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A DSGE model for a SOE with Systematic Interest and Foreign Exchange policies in which policymakers exploit the risk premium for stabilization purposes[1]

Guillermo J. Escudé
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Abstract: This paper builds a DSGE model for a SOE in which the central bank systematically intervenes both the domestic currency bond and the FX markets using two policy rules: a Taylor-type rule and a second rule in which the operational target is the rate of nominal currency depreciation. For this, the instruments used by the central bank (bonds and international reserves) must be included in the model, as well as the institutional arrangements that determine the total amount of resources the central bank can use. The 'corner' regimes in which only one of the policy rules is used are particular cases of the model. The model is calibrated and implemented in Dynare for 1) simple policy rules, 2) optimal simple policy rules, and 3) optimal policy under commitment. Numerical losses are obtained for ad-hoc loss functions for different sets of central bank preferences (styles). The results show that the losses are systematically lower when both policy rules are used simultaneously, and much lower for the usual preferences (in which only inflation and/or output stabilization matter). It is shown that this result is basically due to the central bank's enhanced ability, when it uses the two policy rules, to influence capital flows through the effects of its actions on the endogenous risk premium in the (risk-adjusted) interest parity equation.

JEL classification: E58, F41, O24

Keywords: DSGE models, Small Open Economy, Exchange rate policy, Optimal policy

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1. Introduction

According to John Williamson 'the overwhelming conventional view in the profession is that it is a mistake to try to manage exchange rates' (J. Williamson (2007)), although he does not subscribe this view. After having for a long time recommended a basket, band, and crawl (BBC) regime, Williamson lately confesses to have converted to the cause of inflation targeting, but with some significant additional ingredients: 'most of the time the only monetary policy objective that may merit consideration -other than inflation targeting- is the maintenance of a sufficiently competitive exchange rate to preserve the incentive to invest'... (in tradable sectors). He also argues that 'the government can expect to reduce misalignments by a policy of intervention. The question is how those interventions should be structured: whether they should be ad-hoc or systematic and, if the latter, how the system should be designed.' This paper attempts to deal with these issues in a novel way, integrating the usual 'inflation targeting' (or Taylor rule) approach with a policy of systematic intervention in the foreign exchange market.

In my view there is no justification for having to choose between an inflation target anchor and an exchange rate target anchor. But it is by no means easy to escape this dichotomy in the absence of an accepted and adequate theoretical framework. My hunch is that this absence is due to the pervasive preference of modelers (theoreticians) to 'sweep under the rug' some of the Central Bank 'nuts and bolts' that are necessary to achieve a more general theory. Such 'nuts and bolts' as the Central Bank balance sheet (and the financial assets and liabilities within it), are detailed and analyzed in any IMF Article IV mission report pertaining to developing countries. However, when it comes to modeling the macroeconomy. such aspects are simply omitted in both academic and IMF models. What makes such an omission possible, of course, is that if we accept the dichotomy in question, an argument of system decomposability allows one to focus on the central block of equations. However, if we do not accept the dichotomy, the need to include such 'nuts and bolts' arises merely to ensure a consistent policy model.

This paper, and the model on which it is based, attempts to build such a consistent policy model. Using the model with various policy frameworks (simple policy rules, optimal simple policy rules, optimal policy under commitment) and implementing a first order approximation using Dynare, I find strong evidence that a proper systematic use by Central Banks (CBs) of small open economies (SOEs) of two policy rules, one for the nominal interest rate and another for the rate of nominal depreciation, outperforms the 'corner' regimes of inflation targeting (floating exchange rate) and an exchange rate peg. The basic difference between the model used here and the workhorse DSGE model of the profession is the inclusion of more detail in the modeling of the institutional structure that takes us closer to a formal representation of how most CBs (at least those in developing economies) implement their interest and foreign exchange policies. However, as far as I am aware no CB implements its FX policy the way that it is modeled in this paper. When FX policy is systematic, there tends to be an exchange rate-related target. And when there is an explicit inflation targeting framework, FX policy tends to be highly discretional. One of the conclusions of this paper is that it is perfectly possible to articulate a consistent model which conserves the systematic interest rate policy rule that prevails in the literature (Taylor rule models) yet incorporates

an additional policy rule to represent FX policy. Furthermore, the paper shows that when optimal simple rules or optimal policy under commitment are introduced through an ad-hoc CB loss function, significant gains are obtained using two policy rules (or two control variables) for all the usual CB preferences (i.e. combinations of weights for inflation and output).

The model used for this paper, ARGEMmin (a smaller version of two previous models: Escudé (2008) and Escudé (2009)), can represent the simultaneous (i.e. within the same quarterly period) intervention in the foreign exchange (FX) and the domestic currency bond markets. The simultaneous use of two policy rules is a generalization of standard models that are limited to having either a Taylor rule for the interest rate with a pure currency float or a pure pegged regime in which there is usually no feedback. The fact that most CBs of developing economies intervene regularly in both markets should make this generalization of practical interest.[2] And a model that only adds the essential features that are needed to include foreign exchange policy without excluding interest rate policy should help in obtaining intuition as to why the CB can better achieve its objectives, whatever they may be, by the use of two policy rules instead of one. It is shown that the gains the CB obtains using the two instruments are basically due its increased ability to exploit the foreign investors' risk premium function that constrains the domestic household's optimal foreign debt decision.

The household decision problem delivers the risk-adjusted uncovered interest parity (UIP) equation.[3] The use of an endogenous risk premium function that Rest of the World (RW) agents use to determine the interest rate at which they are willing to purchase the economy's foreign currency bonds plays a fundamental role in the model's dynamics of capital flows. The use of a risk premium for foreign debt has a long history in open economy macroeconomics (see e.g. Bhandari, Ul Haque and Turnovsky (1990)). In the DSGE strand, Schmitt-Grohé and Uribe (2003) note that the simplest SOE models with incomplete asset markets use the assumption that the subjective discount rate equals the average real interest rate and, hence, present equilibrium dynamics that have a random walk component. They present five alternative modifications that have been used to eliminate this random walk component and show that they have quite similar dynamics. Among these modifications is the complete assets market model (i.e., doing away with the incomplete asset markets assumption altogether) and, more relevant for this paper, the use of a risk premium function by which the interest rate on foreign funds responds to the amount of debt outstanding. In the latter variant, combining the non-stochastic steady state (NSS) versions of the Euler and UIP equations gives

[2] IMF (2011), for example, notes that 'on average about on-third of the countries in the region (Latin America) intervened in any given day'. Indeed, their Table 3.1 (Stylized facts of FX Purchases, 2004-10) shows that Colombia and Peru intervened in 32% and 39% of working days, respectively. This table also contains interesting information on other regions: in the same period, Australia and Turkey intervened in 62% and 66% of working days, respectively, while Israel intervened 24% of working days but with a cumulative intervention that represented 22.3% of GDP.

[3] This differs from my two previous (and larger) models, where it was the decision of banks that delivered the model's UIP equation. The simplification in this paper seeks to obtain a model that is sufficiently close to the standard workhorse model that the specific difference in modeling policy is highlighted.

an equation such as $\beta(1+i^*)\varphi_D(d)=\pi$, where β is the intertemporal discount factor, i^* is the RW's NSS real interest rate, π is the SOE's inflation rate, d is the SOE's foreign debt and $\varphi_D(.)$ is a risk premium function. This equation then determines d as a function of model parameters (including those that define the risk premium function $\varphi_D(.)$ and the policy target that defines π). Lubik (2007) adds that even if there is an exogenous risk premium function, to avoid the unit root problem it is necessary that it be fully internalized by the individual households, i.e., that each household take into account that other households' decisions are the same as its own and, hence, that the risk premium it faces is a function of the aggregate (and not its individual) foreign debt. The only significant change that this paper presents with respect to such a risk premium is that $\varphi_D(.)$ is a function of the foreign debt to GDP ratio: ed/Y (where e is the SOE's real exchange rate (RER) and Y is its GDP) and that there is an additional multiplicative shock ϕ^* (giving $\phi^*\varphi_D(.)$) that may represent either an exogenous component of the risk function or an international liquidity shock (or both).[4]

Simply for convenience, I call the policy framework where the CB uses two simultaneous policy rules a Managed Exchange Rate (MER) regime. I explicitly include the instruments that the CB uses for its intervention in the two markets as well as the CB balance sheet that binds them. Hence, the CB balance sheet is one of the model equations. It has cash m_t and CB-issued domestic currency bonds b_t on the liabilities side, and foreign currency reserves r_t on the asset side. To make sure that there are no loose ends, I explicitly consider the CB's flow budget constraint and assume that the institutional framework is such that any 'quasi-fiscal' surplus (or deficit) is handed over (financed) period by period to the Treasury, defining 'quasi-fiscal surplus' as financial flows (specifically, those related to interest earned and capital gains on international reserves, and the interest paid on CB bonds) that could make the CB net worth different from zero. Hence, while there is overall fiscal consistency (since the Treasury is assumed to be able to collect enough lump-sum taxes each period to finance its expenditures in excess of the qusi-fiscal surplus), the CB has a constraint each period on its two instruments (r_t and b_t): $e_t r_t = m_t + b_t$, where e_t and m_t are the real exchange rate (RER) and real cash held by households. This equation implicitly defines how much the CB 'sterilizes' (through the issuance of domestic currency bonds) any unwanted monetary effect of its simultaneous and systematic monetary and exchange policy. However, I avoid the expression 'sterilized intervention' (in the foreign exchange market) because it implicitly gives the exchange rate policy a subordinate role (the undesired effects of which must be 'sterilized' to avoid disrupting the monetary equilibrium that is achieved through the use of conventional monetary policy). Generality is best preserved treating both interventions in a symmetrical way, neither of which 'sterilizes' the effects of the other. When the CB intervenes in both the money and foreign exchange market, it is subject to the set of constraints given by the equations of the model, among which is monetary equilibrium and the assumed institutional constraint

[4] In addition to ϕ^*, there are three more RW shocks that impinge on the SOE: the world nominal riskfree interest rate $1+i^*$ and the rates of inflation of imported and exported goods. There are also two domestic shocks: a transitory productivity shock in the domestic output sector and a government expenditure ratio (to GDP) shock.

that the CB's net worth is kept at zero.[5] Clearly, other similar constraints could be used for the same purpose of endogenizing the CB's 'sterilization' policy. The one I use has the virtue of simplicity. The important point is that the overall means that the CB has available be made explicit. To further ensure consistency, the model includes the balance of payments (where both household foreign debt and CB reserves play relevant roles) and the fiscal equation.

Since the 2008 financial meltdown and the consequent introduction of 'unconventional' monetary policies it has become customary to stress the importance of central bank balance sheets in the sense that huge purchases of financial assets by central banks get reflected in their assets as well as their liabilities. Caruana (2012), e.g., stresses the need to start normalizing the situation before the risk of monetizing debts gets out of hand. In this paper the point is made that inclusion of the central bank balance sheet and its composition is important even in a more 'normal' world with short term interest rates that are above zero and CB assets and liabilities that are closer to normal levels. In this paper, 'normal' levels are given by the long run (i.e., the model's nonstochastic steady state) CB foreign exchange reserves ratio to GDP, and actual CB reserves fluctuate around the corresponding long run level. Hence, a return to normal levels is automatically guaranteed whenever the model is dynamically stable. But the explicit consideration of the CB's balance sheet opens the door for modeling the novel ('unconventional') types of CB monetary policies in which the CB, say, additionally intervenes in a market for long-period bonds in order to deepen its expansionary policy when the short run interest rate is at its zero lower bound. This, however, is for future research.

The rest of the paper has the following structure. In section 2 I set up the model. In section 3 I study the functioning of the model under simple policy rules, optimal simple policy rules, and optimal policy under commitment and full information (as in Svensson and Woodford (2002)) and show that there are indeed gains from using these two simultaneous policy rules instead of only one of the 'corner' regimes. In section 4 I show that such gains are basically due to the central bank's enhanced ability to influence the risk premium in the UIP equation when it uses the two policy rules. Section 5 concludes. Appendix I shows how the model parameters and the NSS were jointly calibrated. Finally, Appendix 2 shows a selection of the impulse response functions for the optimal simple rules and the optimal policy under commitment.

2. The model

2.1. Households

2.1.1 The household optimization problem

Infinitely lived identical households consume a CES bundle of domestic and imported goods and hold financial wealth in the form of domestic currency cash (M_t) and domestic currency denominated one period nominal bonds issued by the CB (B_t) that pay a nominal interest rate i_t. They also issue one period foreign currency bonds (D_t) in the international capital market that pay a nominal (foreign cur-

[5] Notice that the latter can be expressed as an institutional constraint of the CB preserving a 'full backing' of its domestic currency liabilities with (the domestic currency value of) its foreign reserves.

rency) interest rate i_t^D. I assume that the CB fully and credibly insures investors in CB bonds, so the domestic currency nominal rate is considered riskfree. However, foreign investors are only willing to hold the SOE's foreign currency bonds if they receive a risk premium over the international riskfree rate i_t^*. Since I do not model the RW, the premium function is exogenously given. It has an exogenous stochastic and time-varying component ϕ_t^* (that can represent general liquidity conditions in the international market) as well as an endogenous (more country risk-related) component $\tau_D(.)$ that is an increasing convex function of the aggregate foreign debt to GDP ratio. Individual households are assumed to fully internalize the dependence of the interest rate they face on the aggregate (instead of individual) foreign debt based on to their knowledge that all households are (at least in this aspect) identical. The foreign currency gross interest rate households face is:

$$1 + i_t^D = (1 + i_t^*)\phi_t^* \tau_D\left(\gamma_t^D\right), \tag{1}$$

where

$$\gamma_t^D = \frac{S_t D_t}{P_t Y_t} = \frac{e_t d_t}{Y_t}, \qquad e_t \equiv \frac{S_t P_t^*}{P_t}, \qquad d_t \equiv \frac{D_t}{P_t^*}. \tag{2}$$

γ_t^D, e_t, and d_t, are the foreign debt to GDP ratio, the real exchange rate, and real foreign debt (in terms of foreign prices), respectively, S_t is the nominal exchange rate, P_t is the domestic goods price index, P_t^* is the price index of the goods the SOE imports, and Y_t is GDP. I assume that the gross risk premium function $\tau_D\left(\gamma_t^D\right)$ is increasing and convex ($\tau_D \equiv 1 + \overline{\tau}_D > 1$, $\tau_D' > 0$ and $\tau_D'' > 0$).

The household holds cash M_t because doing so reduces its transaction costs. I assume that transaction frictions result in a loss of purchasing power (through the non-utility generating consumption of domestic goods) when households purchase consumption goods, and that this cost can be ameliorated using cash.[6] To purchase quantity C_t of the consumption bundle, households must spend $\tau_M\left(\gamma_t^M\right) P_t^C C_t$, where P_t^C is the price index of the consumption bundle. All price indexes are in monetary units. The gross transactions cost function $\tau_M\left(\gamma_t^M\right)$ is assumed to be a decreasing and convex function ($\tau_M \equiv 1 + \overline{\tau}_M > 1$, $\tau_M' < 0$, $\tau_M'' > 0$) of the cash/consumption ratio γ_t^M:

$$\gamma_t^M \equiv \frac{M_t}{P_t^C C_t} = \frac{m_t}{p_t^C C_t}, \tag{3}$$

where

$$p_t^C \equiv \frac{P_t^C}{P_t}, \qquad m_t \equiv \frac{M_t}{P_t} \tag{4}$$

are the relative price of consumption goods and real cash.

The representative household maximizes an inter-temporal utility function which is additively separable in (constant relative risk aversion subutility functions of) goods C_t and labor N_t:

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{C_{t+j}^{1-\sigma^C}}{1-\sigma^C} - \xi^N \frac{{N_{t+j}}^{1+\sigma^N}}{1+\sigma^N} \right\}, \tag{5}$$

[6] The introduction of money is similar to the theoretical treatment in Montiel (1999), and also to the numerically implemented treatment in Schmitt-Grohé and Uribe (2004). It differs from the latter in that instead of defining velocity I use its inverse (the cash/consumption ratio), and I use a different specification of the transactions cost function.

where β is the intertemporal discount factor, σ^C, and σ^N are the constant relative risk aversion coefficients for goods and labor, respectively, and ξ^N is a parameter.

The household receives income from profits, wages, and interests, and spends on consumption, interests, and taxes. Its nominal budget constraint in period t is:

$$\begin{aligned}\tau_M\left(\gamma_t^M\right) P_t^C C_t + M_t + B_t - S_t D_t = W_t N_t + \Pi_t - Tax_t \\ + M_{t-1} + (1+i_{t-1})\, B_{t-1} - (1+i_{t-1}^D) S_t D_{t-1}\end{aligned} \tag{6}$$

where i_t is the interest rate that CB bonds pay each quarter, W_t is the nominal wage rate, Π_t is nominal profits, and Tax_t is lump sum taxes net of transfers. Introducing (1) in (6) and dividing by P_t, the real budget constraint is:

$$\begin{aligned}\tau_M\left(\gamma_t^M\right) p_t^C C_t + m_t + b_t - e_t d_t = w_t N_t + \frac{\Pi_t}{P_t} - tax_t + \frac{m_{t-1}}{\pi_t} \\ + (1+i_{t-1})\,\frac{b_{t-1}}{\pi_t} - (1+i_{t-1}^*)\phi_{t-1}^* \tau_D\left(\gamma_{t-1}^D\right) e_t \frac{d_{t-1}}{\pi_t^*},\end{aligned} \tag{7}$$

where

$$b_t \equiv \frac{B_t}{P_t}, \qquad w_t \equiv \frac{W_t}{P_t}, \qquad tax_t \equiv \frac{Tax_t}{P_t}, \qquad \pi_t \equiv \frac{P_t}{P_{t-1}}, \qquad \pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$$

are the real stock of domestic currency bonds, the real wage (in terms of domestic goods), real lump sum tax collection, and the gross rates of quarterly inflation for domestic goods and foreign goods, respectively.

The household chooses the sequence $\{C_{t+j}, m_{t+j}, b_{t+j}, d_{t+j}, N_{t+j}\}$ that maximizes (5) subject to its sequence of budget constraints (7) (and initial values for the predetermined variables). The Lagrangian is hence:

$$\begin{aligned}E_t \sum_{j=0}^{\infty} \beta^j \Bigg\{ \frac{C_{t+j}^{1-\sigma^C}}{1-\sigma^C} - \xi^N \frac{{N_{t+j}}^{1+\sigma^N}}{1+\sigma^N} + \lambda_{t+j} \Bigg\{ w_{t+j} N_{t+j} + \frac{\Pi_{t+j}}{P_{t+j}} + \frac{m_{t-1+j}}{\pi_{t+j}} \\ + (1+i_{t-1+j})\,\frac{b_{t-1+j}}{\pi_{t+j}} - (1+i_{t-1+j}^*)\phi_{t-1+j}^* \tau_D\left(\frac{e_{t-1+j} d_{t-1+j}}{Y_{t-1+j}}\right) e_{t+j} \frac{d_{t-1+j}}{\pi_{t+j}^*} \\ - \tau_M\left(\frac{m_{t+j}}{p_{t+j}^C C_{t+j}}\right) p_{t+j}^C C_{t+j} - m_{t+j} - b_{t+j} + e_{t+j} d_{t+j} - tax_{t+j} \Bigg\}\Bigg\}\end{aligned} \tag{8}$$

where $\beta^j \lambda_{t+j}$ are the Lagrange multipliers, and can be interpreted as the marginal utility of real income.[7]

The first order conditions for an optimum are the following:

$$C_t \;:\quad C_t^{-\sigma^C} = \lambda_t p_t^C \varphi_M\left(m_t / p_t^C C_t\right) \tag{9}$$

$$m_t \;:\quad \lambda_t\left[1 + \tau_M'\left(m_t / p_t^C C_t\right)\right] = \beta E_t\left(\lambda_{t+1}/\pi_{t+1}\right) \tag{10}$$

$$b_t \;:\quad \lambda_t = \beta\,(1+i_t)\, E_t\left(\lambda_{t+1}/\pi_{t+1}\right) \tag{11}$$

$$d_t \;:\quad \lambda_t e_t = \beta(1+i_t^*)\phi_t^* \varphi_D\left(e_t d_t / Y_t\right) E_t\left(\lambda_{t+1} e_{t+1}/\pi_{t+1}^*\right) \tag{12}$$

$$N_t \;:\quad \xi^N N_t^{\sigma^N} = \lambda_t w_t \tag{13}$$

[7] There is also a no-Ponzi game condition that I omit for simplicity and yields the transversality condition $\lim_{t\to\infty} \beta^t d_t = 0$ that prevents households from incurring in Ponzi games.

Notice that in (9) and (12) the auxiliary functions φ_M and φ_D have been introduced merely to obtain a more compact notation:

$$\begin{aligned} \varphi_D\left(\gamma^D\right) &\equiv \tau_D\left(\gamma^D\right) + \gamma^D \tau'_D\left(\gamma^D\right), \\ \varphi_M\left(\gamma^M\right) &\equiv \tau_M\left(\gamma^M\right) - \gamma^M \tau'_M\left(\gamma^M\right). \end{aligned} \tag{14}$$

Combining (10) and (11) implicitly gives the demand for cash as a function of the nominal interest rate and consumption expenditure:

$$-\tau'_M\left(m_t/p_t^C C_t\right) = 1 - \frac{1}{1+i_t}, \tag{15}$$

Inverting $-\tau'_M$ gives the explicit demand function for cash as a vehicle for transactions (or 'liquidity preference' function):

$$m_t = \mathcal{L}\left(1+i_t\right) p_t^C C_t, \tag{16}$$

where $\mathcal{L}\left(.\right)$ is defined as:

$$\mathcal{L}\left(1+i_t\right) \equiv \left(-\tau'_M\right)^{-1}\left(1 - \frac{1}{1+i_t}\right), \tag{17}$$

and is strictly decreasing, since:

$$\mathcal{L}'\left(1+i_t\right) = \left[-\tau''_M(\mathcal{L}\left(1+i_t\right))\left(1+i_t\right)^2\right]^{-1} < 0.$$

Under the assumption that the Central Bank always satisfies cash demand, from now on I call (16) the money market clearing condition.

Using (9) to eliminate λ_t from (11) yields a version of the classical Euler equation that reflects the additional influence of the use of money on transactions costs:

$$\frac{C_t^{-\sigma^C}}{\varphi_M\left(m_t/p_t^C C_t\right)} = \beta\left(1+i_t\right) E_t\left(\frac{C_{t+1}^{-\sigma^C}}{\varphi_M\left(m_{t+1}/p_{t+1}^C C_{t+1}\right)} \frac{1}{\pi_{t+1}^C}\right), \tag{18}$$

where $\pi_t^C \equiv P_t^C / P_{t-1}^C$ is the gross rate of inflation of the basket of consumption goods and I have used the identity:

$$\frac{p_t^C}{p_{t-1}^C} = \frac{\pi_t^C}{\pi_t} \tag{19}$$

(based on the definition of p_t^C in (4)) to eliminate the rate of inflation for domestic goods.

The definition of the RER in (2) gives the following identity:

$$\frac{e_t}{e_{t-1}} = \frac{\delta_t \pi_t^*}{\pi_t}, \tag{20}$$

where $\delta_t \equiv S_t / S_{t-1}$ is the rate of nominal depreciation of the domestic currency. Hence, (12) may be written as:

$$1 = \beta(1+i_t^*)\phi_t^* \varphi_D\left(\frac{e_t d_t}{Y_t}\right) E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{\delta_{t+1}}{\pi_{t+1}}\right).$$

Also, multiplying both sides of (11) by δ_{t+1} and applying the expectations operator gives:

$$E_t\delta_{t+1} = \beta\left(1+i_t\right)E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{\delta_{t+1}}{\pi_{t+1}}\right).$$

Combining the last two equations yields the risk-adjusted uncovered interest parity (UIP) equation:

$$1+i_t = (1+i_t^*)\phi_t^*\varphi_D\left(\frac{e_t d_t}{Y_t}\right)E_t\delta_{t+1}. \tag{21}$$

Finally, eliminating λ_t from (13) gives the household's labor supply:

$$N_t = \left(\frac{w_t}{\xi^N p_t^C C_t^{\sigma^C}\varphi_M\left(m_t/p_t^C C_t\right)}\right)^{\frac{1}{\sigma^N}}. \tag{22}$$

2.1.2 Domestic and imported consumption

The consumption index used in the household optimization problem is a constant elasticity of substitution (CES) aggregate consumption index of domestic $\left(C_t^D\right)$ and imported $\left(C_t^N\right)$ goods:

$$C_t = \left(a_D{}^{\frac{1}{\theta^C}}\left(C_t^D\right)^{\frac{\theta^C-1}{\theta^C}} + a_N{}^{\frac{1}{\theta^C}}\left(C_t^N\right)^{\frac{\theta^C-1}{\theta^C}}\right)^{\frac{\theta^C}{\theta^C-1}}, \qquad a_D + a_N = 1. \tag{23}$$

$\theta^C(\geq 0)$ is the elasticity of substitution between domestic and imported goods. Total consumption expenditure is:

$$P_t^C C_t = P_t C_t^D + P_t^N C_t^N, \tag{24}$$

where P_t^N is the domestic currency price of imported goods. Then minimization of (24) subject to (23) for a given C_t, yields the following relations:

$$P_t = P_t^C\left(\frac{C_t^D}{a_D C_t}\right)^{-\frac{1}{\theta^C}} \tag{25}$$

$$P_t^N = P_t^C\left(\frac{C_t^N}{a_N C_t}\right)^{-\frac{1}{\theta^C}}. \tag{26}$$

Introducing these in (23) yields the consumption price index:

$$P_t^C = \left(a_D\left(P_t\right)^{1-\theta^C} + a_N\left(P_t^N\right)^{1-\theta^C}\right)^{\frac{1}{1-\theta^C}}. \tag{27}$$

Dividing (27) through by P_t yields a relation between the relative prices of consumption and imported goods (both in terms of domestic goods):

$$p_t^C = \left(a_D + (1-a_D)\left(p_t^N\right)^{1-\theta^C}\right)^{\frac{1}{1-\theta^C}}, \tag{28}$$

where

$$p_t^N \equiv \frac{P_t^N}{P_t}.$$

For simplicity, I assume that the Law of One Price holds. Hence, the domestic price of (the aggregate of) imported goods is simply:

$$P_t^N = S_t P_t^*.$$

This implies that the domestic relative price of imports is simply the RER:

$$p_t^N = \frac{P_t^N}{P_t} = \frac{S_t P_t^*}{P_t} = e_t. \tag{29}$$

Hence, the relative price of the consumption bundle (28) is:

$$p_t^C = \left(a_D + (1 - a_D)\, e_t^{1-\theta^C}\right)^{\frac{1}{1-\theta^C}}. \tag{30}$$

(25) and (26) show that a_D and $a_N = 1 - a_D$ in (23) are directly related to the shares of domestic and imported consumption in total consumption expenditures. In fact, the shares are:[8]

$$\frac{C_t^D}{p_t^C C_t} = a_D \frac{1}{(p_t^C)^{1-\theta^C}} \tag{31}$$

$$\frac{e_t C_t^N}{p_t^C C_t} = (1 - a_D)\left(\frac{e_t}{p_t^C}\right)^{1-\theta^C} \tag{32}$$

I assume throughout that there is a bias for domestic goods, i.e., $a_D > 1/2 > a_N$, and that $\theta^C > 1$.

C_t^D is a CES aggregate of an infinite number of domestic varieties of goods, each produced by a monopolist under monopolistic competition:

$$C_t^D = \left(\int_0^1 C_t^D(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}, \qquad \theta > 1 \tag{33}$$

where θ is the elasticity of substitution between varieties of domestic goods in household expenditure.

Conditions (25), and (26) are necessary for the optimal allocation of household expenditures across domestic and imported bundles of goods. Similarly, for the optimal allocation across varieties of domestic goods within the first of these classes, use of (33) yields the following necessary conditions:

$$P_t(i) = P_t \left(\frac{C_t^D(i)}{C_t^D}\right)^{-\frac{1}{\theta}}.$$

[8] In the Cobb-Douglas case ($\theta^C = 1$) the shares are a_D and $a_N = 1 - a_D$ (and hence are time invariant). But in this case the relative demand of domestic to imported goods is independent of p_t^N (and hence, the RER), which is something not too desirable. With $\theta^C > 1$ an increase in the relative price of imported goods increases the relative demand for domestic goods.

2.2. Firms

2.2.1 The representative final goods firm

There is perfect competition in the production (or bundling) of final domestic output Q_t, with the output of intermediate firms as inputs. A representative final domestic output firm uses the following CES technology:

$$Q_t = \left(\int_0^1 Q_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \qquad \theta > 1 \tag{34}$$

where $Q_t(i)$ is the output of the intermediate domestic good i. The final domestic output representative firm solves the following problem each period:

$$\max_{Q_t(i)} P_t \left(\int_0^1 Q_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} - \int_0^1 P_t(i) Q_t(i) di, \tag{35}$$

the solution of which is the demand for each type of domestic good (as an input):

$$Q_t(i) = Q_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta}. \tag{36}$$

Introducing (36) in (34) and simplifying, it is readily seen that the domestic goods price index is:

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \tag{37}$$

Also, introducing (36) into the cost part of (35) yields:

$$\int_0^1 P_t(i) Q_t(i) di = P_t Q_t.$$

2.2.2 The monopolistically competitive firms

A continuum of monopolistically competitive firms produce the intermediate domestic goods (that the final goods producer bundles) using homogenous labor, with no entry or exit. The production function of each firm is:

$$Q_t(i) = \epsilon_t N_t(i) \tag{38}$$

where ϵ_t is an industry-wide transitory productivity shock.

Since $N_t(i)$ is firm i's labor demand, using (38) and (36) and integrating yields aggregate labor demand:

$$N_t^D = \int_0^1 N_t(i) di = \int_0^1 \frac{Q_t(i)}{\epsilon_t} di = \frac{1}{\epsilon_t} \int_0^1 Q_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta} di = \frac{Q_t}{\epsilon_t} \Delta_t \tag{39}$$

where (as in Schmitt-Grohé and Uribe (2004) and (2007)) I defined a measure of price dispersion at period t:

$$\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\theta} di \geq 1.$$

Notice that $\Delta_t = 1$ when all prices are the same and $\Delta_t > 1$ otherwise.[9]

Equating labor supply (22) and demand (39) gives the labor market equilibrium real wage (in terms of domestic goods):

$$w_t = \xi^N \left(\frac{Q_t}{\epsilon_t}\Delta_t\right)^{\sigma^N} p_t^C C_t^{\sigma^C} \varphi_M\left(m_t/p_t^C C_t\right) \tag{40}$$

Each firm's cost is $W_t N_t(i) = (W_t/\epsilon_t)\, Q_t(i)$. Hence, its marginal cost is W_t/ϵ_t and its real marginal cost (in terms of domestic goods) is:

$$mc_t = \frac{w_t}{\epsilon_t}. \tag{41}$$

Notice that all firms face the same marginal cost. Also, (40) shows that increases in price dispersion raise the equilibrium real wage and hence the real marginal cost of firms. This is due to the positive effect of increased price dispersion on aggregate labor demand (see (39)) and, given the level of supply, on the equilibrium real wage. Furthermore, tighter monetary conditions increase marginal cost because an increase in i_t makes households economize on cash (see (16)), lowering $m_t/p_t^C C_t$. Because $\varphi'_M = -\gamma^M \tau''_M < 0$, this has a positive effect on φ_M, lowering labor supply (see (22)) and hence increasing the equilibrium real wage.

2.2.3 The dynamics of inflation and price dispersion

Firms make pricing decisions taking the aggregate price and quantity indexes as parametric. Every period, each firm has a probability $1-\alpha$ of being able to set the optimum price for its specific type of good. The firms that can't optimize must leave the same price they had last period. The pricing problem of firms that get to optimize is:

$$\max_{P_t(i)} E_t \sum_{j=0}^{\infty} \alpha^j \Lambda_{t,t+j} Q_{t+j}(i) \left\{\frac{P_t(i)}{P_{t+j}} - mc_{t+j}\right\} \tag{42}$$

subject to the demand they will face until they can again optimize:

$$Q_{t+j}(i) = Q_{t+j}\left(\frac{P_t(i)}{P_{t+j}}\right)^{-\theta}. \tag{43}$$

$\Lambda_{t,t+j}$ is the pricing kernel used by domestic firms for discounting, which, since firms are owned by households and respond to their preferences, is equal to households' intertemporal marginal rate of substitution in the consumption of domestic goods between periods $t+j$ and t:

$$\Lambda_{t,t+j} \equiv \beta^j \frac{U_{C^D,t+j}}{U_{C^D,t}},$$

where $U\left(C_{t+j}, N_{t+j}\right)$ is the function within brackets in (5). Notice that the marginal utility of consuming domestic goods can be obtained from the marginal utility of consuming the aggregate bundle of (domestic and imported) goods. Specifically:

$$U_{C^D,t} = U_{C,t}\frac{dC_t}{dC_t^D} = U_{C,t} a_D^{\frac{1}{\theta^C}} \left(\frac{C_t^D}{C_t}\right)^{-\frac{1}{\theta^C}} = C_t^{-\sigma^C}\frac{P_t}{P_t^C} = \frac{1}{p_t^C C_t^{\sigma^C}},$$

[9] See Schmitt-Grohé and Uribe (2007).

where the second equality is obtained by differentiating (23) with respect to C_t^D, and the third comes from using (25). Hence, the pricing kernel of domestic firms is:

$$\Lambda_{t,t+j} \equiv \beta^j \frac{p_t^C C_t^{\sigma^C}}{p_{t+j}^C C_{t+j}^{\sigma^C}}. \tag{44}$$

Introducing (43) and (44) in (42) (and eliminating irrelevant multiplying terms that refer to time t) gives

$$\max_{P_t(i)} E_t \sum_{j=0}^{\infty} (\beta\alpha)^j \frac{Q_{t+j}}{p_{t+j}^C C_{t+j}^{\sigma^C}} \left\{ \left(\frac{P_t(i)}{P_{t+j}} \right)^{1-\theta} - mc_{t+j} \left(\frac{P_t(i)}{P_{t+j}} \right)^{-\theta} \right\}.$$

Since by symmetry all optimizing firms make the same decision I call the optimum price $\widetilde{P}_t$ and drop the firm index. Hence, the firm's first order condition is the following:

$$0 = E_t \sum_{j=0}^{\infty} (\beta\alpha)^j \frac{Q_{t+j}}{p_{t+j}^C C_{t+j}^{\sigma^C}} \left(\frac{P_{t+j}}{P_t} \right)^{\theta} \left\{ \widetilde{p}_t \frac{P_t}{P_{t+j}} - \frac{\theta}{\theta - 1} mc_{t+j} \right\} \tag{45}$$

where $\widetilde{p}_t \equiv \widetilde{P}_t / P_t$ is the relative price of firms that optimize and the general price level (which includes the prices of both optimizers and non-optimizers). In the Calvo setup, because optimizers (and hence non-optimizers) are randomly chosen from the population, the average price in $t-1$ of non-optimizers (which must keep their price constant) is equal to the overall price index in $t-1$ no matter when they optimized for the last time. Hence, (37) implies the following law of motion for the aggregate domestic goods price index:

$$P_t^{1-\theta} = \alpha \left(P_{t-1} \right)^{1-\theta} + (1-\alpha) \widetilde{P}_t^{1-\theta}. \tag{46}$$

Dividing through by $P_t^{1-\theta}$ and rearranging yields the relative price of optimizers as an increasing function of the inflation rate:

$$\widetilde{p}_t = \left(\frac{1 - \alpha \pi_t^{\theta-1}}{1-\alpha} \right)^{\frac{-1}{\theta-1}} \equiv \widetilde{p}\left(\pi_t\right). \tag{47}$$

Hence, using this in (45) gives the (non-linear) Phillips equation that determines the dynamics of domestic inflation:

$$0 = E_t \sum_{j=0}^{\infty} (\beta\alpha)^j \frac{Q_{t+j}}{p_{t+j}^C C_{t+j}^{\sigma^C}} \left(\frac{P_{t+j}}{P_t} \right)^{\theta} \left\{ \widetilde{p}\left(\pi_t\right) \frac{P_t}{P_{t+j}} - \frac{\theta}{\theta - 1} mc_{t+j} \right\}. \tag{48}$$

In order to implement the Phillips equation in Dynare I express this in a recursive (nonlinear) form. Define:

$$\begin{aligned} \Gamma_t &= E_t \sum_{j=0}^{\infty} (\beta\alpha)^j \frac{Q_{t+j}}{p_{t+j}^C C_{t+j}^{\sigma^C}} \left(\frac{P_{t+j}}{P_t} \right)^{\theta-1} \\ \Psi_t &= \frac{\theta}{\theta - 1} E_t \sum_{j=0}^{\infty} (\beta\alpha)^j \frac{Q_{t+j}}{p_{t+j}^C C_{t+j}^{\sigma^C}} \left(\frac{P_{t+j}}{P_t} \right)^{\theta} mc_{t+j} \end{aligned} \tag{49}$$

and express (48) as:

$$\widetilde{p}\left(\pi_t\right)\Gamma_t = \Psi_t.$$

Now write Γ_t and Ψ_t recursively as follows:

$$\begin{aligned}\Gamma_t &= \left(Q_t/p_t^C C_t^{\sigma^C}\right) + \beta\alpha E_t \pi_{t+1}^{\theta-1}\Gamma_{t+1} \\ \Psi_t &= \frac{\theta}{\theta-1}\left(Q_t/p_t^C C_t^{\sigma^C}\right) mc_t + \beta\alpha E_t \pi_{t+1}^{\theta}\Psi_{t+1}.\end{aligned}$$

Hence, the complicated Phillips equation (with infinite summations) is transformed into these three simple nonlinear equations. Notice that collapsing the log-linear approximations of these equations yields the usual log-linearized Phillips equation:

$$\widehat{\pi}_t = \frac{\left(1-\beta\alpha\right)\left(1-\alpha\right)}{\alpha}\widehat{mc}_t + \beta E_t\widehat{\pi}_{t+1}.$$

Δ_t is an additional variable in the model, which hence needs an additional equation. A recursive equation for the dynamics of this variable is now derived in three steps. First, separate the set of non-optimizing firms N from the set of optimizing firms O and notice that in a given period the latter all set the same price $\widetilde{P}_t$ and have mass $1-\alpha$:

$$\Delta_t \equiv \int_{i\in N}\left(\frac{P_t(i)}{P_t}\right)^{-\theta} di + \int_{i\in O}\left(\frac{P_t(i)}{P_t}\right)^{-\theta} di = \alpha\Delta_t^N + (1-\alpha)\widetilde{p}_t^{-\theta} \tag{50}$$

where I defined the equivalent measure of price dispersion for non-optimizers:

$$\Delta_t^N \equiv \int_{i\in N}\frac{1}{\alpha}\left(\frac{P_t(i)}{P_t}\right)^{-\theta} di.$$

Second, write Δ_t^N recursively using the fact that non-optimizers maintain in t the same price as in $t-1$:

$$\Delta_t^N \equiv \int_{i\in N}\frac{1}{\alpha}\left(\frac{P_{t-1}(i)}{P_{t-1}}\frac{1}{\pi_t}\right)^{-\theta} di = \pi_t^{\theta}\int_{i\in N}\frac{1}{\alpha}\left(\frac{P_{t-1}(i)}{P_{t-1}}\right)^{-\theta} di = \pi_t^{\theta}\Delta_{t-1}^N$$

and use this and (47) in (50) to get:

$$\Delta_t = \alpha\pi_t^{\theta}\Delta_{t-1}^N + (1-\alpha)\widetilde{p}\left(\pi_t\right)^{-\theta}.$$

Finally, since non-optimizers (as well as optimizers) are selected randomly from the set of all firms, the dispersion of non-optimizers in $t-1$ is equal to the dispersion of the population: $\Delta_{t-1}^N = \Delta_{t-1}$. The new model equation is therefore:

$$\Delta_t = \alpha\pi_t^{\theta}\Delta_{t-1} + (1-\alpha)\widetilde{p}\left(\pi_t\right)^{-\theta}. \tag{51}$$

A log-linear approximation of this equation is simply:

$$\widehat{\Delta}_t = \alpha\pi^{\theta}\widehat{\Delta}_{t-1}.$$

Hence, if in the NSS there is price stability and hence no price dispersion, a log-linear approximation of the model will not give any dynamics for $\widehat{\Delta}_t$ if initially there is no price dispersion (see Schmitt-Grohé and Uribe (2007)). Since in this paper I do not go beyond a log-linear approximation of the model and wish to see the dynamics of price dispersion in IRFs (that show the responses of the log-linear deviations of the variables from the NSS values to shocks when they are initially at the NSS), in Appendix I I calibrate a NSS with non-zero inflation.

