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Productive Government Expenditure in Monetary Business Cycle Models

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Productive Government Expenditure in Monetary Business Cycle Models

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Abstract

This paper assesses the transmission of fiscal policy shocks in a New Keynesian framework where government expenditures contribute to aggregate production. It is shown that even if the impact of government expenditures on production is small, this assumption helps to reconcile the models’ predictions about fiscal policy effects with recent empirical evidence. In particular, it is shown that government expenditures can cause a rise in private consumption, real wages, and employment if the government share is not too large and public finance does not solely rely on distortionary taxation. When government expenditures are partially financed by public debt, unit labor costs fall in response to a fiscal expansion, such that inflation tends to decline. Households are willing to raise consumption if monetary policy is active, i.e. ensures that the real interest rate rises with inflation. Otherwise, private consumption can also be crowded-out, as in the conventional case where government expenditures are not productive. The interaction between monetary and fiscal policy is thus decisive for the short-run macroeconomic effects of government expenditure shocks.

JEL classification: E62, E21, E32.

Keywords: Productive government expenditures, private consumption, distortionary taxation, monetary and fiscal policy interaction.

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1 Introduction

The recent literature on the effects of public policies on short-run macroeconomic dynamics shows two remarkable trends. On the one hand, a rather widespread consensus has emerged on the question how monetary policy is transmitted to the economy. The New Keynesian paradigm of intertemporally optimizing agents who are subject to temporary price stickiness has proved to be a workhorse capable of capturing the essential relation between monetary variables and the real economy in a transparent and empirically successful way. The monetary policy literature has consequently moved onwards to study the question of how precisely monetary policy should be conducted in this framework (e.g. Clarida et al., 1999, or Woodford, 2003). On the other hand, there is no such unanimity with respect to the other branch of macroeconomic policies, namely fiscal policy. In particular, the question how the relation between government expenditures and private activity over the business cycle should be modeled is currently open to debate.

The reason is that, unlike with monetary policy, in the case of fiscal policy there is no generally accepted model that is able to capture the way in which variations in government expenditures seem to affect the business cycle. Empirically, positive government demand shocks appear to set forth positive temporary responses of employment, wages, and private consumption (e.g. Blanchard and Perotti, 2002, Fatas and Mihov, 2002, Gali et al., 2004, Canzoneri et al., 2003). This evidence is not easily squared with the notion of intertemporally optimizing household behavior under rational expectations. With optimizing households, the increased public command over available resources that is associated with higher government demand implies an incentive to work and save more to offset the negative consequences to household wealth. Thus, increased fiscal spending should be expected to lead to a reduction in private consumption demand, increased labor supply and thus lowered real wages. In fact, this is the counterfactual theoretical prediction of the flexible price models in e.g. Baxter and King (1993) or Campbell (1994), as well as of the sticky price models in Linnemann and Schabert (2003) or Canzoneri et al. (2003).

The natural conclusion to this mismatch between theory and evidence is that wealth effects can hardly be the only channel through which fiscal policy affects the business cycle. Consequently, the literature has produced a number of ways in which wealth effects are mitigated or compensated by other mechanisms. Ravn et al. (2004a,b) suggest that wage and consumption responses to demand shocks are less likely to be negative in models where there are countercyclical markups, either as a result from habit formation with respect to individual goods or from subsistence points in household’s utility functions. Ludvigson (1996) shows that with distortionary income taxation higher government expenditures can temporarily
raise consumption through an intertemporal substitution effect if they are accompanied by a large enough increase in debt to let the tax rate decline initially, which would however imply a strong negative effect on real wages. An alternative explanation is that households, or a part of them at least, do not optimize intertemporally at all, and therefore do not cut back their consumption in response to higher government expenditures, but instead determine their consumption as a simple reaction function to current disposable income. This assumption has been used by Mankiw (2000) in a growth model, and has recently been applied to business cycle models with monetary and fiscal policy by Gali et al. (2004). They show that if a substantial fraction of households follow the ‘rule of thumb’ of choosing their consumption as a fraction of current disposable income, then higher fiscal spending can be associated with increased private consumption and real wages. However, recent results by Coenen and Straub (2005), who estimate a dynamic general equilibrium model with ‘rule of thumb’-consumers with Euro area data cast doubt on the empirical relevance of this explanation.

The present paper examines another possibility, which allows to retain the assumption of an optimizing representative agent, to argue that the actual reduction in household wealth implied by increased government expenditures may be not that large in fact. In particular, if government spending is not completely wasteful, but rather enhances the productivity of the private production sector, then it can be expected that the reduction of private wealth through taxation can be mitigated by the improvement in production possibilities. We present a sticky price model with government spending entering the private firms’ production functions with an elasticity equal to or larger than the share of government expenditures in output. The central bank’s interest rate policy is assumed to follow an active inflation feedback rule, such that the real interest rate, and thus private consumption, is closely related to inflation. Since in the case of Calvo (1983) style staggered price adjustment inflation depends on real marginal costs, which are in turn influenced by the productive contribution of government spending, the possibility of positive effects of fiscal policy shocks on consumption, wages, and employment arises. The results are shown to depend on the fraction of government expenditures that are financed by distortionary (income) taxation and, correspondingly, on the degree of debt financing. It is further demonstrated that the stance of monetary policy is not irrelevant for the effects of fiscal policy. In particular, a passive interest rate policy can revert the results, while the interaction of monetary and fiscal policy is decisive for equilibrium stability and uniqueness.

