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Working Paper
Go Public or Stay Private: A Theory of Entrepreneurial Choice

Tinbergen Institute Discussion Paper, No. 03-096/2

Provided in Cooperation with:
Tinbergen Institute, Amsterdam and Rotterdam

Suggested Citation: Boot, Arnoud W.A.; Gopalan, Radhakrishnan; Thakor, Anjan V. (2003) : Go Public or Stay Private: A Theory of Entrepreneurial Choice, Tinbergen Institute Discussion Paper, No. 03-096/2, Tinbergen Institute, Amsterdam and Rotterdam

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Go Public or stay Private

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GO PUBLIC OR STAY PRIVATE : A THEORY OF ENTREPRENEURIAL CHOICE

by

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October, 2003

Acknowledgements: We thank Jun Yang, Venky Panchapagesan and seminar participants at Washington University and the World Bank for helpful comments.

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ABSTRACT

In this paper we analyze an entrepreneur/manager’s choice between private and public ownership in a setting in which management needs some “elbow room” or autonomy to optimally manage the firm. In public capital markets, the corporate governance regime in place exposes the firm to exogenous controls, so that management may lack the autonomy it desires. By contrast, private ownership can provide management the desired autonomy due to the possibility of precisely-calibrated private contracting. The disadvantage of private ownership (relative to public ownership) is that it imposes a cost of illiquidity on those who provide financing. We explore this tradeoff between managerial autonomy and the cost of capital in a simple setting and draw a number of new testable implications.
GO PUBLIC OR STAY PRIVATE: A THEORY OF ENTREPRENEURIAL CHOICE

The battle for Safeway raises an old but important question:

is it better to be a private company than to be a public one?

*The Economist*, January 25th 2003, p. 57

1 INTRODUCTION

One of the most important decisions taken by a firm is the choice of ownership structure. There are broadly two ownership structures that firms choose from: public ownership and private ownership. Public ownership is characterized by listed and traded shares with ownership and control among multiple (possibly atomistic) shareholders, and public corporate governance arrangements and reporting requirements. By contrast, private ownership operates without market listing and trading and is characterized by private contracting, typically with concentrated ownership and control resting with a few large investors. These distinctly different attributes of private and public ownership raise the question: how does a firm determine whether to be publicly or privately owned? This is the question we address. A key observation in our analysis is that these two ownership structures also typify two governance paradigms. Public ownership is characterized by market-imposed discipline which is usually not customized to specific firm needs, while private ownership accommodates precise contracting with monitoring by large shareholders, the intensity of which can be customized to the needs of the firm. We focus on the pooling characteristic of a market-based governance structure vis-a-vis the customized governance structure made possible with private ownership, and study the firm’s choice between these two structures.

Corporate governance inevitably brings with it controls on management. These controls manifest themselves in various ways: requirements to disclose information and get decisions approved by the
Board of directors, veto of management decisions by directors, proxy fights, etc. Consequently, management may sometimes be unable to do what it wants because the governance structure put in place by investors stands in the way. In some instances, these controls are designed to deal with agency problems and hence help to minimize self-serving behavior by management. In other instances, however, management may actually be attempting to maximize firm value but investors may disagree with management about how to go about doing this.\footnote{Such “genuine” disagreement may arise due to a variety of reasons, such as non-uniform priors (Allen and Gale (1999)), overconfidence on the part of either management or investors (Daniel, Hirshleifer and Subrahmanyam (1998)), or differences in philosophical bias (Mullainathan and Shleifer (2002)).} It is this form of disagreement that we focus on in this paper. That is, we study the incorporation problem of an entrepreneur who is motivated to maximize firm value in his role as a manager and is concerned about potential disagreement with investors over the best way to do this.\footnote{We do not deny the existence of agency problems, but abstract from them in order to focus on the issue of disagreement with entrepreneurs who are a priori motivated to maximize firm value} The nature of corporate governance and the associated investor intervention and controls will determine how much elbow room or autonomy the manager will have when such disagreement arises.

Management, acting to maximize firm value, will value autonomy \textit{ex ante}. While the term “autonomy” may have various connotations in economics, we use it here to refer to management’s ability to take decisions in the face of disagreement with investors.\footnote{In our future discussion we shall be equating managerial autonomy with governance practices like reporting structure, board composition, voting structures of the firm etc. This is because these choice variables are important determinants of the level of managerial autonomy.} Management attaches a value to autonomy because of the perception that greater autonomy will facilitate decisions that lead to higher firm value. Autonomy comes at a cost because rational shareholders will \textit{ex ante} demand a higher cost of capital to allow management to take actions which in their opinion are value reducing. Thus, there is a natural tradeoff between managerial autonomy and cost of capital for a firm. The benefit of private ownership is that it enables managers to achieve the desired tradeoff between governance structure and cost of capital through private contracting with a few large investors. The investors and manager divide responsibility among them in a manner that optimizes costly intervention from the standpoint of the entrepreneur/manager. Such arrangements are commonplace in venture capital contracts wherein, al-
though the investor takes an active interest in the firm, there is a clear delineation of responsibilities. By contrast, in the case of public ownership much of the governance structure and disciplining mechanisms are externally imposed by the financial market regulators and investors. They are usually designed for the median firm in the economy, so that the combination of the governance structure (management autonomy) and cost of capital is market determined rather than privately negotiated. Balanced against this disadvantage of public ownership is the benefit to investors of being able to trade their holdings in a liquid public market, which lowers the cost of capital to the firm. From the standpoint of the entrepreneur/manager, on the one hand public ownership brings with it a more rigid corporate governance structure while on the other hand it is accompanied by a lower cost of capital than private ownership.

Our theory yields a rich set of predictions, some of which have existing empirical support, while others can be tested to potentially refute the model. We find that the choice of private versus public ownership depends on the stringency of corporate governance for public firms. When corporate governance is extremely lax and permits management considerable autonomy, investors in the public market demand an excessively high rate of return on their capital and firms prefer private ownership. When corporate governance is extremely stringent and leaves management with relatively little autonomy, firms once again prefer private ownership because public-market corporate governance is considered excessively intrusive by the entrepreneur/manager. For intermediate values of corporate governance stringency, firms choose public ownership. The measure of this set of values of corporate governance stringency is affected by the exogenous parameters in the analysis, such as the cost of illiquidity for investors with private ownership as well as the likelihood of disagreement between management and investors.

The rest of the paper is organized as follows. Section 2 gives a brief overview of the related literature, in Section 3 we present our basic model, Section 4 contains our analysis. Robustness issues are discussed in Section 5. Empirical predictions are discussed in Section 6. Section 7 summarizes and concludes. All proofs are in the Appendix.
2 RELATED LITERATURE

Our paper is related to the emerging literature on the liquidity-versus-control tradeoff in the choice between private and public ownership. This literature has focused on the problem that investors face in trading off liquidity of ownership against control over management. Bhide (1993) and Coffee (1991) argue that the dispersed ownership that typically accompanies public incorporation brings with it liquidity benefits for investors. By contrast, private ownership tends to be more concentrated and hence owners have a greater say in management decisions. But this control comes at the expense of liquidity of ownership. While others have pointed out that the dichotomy between private and public ownership need not be as stark in either theory or practice,\(^4\) this fundamental difference in corporate governance between private and public ownership is generally acknowledged.

