holds when

\[
\frac{\beta}{2} > \frac{[(3\beta + \beta^2) + \alpha(3 + 2\beta)(1 - \beta)]^2}{36(1 + \beta)^2} + \frac{\beta[3 - 2\alpha + 2(\alpha - 1)\beta]^2}{36(1 + \beta)}.
\]

As the first term of the RHS is increasing in \(\alpha\) and as \(0 > \alpha > -1/2\), this inequality certainly holds if

\[
\frac{1}{2} > \frac{\beta(3 + \beta)^2}{36(1 + \beta)^2} + \frac{16\beta}{36(1 + \beta)}.
\]

It is straightforward to show that this is always the case.

References