The idea that government spending is a productive input to firms’ production functions has been expounded in a large empirical and theoretical literature. Empirically, a positive output elasticity of public investment or capital has been found in many studies (e.g. As-
Theoretical consequences of this observation have been mainly discussed in the context of growth models, following the seminal contribution of Barro (1990); see Turnovsky (2000) for a survey. From the viewpoint of business cycle theory, the closest predecessor of the current paper is Turnovsky and Fisher (1995), who study the short-run effects of productive government spending in a model with endogenous labor supply and lump-sum taxation. Fisher and Turnovsky (1998) analyze the role of distortionary taxes in a model with inelastic labor supply. None of the earlier studies considers sticky prices.

The results of this paper can be summarized as follows. First, a moderate production elasticity of government expenditures is sufficient, in a model with lump-sum taxes, to generate effects that are qualitatively consistent with the empirically observed pattern of positive employment, wage and consumption responses to a fiscal shock. The result with respect to consumption depends on the relative sizes of the output elasticity of government spending and its share in output, the effect being stronger if the share is smaller than in a long-run optimum, in the sense of Barro (1990). Second, if taxes are distortionary, there is of course a trade-off between the expansionary effects of productive spending and the contractionary ones of taxation. With a continuously balanced budget, the conditions for the spending effect to prevail are extremely restrictive. Third, however, if the empirically plausible assumption of low variations of tax rates at business cycle frequencies is adopted, and fiscal spending changes are in the short-run to a considerable degree reflected in temporary debt accumulation, it appears that the results found for the lump-sum tax case go through for a large set of reasonable parameter values. Fourth, since the mechanism by which these effects occur are tightly linked to the effect of fiscal policy on costs and prices, the results depend crucially on whether the central bank sets interest rates actively or passively in reaction to inflation.

The rest of the paper is organized as follows. Section 2 presents the model. In section 3 we examine the macroeconomic responses to expansionary government expenditure shocks. Therein, we consider first, for analytical convenience, the stylized cases where government expenditures are entirely financed by either lump-sum or labor income taxation. In the final part of section 3 we then turn to the more realistic case where government receipts are jointly raised by income taxation and the issuance of interest bearing debt. In section 4 we briefly assess to role of monetary policy for the transmission of fiscal policy shocks. Section 5 concludes.
2 The model

In this section we present a simple business cycle model where government expenditures are assumed to affect aggregate production. We abstract from considering money demand, which can be interpreted as a specification of an economy which is cashless in the limit (see Woodford, 2003).

Throughout the paper, nominal variables are denoted by upper-case letters, while real variables are denoted by lower-case letters. There is a continuum of households of mass one. They have identical asset endowments and identical preferences, and are infinitely lived. A representative household maximizes the expected sum of a discounted stream of instantaneous utilities, which rises in consumption and leisure. The instantaneous utility function $u$ is further assumed to exhibit constant intertemporal elasticities of substitution and to be additively separable, for convenience. Thus, the households’ utility function is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\sigma}}{1-\sigma} - \gamma \frac{n_t^{1+\vartheta}}{1+\vartheta} \right], \quad \sigma, \gamma > 0, \vartheta \geq 0,$$

where $E_0$ is the expectation operator conditional on the time 0 information set, $\beta \in (0,1)$ is the subjective discount factor, $c_t$ is private consumption, and $n_t$ is working time. Households’ financial wealth is held in form of nominal one period government bonds $B_t$, earning a nominal interest $i_t$ in period $t$.

Households receive labor income $P_tw_tn_t$ (where $P_t$ is the aggregate price level, and $w_t$ is the real wage rate) and dividends $D_{it}$ from monopsonistically competitive firms indexed by $i \in [0,1]$. Further they face lump-sum taxes $\tau_t$ and distortionary taxes with a tax rate $\tau_d^t$ on labor income. The flow budget constraint of a representative household reads

$$B_{t+1} + P_t c_t \leq (1 - \tau_d^t) P_t w_t n_t + (1 + i_t) B_t - P_t \tau_t + \int_0^1 D_{it} di.$$

The household maximizes (1) subject to (2), and a no-Ponzi game condition, for a given initial wealth endowment. The first order conditions for the household’s problem are given by

$$\lambda_t = c_t^{-\sigma},$$

$$(1 - \tau_d^t) w_t \lambda_t = \gamma n_t^{\vartheta},$$

$$\lambda_t = \beta E_t [\lambda_{t+1} R_{t+1}/\pi_{t+1}],$$

where $\lambda_t$ is the Lagrange multiplier on the budget constraint and $\pi_t = P_t/P_{t-1}$ is the gross inflation rate. Further, the transversality condition $\lim_{t \to \infty} E_0 \beta^t c_t^{-\sigma} B_{t+1}/P_t = 0$ is required
to hold. Note that the gross nominal interest rate $R_t = 1 + i_t$ will serve as the monetary policy instrument.