As a starting point for our analysis, we take as a given the greater liquidity associated with the public market. However, our first point of departure is the further observation that corporate governance – the degree of control investors effectively exercise over management – tends to be generic or standardized with public ownership, and can be tailored to specific needs with private ownership. Thus, we build on the idea that the choice of public ownership versus private ownership is a commitment to a particular type of governance structure. This is in consistent with Pagano (1993) who emphasizes the contractibility of corporate governance arrangements with private ownership. Maksimovic and Pichler (2001) focus on the lower flexibility of public ownership, but specifically within the context of increased disclosure requirements.\(^5\)

Our work is also related to the papers that have focused on the possibility of excessive monitoring by

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\(^4\) Bolton and von Thadden (1998) argue that a limited degree of ownership concentration in public capital markets would permit one to combine the benefits of liquidity and control. In particular, they argue that block shareholdings will come about only if there is sufficient aggregate dispersion of ownership to generate liquidity and an exit option for large shareholders. Maug (1998) challenges the view that a liquid stock market dilutes large shareholders' incentive to monitor, and shows that a more liquid stock market leads to better corporate governance.

\(^5\) Zingales (1995) focuses on a different type of commitment in which the ownership structure affects the bargaining power of the (selling) initial owner.
shareholders as part of corporate governance. For example, Pagano and Roell (1998) argue that private companies owned by more than one shareholder may experience excessive monitoring. With sufficiently many investors, the cost of this overmonitoring may be so large that the firm may wish to go public.\(^6\) Similarly, Burkhart, Gromb and Panunzi (1997) suggest that while tight control by shareholders might be ex post efficient, it might ex ante discourage firm-specific investments. Managerial discretion may then have distinct benefits, and as Burkhart, Gromb and Panunzi argue, there might be a trade-off between the gains from shareholder monitoring and those from managerial initiative. While the perspective in these papers is that of shareholders, we explicitly take a \textit{managerial} perspective and ask how an entrepreneur or owner/manager, who intends to run the firm, would choose between private and public ownership. In this respect, the manager views investor monitoring as diminishing his autonomy. Unlike the existing literature, shareholders do not view this as decreasing firm value. However, the entrepreneur views it as decreasing firm value because it inhibits his ability to make value-maximizing decisions.

There are several other papers that have examined the choice between private and public ownership. For example, the venture capital literature has discussed the improved control that comes with private ownership (see, for example, Black and Gilson (1998) and Kaplan and Stromberg (2003)). This is in the spirit of Jensen (1989) who stresses the free-rider (and free cash flows) problems that may be prevalent in publicly-listed corporations. By contrast, because we explicitly assume that the entrepreneur/manager and investors are both interested in maximizing firm value, we have no agency problems \textit{per se}. We later discuss the differences between our results and those that would obtain in an agency setting.

Others have emphasized the diversification and liquidity benefits of public capital markets (see Pagano (1993), Admati, Pfleiderer and Zechner (1994) and Shah and Thakor (1988)).\(^7\) While diversi-

\(^6\)By contrast, Chemannur and Fulghieri (1999) argue that public capital markets involve duplicated monitoring costs. \(^7\)Shah and Thakor (1988) explore the tradeoff between the risk sharing afforded by public ownership and the listing costs of public ownership. Admati, Pfleiderer and Zechner (1994) focus on the tradeoff between the risk-sharing costs and the intervention benefits of concentrated ownership within the context of an individual investor with an exogenously given ownership stake.
fication is not an issue in our analysis because we have universal risk neutrality, the liquidity benefit of
public ownership is present in our analysis as well.

3 MODEL OUTLINE

In this section we describe the key elements of our model, the agents and project possibilities, the
liquidity costs and the managerial autonomy parameter. We conclude this section with the sequence of
events.

3.1 Agents and Project Possibilities

We consider an economy with one firm and several investors. The firm is managed by an entrepreneur
or owner-manager (manager henceforth) who owns 100 percent of the equity. All agents are risk neutral
and the riskfree interest rate is 0. There are three dates 0, 1 and 2, defining two time periods. The
firm has existing assets (whose values are normalized to 0) and gets an investment project at date 0.
The firm has no internal finance and hence requires external financing for the project. The firm raises
financing from investors in the form of equity at date 0. Investment in the project occurs at date 1
and the project realizes final cash flows at date 2. The time between date 0 and date 1 can be thought
of as the period spent in preparing for implementing the project in terms of raising finance, preparing
feasibility reports etc. At every stage, the manager makes decisions to maximize terminal firm value.
Thus, the long-term interests of the manager and outside financiers are aligned.

The firm’s project can be of two types, Good (G) or Bad (B). The commonly-known prior probability
that a project is $G$ is $p$. A $G$ project has positive NPV while a $B$ project has negative NPV. Both
projects require an investment of $1 at date 1. At date 2 project $G$ realizes a cash flow of $X_G$ for
sure, while Project $B$ realizes a cash flow of $-X_B$ for sure. We have $X_G > 1$, and assume that the
appropriate discount rate is zero. Moreover, $pX_G - [1 - p]X_B = 0$, for simplicity.
The project implementation decision at date 1 is state contingent. The manager and the investors decide on the optimal choice based on a public signal, $S$, that they both observe at date 1. The signal value can be $G$ or $B$, indicating project quality. Although the manager and investors observe the same signal, they have different priors on the informativeness of the signal. The prior beliefs about the informativeness of the signal, $q$, are drawn randomly from the set $\{I,U\}$, where $q = I$ represents an “informative” signal and $q = U$ represents an “uninformative” signal. The probability of drawing $I$ is $\theta \in (0,1)$ and of drawing $U$ is $1 - \theta$. When the prior belief is that the signal is informative the agent believes $Pr(\text{Project } G|S = G, q = I) = 1$. And when the prior belief is that the signal is uninformative, the agent believes $Pr(\text{Project } G|S = G, q = U) = p$, i.e. an uninformative signal fails to change the agent’s prior belief about the quality of the project.

If the public signal is $S = B$, then it is clear that a prior belief that the signal is $I$ would lead to a decision to reject the project. Moreover, even if the prior belief is that the signal is $U$, the decision would be to reject the project since its expected value would be $pX_G - [1 - p]X_B = 0$ and its net present value (NPV) would be $pX_G - [1 - p]X_B - 1 < 0$.

If the public signal is $S = G$, and the prior belief is that the signal is $I$, then the project NPV will be $X_G - 1 > 0$, and the decision would be to accept the project. If the prior is $U$, then the project NPV would be assessed as $pX_G - [1 - p]X_B - 1 < 0$, and the decision would be to reject the project.

The differences in the prior beliefs of the manager and investors come about as follows: The manager receives a private signal that is a random draw from $\{I,U\}$ and this signal sets his prior belief about the precision of the public signal $S$. Similarly, investors as a group also randomly draw a prior belief from $\{I,U\}$ about the precision of the public signal. We allow the prior beliefs about signal precision that the manager and investors draw to be correlated. That is, we assume $Pr(q_i = j|q_m = j) = \rho$, $Pr(q_i = j|q_m = k) = 1 - \rho$, where $j \neq k$, $j, k \in \{I,U\}$, $q_i$ represents the signal precision for the investors and $q_m$ the signal precision for the manager.
It is clear that if \( S = B \) is the commonly-observed public signal, the manager and investors will always agree that the project should be rejected, regardless of their prior beliefs about the precision of the signal. If \( S = G \) and investors and the manager have prior beliefs \( q = U \), they will once again agree that the project should be rejected. If \( S = G \) and investors’ prior belief about signal precision is \( q = I \) whereas the manager’s prior belief is \( q = U \), then the manager will wish not to invest even though investors would like to. We assume that no one can force the manager to invest when he does not want to, so disagreement is irrelevant in this case.\(^8\) However, when \( S = G \), the manager’s prior belief about signal precision is \( q = I \) and the investors’ prior belief is \( q = U \), the manager will wish to invest but investors won’t. This disagreement is the focus of our analysis.\(^9\) It follows that, conditional on the manager and investors observing \( G \) and the manager believing that the precision of the public signal is \( q = I \), \( \rho \) is the probability that the investors agree with the manager that investment in the project should occur.\(^{10}\)

The greater is the value of \( \rho \), the greater is the potential agreement between the manager and the investors; \( \rho = 1 \) indicates perfect agreement while \( \rho = 0 \) indicates perfect disagreement. The agreement parameter \( \rho \) can be thought of as being affected by the attributes of the project (or more generally the nature of the firm’s business) as well as management’s experience and track record in managing projects of that type. If the project is one that the manager has dealt with before successfully and investors are familiar with it as well, \( \rho \) will tend to be high. For unfamiliar projects and/or those the manager may not have successfully handled in the past, \( \rho \) can be expected to be low. Hence, there may well be a temporal dimension to the level of agreement between investors and the manager. We now discuss two key features of the model: the liquidity costs faced by investors and the corporate governance (or

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\(^8\)This is a fairly natural restriction. One way to think about it formally is to assume that the probability with which a project arrives at \( t = 1 \) is less than 1 and the manager is the first one to see whether the project has arrived, observes \( S \) and \( q \), and then decides whether to bring the project forward for investor to evaluate. Only then can investors observe \( S \) and their own \( q \). This way the manager can always say there was no project when he does not wish to invest. Adopting this structure would introduce a bit more algebra but would not materially affect the analysis.