2.3. Foreign trade, the public sector, and the balance of payments

Firms in the export sector use domestic goods and 'land' (representing natural resources) to produce an export commodity. Land is assumed to be fixed in quantity, hence generating diminishing returns. I assume that the export good is a single homogenous primary good (a commodity). Firms in this sector sell their output in the international market at the foreign currency price P_t^{*X}. They are price takers in factor and product markets. The price of primary goods in terms of the domestic currency is merely the exogenous international price multiplied by the nominal exchange rate: $S_t P_t^{*X}$.

Let the production function employed by firms in the export sector be the following:

$$X_t^* = \left(Q_t^X\right)^{b^A} Y_t^{1-b^A}, \qquad 0 < b^A < 1, \tag{52}$$

where Q_t^X is the amount of domestic goods used as input in the export sector and Y_t is real GDP. These firms maximize profit $S_t P_t^{*X} X_t^* - P_t Q_t^X$ subject to (52). In terms of domestic goods, they maximize:

$$\frac{\Pi_t^X}{P_t} = e_t p_t^* \left(Q_t^X\right)^{b^A} Y_t^{1-b^A} - Q_t^X$$

where I defined the SOE's external terms of trade (XTT):

$$p_t^* \equiv \frac{P_t^{*X}}{P_t^*},$$

where P_t^* is the price index of the foreign currency price of the SOE's imports. Notice that the XTT is a ratio of two price indexes determined in the RW. Hence, the follow identity relates the rates of foreign inflation of exported and imported goods to the XTT (giving the dynamics of the XTT):

$$\frac{p_t^*}{p_{t-1}^*} = \frac{\pi_t^{*X}}{\pi_t^*}, \qquad \text{where} \quad \pi_t^{*X} \equiv \frac{P_t^{*X}}{P_{t-1}^{*X}}.$$

The first order condition for profit maximization yields the export sector's (factor) demand for domestic goods:

$$Q_t^X = \left(b^A e_t p_t^*\right)^{\frac{1}{1-b^A}} Y_t. \tag{53}$$

Also, inserting the factor demand function in the production function shows that optimal exports vary directly with the product of the RER and the XTT and GDP:

$$X_t^* = \left(b^A e_t p_t^*\right)^{\frac{b^A}{1-b^A}} Y_t. \tag{54}$$

The real value of exports in terms of domestic goods is:

$$X_t = \frac{S_t P_t^{*X} X_t^*}{P_t} = e_t p_t^* X_t^* = e_t p_t^* \left(b^A e_t p_t^*\right)^{\frac{b^A}{1-b^A}} Y_t = \kappa_X \left(e_t p_t^*\right)^{b_X} Y_t \tag{55}$$

where for simplicity of notation I define:

$$b_X \equiv \frac{1}{1-b^A}, \qquad \kappa_X \equiv \left(b^A\right)^{\frac{b^A}{1-b^A}}.$$

Government expenditure is assumed to be a time-varying and stochastic fraction $\overline{G}_t$ of private consumption expenditure. Define the gross government expenditure fraction as: $G_t \equiv 1 + \overline{G}_t$. Hence, using (31) and (55), GDP in terms of domestic goods is:

$$\begin{aligned} Y_t &= \tau_M\left(\gamma_t^M\right) G_t p_t^C C_t + X_t - (1-a_D)\, e_t^{1-\theta^C} \tau_M\left(\gamma_t^M\right) G_t \left(p_t^C\right)^{\theta^C} C_t \\ &= a_D \tau_M\left(\gamma_t^M\right) G_t \left(p_t^C\right)^{\theta^C} C_t + X_t. \end{aligned} \tag{56}$$

In the domestic goods market, the output of domestic firms Q_t must satisfy final demand from households (including the resources for transactions), the government, and the export sector:[10]

$$Q_t = a_D \tau_M\left(\gamma_t^M\right) G_t \left(p_t^C\right)^{\theta^C} C_t + Q_t^X = Y_t - \left(1-b^A\right) X_t. \tag{57}$$

The public sector includes the Government and the CB. The latter issues currency (M_t) and domestic currency bonds (B_t), and holds international reserves (R_t) in the form of foreign currency denominated riskfree bonds issued by the RW. I assume that the CB has no operational costs and that CB bonds are only held by domestic residents. The (flow) budget constraint of the CB is:

$$\begin{aligned} M_t + B_t - S_t R_t &= M_{t-1} + (1+i_{t-1}) B_{t-1} - (1+i_{t-1}^*) S_t R_{t-1} \\ &= \left[M_{t-1} + B_{t-1} - S_{t-1} R_{t-1}\right] - QF_t. \end{aligned} \tag{58}$$

where

$$\begin{aligned} QF_t &= i_{t-1}^* S_t R_{t-1} + (S_t - S_{t-1}) R_{t-1} - i_{t-1} B_{t-1} \\ &= \left[i_{t-1}^* + (1 - 1/\delta_t)\right] S_t R_{t-1} - i_{t-1} B_{t-1} \end{aligned}$$

is the CB's quasi-fiscal surplus, which includes interest earned and capital gains on international reserves minus the interest paid on its bonds. I assume that the CB transfers its quasi-fiscal surplus (or deficit) to the Government every period. Hence, its net wealth is constant. Furthermore, assuming for convenience that the CB's net worth is zero, the following holds for all t:

$$M_t + B_t - S_t R_t = M_{t-1} + B_{t-1} - S_{t-1} R_{t-1} = 0. \tag{59}$$

[10] Notice that intermediate output in the export sector (53) can be written as:

$$Q_t^X = \left(b^A\right)^{\frac{1}{1-b^A}} \left(e_t p_t^*\right)^{b_X} Y_t = b^A \left(b^A\right)^{\frac{b^A}{1-b^A}} \left(e_t p_t^*\right)^{b_X} Y_t = b^A X_t$$

Hence, rearranging the second equality in (57) shows that GDP is the sum of the outputs of the domestic and export sectors, minus the intermediate use of domestic goods in the export sector $Y_t = Q_t + X_t - b^A X_t$.

The CB supplies whatever amount of cash is demanded by households, and can influence these supplies by changing R_t or B_t, i.e. intervening in the foreign exchange market or in the domestic currency bond market. In terms of domestic goods, the CB balance, for all t, is:

$$m_t + b_t = e_t r_t. \tag{60}$$

This equation provides a constraint on the CB's ability to simultaneously intervene in the foreign exchange market (through sales and purchases of foreign reserves r_t) and in the domestic bonds market (through sales and purchases of domestic currency CB bonds b_t).[11]

The Government spends on goods, receives the quasi-fiscal surplus (or finances the deficit) of the CB, and collects taxes. I assume that fiscal policy consists of an exogenous autoregressive path for real government expenditures as a (gross) fraction of private consumption (G_t) and collecting whatever lump-sum taxes are needed to balance the budget each period. The Public Sector flow budget constraint is hence:

$$Tax_t = \overline{G}_t \tau_M \left(\gamma_t^M\right) P_t^C C_t - QF_t. \tag{61}$$

So in real terms:

$$\begin{aligned} tax_t &= \overline{G}_t \tau_M \left(\gamma_t^M\right) p_t^C C_t - qf_t, \\ qf_t &= \left(1 + i_{t-1}^* - 1/\delta_t\right) \frac{e_t r_{t-1}}{\pi_t^*} - \left(\left(1 + i_{t-1}\right) - 1\right) \frac{b_{t-1}}{\pi_t}. \end{aligned} \tag{62}$$

Inserting

$$Y_t = w_t N_t + \frac{\Pi_t}{P_t},$$

in the household budget constraint (7) and consolidating the household, CB and government budget constraints yields the balance of payments equation:

$$r_t - d_t = CA_t + r_{t-1} - d_{t-1},$$

where the current account (in foreign currency) is:

$$CA_t = \left(\frac{1 + i_{t-1}^*}{\pi_t^*} - 1\right) r_{t-1} - \left[\frac{1 + i_{t-1}^*}{\pi_t^*} \phi_{t-1}^* \tau_D \left(\frac{e_{t-1} d_{t-1}}{Y_{t-1}}\right) - 1\right] d_{t-1} + TB_t$$

[11] It is obviously unnecessary to restrict the CB net wealth to zero. Any fixed number would do. Moreover, there is clearly the possibility of adding a degree of freedom for a more general model in which the CB net wealth can vary (perhaps stochastically) or even be used as an additional control variable. The latter would require additional modeling, such as market perceptions of CB risk. For my purpose of modeling the simultaneous use of the interest rate and the rate of nominal depreciation as control variables, the simplest assumption of zero CB net wealth is sufficient.

and, using (32) and (56), the trade balance (in foreign currency) is:

$$\begin{aligned} TB_t &= \frac{1}{e_t}\left[X_t - e_t\tau_M\left(\gamma_t^M\right)G_tC_t^N\right] \\ &= \frac{1}{e_t}\left[X_t - (1-a_D)\,e_t^{1-\theta^C}\left(p_t^C\right)^{\theta^C}\tau_M\left(\gamma_t^M\right)G_tC_t\right] \\ &= \frac{1}{e_t}\left[X_t - \frac{1-a_D}{a_D}e_t^{1-\theta^C}\left(Y_t - X_t\right)\right] \\ &= \frac{1}{a_De_t}\left[\left(p_t^C\right)^{1-\theta^C}X_t - (1-a_D)\,e_t^{1-\theta^C}Y_t\right]. \end{aligned}$$

2.4. Monetary and exchange rate policy

In this paper the CB uses either policy rules or optimal policy under commitment (and full information) (OPC). The policy rules are simple (i.e., respond to a limited number of endogenous variables through constant coefficients) and they may have either exogenous or endogenous and optimal coefficients. Under simple rules with exogenous coefficients, in the case of the rule for the nominal interest rate there is feedback (as in the typical Taylor-like rule) and the simple rule for nominal depreciation may or may not involve feedback. In the case of optimal simple rules, the CB is assumed to minimize a weighted average of the variances of some of the endogenous variables. In the case of OPC, the CB is assumed to minimize the expected discounted value of future losses for a suitably defined quadratic loss function of some of the endogenous variables.

In any of these three cases, the CB can operate under one of three alternative monetary regimes. I use the expression 'monetary regime' broadly. It expresses the combination of the CB's operating procedures concerning the issuance of (base) money, and the intervention it may have in the bond and FX markets to influence the nominal interest rate and the rate of nominal currency depreciation. As shown below, in this paper 'monetary' policy (in the narrow sense) is passive, being money issuance whatever is needed to balance the money market once the other two policies are defined. For convenience, the three alternative monetary regimes are denominated: I) a Managed Exchange Rate (MER) regime, in which the CB uses both rules (or both instruments in the case of OPC), II) a Floating Exchange Rate (FER) regime, in which the CB only uses the Taylor-like rule (or only uses the interest rate as an instrument -in the case of OPC), and III) a Pegged Exchange Rate (PER) regime, in which the CB only uses the rule for the rate of nominal depreciation (or only uses the rate of nominal depreciation as an instrument, in the case of OPC).

In the MER regime, through its regular and systematic interventions in the domestic currency bond (or 'money') market and in the foreign exchange market, the CB aims for the achievement of two operational targets: one for the interest rate i_t, and another for the rate of nominal depreciation δ_t. When there are simple policy rules (whether they are optimal or not), the CB can respond to deviations of the consumption inflation rate (π_t^C) from a target (π^T) which is the NSS value of this variable, to deviations of GDP from its NSS value, and to deviations of the RER from its NSS value. The rate of nominal depreciation can respond to the same variables and additionally to the deviations of the CB's international reserves (IRs) ratio (to GDP) from a long run target (γ^R). There may be history

dependence (or inertia) in one or both of the two simple rules through the presence of the lagged operational target variable. The simple rules are the following:

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i}\right)^{h_0} \left(\frac{\pi_t^C}{\pi_t^T}\right)^{h_1} \left(\frac{Y_t}{Y}\right)^{h_2} \left(\frac{e_t}{e}\right)^{h_3} \tag{63}$$

$$\frac{\delta_t}{\delta} = \left(\frac{\delta_{t-1}}{\delta}\right)^{k_0} \left(\frac{\pi_t^C}{\pi_t^T}\right)^{k_1} \left(\frac{Y_t}{Y}\right)^{k_2} \left(\frac{e_t}{e}\right)^{k_3} \left(\frac{e_t r_t/Y_t}{\gamma^R}\right)^{k_4}, \tag{64}$$

where $h_1 \neq 0$ and $k_4 \neq 0$ and variables without time subscripts denote NSS values. The first of these is used in the MER and FER regimes, and the second is used in the MER and PER regimes. In a floating exchange rate regime (FER), the CB abstains from intervening in the foreign exchange market. Hence, the international reserves that appear in its balance sheet remain constant. For simplicity, I assume that they remain constant at the NSS value r of the general model (with MER regime). In a pegged exchange rate regime (PER), the CB abstains from intervening in the domestic currency bond market. Hence, its stock of bonds remains constant, and I assume that they remain at the NSS value b of the MER regime. In both of the corner cases, one of the policy rules is dropped and one of the endogenous variables is turned into an exogenous parameter. But there is an alternative way of thinking about this issue which is more illuminating, particularly in an optimal control framework.

The FER and PER regimes are extreme cases ('corner regimes') in which the CB chooses not to use one of its potential instruments. In the case of OPC this means that the optimal policy under any one of the 'corner' regimes cannot dominate the optimal policy under the MER regime. One can define these regimes as cases in which the CB imposes an additional restriction on itself ('ties its hands') and relinquishes its use of one of its 'control' variables. Hence that variable turns into a 'non-control' variable.[12]

To obtain a generalization of the standard DSGE monetary policy model, I specify the instruments that the CB uses when it intervenes in each of the two markets and include them in the model. The CB purchases or sells domestic currency bonds, and thus changes its stock of bonds b_t, to intervene with high frequency in this market in order to attain its operational target for the interest rate as determined by (63).[13] And it purchases or sells foreign exchange to intervene in the foreign exchange market, thereby changing its stock of international reserves r_t, in order to attain its operational target for the rate of nominal depreciation as determined by (64). While at high frequency (hours, days, weeks) the CB is active changing b_t and/or r_t, at low frequency (quarters in this paper) these variables passively adapt to accommodate i_t and δ_t as given by the feedback policy rules and the rest of the model equations.

To represent the constraints that the CB faces it is necessary to broaden the usual policy model to include the CB balance sheet (60) and its arrangement with

[12] I hesitate to use the term 'state variable' because in this model both i and δ are non-predetermined (or jump) variables and it is usual to call predetermined variables 'state variables'.

[13] Notice that this high-frequency action may be modeled in different ways. But in the quarterly frequency of the model the instruments, operational target variables, and the rest of the model variables are related through the model equations that any higher frequency model must respect if it is designed to be consistent with the quarterly model.

the rest of the government (Treasury) as to the use of the fiscal dimension of the CB's flow budget constraint (which I called CB quasi-fiscal surplus qf_t above). By assuming, as I do here, that the CB's arrangement with the Treasury is that it hands over its quasi-fiscal surplus (or receives automatic finance for its quasi-fiscal deficit) period by period, the CB balance sheet equation is maintained period by period in the sense that the CB's net worth is constant. This can be seen as a simple device for defining the CB's 'sterilization' policy, i.e. the value of b_t, given the values of m_t ('determined' by money market balance), and the values of e_t and r_t. But it is probably more adequate to think more symmetrically that (60) imposes a constraint on the simultaneous use of b_t and r_t. From this vantage point, one should think of the 'corner' regimes as the imposition of an additional constraint (instead of the dropping of an endogenous variable). In the case of the FER regime, the additional constraint is $r_t = r$ (an equation that replaces (64)). And in the case of the PER regime, the additional constraint is $b_t = b$ (an equation that replaces (63)). In terms of an optimal control framework (as is OPC), any one of the 'corner' regimes imposes an additional constraint on the policymaker and, simultaneously, converts one of the 'controls' (δ_t in the case of the FER regime and i_t in the case of the PER regime) into a non-control variable. Hence, it quite evident that the MER regime cannot be inferior to any of the two 'corner' regimes (in the sense of generating a larger loss). With the same loss function and the same (basic) model equations and endogenous variables, but with one additional constraint (equation) and one less 'control', the expected discounted loss cannot be lower. Indeed, I show below that it is very much higher in all of the usual CB preferences (represented through weights for inflation and output deviations).

The policy framework in this paper is one in which monetary growth is passive.[14] Indeed, defining the rate of money growth $\mu_t \equiv M_t/M_{t-1}$, (16) and (18) imply:

$$\mu_t = \pi_t^C \frac{\mathcal{L}(1+i_t)}{\mathcal{L}(1+i_{t-1})} \left[\beta(1+i_t)\frac{\varphi_M(\mathcal{L}(1+i_t))}{\varphi_M(1+i_{t-1})}\right]^{\frac{-1}{\sigma^C}}. \tag{65}$$

Hence, under the MER or FER regimes, achieving the operational target for the nominal interest rate bearing in mind the need to balance the money market implies that the growth in real money (μ_t/π_t^C) only depends on the current and lagged interest rate. However, (65) is equally valid under the PER regime, where there is no CB policy rule for the interest rate.

2.5. Functional forms for auxiliary functions

For calibrations it is convenient to define the **net** functions:

$$\begin{aligned} \overline{\tau}_D\left(\gamma_t^D\right) &= \tau_D\left(\gamma_t^D\right) - 1, \qquad & \overline{\varphi}_D\left(\gamma_t^D\right) &= \varphi_D\left(\gamma_t^D\right) - 1 \\ \overline{\tau}_M\left(\gamma_t^M\right) &= \tau_M\left(\gamma_t^M\right) - 1, \qquad & \overline{\varphi}_M\left(\gamma_t^M\right) &= \varphi_M\left(\gamma_t^M\right) - 1. \end{aligned} \tag{66}$$

[14] See Olivera (1970).

I use the following functional forms:[15]

$$\overline{\tau}_D\left(\gamma_t^D\right) \equiv \frac{\alpha_1}{1-\alpha_2\gamma_t^D}, \qquad \alpha_1,\alpha_2>0, \tag{67}$$

$$\overline{\tau}_M\left(\gamma_t^M\right) \equiv \frac{\beta_1}{\left(1+\beta_2\gamma_t^M\right)^{\beta_3}}, \qquad \beta_1,\beta_2,\beta_3>0 \tag{68}$$

which, according to definitions (14), give:

$$\begin{aligned}\overline{\varphi}_D\left(\gamma_t^D\right) &= \frac{\alpha_1}{\left(1-\alpha_2\gamma_t^D\right)^2}, \\ \overline{\varphi}_M\left(\gamma_t^M\right) &= \frac{\beta_1}{\left(1+\beta_2\gamma_t^M\right)^{\beta_3}}\left(1+\beta_3\frac{\beta_2\gamma_t^M}{1+\beta_2\gamma_t^M}\right).\end{aligned} \tag{69}$$

The liquidity preference function (17) that results from (68) is:

$$\frac{m_t}{p_t^C C_t} \equiv \gamma_t^M = \boldsymbol{\mathcal{L}}\left(1+i_t\right) = \frac{1}{\beta_2}\left[\left(\frac{\beta_1\beta_2\beta_3}{1-\frac{1}{1+i_t}}\right)^{\frac{1}{\beta_3+1}}-1\right]. \tag{70}$$

And to get a more compact notation in some of the equations the following auxiliary variables and equations are introduced:

$$\begin{aligned}\tau_{M,t} &= 1+\frac{\beta_1}{\left(1+\beta_2\gamma_t^M\right)^{\beta_3}} \\ \varphi_{M,t} &= 1+\left(\tau_{M,t}-1\right)\left(1+\beta_3\frac{\beta_2\gamma_t^M}{1+\beta_2\gamma_t^M}\right).\end{aligned}$$

2.6. The nonlinear system of equations

In this section I put together the model equations for simple feedback rules in a MER regime.

Consumption Euler:

$$\frac{C_t^{-\sigma^C}}{\varphi_{M,t}} = \beta\left(1+i_t\right)E_t\left(\frac{C_{t+1}^{-\sigma^C}}{\varphi_{M,t+1}}\frac{1}{\pi_{t+1}^C}\right)$$

Risk-adjusted uncovered interest parity:

$$1+i_t = (1+i_t^*)\phi_t^*\left[1+\frac{\alpha_1}{\left(1+\alpha_2\gamma_t^D\right)^2}\right]E_t\delta_{t+1} \tag{71}$$

Phillips equations:

$$\begin{aligned}\Gamma_t &= \frac{Q_t}{p_t^C C_t^{\sigma^C}}+\beta\alpha E_t\pi_{t+1}^{\theta-1}\Gamma_{t+1} \\ \Psi_t &= \frac{\theta}{\theta-1}\frac{Q_t}{p_t^C C_t^{\sigma^C}}mc_t+\beta\alpha E_t\pi_{t+1}^{\theta}\Psi_{t+1} \\ \Psi_t &= \left(\frac{1-\alpha\pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{-1}{\theta-1}}\Gamma_t\end{aligned}$$

[15] In calibrating the model parameters I found it important to include a third parameter in the the transactions cost function. Otherwise I could not obtain realistic money demand interest elasticities, and the variability of the instruments was systematically excessive.

Dynamics of price dispersion:

$$\Delta_t = \alpha \pi_t^{\theta} \Delta_{t-1} + (1-\alpha) \left(\frac{1-\alpha \pi_t^{\theta-1}}{1-\alpha} \right)^{\frac{\theta}{\theta-1}}$$

Exports:

$$X_t = \kappa_X \left(e_t p_t^* \right)^{b_X} Y_t$$

Trade Balance:

$$TB_t = \frac{1}{a_D e_t} \left[\left(p_t^C \right)^{1-\theta^C} X_t - (1-a_D) \, e_t^{1-\theta^C} Y_t \right]$$

Current Account:

$$CA_t = \left(\frac{1+i_{t-1}^*}{\pi_t^*} - 1 \right) r_{t-1} - \left[\frac{1+i_{t-1}^*}{\pi_t^*} \phi_{t-1}^* \left(1 + \frac{\alpha_1}{1-\alpha_2 \gamma_{t-1}^D} \right) - 1 \right] d_{t-1} + TB_t.$$

Balance of Payments:

$$r_t - d_t = CA_t + r_{t-1} - d_{t-1}$$

Real marginal cost:

$$mc_t = \frac{w_t}{\epsilon_t}$$

Labor market clearing:

$$w_t = \xi^N p_t^C C_t^{\sigma^C} \varphi_{M,t} N_t^{\sigma^N}$$

Hours worked:

$$N_t = \frac{Q_t}{\epsilon_t} \Delta_t$$

Domestic goods market clearing:

$$Q_t = Y_t - \left(1 - b^A \right) X_t$$

GDP:

$$Y_t = a_D \tau_{M,t} G_t \left(p_t^C \right)^{\theta^C} C_t + X_t$$

Consumption relative price:

$$p_t^C = \left(a_D + (1-a_D) \, e_t^{1-\theta^C} \right)^{\frac{1}{1-\theta^C}}$$

Money market clearing:

$$m_t = \frac{1}{\beta_2} \left[\left(\frac{\beta_1 \beta_2 \beta_3}{1 - \frac{1}{1+i_t}} \right)^{\frac{1}{\beta_3+1}} - 1 \right] p_t^C C_t,$$

CB balance sheet:

$$b_t = e_t r_t - m_t$$

Consumption inflation:

$$\frac{\pi_t^C}{\pi_t} = \frac{p_t^C}{p_{t-1}^C}$$

Real Exchange Rate:

$$\frac{e_t}{e_{t-1}} = \frac{\delta_t \pi_t^*}{\pi_t}$$

External terms of trade:

$$\frac{p_t^*}{p_{t-1}^*} = \frac{\pi_t^{*X}}{\pi_t^*} \tag{74}$$

Tax collection:

$$tax_t = \overline{G}_t \tau_{M,t} p_t^C C_t - qf_t$$

Quasi-fiscal surplus:

$$qf_t = \left(1 + i_{t-1}^* - 1/\delta_t\right) \frac{e_t r_{t-1}}{\pi_t^*} - \left(\left(1 + i_{t-1}\right) - 1\right) \frac{b_{t-1}}{\pi_t}$$

Great ratios:

$$\gamma_t^D = \frac{e_t d_t}{Y_t}, \qquad \gamma_t^M = \frac{m_t}{p_t^C C_t},$$

Auxiliary functions:

$$\tau_{M,t} = 1 + \frac{\beta_1}{(1 + \beta_2 \gamma_t^M)^{\beta_3}}, \qquad \varphi_{M,t} = 1 + (\tau_{M,t} - 1)\left(1 + \beta_3 \frac{\beta_2 \gamma_t^M}{1 + \beta_2 \gamma_t^M}\right).$$

Interest rate feedback rule:

$$\frac{1 + i_t}{1 + i} = \left(\frac{1 + i_{t-1}}{1 + i}\right)^{h_0} \left(\frac{\pi_t^C}{\pi_t^T}\right)^{h_1} \left(\frac{Y_t}{Y}\right)^{h_2} \left(\frac{e_t}{e}\right)^{h_3} \tag{75}$$

Nominal depreciation feedback rule:

$$\frac{\delta_t}{\delta} = \left(\frac{\delta_{t-1}}{\delta}\right)^{k_0} \left(\frac{\pi_t^C}{\pi_t^T}\right)^{k_1} \left(\frac{Y_t}{Y}\right)^{k_2} \left(\frac{e_t}{e}\right)^{k_3} \left(\frac{e_t r_t / Y_t}{\gamma^R}\right)^{k_4} \tag{76}$$

Notice that I am not constraining b_t nor r_t to be non-negative, which may be quite unrealistic. Negative international reserves would mean borrowing from abroad and, in the context of this model, would require a risk premium as in the case of households. And many Central Banks are institutionally constrained in lending to the non-financial private sector, making b_t non-negative. Here, I assume that the Central Bank's target for reserves γ^R is sufficiently high and the household's steady state demand for cash is sufficiently low to ensure that these non-negativity constraints hold for all t and all relevant stochastic shocks.[16]

In addition to these equations there are those that are subject to stochastic shocks, most of which are simple AR(1) processes. The external terms of trade (XTT) is a particularly important external effect for most SOE's. This justified giving the calibration of its components a careful treatment. As a working hypothesis, I assumed that the inflation rates for imported and exported goods are interrelated in such a way that a shock to one of them affects the other through

[16] In the parent model ARGEM, it is banks that invest in domestic currency bonds and usually Central Banks do have the institutional ability to assist banks, though usually with limitations.

the dynamics of the XTT (which is the ratio of the two corresponding foreign price levels). Hence, I assumed:

$$\begin{aligned}
\pi_t^{*X} &= \left(\pi_{t-1}^{*X}\right)^{\rho^{\pi^{*X}}} \left(\pi^{*X}\right)^{1-\rho^{\pi^{*X}}} \left(p_{t-1}^{*}\right)^{\alpha_{\pi^{*X}}} \exp\left(\sigma^{\pi^{*X}} \varepsilon_t^{\pi^{*X}}\right), \qquad (77)\\
\pi_t^{*} &= \left(\pi_{t-1}^{*}\right)^{\rho^{\pi^{*}}} \left(\pi^{*}\right)^{1-\rho^{\pi^{*}}} \left(p_{t-1}^{*}\right)^{\alpha_{\pi^{*}}} \exp\left(\sigma^{\pi^{*}} \varepsilon_t^{\pi^{*}}\right),\\
p_t^{*} &= p_{t-1}^{*} \frac{\pi_t^{*X}}{\left(\pi_t^{*}\right)^{\beta_{\pi^{*}}}}.
\end{aligned}$$

Notice that if the two price indexes are non-stationary, this implies that they are cointegrated. The XTT variable p_t^* plays the role of a cointegration error term, $\alpha_{\pi^{*X}} \leq 0$, $\alpha_{\pi^*} > 0$ are the speeds of adjustment and $(1, -\beta_{\pi^*})$ plays the role of a cointegrating vector, with $\beta_{\pi^*} = 1$ as in the identity (74). In Appendix I, I estimate these equations using data for Argentina and find evidence for the cointegration hypothesis with an additional influence of π_{t-1}^{*X} on π_t^*, as in the equation below.

The equations subject to stochastic shocks are hence the following (where the NSS values ϵ, π^*, π^{*X} are assumed equal to one):

Productivity shock:

$$\epsilon_t = \left(\epsilon_{t-1}\right)^{\rho^{\epsilon}} \exp\left(\sigma^{\epsilon} \varepsilon_t^{\epsilon}\right)$$

Government expenditure shock:

$$G_t = \left(G_{t-1}\right)^{\rho^{G}} G^{1-\rho^{G}} \exp\left(\sigma^{G} \varepsilon_t^{G}\right)$$

Riskfree interest rate shock:

$$1 + i_t^* = \left(1 + i_{t-1}^*\right)^{\rho^{i^*}} \left(1 + i^*\right)^{1-\rho^{i^*}} \exp\left(\sigma^{i^*} \varepsilon_t^{i^*}\right)$$

Financing risk/liquidity shock:

$$\phi_t^* = \left(\phi_{t-1}^*\right)^{\rho^{\phi^*}} \left(\phi^*\right)^{1-\rho^{\phi^*}} \exp\left(\sigma^{\phi^*} \varepsilon_t^{\phi^*}\right)$$

Exports inflation shock:

$$\pi_t^{*X} = \left(\pi_{t-1}^{*X}\right)^{\rho^{\pi^{*X}}} \left(\pi^{*X}\right)^{1-\rho^{\pi^{*X}}} \left(p_{t-1}^{*}\right)^{\alpha_{\pi^{*X}}} \exp\left(\sigma^{\pi^{*X}} \varepsilon_t^{\pi^{**}}\right)$$

Imported inflation shock:

$$\pi_t^{*} = \left(\pi_{t-1}^{*}\right)^{\rho^{\pi^{*}}} \left(\pi^{*}\right)^{1-\rho^{\pi^{*}}} \left(p_{t-1}^{*}\right)^{\alpha_{\pi^{*}}} \left(\pi_{t-1}^{*X}\right)^{\rho^{\pi^{*XN}}} \exp\left(\sigma^{\pi^{*}} \varepsilon_t^{\pi^{*}}\right).$$

3. Numerical solution in Dynare

A detailed calibration of the parameters and derivation of the NSS values of the endogenous variables can be found in Appendix 1. In this section I analyze the stabilizing role of the two policy rules under the different monetary and exchange rate regimes, mainly by studying the volatilities (standard deviations) of the main endogenous variables in the model. I also explore the policy parameter ranges that guarantee the Blanchard-Kahn (BK) stability conditions. Table 1 summarizes the calibrated values of the main model parameters that are used throughout, and also contains some comparisons with parameter values used in two other relevant SOE models.[17]

[17] 'E.S.' denotes 'elasticity of substitution', G_M stands for 'Galí and Monacelli (2005)', and De P for 'De Paoli (2006)'.

Table 1: Calibrated values of main model parameters

	Parameters	This paper	G-M	De P
β	Intertemporal discount factor	0.99	0.99	0.99
σ^C	Relative risk aversion for goods	1.5	1	1
σ^N	Relative risk aversion for labor	0.5	3	0.47
α	Probability of not adjusting price	0.66	0.75	0.66
θ	E.S. between domestic goods	6	6	10
θ^C	E.S. domestic vs. imported goods	1.5	1	3
a_D	Coef. for share of domestic goods	0.86	0.6	0.6
b^A	Coef. in production function for commodities	0.5	1	
ε_D^{φ}	Elasticity of risk function in UIP $\varphi_D(ed/Y)$	2.0		
$\varepsilon_{\mathcal{L}}$	Elasticity of $\mathcal{L}(1+i)$	1.02		

The standard errors and persistence parameters used for the six shock variables are given in Table 2. They were calibrated taking into account the available time series for Argentina and the RW during the period 1994.1-2009.2: public consumption to GDP $\left(\sigma^G, \rho^G\right)$, imported and exported goods inflation as they conform Argentina's XTT $\left(\sigma^{\pi^*}, \sigma^{\pi^{*X}}, \rho^{\pi^*}, \rho^{\pi^{*X}}, \rho^{\pi^{*XN}}\right)$, Libor 3 months $\left(\sigma^{i^*}, \rho^{i^*}\right)$, and balance of payments information on private sector foreign debts and interest payments as well as my own calculation of the spread over Libor 3 months $\left(\sigma^{\phi^*}, \rho^{\phi^*}\right)$. The only cases in which I took the the standard deviations exactly according to the data are the cases of σ^{i^*}, σ^{π}, and $\sigma^{\pi^{*X}}$. The rest were calibrated taking both the data (except for σ^{ϵ}) and the resulting theoretical standard deviation and variance decomposition for GDP with a baseline calibration of the two policy rules ($h_1 = 0.8$, $h_2 = 0.8$, $k_4 = -0.8$, and the rest of the coefficients zero). This implied diminishing the observed standard deviation of G (from 0.054 in a simple AR(1) estimation from which I did use the persistence parameter ρ^G), which seemed to weigh too heavily in the volatility of Y, and increasing the standard deviation of ϕ^* (from 0.0034), which seemed not to weigh enough. The value of σ^{ϵ} was chosen so that the resulting theoretical standard deviation of Y was similar to the data for detrended and s.a. GDP for Argentina leaving out the crisis years 2001/2002.

Table 2: Calibration of shock variables

standard deviations

σ^{ϵ}	σ^G	σ^{i^*}	σ^{ϕ^*}	σ^{π^*}	$\sigma^{\pi^{*X}}$
0.01	0.03	0.0046	0.05	0.0295	0.0424

persistence

ρ^{ϵ}	ρ^G	ρ^{i^*}	ρ^{ϕ^*}	ρ^{π^*}	$\rho^{\pi^{*X}}$	$\rho^{\pi^{*XN}}$
0.8	0.85	0.7	0.3	0.2	0.41	0.18

speeds of adjustment

α_{π^*}	$\alpha_{\pi^{*X}}$
0.181	-0.255

3.1. The effects of the simple policy rule coefficients under the MER regime

I first study some of the general stability properties of the model in relation to the parameters of the two simple policy rules in the MER regime. The coefficients on the policy rules not explicitly mentioned below are made equal to zero. When I

say that a particular configuration of parameters gives stability I mean that all the requirements for determinacy and non-explosiveness are met, including the rank condition. In particular, there are no unit roots.

1) A very general result is that if the coefficient that makes the rate of nominal appreciation respond to central bank deviations from target is zero ($k_4 = 0$) the model has a unit root for any value of the remaining coefficients. Hence, from now on k_4 will always be different from zero in the MER regime. Let $k_4 = -0.8$ until further notice. Observe that with negative values for k_4, when there are insufficient reserves, and hence, $e_t r_t / Y_t < \gamma^R$, i.e. $\widehat{e}_t + \widehat{r}_t - \widehat{Y}_t < 0$, the CB tends to depreciate the currency (more than in the NSS):

$$\widehat{\delta}_t = k_4 \left(\widehat{e}_t + \widehat{r}_t - \widehat{Y}_t \right) > 0.$$

Since a purchase of IR (increase in r_t) expands the money supply (*ceteris paribus*) one tends to associate it with a currency depreciation. But thins are more complex. First, it is the ratio between the real domestic value of IR ($e_t r_t$) to GDP that must increase if in the initial period $e_t r_t / Y_t < \gamma^R$. Second, that increase must take place in the long run, so the direction of movement may be the opposite during a transition period. In fact, I show below that sometimes it is optimal to have a positive k_4.

2) I first look at very streamlined policy rules with no inertia, an interest rate policy rule that only responds to inflation, and a nominal depreciation policy rule that only responds to the deviation from the long run target for international reserves. Hence, in (75) and (76) only h_1 and $k_4 = -0.8$ are non-zero. It is readily seen that there is stability as long as h_1 is greater than one: the classical Taylor Principle. Otherwise there is indeterminacy.

3) Next I introduce interest rate inertia, letting h_0 become positive. I find that, for example, if $h_0 = 0.2$, then h_1 must be at least 0.81 (to two decimal points) for stability. Otherwise there is indeterminacy. The scheme below shows that gradually raising h_0 lowers the minimum value of h_1 required for stability (and viceversa). Hence, there is an 'inertia-inflation-responsiveness frontier' for the interest rate policy rule that is downward sloping and linear. For stability, $h_0 + h_1$ must be greater than one and h_1 must be different from zero. Hence, the 'Taylor Principle' holds in the model (see Woodford (2003), page 255, Proposition 4.4). Also, the higher above one is $h_0 + h_1$, the wider is the range within h_2 can move without impairing stability, with the center of that range in the negative territory. Furthermore, the 'inertia-inflation-responsiveness frontier' (IIRF) is valid for a wide range of values for the remaining coefficients in the second policy rule as long as k_4 is negative. That is, below this frontier there is no way to stabilize the economy using interest rate responsiveness to GDP or the RER, or depreciation rate responsiveness to inflation, GDP or the RER.

h_0	-0.2	0.0	0.2	0.4	0.6	0.8	1.0	1.2	...	10.2	...
h_1	1.21	1.01	0.81	0.61	0.41	0.21	0.01	-0.19	...	-9.19	...

Furthermore, this is true for any other negative value of k_4 as well as positive values greater than 0.22. Hence, it is nice to see that the present generalization of the standard monetary policy framework in DSGE models maintains some of the

key ingredients in the more limited, conventional, model. On the other hand, if k_4 has low positive values (less than or equal to 0.23), there is a reversal in the Taylor Principle: stability requires that $h_0 + h_1$ be less than one. Remarkably, a policy with a positive k_4 less than 0.24 (to two digits) and all the other coefficients in both policy rules equal to zero is stable. Positive values for k_4 will come up below as optimal values for simple policy rules in the MER regime for certain CB styles. This illustrates the fact that the general model (with the MER) is considerably more complex (and richer) than the standard models (with either of the two 'corner' regimes: FER or PER).

4) Next I looked a little closer into the effects of changing one of the two critical coefficients h_0 and h_1 (keeping the rest at baseline values) on the standard deviations of some of the endogenous variables Central Banks typically care for.[18] First I take a fixed value of h_0 starting on the IIRF and find the volatilities (standard deviations) for increasing values of h_1. The results are in the Table 3, where the minimum value in each row is highlighted in bold and the maximum is in italics. The ratio between the maximum and minimum volatility is also shown in the last column. It is interesting to see that some of the volatilities of variables of interest decrease steadily (inflation -piC in the Dynare file-, price dispersion -$DeltaP$ -, the RER -e-, TB, $Utility$, d, δ) while others increase steadily (C, real interest rate, r), and still others at first diminish, reach their minimum, and then increase (Y, N, mc). Maximum volatilities are almost always in the extremes, but minimum volatilities are more scattered.

Although attention is usually focused on the volatility of Y, it is C and N that enter the aggregate utility of households, and their volatilities respond quite differently to increases in h_1. Indeed, while the volatility of C increases steadily with h_1, that of N falls up to $h_1 = 2$ and then starts to increase. The volatility of period utility (Utility), however, steadily diminishes as h_1 increases, as does the volatility of inflation and price dispersion.

[18] Amato and Laubach (2003) do a similar analysis for the case of sticky prices and wages when only an interest rate rule is used.