The final consumption good is an aggregate of differentiated goods produced by monopolistically competitive firms indexed with $i \in [0, 1]$. The CES aggregator of differentiated goods is defined as $y_{it}^{\epsilon} = \int_0^1 y_{it}^{1-\epsilon} \, di$, with $\epsilon > 1$, where $y_t$ is the number of units of the final good, $y_{it}$ the amount produced by firm $i$, and $\epsilon$ the constant elasticity of substitution between any two differentiated goods. Let $P_{it}$ and $P_t$ denote the price of good $i$ set by firm $i$ and the price index for the final good. The demand for each differentiated good is $y_{it} = (P_{it}/P_t)^{-\eta} y_t$, with $P_t = \int_0^1 P_{it}^{1-\epsilon} \, di$. A firm $i$ produces good $y_i$ employing a technology which is homogeneous of degree one in labor and aggregate government expenditures $g_t$,

$$y_{it} = n_{it}^{\eta} g_t^{1-\eta}, \quad \eta \in (0, 1].$$

According to (6), government expenditures raise current production and the marginal productivity of labor if the elasticity $\eta$ is smaller than one. It should further be noted that total government expenditures affect the production of each individual firm in the same way. Thus, there is no congestion with regard to the use of public goods as for example considered by Fisher and Turnovsky (1998). Note that, in contrast to the latter authors, we assume that the flow of expenditures, and not the stock of accumulated government capital, enters the production function. This assumption is made purely for analytical convenience; explicitly considering stock-flow dynamics would primarily affect the timing of responses without opening up different transmission channels.

Profit maximization implies $mc_{it} = w_i/(\eta n_{it}^{1-\eta} g_t^{1-\eta})$, where $mc$ denotes the firm’s real marginal costs. Nominal stickiness is present in form of staggered price setting as developed by Calvo (1983). Each period firms may reset their prices with the probability $1 - \phi$ independently of the time elapsed since the last price setting. The fraction $\phi \in [0, 1)$ of firms are assumed to adopt the previous period’s price $P_{it} = P_{it-1}$. Firms are assumed to maximize their market value, which equals the expected sum of discounted dividends. It is well known from the literature (e.g. Yun 1996) that the first order condition for the optimal price setting of re-optimizing producers can together with the formula for the aggregate price index be log-linearized to produce the New Keynesian Phillips curve

$$\pi_t = \chi m_{ct} + \beta E_t \pi_{t+1},$$

where $\chi = (1 - \phi)(1 - \beta \phi)/\phi$, and for any variable $x_t$ the notation $\pi_t$ denotes a percentage deviation of the value of $x_t$ from its constant steady state value $x$, i.e. $\pi_t = \ln(x_t/x)$.

The public sector consists of the fiscal authority and the central bank. The fiscal authority
issues one-period bonds, and raises income taxes $P_t \tau^d_t w_t n_t$ and lump-sum taxes $P_t \tau_t$ from households, to finance an exogenous expenditure sequence $\{g_t\}_{t=0}^\infty$:

$$B_{t+1} + P_t \tau_t + P_t \tau^d_t w_t n_t = R_t B_t + P_t g_t.$$  \hfill (8)

Throughout the paper, we solely consider fiscal policy regimes that ensure government solvency (for any price level sequence). We further assume that $g_t$ satisfies $g_t = g^{1-\rho} g_{t-1}^\rho \exp(\varepsilon_t)$, where $g \geq 0$ and $\varepsilon_t$ is an i.i.d. random variable with mean zero. The central bank controls the risk-free nominal interest rate $R_t$. Following large parts of the recent monetary business cycle literature, we assume that the central bank sets $R_t$ according to a simple feedback rule, i.e., contingent on current inflation:

$$R_t = R(\pi_t), \quad \partial R_t / \partial \pi_t \geq 0, \quad R_t \geq 1.$$  \hfill (9)

Hence, the nominal interest rate is non-negatively related to the inflation rate through the elasticity $\partial R_t / \partial \pi_t$. A rational expectations equilibrium is a set of sequences satisfying the firms’ first order conditions, the households’ first order conditions, the aggregate resource constraint, a monetary policy (9), and the transversality condition, for a given sequence $\{\varepsilon_t\}_{t=0}^\infty$ and a tax regime (to be discussed below).

3 Productive government expenditures

In this section we assess the macroeconomic responses to productive government expenditures. We are particularly interested in the ability of the model to generate a rise in private consumption, i.e., a ‘crowding-in’ effect. We start with the highly stylized case where the government solely raises lump-sum taxes to finance spending. This case will be useful to disclose the main mechanisms. In the second part of this section, we realistically assume that the government does not have access to lump-sum taxes, and that government expenditures are financed by distortionary taxation. In the last part, we introduce government debt as an additional source of funds. Throughout this section we report fiscal policy effects for the fundamental (minimum state variable) solution of the model. The role of monetary policy for the transmission of fiscal policy shocks and for equilibrium solution uniqueness will be discussed in the subsequent section.

3.1 Lump-sum taxes

In this section we abstract from the issuance of bonds such that government expenditures are entirely tax financed. Consider the case where the government has access to lump-sum taxes and the fiscal policy regime is characterized by a zero tax rate on labor income, $\tau^d_t = 0$, such
that the government budget reads
\[ P_t g_t = P_t \tau_t, \]
and (4) reduces to \( w_t \lambda_t = \gamma n_t^\vartheta. \) To assess macroeconomic effects of productive government expenditures, we have to consider that fiscal policy affects the economy via two main channels: A rise in \( g_t \) tends \( i) \) to increase aggregate demand, and \( ii) \) to raise aggregate production and the marginal productivity of labor. Evidently, both effects will contribute to a rise in total output. The main question we are interested in is, however, if private consumption rises or falls in response to an expansionary fiscal policy shock.