\(^9\)This disagreement is not an agency problem related to a divergence of objectives since the manager and investors both have the same objective. It is also not a problem of asymmetric information or insufficient information aggregation since the manager and investors receive exactly the same signal \( S \).

\(^{10}\)If no investment occurs but funding was raised, the funds remain in the firm till date 2.
managerial autonomy) parameter $\eta$.

### 3.2 Investors’ liquidity cost

After the investment has been made (between dates 1 and 2), investors may suffer a stochastic liquidity shock, i.e. with a probability $\lambda$ investors suffer a liquidity shock while with a probability $(1 - \lambda)$ they do not suffer a shock. When there is a liquidity shock, investors try to sell their assets to raise cash. Thus, if the investors hold shares in the firm, they will try to sell the shares in the market. If the investors are prevented from selling the shares for whatever reason, we assume that they are forced to sell other personal property and suffer a non-pecuniary loss due to such a sale of $L > 0$. For simplicity, we assume that whenever the firm is publicly traded, investors will be able to sell their shares at the market-clearing price, thereby avoiding the non-pecuniary loss $L$. By contrast, if the firm is privately owned, the investors will not be able to sell their shares and will be forced to sell other assets and suffer a loss of $L$; this is what we call the liquidity cost of private ownership. For our results to go through, all we need is that the illiquidity of the ownership stake in a private firm imposes additional costs on the investors when they suffer a liquidity shock. For simplicity we make this extreme illiquidity assumption.

### 3.3 Managerial Autonomy Parameter

Whenever there is disagreement between shareholders and the manager about project quality, there has to be a rule to resolve the disagreement. We model this using the concept of “autonomy” for the manager. Managerial autonomy (parameterized by $\eta$) is the probability that the manager will be able to implement his decision in the face of disagreement with the shareholders. One could also refer to this as the degree of control given to the manager. Thus, $(1 - \eta)$ is the probability with which shareholders can successfully intervene to stop the manager from investing in the project. Higher values of $\eta$ correspond to higher values of managerial autonomy. 

*ceteris paribus* the manager prefers more autonomy to less,
even though he derives no direct utility from autonomy, i.e. there is no exogenous private control benefit in this model. In real world terms, one can think of the manager having greater autonomy by curtailing the rights of shareholders, having more seats on the Board of Directors that can be occupied by directors friendly to the manager, negotiating for fewer information disclosure requirements, etc. The key is that \( \eta \) is determined by the corporate governance regime in place in public markets, while it is an endogenous choice variable for the manager in the private market.\footnote{The assumption that corporate governance with public ownership is exogenously fixed should be viewed as an approximation rather than being taken literally. Clearly, the stringency of corporate governance varies in the cross-section of public firms. The idea we want to capture is that there is a greater ability to calibrate the governance structure to the needs of the firm (as perceived by management) in the case of private ownership due to the absence of an exogenous set of governance requirements imposed by the securities exchange in the case of public firms. As we indicate in Section 5, our results will go through even if \( \eta \) is partially endogenized with public ownership but it is more “rigid” than with private ownership.}

### 3.4 Sequence of Events

The time line is as follows. At date 0, the firm decides whether to be privately or publicly owned and raises financing required for the project in the form of outside equity. Between dates 0 and 1, investors and the manager receive their common public signal about project quality and also draw their private priors about the precision of this public signal. At date 1, the project investment decision is made. As indicated earlier, the only state in which there is decision-relevant disagreement between investors and the manager is when the public signal is \( G \), investors draw a prior belief about its precision that it is \( q = U \) and the manager draws a prior belief about precision that it is \( q = I \). In this case, the manager wants to invest in the project but investors don’t. We assume that investors can block management with probability \( 1 - \eta \); with probability \( \eta \), management is able to invest in the project despite investor objections.

After the project investment decision is made (between dates 1 and 2), investors could suffer a liquidity shock. If the firm is publicly traded, investors sell their shares in the market at the market-clearing price. If the firm is private, investors are unable to sell their shares in the market and suffer a
loss $L$. Terminal cash flows are realized at date 2. This sequence of events is shown in Figure 1.

4 ANALYSIS

The only decision to be made by the manager is the ownership choice at date 0: public or private. With this the choice of the financing source (private versus public equity) is determined. The manager chooses the ownership structure to maximize his expected wealth at date 2. The manager owns all of the firm’s shares at date 0 (prior to financing). The manager’s payoffs under alternative ownership structures are analyzed in this section.

4.1 Public Ownership - IPO

When the manager raises investment finance through an IPO at date 0, the manager offers the (new) investors a fractional ownership of $(1 - \alpha)$ of date 2 cash flows in return for the investment. The value of the firm at date 0 (after the $1 is raised from investors), as estimated by the public market (shareholders), is given by:

$$V_{pub} = Pr(s = G)Pr(q_m = I)Pr(q_i = I|q_m = I)[X_G - 1]$$

$$+ Pr(s = G)Pr(q_m = I)Pr(q_i = U|q_m = I)\eta[-1] + 1$$

(1)

Note that the first term is the net present value (NPV) of the project when both the investors and the manager see a public signal $S = G$ and agree that it is informative. The second term is the project NPV when a public signal $S = G$ is observed, the manager’s prior belief is that the signal is informative and the investors’ prior belief is that it is uninformative. In this case the manager is able to invest in the project nonetheless (with a probability $\eta$), and the investors assess the project NPV as -1. The last term is simply the $1 in external financing raised at date 0. Note that in all states other than those reflected in the first two terms, the NPV of the project is zero because there is no investment. For
notational convenience, we will write \( p \theta = \hat{p} \). Thus,

\[
V_{pub} = \hat{p}[X_G - 1] - \hat{p}[1 - \rho]\eta + 1
\]

\[
= \hat{p}[\rho A + [1 - \rho]\eta D] + 1
\]

(2)

where \( A = X_G - 1 \) and \( D = -1 \).

Here \( A \) and \( D \) represent the NPV in the agreement and disagreement states. When investors are given a fraction \( (1 - \alpha) \), the value as perceived by them is \( V_{pa} = (1 - \alpha)V_{pub} \). The minimum fraction required to be sold to the investors to raise an amount of $1 is given by the individual rationality (IR) constraint of the investors as:

\[
1 - \alpha_{pub} = \frac{1}{V_{pub}}
\]

(3)

The exogenous parameters characterizing the public market determine the fraction of equity to be sold to the public investors.

**Lemma 1**

When the manager raises financing from the public market, the cost of capital (the fraction \( 1 - \alpha_{pub} \) that must be sold to investors to raise $1) is decreasing in the agreement parameter \( \rho \) and increasing in the autonomy parameter \( \eta \).