Table 3: Means and standard deviations of main variables for different values of h_1

h_0=0.4	MEAN	STANDARD DEVIATION								
VARIABLE		h_1=0.61	h_1=0.8	h_1=1.0	h_1=1.5	h_1=2	h_1=3	h_1=4	h_1=5	max/min
piC	1.0150	*0.0190*	0.0155	0.0143	0.0126	0.0114	0.0095	0.0082	**0.0072**	2.64
DeltaP	1.0051	*0.0082*	0.0048	0.0038	0.0030	0.0027	0.0023	0.0022	**0.0022**	3.73
Y	1.4430	*0.0742*	0.0720	0.0709	0.0694	**0.0690**	0.0696	0.0709	0.0723	1.08
C	1.3108	**0.0236**	0.0272	0.0298	0.0351	0.0396	0.0466	0.0518	*0.0557*	2.36
N	1.3220	*0.0731*	0.0663	0.0637	0.0612	**0.0606**	0.0614	0.0630	0.0647	1.21
real_ii	1.0101	**0.0111**	0.0131	0.0154	0.0206	0.0248	0.0312	0.0359	*0.0394*	3.55
mc	0.8302	0.0166	**0.0135**	0.0135	0.0175	0.0227	0.0317	0.0386	*0.0439*	3.25
e	0.5951	*0.0496*	0.0494	0.0493	0.0490	0.0488	0.0485	0.0484	**0.0482**	1.03
TB	0.0082	*0.0608*	0.0604	0.0601	0.0596	0.0593	0.0588	0.0584	**0.0582**	1.04
d	1.2125	*0.1020*	0.1016	0.1013	0.1011	0.1009	0.1009	0.1008	**0.1008**	1.01
m	0.1154	0.0032	**0.0032**	0.0034	0.0042	0.0050	0.0062	0.0072	*0.0080*	2.50
Utility	-2.2744	*0.0574*	0.0551	0.0543	0.0536	0.0533	0.0531	0.0529	**0.0529**	1.09
ii	1.0253	0.0153	**0.0148**	0.0165	0.0214	0.0256	0.0320	0.0366	*0.0401*	2.71
b	0.0722	0.0160	*0.0162*	0.0161	0.0159	0.0157	0.0153	0.0151	**0.0149**	1.09
delta	1.0150	*0.0711*	0.0695	0.0685	0.0667	0.0653	0.0632	0.0617	**0.0606**	1.17
r	0.3152	**0.0432**	0.0439	0.0443	0.0453	0.0460	0.0470	0.0478	*0.0484*	1.12

The remaining rows in this table focus on the volatility of the CB intermediate targets and instruments. While the volatility of i (ii in the Dynare file) is non-monotonic, decreasing at first and then increasing, the volatility of the second operational target δ (*delta* in the Dynare file) is steadily decreasing. Furthermore, the volatility of the variables that the CB actually uses as instruments on a day by day basis, b and r, behave quite differently. The volatility of b varies in the opposite direction to i as h_1 increases. To achieve a substantial reduction in the volatility of the operational target (from 0.040 to 0.015) it is only necessary to increase the volatility of the instrument 9%. The volatility of r also varies in the opposite direction to that of δ as h_1 increases, reaching its maximum where the volatility of of δ reaches its minimum. But in this case not much reduction in volatility of δ (from 0.071 to 0.061) is achieved with a 12% increase in the volatility of r.

Table 4 shows a similar exercise except that h_1 is now fixed and it is h_0 that increases. The volatilities of π^C, Δ, Y, N, and *Utility* are highest for the lowest value of h_0, fall to a minimum and then start increasing. As in the previous table, the volatilities of C and the real interest rate increase steadily. But now the volatilities of e and TB increase steadily as h_0 increases, though, as in the previous case, they do not vary much. As to the intermediate targets and the instruments, increases in h_0 are very efficient in reducing the volatility of i: with a 9% increase in the volatility of b a substantial reduction in the volatility of i is achieved (from 0.015 to 0.002). In order to implement an increasing 'inertia' for its interest rate operational target, the CB must use its corresponding instrument with only moderately higher volatility. On the other hand, it is clear that h_0 is not efficient for reducing the volatility of δ.

Table 4: Means and standard deviations of main variables for different values of h_0

h_1=0.61	MEAN	STANDARD DEVIATION								
VARIABLE		h_0=0.4	h_0=0.6	h_0=0.8	h_0=1.0	h_0=2	h_0=3	h_0=4	h_0=5	max/min
piC	1.0150	*0.0190*	0.0142	0.0126	**0.0118**	0.0123	0.0137	0.0146	0.0152	1.61
DeltaP	1.0051	*0.0082*	0.0033	0.0015	**0.0007**	0.0040	0.0056	0.0064	0.0070	11.71
Y	1.4430	*0.0742*	0.0721	0.0710	**0.0705**	0.0705	0.0710	0.0713	0.0716	1.05
C	1.3108	**0.0236**	0.0272	0.0298	0.0320	0.0381	0.0405	0.0417	*0.0424*	1.80
N	1.3220	*0.0731*	0.0649	0.0620	**0.0608**	0.0612	0.0625	0.0633	0.0639	1.20
real_ii	1.0101	**0.0111**	0.0119	0.0129	0.0137	0.0158	0.0165	0.0169	*0.0171*	1.54
mc	0.8302	0.0166	0.0111	**0.0094**	0.0105	0.0198	0.0241	0.0263	*0.0276*	2.94
e	0.5951	**0.0496**	0.0497	0.0500	0.0503	0.0512	0.0515	0.0517	*0.0519*	1.05
TB	0.0082	**0.0608**	0.0610	0.0615	0.0620	0.0636	0.0643	0.0647	*0.0649*	1.07
d	1.2125	**0.1020**	0.1031	0.1045	0.1058	0.1094	0.1107	0.1113	*0.1117*	1.10
m	0.1154	*0.0032*	0.0027	0.0026	0.0025	0.0025	0.0025	0.0025	**0.0025**	1.28
Utility	-2.2744	*0.0574*	0.0543	0.0533	0.0529	**0.0523**	0.0523	0.0523	0.0523	1.10
ii	1.0253	*0.0153*	0.0111	0.0096	0.0085	0.0051	0.0035	0.0027	**0.0022**	6.95
b	0.0722	**0.0160**	0.0166	0.0168	0.0169	0.0172	0.0174	0.0174	*0.0174*	1.09
delta	1.0150	*0.0711*	0.0698	0.0694	**0.0692**	**0.0692**	0.0695	0.0697	0.0699	1.03
r	0.3152	**0.0432**	0.0440	0.0445	0.0447	0.0451	0.0451	0.0451	*0.0452*	1.05

5) Now I look at what happens when the CB changes the speed with which it seeks to attain its long run target for international reserves through its nominal depreciation response. For this I keep h_0 and h_1 constant at values in the interior of the IIRF ($h_0 = 0.4$ and $h_1 = 0.8$) while k_4 gets increasingly negative, starting from -0.1.

Table 5: Means and standard deviations of main variables for different values of k_4

h_0=0.4, h_1=0.8		STANDARD DEVIATION								
VARIABLE	MEAN	k_4=-0.1	k_4=-0.4	k_4=-0.7	k_4=-1	k_4=-2	k_4=-3	k_4=-4	k_4=-5	max/min
piC	1.0150	**0.0135**	0.0148	0.0154	0.0157	0.0161	0.0162	*0.0163*	*0.0163*	1.21
DeltaP	1.0051	*0.0053*	0.0049	0.0048	0.0048	0.0048	0.0048	0.0048	**0.0048**	1.10
Y	1.4430	*0.0778*	**0.0719**	0.0720	0.0721	0.0724	0.0725	0.0725	0.0726	1.08
C	1.3108	*0.0282*	**0.0272**	**0.0272**	**0.0272**	0.0273	0.0273	0.0273	0.0273	1.04
N	1.3220	*0.0689*	0.0664	**0.0663**	0.0664	0.0664	0.0665	0.0665	0.0665	1.04
real_ii	1.0101	**0.0102**	0.0121	0.0130	0.0134	0.0141	0.0144	0.0146	*0.0147*	1.44
mc	0.8302	*0.0182*	0.0140	0.0136	0.0134	0.0132	0.0131	0.0131	**0.0131**	1.39
e	0.5951	0.0496	**0.0492**	0.0494	0.0495	0.0497	0.0498	0.0499	*0.0499*	1.01
TB	0.0082	*0.0722*	**0.0593**	0.0601	0.0608	0.0619	0.0624	0.0626	0.0628	1.22
d	1.2125	*0.1353*	0.1101	0.1029	0.0997	0.0960	0.0948	0.0942	**0.0938**	1.44
m	0.1154	*0.0033*	0.0032	0.0032	0.0032	0.0032	0.0032	0.0032	**0.0032**	1.03
Utility	-2.2744	*0.0557*	**0.0549**	0.0551	0.0552	0.0553	0.0554	*0.0554*	0.0554	1.01
ii	1.0253	*0.0151*	**0.0147**	0.0148	0.0149	0.0149	0.0150	0.0150	0.0150	1.03
b	0.0722	*0.0919*	0.0282	0.0178	0.0140	0.0110	**0.0107**	0.0108	0.0109	8.59
delta	1.0150	**0.0504**	0.0640	0.0686	0.0709	0.0740	0.0751	0.0757	*0.0761*	1.51
r	0.3152	*0.1700*	0.0645	0.0468	0.0397	0.0319	0.0295	0.0284	**0.0278**	6.12

Several of the variables of interest have minimum volatilities for k_4 in the range $-0.1/-0.7$. On the other hand, Δ, mc, d, and m have lowest volatilities at $k_4 = -5$. As k_4 gets less negative (going from right to left in Table 5) an increasingly volatile use of the instrument (r) progressively reduces the volatility of the operational target (δ). It also has the effect of reducing the volatility of inflation. Surprisingly, it also implies a slight *increase* in the volatility of price dispersion. Furthermore, the volatility of the other instrument (b) also increases, without a significant effect on the volatility of the other operational target (i).

6) To get a feeling for the range within I could move each policy rule coefficient, I started from a baseline calibration for the coefficients in the two policy feedback rules well within the IIRF frontier and looked for the intervals within which each of the coefficients could be moved individually (leaving the rest at the baseline value) without impairing stability. I restricted my search to two decimal points accuracy and only checked for parameter values below 10 in absolute value. The following is the baseline calibration for this exercise:

Baseline calibration

h_0	h_1	h_2	h_3	k_0	k_1	k_2	k_3	k_4
0.8	0.8	0.0	0.0	0.0	0.0	0.0	0.0	−0.8

The results for the three policy regimes are in the Table 6. Starting with the general MER regime, both of the inertial coefficient intervals of stability are quite wide, both going into high superinertial levels (of 10 and 4.48 for the interest rate and depreciation rate rules, respectively). Because unity is included in the feasible intervals for h_0 and k_0, one or both of the simple policy rules can be implemented as the feedback response of the first difference (in the interest rate or the depreciation rate, respectively) to the various arguments on the r.h.s. In the case of the interest rate rule, there are no upper bounds for the reactions to inflation or the RER, but, perhaps surprisingly, there is an upper bound of only 1.04 for the response to GDP. There is much more room for an accommodating policy of diminishing (raising) the interest rate (up to -3.69) when GDP is high (low). In the case of the nominal depreciation rule, there are no upper or lower bounds for the reactions to inflation or GDP, and an upper bound of 9.12 for the reaction to the RER. In the case of k_4, the only restriction is that it must be outside of a small interval around zero, which is mostly on the positive side. The fact that there is a comparatively low upper bound for the interest rate response to GDP while there is no bound for the nominal depreciation response to the same variable is quite interesting, since the stabilization of GDP is, of course, of primary interest in most CBs (along with the stabilization of inflation).

Table 6: Stability ranges for individual coefficients of policy rules

	MER	FER	PER
	Interest rate rule		
$h_0 \in$	$[0.21, 10]$	$[0.21, 10]$	
$h_1 \in$	$[0.21, 10]$	$[0.21, 10]$	
$h_2 \in$	$[-3.69, 1.04]$	$[-3.54, 1.03]$	
$h_3 \in$	$[-8.14, 10]$	$[-6.89, 4.63]$	
	Nominal depreciation rule		
$k_0 \in$	$[-4.55, 4.48]$		$[-1.32, 0.67]$
$k_1 \in$	$[-10, 10]$		$[-10, 10]$
$k_2 \in$	$[-10, 10]$		$[-1.16, 1.67]$
$k_3 \in$	$[-10, 9.12]$		$[-1.77, 2.82]$
$k_4 \in$	$[-10, -0.01] \cup [0.23, 10]$		$[-0.95, 2.44]$

The FER regime shows stability ranges that are very similar to those of the first policy rule of the MER regime. There is a narrowing of the range in the case of h_3.

The narrowing of the range of stability is more significant in the case of the PER regime, especially in the cases of k_2, k_3, and k_4. On the other hand, in the PER regime the stability range for k_4 includes 0, indicating that the need to respond to a target for international reserves is only valid in the more general MER regime.

Because GDP is typically available with a significant lag, it is of interest to see how these stability ranges are altered when the policy rules respond to output with a one quarter lag. Hence, the exercise was repeated by replacing Y_t with Y_{t-1} in both simple policy rules (including the IRs ratio to GDP). The resulting stability ranges are quite similar. For both the MER and FER regimes there is again no upper bound for h_0 and h_1, and in this case there is no lower bound for h_2, whereas the same upper bound subsists. There is no lower bound for h_3 in the case of the MER regime, and an increase in the upper bound to 5.99 in the FER regime. The stability ranges for the k_i remain almost unaltered in the case of the MER regime. In the PER regime, however, k_1 is bounded above (by 0.67) whereas k_2 is not. Also, the stability range for k_3 is widened to $[-1.77, 2.82]$ and the lower bound for k_4 becomes -2.45.

Leaving behind the baseline calibration, it is interesting to verify that in the PER case there is stability when all the coefficients are zero ($k_j = 0, j = 0, 1, 2, 3, 4$). In this case the policy rule is to intervene in the FX market sufficiently to maintain the nominal exchange rate fixed at the existing level, letting the economy run its course, and not worrying about international reserves.[19]

7) The relatively narrow range of stability for the coefficient on the interest rate response to GDP deviations (h_2) in the MER case, along with the boundless range of stability for the corresponding coefficient in the second policy rule (k_2), naturally raises the question of the effects of the latter coefficient on the volatilities. Table 7 shows these effects. Most the variables reach minimum volatilities for non-positive values of k_2. And for a number of very significant variables such as Δ, Y, C, N, m, and $Utility$, the minimum is reached for highly negative values of k_2 (-10 or -8). Indeed, the lowest volatility of Y, N, and $Utility$ is lower than the lowest volatility they achieve, respectively, in any of the analogous tables above. Hence, reducing the rate of nominal depreciation (or perhaps even appreciating the currency) when GDP is above its NSS level has a very important stabilizing role for most of the variables of interest. Notice that this implies using both instruments with high volatility.

[19] However, one must bear in mind that here the nominal and real exchange rates are (in spirit) multilateral. If we modeled a multicountry RW, the nominal exchange rate would be the domestic currency price of a basket of the nominal exchange rates of the SOE's trade partners, with weights equal to the shares in trade. Hence, our peg is completely different from pegging against the currency of a country with which only a small part of the SOE's trade is done (as was the case of Argentina's ill fated 'Convertibility').

Table 7: Means and standard deviations of main variables for different values of k_2

h_0=0.4,h_1=0.8,k_4=-0.8		STANDARD DEVIATION								
VARIABLE	MEAN	k_2=-10.0	k_2=-8.0	k_2=-4.0	k_2=-2.0	k_2=0.0	k_2=2.0	k_2=4.0	k_2=6.0	max/min
piC	1.0150	0.0119	0.0117	**0.0116**	0.0117	0.0120	0.0128	0.0147	*0.0183*	1.58
DeltaP	1.0051	**0.0013**	**0.0013**	**0.0013**	***0.0013***	***0.0013***	***0.0013***	*0.0014*	*0.0014*	1.08
Y	1.4430	**0.0596**	0.0610	0.0645	0.0669	0.0700	0.0741	0.0800	*0.0886*	1.49
C	1.3108	**0.0318**	**0.0318**	0.0319	0.0320	0.0322	0.0325	0.0332	*0.0346*	1.09
N	1.3220	**0.0545**	0.0554	0.0576	0.0590	0.0608	0.0631	0.0662	*0.0706*	1.30
real_ii	1.0101	0.0147	0.0146	**0.0144**	0.0145	0.0148	0.0158	0.0180	*0.0226*	1.57
mc	0.8302	0.0112	0.0111	**0.0109**	0.0109	0.0109	0.0110	0.0114	*0.0122*	1.12
e	0.5951	0.0509	0.0503	0.0495	**0.0494**	0.0499	0.0511	0.0539	*0.0592*	1.20
TB	0.0082	0.0658	0.0637	0.0604	**0.0600**	0.0613	0.0655	0.0745	*0.0907*	1.51
d	1.2125	*0.1061*	0.1059	0.1055	0.1051	0.1045	0.1037	**0.1034**	0.1057	1.03
m	0.1154	**0.0028**	**0.0028**	**0.0028**	0.0029	0.0029	0.0029	0.0030	*0.0031*	1.11
Utility	-2.2744	**0.0486**	0.0492	0.0509	0.0519	0.0532	0.0549	0.0572	*0.0607*	1.25
ii	1.0253	0.0114	0.0114	**0.0113**	0.0114	0.0116	0.0121	0.0130	*0.0146*	1.29
b	0.0722	0.0908	0.0732	0.0369	0.0198	**0.0167**	0.0355	0.0616	*0.0940*	5.63
delta	1.0150	0.0669	0.0664	**0.0659**	0.0665	0.0687	0.0737	0.0843	*0.1048*	1.59
r	0.3152	*0.1622*	0.1340	0.0778	0.0538	**0.0448**	0.0629	0.0988	0.1467	3.62

3.2. Optimal simple rules

In view of these results, it is worthwhile to enquire what the optimal simple policy rules coefficients are when using an objective function that represents the CB's priorities with respect to to the volatilities it wants to minimize. In this subsection a (loss) function is defined that the CB wants to minimize and is defined using weights that reflect the CB's priorities. It is an ad-hoc function, since it is not based directly on the maximization of household utility (or its second order approximation).

1) First I used Dynare's 'osr' ('optimal simple rule') command to obtain the policy coefficients that minimized the variance of aggregate household Utility. In the case of the MER regime:

$$\begin{aligned} & \arg\min_{h_i,k_i} \left\{\omega_U Var\left(Utility_t\right)\right\} \\ = \; & \arg\min_{h_i,k_i} \lim_{\beta\to 1} E_0 \sum_{t=1}^{\infty} \left(1-\beta\right)\beta^t \left\{\omega_U \left(Utility_t - Utility\right)^2\right\}. \end{aligned}$$

A coefficient ω_U =1000 was used in the loss function. The use of large coefficients in the objective function is motivated by the need to have 'osr' effectively search the parameter space before settling on the optimal coefficients. When I used low coefficients (in the order of 1) the search was very short and I had to iterate the command (after putting the resulting coefficients as the initial ones) many times before converging to the truly optimal ones. I obtained the following optimal coefficients for the two simple policy rules (rounding off to two digits):

h_0	h_1	h_2	h_3	k_0	k_1	k_2	k_3	k_4
0.97	1.53	2.59	−0.06	2.41	−0.04	−0.80	−3.60	−0.44

Note that k_4 is negative, and the sum of h_0 and h_1 is above one.[20] Also, the optimal value of k_2 is -0.8. These values are in accordance with what was

[20] Some of these values are outside the stability ranges shown in the table above. However,

obtained above. Under our simple rules, and assuming that the policymakers wish to reduce the variance of the utility of households, it is optimal to react strongly and positively to inflation and GDP in the interest rate rule and strongly and negatively to the RER in the nominal depreciation rule. A deviation of 1% in inflation above its target value here commands an increase of 1.5 p.p. in the interest rate (assuming it was at the NSS level the previous period) and a slight reduction of 0.04 p.p. in the rate of nominal depreciation (assuming it was at the NSS level the previous period). And a deviation of 1% in GDP above its NSS value commands an increase in the interest rate of 2.6 p.p. and a reduction of 0.8 p.p. in the rate of nominal depreciation. On the other hand, a deviation of 1% in the RER above its NSS value commands a tiny reduction in the interest rate (of 0.06 p.p.) and a significant reduction in the rate of nominal depreciation of 3.6 p.p. The latter seems quite natural: if the currency is weak in real terms (e is high), it is optimal to depreciate less. Finally, it is optimal to make strong use of policy inertia in both rules (superinertial in the case of the second policy rule) even though no CB preference for such policies has been assumed.

Table 8 shows the standard deviations of the main endogenous variables when using these optimal simple rules.

Table 8: Means and standard deviations of main variables under optimal simple rules that minimize the variance of Utility

VARIABLE	MEAN	Std.Dev.	Std.Dev./Mean
piC	1.015	0.065	0.06
DeltaP	1.005	0.044	0.04
Y	1.443	0.052	0.04
C	1.311	0.036	0.03
N	1.322	0.033	0.02
real_ii	1.010	0.020	0.02
mc	0.830	0.037	0.04
e	0.595	0.037	0.06
TB	0.008	0.058	7.11
d	1.213	0.131	0.11
m	0.115	0.005	0.05
Utility	-2.274	0.032	-0.01
ii	1.025	0.053	0.05
b	0.072	0.102	1.41
delta	1.015	0.079	0.08
r	0.315	0.182	0.58

First, notice how small the standard deviation of Utility is. While the four tables above all had standard deviations above 0.0486, the 'osr' routine reduced it to 0.032. Second, it is noteworthy that minimizing the volatility of Utility actually implies having substantial volatilities in many of the variables that ad-hoc CB loss functions usually try to minimize. While the highest s.d. of consumer inflation in the above four tables was 0.019, it is 0.065 when this optimal simple policy rule is

that range was obtained keeping all the other coefficients at their baseline levels, which is not done here. Dynare's osr search changes direction whenever it goes into parameter values that do not comply with the Blanchard and Kahn conditions. In some cases, the search engine tended to obtain minor gains in loss with exceedingly high coefficients (in absolute value, in the hundreds or thousands) when using some initializations. I always ignored such gains and kept to the more moderate coefficient values.

used. The contrast with the price dispersion variable is even greater. The highest above was 0.008 and now it is 0.044.

2) Few CBs actually use models in which the explicit goal of the policymaker has to do with household utility. This is probably due to the fact that most models misrepresent reality in ways that CBs cannot take for granted: they assume homogenous households (except possibly for the heterogeneity derived from wage setting in a monopolistically competitive setting). The usual target variables of CB loss functions are inflation and GDP, and there is usually some explicit distaste for excessive movement in the operational target variable (the interest rate). This, of course, also brushes away, though in a different way, the incidence of CB actions on different sectors of the economy and different factor incomes. However, whereas aggregate household utility is an abstract concept because it is known that the model is misspecified in the dimension of household heterogeneity, variables like inflation, GDP, or the RER, have clear empirical counterparts that are very present in the minds of policymakers when they make decisions. Hence, I now repeat the above exercise assuming that the CB minimizes a linear combination of the variances of its target variables:

$$\arg\min_{h_i,k_i} \left\{ \omega_\pi Var\left(\pi_t^C\right) + \omega_Y Var\left(Y_t\right) + \omega_e Var\left(e_t\right) + \omega_r Var\left(r_t\right) \right.$$
$$\left. + \omega_{\Delta i} Var\left(\Delta i_t\right) + \omega_{\Delta\delta} Var\left(\Delta\delta_t\right) \right\}$$

Aside from the usual terms (with weights ω_π, ω_Y, $\omega_{\Delta i}$), this loss function also allows for CB preferences with respect to the variances of the RER, of the CBs IRs, and of changes in the rate of nominal depreciation (with weights ω_e, ω_r, $\omega_{\Delta\delta}$). In Table 9 I define six different CB styles (or preferences: A-F) according to the combinations of weights in each. In all of them I have given the same weight to the changes in each of the operational targets (50), and avoided zeros giving a weight of 1 to target variables with no importance. Hence, in style A only inflation matters and in style B only GDP matters, whereas both matter equally in style C. In style D (F) the real exchange rate (international reserves) matters as much as inflation and GDP. Finally, in style F inflation, GDP, the RER and the IRs all matter equally.

Table 9: Definition of CB styles

Weights	Styles					
	A	*B*	*C*	*D*	*E*	*F*
ω_π	100	1	100	100	100	100
ω_Y	1	100	100	100	100	100
ω_e	1	1	1	100	1	100
ω_r	1	1	1	1	100	100
$\omega_{\Delta i}$	50	50	50	50	50	50
$\omega_{\Delta\delta}$	50	50	50	50	50	50

With Dynare's 'osr' command I obtained the optimal simple policy rules for each of the CB styles in each of the interest and exchange regimes. The coefficients are shown in Table 10:

Table 10: Optimal simple policy rules for different CB styles and regimes

	OPTIMAL SIMPLE POLICY RULES																	
	A			B			C			D			E			F		
	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER
h_0	0.33	0.01		1.86	1.17		1.63	1.28		1.71	1.28		1.94	1.28		1.97	1.28	
h_1	1.26	2.04		-1.01	-0.39		1.92	-0.20		-2.09	-0.22		0.56	-0.20		0.49	-0.22	
h_2	0.02	-0.05		4.34	-3.72		1.43	-0.34		2.56	-0.34		-5.24	-0.34		-5.37	-0.34	
h_3	0.12	0.04		-0.21	-0.40		0.82	-0.02		-0.05	-0.01		0.14	-0.02		0.14	-0.01	
k_0	-0.03		-0.26	3.08		-0.47	0.44		-0.37	3.39		-0.43	0.26		-0.39	0.34		-0.41
k_1	-0.07		-2.08	-3.92		-2.13	-1.31		-2.82	-1.18		-3.70	1.28		-4.46	1.25		-3.71
k_2	-0.08		-2.36	-2.28		-4.66	-0.12		-3.85	-3.05		-3.92	-2.04		-5.09	-2.16		-4.32
k_3	-0.43		-1.09	1.18		0.46	-0.91		-0.10	1.40		0.08	0.16		0.11	-0.04		0.22
k_4	-0.08		-2.18	0.40		-0.87	-0.06		-2.57	0.13		-2.35	-0.74		-3.29	-0.84		-2.74

The inertial coefficient for the interest rate (h_0) is superinertial in all styles except A, in which only inflation matters. In the MER regime, the interest rate response to inflation deviations (h_1) is greater than one in styles A and C, in both of which inflation matters. However, it is negative for styles B and D. Interestingly, with the latter styles the optimal nominal depreciation rule in the MER regime has $k_4 > 0$. Furthermore, in both cases $h_0 + h_1$ is less than one and h_1 is negative, and yet there is Blanchard-Kahn stability since in both of these cases the interest rate response to GDP is sufficiently high (4.3 and 2.6, respectively). This is again in line with the Taylor Principle (see Woodford (2003), Proposition 4.4). Also, the depreciation rate response to to inflation and GDP are highly negative (-2.3 and -3.1, respectively).

In the FER regime, central banks of style A practically respond only to inflation, with a coefficient greater than two. However, in all the rest of the styles, h_0 is superinertial and h_1 is negative. The inequality $h_0 + h_1 > 1$ is valid in all CB styles except B where only GDP stabilization matters. Curiously, here it is a very negative interest rate reaction to GDP deviations maintains stability, quite the opposite from the MER regime case. And in the PER regime, all the coefficients are negative except for k_3, which is positive for styles B, D, E, and F. Hence, under the PER regime, high inflation and high GDP imply lowering the rate of nominal depreciation (or appreciating), and the previous' period rate of nominal depreciation affects the present rate negatively. Finally, in the PER regime k_4 is negative for all the CB styles considered, and the highest coefficient in absolute value is always k_2. The latter means that responses to deviations of GDP tend to very firm, regardless of the particular CB preferences.

To see if the relatively high preference for inertia in the operational targets ($\omega_{\Delta i} = \omega_{\Delta\delta} = 50$) in the definitions of the CB styles is the reason for the high superinertial coefficients in Taylor rule under styles B-F, I made the same calculations using a much lower preference for inertia: $\omega_{\Delta i} = \omega_{\Delta\delta} = 10$.[21] Table 11 shows that the broad outline of the optimal policy rules remain very similar to the previous table. Paradoxically, h_0 actually increases in six of the twelve cases, and quite substantially for some, showing that it is definitely not the preference of avoiding changes in the operational targets that are behind the high inertial coefficients in the Taylor rule.

[21] For notational simplicity I maintain the same names for the alternative CB preferences as in Table 9 although the last two rows of that table are modified.

Table 11: Optimal simple policy rules for alternative CB styles with $\omega_{\Delta i} = \omega_{\Delta \delta} = 10$

	OPTIMAL SIMPLE POLICY RULES																	
	A			B			C			D			E			F		
	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER
h_0	0.79	0.29		1.13	1.38		1.14	1.28		5.33	1.27		4.11	1.28		3.59	1.27	
h_1	1.53	3.01		-0.21	0.27		-0.66	-0.11		-2.54	-0.12		-0.15	-0.11		0.05	-0.12	
h_2	-0.03	0.01		4.10	-4.66		1.58	-0.23		-4.80	-0.23		-2.25	-0.23		-2.94	-0.23	
h_3	0.08	0.22		-0.19	0.32		0.02	-0.01		0.81	0.00		0.37	-0.01		0.52	0.00	
k_0	-0.09		-0.38	2.88		-0.82	1.26		-1.11	-0.64		-0.91	0.09		-1.26	0.25		-1.16
k_1	-0.01		-10.58	-2.13		-1.64	1.72		-7.97	1.87		-5.43	-0.90		-8.80	-0.58		-8.08
k_2	-0.07		-3.66	-2.06		-4.93	-7.32		-7.57	-2.17		-5.38	-1.98		-9.49	-1.94		-7.76
k_3	-0.39		-1.52	1.00		0.28	-0.02		0.45	-1.97		0.53	0.71		0.75	-0.77		1.02
k_4	-0.15		-3.24	0.34		-1.52	0.02		-3.73	-0.18		-2.59	-3.53		-5.26	-4.65		-3.87

Table 12 shows the standard deviations of the main endogenous variables in each regime and CB style, as well as the total and relative losses. As expected, the loss is always lowest with the MER regime. For CB styles A and B the losses with the FER and PER regimes are between six and eleven times higher than with the MER regime. In style C, where both inflation and GDP matter, the losses in the FER and PER regimes are 3 and 2.4 times the loss with the MER regime. The differences in the losses are lowest with CB styles E and F (where IRs matter). But the corner regimes still have losses that are between 30% and 70% greater than in the MER regime.

It should be emphasized that the PER regime here is not the usual pegged exchange regime. The simple rule in the PER regime includes the typical peg, which has no feedback. But this section shows that it is in general optimal to operate the PER regime with feedback. And the feedback coefficients are in general quite high (in absolute value). Hence, it is optimal to operate a very active peg for any of the CB styles.

Table 12: Standard deviations of main variables and losses under optimal simple rules

		OPTIMAL SIMPLE POLICY RULES																	
	MEAN	STANDARD DEVIATION																	
		A			B			C			D			E			F		
		MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER
piC	1.015	0.006	0.016	0.011	0.070	0.157	0.045	0.033	0.039	0.017	0.039	0.039	0.017	0.037	0.039	0.017	0.037	0.039	0.018
DeltaP	1.005	0.004	0.004	0.007	0.050	0.109	0.029	0.019	0.022	0.009	0.027	0.022	0.009	0.024	0.022	0.009	0.024	0.022	0.009
Y	1.443	0.073	0.070	0.078	0.012	0.014	0.056	0.040	0.042	0.063	0.027	0.043	0.063	0.017	0.042	0.063	0.017	0.043	0.063
C	1.312	0.037	0.039	0.074	0.086	0.089	0.053	0.078	0.072	0.048	0.077	0.072	0.049	0.082	0.072	0.044	0.082	0.072	0.045
N	1.322	0.063	0.064	0.079	0.072	0.138	0.037	0.036	0.054	0.054	0.061	0.054	0.053	0.050	0.054	0.053	0.049	0.054	0.053
real_ii	1.010	0.011	0.035	0.052	0.017	0.035	0.035	0.028	0.022	0.035	0.023	0.021	0.036	0.032	0.022	0.032	0.031	0.021	0.032
mc	0.830	0.026	0.025	0.069	0.079	0.092	0.045	0.072	0.071	0.036	0.072	0.070	0.036	0.076	0.071	0.031	0.076	0.070	0.032
e	0.594	0.038	0.052	0.048	0.037	0.052	0.048	0.044	0.055	0.048	0.034	0.055	0.048	0.050	0.055	0.049	0.049	0.055	0.048
TB	0.007	0.060	0.067	0.058	0.052	0.068	0.058	0.080	0.073	0.059	0.068	0.073	0.058	0.060	0.073	0.059	0.059	0.073	0.059
d	1.214	0.135	0.081	0.099	0.123	0.072	0.094	0.144	0.087	0.094	0.126	0.086	0.094	0.094	0.087	0.094	0.095	0.086	0.093
m	0.115	0.003	0.006	0.011	0.014	0.019	0.007	0.007	0.008	0.007	0.012	0.009	0.007	0.011	0.008	0.006	0.011	0.009	0.006
Utility	-2.273	0.050	0.055	0.054	0.074	0.105	0.041	0.051	0.057	0.050	0.064	0.058	0.049	0.060	0.057	0.050	0.060	0.058	0.050
ii	1.025	0.011	0.036	0.048	0.076	0.136	0.049	0.026	0.033	0.036	0.055	0.034	0.038	0.042	0.033	0.034	0.042	0.034	0.035
b	0.072	0.106	0.021	0.000	0.116	0.022	0.000	0.163	0.018	0.000	0.163	0.018	0.000	0.014	0.018	0.000	0.014	0.018	0.000
delta	1.015	0.029	0.081	0.058	0.066	0.166	0.078	0.044	0.093	0.066	0.036	0.092	0.066	0.073	0.093	0.068	0.072	0.092	0.067
r	0.316	0.197	0.000	0.042	0.196	0.000	0.035	0.277	0.000	0.035	0.282	0.000	0.035	0.038	0.000	0.034	0.041	0.000	0.034
Loss		***0.08***	*0.87*	*0.53*	***0.10***	*0.65*	*0.91*	***0.41***	*1.23*	*0.97*	***0.44***	*1.53*	*1.20*	***0.74***	*1.23*	*1.09*	***0.99***	*1.53*	*1.32*
Relative Loss			***10.9***	***6.6***		***6.5***	***9.1***		***3.0***	***2.4***		***3.5***	***2.7***		***1.7***	***1.5***		***1.5***	***1.3***

3.3. Optimal policy under commitment

In this section I use Dynare's 'ramsey' command to obtain the optimal policy under commitment, i.e., the policy functions that yield the minimum expected value (conditional on the information at $t = t_0$, including given initial conditions

for the predetermined variables) of the discounted ad-hoc loss function:

$$\mathcal{L}_{t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} L_t, \tag{78}$$

where the period loss function L_t is given by:

$$\begin{aligned} L_t &= \omega_\pi \left(\pi_t^C - \pi^T\right)^2 + \omega_Y \left(Y_t - Y\right)^2 + \omega_e \left(e_t - e\right)^2 + \omega_r \left(r_t - r\right)^2 + \omega_{\Delta i} \left(\Delta i_t\right)^2 \\ &\quad + \omega_{\Delta\delta} \left(\Delta\delta_t\right)^2, \end{aligned} \tag{79}$$

subject to all the model equations (except, of course, the simple policy rules). I maintain the same definition of CB styles as in the previous section (with $\omega_{\Delta i} = \omega_{\Delta\delta} = 50$). Also, for simplicity I assume that the planner has the same intertemporal discount rate as households ($\beta = 0.99$). In Table 13 I report the standard deviations of the main variables as well as the expected loss for the alternative CB styles (A-F) and the alternative policy regimes (MER, FER, PER).

Table 13: Standard deviations of main variables and losses under optimal policy under commitment

OPTIMAL POLICY UNDER COMMITMENT																				
VARIABLE	MEAN	STANDARD DEVIATION																		
		A			B			C			D			E			F		
		MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER
piC	1.015	0.007	0.035	0.020	0.052	0.145	0.118	0.025	0.036	0.031	0.025	0.036	0.031	0.032	0.036	0.032	0.032	0.036	0.031
DeltaP	1.005	0.004	0.019	0.012	0.037	0.100	0.082	0.015	0.022	0.020	0.015	0.023	0.020	0.020	0.022	0.019	0.020	0.023	0.019
Y	1.443	0.072	0.096	0.070	0.008	0.021	0.019	0.018	0.024	0.023	0.019	0.026	0.024	0.022	0.024	0.024	0.023	0.026	0.025
C	1.312	0.043	0.058	0.060	0.080	0.090	0.089	0.073	0.078	0.077	0.068	0.077	0.077	0.075	0.078	0.072	0.074	0.077	0.073
N	1.322	0.063	0.090	0.068	0.054	0.129	0.110	0.038	0.047	0.046	0.038	0.047	0.046	0.044	0.047	0.044	0.044	0.047	0.044
real_ii	1.010	0.012	0.036	0.047	0.017	0.050	0.043	0.023	0.044	0.037	0.020	0.045	0.038	0.033	0.044	0.035	0.033	0.045	0.035
mc	0.830	0.034	0.075	0.053	0.076	0.091	0.087	0.068	0.070	0.070	0.065	0.071	0.070	0.068	0.070	0.066	0.068	0.071	0.066
e	0.594	0.039	0.049	0.048	0.037	0.051	0.050	0.038	0.052	0.049	0.028	0.051	0.049	0.049	0.052	0.049	0.048	0.051	0.049
TB	0.007	0.051	0.057	0.056	0.047	0.063	0.060	0.052	0.065	0.058	0.052	0.064	0.058	0.058	0.065	0.059	0.056	0.064	0.058
d	1.214	0.135	0.071	0.091	0.121	0.066	0.082	0.129	0.073	0.089	0.112	0.071	0.087	0.091	0.073	0.088	0.090	0.071	0.087
m	0.115	0.004	0.011	0.009	0.012	0.019	0.016	0.009	0.010	0.010	0.009	0.011	0.010	0.009	0.010	0.009	0.009	0.011	0.009
Utility	-2.273	0.050	0.055	0.053	0.064	0.099	0.089	0.054	0.058	0.057	0.053	0.058	0.057	0.057	0.058	0.056	0.056	0.058	0.056
ii	1.025	0.010	0.057	0.044	0.058	0.125	0.103	0.032	0.046	0.040	0.031	0.048	0.042	0.037	0.046	0.038	0.038	0.048	0.039
b	0.072	0.100	0.015	0.000	0.100	0.023	0.000	0.116	0.022	0.000	0.149	0.022	0.000	0.013	0.022	0.000	0.013	0.022	0.000
delta	1.015	0.027	0.076	0.059	0.051	0.150	0.123	0.036	0.080	0.068	0.034	0.080	0.067	0.067	0.080	0.069	0.066	0.080	0.068
r	0.316	0.188	0.000	0.036	0.178	0.000	0.036	0.206	0.000	0.036	0.262	0.000	0.036	0.037	0.000	0.034	0.039	0.000	0.034
Loss		***115.7***	*450.9*	*403.1*	***59.0***	*159.7*	*173.8*	***179.6***	*492.8*	*476.7*	***224.5***	*519.9*	*501.7*	***417.6***	*492.8*	*492.2*	***441.5***	*519.9*	*517.4*
Relative Loss			*3.90*	*3.48*		*2.71*	*2.95*		*2.74*	*2.65*		*2.32*	*2.24*		*1.18*	*1.18*		*1.18*	*1.17*

As expected, the MER regime always dominates the two 'corner' regimes. Under CB styles A, B, and C, the losses under the FER and PER regimes are between 2.65 and 3.90 times the corresponding losses under the MER regime. Under CB styles E and F, where IRs matter for the CB, the losses under the FER and PER regimes are 'only' 17/18% higher than in the MER regime. In CB style B, in which only GDP matters, the FER regime achieves a significantly lower cost than the PER regime. In CB styles A, C, and D, it is the PER regime that is second best. And in CB styles E and F, the two 'corner' regimes obtain losses that are approximately the same.

