Hartog, Joop; van Ophem, Hans; Bajdechi, Simona Maria

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How Risky is Investment in Human Capital?

Joop Hartog
Hans van Ophem
Simona Maria Bajdechi

Department of Economics, Universiteit van Amsterdam, and Tinbergen Institute.
Tinbergen Institute
The Tinbergen Institute is the institute for economic research of the Erasmus Universiteit Rotterdam, Universiteit van Amsterdam, and Vrije Universiteit Amsterdam.

Tinbergen Institute Amsterdam
Roetersstraat 31
1018 WB Amsterdam
The Netherlands
Tel.: +31(0)20 551 3500
Fax: +31(0)20 551 3555

Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31(0)10 408 8900
Fax: +31(0)10 408 9031

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Abstract: The risk of investment in schooling has largely been ignored. We assess the variance in the rate of return by surveying the international empirical literature from this fresh perspective and by simulating risky earnings profiles in alternative options, choosing parameters on basis of the very limited evidence. The distribution of rates of return appears positively skewed. Our best guess of ex ante risk in university education is a coefficient of variation of about 0.3, comparable to that in a randomly selected financial portfolio with some 30 stocks. Allowing for stochastic components in earnings also markedly affects expected returns.

Keywords: education, return, earnings dispersion, risk.

JEL classification: I2, J3.
1. Introduction

A remarkable flaw in the analysis of the investment in human capital is the failure to account for risk. When deciding on (additional) schooling an individual will not only be interested in the expected returns but also in the corresponding risk. In fact, the perceived risk of the investment may well be a dominant concern in the decision making process.

In terms of *ex ante* risk, the first risk is that on educational performance: how well will the individual do in school. This depends on abilities, only partly known when entering school, on effort (which may also be imperfectly anticipated) and on the match between curriculum and individual. With hurdles like final exams and thresholds for passing on to the next class, an individual may not even finish the school she entered. All these factors may be summarized as uncertainty as to where in the educational performance distribution the individual will end up. As performance in school is not the same as performance in the labor market, the second source of risk is uncertainty about the relative position in the post-school earnings distribution. A third source of risk is market risk. The value of an education, or associated occupation, may shift over time in response to changes in technology, product demand patterns or relative supply.

Surprisingly little is known empirically about the dispersion in returns to education. Even though heterogeneity among individuals and hence in their returns has been stressed in several contributions, such as Willis and Rosen (1979) and Card (1995), this has not led to a focus on the risk associated with human capital investment. We search the empirical literature for information on the variation in rates of return, to get some feel for the universe from which returns may be drawn. To assess *ex ante* risk, we design a simulation model to mimic the situation facing an individual about to decide on investment in education. This model is simply the basic human capital investment model that compares two future earnings streams.
Ex post variability in returns is not the same as ex ante risk. Selectivity, extensively highlighted in the recent literature, is an important cause of deviations. It’s also an interesting question to what extent heterogeneity coincides with risk for the individual investor. If the individuals themselves are imperfectly informed on their abilities, future efforts, job opportunities etc, heterogeneity comes close to ex ante risk. The distinction between heterogeneity and risk is not relevant for the structure of our simulation model, as it can accommodate both foreseeable heterogeneity (variation between individuals) and risk. The distinction is mostly relevant when it comes to selecting the parameter values in the simulations. We take key parameters from a survey of the empirical literature, without paying much attention to this distinction.

As the relation between risk and return is at the heart of financial investment theory, we may turn to that literature for some benchmark information. In a widely used textbook on finance, Bodie, Kane and Marcus (1999) refer to Fisher and Lorie (1970) who give an overview of returns to portfolio’s on the New York Stock Exchange. They calculated one-year mean returns and standard deviations for randomly selected portfolios differing in size. All portfolios had a mean return of 28.2%. But with the portfolio size increasing from 1 to 8 and then further to 32 and 128, increasing diversification led to a drop in the coefficient of variation, from 1.45 through 0.51 and 0.25 to 0.12. The results by Fisher and Lorie are an interesting reference, to infer whether schooling is like a single asset (just school years) or more like a portfolio with several skills; we will return to this in the concluding section. First, in section 2, we survey the literature for ex-post variation in the rate of return, and in section 3 we simulate to assess ex-ante risk. Conclusions are collected in the last section.
2. Ex Post Variability in Returns to Education: Scanning the Literature

2.1. Earnings Dispersion by Education and Experience

Earnings distributions by education can tell us whether schooling moves individuals to distributions with different variances. If individuals cannot condition these distributions on variables they know when considering entering an education, the distributions provide a crude indication of differences in risk. One may differentiate by age or experience, to consider different risk profiles over the career. One may either consider the distribution of earnings itself or consider residuals from an earnings function, conditioning on an imposed structure of returns to schooling and age/experience. Some authors have analyzed the former, some the latter and we will just report their outcomes.