This lemma shows the tradeoff between managerial autonomy and the cost of capital. Whenever there is disagreement between investors and the manager, managerial autonomy is costly for the investors. This is because at any non-zero level of managerial autonomy \( (\eta > 0) \), the manager sometimes succeeds in undertaking a project the investors think is a bad bet. Hence, the higher is the autonomy offered to the manager, the higher is the return demanded by the investors. Further, this cost is a decreasing function of the level of agreement between the manager and investors. Hence, the greater the agreement between investors and management, the lower is the perceived cost of managerial autonomy. This lemma also highlights a point with development implications. In developing countries,
securities laws and disclosure requirements are not very well developed. This provides managers with
greater autonomy from investors and is equivalent to a high $\eta$ in the market. Under such situations,
this lemma clarifies that only managers with very good track records (high $\rho$) will be able to raise
equity. For other managers, the cost of capital demanded by the market will be so high as to preclude
financing. Thus, this lemma offers a strong argument for limiting managerial autonomy through more
stringent corporate governance even in the absence of a divergence of objectives between management
and investors.

The total value as perceived by the manager with public ownership is given by:

$$V_{m}^{\text{pub}} = \alpha_{\text{pub}}\{Pr(S = G)Pr(q_{m} = I)Pr(q_{i} = I|q_{m} = I)|X_{G} - 1]\}
+ \alpha_{\text{pub}}\{Pr(S = G)Pr(q_{m} = I)Pr(q_{i} = U|q_{m} = I)|X_{G} - 1] + 1\}
= \alpha_{\text{pub}}[\hat{p}A\{\rho + (1 - \rho)\eta\} + 1] \tag{4}$$

Substituting for $\alpha_{\text{pub}}$ from (3) we have :

$$V_{m}^{\text{pub}} = \left[1 - \frac{1}{V_{\text{pub}}^{\text{pub}}}\right][\hat{p}A\{\rho + (1 - \rho)\eta\} + 1] \tag{5}$$

At date 0 the manager chooses between private and public ownership, by comparing the maximum
value that he gets with public ownership, $V_{m}^{\text{pub}}$, with the maximum value he gets from having private
ownership, which we shall derive next.

4.2 Private Ownership : Private placement

When the manager raises private equity, we assume that he raises money from a single investor,
who can be thought of as a VC or an institutional investor. This is in line with the evidence in
Wruck[1989] that private placement of equity takes place predominantly with a single investor. The
principal advantage of private ownership is that, since ownership is concentrated in the hands of only
one investor, the manager and investor can, at the time of financing, contractually fix the optimal level
of autonomy to be given to the manager. This can be thought of as a contract specifying the rights of
the investor and also the reporting structure and division of responsibility between the manager and
the investor. The main cost of private ownership is the liquidity loss imposed on the investor. Now
we shall solve for the optimal degree of managerial autonomy and the associated cost of capital with
private ownership.

The two decision variables the manager fixes when raising the financing at date 0 from a private
investor are $\alpha$ and $\eta$. The initial analysis parallels the earlier analysis of public ownership. The value
of the firm at date 0 as estimated by the private investor for a given level of $\eta$ is given by:

$$V_{pr} = \hat{p}\rho X_G + [1 - \hat{p}(\rho + (1 - \rho)\eta)] - L\lambda$$

This is similar to equation (2) with public ownership except for the $L$ term. We can rewrite $V_{pr}$ in
a more convenient form as:

$$V_{pr} = \hat{p}(\rho A + (1 - \rho)\eta D) + 1 - L\lambda$$

where $A = X_G - 1$ and $D = -1$ as before.

Observe that $\hat{p}(\rho A + (1 - \rho)\eta D)$ is the NPV of the project as perceived by the investor; call it
$NPV_i$. Thus, when the investor is given a fraction $(1 - \alpha)$, the value as perceived by her is $V_{pr}(\alpha) =
(1 - \alpha)(NPV_i + 1) - L\lambda$. It is clear that the investor’s valuation is strictly decreasing in $\eta$ (since $D < 0$)
and increasing in $\alpha$. The investor will agree to invest in the project as long as $V_{pr}(\alpha) \geq 1$.

The manager’s objective is to maximize the expected value of his wealth. After raising external
financing, the manager will own a fraction $\alpha$ of the firm. Thus, his expected wealth is:

$$V_{pr}^m = \alpha[\hat{p}\{\rho + (1 - \rho)\eta\} + 1]$$

$$= \alpha[NPV_m + 1]$$

where $\hat{p}[\rho A + (1 - \rho)\eta] \equiv NPV_m$ is the NPV of the project as estimated by the manager. It is clear
that $V_{pr}^m$ is increasing in $\eta$ and decreasing in $\alpha$. Thus, $V_{pr}^m$ and $V_{pr}(\alpha)$ move in opposite directions with
changes in $\alpha$ and $\eta$. This is what sets up the tradeoff between autonomy $\eta$ and the cost of capital measured by $(1 - \alpha)$.

The manager’s problem in the case of private ownership can be formulated as:

\[
\max_{\alpha, \eta} \quad \alpha[NPV_m + 1] \quad (9)
\]
\[
s.t. \quad V_{pr}(\alpha) \geq 1 \quad (10)
\]
\[
0 \leq \alpha \leq 1, \quad 0 \leq \eta \leq 1 \quad (11)
\]

We can now derive Lemma 2.

**Lemma 2**

The optimal value of $\eta^*$ at the local maximum is:

\[
\eta^* = \frac{-A[1 + \hat{\rho}A] + t}{\hat{\rho}(1 - \rho)DA} \quad (12)
\]

where $t \equiv \sqrt{A(1 + L\lambda)(A - D)(1 + A\hat{\rho})}$

The values of $\alpha^*$, $NPV_i$ and $V_{pr}^m$ are given by:

\[
\alpha^* = \frac{t - A[1 + L\lambda]}{t} \quad (13)
\]
\[
NPV_i^* = \frac{t - A}{A} \quad (14)
\]
\[
V_{pr}^m = [A + 1][1 + \hat{\rho}A] + A[1 + L\lambda] + \frac{A[1 + L\lambda]}{t} - 2t - 1 \quad (15)
\]

This lemma says that there is an interior optimum with respect to the degree of managerial autonomy, $\eta^*$, and that this is jointly determined with the cost of capital to the entrepreneur, as represented by $\alpha^*$, the ownership of the firm that he is able to retain after raising external financing. We shall now examine the comparative statics properties of the equilibrium in the private equity market.
Lemma 3

When the manager raises equity from the private investor, the fractional ownership to be sold to the private investor (and consequently the cost of capital) is decreasing in the agreement parameter $\rho$ and increasing in the liquidity cost $L$.

Thus, we find that even in the case where managerial autonomy is endogenously determined, the fractional ownership sold to the investors decreases in the level of agreement between the investor and the manager. Managers with well-established track records in having successfully managed similar projects in the past (high $\rho$) need to sell smaller fractions of their firms for raising the same amount of financing as managers operating newer projects that both they and investors are relatively less familiar with (low $\rho$). Further, the greater is the liquidity loss suffered by the private investor, the greater will be the fractional ownership demanded by the investor.

Now we shall look at how $\eta^*$ varies with $\rho$. Intuitively, it should be clear that $\eta^*$ should increase with $\rho$ because the marginal cost of a higher $\eta$ for shareholders is decreasing in $\rho$. The following lemma shows that this intuition is correct.

Lemma 4

In the interior of $(0,1)$, the optimal autonomy $\eta^*$ is strictly increasing in the agreement parameter $\rho$.

This lemma says that investors give greater autonomy to managers with whom they agree more. Along with Lemma 3, this lemma shows that with private ownership greater managerial autonomy goes along with higher firm valuation. This result is different from what one would expect given the basic set-up of our model, since ceteris paribus greater managerial autonomy results in lower valuation by
investors. But this lemma is talking about the *endogenously-determined* equilibrium level of autonomy. Investors who are ready to assign a higher valuation to a firm’s shares are basically indicating greater confidence in the firm’s managers as reflected in a greater propensity to agree with the manager. The manager gets greater autonomy (higher $\eta^*$) from such investors because the marginal cost of the autonomy is lower.