As is well known, the equilibrium behavior of private consumption is mainly governed by the consumption Euler equation
\[ 1/\beta = E_t \left[ (c_t/c_{t+1})^{\sigma} (R_{t+1}/\pi_{t+1}) \right] \]
when financial markets are frictionless. It predicts that the growth rate of consumption rises with the real interest rate. The behavior of the latter is known to depend on monetary policy (9), which decides on whether the real interest rate rises or falls with a change in the expected rate of inflation. Given that the focus in this paper is on the impact of fiscal policy shocks, we keep the monetary stance constant in the analysis. Throughout this section, we assume that the central bank sets the nominal interest rate according to
\[ \psi \equiv \frac{\partial R_t}{\partial \pi_t} \frac{\pi_t}{R_t} > 1. \] (10)
Condition (10) ensures that monetary policy is active, i.e., satisfies the so-called Taylor-principle (see Woodford, 2003). This assumption is introduced for several reasons. First, this kind of central bank behavior has been shown by various studies to be a reasonable strategy if there exists non-negligible distortions due to price rigidities (see Woodford, 2003, or Schmitt-Grohe and Uribe, 2004). Second, it is broadly consistent with empirical evidence on recent central bank behavior (see e.g. Clarida et al., 1998, or Orphanides, 2004). Though empirical specifications of interest rate feedback rules additionally consider lagged interest rates as determinants of current interest rates, the essential common property is that the nominal interest rate is raised (in the long-run) by more than one for one with inflation.3 Third, condition (10) is helpful to deal with the problem of equilibrium solution multiplicity.

Once interest rate policy is restricted to satisfy (10), the consumption Euler equation predicts that the dynamic behavior of consumption is linked to the equilibrium sequence of inflation: Whenever inflation rises, the real interest rate and, thus, consumption growth increase, which requires current consumption to fall, i.e., to jump below its steady state value. Thus, to assess the response of private consumption to a fiscal policy shock, we have to un-

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3Note that weak interest rate responses to changes in the output-gap, which are typically found in empirical studies, do not affect the main results derived in this paper.
derstand how government spending affects the firms’ price setting behavior, which decides on the endogenous reaction of the real interest rate. This is the point where the aforementioned two channels come into play, since they exert opposite effects on firms’ marginal costs.

If government spending were not productive, the expansionary impact of government expenditures on aggregate demand could be expected to crowd-out private consumption according to standard models. In the context of dynamic general equilibrium models, the impact of government expenditures on private consumption is governed by the so-called wealth effect (see Baxter and King, 1993). An expansionary fiscal policy shock tends to raise production and, therefore, to decrease leisure. This, however, causes households to reduce consumption expenditures, as long as leisure and consumption are not assumed to be Edgeworth-substitutes. This central mechanism has been shown to be responsible for a crowding-out even if prices are sticky (see Linnemann and Schabert, 2003). Thus, regardless of the extent to which fiscal policy raises aggregate output, households will accommodate higher labor demand, which in turn tends to lower their willingness to consume.

In our sticky price model, the rise in the costs of labor, induced by higher real wages necessary to compensate households for forgone leisure (see 3 and 4), causes monopolistically competitive firms to raise their prices (see 7). Hence, in equilibrium inflation and the real interest rate will rise in response to a fiscal expansion, implying that consumption falls and returns to its initial level in the subsequent periods. If, however, government expenditures have a cost alleviating effect, it might be possible that private consumption is crowded-in. In fact, the supply effect, which is due to the assumption that government expenditures are productive and contribute to the marginal productivity of labor, can be sufficient for this. To see how government expenditures affect firms’ costs, we log-linearize the firms’ profit maximizing labor demand at the steady state,

$$\tilde{m}c_t = \tilde{w}_t + (1 - \eta)\tilde{n}_t - (1 - \eta)\tilde{g}_t.$$  \hspace{1cm} (11)

Evidently, (11) predicts that productive government expenditures have a negative partial effect on real marginal costs of firms. Yet, the total effect of government expenditures on marginal costs is not unambiguous, since any rise in aggregate demand tends to raise firms’ demand for labor and thus their costs. Therefore, the consumption response crucially depends i) on the output share of government spending $g/y$, which weights the demand effect, and ii) on the production elasticity $1 - \eta$, which weights the supply effect.

The preceding argument can more precisely be shown to hold. Log-linearizing the firms’ first order conditions at the steady state, and eliminating marginal costs (by 11) and wages...
by (3) and (4) (which imply \( \hat{w}_t = \sigma \hat{c}_t + \vartheta \hat{m}_t \)) gives the following aggregate supply constraint:

\[
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \Delta_1 \hat{c}_t + \Delta_2 \hat{g}_t, \tag{12}
\]

where \( \Delta_1 = \theta [\sigma + \vartheta + (1 - \eta)] \eta^{-1} c/y > 0, \)

\[
\Delta_2 = \theta \{ \eta^{-1} (\vartheta + 1 - \eta) [(g/y) - (1 - \eta)] - (1 - \eta) \},
\]

and \( \theta = (\vartheta + \sigma)(1 - \phi)(1 - \beta \phi)/\phi > 0. \) The coefficient \( \Delta_2 \) in the aggregate supply constraint (12) reveals that a rise in government expenditures can have a positive or a negative impact on current inflation. In particular, if the steady state government share \( g/y \) is not larger than the elasticity of aggregate production with regard to government expenditures \( 1 - \eta \), then the aggregate supply effect dominates the aggregate demand effect. In this case, inflation tends to decline with government spending, which stimulates current private consumption, given (10), as can be seen from the log-linearized consumption Euler equation \( \sigma \hat{c}_t = \sigma E_t \hat{c}_{t+1} - (\psi - 1) E_t \hat{\pi}_{t+1} + E_t \hat{\pi}_{t+1}. \) The precise condition for a crowding-in is presented in the following proposition.