Our present limited survey reveals that there is no unequivocal pattern of earnings dispersion by education level or by experience (Table 1). There are very few robust “stylized facts”, and earnings variance apparently may increase, decrease or have no relation at all with education or experience. Clearly, there is scope for basic descriptive work to check the robustness of this conclusion.\(^1\) Differences in patterns between countries might point to very different effects of education systems, through differences in school admission rules and curriculum structures (e.g. broad versus specialized educations).

2.2. Variation in Mincer coefficients across time and place

The project PURE (Harmon, Walker and Westergaard-Nielsen 2001) generated private returns to education across Europe from a standard Mincer earnings equation. Minimum rates over the sample period varied between countries from 4.0 to 10.7, maximum rates between 6.2 and 11.5 percent (see Table 2). Trostel, Walker and Woolley (2002) use data for 28 countries covering the period 1985-1995, from a common questionnaire applied in all

\(^1\) We are working on a standardized international comparison, using the LIS data set.
countries. Averaged over the 28 separate country estimates, the mean return is 5.8% for men, with an unweighted standard deviation of 3.5%. For women, the mean return is 6.8%, with standard deviation 3.9%.

Ashenfelter et al (1999) performed a meta-analysis to 96 estimated returns obtained from 27 studies covering 9 countries. The mean return was 7.9% with a standard deviation of 3.6%. Regressing these returns on controls like region (within a country), time, ability, estimation method, left an intercept of 3% with a standard deviation of 1.6%.

Repeated estimates of Mincer equations over time, within a country, can provide some indication of the risk that is associated with shifting market value of schooling, as a consequence of changes in supply and demand conditions. In Holland (Hartog et al. 1993) the return to human capital has fallen steadily from 13% in 1962 to 7% in 1985 and then has risen slightly until 1989. Dutch experience differs from that of US, UK and Australia where the return to human capital increased during the eighties. In the U.S., Welch (see Willis, 1991) found that the rate of return to college education stayed within a narrow range of 8 to 9 percent from 1967 to 1981 and rose to little over 10.2 percent in 1982. Heckman, Lochner and Todd (2003) find a modest variation of Mincer returns for white men between 1940 and 1990 (between 10.2 and 12.9 percent); for black men, the variation is larger, with the return increasing from 8.7 percent in 1940 to 15.2 in 1990.

The project PURE (o.c.) finds that in Austria and Sweden, the rates of return decreased by about three percent points, whereas in Denmark, Netherlands, Portugal, UK, Ireland and Italy returns are upward trended (Table 2). The returns in Germany, France, Norway, Finland, Spain, Switzerland and Greece indicate no obvious trend.
2.3. Variation in Mincer coefficients across individuals

Harmon, Hogan and Walker (2003) treat the return to education on a sample of U.K data as a random coefficient. They specify earnings for individual $i$ as:

$$\ln w_i = (\beta + \varepsilon_i) x_i + \gamma X_i + \eta_i$$

where $X$ is a vector of explanatory variables (including age) and $\eta$ is the residual, with standard deviation $\sigma$. This is a heteroscedastic model, with error variance

$$E[(\varepsilon_i, s_i + \eta_i)^2] = \theta s_i^2 + \sigma^2$$

Harmon et al. find an estimated mean return of 4% for men and 7% for women, with dispersions of 4% and 3.3%. 95% of the men have returns in the +/- 7.8% interval around the mean, 95% of the women are in the +/- 6.5% around their mean. Thus the dispersion is large even after allowing for several observable individual characteristics, and some individuals even have negative returns. The results by Harmon et al for the UK and two studies they cite (Finland and the USA) all have coefficients of variation in the range 0.4 to 0.6. This exactly coincides with the values found by Ashenfelter et al (1999) when they allow for heterogeneous returns.

Pereira and Martins (2001) measure risk as the difference in returns between the ninth and the first decile from a quantile regression estimation of the Mincer equation. Across 16 countries the risk lies between -1.95% and 8.9%, at an average unweighted OLS return of 7.8 percent. Note that in a normal distribution, the difference between the value at $P(90)$ and
$P(10)$ would cover 2.56 standard deviations. The risk measure would then indicate a standard deviation between 0.76 and 3.47 percent.²

### 2.4. What have we learned?

We did not find robust stylized facts on the relation between earnings dispersion and education or experience (age). This suggests that the education systems in different countries function quite differently in segmenting the labour force. If individuals cannot perfectly predict their position in post-school earnings distributions, this suggests that the risk in educational investment varies across countries.