Tables 14 and 15 show the coefficients of the policy functions in the reduced form (or 'solution' of the DSGE model) corresponding to the instrument variables (in the sense of optimal control theory), i.e., the operational targets (in the economic sense), for the three alternative regimes. These variables[22] are linear functions of

[22] Notice that we show the variables in the tables as they appear in the Dynare output. However, it is necessary to 'read' the variables (contemporaneous or lagged) as their log-linear deviations with respect to their NSS values.

the 9 non-shock predetermined variables (i, δ, r, e, Y, d, Δ, p^C, p^*), the 6 shock variables[23], and the Lagrange multipliers corresponding to the 5 equations with forward-looking terms (the UIP equation, the two dynamic Phillips equations, the consumption Euler equation, and the real interest rate equation). In all of the CB styles there is substantial inertia in the interest rate policy function (between 0.28 and 0.69) and in the nominal depreciation policy function (between 0.1 and 0.6). This is hardly surprising since all these CB styles have been defined to show a significant preference for policy inertia. What is perhaps surprising is the dispersion in the inertial coefficients, given that they all have the same weight for preference for inertia (50). The coefficients on the Lagrange multipliers are relatively small, implying that the policy function coefficients (for the rest of the variables) do not vary much from quarter to quarter when these effects are cumulated (attributable to the commitment to never again re-optimize). The largest of these coefficients correspond to the Lagrange multipliers for the Phillips equations under CB style B, where only GDP matters.

Table 14: Reduced form policy functions under optimal policy under commitment and MER regime

OPTIMAL POLICY UNDER COMMITMENT												
MER												
STYLES:	**A**		**B**		**C**		**D**		**E**		**F**	
	ii	*delta*	ii	*delta*	ii	*delta*	ii	*delta*	ii	*delta*	ii	*delta*
Constant	1.025	*1.015*	1.025	*1.015*	1.025	1.015	1.025	*1.015*	1.025	*1.015*	1.025	*1.015*
ii(-1)	0.689	0.011	0.551	0.325	0.387	0.106	0.376	0.098	0.307	0.040	0.307	0.040
delta(-1)	0.011	0.358	0.325	0.604	0.106	0.302	0.098	0.240	0.040	0.131	0.040	0.130
r(-1)	-0.021	-0.070	0.044	-0.044	0.022	-0.073	0.012	-0.054	-0.227	-0.353	-0.230	-0.344
e(-1)	0.029	-0.564	0.507	-0.484	0.372	-0.685	0.321	-0.883	-0.004	-1.314	-0.002	-1.322
Y(-1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
d(-1)	0.021	0.070	-0.044	0.044	-0.022	0.073	-0.012	0.054	0.227	0.354	0.231	0.345
DeltaP(-1)	0.009	0.007	-0.019	0.016	-0.009	0.031	-0.006	0.042	0.014	0.069	0.014	0.069
pC(-1)	-0.177	0.364	0.008	0.012	0.119	0.195	0.115	0.208	0.105	0.218	0.105	0.218
pStar(-1)	-0.017	-0.182	0.194	-0.184	0.158	-0.235	0.148	-0.237	-0.141	-0.631	-0.145	-0.619
z_piStar(-1)	-0.004	-0.106	0.073	-0.071	0.036	-0.123	0.024	-0.138	-0.058	-0.261	-0.059	-0.260
z_piStarX(-1)	-0.021	-0.126	0.110	-0.106	0.069	-0.163	0.055	-0.136	-0.112	-0.412	-0.115	-0.405
z_G(-1)	0.012	-0.052	0.154	-0.124	0.115	-0.162	0.108	-0.216	-0.021	-0.404	-0.021	-0.402
z_epsilon(-1)	-0.029	-0.031	0.066	-0.053	0.018	-0.114	0.008	-0.161	-0.069	-0.273	-0.069	-0.273
z_iStar(-1)	0.045	0.169	-0.117	0.117	-0.071	0.178	-0.044	0.136	0.390	0.725	0.397	0.708
z_phiStar(-1)	0.035	0.120	-0.075	0.075	-0.038	0.124	-0.022	0.093	0.360	0.583	0.365	0.567
mult_8(-1)	0.000	0.004	0.003	0.006	0.001	0.003	0.001	0.002	0.000	0.001	0.000	0.001
mult_15(-1)	0.005	0.026	0.123	0.165	0.016	0.021	0.016	0.023	0.019	0.027	0.019	0.027
mult_16(-1)	-0.003	0.038	0.161	0.216	0.020	0.025	0.021	0.029	0.024	0.036	0.024	0.036
mult_22(-1)	0.007	-0.002	-0.002	-0.004	0.001	0.001	0.001	0.000	0.001	-0.001	0.001	-0.001
mult_30(-1)	0.001	-0.002	-0.004	-0.005	-0.001	-0.001	-0.001	-0.001	0.000	-0.001	0.000	-0.001
eps_epsilon	-0.036	-0.039	0.083	-0.066	0.022	-0.143	0.011	-0.201	-0.087	-0.341	-0.087	-0.341
eps_G	0.014	-0.061	0.181	-0.145	0.135	-0.191	0.127	-0.254	-0.024	-0.476	-0.025	-0.473
eps_iStar	0.037	0.151	-0.111	0.111	-0.074	0.161	-0.048	0.125	0.266	0.582	0.271	0.570
eps_phiStar	-0.033	-0.114	0.075	-0.075	0.041	-0.120	0.024	-0.089	-0.283	-0.513	-0.288	-0.500
eps_piStar	0.025	-0.255	0.121	-0.123	0.029	-0.257	0.001	-0.391	-0.045	-0.399	-0.041	-0.412
eps_piStarX	-0.052	-0.306	0.269	-0.259	0.168	-0.398	0.135	-0.332	-0.273	-1.006	-0.281	-0.987

[23]Notice that the shock variables appear here because Dynare automatically expresses the transition or policy functions of all variables (including those that are jump variables) in terms of lagged predetermined variables. If, as in Klein (2000) we were to express jump variables in terms of contemporaneous predetermined variables the shocks would not appear in the tables.

In order to study the sensitivity of the expected discounted loss under Ramsey to different parameter values, it is useful to know the structural parameter ranges under which (under a MER regime and Ramsey optimal policy rules) there is stability. A simple way to approach this is to start from the baseline set of parameters used above, and vary each parameter individually using a specific CB style until stability is impaired. I used CB style C. Table 16 shows that there are remarkably wide ranges within which the parameters can be moved individually while maintaining stability. Obviously, in some cases I did not bother to find the actual extremes.

Table 15: Reduced form policy functions under optimal policy under commitment and FER and PER regimes

OPTIMAL POLICY UNDER COMMITMENT												
Regimes:	**FER**						**PER**					
STYLES:	**A**	**B**	**C**	**D**	**E**	**F**	**A**	**B**	**C**	**D**	**E**	**F**
	ii	ii	ii	ii	ii	ii	*delta*	*delta*	*delta*	*delta*	*delta*	*delta*
Constant	1.025	1.025	1.025	1.025	1.025	1.025	1.015	1.015	1.015	1.015	1.015	1.015
ii(-1)	0.635	0.523	0.279	0.277	0.279	0.277	-0.139	0.351	0.002	0.003	0.002	0.003
delta(-1)	-0.086	0.352	0.015	0.016	0.015	0.016	0.185	0.571	0.105	0.105	0.104	0.104
r(-1)	-0.178	-0.014	-0.342	-0.354	-0.342	-0.354	-0.378	-0.056	-0.376	-0.375	-0.379	-0.379
e(-1)	-0.322	0.514	-0.156	-0.153	-0.156	-0.153	-1.259	-0.593	-1.448	-1.450	-1.452	-1.453
Y(-1)	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
d(-1)	0.178	0.014	0.342	0.355	0.342	0.355	0.379	0.057	0.377	0.376	0.380	0.380
DeltaP(-1)	0.032	-0.019	0.022	0.023	0.022	0.023	0.043	0.020	0.074	0.074	0.074	0.074
pC(-1)	-0.176	0.008	0.102	0.102	0.102	0.102	0.404	0.013	0.216	0.216	0.215	0.215
pStar(-1)	-0.237	0.153	-0.273	-0.288	-0.273	-0.288	-0.599	-0.229	-0.676	-0.675	-0.681	-0.680
z_piStar(-1)	-0.084	0.065	-0.098	-0.100	-0.098	-0.100	-0.249	-0.088	-0.272	-0.273	-0.272	-0.273
z_piStarX(-1)	-0.167	0.091	-0.187	-0.197	-0.187	-0.197	-0.366	-0.134	-0.415	-0.416	-0.415	-0.416
z_G(-1)	-0.104	0.149	-0.077	-0.081	-0.077	-0.081	-0.308	-0.155	-0.491	-0.490	-0.505	-0.505
z_epsilon(-1)	-0.112	0.068	-0.101	-0.102	-0.101	-0.102	-0.165	-0.068	-0.290	-0.290	-0.292	-0.292
z_iStar(-1)	0.352	-0.023	0.600	0.622	0.600	0.622	0.737	0.151	0.745	0.745	0.745	0.746
z_phiStar(-1)	0.293	0.014	0.542	0.562	0.542	0.562	0.614	0.099	0.615	0.613	0.619	0.618
mult_9(-1)	-0.001	0.004	0.000	0.000	0.000	0.000	0.002	0.006	0.001	0.001	0.001	0.001
mult_16(-1)	0.009	0.121	0.021	0.021	0.021	0.021	0.035	0.176	0.027	0.027	0.027	0.027
mult_17(-1)	0.003	0.158	0.026	0.026	0.026	0.026	0.052	0.232	0.036	0.036	0.036	0.036
mult_23(-1)	0.006	-0.002	0.001	0.001	0.001	0.001	-0.004	-0.005	-0.001	-0.001	-0.001	-0.001
mult_31(-1)	0.001	-0.004	0.000	0.000	0.000	0.000	-0.002	-0.006	-0.001	-0.001	-0.001	-0.001
eps_epsilon	-0.140	0.085	-0.126	-0.128	-0.126	-0.128	-0.207	-0.085	-0.363	-0.363	-0.364	-0.365
eps_G	-0.122	0.175	-0.090	-0.095	-0.090	-0.095	-0.362	-0.182	-0.578	-0.577	-0.594	-0.594
eps_iStar	0.275	-0.051	0.418	0.435	0.418	0.435	0.567	0.144	0.581	0.582	0.578	0.579
eps_phiStar	-0.256	0.013	-0.426	-0.442	-0.426	-0.442	-0.515	-0.103	-0.526	-0.526	-0.528	-0.528
eps_piStar	-0.053	0.127	-0.076	-0.066	-0.076	-0.066	-0.445	-0.145	-0.452	-0.453	-0.451	-0.451
eps_piStarX	-0.406	0.222	-0.457	-0.480	-0.457	-0.480	-0.892	-0.328	-1.011	-1.014	-1.012	-1.016

The degree of price stickiness (α) in the New Keynesian Phillips equation is often considered an important factor in determining the desirability of alternative exchange regimes. Table 17 shows the losses under each CB style and exchange rate regime for six alternative degrees of price stickiness, which go from practically no price stickiness (α=0.01) to very high price stickiness (α=0.90). As expected, for each CB style and value of α, the MER regime does better and in most cases much better. CB styles E and F are the ones for which the advantage of the MER regime is smallest, especially when there is little price stickiness: for α=0.01, 0.10 and 0.30, the PER regime has a loss which is only 3-5% higher than in the MER regime. This is probably because the CB preference for stabilizing IRs makes it

behave similarly in MER and PER regimes. In the FER regime the excess loss is in the 7-10% range for CB styles E and F. However, for CB styles A, B, and C, the corner regimes have losses between 20% and 260% higher. The highest relative advantage for the MER regime is obtained for high degrees of price stickiness. In general, the PER regime is second best for low degrees of price stickiness ($\alpha \leq 0.30$). For α=0.50, the FER regime is second best only for CB style B (where only GDP matters). And for higher values of α, the FER regime is second best for CB styles A, B, E, and F. Another interesting feature is that the (absolute) losses are not always strictly increasing with α. For example, under CB style B and regime MER, the loss reaches a peak for α=0.30. For the same CB style but regimes FER and PER, the loss does increase monotonously with α. But for CB style A, these regimes reach a peak at α=0.70, while the MER regime has its loss increasing monotonously throughout.

Table 16: Stability ranges for individual non-policy parameters with optimal policy under commitment, MER regime, and CB style C

	Parameter	*Baseline value*	*Stability range*
π^T	piT	1.015	<0.8 - 1.07
β	betta	0.99	<0.8 - 0.999999
σ^C	sigmaC	1.5	0.01 - 50
σ^N	sigmaN	0.5	0.01 - 50
a_D	a_D	0.86	0.35 - 0.99
θ	thetta	6	1.01 - 27
θ^C	thettaC	1.5	0.01 - 0.99 and 1.01 - 50
b^A	bA	0.5	0.01 - 0.99
α	alpha	0.66	0.01 - 0.91
ε_D^{φ}	varepsvarphi_D	2	0.01 - 10000
$\varepsilon_{\mathcal{L}}$	varepsilon_L	1.02	0.3 - 100
γ^D	gammaD	0.5	0.01 - 50

Summing up, with or without price stickiness there is a gain from intervening in the FX market in the sense that the CB can better stabilize its target variables. The advantage is greater when the CB only cares about stabilization inflation and/or GDP (CB styles A, B, or C) and the degree of price stickiness is high (around 0.70 in CB style A, and around 0.90 in CB style B).

Table 17: CB losses with optimal policy under commitment for different values of α

SENSITIVITY TO PRICE STICKINESS UNDER OPTIMAL POLICY UNDER COMMITMENT																		
STYLE	LOSS																	
	alpha=0.01			alpha=0.10			alpha=0.30			alpha=0.50			alpha=0.70			alpha=0.90		
	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER
A	**78.2**	144.1	131.3	**78.7**	149.5	134.9	**82.9**	182.0	157.2	**93.8**	295.1	234.1	**125.8**	432.6	437.5	**168.1**	230.9	249.3
B	**50.0**	74.7	64.7	**52.8**	75.4	65.6	**74.3**	97.7	89.0	**73.3**	139.6	141.8	**57.4**	163.5	179.8	**60.6**	193.5	218.4
C	**148.4**	211.3	194.8	**154.9**	218.9	201.9	**186.8**	279.5	256.6	**211.3**	432.9	399.1	**172.2**	490.3	480.1	**183.1**	520.9	520.4
D	**162.1**	242.8	221.8	**169.0**	250.0	228.6	**202.9**	309.7	282.4	**238.9**	462.7	425.0	**219.4**	516.6	504.6	**235.9**	545.7	543.6
E	**198.2**	211.3	204.0	**205.4**	218.9	211.7	**256.3**	279.5	268.4	**373.3**	432.9	412.9	**416.3**	490.3	496.2	**447.6**	520.9	536.6
F	**225.2**	242.8	231.1	**232.4**	250.0	238.5	**282.8**	309.7	294.3	**398.7**	462.7	439.0	**439.9**	516.6	520.9	**470.7**	545.7	559.8
	RELATIVE LOSS																	
A		*1.84*	***1.68***		*1.90*	***1.71***		*2.19*	***1.90***		*3.15*	***2.50***		***3.44***	*3.48*		***1.37***	*1.48*
B		*1.49*	***1.29***		*1.43*	***1.24***		*1.31*	***1.20***		***1.90***	*1.93*		***2.85***	*3.13*		***3.19***	*3.60*
C		*1.42*	***1.31***		*1.41*	***1.30***		*1.50*	***1.37***		*2.05*	***1.89***		*2.85*	***2.79***		*2.84*	***2.84***
D		*1.50*	***1.37***		*1.48*	***1.35***		*1.53*	***1.39***		*1.94*	***1.78***		*2.35*	***2.30***		*2.31*	***2.30***
E		*1.07*	***1.03***		*1.07*	***1.03***		*1.09*	***1.05***		*1.16*	***1.11***		***1.18***	*1.19*		***1.16***	*1.20*
F		*1.08*	***1.03***		*1.08*	***1.03***		*1.10*	***1.04***		*1.16*	***1.10***		***1.17***	*1.18*		***1.16***	*1.19*

4. Monetary and exchange rate policy and capital flows in the SOE

We have seen that in general the CB can better achieve its goals when it uses two policy rules instead of one. What aspects of the model explain this, and why the differences in losses can be so large, remain to be seen. I start by conjecturing the gain in using two policy rules is related to the CB's ability to influence, to a certain extent, households' foreign debt ratio. The latter determines the endogenous risk premium that foreign agents charge over the international interest rate, which is a primary ingredient in determining the relation between the interest rate differential and the capital flows in and out of the SOE (through the UIP equation). To get some intuition as to why this may be so under simple policy rules, let us first take the log-linear approximations of the UIP equation and the two simple policy rules equations under the MER regime :

$$\widehat{i}_t = E_t\widehat{\delta}_{t+1} + \widehat{i}_t^* + \widehat{\phi}_t^* + \varepsilon_D^{\varphi}\left(\widehat{d}_t + \widehat{e}_t - \widehat{Y}_t\right) \tag{80}$$

$$\widehat{i}_t = h_0\widehat{i}_{t-1} + h_1\widehat{\pi}_t^C + h_2\widehat{Y}_t + h_3\widehat{e}_t \tag{81}$$

$$\widehat{\delta}_t = k_0\widehat{\delta}_{t-1} + k_1\widehat{\pi}_t^C + k_2\widehat{Y}_t + k_3\widehat{e}_t + k_4\left(\widehat{r}_t + \widehat{e}_t - \widehat{Y}_t\right). \tag{82}$$

Leading the third equation, subtracting the resulting equation from the second, and using the first, gives the following equation:

$$\widehat{i}_t^* + \widehat{\phi}_t^* + \varepsilon_D^{\varphi}\left(\widehat{d}_t + \widehat{e}_t - \widehat{Y}_t\right) = \left(h_0\widehat{i}_{t-1} - k_0\widehat{\delta}_t\right) + \left(h_1\widehat{\pi}_t^C - k_1E_t\widehat{\pi}_{t+1}^C\right) \tag{83}$$
$$+ \left(h_2\widehat{Y}_t - k_2E_t\widehat{Y}_{t+1}\right) + \left(h_3\widehat{e}_t - k_3E_t\widehat{e}_{t+1}\right) - k_4\left(E_t\widehat{r}_{t+1} + E_t\widehat{e}_{t+1} - E_t\widehat{Y}_{t+1}\right).$$

On the l.h.s. is the log-linear deviation (from the NSS) of the risk/liquidity premium in the UIP (both the exogenous and endogenous parts). On the r.h.s. is a complex term that exclusively depends on the log-linear deviations of the variables the CB uses for its simple policy rules and the exogenous coefficients in the simple policy rules. Changes in the coefficients on the CB policy rules can thus modify a crucial relation between the (deviations in the) present and next (or preceding, in the case of the interest rate) period endogenous variables whose deviations the CB responds to and the deviation in the households' foreign debt ratio. The policy coefficients thus have an important role in determining what households' foreign debt is in each period, given the values of the international interest rate and risk/liquidity premium $(\widehat{i}_t^* + \widehat{\phi}_t^*)$, both exogenous. For example, when one of the latter is shocked, the policy coefficients help in determining the effects on the households' foreign debt and, hence, international capital flows. The constraints that the respective 'corner' regimes impose (the constancy of one of the potential CB instruments: either $b_t = b$, $\forall t$, or $r_t = r$, $\forall t$, each replacing one of the simple policy rules), imply that the CB has less leeway in affecting international capital flows in the direction that helps it stabilize the economy according to its preferences (or style).

Under the FER regime, in which (82) is replaced by $\widehat{r}_t = 0$, instead of (83) we have:

$$\widehat{i}_t^* + \widehat{\phi}_t^* + \varepsilon_D^{\varphi}\left(\widehat{d}_t + \widehat{e}_t - \widehat{Y}_t\right) = h_0\widehat{i}_{t-1} + h_1\widehat{\pi}_t^C + h_2\widehat{Y}_t + h_3\widehat{e}_t - E_t\widehat{\delta}_{t+1}$$

and under the PER regime, in which (81) is replaced by $\widehat{b}_t = 0$, we have:

$$\widehat{i}_t^* + \widehat{\phi}_t^* + \varepsilon_D^{\varphi}\left(\widehat{d}_t + \widehat{e}_t - \widehat{Y}_t\right) = \widehat{i}_t - k_0\widehat{\delta}_t - E_t\left[k_1\widehat{\pi}_{t+1}^C + k_2\widehat{Y}_{t+1} + k_3\widehat{e}_{t+1} + k_4\left(\widehat{r}_{t+1} + \widehat{e}_{t+1} - \widehat{Y}_{t+1}\right)\right].$$

In both of these corner cases, the CB affects the foreign debt ratio through its interest rate or exchange rate policy, respectively. It therefore also affects the the endogenous part of the risk/liquidity premium, and hence the (domestic) foreign currency interest rate that impinges on the economy. Note that in the particular PER regime in which there is no feedback, the r.h.s. of the last equation is simply $\widehat{i}_t - k_0\widehat{\delta}_t$, and in the fixed exchange rate policy it reduces to $\widehat{i}_t$. The flexibility that the CB achieves by using two simultaneous policy rules generates gains that, at least for the most usual CB styles, can be substantial. Such gains have been measured above, in the context of this particular model, as the reductions in expected loss obtained from using the MER regime instead of any of the corner regimes. Although this argument is more clearly valid in the case of optimal simple rules, in which the optimal coefficients in the policy rules are obtained, given a CB style, through a search in the parameter space of the simple policy rules, it would seem to be also valid for the Ramsey case, since the additional constraints (either $b_t = b$, $\forall t$, or $r_t = r$, $\forall t$) which the corner regimes impose also imply less leeway for CB optimal action.

To see if this conjecture can be validated (or refuted) I now use the optimal policy under commitment framework to study the sensitivity of the expected intertemporal loss to the elasticity of $\varphi_D\left(ed/Y\right)$, i.e., ε_D^{φ}, for each CB style and regime. This elasticity measures how much changes in the foreign debt ratio impact on the domestic interest rate through the arbitrage relation given by the UIP. (114) in Appendix 1 shows that it is linearly related to the elasticity of foreign investor's risk premium function. Table 18 shows that, as conjectured, ε_D^{φ} is extremely relevant in the determination of the relative excess loss which the two corner regimes generate. For each CB style, 1) the corner regimes imply higher losses than the MER regime; 2) the lower is ε_D^{φ}, the lower is the excess loss which the corner regimes imply, and the losses are practically the same for all three regimes when the elasticity is very low ($\varepsilon_D^{\varphi} = 0.01$), at least for styles A-D, and also style E in the FER regime; 3) when $\varphi_D\left(\gamma^D\right)$ is unit elastic or more, the losses under the corner regimes are more than twice as high as in the MER regime under styles A-D; 4) the effect of changes in the elasticity on the expected loss is dramatically different for the two corner regimes in comparison to the MER regime. While in the corner regimes the loss is monotonously decreasing with ε_D^{φ}, the expected loss is monotonously increasing in the MER regime for CB styles B-D, has a maximum at $\varepsilon_D^{\varphi} = 0.5$ for style A and a maximum at $\varepsilon_D^{\varphi} = 1$ for styles E and F.

This exercise confirms the conjecture that the ability of the CB to better affect household indebtedness behavior in order to get nearer to its objectives is considerably greater under the MER regime, at least for the most usual CB styles. On the other hand, there is a much smaller increase in the CB's ability to achieve its objectives through a MER regime when a desired ratio of IRs is an important determinant of CB preferences. In such cases (styles E and F), the excess loss for the corner regimes is in the 17%-18% range when $\varepsilon_D^{\varphi} = 2$.

Table 18: CB losses with optimal policy under commitment for different values of ε_D^{φ}

SENSITIVITY OF LOSS TO ELASTICITY OF UIP UNDER OPTIMAL POLICY UNDER COMMITMENT																		
STYLE	LOSS																	
	varepsvarphiD=2;			varepsvarphiD=1;			varepsvarphiD=0.5;			varepsvarphiD=0.1;			varepsvarphiD=0.05;			varepsvarphiD=0.01;		
	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER
A	115.7	450.9	403.1	122.0	312.3	267.8	130.9	201.4	189.2	129.8	133.5	132.8	124.2	125.1	124.9	116.5	116.6	116.6
B	59.0	159.7	173.8	67.1	143.5	154.0	83.4	134.9	142.5	120.9	128.0	131.2	124.9	127.2	129.5	126.8	127.0	128.1
C	179.6	492.8	476.7	199.3	465.6	455.5	244.5	440.0	435.9	381.0	410.3	410.1	396.2	404.9	405.0	398.7	399.1	399.2
D	224.5	519.9	501.7	237.1	489.9	478.1	270.9	460.8	455.8	393.8	425.6	425.2	409.1	419.0	419.0	411.3	411.9	412.0
E	417.6	492.8	492.2	439.3	465.6	470.3	432.8	440.0	449.2	409.9	410.3	420.6	404.9	404.9	414.8	399.1	399.1	408.1
F	441.5	519.9	517.4	461.8	489.9	493.1	453.1	460.8	469.3	425.2	425.6	435.8	418.9	419.0	428.8	411.9	419.0	420.7
	RELATIVE LOSS																	
A		3.897	*3.483*		2.560	*2.196*		1.539	*1.446*		1.028	*1.023*		1.007	*1.006*		*1.000*	1.001
B		*2.708*	2.946		*2.140*	2.297		*1.617*	1.708		*1.059*	1.085		*1.019*	1.036		*1.001*	1.010
C		2.744	*2.655*		2.336	*2.286*		1.799	*1.783*		1.077	*1.076*		*1.022*	1.022		*1.001*	1.001
D		2.316	*2.235*		2.066	*2.017*		1.701	*1.682*		1.081	*1.080*		1.024	*1.024*		*1.001*	1.002
E		1.180	*1.179*		*1.060*	1.071		*1.017*	1.038		*1.001*	1.026		*1.000*	1.025		*1.000*	1.022
F		1.177	*1.172*		*1.061*	1.068		*1.017*	1.036		*1.001*	1.025		*1.000*	1.024		*1.017*	1.021

For additional confirmation of my conjecture I also studied the sensitivity of the expected loss to the standard deviation of the RW risk/liquidity shock (σ^{ϕ^*}). The results can be seen in Table 19. For a range of values of σ^{ϕ^*} that go from 0.01 to 0.8, I calculated the expected discounted intertemporal loss for the three alternative regimes and six alternative CB styles. As expected, the losses are monotonously increasing in σ^{ϕ^*}. But so are the excess losses of the two corner regimes under styles A-D and in the FER regime for all styles. The case of the excess loss in the PER regime is radically different under styles E and F: the excess loss is decreasing with σ^{ϕ^*}. The excess losses under styles A-D are all more than 400% (higher in the corner regimes than in the MER regime) when the standard deviation is high ($\sigma^{\phi^*} = 0.8$). Also, for standard deviations greater or equal to 0.1, the PER regime is second best (shown in italics and red in the 'relative loss' section) for all styles. The FER regime is second best only for style B and $\sigma^{\phi^*} \leq 0.05$ and for styles B, E and F and $\sigma^{\phi^*} = 0.01$.

Table 19: CB losses with optimal policy under commitment for different values of σ^{ϕ^*}

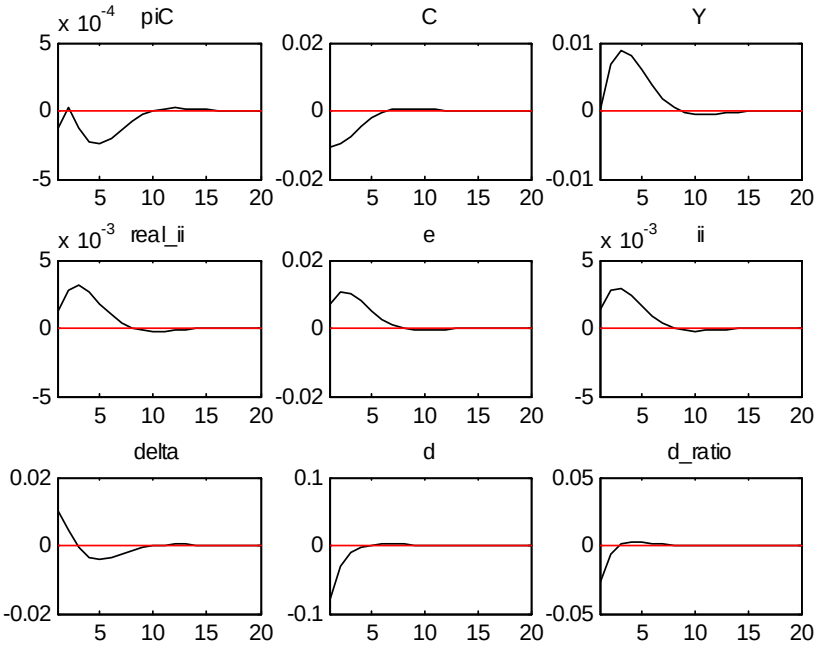
SENSITIVITY OF LOSS TO ST.DEV. OF EXOGENOUS RISK/LIQ. SHOCK UNDER OPTIMAL POLICY UNDER COMMITMENT																		
STYLE	LOSS																	
	sterr(eps_phiStar)=0.8;			sterr(eps_phiStar)=0.5;			sterr(eps_phiStar)=0.3;			sterr(eps_phiStar)=0.1;			sterr(eps_phiStar)=0.05;			sterr(eps_phiStar)=0.01;		
	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER	MER	FER	PER
A	491.6	6528.0	4612.9	261.7	2810.3	2037.5	167.3	1285.0	980.9	120.1	522.4	452.6	115.7	450.9	403.1	114.3	428.1	387.2
B	491.6	5446.0	3596.3	226.9	2212.0	1502.5	118.4	885.3	643.5	64.1	221.9	214.0	59.0	159.7	173.8	57.4	139.8	160.9
C	708.3	8285.6	5157.2	384.8	3518.2	2293.9	252.1	1562.4	1119.1	185.8	584.4	531.8	179.6	492.8	476.7	177.6	463.4	459.1
D	1240.3	10083.7	6674.9	618.8	4232.9	2898.4	363.9	1832.6	1349.0	236.4	632.4	574.4	224.5	519.9	501.7	220.7	483.9	478.5
E	5950.1	8285.6	6191.9	2565.5	3518.2	2705.1	1176.9	1562.4	1274.5	482.7	584.4	559.3	417.6	492.8	492.2	396.7	463.4	470.8
F	7438.6	10083.7	7716.8	3158.0	4232.9	3312.5	1401.9	1832.6	1505.6	523.8	632.4	602.1	441.5	519.9	517.4	415.2	483.9	490.3
	RELATIVE LOSS																	
A		13.279	*9.383*		10.740	*7.787*		7.681	*5.863*		4.349	*3.767*		3.897	*3.483*		3.745	*3.388*
B		11.079	*7.316*		9.748	*6.621*		7.480	*5.437*		3.463	*3.340*		*2.708*	2.946		*2.438*	2.805
C		11.698	*7.281*		9.142	*5.961*		6.196	*4.438*		3.146	*2.862*		2.744	*2.655*		2.610	*2.585*
D		8.130	*5.382*		6.840	*4.683*		5.036	*3.707*		2.675	*2.429*		2.316	*2.235*		2.193	*2.168*
E		1.393	*1.041*		1.371	*1.054*		1.327	*1.083*		1.211	*1.159*		1.180	*1.179*		*1.168*	1.187
F		1.356	*1.037*		1.340	*1.049*		1.307	*1.074*		1.207	*1.149*		1.177	*1.172*		*1.165*	1.181

Finally, I illustrate the advantage of the MER regime by showing some of the IRFs corresponding to a positive shock to ϕ^* (i.e., an adverse liquidity/risk shock) in the case of optimal simple policy rules and CB style A (in which only stabilizing inflation matters). The three graphs below show IRFs for each of the three policy regimes when the CB uses the optimal simple rules found in Table 10 for style A.

Notice that under the MER regime the optimal policy achieves a significantly smaller volatility in inflation, the only target variable in this CB style, than in any

of the 'corner' regimes as a response to the shock.[24] The volatility of consumption and output are also significantly lower in the MER regime, even though GDP is not a target variable.[25] The same can be said of the real interest rate, which increases much less in the MER regime. Finally, optimal policy makes the debt ratio, and hence the endogenous risk premium, fall in all three regimes. Also, the fall in the foreign debt ratio (d_ratio), and hence in the endogenous risk premium, is much greater in the MER regime, resulting in a much smaller increase in the domestic nominal and real interest rate.

Figure 1: lRFs for a positive shock to ϕ^* under optimal simple rules, MER, and style A



[24] Notice that this is only the dynamic response to a single shock (a deterministic exercise), which is much simpler than the exercises above in which the full set of shocks (and their variances) were considered (a stochastic exercise).

[25] The expansionary effect on output is certainly not realistic. It is due to the fact that in this simple model the effect of the RER on exports is contemporaneous instead of lagged (as in the more complicated model in Escudé (2009). Under the calibrations used, the expansionary effect of the adverse shock on exports more than compensates for the contractionary effect on consumption.

Figure 2: lRFs for a positive shock to ϕ^* under optimal simple rules, FER, and style A

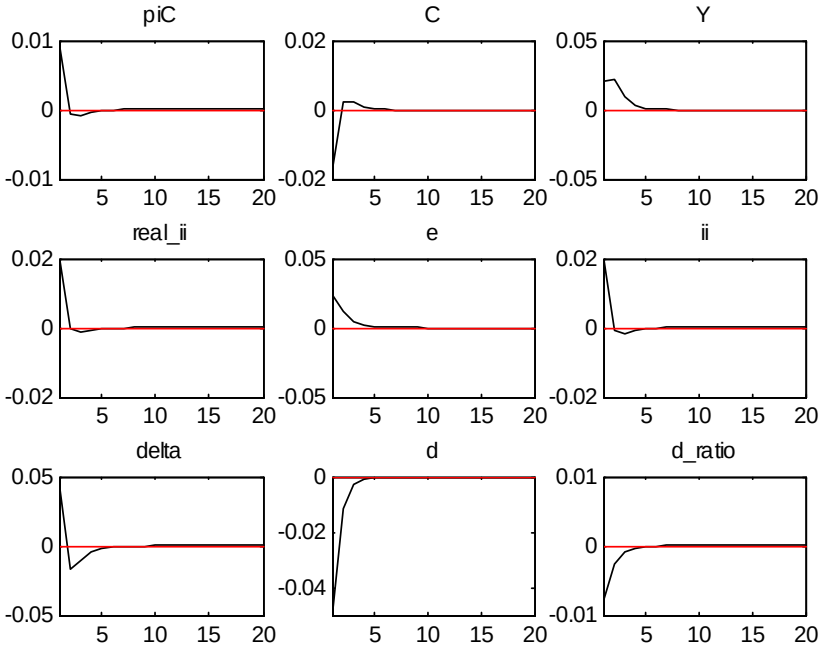
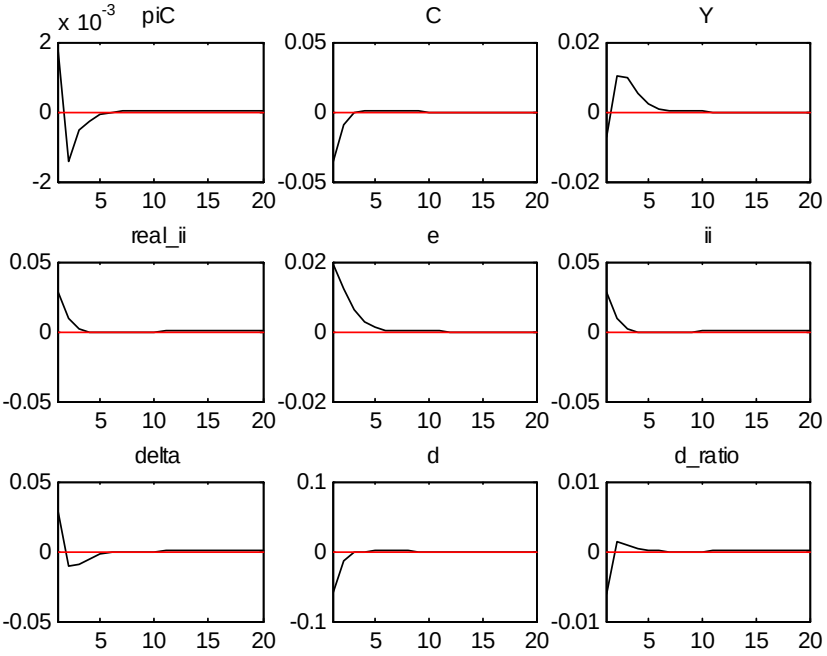


Figure 3: lRFs for a positive shock to ϕ^* under optimal simple rules, PER, and style A



5. Conclusion

This paper tries to bridge the gap between the fact that many central banks systematically intervene both in the domestic bond market (trying to impose a nominal interest rate and often indirectly trying to approach an inflation target) and in

the foreign exchange market, and the absence of any generally accepted model for the representation and analysis of this practice. The paper builds a model that can represent a policy framework in which the CB can simultaneously intervene in the foreign exchange and bond markets, varying its outstanding bond liabilities and reserve assets in order to achieve two operational targets: one for the interest rate and another for the rate of nominal depreciation. For this, the DSGE model includes financial variables and institutional practices ('nuts and bolts' of central banking) that are left out of conventional modeling in which only the extreme policy regimes of a pure float or a pure peg are considered, but cannot be left out when trying build a more general model. The resulting model has a core that is little more than the typical DSGE workhorse of the profession, but extends it in directions which allow for a richer policy framework. The model parameters and steady state values of endogenous variables are calibrated, and the model is implemented in Dynare.

Three alternative policy regimes are considered: the general, two rules regime (denominated Managed Exchange Rate regime), and the two 'corner' regimes of Floating Exchange Rate and Pegged Exchange Rate (both of which use a single simple policy rule or a single control variable). The alternative policy regimes are studied under simple policy rules, optimal simple policy rules (where the coefficients are obtained by minimizing a linear combination of the variances of the target variables), and optimal policy in a linear-quadratic optimal control framework under commitment. First there is a study of the effects of moving individual coefficients of the simple policy rules on the standard deviations of the typical target variables under the MER regime. Then the minimum losses are obtained in the two optimal policy frameworks for a wide range of alternative ad-hoc CB preferences and the three alternative policy regimes. It is shown that the use of two policy rules (or control variables) systematically outperforms any of the corner regimes. For all central bank styles usually considered (that seek low variability of inflation and/or output) substantially better results are achieved when two control variables are used. The reason for this outperformance is shown to derive from the added leverage the CB obtains in exploiting capital flows through its influence on the risk premium function in the UIP equation. By using its interventions to obtain operational targets for both the domestic interest rate and the (actual and expected) rate of nominal depreciation, the CB has greater influence on the foreign debt ratio that determines (endogenous part of) the risk premium in the UIP equation. The CB can get a lower loss when it intervenes in both markets instead of one by manipulating the factor that determines private foreign indebtedness.

We conclude that a policy of systematically intervening in the foreign exchange market through a feedback rule is a valuable complement to any interest rate policy rule framework, and that there are good reasons for defending a managed exchange rate regime as the baseline in any SOE modeling framework. Analogous results are expected in a context of economies that are not small, as long as there are endogenous risk premiums that depend on debt levels. In that case, however, a policy game must be considered, which is left for future research. The Appendixes detail the calibrations used and show some of the IRFs generated by Dynare.

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Appendix 1. Calibration of parameters and derivation of the corresponding non-stochastic steady state

In this Appendix I obtain the calibrated values for the model's parameters and the corresponding non-stochastic steady state (NSS) values of the model variables. There are always many ways of doing this. I calibrate some of the parameters, some ratios and some NSS values of endogenous variables, and obtain the rest sequentially from the static nonlinear equations so that a computer code can follow the same steps if one changes some of the calibrated values or estimates some of them from the data.