### 4.3 Choice Between Private and Public Ownership

Based on our analysis of private ownership, we have shown that both the autonomy offered to the manager and the fractional ownership retained by the manager are increasing functions of $\rho$. We shall now compare the private and public ownership scenarios and characterize the manager’s choice between the two.

**Proposition 1**

There exist cut-off values of autonomy $\eta$ in the public market, given by $\eta_1$ and $\eta_2$, such that a manager prefers public ownership for all $\eta \in [\eta_1, \eta_2]$. For $\eta \notin [\eta_1, \eta_2]$ the manager prefers private ownership.

The basic intuition is as follows. When $\eta^* \in (0, 1)$, the manager’s objective is uniquely maximized at $\eta = \eta^*$. For a given $\rho$, the manager’s objective monotonically decreases as $\eta$ moves away from $\eta^*$. The manager will be willing to accept the sub-optimal $\eta$ of the public market as long as the loss due to this sub-optimality is less than the cost due to the illiquidity of private ownership. While the illiquidity cost is independent of $\eta$, the loss due to a sub-optimal choice of $\eta$ increases in the distance between $\eta^*$ and $\eta$. Thus, for $\eta$ sufficiently close to $\eta^*$, the illiquidity cost dominates, while for $\eta$ sufficiently far away from $\eta^*$, the cost due to sub-optimality dominates. This yields the result that, for values of $\eta$ close to $\eta^*$, the manager prefers public ownership, while for values of $\eta$ sufficiently different from $\eta^*$, he prefers private ownership.
This proposition illustrates the tradeoff involved in the choice of ownership mode. As mentioned earlier, market governance systems cannot be tailored for individual firm needs. So whenever there is a greater disparity between the governance dictated by the public market and the individual needs of the firm, the firm chooses to forego the liquidity provided by the market and chooses private ownership. When the disparity between what market-based corporate governance dictates and what the manager would like is not too large, the firm chooses public ownership to avoid the higher cost of capital associated with private ownership. Further, since we know that \( \eta^* \) is increasing in \( \rho \), private ownership is optimal when the public market determined \( \eta \) is misaligned with a firm’s \( \rho \). Thus both for high \( \rho \) and low \( \eta \) (Case a) and for low \( \rho \) and high \( \eta \) (Case b) private ownership is preferred. In Case a, we have a situation in which the public market imposes relatively restrictive governance and reporting requirements. A manager whose track record in managing similar projects previously is such that it has led to a relatively high \( \rho \) will find the low \( \eta \) associated with restrictive public-market corporate governance to be unacceptable. Even though the high \( \rho \)-low \( \eta \) combination means that the firm could go public at a very high stock price, the manager chooses private ownership. That is, despite offering the firm a very low cost of capital, the public capital market is unable to attract the firm. Case b corresponds to a situation in which the firm is investing in an unfamiliar project with an inadequate managerial track record in managing similar projects in the past, so that \( \rho \) is low; moreover, the corporate governance imposed by the public market is not particularly restrictive. In this case, the manager chooses to remain private because the cost of capital in the public market is exorbitantly high.

We now present some comparative statics results.

**Proposition 2**

The measure of the autonomy interval \( [\eta_1, \eta_2] \) within which public ownership is preferred is increasing in the liquidity cost \( L \), the probability of liquidity shock \( \lambda \) and the agreement parameter \( \rho \).

The intuition is as follows. The interval \( [\eta_1, \eta_2] \) represents the set of managerial autonomy parameters
for which the manager prefers public ownership. As the liquidity cost, \( L \), for private ownership increases, so does the effective difference in the costs of capital for private versus public ownership, and the firm goes public for a bigger set of autonomy parameters or public-market governance regimes. Similar intuition applies to \( \lambda \). Finally, an increase in \( \rho \) means that investors are more likely to endorse managerial decisions. This makes any market-determined corporate governance regime less unattractive to the manager since the likelihood of disagreement with investors declines as \( \rho \) increases. Thus, the measure of \([\eta_1, \eta_2]\) increases with \( \rho \).

**Proposition 3**

Whenever there is uncertainty about the value of autonomy \( \eta \) in the public market, the probability of the firm choosing public ownership decreases.

The intuition behind this proposition is that the manager’s objective function is concave in the autonomy parameter \( \eta \). Thus, the manager behaves as if he is risk averse with respect to \( \eta \), and abhors uncertainty about \( \eta \). Since in the case of private ownership, the owner/firm relationship is more of a permanent nature (ownership in public markets is more fluid, changes often), the manager may perceive less uncertainty about \( \eta \). This would imply that uncertainty about \( \eta \) is particularly important with public ownership, and that an increase in this uncertainty reduces the attractiveness of public ownership for the manager. To the extent that Sarbanes-Oxley increases the uncertainty about the future values of \( \eta \) with public ownership – due to uncertainty about how the new governance requirements will affect management autonomy – we should expect the attractiveness of private ownership to increase.

**5 MODEL ROBUSTNESS**

In this section we discuss some of the key assumptions in the analysis and the robustness of our results to relaxing those assumptions.
Consider first the assumption that the autonomy parameter $\eta$ is exogenously fixed with public ownership. As we indicated earlier, we could permit $\eta$ to be partially endogenized by choosing it say to be the optimal $\eta$ corresponding to the agreement parameter $\rho$ of the median firm. This will not affect the results as long as the public-ownership $\eta$ for each firm cannot be chosen to be precisely optimal for that firm’s agreement parameter $\rho$. The notion that the tradeoff between the potential suboptimality of exchange-imposed disclosure requirements (and hence an associated governance structure) and the liquidity of public equity markets is an important determinant of the choice between private and public ownership is illustrated by the empirical evidence provided by Bushee and Lenz (2003). They examine the impact of the recent regulatory change mandating firms on the OTC Bulletin Board (OTCBB) to comply with the reporting requirements under the Securities Exchange Act of 1934. They find that this change resulted in a number of firms opting to not comply with these requirements and thus accepting removal from the OTCBB; the consequence is that these firms went private or traded in the Pink Sheets, where SEC filing is not required. Only a small number of firms found it optimal to adopt the new disclosure requirements to continue trading on the OTCBB. These firms experienced a permanent increase in market liquidity.

A related issue has to do with our assumption that the liquidity cost, $L$, and the probability of a liquidity shock, $\lambda$, are the same for all investors. In reality, these will vary cross-sectionally, so that a clientele effect in ownership is likely to emerge; investors with longer investment horizons and lower probabilities of experiencing interim liquidity shocks are more likely to become private equity investors. However, this clientele effect will not materially affect the analysis.

The third robustness issue is related to our assumption that there is no divergence of objectives between the manager and investors. Since the typical justification for having corporate governance restrictions on managers is the presence of agency problems, it is interesting to ask what introducing agency concerns would do. Our analysis predicts that more stringent corporate governance (lower $\eta$) will increase the attractiveness of public ownership for the low-$\rho$ (low-valuation) firms and decrease
it for the high-$\rho$ firms. By contrast, with an agency framework, all firms find public ownership more attractive as corporate governance becomes stricter. This is because stricter corporate governance leads to lower agency costs and makes the firm/entrepreneur better off since agency costs are absorbed *ex ante* by the entrepreneur. Thus, an agency perspective would suggest that limiting case of $\eta = 0$ as optimal with private ownership and would also suggest that regulators of publicly-traded firms would wish to move in the direction of $\eta = 0$.