The Mincerian rate of return in one country may easily be 2 to 3 times the returns in another country. Across countries, the coefficient of variation may be something like 0.5. Within countries, there is generally a fair amount of stability over several decades. The large changes in The Netherlands and for Black men in the US seem exceptional. Over time, within countries the differences between the minimum and the maximum rate seem generally perhaps no more than a third of the minimum rate.

On differences in returns between individuals there is even less information. Available studies suggest a coefficient of variation between 0.4 and 0.6.

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² Pereira and Martins use their estimates to test for compensation of earnings risk in wages. A few other studies have also found risk compensation in wages (Hartog et al, 2003; Hartog and Vijverberg 2002, and the references cited there).
3. Assesing risk

3.1 Analytical solutions

By definition, the internal rate of return to education is the rate of discount, \( \delta \), that equates the present values of lifetime earnings for two different educational levels, \( s_0 \) and \( s_1 \), i.e., the interest rate that solves the equation:

\[
\int_{s_0}^{s_0+T_0} f(s_0, (t-s_0)) e^{-\delta t} dt = \int_{s_1}^{s_1+T_1} f(s_1, (t-s_1)) e^{-\delta t} dt
\]

(1)

The earnings functions are \( f(s_0, (t-s_0)) \) and \( f(s_1, (t-s_1)) \), with \( s_0, s_1 \) years of schooling and with \( t-s_0 \) to \( t-s_1 \) years of work experience. \( T_0 \) and \( T_1 \) are the durations of the working life after graduation. Note that this is a quite general framework, though not without limitations. We compare two investments, of different lengths, with a binding commitment up front. This might apply to different types of education, possibly but not necessarily differing in length (e.g., a 3 year education in economics or a 4 year education in law). It can be reduced to the basic Mincer model by setting \( s_0 = 0 \) and \( s_1 \) at the relevant value for a particular education (high school, university), or at \( s_0 = 1 \) to study marginal investments. But it excludes the option value of education, a worthy target for future work.

Let us start simply by computing the internal rate of return to \( s_1 \) rather than \( s_0 \) years of schooling, for an infinitely lived individual \( (T = \infty) \), with potential earnings functions that include independent stochastic components \( u_0 \sim (0, \sigma_0^2) \), \( u_1 \sim (0, \sigma_1^2) \). We assume only one lifetime shock. The amount of human capital produced in school is unknown when entering but revealed at the predetermined time of leaving and then determines annual earnings for the rest of working life. The profiles differ in returns to schooling and to experience. For the moment we ignore the usual quadratic term in experience. The internal rate of return then follows from:
Working out equation (2) we obtain:

$$E\int_{x_0}^{\infty} e^{\beta_1x_1 + \beta_2 (x_1-x_0) + x_0} e^{-\delta t} dt = E\int_{x_1}^{\infty} e^{\beta_2x_2 + \beta_3 (x_2-x_1) + x_1} e^{-\delta t} dt$$

(2)

If $E(e^{x_0}) = E(e^{x_1})$ we can rewrite equation (3) as:

$$\frac{E(e^{\beta_1x_1} e^{(\beta_2 - \delta)x_0})}{\delta - \beta_2} = \frac{E(e^{\beta_3x_3} e^{(\beta_4 - \delta)x_1})}{\delta - \beta_4}$$

(3)

Equation (4) is a generalization of the Mincer specification. With equal means in shock exponentials, identical experience profiles and with minimum level of education zero ($s_0 = 0$), we obtain $\delta = \beta_2$: the coefficient in the earnings function equals the internal rate of return.

Generally, in equation (4), the internal rate of return is the return for selecting a longer education with a different reward per year of schooling and per year of experience. The latter feature is routinely neglected. If higher education brings more earnings growth this boosts the returns to education. Note that $\beta_3$ measures the average return per school year for $s_1$ years of schooling and $\beta_4$ measures the average return per school year for $s_0$ years of schooling. In empirical earnings functions with dummies for different levels of education, average returns for longer educations are often lower than for shorter education. This depresses the internal rate of return $\delta$.

Equation (4) has to be solved numerically for $\delta$. If $s_0 = 0$ or $\beta_1 = \beta_3$, $\delta$ will be given by the transcendent equation:

$$\delta = \frac{\beta_3 s_1 - \beta_1 s_0}{s_1 - s_0} + \ln \frac{\delta - \beta_2}{\delta - \beta_4}$$

(4)

3 The same point is made by Heckman, Lochner and Taber (1999, p 331). Brunello and Comi (2000) documents steeper profiles for the higher educated in Europe.
\[
\delta = \beta_1 + \ln \frac{\delta - \beta_2}{\delta - \beta_4}
\] (5)

The effect of differential experience profiles (\(\beta_2 \neq \beta_4\)) can be substantial. Suppose, the return to school years is 0.065 and the experience growth differs by one percent point: \(\beta_2 = 0.05\) for \(s_0 = 0\) and \(\beta_4 = 0.06\) for \(s_1 = 4\). Then the internal rate of return \(\delta = 0.160\). If \(\beta_2 = 0.01\) for \(s_0 = 0\) and \(\beta_4 = 0.015\) for \(s_1 = 4\) then the internal rate of return \(\delta = 0.114\).