A.1.1. Calibration of the components of the external terms of trade

The terms of trade is a particularly important variable for any SOE. Hence, I made a preliminary investigation of the data pertaining to Argentina. To confront (77) with the data, notice that the first two of these equations can be written in terms of the (logs of) price indexes:

$$\begin{aligned}
\Delta \log P_t^{*X} &= \rho^{\pi^{*X}} \Delta \log P_{t-1}^{*X} + \left(1-\rho^{\pi^{*X}}\right) \log \pi^{*X} + \alpha_{\pi^{*X}} \left(\log P_{t-1}^{*X} - \log P_{t-1}^{*N}\right) \\
&\quad + \sigma^{\pi^{*X}} \varepsilon_t^{\pi^{*X}}, \\
\Delta \log P_t^{*N} &= \rho^{\pi^{*}} \Delta \log P_{t-1}^{*N} + (1-\rho^{\pi *}) \log \pi^{*N} + \alpha_{\pi^{*}} \left(\log P_{t-1}^{*X} - \log P_{t-1}^{*N}\right) \\
&\quad + \sigma^{\pi^{*}} \varepsilon_t^{\pi^{*}}.
\end{aligned}$$

A quick estimation for cointegration of Argentina's trade price indexes during 1993Q3-2009Q2 gave the results in the table below (the notation should be obvious). Although empirically I was not able to impose a coefficient of negative one for the second coefficient in the cointegrating relation, I did impose it in the calibration to be consistent with the definition of the terms of trade. I also ignored the small deterministic trend in the cointegrating relation, the two time dummies (first and fourth quarters of 2008) that made the residuals normal, homoscedastic and devoid of serial correlation, as well as the non-significant coefficients. Hence, I use the following specification in the model:

$$\begin{aligned}
\Delta \log P_t^{*X} &= 0.41 \Delta \log P_{t-1}^{*X} + (1-0.41) \log \pi^{*X} - 0.25 \left(\log P_{t-1}^{*X} - \log P_{t-1}^{*N}\right) \\
&\quad + 0.0424 \varepsilon_t^{\pi^{**}}, \\
\Delta \log P_t^{*N} &= 0.20 \Delta \log P_{t-1}^{*N} + (1-0.20) \log \pi^{*N} + 0.18 \left(\log P_{t-1}^{*X} - \log P_{t-1}^{*N}\right) \\
&\quad + 0.18 \Delta \log P_{t-1}^{*X} + 0.0295 \varepsilon_t^{\pi^{*}},
\end{aligned}$$

where, using the notation in (77), $\beta_{\pi^*} = 1$, and $\rho^{\pi^{*XN}} = 0.18$ is added for the effect of $\Delta \log P_{t-1}^{*X}$ on $\Delta \log P_t^{*N}$ (which did not appear in the original specification). Hence, the final specification of the XTT block (77) is:

$$\begin{aligned}
\pi_t^{*X} &= \left(\pi_{t-1}^{*X}\right)^{0.41} \left(\pi^{*X}\right)^{1-0.41} \left(p_{t-1}^{*}\right)^{-0.25} \exp\left(0.0424 \varepsilon_t^{\pi^{**}}\right), \\
\pi_t^{*} &= \left(\pi_{t-1}^{*}\right)^{0.20} \left(\pi^{*}\right)^{1-0.20} \left(p_{t-1}^{*}\right)^{0.18} \left(\pi_t^{*X}\right)^{0.18} \exp\left(0.0295 \varepsilon_t^{\pi^{*}}\right), \\
p_t^{*} &= p_{t-1}^{*} \frac{\pi_t^{*X}}{\pi_t^{*}}.
\end{aligned}$$

Vector Error Correction Estimates
Sample (adjusted): 1993Q3 2009Q2
Included observations: 64 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
LPSTARXLEVEL(-1)	1.0000	
LPSTARNLEVEL(-1)	-1.4924	
	0.1263	
	[-11.8125]	
@TREND(93Q1)	-0.0044	
C	2.3074	
Error Correction:	D(LPSTARXLEVEL)	D(LPSTARNLEVEL)
CointEq1	-0.25543	0.18115
	0.09767	0.06597
	[-2.61520]	[2.74597]
D(LPSTARXLEVEL(-1))	0.40776	0.17699
	0.13273	0.08965
	[3.07203]	[1.97414]
D(LPSTARNLEVEL(-1))	0.15719	0.20080
	0.17834	0.12046
	[0.88142]	[1.66697]
C	-0.00273	-0.00498
	0.00838	0.00566
	[-0.32536]	[-0.87938]
@TREND(93Q1)	0.00021	0.00018
	0.00023	0.00015
	[0.95374]	[1.17769]
D081	0.08543	0.00287
	0.03638	0.02457
	[2.34827]	[0.11686]
D084	-0.15245	-0.12326
	0.03296	0.02226
	[-4.62518]	[-5.53617]
R-squared	0.48888	0.52026
Adj. R-squared	0.43508	0.46976
Sum sq. resids	0.05778	0.02636
S.E. equation	0.03184	0.02151
F-statistic	9.08656	10.30235
Log likelihood	133.50707	158.62000
Akaike AIC	-3.95335	-4.73813
Schwarz SC	-3.71722	-4.50200
Mean dependent	0.00581	0.00029
S.D. dependent	0.04236	0.02953
Determinant resid covariance (dof adj.)		0.00000045
Determinant resid covariance		0.00000035
Log likelihood		293.76131
Akaike information criterion		-8.68004
Schwarz criterion		-8.14032

A.1.2. The NSS relations between parameters and endogenous variables

Eliminating time indexes from the model equations and simplifying gives a set of nonlinear equations that involve both the parameters and NSS values of the endogenous variables. I assume that in the NSS $\epsilon = 1$. I also use the target value for the CB reserves ratio $\gamma^R = er/Y$, the NSS household foreign debt ratio $\gamma^D = ed/Y$ and money ratio $\gamma^M = m/\left(p^C C\right)$. In some cases I divided the equation through by GDP.

Consumption:

$$\frac{1+i}{\pi^C} = \frac{1}{\beta} \tag{84}$$

Risk-adjusted uncovered interest parity:

$$1+i=(1+i^{*})\phi^{*}\varphi_{D}\left(\gamma^{D}\right)\delta \tag{85}$$

Phillips inflation equations:

$$\Gamma = \frac{Q/\left(p^{C}C^{\sigma^{C}}\right)}{1-\beta\alpha\pi^{\theta-1}} \tag{86}$$

$$\Psi = \frac{\theta}{\theta-1}mc\frac{Q/\left(p^{C}C^{\sigma^{C}}\right)}{1-\beta\alpha\pi^{\theta}} \tag{87}$$

$$\frac{\Gamma}{\Psi} = \left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha}\right)^{\frac{1}{\theta-1}}=\widetilde{p}\left(\pi\right)^{-1} \tag{88}$$

Dynamics of price dispersion:

$$\Delta=\left(\frac{1-\alpha}{1-\alpha\pi^{\theta}}\right)\left(\frac{1-\alpha\pi^{\theta-1}}{1-\alpha}\right)^{\frac{\theta}{\theta-1}}. \tag{89}$$

Exports:

$$X=\kappa_{X}\left(ep^{*}\right)^{b_{X}}Y \tag{90}$$

Trade Balance:

$$TB\frac{e}{Y}=\frac{1}{a_{D}}\left[\left(p_{t}^{C}\right)^{1-\theta^{C}}\frac{X}{Y}-\left(1-a_{D}\right)e^{1-\theta^{C}}\right] \tag{91}$$

Current Account:

$$CA\frac{e}{Y}=\left(\frac{1+i^{*}}{\pi^{*}}-1\right)\gamma^{R}-\left[\frac{1+i^{*}}{\pi^{*}}\phi^{*}\tau_{D}\left(\gamma^{D}\right)-1\right]\gamma^{D}+TB\frac{e}{Y} \tag{92}$$

Balance of Payments:

$$CA=0 \tag{93}$$

Real marginal cost:

$$mc=w \tag{94}$$

Labor market clearing:

$$w=\xi^{N}p^{C}C^{\sigma^{C}}\varphi_{M}\left(\gamma^{M}\right)N^{\sigma^{N}} \tag{95}$$

Hours worked:

$$N=Q\Delta \tag{96}$$

Domestic goods market clearing:

$$\frac{Q}{Y}=1-\left(1-b^{A}\right)\frac{X}{Y} \tag{97}$$

GDP:

$$1=a_{D}\frac{\tau_{M}\left(\gamma^{M}\right)G}{\left(p^{C}\right)^{1-\theta^{C}}}\frac{p^{C}C}{Y}+\frac{X}{Y} \tag{98}$$

Consumption relative price:

$$p^C = \left(a_D + (1 - a_D)\, e^{1-\theta^C}\right)^{\frac{1}{1-\theta^C}} \tag{99}$$

Money market balance:

$$m = \mathcal{L}\,(1 + i)\, p^C C, \tag{100}$$

CB balance sheet:

$$\frac{b}{Y} = \gamma^R - \gamma^M \frac{p^C C}{Y} \tag{101}$$

Consumption inflation:

$$\pi^C = \pi \tag{102}$$

Real Exchange Rate:

$$\delta\pi^* = \pi \tag{103}$$

External terms of trade:

$$\pi^{*X} = \pi^* \tag{104}$$

Tax collection:

$$tax = \overline{G} p^C C - qf$$

Quasi-fiscal surplus:

$$qf = (1 + i^* - 1/\delta)\, \frac{er}{\pi^*} - ((1 + i) - 1)\, \frac{b}{\pi}$$

Interest rate feedback rule:

$$1 = \left(\frac{\pi^C}{\pi^T}\right)^{h_1} \tag{105}$$

Nominal depreciation feedback rule:

$$1 = \left(\frac{\pi^C}{\pi^T}\right)^{k_1} \left(\frac{er/Y}{\gamma^R}\right)^{k_4} \tag{106}$$

Exports inflation shock

$$1 = (p^*)^{\alpha_{\pi^{**}}} \tag{107}$$

Imported inflation shock

$$1 = (p^*)^{\alpha_{\pi^*}} \left(\pi^{*X}\right)^{\rho^{\pi^{*XN}}}. \tag{108}$$

I now show one way in which the EENE values of the model's variables and the calibrated values of parameters can be obtained sequentially.

(105) implies $\pi^C = \pi^T$, since $h_1 \neq 0$ is assumed. Inserting this in (102) yields $\pi = \pi^T$. Also, (107) implies that the XTT is $p^* = 1$, and hence (108) implies that $\pi^{*X} = 1$, and (104) that $\pi^* = 1$. Therefore, (103) implies $\delta = \pi^T$. Summing up, we have:

$$\pi = \delta = \pi^C = \pi^T, \quad \text{and} \quad \pi^* = \pi^{*X} = p^* = 1.$$

Hence, (84) gives the nominal interest rate: $1 + i = \pi^T/\beta$ and (106) yields $er/Y = \gamma^R$, since it is assumed that $k_4 \neq 0$, which implies that the CB's target ratio of international reserves to GDP is attained in the NSS.

I assume $\beta = 0.99$. For illustrative purposes I use as Argentina's NSS GDP its 2010 level (at 2010 prices and in trillions of pesos): $Y = 1.443$. The gross exogenous risk/liquidity premium for households and the RW gross interest rate are assumed to be $\phi^* = 1.005^{0.25}$ and $1 + i^* = 1.03^{0.25}$, respectively. Also, the household ratios are $\gamma^D \equiv ed/Y = 0.5$, $\gamma^M \equiv m/p^C C = 0.095522$, and Government to household consumption ratio is assumed to be $G = 1.19$.

The home bias parameter (or share of domestic goods) in household consumption is calibrated to $a_D = 0.86$. The constant relative risk aversion for labor (which is also the inverse of the elasticity of labor supply with respect to the real wage) and consumption are: $\sigma^N = 0.5$ and $\sigma^C = 1.5$, respectively. Finally, I assume that the elasticity of substitution between varieties of domestic goods is $\theta = 6$ and the elasticity of substitution between the bundles of domestic and imported goods is $\theta^C = 1.5$. Assuming that the exogenous parameter for exports demand is $b_A = 0.5$, yields $b_X \equiv \left(1 - b^A\right)^{-1} = 2$ and $\kappa_X \equiv \left(b^A\right)^{b^A b^X} = 0.5$.

I now focus on the NSS values of the remaining endogenous variables and parameters.

A.1.2.1 The endogenous risk premium

Using (84), (102), and (103) in the UIP equation (85) gives the household foreign debt to GDP ratio as a function of parameters which I have already calibrated:

$$\gamma^D \equiv \frac{ed}{Y} = \varphi_D^{-1}\left(\frac{1/\beta}{\phi^*\left(1+i^*\right)/\pi^*}\right) = \varphi_D^{-1}\left(\frac{1}{\beta\phi^*\left(1+i^*\right)}\right).$$

However, calculating this requires the values of the exogenous parameters α_1 and α_2 which help define the function φ_D. I now seek to calibrate them in terms of the more intuitive elasticity of the risk premium function in the UIP (which plays a critical role in the present research). First, notice that the elasticity $\overline{\varepsilon}_D$ of $\overline{\tau}_D$ is

$$\overline{\varepsilon}_D\left(\gamma_t^D\right) \equiv \frac{\alpha_2 \gamma_t^D}{1 - \alpha_2 \gamma_t^D}. \tag{109}$$

$\overline{\tau}_D$ and $\overline{\varphi}_D$ are related to $\overline{\varepsilon}_D$ by (see (67) and (69)):

$$\begin{aligned} \overline{\tau}_D\left(\gamma_t^D\right) &= \alpha_1\left[1 + \overline{\varepsilon}_D(\gamma_t^D)\right], \\ \overline{\varphi}_D\left(\gamma_t^D\right) &= \alpha_1\left[1 + \overline{\varepsilon}_D(\gamma_t^D)\right]^2. \end{aligned} \tag{110}$$

Hence, if the NSS values of $\overline{\varepsilon}_D$ and γ^D are calibrated, (109) gives the value of α_2:

$$\alpha_2 = \frac{1}{\gamma^D\left(\frac{1}{\overline{\varepsilon}_D} + 1\right)}. \tag{111}$$

Also, using (110), (66), and (84) in (85) yields:

$$\overline{\varphi}_D\left(\gamma^D\right) = \frac{1+i}{(1+i^*)\delta\phi^*} - 1 = \frac{1}{\beta\left(1+i^*\right)\phi^*} - 1 = \alpha_1\left(1 + \overline{\varepsilon}_D\right)^2, \tag{112}$$

which gives the value of α_1:

$$\alpha_1 = \frac{1}{(1+\overline{\varepsilon}_D)^2}\left(\frac{1}{\beta\,(1+i^*)\,\phi^*}-1\right) = \left(1-\alpha_2\gamma^D\right)^2\left(\frac{1}{\beta\,(1+i^*)\,\phi^*}-1\right), \quad (113)$$

where the second equality is derived from (111).

However, because of the critical role of the derived function φ_D in the UIP equation (71) it is perhaps more intuitive in calibrations to start with the value of the elasticity of φ_D, which I denote as ε_D^{φ}, along with γ^D, and derive the value of $\overline{\varepsilon}_D$. It is straightforward to prove that ε_D^{φ} and $\overline{\varepsilon}_D$ are related by:

$$\varepsilon_D^{\varphi} = \overline{\varepsilon}_D \frac{2}{\gamma^D}\frac{\overline{\varphi}_D}{1+\overline{\varphi}_D} = \overline{\varepsilon}_D \frac{2}{\gamma^D}\left[1-\beta\,(1+i^*)\,\phi^*\right], \quad (114)$$

where the second equality uses (112). Hence, using (111), (113) and (109):

$$\begin{aligned}
\alpha_2 &= \frac{1}{\frac{2}{\varepsilon_D^{\varphi}}\left[1-\beta\,(1+i^*)\,\phi^*\right]+\gamma^D} \\
\alpha_1 &= \left(1-\alpha_2\gamma^D\right)^2\left(\frac{1}{\beta\,(1+i^*)\,\phi^*}-1\right).
\end{aligned}$$

If, say, $\varepsilon_D^{\varphi} = 2$ then

$$\begin{aligned}
\alpha_2 &= \frac{1}{\left(1-0.99\left(1.03^{0.25}\right)1.005^{0.25}\right)+0.5} = 1.9944 \\
\alpha_1 &= (1-1.9944*0.5)^2\left(\frac{1}{0.99\left(1.03^{0.25}\right)1.005^{0.25}}-1\right) = 1.1092\times 10^{-8}
\end{aligned}$$

and hence:

$$\begin{aligned}
\overline{\tau}_D &= \frac{1.1092\times 10^{-8}}{1-1.9944*0.5} = 3.9614\times 10^{-6} \\
\overline{\varphi}_D &= \frac{1.1092\times 10^{-8}}{(1-1.9944*0.5)^2} = 1.4148\times 10^{-3}.
\end{aligned}$$

A.1.2.2 The balance of payments

Using the previous calibrations, (93) and (92) give the trade balance to GDP ratio necessary to sustain net interest payments abroad:

$$\begin{aligned}
TB\frac{e}{Y} &= \left[\frac{1+i^*}{\pi^*}\phi^*\tau_D-1\right]\gamma^D-\left(\frac{1+i^*}{\pi^*}-1\right)\gamma^R \\
&\quad \left(\frac{1.03^{0.25}}{1}1.005^{0.25}\left(1.0000039368\right)-1\right)0.5-\left(\frac{1.03^{0.25}}{1}-1\right)0.13 \\
&= 0.00337476
\end{aligned}$$

Then, using (91), (90), and (99), one can obtain the RER necessary to generate this trade surplus:

$$\begin{aligned}
\kappa_X \left(ep^*\right)^{b_X} \left[a_D + (1-a_D)\, e^{1-\theta^C}\right] - (1-a_D)\, e^{1-\theta^C} &= a_D TB \frac{e}{Y} \\
0.5e^2 \left[0.86 + (1-0.86)\, e^{1-1.5}\right] - (1-0.86)\, e^{1-1.5} &= 0.86\,(0.00337476) \\
e &= 0.595055
\end{aligned}$$

and hence the exports to GDP ratio and p^C:

$$\begin{aligned}
\frac{X}{Y} &= \kappa_X \left(ep^*\right)^{b_X} = 0.5\,(0.595055)^2 = 0.177\,045, \\
p^C &= \left(0.86 + (1-0.86)\,(0.595055)^{1-1.5}\right)^{\frac{1}{1-1.5}} = 0.921915
\end{aligned}$$

A.1.2.3 The transactions cost function and money demand

The elasticity of $\mathcal{L}(1+i)$ (see (70)) can be shown to satisfy the following relation:

$$\varepsilon_{\mathcal{L}}\left(\gamma^M\right) = \frac{1}{(\beta_3+1)\, i}\left(1 + \frac{1}{\beta_2 \gamma^M}\right), \tag{115}$$

from which we obtain:

$$\beta_2 = \frac{1}{\gamma^M} \frac{1}{(\beta_3+1)\, \varepsilon_{\mathcal{L}} i - 1}.$$

Also, reshuffling (70) gives:

$$\beta_1 = \frac{\left(1+\beta_2\gamma^M\right)^{\beta_3+1}}{\beta_2\beta_3}\left(1 - \frac{1}{1+i}\right).$$

So using the last two expressions in (68) to eliminate β_1 and β_2 gives:

$$\overline{\tau}_M\left(\gamma^M\right) = \left(1 + \frac{1}{\beta_3}\right)\left(1 - \frac{1}{1+i}\right) i\varepsilon_{\mathcal{L}}\left(\gamma^M\right)\gamma^M. \tag{116}$$

Since transaction costs are dependent on the inflation rate (through the nominal interest rate) I cannot calibrate the three parameters β_1, β_2, and β_3 without first calibrating the inflation rate. I assume that the target inflation rate is $\pi^T = 1.015$. Hence, the nominal interest rate is given by (84): $1+i = 1.015/0.99 = 1.0253$. Next, calibrate the value of the interest elasticity of money demand to, say, $\varepsilon_{\mathcal{L}} = 1.02$. We also have $\gamma^M = 0.095522$. Notice that to have β_2 positive, β_3 must be sufficiently high (and hence $\overline{\tau}_M$ sufficiently low):[26]

[26] Although this level of transaction costs may seem unrealistically low, we really do not care much about transaction costs per se but only their effect on money demand. To have more realistic levels of transaction costs we would need a different transaction costs function. As long as we are confortable with the resulting interest elasticity of money demand and the assumed stock of money, we can hold on to the present function.

$$\begin{aligned}\beta_3 &> \frac{1}{\varepsilon_{\mathcal{L}} i} - 1 = \frac{1}{1.02\,(1.015/0.99 - 1)} - 1 = 37.82352941\\ \overline{\tau}_M &< \left(1 + \frac{1}{37.82352941}\right)\left(1 - \frac{1}{1.015/0.99}\right)\left(\frac{1.015}{0.99} - 1\right) 0.095522 * 1.02\\ &= 0.00006220357.\end{aligned}$$

If, say, $\beta_3 = 160$, then:

$$\begin{aligned}\beta_2 &= \frac{1}{0.09553}\frac{1}{(160+1)\,1.02\left(\frac{1.015}{0.99} - 1\right) - 1} = 3.32635\\ \beta_1 &= \frac{(1 + 3.32635 * 0.095522)^{160+1}}{3.32635 * 160}\left(1 - \frac{0.99}{1.015}\right) = 9.07697 \times 10^{14}.\end{aligned}$$

Hence:

$$\begin{aligned}\overline{\tau}_M &= \frac{\beta_1}{(1 + \beta_2 \gamma^M)^{\beta_3}} = \frac{9.07697 \times 10^{14}}{(1 + 3.32635 * 0.095522)^{160}} = 6.0984 \times 10^{-5}\\ \overline{\varphi}_M &= \overline{\tau}_M \left(1 + \beta_3 \frac{\beta_2 \gamma^M}{1 + \beta_2 \gamma^M}\right) = 6.0984 \times 10^{-5}\left(1 + 160\frac{3.32635 * 0.095522}{1 + 3.32635 * 0.095522}\right)\\ &= 2.41374 \times 10^{-3}.\end{aligned}$$

Finally, using (98), the consumption to GDP ratio is:

$$\begin{aligned}\frac{p^C C}{Y} &= \frac{\left(p^C\right)^{1-\theta^C}}{a_D \tau_M \left(\gamma^M\right) G}\left[1 - \kappa_X \left(ep^*\right)^{b^X}\right]\\ &= \frac{(0.921915)^{1-1.5}}{0.86 * 1.00006098 * 1.19}\left(1 - 0.177\,045\right) = 0.837457.\end{aligned}$$

Hence, C and Q can be obtained:

$$\begin{aligned}C &= \frac{p^C C}{Y}\frac{Y}{p^C} = 0.837457 * \frac{1.443}{0.921915} = 1.\,310\,8\\ Q &= \left[1 - \left(1 - b^A\right)\frac{X}{Y}\right] Y = \left[1 - (1 - 0.5)\,0.177\,045\right] 1.443 = 1.\,315\,26.\end{aligned}$$

A.1.2.4 Inflation, price dispersion and marginal cost

(89) shows NSS price dispersion as a function of the NSS inflation rate. It is easy to check that this function has a local minimum at $\pi = 1$, where there is price stability and no price dispersion ($\Delta = 1$). Given the above calibrations, the NSS value of price dispersion is:

$$\Delta = \frac{1 - 0.66}{1 - 0.66\,(1.015)^6}\left(\frac{1 - 0.66\,(1.015)^{6-1}}{1 - 0.66}\right)^{\frac{6}{6-1}} = 1.0051.$$

Hence, (97) gives the value of hours worked:

$$N = Q\Delta = 1.31526 * 1.0051 = 1.32197,$$

(86) gives the value of Γ:

$$\Gamma = \frac{Q/\left(p^C C^{\sigma^C}\right)}{1-\beta\alpha\pi^{\theta-1}} = \frac{1.315\,26/\left(0.921915 * 1.310\,8^{1.5}\right)}{1-0.99*0.66*1.015^{6-1}} = 3.210508,$$

(88) gives the value of Ψ:

$$\Psi = \Gamma\left(\frac{1-\alpha}{1-\alpha\pi^{\theta-1}}\right)^{\frac{1}{\theta-1}} = 3.210508\left(\frac{1-0.66}{1-0.66*1.015^{6-1}}\right)^{\frac{1}{6-1}} = 3.31659,$$

and (87) gives the value of mc:

$$\begin{aligned} mc &= \Psi \Big/ \left(\frac{\theta}{\theta-1}\frac{Q/\left(p^C C^{\sigma^C}\right)}{1-\beta\alpha\pi^{\theta}}\right) \\ &= 3.316\,59 \Big/ \left(\frac{6}{6-1}\frac{1.315\,26/\left(0.921915 * 1.310\,8^{1.5}\right)}{1-0.99*0.66*1.015^{6}}\right) = 0.830\,17. \end{aligned}$$

Finally, (94) and (95) give the value of ξ^N:

$$\begin{aligned} \xi^N &= mc/\left(p^C C^{\sigma^C}\varphi_M N^{\sigma^N}\right) \\ &= 0.83017\Big/\left(0.921915 * 1.3108^{1.5} 1.0024137 * 1.32197^{0.5}\right) = 0.520612, \end{aligned}$$

and the NSS value of period aggregate utility is:

$$\text{Utility} = \frac{1.3108^{1-1.5}}{1-1.5} - 0.520612\frac{1.32197^{1+0.5}}{1+0.5} = -2.2744.$$

The fact that it is negative is irrelevant, since utility has only ordinal, not cardinal, significance.

Appendix 2. Impulse Response Functions for Optimal Policies in the MER regime under CB styles A, B, and C

All shocks in the IRFs below are positive and of 1 standard deviation. Shock variables are in logs in the nonlinear model.

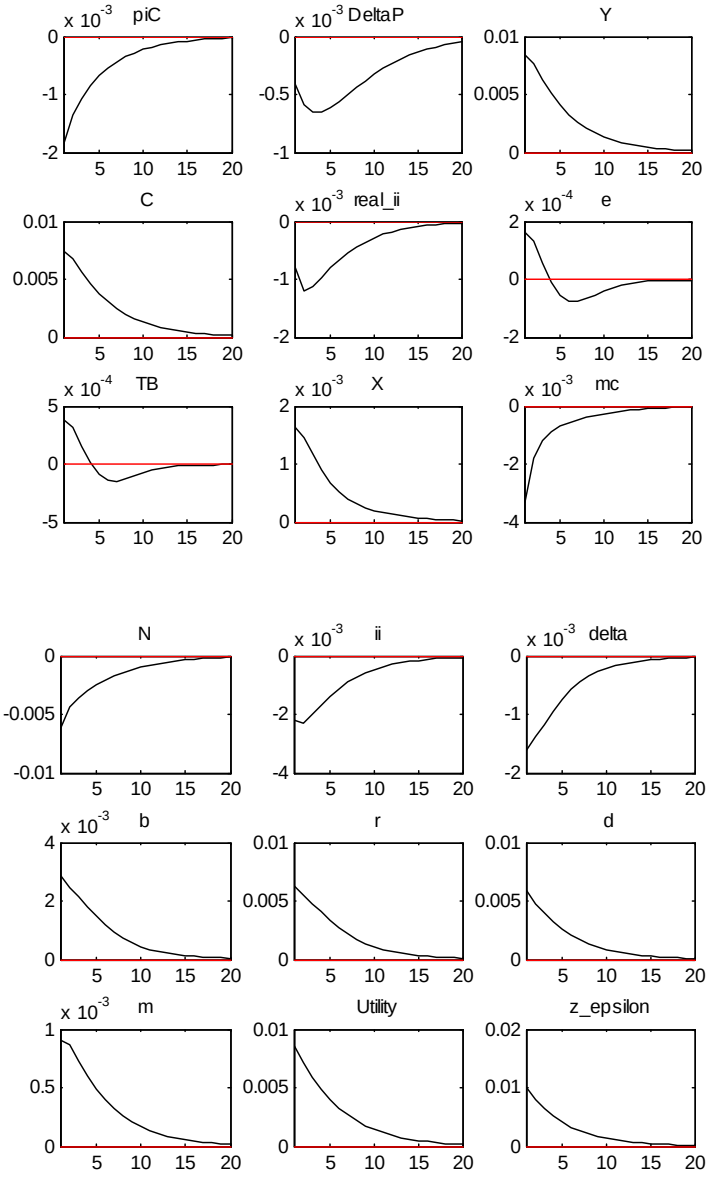
A.2.1 Optimal Simple Policy Rules

A.2.1.1 Central Bank style A

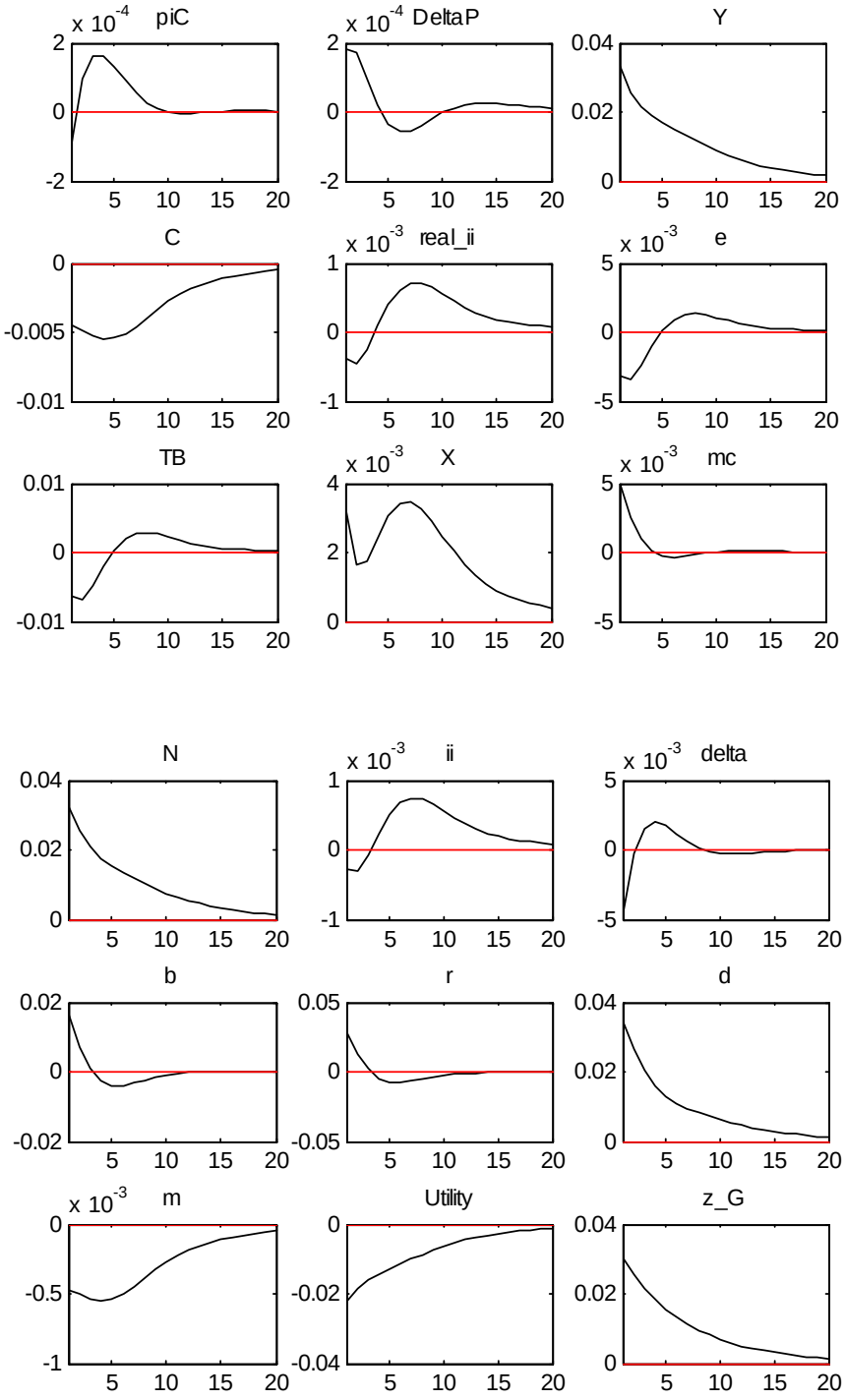
$$\omega_\pi = 100, \quad \omega_Y = 1, \quad \omega_e = 1, \quad \omega_r = 1, \quad \omega_{\Delta i} = 50, \quad \omega_{\Delta\delta} = 50$$

h_0	h_1	h_2	h_3	k_0	k_1	k_2	k_3	k_4
0.33	1.26	0.02	0.12	−0.03	−0.07	−0.08	−0.43	−0.08

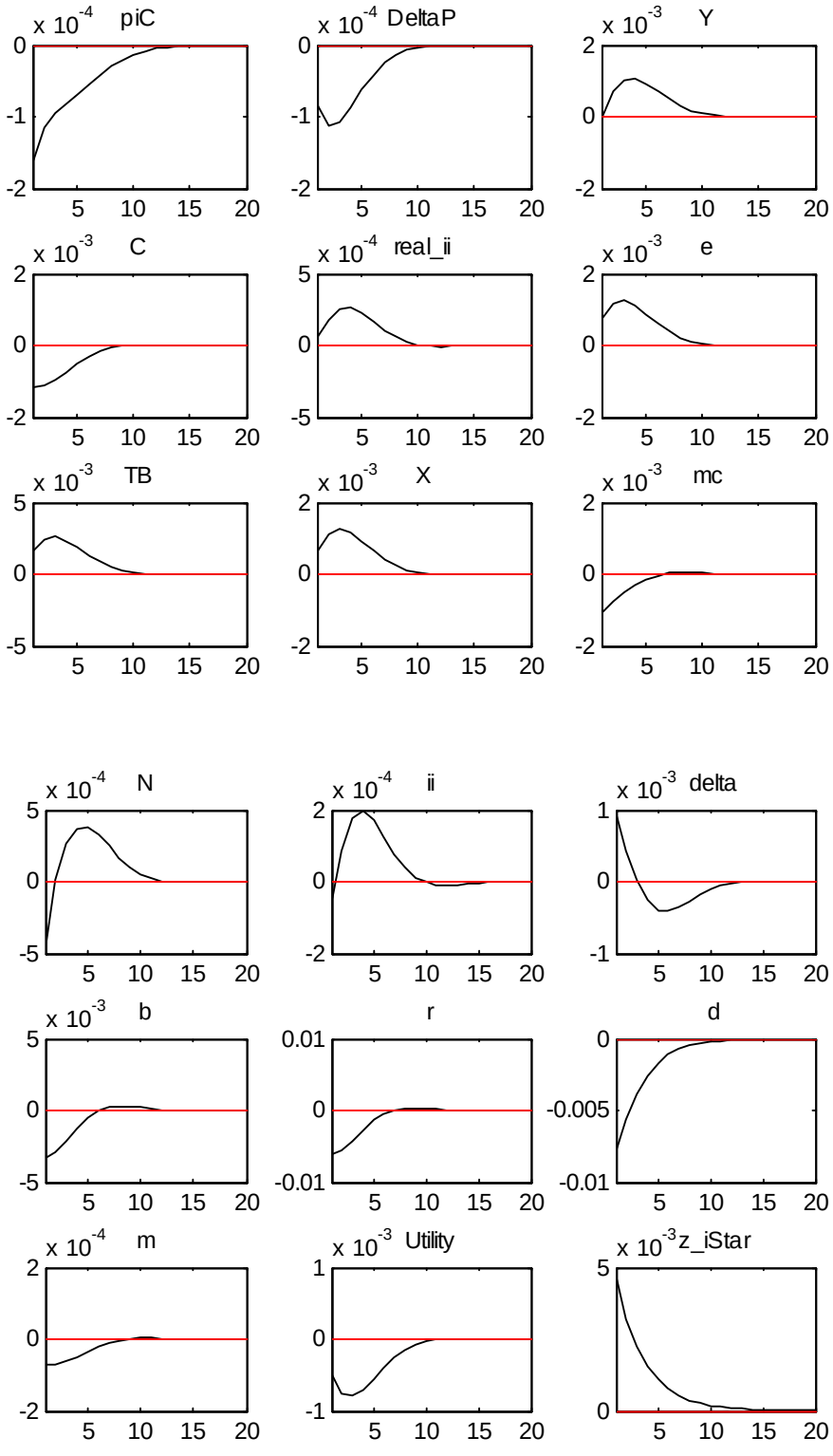
Response to a positive shock to domestic sector productivity: ϵ



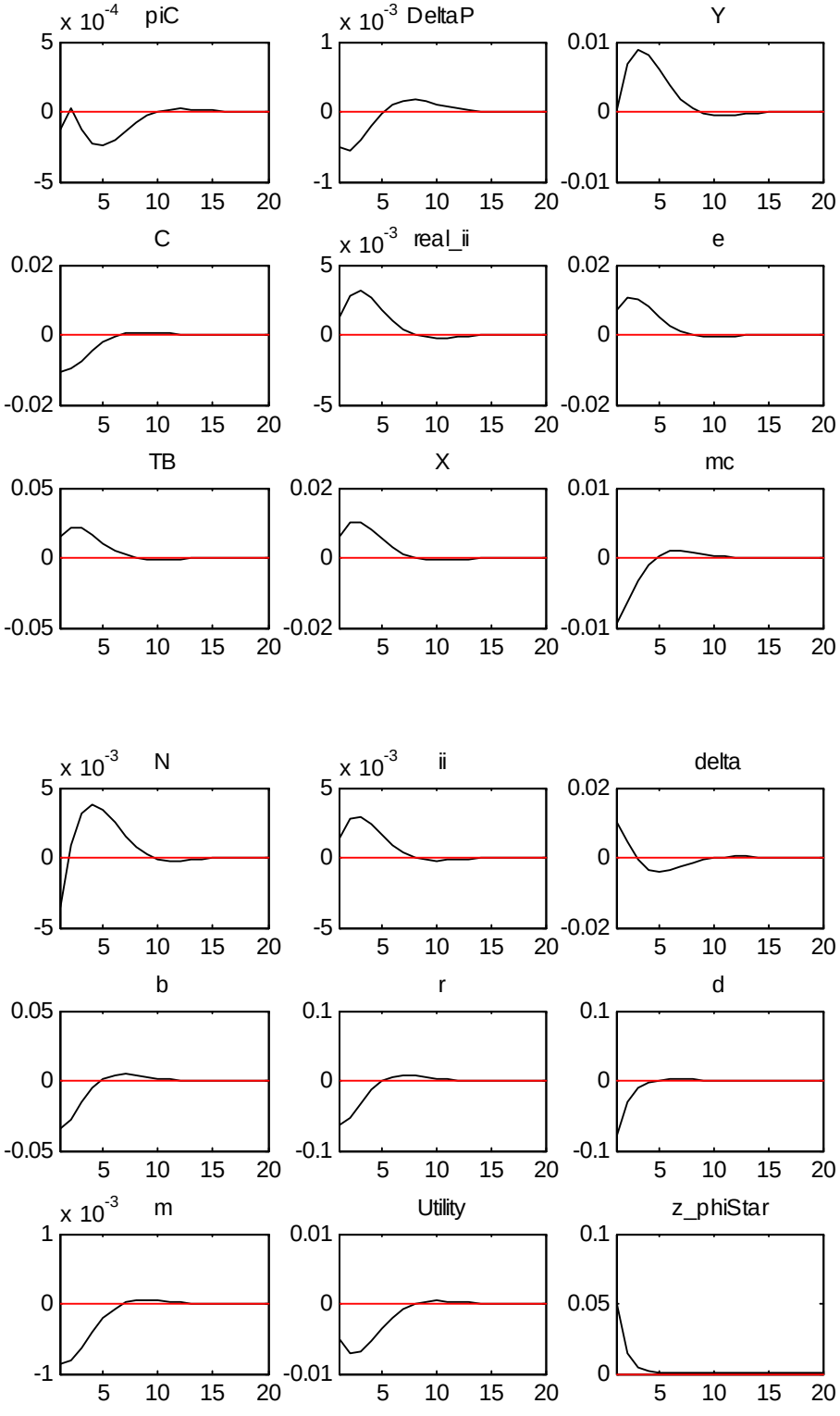
Response to a positive shock to government expenditures: G