A fourth assumption in our analysis is that all financing is done with equity, including the financing of private firms. Given the fact that private debt markets are significantly bigger than private equity markets, it would be interesting to consider the effect of debt. If debt is the only security with which external finance can be raised with both private and public ownership, little of consequence changes in our analysis; public versus private debt will lead to a similar trade-off. If debt and equity are both used for external financing, then the analysis becomes substantially more complicated since the security-issuance and capital structure choices now have to be endogenized, and these choices may vary across private and public ownership. Issues related to collateral-eligible assets and how they affect the preference for debt over equity would also enter the analysis.

The final point has to do with the fact that there is no investment opportunity for the firm after the initial investment at $t=1$. If the firm were to suffer subsequent shocks that would randomly determine its future investment needs and the availability and/or cost of capital in the private capital market were uncertain, the attractiveness of public ownership would go up. That is, a shock to the firm’s need for capital would act in a manner similar to the liquidity shock for investors.\footnote{One interesting avenue of future research is to analyze the stability of $\rho$ over time. Arguably, with private ownership $\rho$ is more stable, while with public ownership shareholders may sell their holdings (for liquidity reasons) and might be replaced by investors with a different $\rho$. This allows for a dynamic analysis of optimal ownership structure.}
6 EMPIRICAL PREDICTIONS

The main empirical predictions of our model are summarized here.

1. When the firm privately places equity, corporate governance is less restrictive for the manager when the firm is more highly valued. This follows from Lemmas 3 and 4 which assert that the firm’s valuation is higher when the agreement parameter \( \rho \) is higher (Lemma 3) and \( \eta^* \) is higher (corporate governance is less restrictive) when \( \rho \) is higher (Lemma 4).

2. From Proposition 1 we know that when the agreement parameter between the manager and investors is low and corporate governance in the public capital market is not particularly restrictive (high \( \eta \)), the manager prefers private ownership to public ownership. This implies that in countries where reporting requirements and corporate governance are relatively lax and the firm is venturing into a new business design or technology, private ownership will be preferred.

3. We know from Lemma 4 that \( \eta^* \), the autonomy that is optimal for the entrepreneur, is increasing in \( \rho \). Moreover, from Proposition 1, we know that firms will prefer public ownership when the public-market \( \eta \in [\eta_1, \eta_2] \). When public-market corporate governance is relatively lax, \( \eta \) will be relatively high. And when \( \rho \) is relatively high, \( \eta^* \) will also be high. So, a combination of lax corporate governance and a high firm valuation (high \( \rho \)) will produce a preference for public ownership since \( \eta \) and \( \eta^* \) will be relatively close. By contrast, when public-market corporate governance is restrictive (low \( \eta \)), a firm with a sufficiently high \( \rho \) will choose to remain private because \( \eta^* \) will be relatively far from the public-market \( \eta \). This means that a combination of stringent corporate governance and a high firm valuation will cause a preference for private ownership. Alternatively, if these firms were to be induced to remain public, they would have a preference for relatively lax corporate governance. Note that this prediction is strikingly different from the one that would emerge from an asymmetric information model in which the most valuable firms would prefer greater stringency in reporting requirements and corporate governance, as this would lead to less
pooling with lower-quality firms.\textsuperscript{13}

4. An increase in the restrictiveness of corporate governance in the public market increases the attractiveness of public ownership for the low-valuation firms and decreases it for the high-valuation firms (see the discussion in Section 5). This is in sharp contrast to the prediction that would come from an agency-costs model wherein all firms would find public ownership more attractive as the stringency of corporate governance with public ownership increases.

5. From Proposition 2 we know that an increase in $\rho$ increases the interval $[\eta_1, \eta_2]$ of managerial autonomy parameters for which the firm goes public. This implies that as a business design or technology becomes more familiar to investors and managers (so that disagreement over optimal decision diminishes), more firms deploying that business design or technology will go public. That is, the percentage of publicly-traded firms will be larger in older, more well-established industries.\textsuperscript{14}

6. Our analysis shows that an increase in $\rho$ increases $\eta^*$, the optimal autonomy with private ownership (Lemma 4) and the range of autonomy parameters, $[\eta_1, \eta_2]$, for which the firm goes public (Proposition 1). Since a high $\rho$ is synonymous with a high valuation in our model, the implication is that the combination of lax corporate governance (high $\eta$) and a bull market (high $\rho$) will cause an increase in firms going public. The effect of a bull market on firms going public will be weaker with stringent public-market corporate governance since the public market $\eta$ will be low in this case and a higher $\rho$ leads to higher $\eta^*$ and moves it further away from the public market $\eta$. Thus, the empirical prediction is that a runup in the market will cause a larger percentage of firms to go public in capital markets with relatively lax corporate governance than in capital markets with relatively stringent corporate governance. A related prediction emerging from Proposition 2 is that firms with a relative high market-to-book ratio within their industry are most likely to go public. The high valuation would signal relatively high values of $\rho$.

7. From Proposition 3 we know that, even with universal risk neutrality, an increase in uncertainty

\textsuperscript{13}See for example, Boot and Thakor (2001).
\textsuperscript{14}In other words, familiarity breeds public ownership.
about the restrictiveness of corporate governance in the public market leads to a lower likelihood of the firm going public. The prediction is that in countries where the rules of corporate governance (including information disclosure requirements) change frequently (leading to greater uncertainty), fewer firms go public.

We believe all of the above predictions are novel in the context of the existing theoretical literature. Two of the predictions are consistent with existing stylized facts. Pagano, Panetta and Zingales (1998), using data for Italian firms, present evidence that the probability of an IPO is increasing in the industry’s market-to-book ratio. Prediction 3 from our analysis says that this is what we should expect to find in countries with lax corporate governance since in such countries it is the high-valuation firms that go public. But Prediction 3 offers a more refined hypothesis to test, with the stringency of corporate governance as a control variable. In particular, while the partial derivative of the probability of an IPO with respective to the industry market-to-book may well be positive in all countries, as Pagano-Panetta-Zingales found for Italy, our analysis predicts that the cross-partial of this with respect to the stringency of corporate governance should be negative. Moreover, if one interprets firms in older industries as also being larger, then Prediction 5 is consistent with the finding of Pagano, Panetta and Zingales (1998) that the likelihood of an IPO is increasing in firm size. The rest of our predictions remain to be tested.

7 CONCLUSION

We have examined an entrepreneur’s choice of whether to go public or stay private. In either case, after raising external capital, the entrepreneur operates as an owner-manager. Hence, the firm’s initial choice of public versus private ownership is driven by the costs and benefits the entrepreneur perceives in his role as manager and part owner. This managerial perspective on the choice of ownership structure leads to the observation that the entrepreneur will care about the degree of autonomy that he will have after he raises external financing. This autonomy matters to the entrepreneur not because he has any innate preference for independence or any private benefits of control. Rather, it matters because it
determines his ability to make decisions that he views as maximizing firm value when investors disagree with him. Possible disagreement arises not from a divergence of objectives (or agency problems) between the manager/entrepreneur and the investors, but from a difference of opinions.

We build on this idea by joining it with two other aspects of ownership structure. One is that corporate governance can be privately contracted upon with private ownership, allowing the entrepreneur to choose the ex ante value-maximizing corporate governance structure, whereas a standardized structure – one dictated by the reporting and other requirements for publicly-listed firms – must be adopted with public ownership. The second aspect is that those who provide financing have more liquid ownership stakes if the firm is publicly traded than if it is private. Consequently, in making his choice, the entrepreneur trades off the greater ability he has with private ownership to calibrate the stringency of corporate governance to what he perceives as the needs of the firm against the greater liquidity and lower cost of capital that he has with public ownership.