In the more general case with different means of the exponential shocks and correlation \(\rho\) between shocks, we can solve from (3) to get:

\[
E\delta = E(\ln \frac{e^{u_1}}{e^{u_0}}) + \beta_3 s_1 - \beta_3 s_0 + \ln \left(\frac{\delta - \beta_2}{\delta - \beta_4}\right)
\] (6)

The correlation coefficient does not affect the expected rate of return. With \(\beta_2 = \beta_4\) and \(u_0, u_1\) normally distributed with means and variances \(\mu_0, \sigma_0^2\) and \(\mu_1, \sigma_1^2\) respectively, the mean of the internal rate of return can be approximated\(^4\) by:

\[
E\delta \approx \frac{\mu_1 - \mu_0}{(s_1 - s_0)} + \frac{\sigma_1^2 - \sigma_0^2}{2(s_1 - s_0)} + \frac{\beta_3 s_1 - \beta_3 s_0}{s_1 - s_0}
\] (7)

Clearly, with \(\sigma_0^2 = \sigma_1^2\), \(\mu_0 = \mu_1\), and \(\beta_2 = \beta_4\) or \(s_0 = 0\), we have equivalence to the standard Mincer world: \(E\delta = \beta_3\). If \(\sigma_0^2 \neq \sigma_1^2\), but the rest of the previous conditions are fulfilled, then the expected value of the internal rate of return is affected by the difference in the errors variances. In this case, part of the return to education derives from a difference in the stochastic processes. If additional schooling gives access to a wider earnings distribution, expected returns go up.

\(^4\)We approximate \(E \ln (\exp x)\) by \(\ln E (\exp x)\).
We can approximate the variance of the internal rate of return as:

$$V\delta = e^{2\sigma_1^2 + \sigma_2^2 - 2\rho \sigma_1 \sigma_2} \frac{(e^{2\sigma_1^2 + \sigma_2^2 - 2\rho \sigma_1 \sigma_2} - 1)}{(s_1 - s_0)^2}$$

(8)

The variance of the rate of return is affected by the sum of the two variances whereas the expectation is affected by the difference in the variances. Positive correlation in the shocks reduces the variance, negative correlation increases it. With perfectly positively correlated shocks and identical variances, the internal rate of return has zero variance: with equal shocks for both schooling options, the shocks have become irrelevant.

### 3.2. A framework for simulation

If we add a quadratic experience term, as commonly estimated, and allow for annual shocks instead of a single lifetime shock, possibly correlated over time, the solution can no longer be derived analytically. We must then resort to numerical solutions. We evoke the flavor of real life choices by performing simulations for individuals who may leave school after completing high school or continue their education in college. The earnings functions are:

$$\ln w_{HS,t} = \beta_{1,HS} s_{HS} + \beta_{2,HS} (t - s_{HS}) + \beta_{3,HS} (t - s_{HS})^2 + u_{HS,t}$$

(9)

$$\ln w_{C,t} = \beta_{1,C} s_{C} + \beta_{2,C} (t - s_{C}) + \beta_{3,C} (t - s_{C})^2 + u_{C,t}$$

(10)

$\beta_{1,HS}$, $\beta_{1,C}$ are the average rates of return to $s_{HS}$ and $s_{C}$ years of schooling respectively. $\beta_{2,HS}$ $\beta_{3,HS}$ and $\beta_{2,C}$, $\beta_{3,C}$ determine the effects of experience and experience squared for an individual with $s_{HS}$ and $s_{C}$ years of schooling respectively. The errors follow AR (1) processes of the form:

$^5$We approximate $V \ln (\exp x)$ by $[\ln E (x)]^2 \ V(\exp x)$. 

11
\begin{align*}
u_{HS,t} &= \gamma_{HS} u_{HS,t-1} + \eta_{HS,t} \\
u_{C,t} &= \gamma_{C} u_{C,t-1} + \eta_{C,t}
\end{align*}

We suppress the individual subscript as we only deal with the perspective of a single individual. We assume that $\eta_{t,HS}$ is i.i.d. $N(0, \sigma_{HS}^2)$ and $\eta_{t,C}$ is i.i.d. $N(0, \sigma_{C}^2)$. We study the case when $u_{t,HS}$ and $u_{t,C}$ are uncorrelated at any $t$, as well as the case when $u_{t,HS}$ and $u_{t,C}$ correlate at $\rho_{HS,C}$ for equal experience $t=t'$ and at zero otherwise.