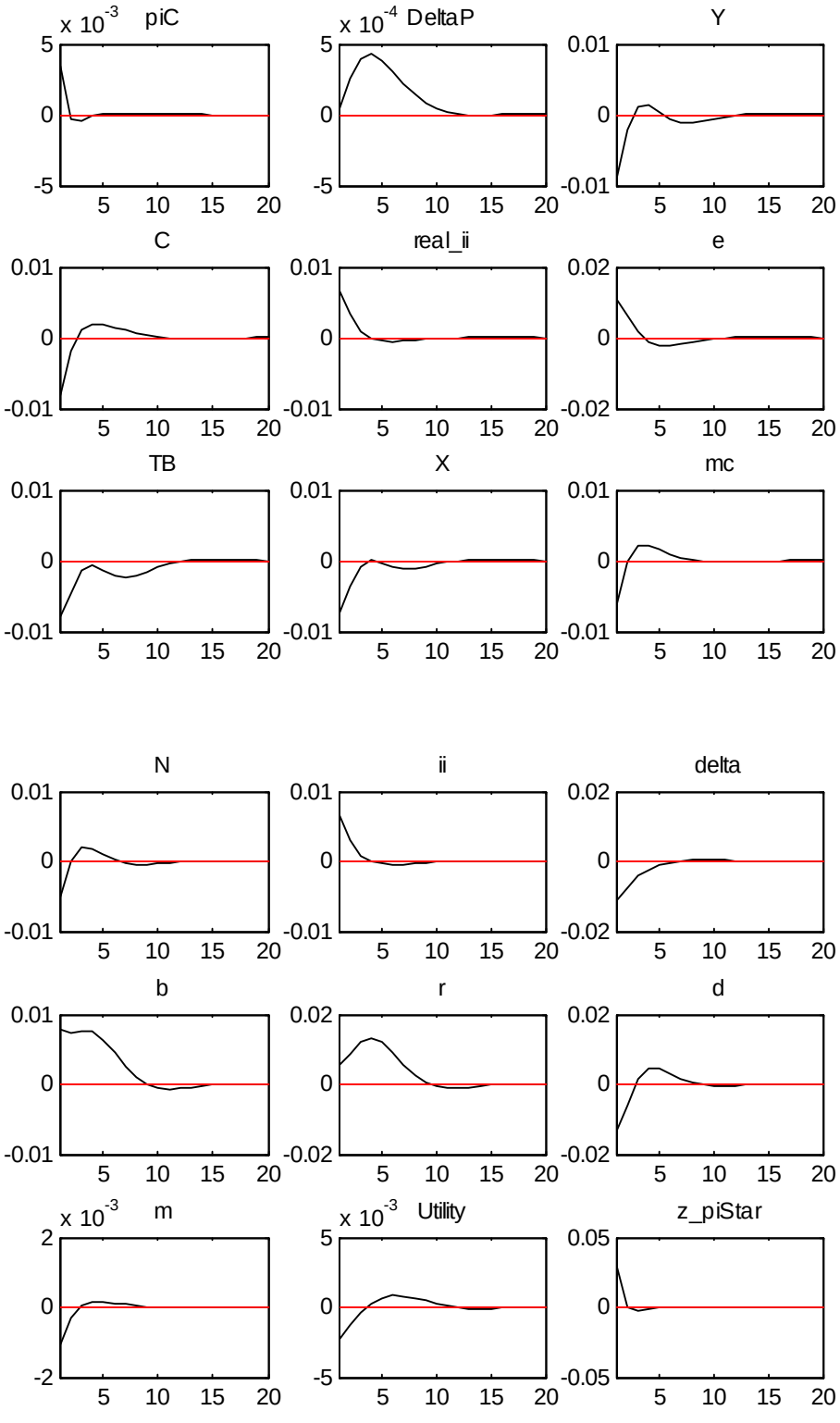
Response to a positive shock to the RW interest rate: i^*



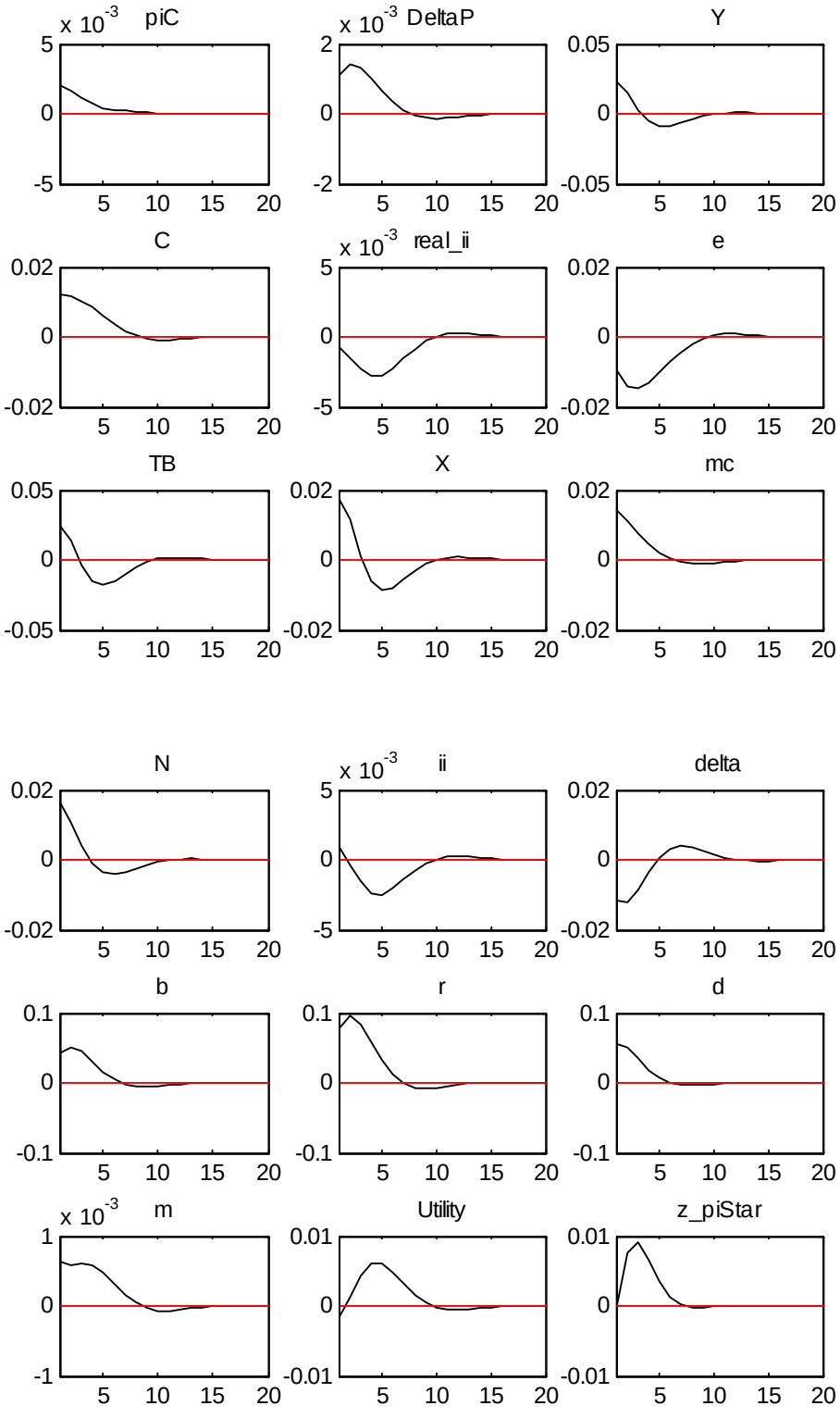
Response to a positive shock to the SOE's exogenous risk/liquidity premium: ϕ^*



Response to a positive shock to imports inflation: π^*



Response to a positive shock to exports inflation: π^{*X}

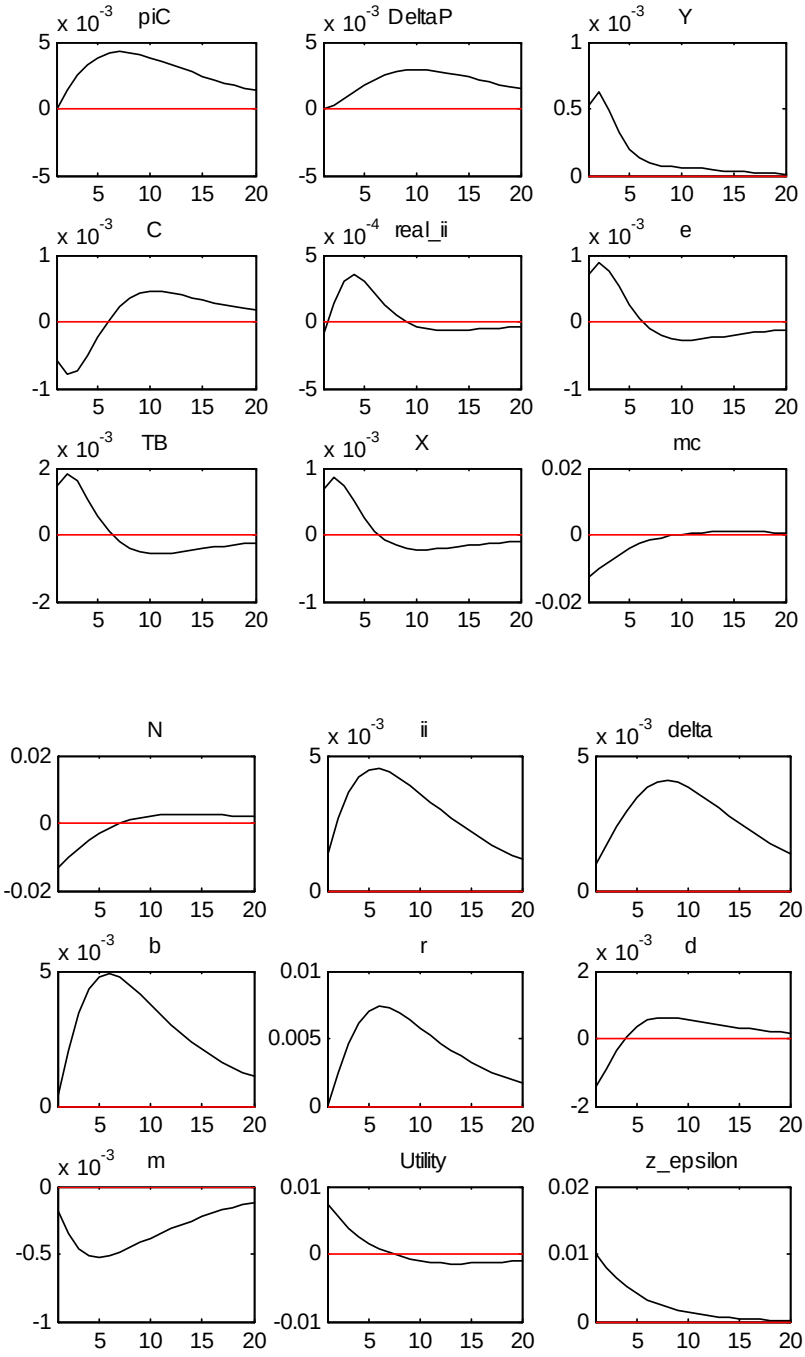


A.2.1.2 Central Bank style B

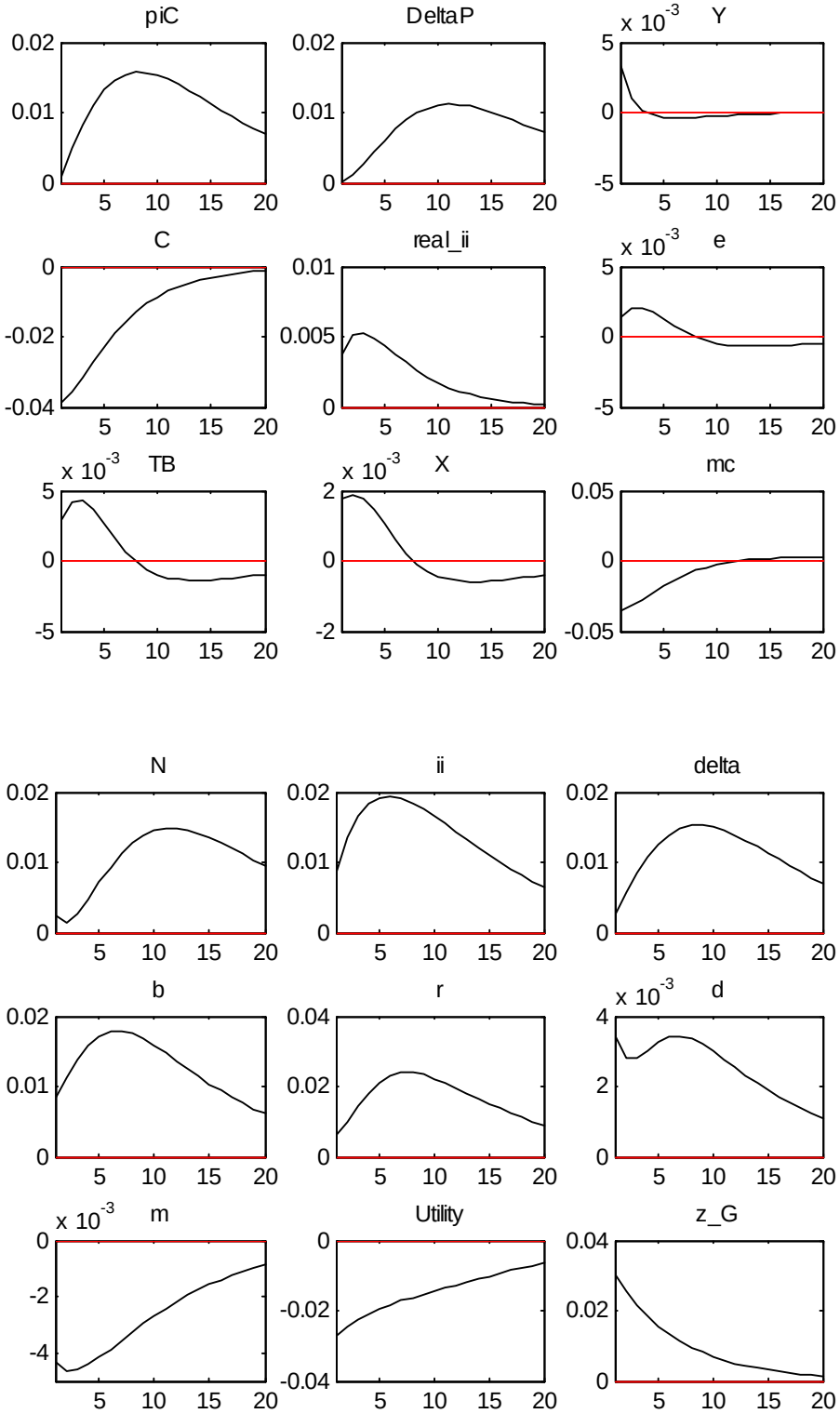
$$\omega_\pi = 1, \quad \omega_Y = 100, \quad \omega_e = 1, \quad \omega_r = 1, \quad \omega_{\Delta i} = 50, \quad \omega_{\Delta\delta} = 50$$

h_0	h_1	h_2	h_3	k_0	k_1	k_2	k_3	k_4
1.86	−1.01	4.34	−0.21	3.08	−3.92	−2.28	1.18	0.40

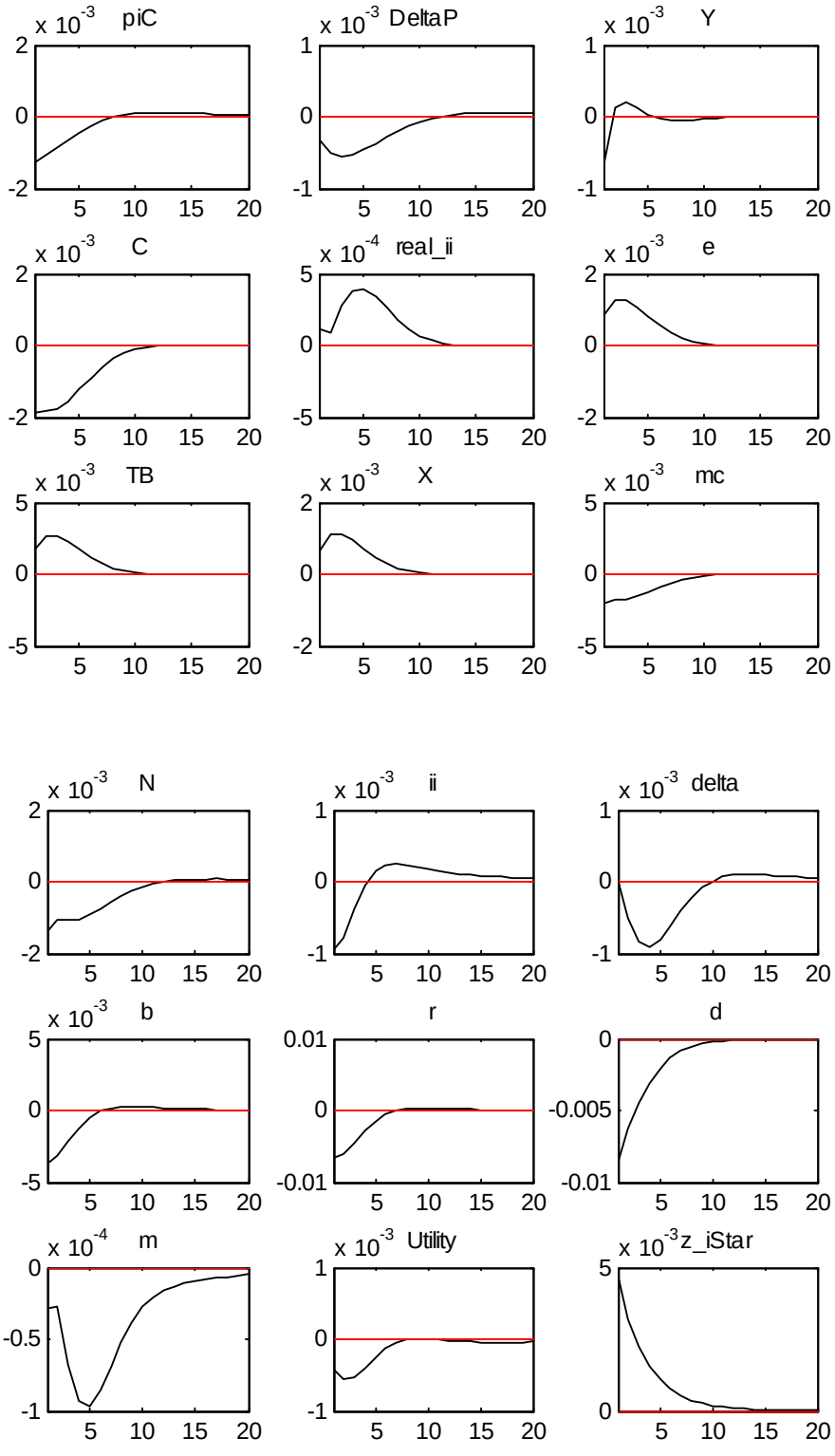
Response to a positive shock to domestic sector productivity: ϵ



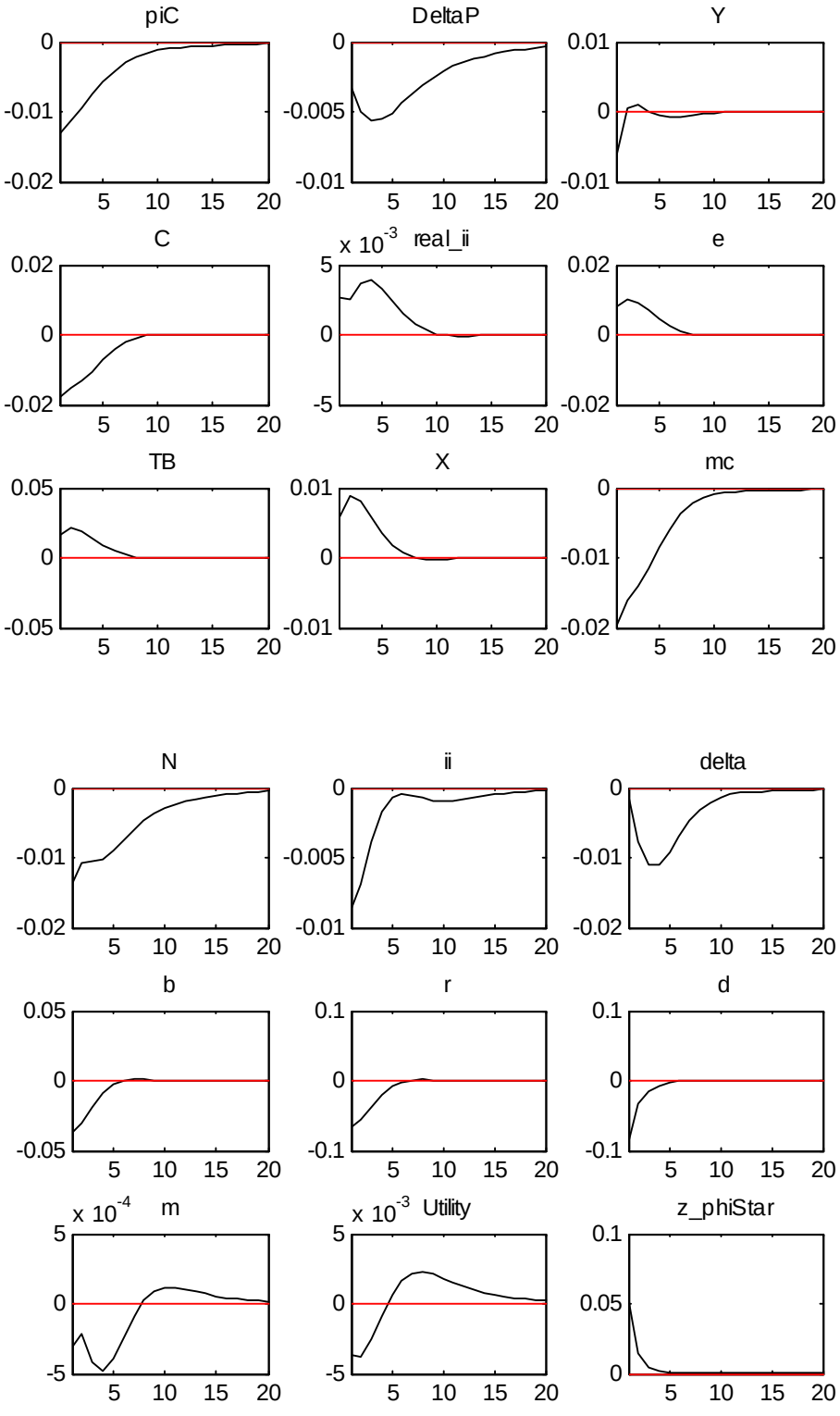
Response to a positive shock to government expenditures: G



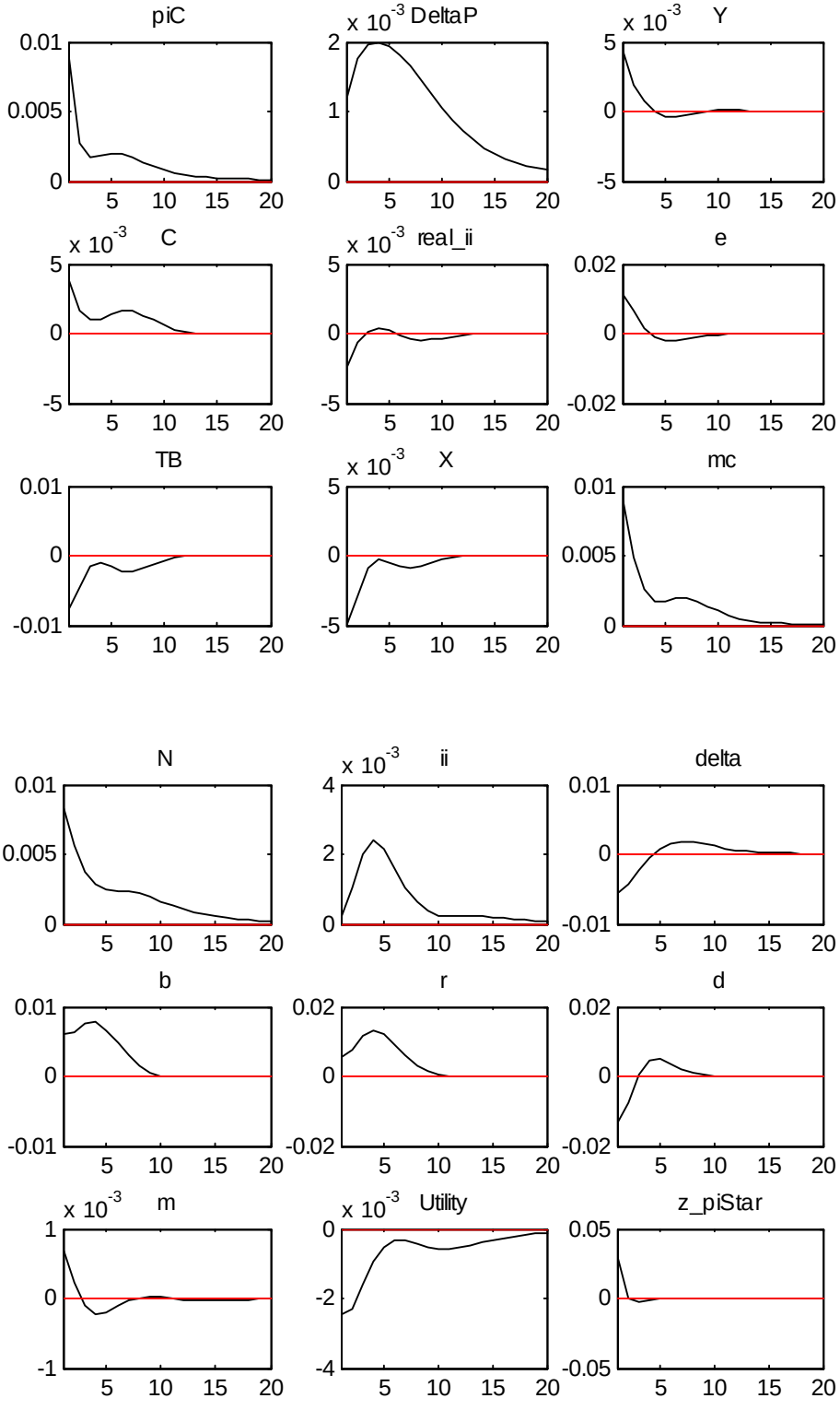
Response to a positive shock to the RW interest rate: i^*



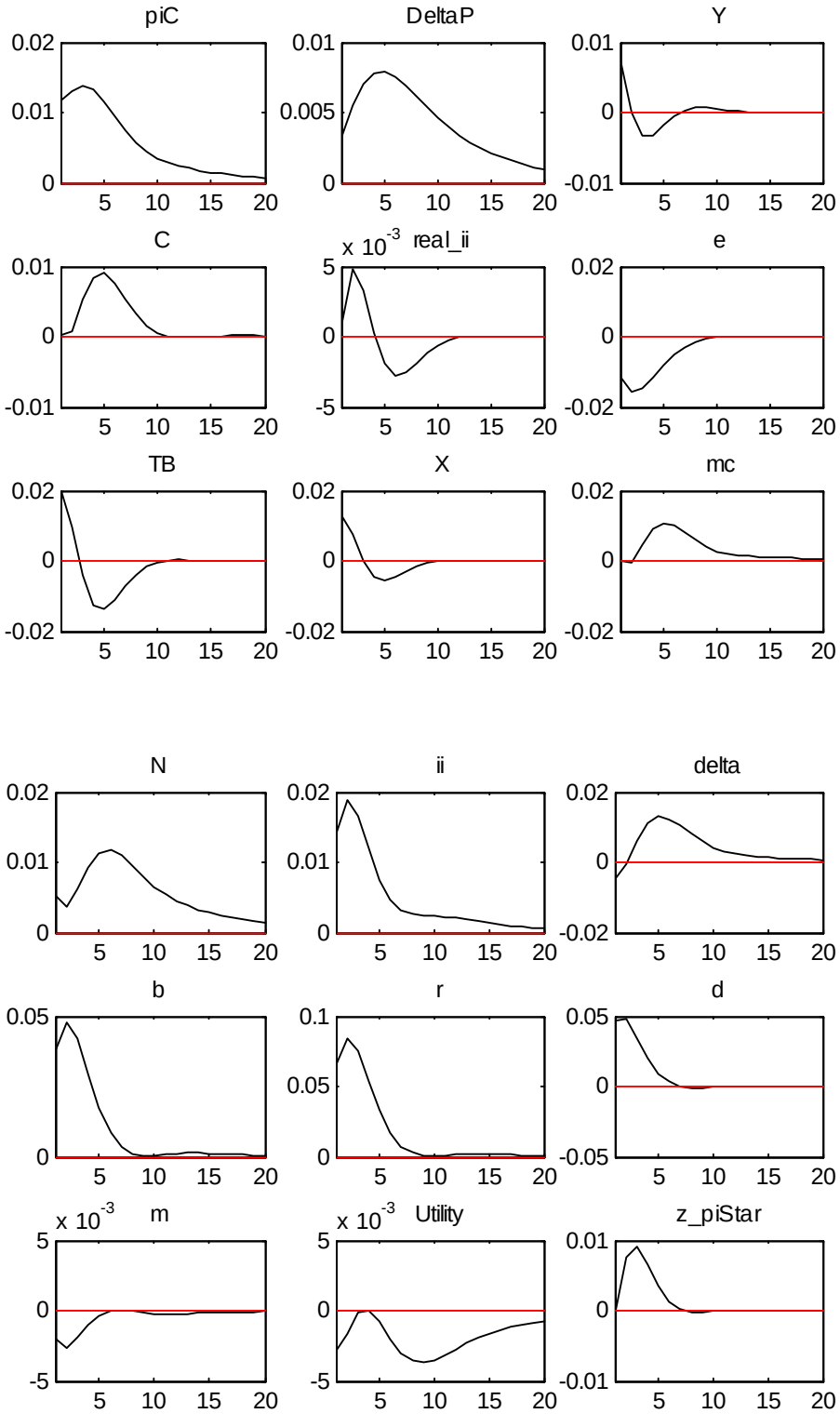
Response to a positive shock to the SOE's exogenous risk/liquidity premium: ϕ^*



Response to a positive shock to imports inflation: π^*



Response to a positive shock to exports inflation: π^{*X}

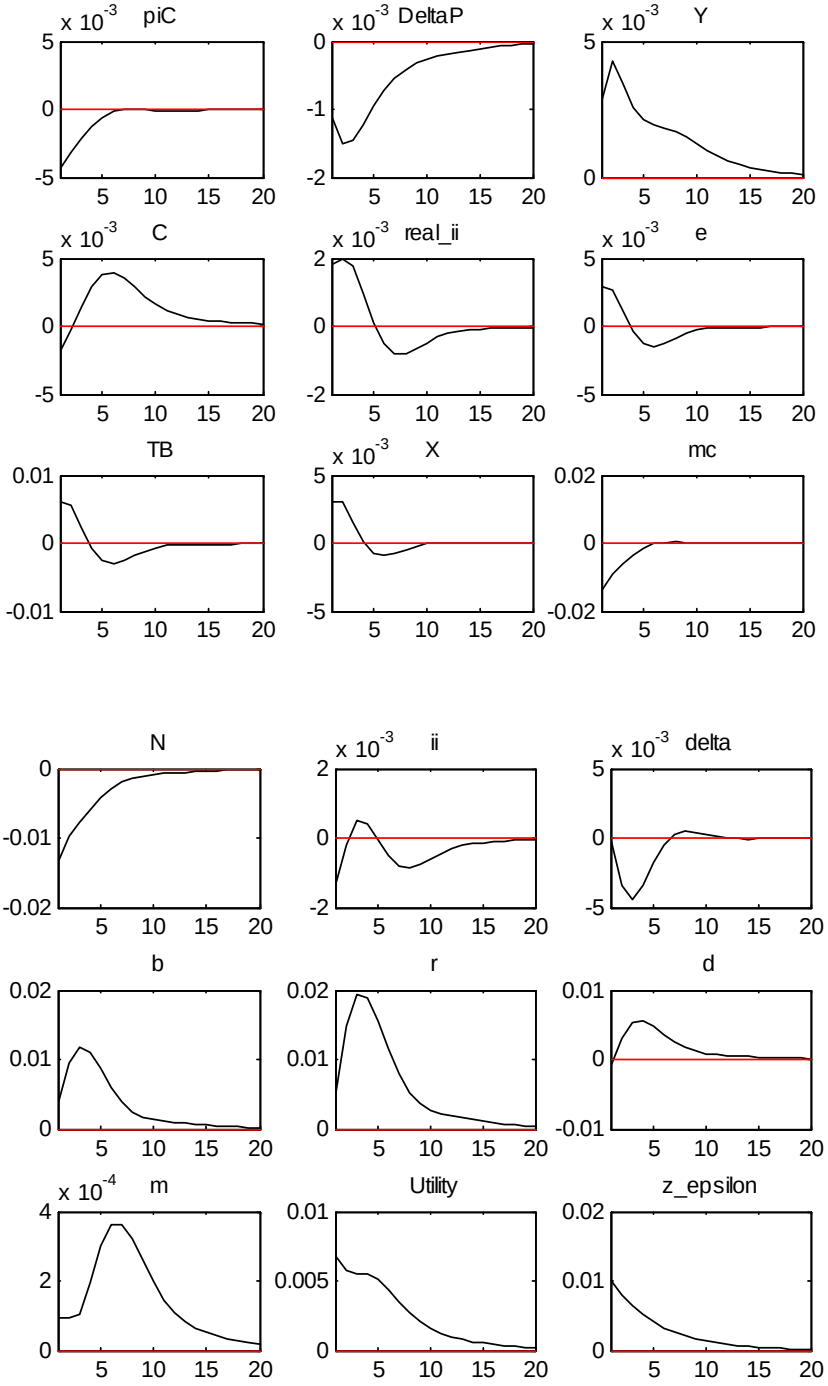


A.2.1.3 Central Bank style C

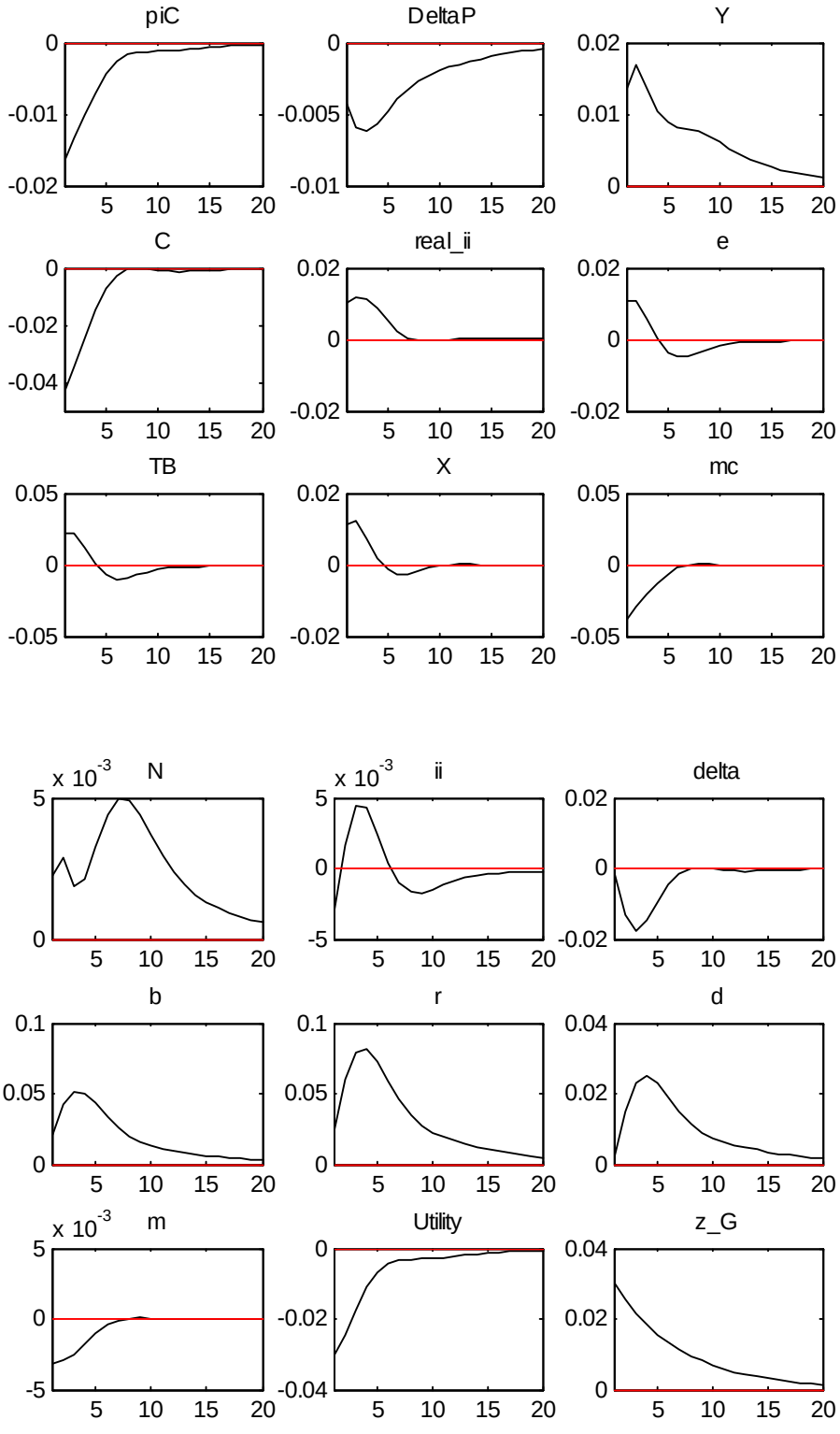
$$\omega_\pi = 100, \quad \omega_Y = 100, \quad \omega_e = 1, \quad \omega_r = 1, \quad \omega_{\Delta i} = 50, \quad \omega_{\Delta\delta} = 50$$

h_0	h_1	h_2	h_3	k_0	k_1	k_2	k_3	k_4
1.63	1.92	1.43	0.82	0.44	−1.31	−0.12	−0.91	−0.06

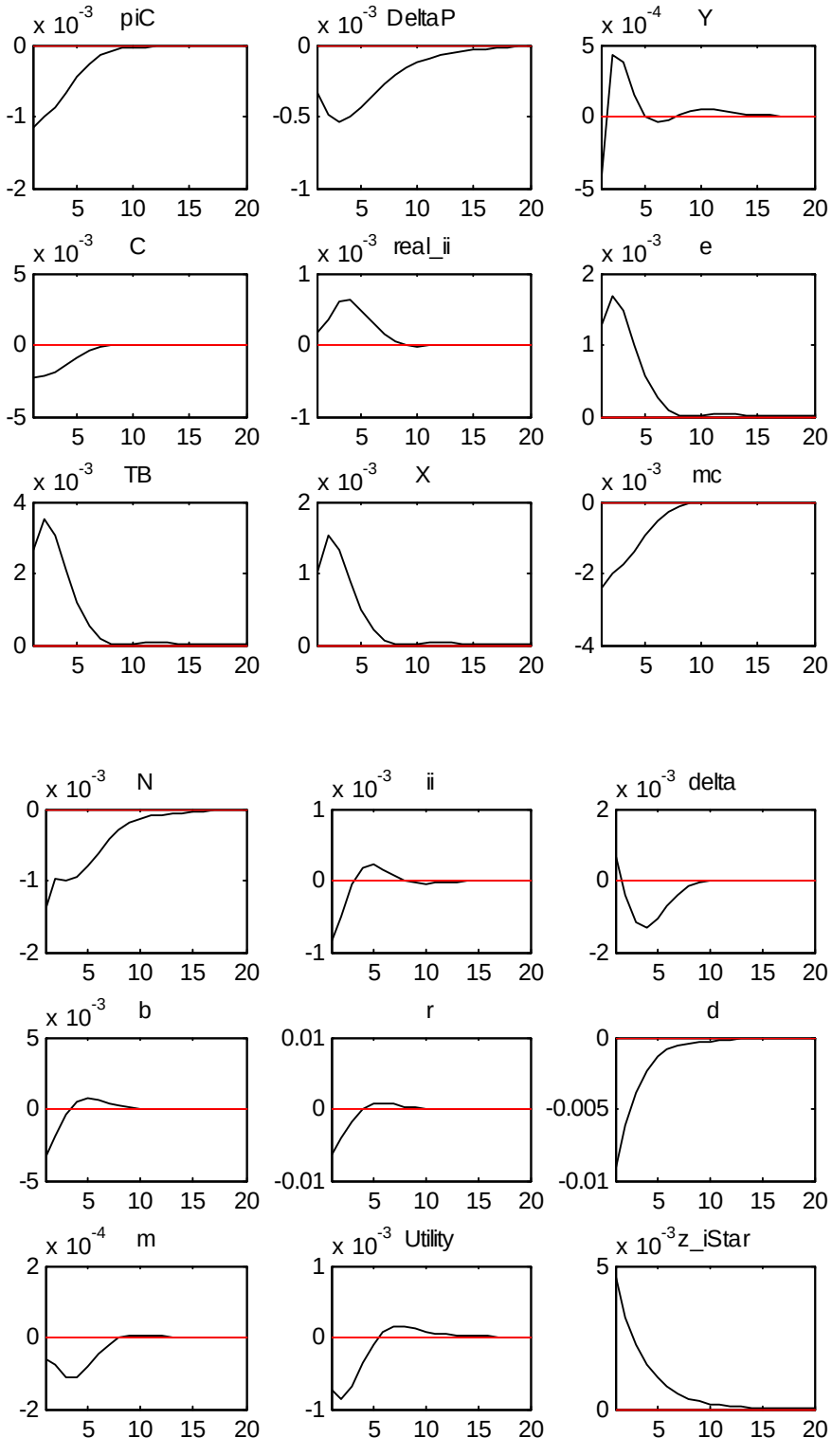
Response to a positive shock to domestic sector productivity: ϵ