We find that both excessively stringent and excessively lax corporate governance structures with public ownership encourage firms to stay private. As spectacular instances of corporate abuses create a momentum in favor of more restrictive corporate governance for public companies, our analysis provides an argument for caution. Future research could be directed at the public policy implications arising from the tension this suggests between the policymaker’s desire on the one hand to encourage greater investor participation and lower firm’s cost of capital through more restrictive corporate governance and the policymaker’s desire on the other hand to attract more firms to go public. As shown in section 4.3, the effect of the strengthening of corporate governance on the going-public decision of firms is positive for low-\( \rho \) firms (less established firms) but possibly negative for established (high-\( \rho \)) firms.
A APPENDIX

A.1 Proof of Lemma 1

Here a measure of the cost of capital is the fraction of ownership, \(1 - \alpha_{pu}\), demanded by the shareholders in return for investment. It is evident that \(1 - \alpha_{pu}\) is decreasing in \(\rho\) and increasing in \(\eta\). Q.E.D.

A.2 Proof of Lemma 2

With a strict tradeoff between the payoffs of the manager and the investor in equilibrium, the individual rationality (IR) constraint of the investors in (10) would bind. Hence, substituting for \(\alpha\) from (10) into the manager’s objective (use 9) we get:

\[
\max_{\eta} \left[ 1 - \frac{1 + L\lambda}{(1 + NPV_i)} \right] [NPV_m + 1] \tag{A-1}
\]

s.t. \(0 \leq \eta \leq 1\)

We will maximize the function in (A-1) with respect to \(\eta\). Initially we shall ignore the boundary conditions on \(\eta\) and later derive the necessary parameter restrictions for the boundary conditions to be satisfied. The first-order condition of the problem is given by:

\[
\frac{(\hat{p}(1 - \rho)D)(1 + L\lambda)}{(1 + NPV_i)^2} (NPV_m + 1) + (1 - \frac{1 + L\lambda}{(NPV_i + 1)})\hat{p}A(1 - \rho) = 0
\]

\[
D(NPV_m + 1)(1 + L\lambda) + (NPV_i - L\lambda)(NPV_i + 1)A = 0 \tag{A-2}
\]

Substituting for \(NPV_m\) and \(NPV_i\) in (A-2) we get a quadratic in \(\eta\), which has two positive real roots, with one referring to a maximum and the other to a minimum. The equilibrium value of \(\eta^*\) follows from (A-2), and is given in (12). To ensure that the boundary conditions on \(\eta^*\) are satisfied, we need:
Assumption 1

\[ \rho > \frac{L\lambda [A - D] - D}{\hat{p}A^2} \]  \hspace{1cm} (A-3)

\[ t - A[1 + \hat{p}A\rho] < \hat{p}[1 - \rho]AD \]  \hspace{1cm} (A-4)

If (A-3) is violated, the optimized value of \( \eta^* \) is equal to 0, while if (A-4) is violated, the optimized value of \( \eta^* \) is equal to 1. \( \alpha^* \) in (13) can be obtained by using (10) as an equality in conjunction with (7) and substituting for \( \eta^* \) from (12). Similarly, \( NPV_i^* \), in (14) can be obtained by substituting for \( \eta^* \) in \( NPV_i = \hat{p}[\rho A + [1 - \rho]\eta^* D] \) and (15) can be obtained by substituting for \( D \) in \( V_{pr}^m = t - \frac{A[1 + L\lambda]}{D} \left\{ \frac{t + [D - A][1 + \hat{p}A\rho]}{D} - 1 \right\} \). Q.E.D.

A.3 Proof of Lemma 3

The fractional ownership sold to the investors is given by \( 1 - \alpha = \frac{A(1 + L\lambda)}{t} \). Since \( t \) is increasing in \( \rho \) it is clear that that \( 1 - \alpha \) is decreasing in \( \rho \). Further, from the expression for \( 1 - \alpha \) it is clear that it is increasing in \( L \). Q.E.D.

A.4 Proof of Lemma 4

At the equilibrium, the investor’s individual rationality (IR) constraint is binding. Hence, if there is a marginal increase in \( \rho \) from the equilibrium level, then both \( \eta^* \) and \( \alpha^* \) adjust so as to get back to equilibrium, since in equilibrium the marginal benefits of \( \alpha^* \) and \( \eta^* \) for the manager are equal. This means that with an increase in \( \rho \), both \( \eta^* \) and \( \alpha^* \) increase so as to get back to equilibrium. To see this, differentiate, (12), the expression for \( \eta^* \), to get:

\[ \frac{\partial \eta^*}{\partial \rho} = \frac{D[(1 - \rho)(t_{\rho} - A^2\hat{p}) + (t - A(1 + \hat{p}A))]}{(D(1 - \rho))^2} \]

We see that \( \frac{\partial \eta^*}{\partial \rho} > 0 \) if

\[ (1 - \rho)(t_{\rho} - A^2\hat{p}) + (t - A(1 + \hat{p}A)) < 0, \text{ since } D < 0 \]  \hspace{1cm} (A-5)
Thus, we need to show that

\[(1 - \rho)(\frac{\hat{p}A^2(A - D)(1 + L\lambda)}{2t} - A^2\hat{p}) + (t - A(1 + \hat{p}\rho)) < 0\]

That is, we need to show that:

\[(1 - \rho)(\frac{\hat{p}A^2(A - D)(1 + L\lambda)}{2t}) + t < A(1 + \hat{p}A)\]

which means showing that:

or 

\[(1 - \rho)(\hat{p}A^2(A - D)) (1 + L\lambda) + 2t^2 < 2tA(1 + \hat{p}A)\]

or 

\[(A - D)(1 + L\lambda)A[\hat{p}A(1 - \rho) + 2(1 + \hat{p}\rho)] < 2t(1 + \hat{p}A)\]

\[(A - D)(1 + L\lambda)[(1 + \hat{p}A) + (1 + \hat{p}\rho)] < 2t(1 + \hat{p}A)\]

Since \(\rho < 1\), it is sufficient to prove that

\[(A - D)(1 + L\lambda)2(1 + \hat{p}A) < 2t(1 + \hat{p}A)\]

or equivalently,

\[(A - D)(1 + L\lambda) < t\]

Substituting for \(t\) and simplifying, this is equivalent to:

\[\rho > \frac{L\lambda(A - D) - D}{A^2\hat{p}}\]  \hspace{1cm} (A-6)

But this is exactly our condition (A-3) in Assumption 1. Thus, as long as \(\eta^* > 0\), \(\eta^*\) is increasing in \(\rho\). Q.E.D.

### A.5 Proof of Proposition 1

The total payoff for the manager through private ownership is given by \(V_{pr}^m\) while the total payoff from public ownership is given by \(V_{pub}^m\). The manager will prefer public ownership iff \(V_{pub}^m > V_{pr}^m\). For a given \(\eta\), this translates into the following.

\[\frac{NPV^*_v - L\lambda}{1 + NPV^*_v} (1 + NPV^*_m) - \frac{NPV^*_v}{1 + NPV^*_v} (1 + NPV^*_m) \leq 0\]

This implies:

\[\frac{(1 + NPV^*_v)(NPV^*_v + 1)(NPV^*_v - L\lambda) - (NPV^*_v + 1)(NPV^*_m + 1)NPV^*_v}{(1 + NPV^*_v)(1 + NPV^*_v)} \leq 0\]
which means:

$$(1 + \text{NPV}_m^*)(\text{NPV}_i + 1)(\text{NPV}_i^* - L\lambda) - (\text{NPV}_i^* + 1)(\text{NPV}_m + 1)\text{NPV}_i \leq 0$$

This inequality can be expressed as:

$$(\text{NPV}_i + 1)(\text{NPV}_i^* - L\lambda)(\text{NPV}_m^* - \text{NPV}_m) + (\text{NPV}_m + 1)(\text{NPV}_i + 1)(\text{NPV}_i^* - L\lambda) - (1 + \text{NPV}_m^*)\text{NPV}_i \leq 0$$

which is:

$$(\text{NPV}_i + 1)(\text{NPV}_i^* - L\lambda)(\text{NPV}_m^* - \text{NPV}_m) + (\text{NPV}_m + 1)(1 + L\lambda)(\text{NPV}_i^* - \text{NPV}_i) - (1 + \text{NPV}_m^*)(1 + \text{NPV}_i^*)\text{NPV}_i \leq 0$$