The inter-temporal correlations are:

\begin{align*}
\rho(u_{HS,t}, u_{HS,t-1}) &= \gamma_{HS}^2 \\
\rho(u_{C,t}, u_{C,t-1}) &= \gamma_{C}^2
\end{align*}

It may seem that with this specification we only allow for transient shocks during working life and not for permanent shocks emanating from effectiveness in school. As in section 3.1, one might think of a specification with uncertain effectiveness of schooling reflected in a single lifetime shock revealed upon completion of schooling, combined with annual earnings shocks (cf. Chen, 2003). However, only the lifetime shock from one schooling level relative to the other is relevant. The shocks may be perfectly correlated indicating that the individual would do as well in one education as in the other, as in a model with hierarchical ability. Or they may be perfectly negatively correlated reflecting perfect comparative advantage: being the best in one education would concur with being the worst in the other. The essence of such cases can be caught in the correlation between annual innovations $\eta$: comparative advantage would be reflected in negative correlation and hierarchical ability in positive correlation. Our specification can therefore describe the options to a large extent. If $\rho_{HS,C} = +1$ talent is something like a general ability that puts an individual in the same performance rank with every education he pursues, whereas at $\rho_{HS,C} = -1$ two different educations completely reverse the individual’s standing.
We consider a working life span $T$ of 40 years, independent of the length of schooling. An individual record consists of 40 draws of the disturbance term $u_{t,HS}$ used to predict earnings with $s_{HS}$ years of education for fixed values of $\beta_{i,HS}(i \in \{1,2,3\})$, and 40 draws of the disturbance term $u_{t,C}$ from an alternative distribution with $s_C$ years of education, added to predicted earnings from the associated $\beta_{i,C}(i \in \{1,2,3\})$ for that education. For such an individual record, we solve numerically for the internal rate of return $\delta$. This process is repeated 100,000 times, with 100,000 new sets of draws for the two earnings profiles. We then calculate mean and standard error of $\delta$ from the 100,000 runs. We repeat this for several sets of parameter values. As explained in the Appendix, we rewrote the stochastic specification for easier computation.

### 3.3. Parameter values

In an Appendix we scan the empirical literature for the possible magnitudes of our parameters. For the return to a year of schooling we assume a rate of 0.065 throughout, without alternatives. This implies that our benchmark internal rate of return is 0.065, the rate that would result in a Mincer world. For the experience profile we take a linear term of 0.05 and a quadratic term of -.001 as our reference values. As an alternative, we set the quadratic term for high school at -.002, maintaining the college quadratic term at -.001; this means that the decline of earnings growth with experience for college education is half the decline for high school education.

Our reading of the evidence indicates that residual earnings standard deviations are generally between 0.25 and 0.65; we take that as our range of variation, with the basic reference value in the middle: 0.45. We allow the residual variance for college earnings to be larger than for high school earnings, not smaller. For the persistence term $\gamma$ we use 0.6 as our preferred value; we will allow variation to vary the relative weight of the innovation in the earnings

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6 Available on our website: http://www1.fee.uva.nl/scholar/mdw/hartog/main.htm
process. We are fairly confident that these are reasonable values, based upon our reading of the empirical literature\(^7\). We are least confident about the correlation across educations, simply because there is no empirical evidence to guide us, in spite of all the emphasis it gets in the self-selection literature. Willis and Rosen (1979), who started this literature, could not identify the correlation. Carneiro, Hansen and Heckman (2003) provides probably the first empirical evidence and their results suggest modest positive correlation for college compared to high-school. We opt for 0.5 as our preferred value, but also consider the extremes of –1 and + 1. We set \( s_{HS}=0 \) and \( s_{C}=4 \), thus calculating the return to university education after completing high school.

### 3.4. Simulation results

In \emph{Table 3}, we compare our analytical approximations for the simplified case (linear experience profile, single lifetime shock) with simulation results. As the table shows, there is a very small difference between the approximation of \( E(\delta) \) from (7) and the value found in the simulations. The difference results from the finite length of the working life \( T \) in the simulations, compared with the assumption of infinite working lifetime in formula (7). The results bear out that the expected rate of return is sensitive to differences in dispersion between alternative educations. \( E(\delta) \) neatly increases in step with the difference between the variances. Equation (8) suggests that the dispersion in the rate of return is more sensitive to the level of earnings dispersions than to the difference in the earnings dispersion. This is indeed what the simulations also show. However, the approximation in (8), based on a first-order Taylor expansion appears quite crude, and unreliable to indicate the magnitude of the dispersion.