**Proposition 1** Suppose that government expenditures are financed by lump-sum taxes and that monetary policy satisfies (10). Then, an unanticipated rise in government expenditures leads to a rise in private consumption and a decline in inflation if and only if

\[
(1 - g/y)(1 - \eta) + \vartheta (1 - \eta - g/y) > 0. \tag{13}
\]

**Proof.** Using the log-linearized interest rate rule \( \hat{R}_t = \psi \hat{\pi}_t \), the equilibrium conditions can be reduced to a set of sequences \( \{ \hat{c}_t, \hat{\pi}_t \}_{t=0}^{\infty} \) satisfying \( \sigma \hat{c}_t = \sigma E_t \hat{c}_{t+1} - (\psi - 1) E_t \hat{\pi}_{t+1} \) and (12), given \( \{ \varepsilon_t \}_{t=0}^{\infty} \) and \( \hat{g}_t = \rho \hat{g}_{t-1} + \varepsilon_t \). The restriction \( \psi > 1 \) implies that there is a unique equilibrium solution satisfying \( \hat{c}_t = \delta_1 \hat{g}_t \) and \( \hat{\pi}_t = \delta_2 \hat{g}_t \) (see Woodford, 2003). Applying the method of undetermined coefficients, the solution coefficients \( \delta_1 \) and \( \delta_2 \) can be shown to satisfy \( \delta_1 = (\psi - 1) \rho \theta \Delta_3/\Delta_4 \) and \( \delta_2 = - (1 - \rho) \sigma \theta \Delta_3/\Delta_4 \), where \( \Delta_3 = (y - g) (1 - \eta) - \vartheta(y \eta - y + g) \) and \( \Delta_4 = c \theta \rho (1 - \eta + \vartheta + \sigma) (\psi - 1) + \sigma y \eta (1 - \rho) (1 - \beta \rho) > 0. \) Hence, \( \partial \hat{c}_t/\partial \hat{g}_t = \delta_1 > 0 \) and \( \partial \hat{\pi}_t/\partial \hat{g}_t = \delta_2 < 0 \) if and only if (13) is satisfied. 

According to condition (13), a sufficient condition for government spending to crowd in private consumption is that \( 1 - \eta > g/y \), i.e. if the productive contribution of fiscal policy is large relative to its impact on aggregate demand. Otherwise, crowding-in can still occur if the intertemporal substitution elasticity of labor \( 1/\vartheta \) is sufficiently large. Then, households are willing to increase labor supply even for small changes in the real wage rate, such that the impact of a rise in labor demand on firms’ costs is mitigated. If \( \vartheta \) is small enough such that the cost pressure induced by the aggregate demand effect is smaller than the cost alleviating impact of the aggregate supply effect, then inflation falls and consumption rises. Evidently,
if the marginal productivity of government expenditures is equal to zero, $\eta = 1$, (13) is violated and the aggregate demand effect dominates the macroeconomic responses to a fiscal expansion. In that case, consumption falls and inflation rises.

It should be noted that the consumption and inflation responses generally have opposite signs for active interest rate policy. This result is consistent with the empirical evidence that higher government demand does not increase inflation (Fatas and Mihov, 2002, find a negative price level response). These results and accompanying responses of other macroeconomic variables are displayed in figure 1. The impulse responses are computed for parameter values which are fairly standard in the business cycle literature: $\beta = 0.99$, $n = 1/3$, $\phi = 0.75$, $\sigma = 1$, $\vartheta = 0$, $\psi = 1.5$, $\epsilon = 6$, and $\rho = 0.9$. The steady state share of government spending in output is set at the value $g/y = 0.2$, which is the sample average of the respective variable in post-war U.S. macroeconomic data. The solid lines displays the responses for a production elasticity of government expenditures $1 - \eta$ equal to 0.2 (implying that the government share in output is equal to its long-run optimum), while the dashed line refers to the case $1 - \eta = 0.1$. For comparison, the dotted lines show the case of zero production elasticity of government spending ($\eta = 1$).
For $1 - \eta$ equal to 0.2 or 0.1, the condition in (13) is satisfied for our parameterization. Accordingly, figure 1 displays a rise in private consumption by 0.14% in response to a 1% increase in government expenditures when the steady state government share equals its optimal value $1 - \eta = g/y = 0.8$ (see Barro, 1990, for the long-run optimality analysis). If government expenditures are not productive, they do not affect marginal costs, since the labor supply elasticity is assumed to be infinite, $\vartheta = 0$. Hence, marginal costs, real wages, inflation, and consumption remain at their steady state value in this case, as can be seen from the dotted lines. In any case, employment and output rise according to the expansionary impact of the demand and the supply effect. At the same time, the real wage rate does not need to fall, because despite higher employment the marginal product of labor is increased through the productivity effect of fiscal spending.

3.2 Balanced budget

In the former section it has been shown that government expenditures can reduce inflation and raise consumption alongside with employment and wages if they are productive. The reason for these macroeconomic effects, which would not obtain if government expenditures were not productive, is that firms’ marginal costs are reduced since any increase in government spending raises the productivity of labor. This result has been derived for the case where government finance is neutral. If, however, the government does not have access to lump-sum taxation, then any rise in government expenditures must be associated with higher distortionary tax rates, such that factor prices will rise. Therefore, government expenditures will have an additional cost raising potential when lump-sum taxes are unavailable.