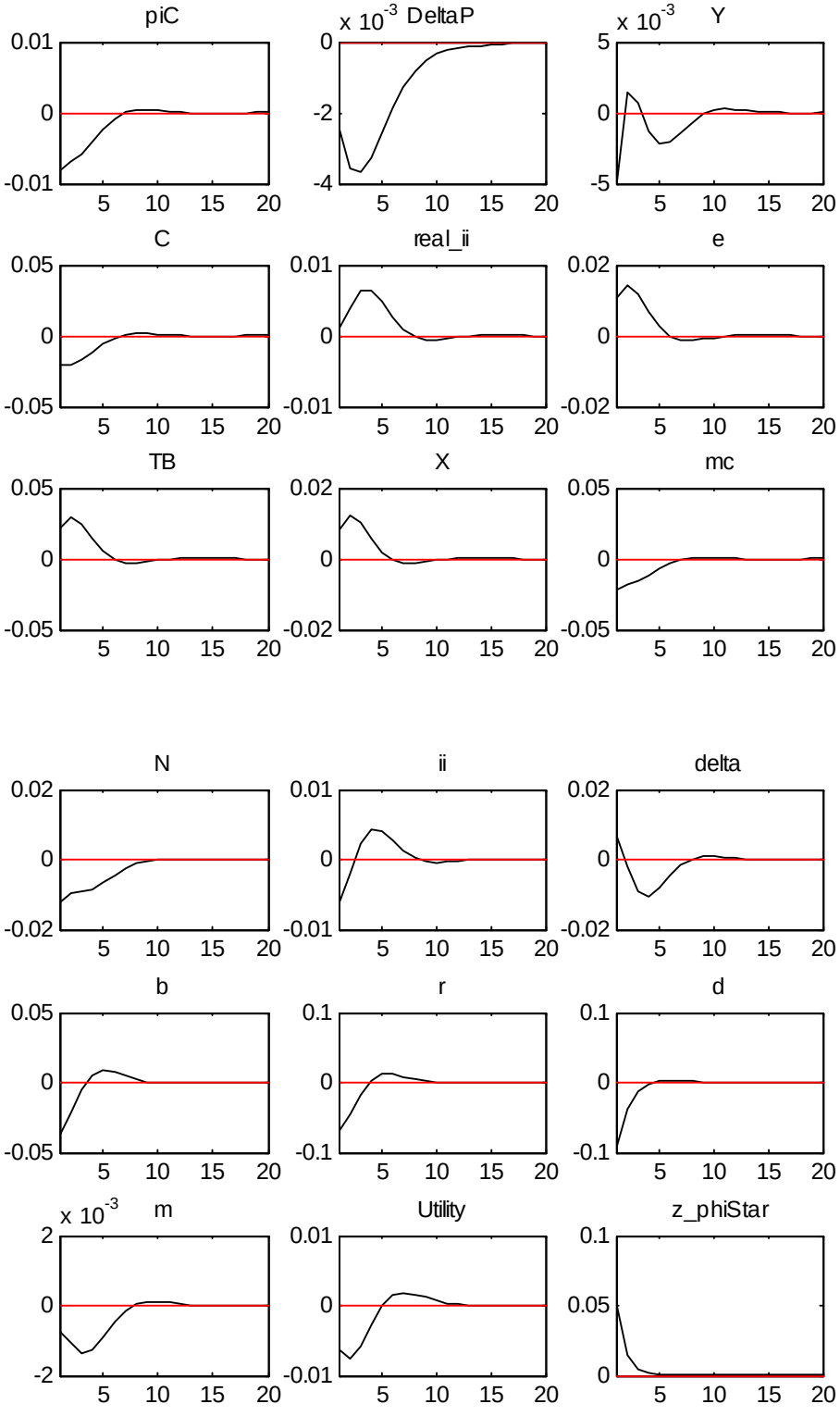
Response to a positive shock to government expenditures: G



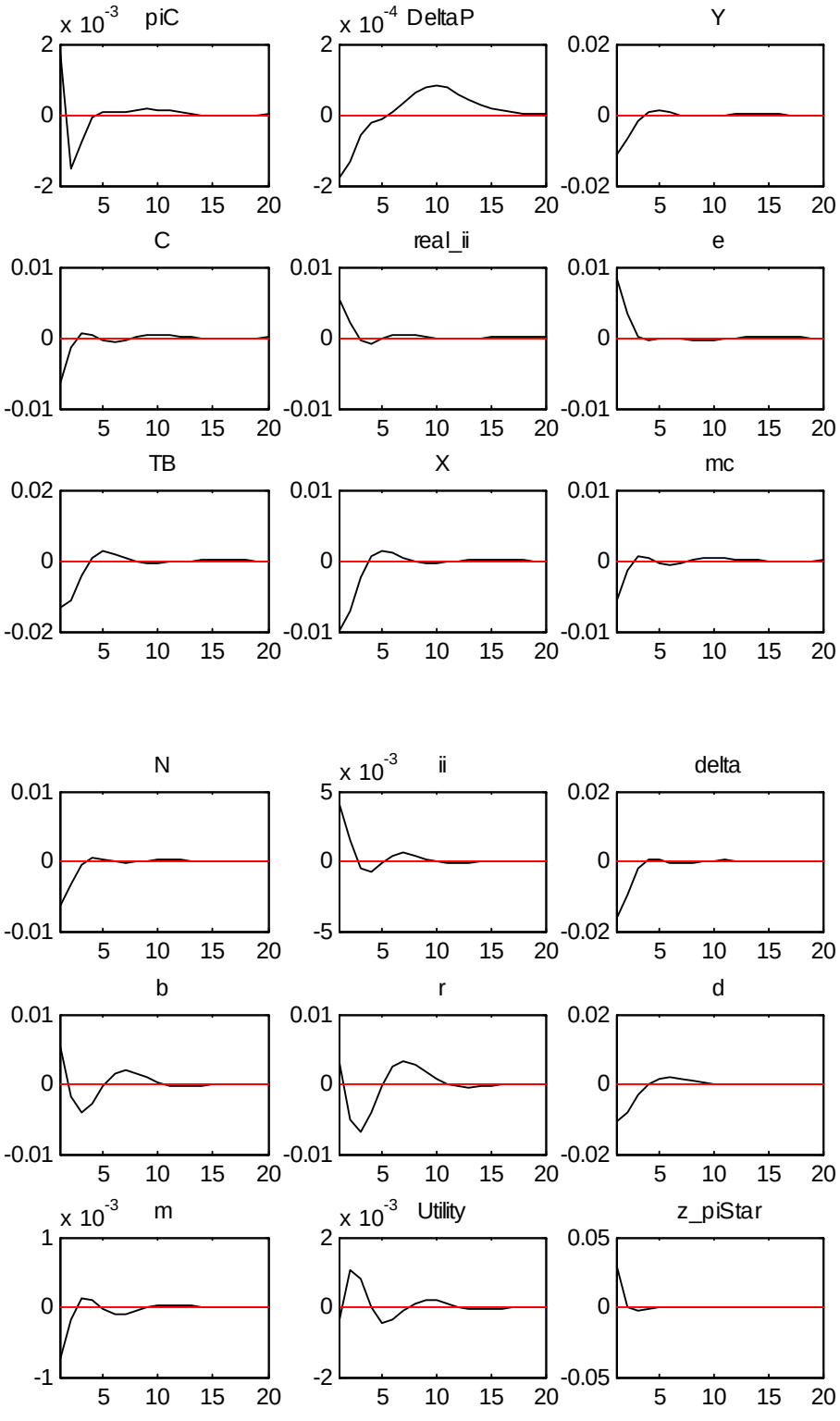
Response to a positive shock to the RW interest rate: i^*



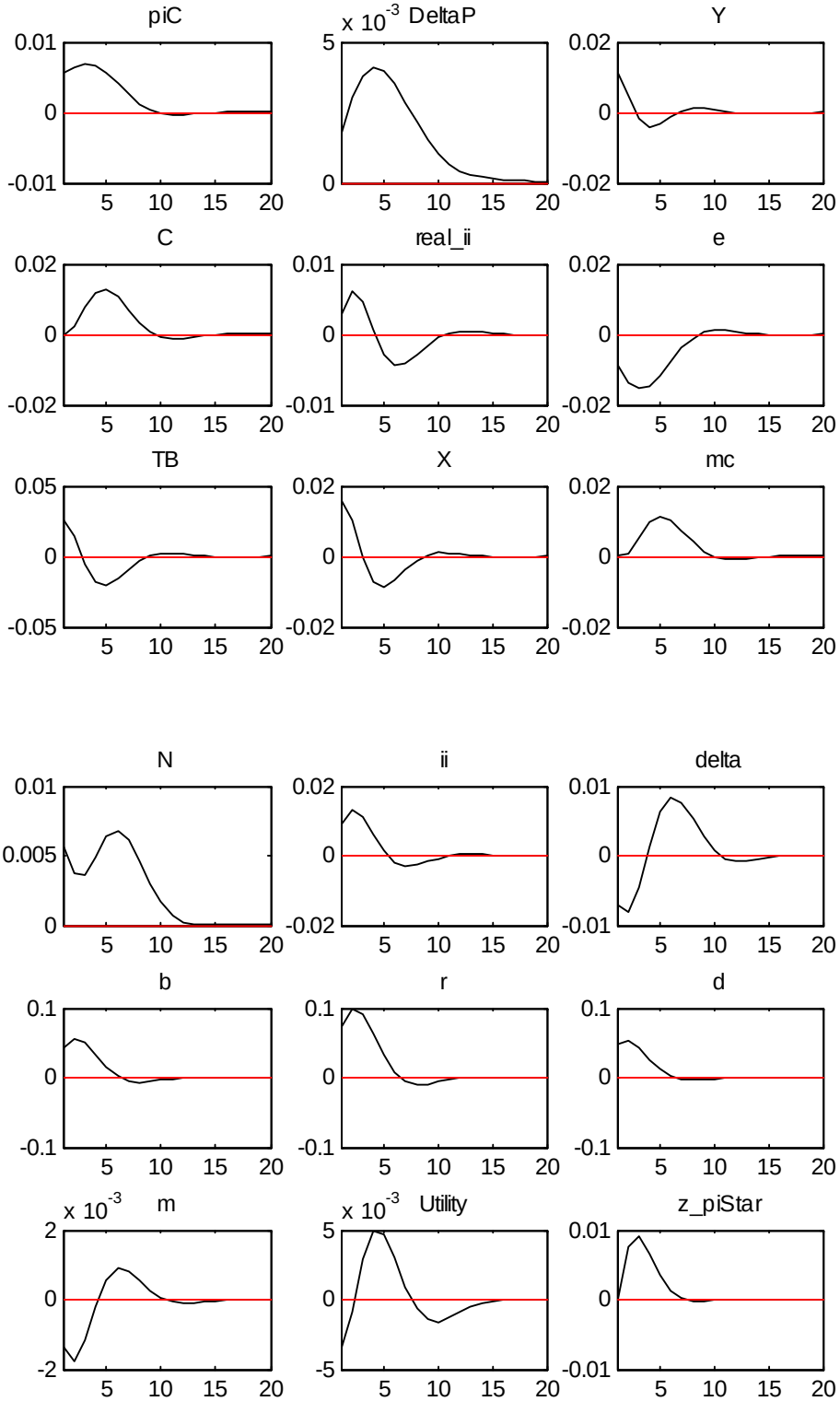
Response to a positive shock to the SOE's exogenous risk/liquidity premium: ϕ^*



Response to a positive shock to imports inflation: π^*



Response to a positive shock to exports inflation: π^{*X}



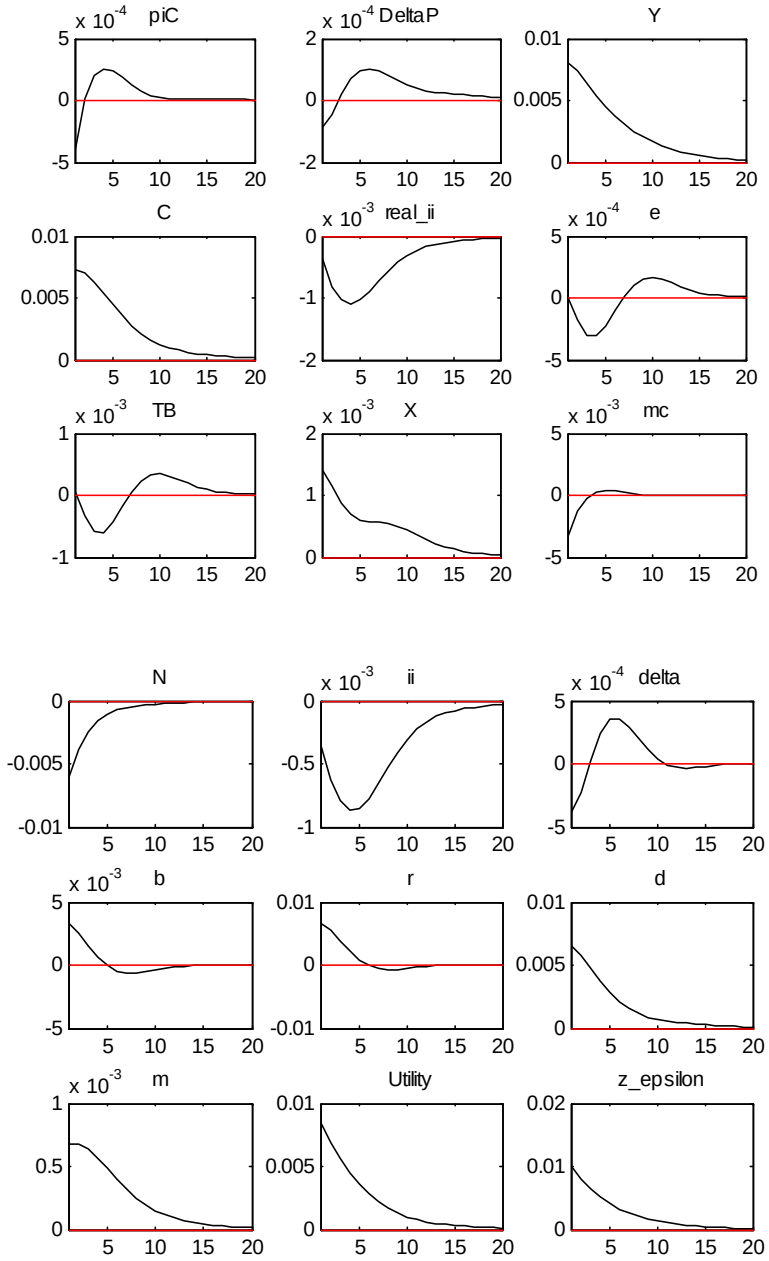
A.2.2 Optimal policy under commitment

The IRFs below correspond to the specified weights on the loss function:

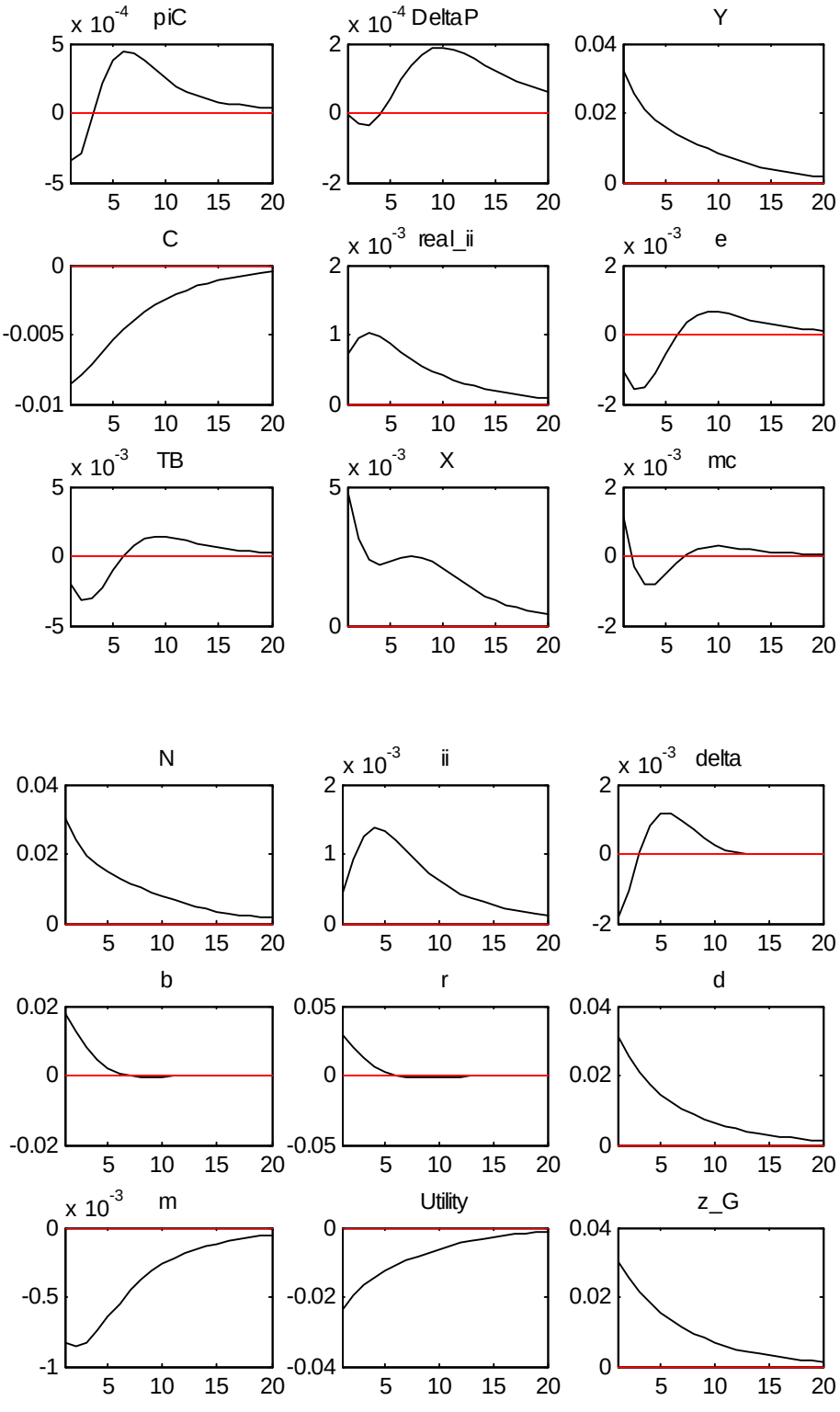
A.2.2.1 Central Bank style A

$$\omega_\pi = 100, \quad \omega_Y = 1, \quad \omega_e = 1, \quad \omega_r = 1, \quad \omega_{\Delta i} = 50, \quad \omega_{\Delta\delta} = 50.$$

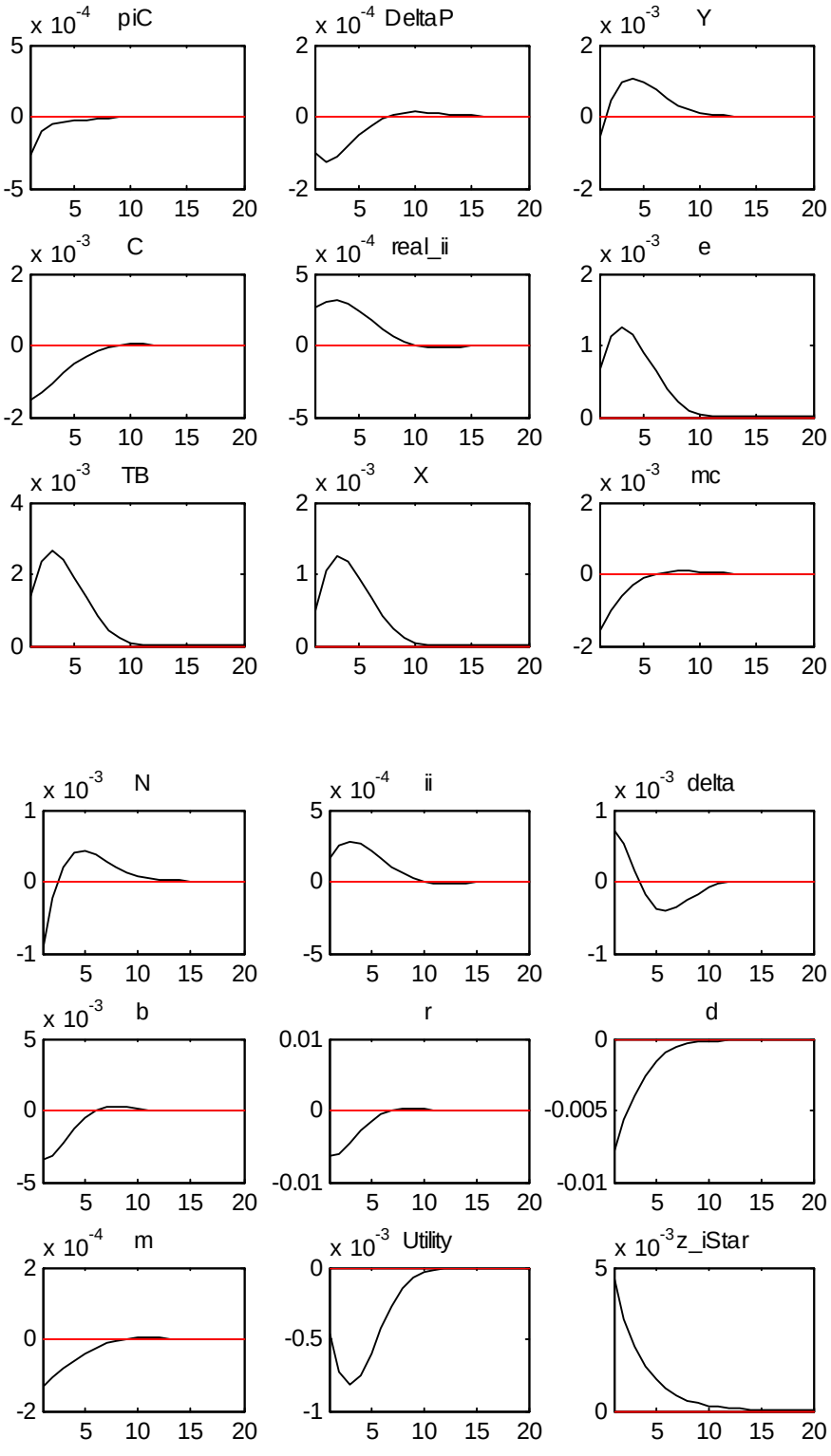
Response to a positive shock to domestic sector productivity: ϵ



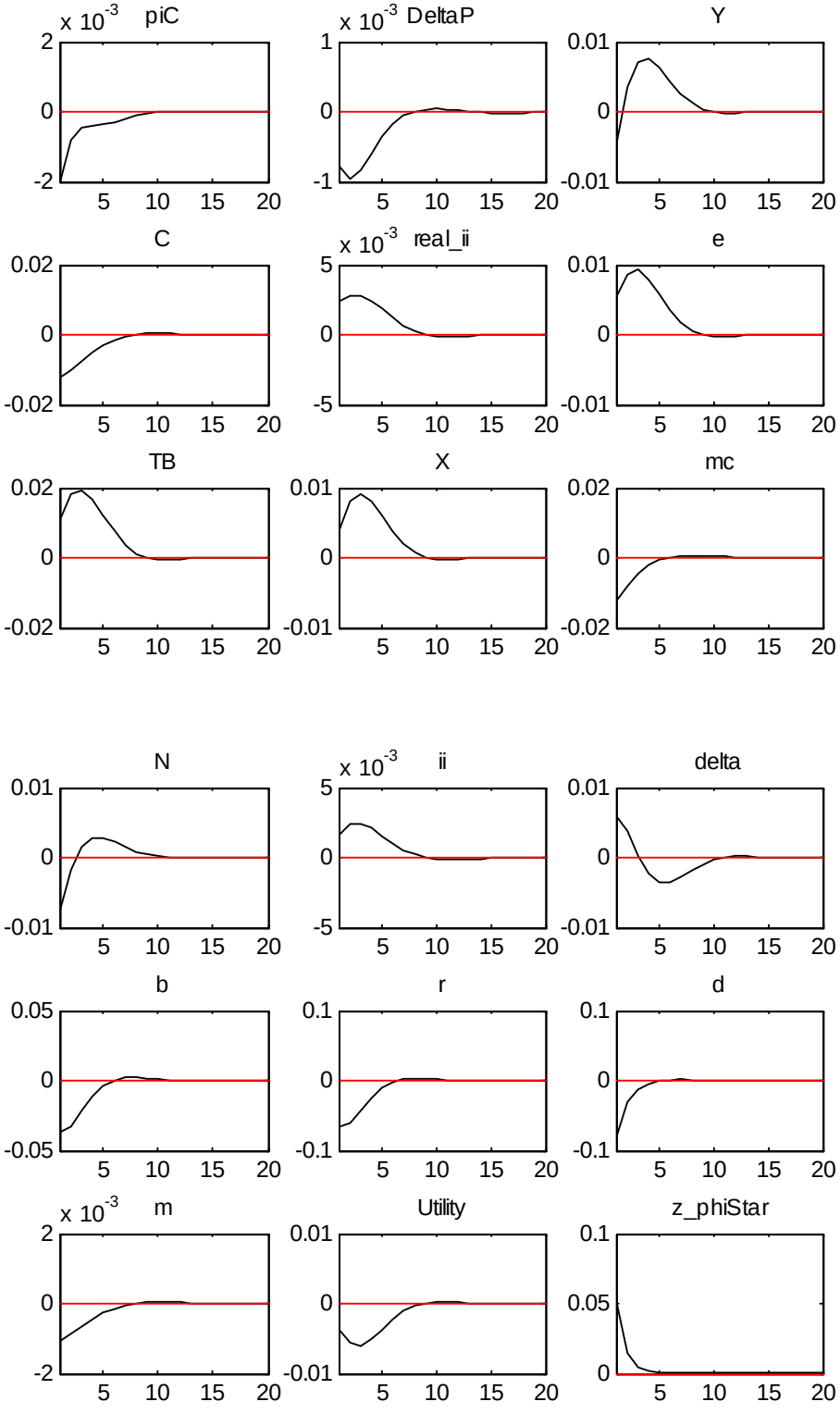
Response to a positive shock to government expenditures: G



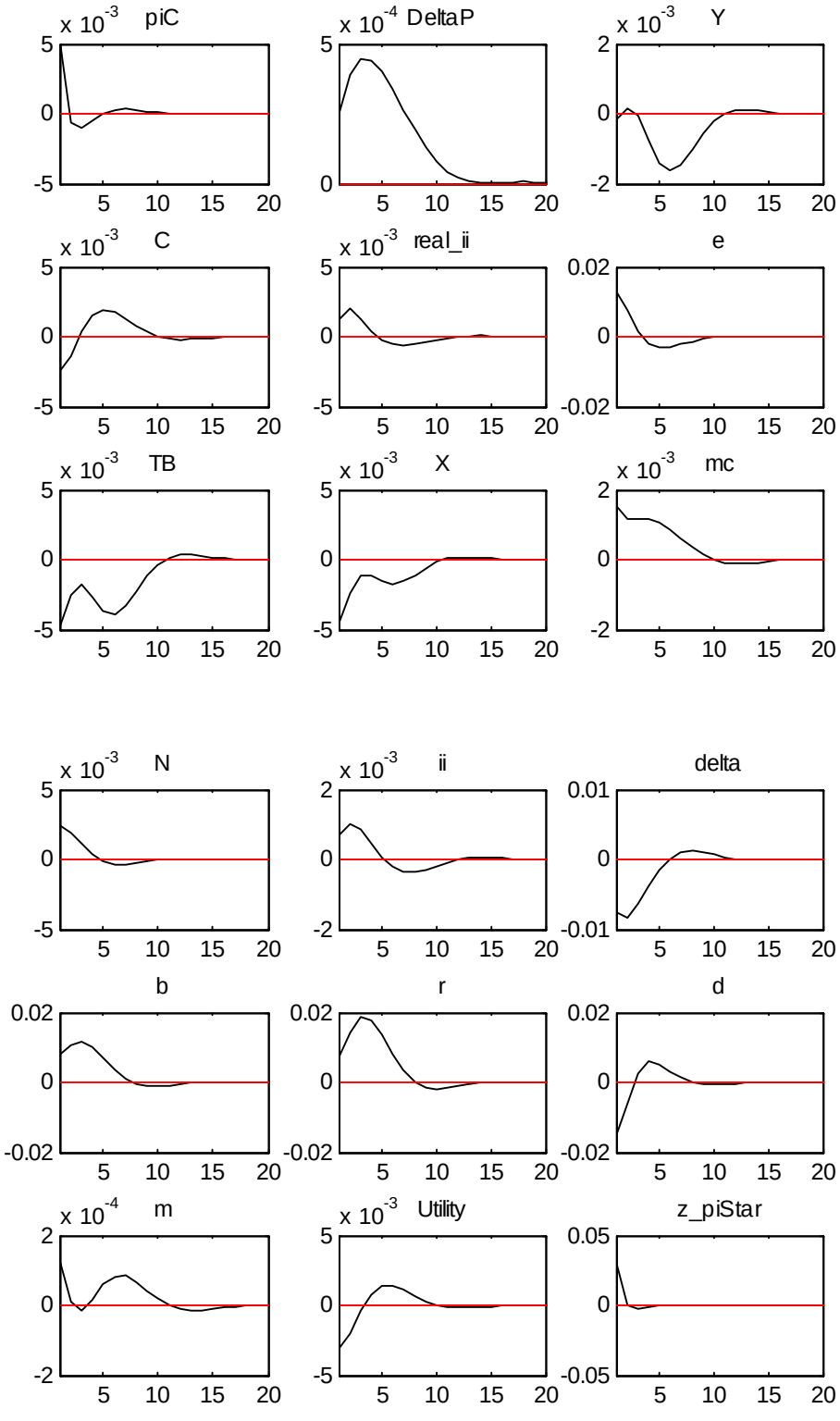
Response to a positive shock to the RW interest rate: i^*



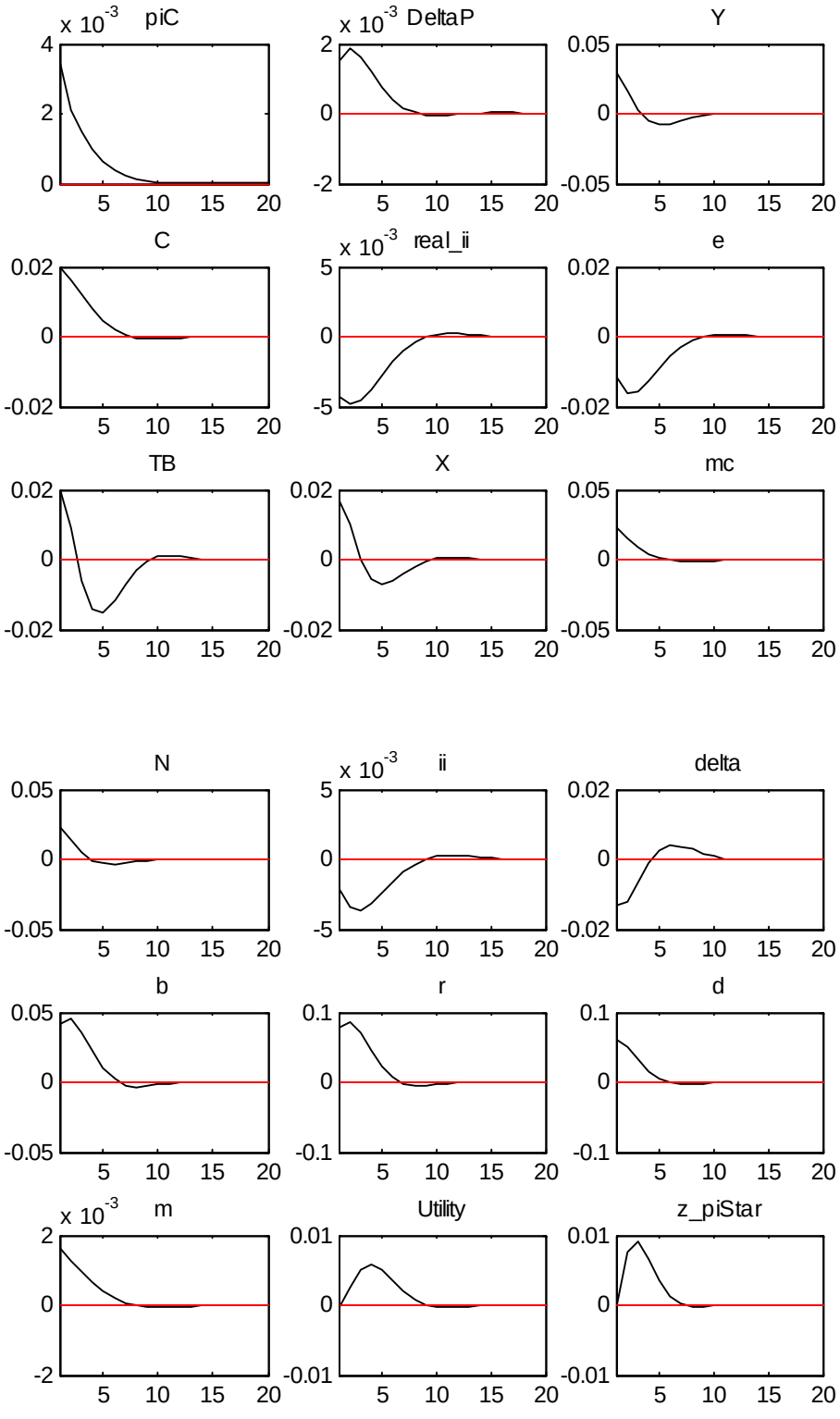
Response to a positive shock to the SOE's exogenous risk/liquidity premium: ϕ^*



Response to a positive shock to imports inflation: π^*



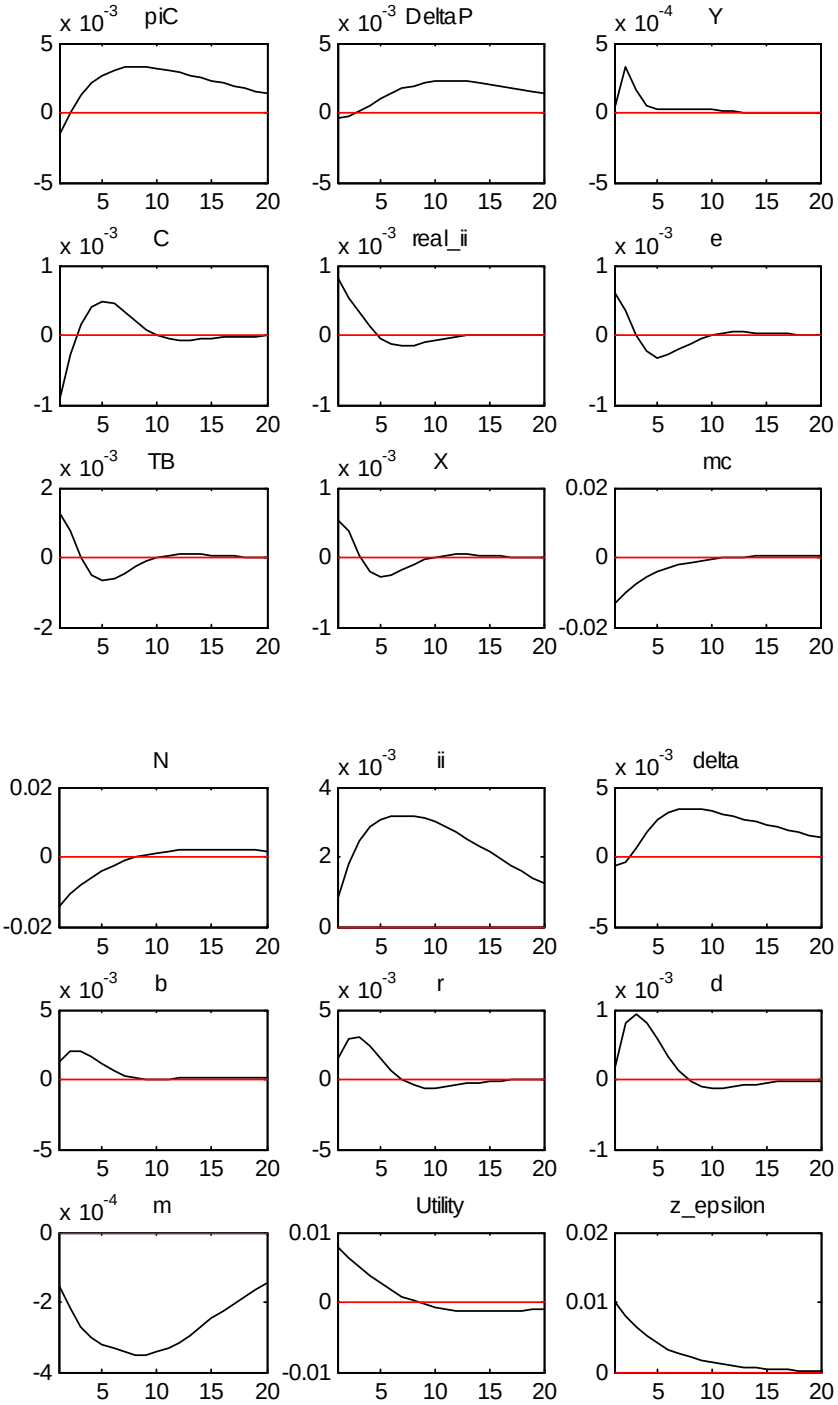
Response to a positive shock to exports inflation: π^{*X}