\(^7\) By taking the parameter values as we found them in the literature, without correction for selectivity or heterogeneity, we assume full ignorance on the position in future distributions. If individuals have better information, their risk will be reduced. This may be reflected in variances near the low end of the intervals, and possibly even lower (as the observed values would be biased).
The core results are collected in Table 4. The first row gives the reference case we defined above: 0.6 persistence, 0.5 cross-education correlation, and identical residual correlations of 0.45. Moving from a risk-less world to stochastic earnings profiles increases the expected rate of return, from 0.065 to 0.071 in our benchmark parameter set, and generates a standard deviation of 0.031, i.e. a coefficient of variation just under 0.5. In Figure 1a, we have graphed the entire frequency distribution of 100,000 draws. Interestingly, the distribution is skewed to the right, with an elongated upper tail. With individuals generally not only caring for risk but also for skewness, this is an interesting observation (cf. Hartog and Vijverberg, 2002). The degree of skewness varies with the parameters. In Figure 1b we show the case with the most skewed distribution in our parameter set, obtained when we set the coefficient of correlation at −1 rather than our reference value of 0.5.

**Figure 1.** The distribution of internal rates of return

\[ \alpha_{\text{HS}}=0.5, \sigma_{\text{HS}}=0.45, \gamma_{\text{HS}}=0.6; \beta_{1,\text{HS}}=\beta_{1,C}=0.065, \beta_{2,\text{HS}}=\beta_{2,C}=0.05, \beta_{3,\text{HS}}=\beta_{3,C}=-0.001 \]
As anticipated in equation (4), differences in earnings profiles have a strong effect on expected returns. A percentage point difference in the linear term boosts the return by almost three percents, cutting the decline in earnings growth for college in half relative to high school boosts it by almost four percents, in both cases without noticeable effect on the dispersion.

Increasing the standard deviations in both earnings profiles simultaneously has a smaller effect on expected returns than creating a difference between them. If both standard deviations increase by 0.20, from 0.45 to 0.65, the expected return increases by 0.008. If college standard deviation surpasses high school standard deviation by 0.20, as in the case (0.45; 0.25), returns are 0.014 higher than in the standard case. This reflects the conclusion from equation (7) that expected returns are sensitive to the difference in variances. Also in line with this approximation we see that increasing the base standard deviation (high school) reduces the expected return, while increasing the standard deviation in the extended education (college) increases the expected return. But the results in the variance panel of Table 4 also indicate that the approximation in equation (7), based on linear profiles and infinite lives, is too simple. At the same variance difference as in the case (0.45; 0.25), the case (0.65; 0.45) generates a much higher expected return, of 0.101 rather than 0.085. This brings out an important result: stochastic properties of earnings profiles have a strong impact on expected returns. Of course, earnings variances also markedly influence the dispersion of
the rate of return. The variance rule brought out in the approximation of equation (8) holds up quite well: the variance of the rate of return increases in the sum of the variances of the earnings profiles. In Table 4, the standard deviation of the rate of return increases monotonically from 0.014 in the case (0.25; 0.25), at a sum of variances of 0.15, to 0.057 in the case (0.65; 0.65), at a sum of 0.85, with the cases of unequal standard deviations smoothly fitting in. The effect of the earnings dispersions is quite strong: increasing both standard deviations from 0.25 to 0.65 increases the standard deviation of the return more than fourfold.

If correlations over time (persistence $\gamma_i$, $i\in\{HS,C\}$) increase jointly, expected returns go up but the dispersion increases non-negligibly. This reflects that although the variance of $u$ is itself unaffected (we constrain it to be constant), the conditional variance (conditional on the past draw) goes up. If we only vary one of the inter-temporal covariances, the dispersion of the rate of return increases in either case. But the expected return reacts in opposite ways, increasing with high school correlation but decreasing with college correlation.

Correlation across educations has a monotonic effect on expected return and dispersion. Both decline when the correlation increases from $-1$ to $+1$. But the effect on dispersion is much stronger than on expected return. Positive correlation dampens stochastic differences, negative correlation widens them. With perfect positive correlation, the standard deviation is about half that in our reference case (correlation 0.5), with perfect negative correlation the standard deviation is almost double that in the reference case.

Now let’s assess the likely magnitude of risk involved in investing in schooling. In our basic Mincer case the internal rate of return is 0.065, with zero dispersion. In what we consider a realistic case, college education would give access to steeper experience profile (earnings growth slope of $-0.001$ instead of $-0.002$), annual shocks would have a dispersion of 0.45 for both educations, persistence would be 0.60 in both educations and the shocks would correlate at 0.50. This would generate an expected rate of return of 0.110, with standard deviation
0.031 (coefficient of variation 0.28). Hence for the case of college versus high school education, we consider a coefficient of variation in the rate of return of about 0.3 our most reasonable guess. But given the uncertainty about parameter values, we have to admit a wide range of possible outcomes. In our simulations, the standard deviation lies between 0.014 and 0.084. The lowest value is obtained when the innovations in the earnings have both minimum standard deviation (0.25), the highest value is obtained when the persistence in both earnings shocks is at the high value of 0.8. In the former case, at the lowest dispersion, the coefficient of variation is 0.21, in the latter case it is 1.14.