To get an idea about the role of distortionary taxation on the macroeconomic effects of fiscal policy, we now consider the extreme case where government expenditures are solely financed by taxes on labor income, implying $\tau_t = B_{t+1} = B_t = 0$ and thus $g_t = \tau^d_t w_t n_t$. Recall that the households’ first order condition then reads $(1 - \tau^d_t)w_t c_t^{-\sigma} = \gamma n_t^d$, implying that households demand a higher real wage rate when taxes rise. Under a balanced budget regime, the tax rate $\tau^d_t$ further increases with government expenditures, $\tilde{\tau}_t = \tau^d_t + \tilde{w}_t + \tilde{n}_t$. Hence, government expenditures exert a positive effect on real marginal costs (11). This cost raising effect causes the coefficients in the aggregate supply constraint to change to

$$\tilde{\pi}_t = \beta E_t \pi_{t+1} + \Delta^d_1 \tilde{c}_t + \Delta^d_2 \tilde{g}_t,$$

where

$$\Delta^d_1 = \theta \left[ \left( (1 - \tau^d) (\sigma + \vartheta) - \tau^d + (1 - \eta) \right) \eta^{-1} c/y, \right.$$

$$\Delta^d_2 = \theta \left\{ \tau^d - \left[ \left( (1 - \tau^d) \vartheta - \tau^d + (1 - \eta) \right) \eta^{-1} \left[ (1 - \eta) - g/y \right] - (1 - \eta) \right] \right\}. $$

As can be seen from the definition of $\Delta^d_2$, government expenditures can have a positive partial
impact on current inflation even if (13) is satisfied. When, in particular, the average (steady state) tax rate \( \tau^d \) is sufficiently high, a rise in government expenditures can lead to strong adverse effects on households’ labor supply. As a consequence, the required compensation for labor input dominates the cost relieving effect of government expenditures on labor productivity, causing firms to raise their prices. Given that the aggregate demand condition, i.e., the consumption Euler equation, is unaffected by the existence of distortionary taxes, consumption declines accordingly. This result is summarized in the following proposition.

**Proposition 2** Suppose that monetary policy satisfies (10), that government expenditures are solely financed by income taxes, and that (13) is satisfied. Then, an unanticipated rise in government expenditures leads to a rise in private consumption and a decline in inflation if

\[
\tau^d < \min \{ \tilde{\tau}_1, \tilde{\tau}_2 \},
\]

where \( \tilde{\tau}_1 = 1 - \frac{(c/y)\rho(\psi - 1) - \sigma(1 - \rho)(1 - \beta \rho)}{(c/y)\rho(\psi - 1)(\sigma + \theta + 1)} \eta \), and \( \tilde{\tau}_2 = 1 - \frac{(1 - g/y)(\psi - 1)(\sigma + \theta + 1) - \eta}{\sigma + \theta + 1} \).

**Proof.** The structure of the problem corresponds to the one in proposition 1 and the solution reads \( \delta_1 = (\psi - 1) \rho \theta \Delta^d_1 / \Delta_1^d \) and \( \delta_2 = - (1 - \rho) \theta \sigma \Delta^d_1 / \Delta_1^d \), where \( \Delta^d_1 = - (y - g) (1 - \tau) (\psi - 1) + ((1 - \tau) y \theta + (y - g)) \eta \) and \( \Delta^d_1 = - \rho \theta \epsilon (1 - \tau) (\psi - 1) (\sigma + \theta + 1) - \sigma \eta (1 - \rho) (1 - \beta \rho) + c \theta \rho \eta (\psi - 1) \). Given that (13) is assumed to be satisfied, \( \Delta^d_1 < 0 \) if \( \tau^d < \tilde{\tau}_1 \) and \( \Delta^d_2 < 0 \) if \( \tau^d < \tilde{\tau}_2 \). Thus, (15) is sufficient for \( \delta_1 > 0 \) and \( \delta_2 < 0 \). \( \blacksquare \)

According to condition (15), a crowding-in requires the average tax rate \( \tau^d \) to be sufficiently low. The bounds on the tax rate thereby rise with the production elasticity of government expenditures \( 1 - \eta : \partial \tilde{\tau}_1(\eta) / \partial \eta > 0 \) and \( \partial \tilde{\tau}_2(\eta) / \partial \eta > 0 \). It should be noted that the value for \( \tau^d \) is related to the government share by \( \tau^d = (g/y) \eta^{-1} mc \), where \( mc = \frac{\tau_1 - \tau}{\tau} < 1 \). Thus, the tax rate \( \tau^d \) can be smaller or larger than \( g/y \) depending on the production elasticity \( \eta \) and the average price mark-up \( mc^{-1} \). Applying the parameter values introduced in the subsequent section leads for \( \eta = 0.2 \) to \( \tau^d = 0.3 \). Since \( \tau^d \) is then not smaller than both bounds, \( \tilde{\tau}_1 = 0.74 \) and \( \tilde{\tau}_2 = 0.2 \), consumption declines in that case. If, however, the production elasticity of government expenditures is extremely high \( 1 - \eta = 0.5 \), then \( \tilde{\tau}_1 = 0.84 \) and \( \tilde{\tau}_2 = 0.5 \) and the steady state tax rate \( \tau^d = 0.48 \) satisfies (15). Figure 2 shows the impulse responses of macroeconomic aggregates under a balanced budget regime, where the baseline parameters are the same as are underlying figure 1.