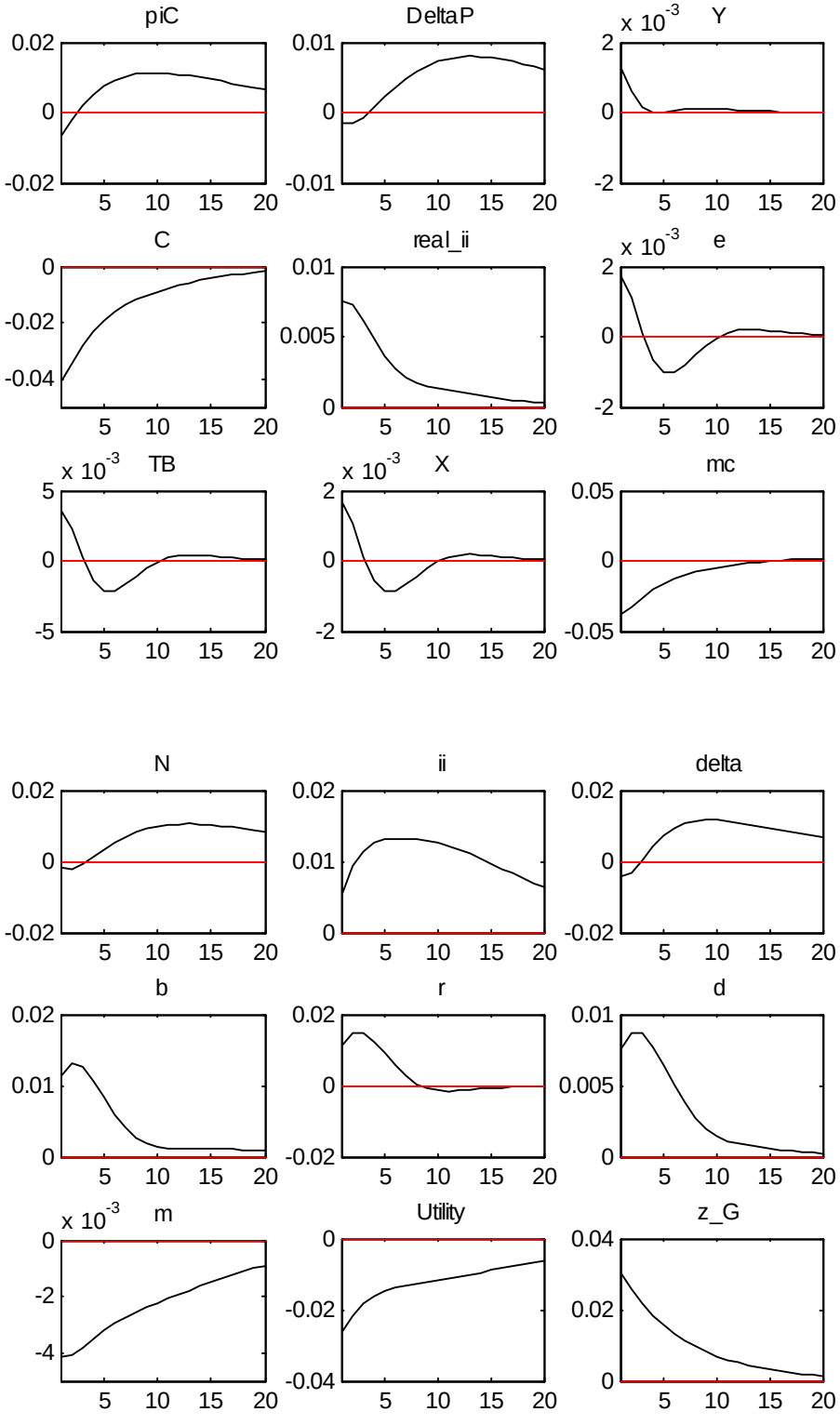
A.2.2.2 Central Bank style B

$$\omega_\pi = 1, \quad \omega_Y = 100, \quad \omega_e = 1, \quad \omega_r = 1, \quad \omega_{\Delta i} = 50, \quad \omega_{\Delta\delta} = 50.$$

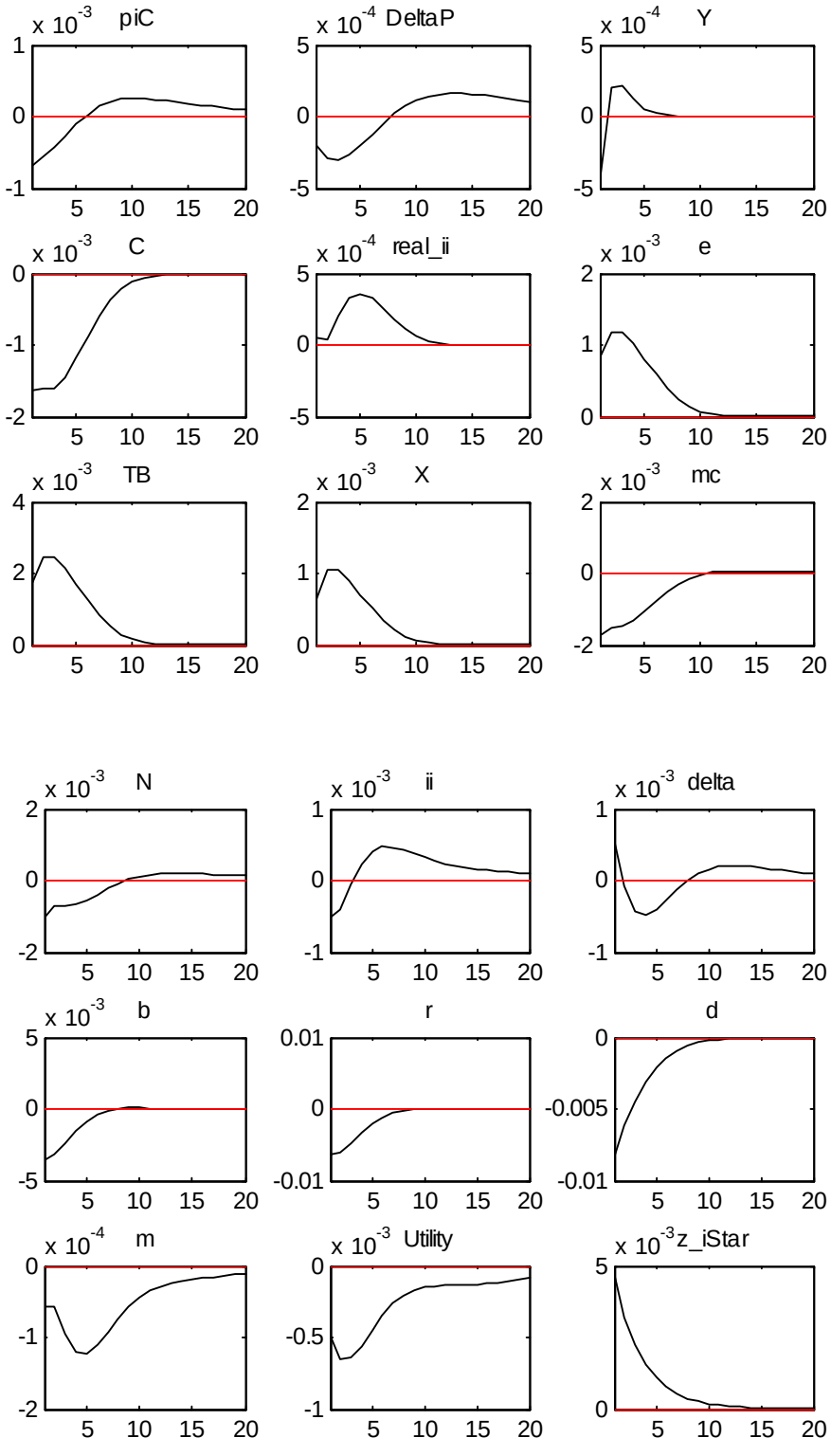
Response to a positive shock to domestic sector productivity: ϵ



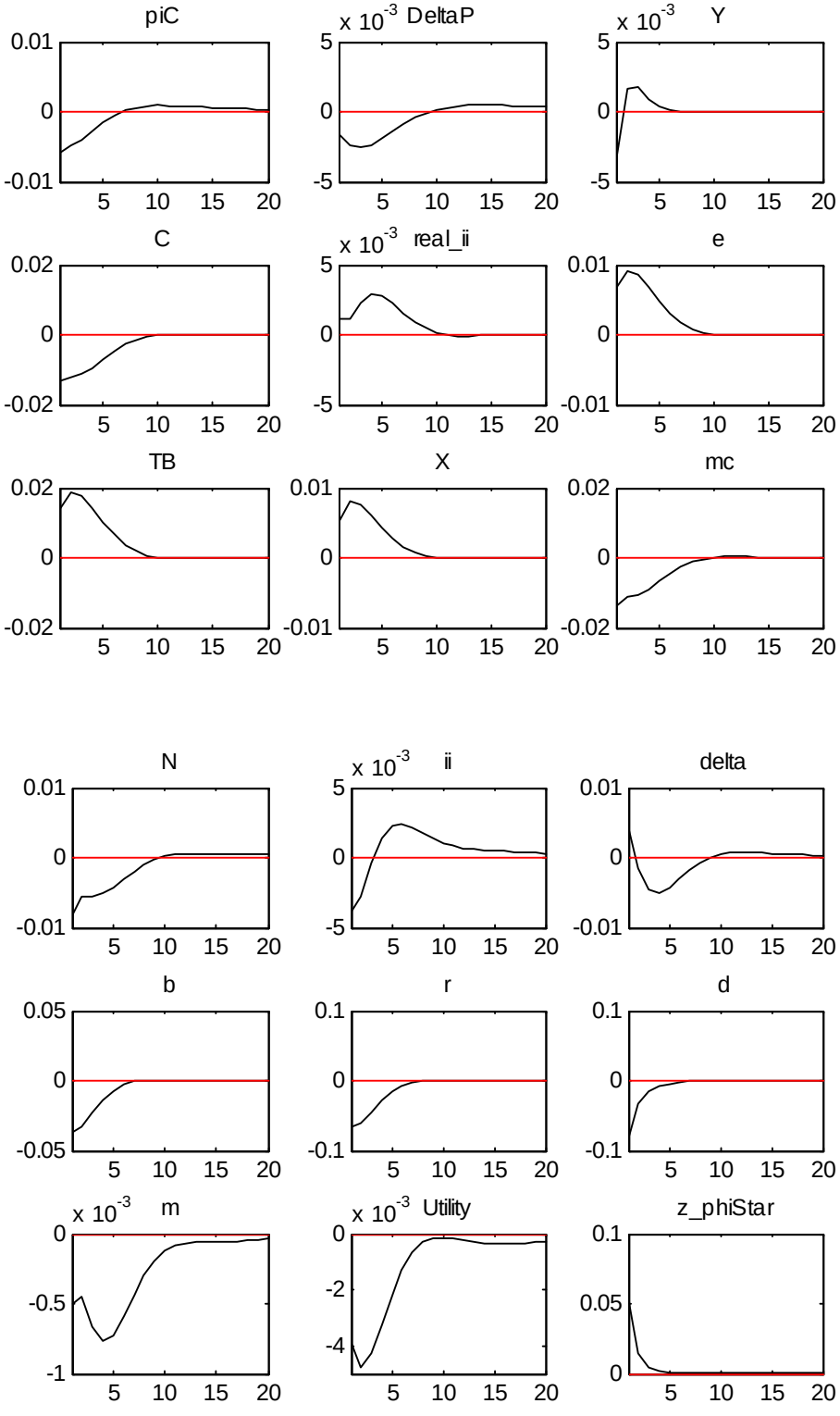
Response to a positive shock to government expenditures: G



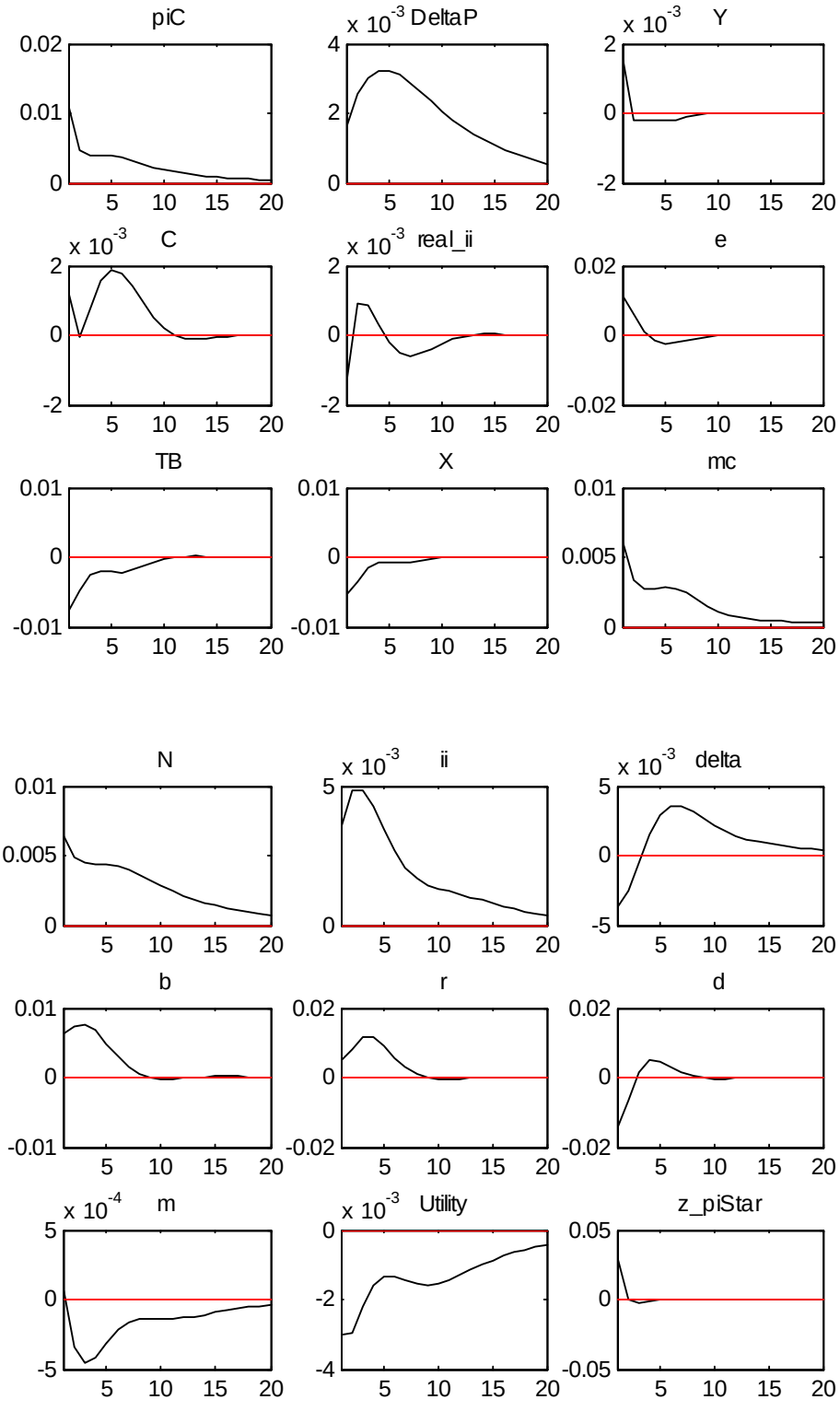
Response to a positive shock to the RW interest rate: i^*



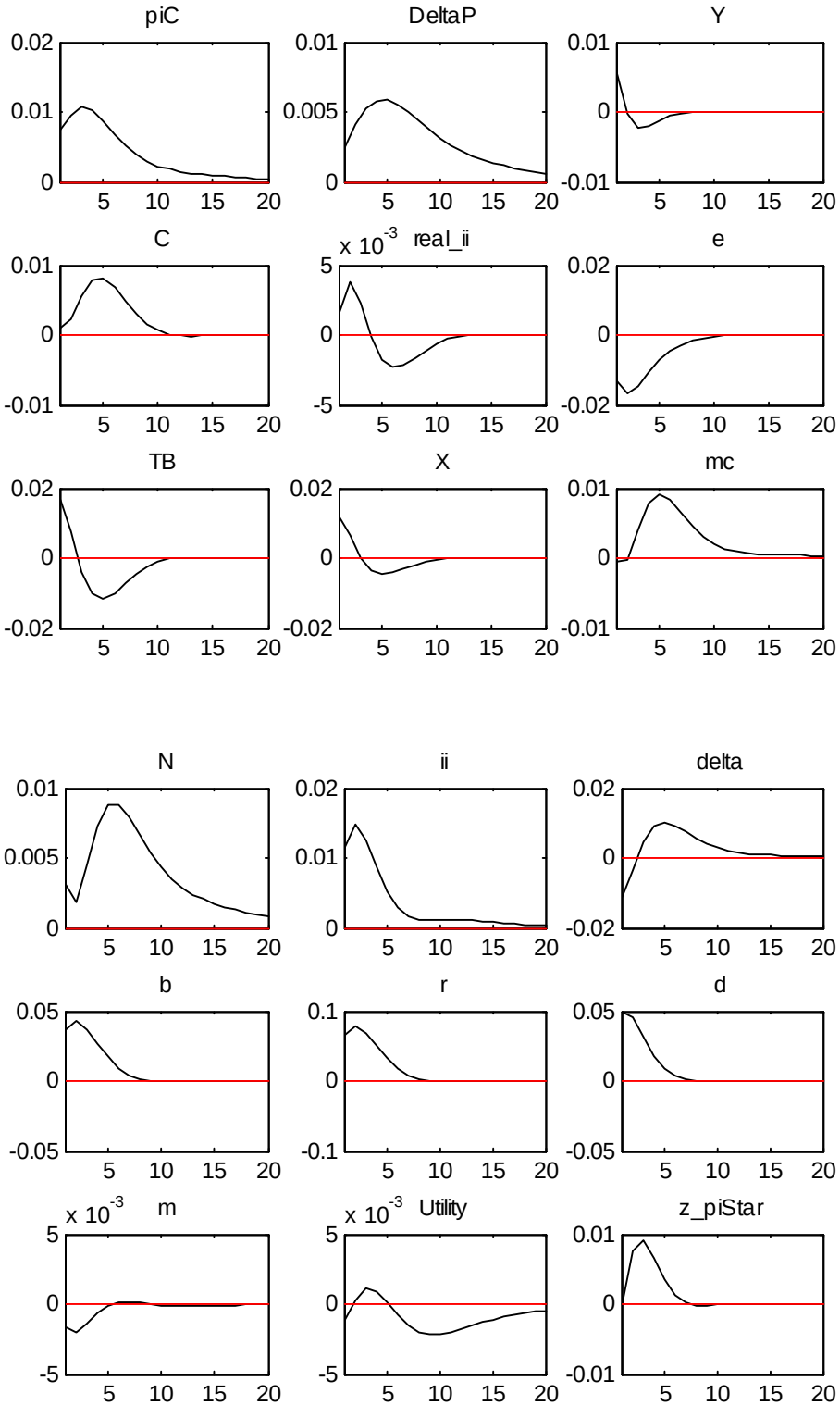
Response to a positive shock to the SOE's exogenous risk/liquidity premium: ϕ^*



Response to a positive shock to imports inflation: π^*



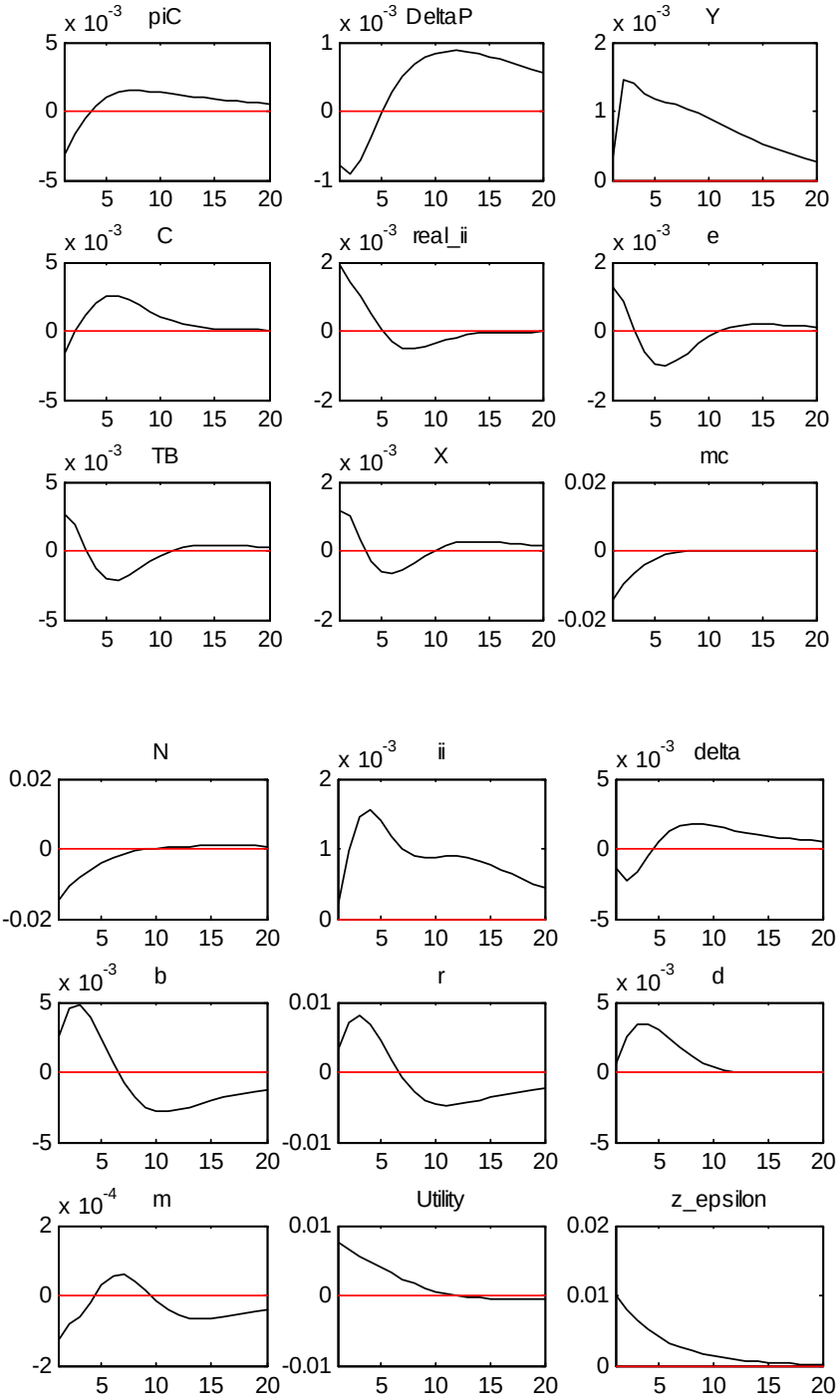
Response to a positive shock to exports inflation: π^{*X}



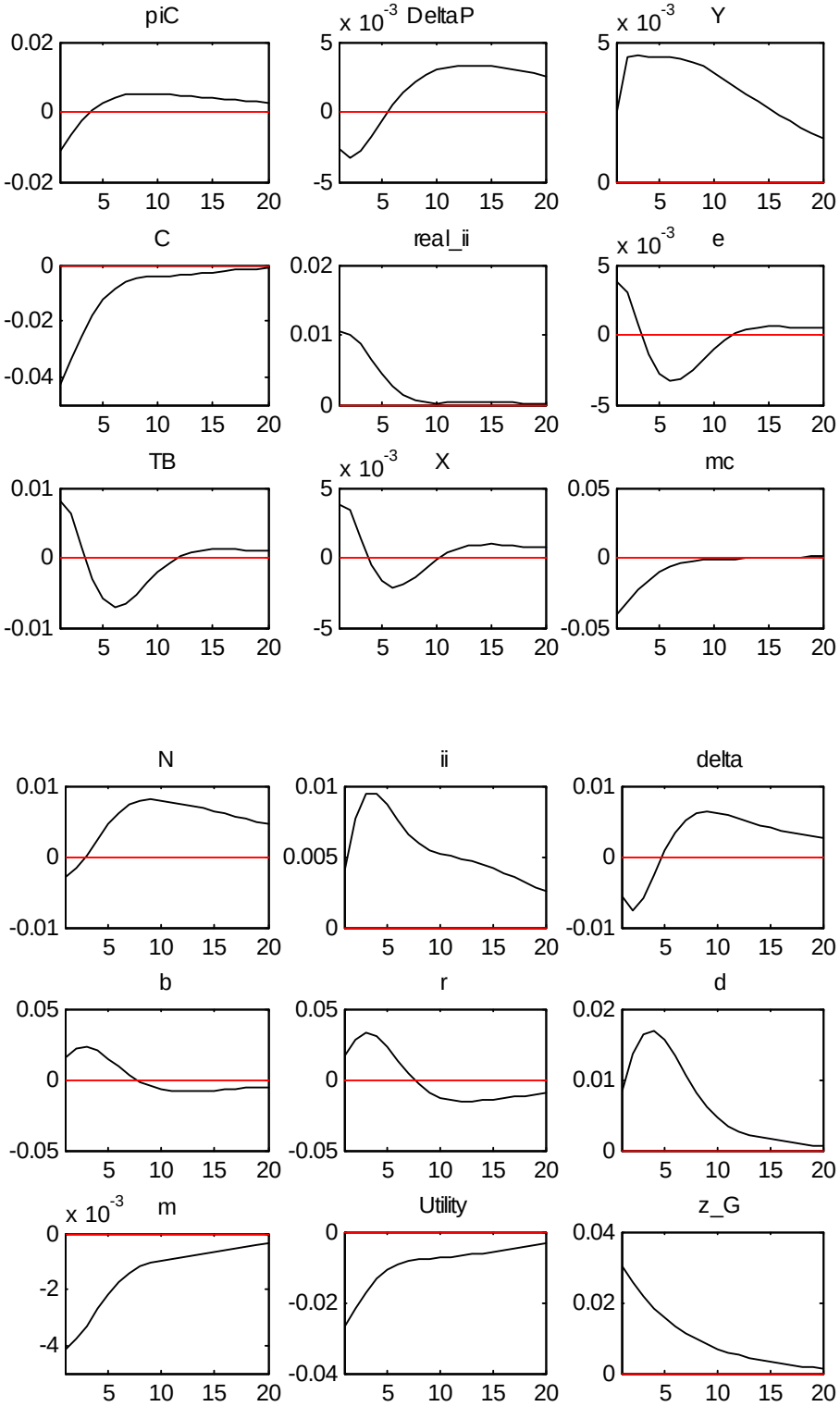
A.2.2.3 Central Bank style C

$$\omega_\pi = 100, \quad \omega_Y = 100, \quad \omega_e = 1, \quad \omega_r = 1, \quad \omega_{\Delta i} = 50, \quad \omega_{\Delta\delta} = 50.$$

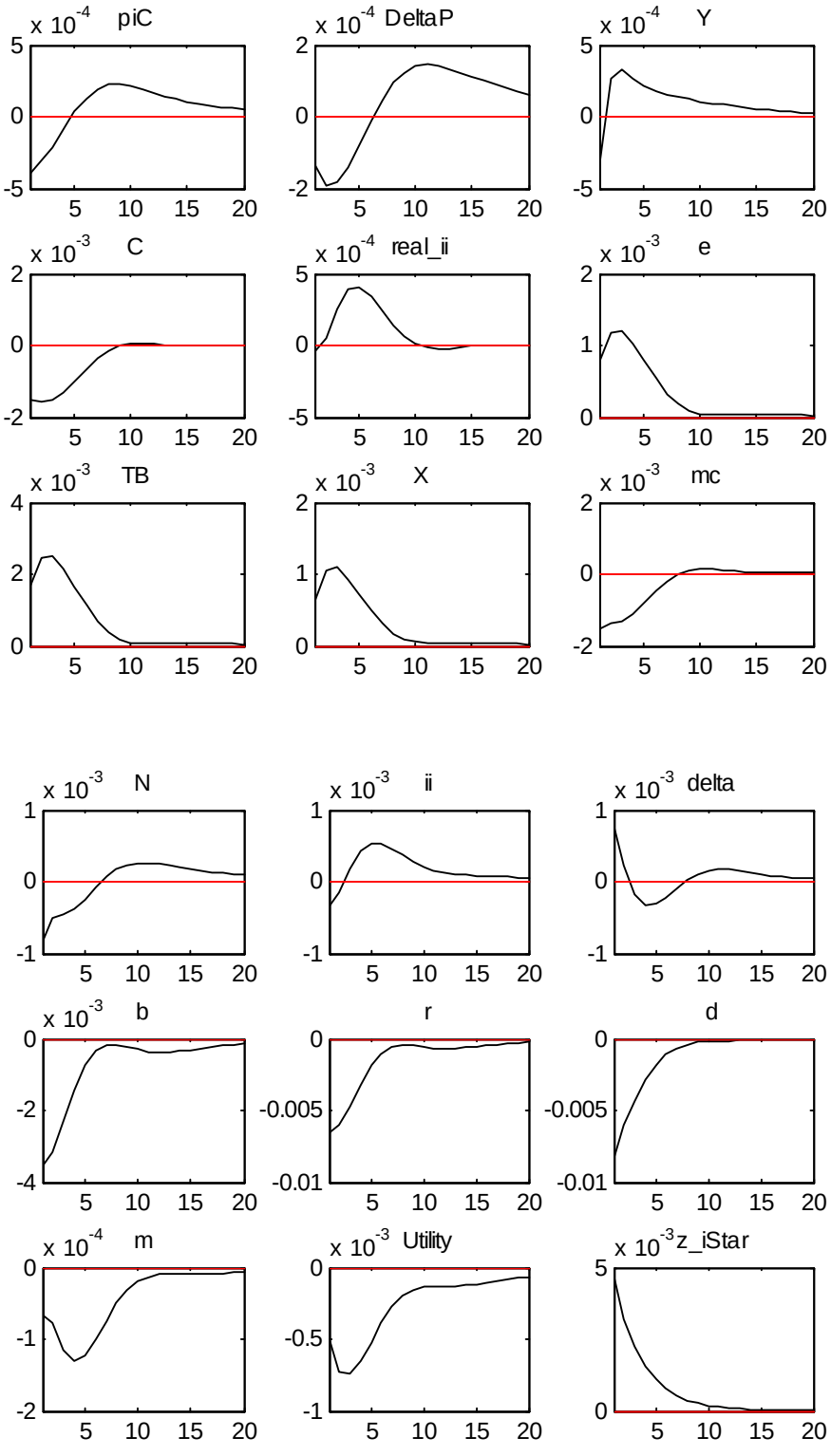
Response to a positive shock to domestic sector productivity: ϵ



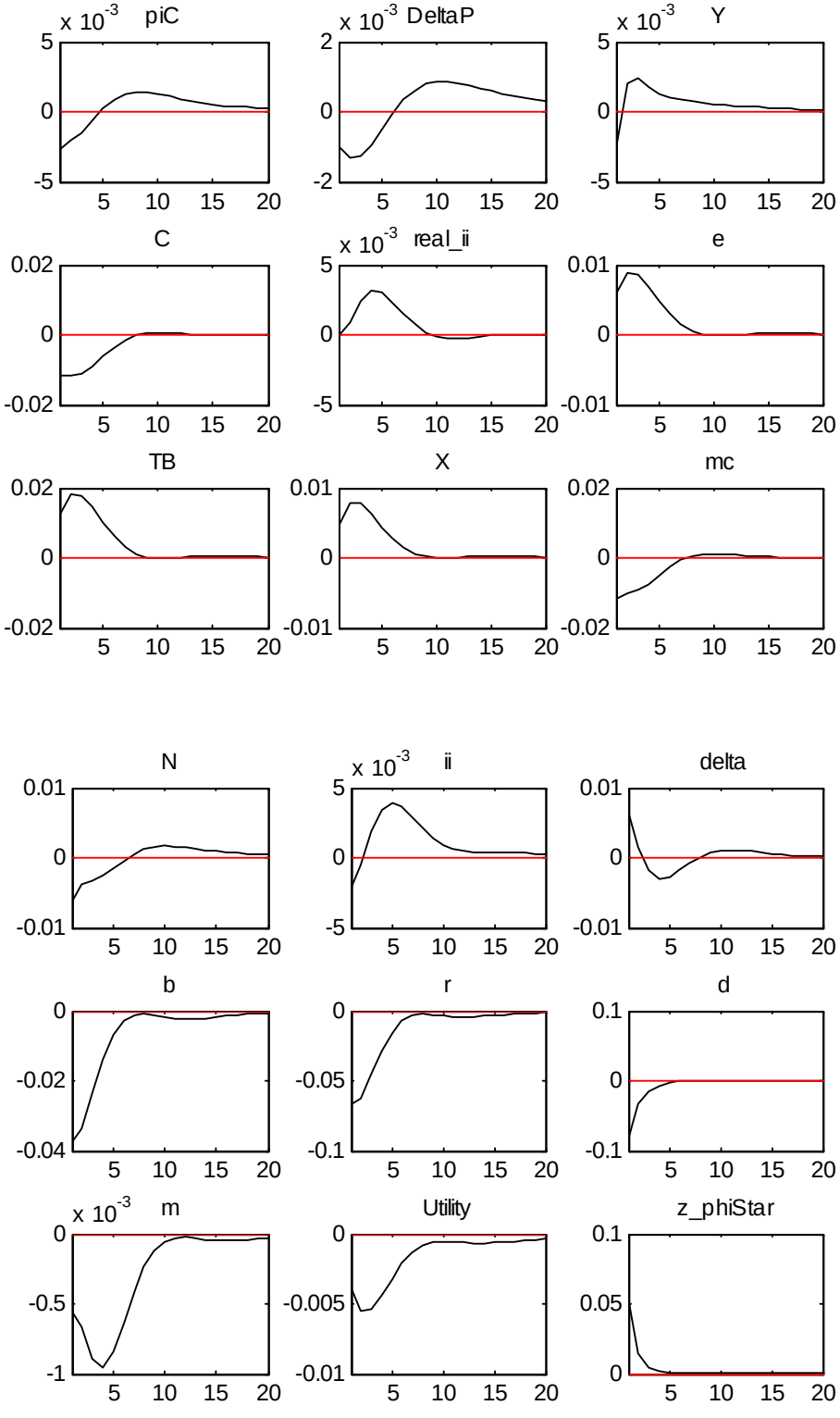
Response to a positive shock to government expenditures: G



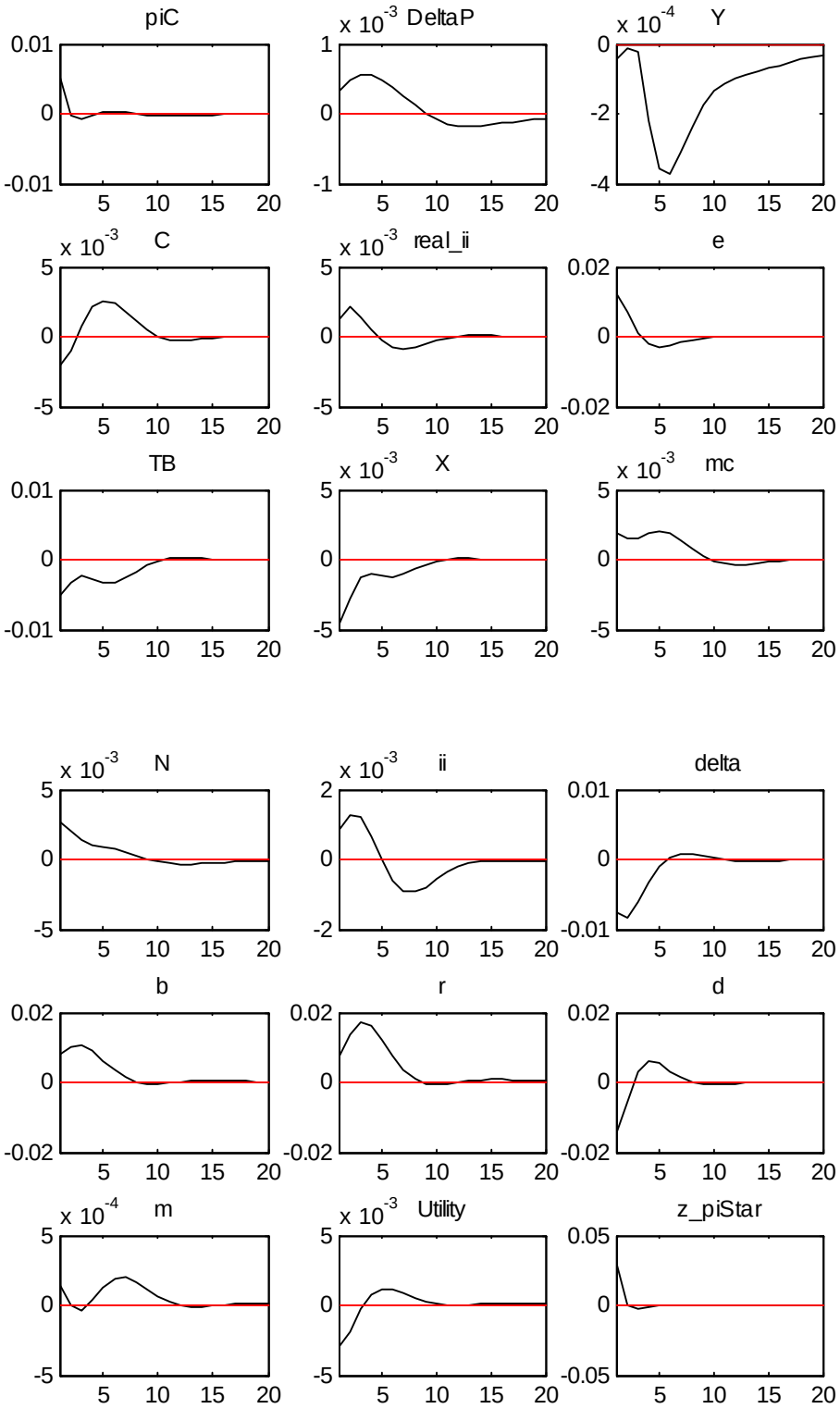
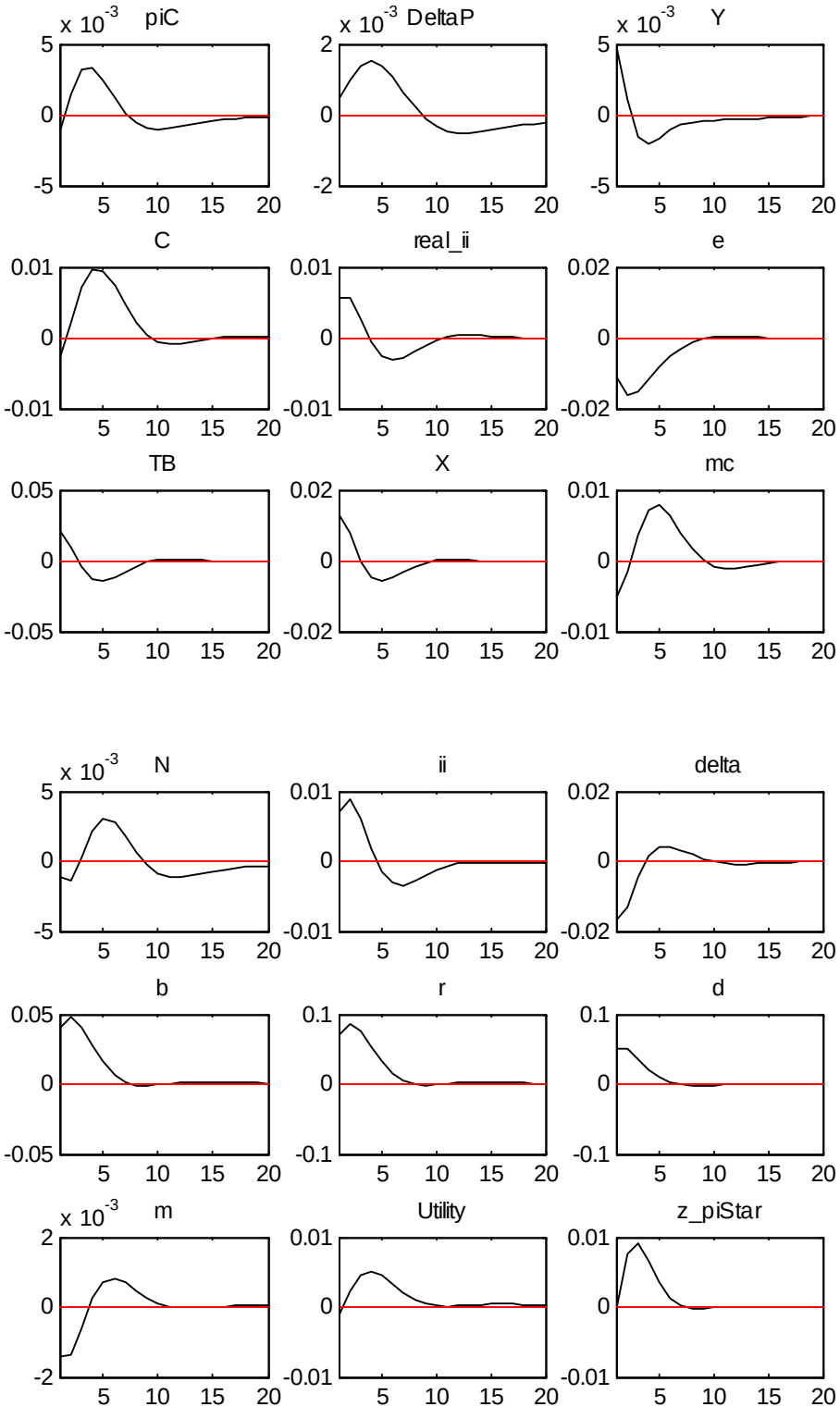
Response to a positive shock to the RW interest rate: i^*



Response to a positive shock to the SOE's exogenous risk/liquidity premium: ϕ^*



Response to a positive shock to imports inflation: π^*



Response to a positive shock to exports inflation: π^{*X}