This can be written as:

$$(\text{NPV}_i + 1)(\text{NPV}_i^* - L\lambda)(\text{NPV}_m^* - \text{NPV}_m) + (\text{NPV}_m + 1)(1 + L\lambda)(\text{NPV}_i^* - \text{NPV}_i) - (\text{NPV}_m + 1)(1 + \text{NPV}_i^*)L\lambda \leq 0$$

which we can express as:

$$(\eta^* - \eta)p(1 - \rho)\text{FOC} - (\text{NPV}_m + 1)(1 + \text{NPV}_i^*)L\lambda \leq 0$$

where $\text{FOC} \equiv D(\text{NPV}_m + 1)(1 + L\lambda) + (\text{NPV}_i - L\lambda)(\text{NPV}_i + 1)A$. The last inequality has been written to highlight the tradeoff involved. When $\eta = \eta^*$, the first term is 0 and the inequality is satisfied. In this case, the manager clearly prefers public ownership as he gets the optimal tradeoff in public ownership without the liquidity penalty for investors. When $\eta > \eta^*$, the difference $\eta^* - \eta$ is negative as is the FOC. Hence, the first term is positive. And the first term increases with $\eta$. Although the second term also increases with $\eta$, it can be easily shown that the rate of increase is greater for the first term, so that for a sufficiently high $\eta = \eta_2$, the left-hand side becomes positive and thus the preference of the manager switches to private ownership. Similarly, when $\eta < \eta^*$, the difference $\eta^* - \eta$ is positive, and the FOC is also positive. In this case, as $\eta$ decreases, so does the second term, and hence, for a sufficiently low $\eta = \eta_1$, the manager again prefers private ownership. This means that for some $\eta_1$ and $\eta_2$ such that $\eta \in [\eta_1, \eta_2]$, the manager prefers public ownership. Q.E.D.
A.6 Proof of Proposition 2

We show the first part of the proposition by rewriting the condition of Proposition 1 as follows:

\[
\frac{NPV_i^* - L\lambda}{1 + NPV_i^*} (1 + NPV_m^*) - \frac{NPV_i}{1 + NPV_i} (1 + NPV_m) \leq 0,
\]

Which is equivalent to:

\[
\frac{NPV_i^*}{1 + NPV_i^*} (1 + NPV_m^*) - \frac{NPV_i}{1 + NPV_i} (1 + NPV_m) \leq \frac{L\lambda}{1 + NPV_i^*} (1 + NPV_m^*)
\]

It is clear that the right-hand side of the above inequality is increasing in \(L\) and \(\lambda\). Further, \(\eta_1\) and \(\eta_2\) are the roots of this equation. And since the second term in the left-hand side has an unique local maximum at \(\eta = \hat{\eta} \in (0, 1)\), the left-hand side has an unique minimum at \(\eta = \hat{\eta}\) and is increasing in \(\eta\) for all \(\eta > \hat{\eta}\) and decreasing in \(\eta\) for all \(\eta < \hat{\eta}\). Hence, when there is an increase in \(L\) or \(\lambda\), the right-hand side increases. This results in an increase in the larger root and a decrease in the smaller root to maintain equality, leading to the result that the interval (which is defined by the roots of this equation) is increasing in \(L\) and \(\lambda\). For the second part of the proposition, we can rewrite the condition for Proposition 1 as follows:

\[
\frac{NPV_i^*}{1 + NPV_i^*} (1 + NPV_m^*) - \frac{L\lambda}{1 + NPV_i^*} (1 + NPV_m^*) \leq \frac{NPV_i}{1 + NPV_i} (1 + NPV_m)
\]

The left-hand side is nothing but \(V_{pr}^m\). Making this substitution and rewriting the equation we have:

\[
V_{pr}^m - \frac{NPV_i}{1 + NPV_i} (1 + NPV_m) \leq 0, \text{ which implies}
\]

\[
NPV_i NPV_m + NPV_i - V_{pr}^m (1 + NPV_i) \geq 0
\]

We know \(\eta_1\) and \(\eta_2\) are the roots of the above quadratic equation. We need to prove that \(\eta_2 - \eta_1\) is increasing in \(\rho\) (assuming \(\eta_2 > \eta_1\)). The above inequality can be written as an equality of the form

\[a\eta^2 + b\eta + c = 0\]

where \(a \equiv p^2(1 - \rho)^2 AD\), \(b \equiv p^2\rho(1 - \rho)(A^2 + AD) + (1 - V_{pr}^m)p(1 - \rho)D\) and \(c \equiv ppA(1 - V_{pr}^m) - V_{pr}^m + p^2\rho^2 A^2\). Hence, to prove \(\eta_2 - \eta_1\) is increasing in \(\rho\), we need to prove \(\frac{-\sqrt{b^2 - 4ac}}{2a}\) is increasing in \(\rho\). Substituting and simplifying we get:

\[
\frac{\sqrt{b^2 - 4ac}}{a} = Diff = \frac{\sqrt{(ppA(A + 1) + 1 - V_{pr}^m)^2 - 4AV_{pr}^m}}{p(1 - \rho)A}
\]
Substituting for $V_{pr}^m$ from (15) and simplifying we have:

$$Diff = \sqrt{\frac{(2 - A(1 + L\lambda) - (A + 1) + 2t - \frac{A(1 + L\lambda)}{t})^2 - 4AV_{pr}^m}{p(1 - \rho)A}}$$  \hspace{1cm} (A-7)

It is clear that the denominator is decreasing in $\rho$ and the first term in the numerator is increasing in $\rho$. Although the second term is decreasing in $\rho$, it can be easily shown that the numerator is increasing in $\rho$ and hence $Diff$ is increasing in $\rho$. Q.E.D.

### A.7 Proof of Proposition 3

When $\eta = \eta_1$ for the public market from Proposition 1, we know that the manager is indifferent between public and private ownership. Hence, the following condition holds:

$$\frac{NPV_i^*}{1 + NPV_i^*}(1 + NPV_m^*) - \frac{L\lambda}{1 + NPV_i^*}(1 + NPV_m^*) = \left[ \frac{NPV_i(\eta_1)}{1 + NPV_i(\eta_1)}(1 + NPV_m(\eta_1)) \right]$$

Now consider applying a mean-preserving spread to the market flexibility, i.e. $\tilde{\eta} = \eta + \tilde{\epsilon}$, where $E(\tilde{\epsilon}) = 0$ and $V(\tilde{\epsilon}) = \sigma^2$. This yields:

$$\frac{NPV_i^*}{1 + NPV_i^*}(1 + NPV_m^*) - \frac{L\lambda}{1 + NPV_i^*}(1 + NPV_m^*) = \left[ \frac{NPV_i(\eta_1)}{1 + NPV_i(\eta_1)}(1 + NPV_m(\eta_1)) \right] < E\left( \left[ \frac{NPV_i}{1 + NPV_i}(1 + NPV_m) \right] \right)$$

We see that the manager now strictly prefers private ownership. Further, if we do a Taylor series expansion of the last term, it is be clear that the difference between $\left[ \frac{NPV_i(\eta_1)}{1 + NPV_i(\eta_1)}(1 + NPV_m(\eta_1)) \right]$ and $E\left( \left[ \frac{NPV_i}{1 + NPV_i}(1 + NPV_m) \right] \right)$ is an increasing function of the variance of $\eta$. Q.E.D.
REFERENCES


Figure 1: Sequence of Events

Manager and investors observe the common signal about project quality and draw their private priors about precision of common signal.

Investor's suffer a liquidity shock with a probability $\lambda$.

Dates 0 1 2

Manager chooses ownership structure and raises financing from investors.

Project investment decision is made.

Cash Flows are realized and all agents consume.