Real wages (and therefore marginal costs and inflation) now always rise due to the distortionary effect of income taxes. As a consequence, consumption declines in response to a fiscal expansion, regardless whether government expenditures are productive or not. Notably, a rise in aggregate output can, for \( 1 - \eta > 0 \), even be associated with a decline in employment due to higher income taxes.
3.3 Debt financed expenditures

In the previous sections, we assessed fiscal policy effects under two extreme government finance regimes, namely lump-sum taxes on the one hand and proportional labor income taxation under a continuously balanced budget on the other. These specifications have been found useful to unveil the main mechanisms and to facilitate the derivation of analytical results. However, none of them is empirically realistic, and the results obtained so far might therefore be considered as not applicable with respect to the analysis of real world public spending and financing behavior.

To address this issue, we now analyze the model under the assumption that lump-sum taxes are unavailable, and that the budget is not balanced in each period, but only in the long run. That is, government finance consists of two potential sources in the short run, namely a proportional labor income tax and debt. The assumption of intertemporal budget balance means that while the government may incur budget deficits over the business cycle, the level of outstanding debt must eventually be returned to its constant steady state value. Long-run debt stability requires that there must be some positive feedback between the tax rate and
the stock of debt, a condition which we interpret as a fiscal policy rule that determines the split between tax revenues and deficit finance. This formulation of the government’s options is useful as it allows to combine two aspects of government finance which should be fulfilled for empirical plausibility: first, the unavailability of lump-sum taxes, and second, the fact that income tax rates do not fluctuate greatly, at least not at business cycle frequencies.

To be explicit, consider the real version of the government budget identity, $b_{t+1} = R_t \pi_t^{-1} b_t + g_t - \tau^d_t w_t n_t$, where real government debt is $b_t = B_t / \pi_t$. The tax rate $\tau^d_t$ is assumed to be adjusted such that the real value of debt is a constant in the steady state, while in the short-run in terms of deviations from the steady state

$$\hat{\tau}^d_t = \xi \hat{b}_t, \quad \xi > 0,$$

is assumed to hold (that is, letting symbols without subscript denote steady state values, the tax rate obeys $\tau^d_t = \xi \frac{\pi}{\psi}(b_t - b) + \frac{(R/\pi - 1)b + g}{wn} \psi - (\tau^d wn / b)(\hat{\tau}_t + \hat{w}_t + \hat{n}_t)$. Though this specification is still highly stylized, it allows to examine fiscal policy effects under public finance regimes which range between the two polar cases discussed in the previous section. The feedback parameter $\xi$ thereby determines how an unanticipated increase in government expenditures is financed. A higher value for $\xi$ raises the short-run tax financed fraction of government spending, and implies that the debt level returns to its steady state at a higher speed. Thus, the feedback coefficient $\xi$ decides on whether a particular fiscal policy qualitatively behaves more like the lump-sum tax regime or the balanced budget regime. In the case of a low $\xi$ coefficient, an increase in government spending is initially largely reflected in deficits, and thus leads to temporary debt accumulation, while the tax rate response is initially low. Eventually, however, the debt level is brought back to its steady state, such that the tax rate must be higher for a prolonged period of time. This case of relatively weak and sluggish tax rate movements coupled with large deficit fluctuations is arguably more realistic from an empirical point of view than the previous section’s assumption of a continuously balanced budget.

In contrast to the previous government finance regimes, the evolution of public debt has to be taken into account in equilibrium. Thus, the log-linearized government budget,

$$\hat{b}_{t+1} = (R/\pi) \hat{b}_t + (R/\pi)(\psi - 1) \hat{\pi}_t + (g/b) \hat{g}_t - (\tau^d wn / b)(\hat{\tau}_t + \hat{w}_t + \hat{n}_t),$$

is added to the equilibrium conditions. Since the set of equilibrium conditions cannot be reduced further than to three equilibrium difference equations, i.e., to (14), (17) and the consumption Euler equation, we apply numerical methods to assess the responses to fiscal policy shocks. It should be noted that there is no simple relation between inflation and consumption in this case, since the model exhibits an endogeneous state variable, i.e., real government debt.
Due to this property a decline in the consumption growth rate, which is induced by a decline in the inflation rate and thus the real interest rate, does not necessarily imply current consumption to rise. The latter has been a requirement for a stable consumption sequence in the previous cases, where the fundamental solutions do not exhibit an endogenous state variable. Figure 3 shows the responses of the model’s endogenous variables to a government spending shock for the case where the feedback parameter in the tax rule $\xi$ is set equal to 0.1 (this is roughly the smallest value that prevents the explosion of debt, and therefore implies the lowest possible tax rate response that is still consistent with debt stability). Evidently, the responses are closely related to the case, where government expenditures are financed by lump-sum taxes. In particular, the impact response of consumption to a one percent rise in government expenditures equals 13%, while it equals 14% in the lump-sum tax case. It should be noted that the impact response of consumption for the case where government expenditures are not productive, $\eta = 0$, is slightly negative, even though inflation declines on impact. Nevertheless, the associated decline in the real interest rate induces a decline in the consumption growth rate.