4. Conclusions

As individuals commonly care not only about the expected return to an investment in education, but also about its risk, we have investigated what the magnitude of this risk may be. The existing literature does not point to a simple universal relation between earnings variance and level of education. To the extent that (residual) earnings variance by education reflects an individual’s ex ante ignorance on her post-school position in an earnings distribution, we cannot say whether continued education increases or decreases “ignorance”, or risk. Ex post realizations of Mincerian rates of return to education show fairly wide variation across countries (up to double or triple in one country relative to another, with coefficient of variation of perhaps 0.5), modest variation over time within countries (with a country’s maximum generally not more than a third above it’s minimum, in a time series) and coefficient of variation across individuals within a country of perhaps 0.5. To the extent that the results also reflect individual heterogeneity, and individuals are better informed about their potential, individual risk may be smaller than reflected in these ex post realisations.

From our simulations of ex ante risk we conclude that a coefficient of variation of about 0.3 is a reasonable guess. Comparing to the NYSE portfolio returns mentioned in the introduction, this suggests that investment in a college education is similar to investing in the stock market,
with a portfolio of some 30 randomly selected stocks. The distribution of the internal rate of return is skewed to the right, with an elongated upper tail.

The standard deviation of the rate of return is quite sensitive to the sum of the variances of the alternative earnings profiles and to the correlation in the shocks. If the standard deviations in the earnings innovations increase from their joint low of 0.25 to their joint high of 0.65, the standard deviation of the rate of return increases fourfold. If the correlation increases from −1 to +1, the standard deviation increases more than threelfold.

Unintentionally, we have found substantial effects on the expected rate of return. Differences in earnings growth rates for different educations can easily bring an extra 4 percent return. While obvious, this effect is routinely overlooked. Less obvious, just introducing stochastic components in earnings profiles has a marked effect on the expected rate of return. When in the risk-less Mincer world the rate of return would be 0.065, in our reference case it has an expected value of 0.071. With increasing differences in shock distributions between the alternatives, the difference can easily increase to several percentage points.

We conclude that indeed investment in human capital carries a substantial risk and therefore, risk aspects in human capital are worthy of further research. Since we also know that individuals are generally risk-averse, in different degrees (Hartog, Ferrer-i-Carbonell and Jonker, 2002), it’s clear that an interesting agenda is waiting.
References:


Appendix. The simulation procedure

Our simulation problem is stated for certain values of the variance in errors: $\sigma^2_{u_{HS}}$, $\sigma^2_{u_{c}}$, correlation between errors: $\rho_{HS,c}$, and inter-temporal correlations: $\gamma_{HS}$, $\gamma_{C}$. Therefore, firstly we set the values of these parameters in a vector $p = (\sigma^2_{u_{HS}}, \sigma^2_{u_{c}}, \rho_{HS,c}, \gamma_{HS}, \gamma_{C})$.

Given this targeted structure, we construct the errors $u_{HS,t}$ and $u_{c,t}$ from 0 to 40, as follows:

-at time $t=0$:

$$u_{HS,0} = \eta_{HS,0}, u_{C,0} = \eta_{C,0}$$

where $\eta_{HS,0}, \eta_{C,0}$ independent and normally distributed $N(0, \sigma^2_{u_{HS}})$ and $N(0, \sigma^2_{u_{c}})$, respectively.

-at time $t$, $1 \leq t \leq 40$,

$$u_{HS,t} = \gamma_{HS} u_{HS,t-1} + \eta_{HS,t} \quad (I)$$
$$u_{C,t} = \gamma_{C} u_{C,t-1} + \eta_{C,t} \quad (II)$$

where the innovations $\eta_{HS,t}, \eta_{C,t}$ are independent, normally distributed $N(0,\sigma^2_{u_{HS}})$ and $N(0,\sigma^2_{u_{c}})$ respectively. The variances of the innovations are obtained from $\sigma^2_{u_{HS}} = (1 - \gamma^2_{HS})\sigma^2_{u_{HS}}$ and $\sigma^2_{u_{c}} = (1 - \gamma^2_{C})\sigma^2_{u_{c}}$. The correlation between the errors $u_{HS,t}$ and $u_{c,t}$ can be set by controlling the correlation between the innovations $\eta_{HS,t}, \eta_{C,t}$. In order to do this, the innovations are generated using four scalars $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and three independent random variables $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ normally distributed $N(0, \sigma^2_{\varepsilon_i})$, $i = 1,3$, such that:

$$\eta_{HS,t} = \lambda_1 \varepsilon_1 + \lambda_2 \varepsilon_2 \quad (III)$$
$$\eta_{C,t} = \lambda_1 \varepsilon_1 + \lambda_2 \varepsilon_3 \quad (IV)$$

Taking variances in (I), (II), (III), and (IV) we write the equations system (we dropped the time subscripts):

23
\[
\begin{cases}
\sigma_{u_m}^2 = \gamma_{HS}^2 \sigma_{u_m}^2 + \sigma_m^2 \equiv \gamma_{HS}^2 \sigma_{u_m}^2 + \lambda_1^2 \sigma_{\varepsilon_1}^2 + \lambda_2^2 \sigma_{\varepsilon_2}^2 \\
\sigma_{u_c}^2 = \gamma_{C}^2 \sigma_{u_c}^2 + \sigma_c^2 \equiv \gamma_{C}^2 \sigma_{u_c}^2 + \lambda_3^2 \sigma_{\varepsilon_2}^2 + \lambda_4^2 \sigma_{\varepsilon_3}^2 \\
\rho_H s_{u_m} \sigma_{u_c} = \gamma_{HS} \gamma_{C} \rho_{HS,C} \sigma_{u_m} \sigma_{u_c} + \lambda_2 \lambda_3 \sigma_{\varepsilon_2}^2 
\end{cases} \quad (V)
\]

where the correlation in innovations, let us denote it \( \rho(\eta_{HS},\eta_{C}) \), satisfies

\[
\rho(\eta_{HS},\eta_{C}) \sigma_{u_m} \sigma_{u_c} = \lambda_2 \lambda_3 \sigma_{\varepsilon_2}^2.
\]

With the parameters initially set in the vector \( p = (\sigma_{u_m}, \sigma_{u_c}, \rho_{HS,C}, \gamma_{HS}, \gamma_{C}) \), we want to find

the values for \( \sigma_{\varepsilon_i}, i \in \{1,2,3\}, \lambda_j, j \in \{1,2,3,4\} \) that satisfy the constraints in equations system (V).

Hence, we have to solve an over-determined equations system that has three equations and

seven unknowns \( \lambda_1, \lambda_2, \lambda_3, \lambda_4, \sigma_{\varepsilon_1}, \sigma_{\varepsilon_2}, \sigma_{\varepsilon_3} \). We have to set the four freedom degrees

(for instance \( \lambda_1, \sigma_{\varepsilon_1}, \sigma_{\varepsilon_2}, \sigma_{\varepsilon_3} \)) and solve the system for the remaining three unknowns \( \lambda_2, \lambda_3, \lambda_4 \).
### Table 1. Earnings dispersion by education and experience (age)

<table>
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<tr>
<th>Author</th>
<th>Country and Sample</th>
<th>Year of the sample</th>
<th>Measure of Income Variation</th>
<th>Education</th>
<th>Experience</th>
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<td>1972</td>
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<tr>
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<td>U.S.</td>
<td>1968-1979</td>
<td>σ(lny)</td>
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<td>U.S.</td>
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<td>1990</td>
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<td>Belzil &amp;</td>
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<td>1979</td>
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<td>Hansen</td>
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</tbody>
</table>

*Notes:* ln(y) log wages; c.v. coefficient of variation; ∪ U-shape; ∩ inverse-U-shape; +/- positive/negative effect; 0 no clear positive or negative effect; n.a. not available.
Table 2. Variation in national rates of return over time

<table>
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<tr>
<th>Country</th>
<th>Minimum Rate of Return to years of schooling and the corresponding year</th>
<th>Trend</th>
<th>Maximum Rate of Return to years of schooling and the corresponding year</th>
<th>Gap</th>
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<td>0.103 (1981)</td>
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</table>

Source: PURE studies.
Table 3. Internal Rate of Return: Analytical and Simulated Solutions for the Linear Profile: $\beta_{1,HS} \neq \beta_{1,C} = 0.065$, $\beta_{2,HS} \neq \beta_{2,C} = 0.050$, $\beta_{3,HS} \neq \beta_{3,C} = 0.00$

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<th>$\gamma_{HS}$</th>
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<th>$\sigma_{HS}$</th>
<th>$\sigma_{C}$</th>
<th>$\rho_{HS,C}$</th>
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<th>$E(\delta)$ sim.</th>
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Notes: $E(\delta)$ and $\sigma(\delta)$ stand for mean and standard deviation of the internal rate of return $\delta$. 
### Table 4. Simulation Results

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*Note: parameters have the reference value stated in the top row, unless a different value is stated.*