To get an impression about the role of the fiscal feedback parameter, figure 4 shows the
effect of varying $\xi$ between 0.1 to 0.4. As can be seen from figure 4, such a variation does not qualitatively affect the impulse responses. As can be expected from the results in the previous sections, a rise in the feedback from debt to taxes lowers the expansionary impact on private consumption, since a larger fraction of government expenditures is then financed by distortionary taxation. It should further be noted that changes in the feedback parameter $\xi$ are not harmless with regard to equilibrium determination. As shown by Linnemann (2005), the likelihood for equilibrium solution multiplicity increases with $\xi$. The issue will be briefly addressed in the subsequent section; here, it suffices to say that the responses in the figure are obtained as the model’s minimum state variable solution in each case.

4 Interactions with monetary policy

In the preceding sections, we abstracted from the impact of monetary policy on the transmission of fiscal policy shocks. In particular, we assumed that the central bank is active, i.e., satisfies (10). This assumption necessarily implies the impact responses of inflation and consumption to be inversely related in the case where government expenditures are entirely tax financed. If, however, monetary policy is allowed to be passive $\psi < 1$, there is no unambiguous link between inflation and consumption. According to the fundamental equilibrium
solution, which has been the unique solution under an active policy (see Woodford, 2003),
consumption and inflation can also be positively related. A rise in inflation is then associated
with a decline in the real interest rate, inducing households to reduce savings and to increase
current consumption.

Remark Suppose that government expenditures are financed by lump-sum taxes and that (13)
is satisfied. If monetary policy is passive, \( \psi < 1 \), then a unanticipated rise in government
expenditures can lead to a decline in private consumption and to a rise in inflation.

The fundamental solution under a lump-sum tax regime, is characterized by \( \delta_1 = - (1 - \psi) \rho \theta \Delta_3 / \Delta_4 \)
and \( \delta_2 = - (1 - \rho) \sigma \theta \Delta_3 / \Delta_4 \) (see proof of proposition 1), implying that the impact response
of consumption and inflation are positively related for a passive policy \( \psi < 1 \). For (13)
\( \Delta_3 > 0 \), while \( \Delta_4 = c \theta \rho (1 - \eta + \vartheta + \sigma) (\psi - 1) + \sigma \eta (1 - \rho) (1 - \beta \rho) \) can be negative if
\( \rho \) is sufficiently high. If \( \rho \) is small enough such that \( \Delta_4 > 0 \), then consumption and in-
flation decline in response to a fiscal policy shock. Otherwise, \( \sigma \eta (1 - \rho) (1 - \beta \rho) / \rho < c \theta (1 - \eta + \vartheta + \sigma) (1 - \psi) \), consumption and inflation rise.

Thus, a passive monetary policy can in principle revert the results derived in the previ-
ous section. Yet, the fundamental solution is not the only stable equilibrium solution under
a passive interest rate policy. In particular, there exist multiple solutions featuring an ex-
traneous endogenous state variable. For example, there exist stable solutions which allow
for non-fundamental shocks to affect the allocation and the equilibrium price system, as
for example demonstrated by Clarida et al. (2000). In any case, the relation between the
responses of inflation and private consumption to fiscal policy shocks is in general not unam-
biguous within the set of feasible equilibrium solutions. Exact predictions for the response of
macroeconomic aggregates to fiscal policy shocks, therefore, rely on the existence of a unique
equilibrium solution.

In the case of distortionary taxation and temporary debt finance, uniqueness of stable
equilibrium sequences does not only depend on the conduct of monetary policy. Since ag-
gregate supply is affected by government finance, the stance of fiscal policy also matters for
equilibrium determination. Thus, the interaction between the monetary and fiscal policy is
decisive for equilibrium determinacy.\( ^4 \) Generally, the larger the tax rate reaction to debt (\( \xi \in \) our notation) is, the more active must monetary policy be, to choke off any inflationary
tendencies that emerge from rising factor prices in the event that tax rates increase to stabil-
ze the debt level (see Linnemann, 2005). This implies that the fundamental solution, which

\footnote{It should be noted that this form of interaction does not rely on so-called non-Ricardian fiscal policy
regimes, which are for example applied in Woodford (1994) and Sims (1994) for analysis of price level deter-
mination, and in Benhabib et al. (2001) for equilibrium determination.}
we applied throughout the analysis, might not be the unique stable solution for high value for $\xi$.

5 Conclusion

Are government expenditures just a pure loss of resources to the private sector? If this were true, it would be difficult to explain why periods of unexpectedly high fiscal spending are empirically associated with increases in private consumption and real wages. The present paper has argued that if government expenditures are channelled into socially productive uses in the way that they increase the production possibilities of private firms, the empirical evidence on the effects of fiscal shocks is easy to explain. In particular, the productive contribution of government spending can raise labor productivity and thus lead to cost savings on the part of firms to an extent that a fiscal shock lowers inflation. If this is the case, an active central bank would lower the nominal interest rate enough to decrease the real rate, such that private consumption could increase. Interestingly, this effect is stronger if the output elasticity of government spending is larger than the steady state value of government’s share in output. Since equality between these two parameters is an efficiency condition that marks the equality of the social marginal costs and benefits of government expenditures, it follows that - given the aggregate production function - fiscal policy would be more effective in a situation with an inefficiently small scale government sector.

Of course, the empirical evidence on the effects of fiscal spending shocks can also be explained through a variety of other mechanisms. For example, the assumption that a sizeable number of households abstain from intertemporal optimization and consume out of current income has been shown by Gali et al. (2004) to lead to qualitatively identical predictions with the model of productive government spending presented here. Thus, a priority for future research should be to apply empirical methods to discriminate between the competing theories, or to determine the degree to which these approaches (and possibly others) reinforce each other.
6 References


